École polytechnique de Louvain

# Application of convex hull pricing to the joint pricing of reserve and energy

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Academic year 2020–2021
Master [120] in Mathematical Engineering

# Acknowledgements

First and foremost, I would like to express my sincere gratitude to my supervisors Professor Anthony Papavasiliou and Dr. Mehdi Madani, for supervising this thesis, as well as for the continuous support, feedback and all the guidance to reach the goal of this work. I would also like to thank Dr. Madani for proposing this thesis topic.

I also want to thank Professor Yves Smeers for agreeing to be part of this thesis jury as a reader.

Finally, this thesis is the conclusion of all my academic years. This is why I want to thank my parents for giving me the opportunity to study at university. I also wish to express my gratitude to my entire family, friends and my better half for their continuous support, love, and encouragement throughout my studies.

# Contents

# Chapter 1

# Introduction

Today's society is dependent on energy. No-one could think of spending one day without electricity, and the amount consumed every year is significantly increasing, almost reaching a global consumption of twenty-five thousand TWh in 2018[1]. On the other hand, it has been quite some time that a lot of attention has been put on global warming. Europe wants to become climate neutral by 2050[2] by increasing the share of renewable as well as the number of electrical vehicles. This share of renewables energy has never been so high, reaching almost 23%[3] of the global production in 2018.

This source of energy is better for the environment, but is one of the causes of the increasing complexity of the energy markets. Consumption can now be met by a lot of different kinds of production plants (coal, gas, nuclear, wind, solar, hydro,...), each of them having its own specific characteristics.

Indeed, on the one hand, coal, gas and nuclear plants have significant start up and maintenance costs, as well as in some cases minimum power output levels. They are also in general more expensive than renewable energies, and some of them are very slow to be started up (for example, it takes on average more than twelve hours to start up a nuclear plant[4]), while others such are very flexible as combustion turbines.

On the other hand, wind, solar and hydro production plants are cheaper than the others and cleaner for the environment. However, they are dependent on the weather conditions and forecasts, hence uncontrollable and a lot of unexpected events can happen due to those plants. A sudden drop in sunshine or wind on a specific day can lead to a considerable decrease of the amount of available production plants. Those unexpected events can also happen to other agents of the system. Indeed, for example, some event can lead to a significant drop of the amount of energy consumed.

[1] From IEA Data Services, `https://www.iea.org/fuels-and-technologies/electricity`

[2] European Commission, `https://ec.europa.eu/clima/policies/strategies/2050_en`

[3] See footnote 1

[4] U.S. Energy Information Administration, `https://www.eia.gov/todayinenergy/detail.php?id=45956`

In order to cope with those uncertainties, a market offering ancillary services must be proposed. On top of this, in most situations, the modeling of units with features such as start-up costs and minimum run capacities necessitates the inclusion of binary variables, which cause non-convexities in the optimization process and as a result, pricing rules must be created from the ground up. This work aims at

- proposing a unit commitment model that co-optimizes energy and reserve in a non-convex market.
- applying two pricing schemes to this model in order to obtain energy and reserve prices.
- comparing those two prices for each agent on the market, and assess their advantages and disadvantages.

Chapter 2 develops the energy market without any trading of reserve. After that, chapter 3 focuses on the integration of reserve capacities into the models of the previous chapter. Chapter 4 discusses the introduction of a transmission system and what it implies. Last but not least, chapter 5 applies these models to the a database of Belgium.

# Nomenclature

### Sets and index

| | | |
|---|---|---|
| $\mathcal{T}, t$ | Set and index of time steps. | |
| $\mathcal{G}, g$ | Set and index of generators. | |
| $\mathcal{N}, n$ | Set and index of nodes in the transmission system. | |
| $\mathcal{L}, l$ | Set and index of lines in the transmission system. | |
| $\mathcal{G}_n, g$ | Set and index of generators of a given node $n$. | |
| $\mathcal{C}, c$ | Set and index of consumers. | |

### Variables

| | | |
|---|---|---|
| $x_{g,t}$ | Electricity production generated by unit $g$ during time step $t$. | [MW] |
| $r_{g,t}$ | Amount of reserve produced by unit $g$ during time step $t$. | [MW] |
| $u_{g,t}$ | ON-OFF variable of unit $g$ during time step $t$. This variable takes 1 as value if unit $g$ is ON at time step $t$, and 0 if unit $g$ is OFF at this time step. | [–] |
| $v_{g,t}$ | Start-up variable of unit $g$ during time step $t$. This variable takes 1 as value if unit $g$ has started up at time step $t$. | [–] |
| $z_{g,t}$ | Turn-off variable of unit $g$ during time step $t$. This variable takes 1 as value if unit $g$ has been shut down at time step $t$. | [–] |
| $d_{c,t}$ | Electricity consumed by consumer $c$ during time step $t$. | [MW] |
| $\tilde{r}_{c,t}$ | Amount of reserve consumed by consumer $c$ during time step $t$. | [MW] |
| $I_{n,t}$ | Injection variable to node $n$ during time step $t$. | [MW] |
| $f_{l,t}$ | Flow variable through line $l$ during time step $t$. | [MW] |

$\Delta f_{l,t}$ Flow of reserve through line $l$ during time step $t$. [MW]

**Parameters**

$C_g$ Marginal cost of producing electricity of unit $g$. [€/MWh]

$K_g$ No-load cost consumption of producing electricity of unit $g$. [€/MWh]

$S_g$ Start-up cost of unit $g$. [€]

$\overline{P_g}$ Maximum power generation of unit $g$. [MW]

$\underline{P_g}$ Minimum power generation of unit $g$. [MW]

$R_g$ Maximum amount of power of unit $g$ that can be produced for reserve per time step. [MW]

$D_t$ Fixed demand of electricity during time step $t$. [MW]

$D_{n,t}$ Fixed demand of electricity of node $n$ during time step $t$. [MW]

$R$ Amount of reserve that is required in the system. [MW]

$R_n$ Amount of reserve that is required in node $n$. [MW]

$\omega_c$ Willingness to pay energy of consumer $c$. [€/MWh]

$\omega_c^R$ Willingness to pay reserve of consumer $c$. [€/MWh]

$L_{g,n}$ Link-matrix, describes which generators are linked to which node. This parameter takes 1 as value if generator $g$ is linked to node $n$, and 0 if it is not. [–]

$B_{l,n}$ Incidence matrix between lines and nodes. This parameter takes 1 as value if line $l$ goes in node $n$, -1 as value if line $l$ goes out of node $n$, and 0 if the line does not connect to node $n$. [–]

$\overline{f}_l$ Maximum flow through line $l$. [MW]

$\underline{f}_l$ Minimum flow through line $l$. [MW]

$UT_g$ Minimum up time of generator $g$. [h]

$DT_g$ Minimum down time of generator $g$. [h]

# Chapter 2

# The energy-only market

Before diving into the main matter of this work, it is important to study the energy-only model on which the rest of this paper will be based. This chapter will explain the optimization of a non-convex electric market trading only energy.

Section 2.1 provides a description of the unit commitment model, as well as its optimization problem. Why pricing models are necessary and how to compute them is presented in section 2.2. Section 2.3 discusses uplifts, the side payment needed with those pricing models, why they are needed and how to compute them. Finally, section 2.4 demonstrate the potential (non-)uniqueness of convex hull prices.

## 2.1 Unit commitment model

This first part of this thesis will only consider transactions between two agents trading energy: producers and consumers. This model will dispatch consumption and production of energy in the system, while taking into account costs of producers and consumers' willingness to pay for energy.

### 2.1.1 Consumers

Consumers need energy, only if the price of energy is below the amount of money they think energy is worth. This can be modeled either by

- a fixed and inelastic demand of energy $D_t$ for every time step that must be met by producers,
- or by an elastic consumption of energy represented by consumers $c \in \mathcal{C}$ with variable $d_{c,t}$ as its energy consumption, which is a continuous variable in MW.

The latter way of modeling consumers assumes that the maximum amount of energy that can be consumed is equal to the inelastic demand $D_t$. This method has been used for the entire thesis. Those consumers show some willingness to pay (or valuation) for energy $\omega$

(€/MWh). This method will allow consumers to be price-sensitive, i.e. they will only consume energy if its price is below their valuation of energy.

Variable $d$ is non-negative. Each consumer has a specific fixed fraction $\kappa_c$ that represent the portion of the total inelastic demand $D_t$ it can consume. For example, in a system with two consumers, the ratios $\kappa_1 = 1/3, \kappa_2 = 2/3$ represent that consumer one (two) can consume up to one third (two third) of the total inelastic demand.

### 2.1.2 Producers

Producers, denoted by $g \in \mathcal{G}$, want to sell energy only if they earn money. They also want to maximize their profits. They are described by several parameters:

1. they produce energy at a certain cost per MWh $C_g$.
2. they have a start-up cost $S_g$.
3. they have a no-load consumption cost $K_g$. This fixed cost represents an amount of fuel that is consumed when the unit is running even though it is not producing energy.
4. they have minimum and maximum run capacities $\underline{P_g}, \overline{P_g}$, which indicate the minimum and maximum output one can produce.
5. they have minimum up and down times $UT_g$ and $DT_g$.

Since not only the cost of production is taken into account, but also the other costs described above, it has been decided to use 3-*bin* models, which means every generator will be described by three binary variables:

- $u_{g,t}$ will describe which generator $g$ is ON (= 1) or OFF (= 0) at time step $t$.
- $v_{g,t}$ will describe which generator $g$ is starting up (= 1) or not (= 0) at time step $t$.
- $z_{g,t}$ will describe which generator $g$ is shutting down (= 1) or not (= 0) at time step $t$.

The initial conditions of the system are fixed to be

$$u_{g,0} = 0 \qquad v_{g,0} = 0 \qquad z_{g,0} = 0 \qquad \forall g \in \mathcal{G}$$

Producers will be modeled by the three binary variables and by a continuous variable $x$ which represents the amount of production of each generator and time step in MW.

### 2.1.3 Complete model

Model (2.1) is the complete unit commitment model in a context of energy only. It takes every cost of each generator into account, and minimizes costs of production. One other way to write this model is to maximize welfare, i.e. maximize the consumption of consumers minus the total cost of generation.

**Unit commitment model optimizing energy**

$$\min_{x,u,v,z,d} \sum_{t\in\mathcal{T}} \left( \sum_{g\in\mathcal{G}} (x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum_{c\in\mathcal{C}} d_{c,t}\omega \right) \tag{2.1a}$$

$$\text{s.t. } x_{g,t} \leq u_{g,t}\overline{P}_g \quad [\nu_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.1b}$$

$$x_{g,t} \geq u_{g,t}\underline{P}_g \quad [\mu_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.1c}$$

$$\sum_{g\in\mathcal{G}} x_{g,t} = \sum_{c\in\mathcal{C}} d_{c,t} \quad [\pi_t] \quad \forall t \in \mathcal{T} \tag{2.1d}$$

$$u_{g,t} = v_{g,t} - z_{g,t} \quad [\xi_{g,t}] \quad \forall g \in \mathcal{G}, t = 1 \tag{2.1e}$$

$$u_{g,t} - u_{g,t-1} = v_{g,t} - z_{g,t} \quad [\chi_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T}\backslash\{1\} \tag{2.1f}$$

$$0 \leq d_{c,t} \quad [\eta_{c,t}] \quad \forall c \in \mathcal{C}, t \in \mathcal{T} \tag{2.1g}$$

$$d_{c,t} \leq \kappa_c D_t \quad [\zeta_{c,t}] \quad \forall c \in \mathcal{C}, t \in \mathcal{T} \tag{2.1h}$$

$$\sum_{\tau=t-UT_g+1}^{t} v_{g,\tau} \leq u_{g,t} \quad \forall g \in \mathcal{G}, t \in \{UT_g, ..., |\mathcal{T}|\} \tag{2.1i}$$

$$\sum_{\tau=t-DT_g+1}^{t} z_{g,\tau} \leq 1 - u_{g,t} \quad \forall g \in \mathcal{G}, t \in \{DT_g, ..., |\mathcal{T}|\} \tag{2.1j}$$

$$x_{g,t} \geq 0 \quad [\delta_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.1k}$$

$$u_{g,t}, v_{g,t}, z_{g,t} \in \{0,1\} \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.1l}$$

In this model, constraints (2.1b) and (2.1c) are limiting the minimum and maximum run capacities of each generator for each time step, constraint (2.1d) is the market clearing constraint (balance condition between production and consumption), of which the dual variable $\pi$ gives the price of energy for each time step. Next, constraints (2.1e) and (2.1f) are constraints linking variables $u, v, z$ together, which ensure that no unit is starting up while shutting down, shutting down while starting up or that a unit is ON before start up or ON after shut down.

Constraints (2.1g)-(2.1h) define the amount of minimum and maximum amount of energy each consumer can consume, which is non-negative but lower than a fraction of the demand, which is specific for each consumer, while constraints (2.1i) and (2.1j) are the constraints that define minimum up and down times for each generator. Finally, constraint (2.1k) ensures the non-negativity of the energy production variable $x$ and (2.1l) enforces variables $u, v, z$ to be binary.

**Example 2.1** (Dispatch of example 2.1)**.**
*Let us consider the set of producers with characteristics displayed in table 2.1 and apply*

*model 2.1 to this data. This model will have only one period and one consumer, whose valuation for energy is equal to 300€/MWh. Since there is only one period, constraints (2.1i) and (2.1j), which respectively represent the minimum up and down times of each generator, will be left out.*

| | Generator 1 | Generator 2 | Generator 3 |
|---|---|---|---|
| $\overline{P_g}$ | 40 | 30 | 20 |
| $\underline{P_g}$ | 20 | 0 | 0 |
| $S_g$ | 200 | 0 | 0 |
| $K_g$ | 0 | 0 | 0 |
| $C_g$ | 60 | 30 | 90 |

Table 2.1: Characteristics of producers of example 1.

*The model of this small system will be the following:*

$$
\begin{aligned}
\min_{x,u,v,z,d} \; & 60x_{1,1} + 30x_{2,1} + 90x_{3,1} + 200v_{1,1} - 8300d_{1,1} & & \\
s.t. \; & 20u_{1,1} \leq x_{1,1} \leq 40u_{1,1} & & 0 \leq x_{2,1} \leq 30u_{2,1} \\
& 0 \leq x_{3,1} \leq 20u_{1,1} & & x_{1,1} + x_{2,1} + x_{3,1} = d_{1,1} \\
& 0 \leq d_{1,1} \leq D_1 & & u_{1,1} = v_{1,1} - z_{1,1} \\
& u_{2,1} = v_{2,1} - z_{2,1} & & u_{3,1} = v_{3,1} - z_{3,1} \\
& u_{1,1}, v_{1,1}, z_{1,1} \in \{0,1\} & & u_{2,1}, v_{2,1}, z_{2,1} \in \{0,1\} \\
& u_{3,1}, v_{3,1}, z_{3,1} \in \{0,1\} & &
\end{aligned}
$$

*The maximum possible consumption is equal to the sum of all maximum run capacities, which is equal to 90MW. The dispatch for a demand from 0MW to 90MW is displayed in figure 2.1. Generator number two has the cheapest marginal cost, and is thus producing energy first up to a demand of 30MW, which is its maximum run capacity. Then, two options are available. Either use generator one, but since it has a minimum run capacity of 20MW, the dispatch must change, or add production from generator three.*
*The optimal solution is that, from a demand equal to 30MW to a demand equal to 43MW, it is cheaper to use generator two and three than to turn* ON *unit one. Between 43MW and 70MW, it is now less expensive to use only generators one and two. Indeed, even though generator one has a start up cost, it has a lower marginal cost than generator three. The remaining 20MW from 70W to 90MW are produced by generator three, since the other two producers are producing at their maximum run capacity.*

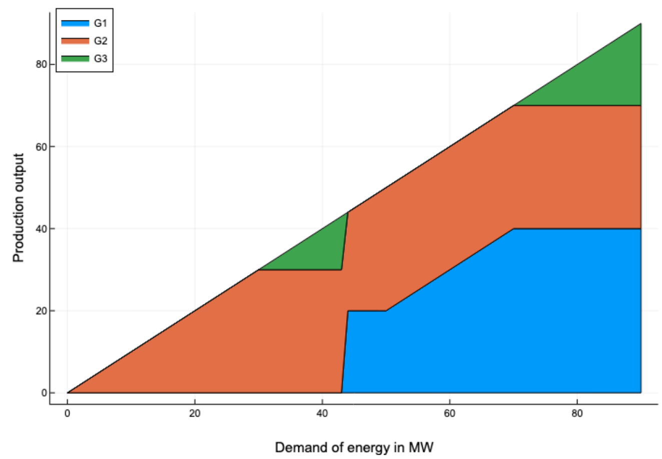



Figure 2.1: Solution of the unit commitment solution (cumulative production).

Integer variables are used in model (2.1), which makes it non-convex. From [Mad+18], it is known that:

- the consequence of this is that a market equilibrium supported by uniform prices in presence of those non-convexities may not exist.
- uniform prices are prices obtained that are the same for the whole market (for each agent) and specific for each time step without any support from side payments.
- near-equilibrium prices can be computed with additional side payments, called uplifts.

**Example 2.2** (Potential non-existence of uniform prices with non-convexities).
*To illustrate these statements, let us pick up on the previous example. If a demand of $45MW$ is considered, and if the system uses a marginal pricing method, the price will be equal to $\pi = 30$ €$/MWh$ since the cost of producing an additional $MW$ would be the marginal cost of generator two. At this price, generator two would make no profit since the price is exactly equal to its production marginal cost. Generator one on the other side, would be producing at a loss, because the price is lower than its marginal cost. The loss would be equal to $(\pi - C_g)x_g - S_g = (30 - 60)25 - 200 = -950$ €, hence generator one would prefer not to be dispatched. As a result, this marginal pricing rule does not provide every market player the appropriate motivation to produce the required amount of power.*

## 2.2 Pricing models

Example 2.2 enlightened the fact that a marginal pricing rule may not provide uniform prices. This section provides two pricing rules that are good candidates to find near-equilibrium prices. To provide the proper motivation for every agent in the system to follow the unit commitment solution, these rules must be backed up by side payments, which are explained in section 2.3.

The pricing rules that are of interest for this thesis are the following:

- Convex hull pricing.
- IP pricing (also called O'Neill pricing).

These pricing rules will modify the unit commitment model in order to obtain a convex model, where calculable prices are characterised as near-equilibrium prices.

Let us note that only prices are of importance in the results of those new models, and that prices are obtained by computing the dual variables of the market clearing constraint (2.1d), and that for this chapter, prices are the same for everybody (one price for the whole system per time step).

### 2.2.1 Convex hull pricing

Sometimes called Extended Locational Marginal Prices (ELMP), this rule has first been proposed in [GHP07]. These prices are the optimizer of the Lagrangian dual of the unit commitment model where the market clearing constraint has been dualized. The Lagrangian dual of unit commitment model (2.1), where balance constraint (2.1d) is dualized, is model (2.2).

**Lagrangian dual of unit commitment model 2.1**

$$\mathcal{O}(\pi) = \begin{cases} \min\limits_{x,u,v,z,d} \sum\limits_{t\in\mathcal{T}} \left( \sum\limits_{g\in\mathcal{G}} (x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum\limits_{c\in\mathcal{C}} d_{c,t}\omega \right) \\ \qquad - \sum\limits_{t\in\mathcal{T}} \pi_t \left( \sum\limits_{g\in\mathcal{G}} x_{g,t} - \sum\limits_{c\in\mathcal{C}} d_{c,t} \right) \\ \text{s.t.} \qquad (2.1b) - (2.1c) \text{ and } (2.1e) - (2.1l) \end{cases} \tag{2.2}$$

Convex hull prices $\pi$ are therefore obtained by solving the following problem:

$$\max_{\pi} \mathcal{O}(\pi) \tag{2.3}$$

[HB17] shows that the solution of the unit commitment model's continuous relaxation is the reduction of the solution of the Lagrangian dual of the same unit commitment model (where the market clearing constraint is dualized) if and only if the continuous relaxation of the primal is equivalent to the unit commitment problem where the feasible set is restrained to its convex hull and the objective function is restrained to its convex envelope over the feasible set. This method is proposed since solving (2.3) can be computationally expensive.

Convex hull prices can, for this reason, be computed with the continuous relaxation of the unit commitment model. The convex envelope of the objective function must be computed, but as the objective function (2.1a) of unit commitment model (2.1) is linear

(every parameter is constant), its convex envelope is exactly equal to this function. The feasible set must also be restrained to its convex hull.
Theorem 2 from [Mad+18] shows that:

**Theorem 2.1** (Convex hull relaxation).
*Consider the market participant's feasible set $X_g$ described by*

$$u_g \in \mathbb{Z} \qquad x_g \leq \overline{P_g} u_g \qquad x_g \geq \underline{P_g} u_g \qquad u_g \leq 1 \qquad u_g \geq 0$$

*Then conv($X_g$) is described by the continuous relaxation of $X_g$, i.e. by*

$$x_g \leq \overline{P_g} u_g \qquad x_g \geq \underline{P_g} u_g \qquad u_g \leq 1 \qquad u_g \geq 0$$

In the case of unit commitment model (2.1), the convex hull of the feasible set is described by its continuous relaxation, leading to the constraint (2.1l) relaxed into:

$$u_{g,t}, v_{g,t}, z_{g,t} \geq 0 \qquad [\alpha^u_{g,t}, \alpha^v_{g,t}, \alpha^z_{g,t}] \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.4a}$$

$$u_{g,t}, v_{g,t}, z_{g,t} \leq 1 \qquad [\psi^u_{g,t}, \psi^v_{g,t}, \psi^z_{g,t}] \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.4b}$$

The complete convex hull pricing model is model (2.1) with constraints (2.1l) relaxed into equations (2.4a)-(2.4b). If this model is solved again, now the dual variable of constraint (2.1d) can be computed, which are the prices of energy. Convex hull prices can also be computed by solving (2.3).

**Example 2.3** (Convex hull prices of example 2.1).
*To have a clear view on the effect of the application of convex hull pricing to the system of example 1, let us consider a fixed demand, hence there is no consumer but only a fixed amount of energy to meet (which is taken to be equal from 1MW to 90MW). The model of example 1 becomes:*

$$\begin{aligned}
\min_{x,u,v,z}\ & 60x_{1,1} + 30x_{2,1} + 90x_{3,1} + 200v_{1,1} \\
s.t.\ & 20u_{1,1} \leq x_{1,1} \leq 40u_{1,1} \qquad && 0 \leq x_{2,1} \leq 30u_{2,1} \qquad && 0 \leq x_{3,1} \leq 20u_{1,1} \\
& x_{1,1} + x_{2,1} + x_{3,1} = D_1 && x \geq 0 && \\
& u_{1,1} = v_{1,1} - z_{1,1} && u_{2,1} = v_{2,1} - z_{2,1} && u_{3,1} = v_{3,1} - z_{3,1} \\
& u_{1,1}, v_{1,1}, z_{1,1} \in \{0,1\} && u_{2,1}, v_{2,1}, z_{2,1} \in \{0,1\} && u_{3,1}, v_{3,1}, z_{3,1} \in \{0,1\}
\end{aligned}$$

*If convex hull pricing is applied to the system above, it can be observed in figure 2.2 that the total cost curve of the system is restrained to its convex hull.*

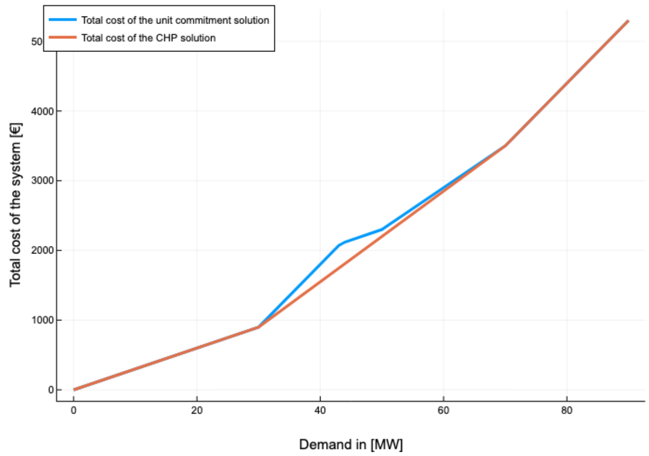



Figure 2.2: Total cost curve of the unit commitment solution and the convex hull pricing solution.

*The convex hull prices are available in figure 2.3. For example, with a demand of 45MW, the convex hull price is equal to 65€/MWh. One can see that the price is different from the marginal costs of producers when the set is not equal to its convex hull set. From a demand equal to 30MW to a demand equal to 70MW, the set is not equal to its convex hull set, neither is the total cost curve, thus the price is not equal to one of the marginal costs of producers (the price is equal to 65€/MWh).*

*One can also note that these prices are clearly monotone, i.e. a price increasing with the load.*

### 2.2.2 IP pricing

IP pricing, often also called O'Neill pricing, was first proposed in [One+05]. The concept consists in computing prices while dealing only with the convex part of the unit commitment model. The following methodology is proposed:

1. solve the unit commitment model,
2. solve the problem one more time but this time by fixing all binary variables to the optimal values obtained in the solution of the unit commitment model.

By doing so, the unit commitment model with the binary variables fixed to their optimal values is convex, since the source of non-convexities comes from the integer part of the model, which is fixed.

[Mad+18] says that IP prices are set by the marginal units in the chosen unit commitment and dispatch. In other words, IP prices are set according to which generators is ON without producing at full capacity. This results in more volatile prices as mentioned in [RH03], since a small increase or decrease in the loads can affect the decision as to which unit is marginal and gives the price.

Variables $u_{g,t}, v_{g,t}, z_{g,t}$ are thus fixed to the unit commitment model solution $u^*_{g,t}, v^*_{g,t}, z^*_{g,t}$, which leads to constraint (2.1l) being modified into:

$$u_{g,t} = u^*_{g,t} \quad [\beta^u_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.5a}$$
$$v_{g,t} = v^*_{g,t} \quad [\beta^v_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.5b}$$
$$z_{g,t} = z^*_{g,t} \quad [\beta^z_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.5c}$$

The complete IP pricing model is model (2.1) with constraints (2.1l) modified into equations (2.5a)-(2.5c), which allows the dual variable $\pi$ of balance constraint (2.1d) representing the price of energy to be computed.

**Example 2.4** (IP prices of example 2.1)**.**
*If IP pricing is applied to the same model used for the example of the convex hull pricing scheme, as the non-convex part of the system is fixed, this will result in the same costs as the unit commitment model.*
*It can be seen on figure 2.3 that IP prices are more volatile than convex hull prices, because IP prices are marginal prices. Since there are only three producers in the system, IP prices can only take three different values, which are the marginal costs of each producer. One can also directly see that IP prices are indeed more volatile.*

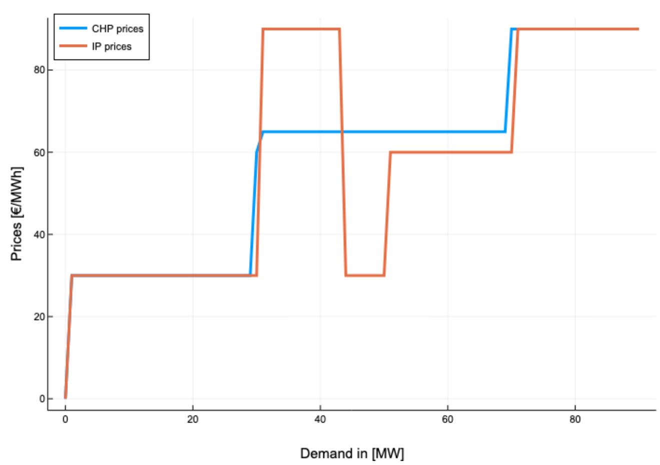



Figure 2.3: IP prices and convex hull prices of energy for several demands.

## 2.3 Uplifts

The pricing rules explained above only give near-equilibrium prices. Indeed, as mentioned in the previous section, it is impossible to find a price with which every agent is maximizing their utilities, while respecting the market clearing constraint in a non-convex optimization problem. This introduces the concept of uplifts, which is a side payment.

For example, for a producer, this is needed because, at a given price $\pi_t$ and unit commitment solution $x^*_{g,t}, u^*_{g,t}, v^*_{g,t}, z^*_{g,t}$, two situations are possible.

- The unit commitment solution makes the generator produce energy although it is producing at loss, i.e. $x^*_{g,t}(C_g - \pi_t) + u^*_{g,t}K_g + v^*_{g,t}S_g < 0$.
- The unit commitment solution makes the generator produce energy although it would want to produce more at this given price, i.e. it is not producing at loss but does not maximise its utility.

Uplifts are the answers to these lost opportunity costs. These can either be paid by an external agency of the market or by consumers. In the first case, each agent is compensated and receives their uplift, however in the second scenario, consumers must account for the uplifts of every other agent in the market as well as theirs.

### 2.3.1 Methodology to compute uplifts

To compute the uplift of one specific agent, it is needed to :

- solve the selfish maximisation problem of this agent,
- then to subtract the objective value of the same problem evaluated at the unit commitment solution (which is the surplus with the market price and welfare maximizing solution) from the optimal objective value of the selfish maximisation problem.

The term that is subtracted from the optimal objective value of the selfish maximisation problem of this agent is exactly what the agent is earning or losing at the unit commitment dispatch.

In order to find the selfish maximisation problem, one must find every constraint that involves more than one agent and dualize them. This results in a Lagrangian relaxed model where the Lagrangian multipliers are the dual variables of those constraints. The selfish maximisation problem of one agent is obtained by keeping from this Lagrangian relaxed problem only constraints and terms in the objective function related to this specific agent.

Following this method with the unit commitment model (2.1), one can find that only constraint (2.1d) involves multiple agents, i.e. multiple generators and multiple consumers. If we follow our method, and dualize this constraint, the new model is exactly model (2.2). This model can be transformed in a maximization model:

$$\begin{aligned} \max_{x,u,v,z,d} \sum_{t\in\mathcal{T}} \pi_t &\left(\sum_{g\in\mathcal{G}} x_{g,t} - \sum_{c\in\mathcal{C}} d_{c,t}\right) \\ - \sum_{t\in\mathcal{T}} &\left(\sum_{g\in\mathcal{G}} (x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum_{c\in\mathcal{C}} d_{c,t}\omega\right) \end{aligned} \tag{2.6a}$$

$$\text{s.t. } (2.1b) - (2.1c) \text{ and } (2.1e) - (2.1l) \tag{2.6b}$$

From this Lagrangian model (2.6), the selfish maximisation problem of one specific agent (each generator and each consumer) can be obtained by only keeping terms that involve this specific agent. Once solved, uplifts can be computed by taking the difference between the optimal objective value of problem (2.6) and this same objective function evaluated at the unit commitment solution.

For each agent, uplifts are always positive. To prove this, let us denote $\overline{\rho}$ the amount of money an agent wants to earn selfishly, and $\rho^*$ the amount of money this agent is earning at the unit commitment solution.

- $0 < \rho^* \leq \overline{\rho}$: this happens when the profit the agent is earning is smaller than what it would do selfishly, hence the result will be positive.
- $0 = \rho^* \leq \overline{\rho}$: in this case, the agent is dispatched to do nothing, hence no profit, but selfishly it would want to do at least better than what it is doing in the unit commitment model (to do nothing or to do something that would earn it some money selfishly). The uplift would be equal to $\overline{\rho} - \rho^* = \overline{\rho} \geq 0$.
- $\rho^* < 0 = \overline{\rho}$: finally, selfishly, the agent would want to do nothing, while the unit commitment solution dispatched it to do something at a loss, which gives uplift $= \overline{\rho} - \rho^* = -\rho^* > 0$.

Theorem 1 cited in [Mad+18] states that:

**Theorem 2.2** (Uplifts minimization)**.**
*Let $\pi^*$ solve the Lagrangian dual of the unit commitment model where the balance constraint(s) have been dualized. Then, $\pi^*$ solves:*

$$\min_{\pi} \textit{uplift}(u_c^*, x_c^*)(\pi)$$

These prices $\pi^*$ are convex hull prices. Hence, convex hull prices are prices that minimize the sum of uplifts of the model. Model (2.3) shows how to compute convex hull prices based on the Lagrangian dual of the unit commitment model which minimizes costs of production. If this optimization problem would instead maximize welfare, i.e. model (2.6), convex hull prices will be obtained by minimizing this problem (i.e. model (2.7)), hence the Lagrangian dual of the maximizing welfare unit commitment model is equal to the sum of uplifts.

$$\min_{\pi} (2.6) \tag{2.7}$$

### 2.3.2 Producers

The selfish maximisation problem of each generator $g \in \mathcal{G}$ can be obtained by taking only constraints and objective terms that are related to this generator :

$$
\begin{aligned}
\max_{x,u,v,z} \; & \sum_{t \in \mathcal{T}} (\pi_t - C_g) x_{g,t} - u_{g,t} K_g - v_{g,t} S_g && && (2.8a)\\
\text{s.t. } & x_{g,t} \le u_{g,t} \overline{P}_g && \forall t \in \mathcal{T} && (2.8b)\\
& x_{g,t} \ge u_{g,t} \underline{P}_g && \forall t \in \mathcal{T} && (2.8c)\\
& u_{g,t} = v_{g,t} - z_{g,t} && t = 1 && (2.8d)\\
& u_{g,t} = u_{g,t-1} + v_{g,t} - z_{g,t} && \forall t \in \mathcal{T} \backslash \{1\} && (2.8e)\\
& \sum_{\tau = t - UT_g + 1}^{t} v_{g,\tau} \le u_{g,t} && \forall t \in \{UT_g, ..., |\mathcal{T}|\} && (2.8f)\\
& \sum_{\tau = t - DT_g + 1}^{t} z_{g,\tau} \le 1 - u_{g,t} && \forall t \in \{DT_g, ..., |\mathcal{T}|\} && (2.8g)\\
& x_{g,t} \ge 0 && \forall t \in \mathcal{T} && (2.8h)\\
& u_{g,t}, v_{g,t}, z_{g,t} \in \{0, 1\} && \forall t \in \mathcal{T} && (2.8i)
\end{aligned}
$$

Given prices $\pi$ and solutions to both the selfish maximisation problem $\overline{x}_{g,t}, \overline{u}_{g,t}, \overline{v}_{g,t}, \overline{z}_{g,t}$ and the unit commitment model solution $x^*_{g,t}, u^*_{g,t}, v^*_{g,t}$, the uplift for each generator $g \in \mathcal{G}$ is thus equal to:

**Uplift computation for producer $g$**

$$
\text{uplift}_g = \underbrace{\sum_{t \in \mathcal{T}} \left( (\pi_t - c_g) \overline{x}_{g,t} - K_g \overline{u}_{g,t} - S_g \overline{v}_{g,t} \right)}_{\text{what the producer wants to do selfishly}} - \underbrace{\sum_{t \in \mathcal{T}} \left( (\pi_t - c_g) x^*_{g,t} - K_g u^*_{g,t} - S_g v^*_{g,t} \right)}_{\substack{\text{profit/loss producer is having} \\ \text{at the unit commitment solution}}} \quad (2.9)
$$

**Example 2.5** (Uplift of producers of example 2.1)**.**
*As an example, the uplift of unit one of example 1 can be computed. Let us consider a demand of 45MW. The selfish maximisation problem would have the following solutions with IP pricing and convex hull pricing ($\pi^{IP} = 30$ €/MWh, $\pi^{CHP} = 65$ €/MWh):*

$$
\begin{aligned}
x^{CHP} &= 40 & \qquad x^{IP} &= 0\\
u^{CHP} &= 1 & \qquad u^{IP} &= 0\\
v^{CHP} &= 1 & \qquad v^{IP} &= 0
\end{aligned}
$$

*Indeed, while with IP prices, unit one would be producing at a loss because the price is lower than its marginal cost, with convex hull prices it brings in 5€ per MWh produced. This producer would want to produce only if it could recover its start-up cost and make more money. If it is thus producing at maximum run capacity (i.e. $x_{1,1} = 40MW$), it can receive 225€, earning thus 25€ after paying its start up cost.*

*Now that the selfish maximisation problem is solved, one must look into the unit commitment solution ($x^*_{1,1} = 20, u^*_{1,1} = 1, v^*_{1,1} = 1$), i.e. what the unit is actually earning. The solution provides the following benefits for each pricing rule:*

$$\begin{aligned} \textit{profit}_1^{CHP} &= (65-60) \times 20 - 200 = -100\,€ \\ \textit{profit}_1^{IP} &= (30-60) \times 20 - 200 = -800\,€ \end{aligned}$$

*These results confirm that both pricing rules without any side payments do not provide the right motivation to follow the unit commitment solution, since both units produce at a loss.*

*Finally, equation (2.9) must be used with both the selfish maximisation solution and the unit commitment solution, the uplift is equal to:*

$$\begin{aligned} \textit{uplift}_1^{CHP} &= ((65-60) \times 40 - 200) - ((65-60) \times 20 - 200) = 100\,€ \\ \textit{uplift}_1^{IP} &= (0) - ((30-60) \times 20 - 200) = 800\,€ \end{aligned}$$

*This uplift is greater than zero because unit one is dispatched to produce less than what it would want to. One can already see the uplift minimizer characteristic of convex hull pricing, which gives an uplift eight times lower than IP pricing.*

### 2.3.3 Consumers

Following the same methodology as for producers, the selfish maximisation of each consumer $c \in \mathcal{C}$ is the following:

$$\max_{d} \sum_{t \in \mathcal{T}} d_{c,t}(\omega - \pi_t) \tag{2.10a}$$

$$\text{s.t. } 0 \leq d_{c,t} \leq \kappa_c D_t \qquad \forall t \in \mathcal{T} \tag{2.10b}$$

Given prices $\pi$ and solutions of both the selfish maximisation problem $\overline{d}$ and the unit commitment model $d^*$, the uplift for each consumer $c \in \mathcal{C}$,

**Uplift computation for consumer $c$**

$$\text{uplift}_c = \underbrace{\sum_{t \in \mathcal{T}} \overline{d}_{c,t}(\omega - \pi_t)}_{\text{what the consumer wants to do selfishly}} - \underbrace{\sum_{t \in \mathcal{T}} d^*_{c,t}(\omega - \pi_t)}_{\substack{\text{profit/loss consumer is having} \\ \text{at the unit commitment solution}}} \tag{2.11}$$

IP pricing fixes the non-convex part of the unit commitment model in order to obtain prices. Since this part is fixed, the dispatch of the unit commitment solution and the IP pricing solution will be the same. In a convex model, equilibrium prices are available and can ensure that no one needs any uplifts. Since IP pricing only deals with a convex model, the convex agents (i.e. consumers) are guaranteed to maximize their utility. Hence, IP pricing ensures that consumers are maximizing their utility, which also ensures that they will not be given any uplifts.

On the other hand, this is not true for the convex hull pricing model since for this model, a relaxation is applied to the unit commitment model, and only prices matter in its solution. This relaxation does not ensure that convex agents maximize their utilities, which consequently, in some cases, gives uplifts to consumers with this pricing scheme.

**Example 2.6** (Uplift of consumer of example 2.1)**.**
*The uplift of the single consumer of example 1 can be computed. As its valuation for energy is way higher than both the prices of convex hull pricing and IP pricing, the consumer would want to consume as much as possible, i.e. the selfish maximisation solution will be equal to the maximum consumption possible.*

*The solution of the unit commitment is exactly equal to the selfish maximisation solution. There will thus be no uplift since he can not do better than what the unit commitment solution is forcing him to do.*

## 2.4 Uniqueness of convex hull prices

The unit commitment model (2.1) could have multiple optimum solutions. Consequently, there could also be multiple convex hull prices.

In order to determine whether it could be possible that these are unique or not, the following methodology will be used:

1. Consider a basic, simple, primal unit commitment model, and find its convex hull relaxation.
2. Compute the dual problem of this convex hull relaxation.
3. Create a new model that minimizes (or maximizes) convex hull prices with the feasible set taking the primal feasible set into account, the dual feasible set, and strong duality, which ensures that the optimal objective value of the primal model and dual model are equal, and then solve it.

To follow this methodology, model (2.1) can be taken and can be a bit simplified by removing minimum up and down time constraints (2.1i) and (2.1j). The convex hull pricing model that will be considered will thus be a model that has (2.1a) as objective value, and (2.1b)-(2.1h) and (2.1k) as constraints, while having constraint (2.1l) relaxed into 2.4.

It is now needed to find the dual of this convex hull model. In general, the corresponding primal-dual problems are the following:

$$
\begin{array}{llllll}
\text{Primal:} & \max\limits_{x} \; c^t x & & & \text{Dual:} & \min\limits_{y} \; b^t y \\
 & \text{s.t. } Ax = b & [y] & \iff & & \text{s.t. } A^t y \geq c \\
 & \quad x \geq 0 & & & & \quad y \text{ free}
\end{array}
$$

The dual variable of each constraint is displayed in model (2.1), and the corresponding dual problem of the convex hull pricing model is model (2.14).

Now that the dual problem has been found, another optimization problem must be solved. This problem will have the convex hull prices as objective function (either maximisation or minimization), and have both constraints of the primal convex hull pricing model, of the dual convex hull pricing model, as well as the strong duality constraint that ensures that the objective values of those two models are equal. This is model (2.12).

$$
\min_{\pi} / \max_{\pi} \sum_{t \in \mathcal{T}} \pi_t \tag{2.12a}
$$

$$
\text{s.t. } (2.1b) - (2.1h), (2.1k), (2.4) \tag{2.12b}
$$

$$
(2.14b) - (2.14n) \tag{2.12c}
$$

$$
(2.1a) = (2.14a) \tag{2.12d}
$$

Model (2.12) can be applied to example 2.1. Let us consider a system with three periods with a demand of energy (in MW) equal to $[69, 70, 71]$. Table 2.2 shows results of four problems: the primal convex hull pricing model, the dual convex hull pricing model (2.14) and finally model (2.12) which either minimizes or maximizes convex hull prices.

| | Primal solution | Dual solution | Maximisation of (2.12) | Minimisation of (2.12) |
|---|---|---|---|---|
| $\pi$ | $[60, 65, 90]$ | $[60, 60, 90]$ | $[60, 90, 90]$ | $[60, 60, 90]$ |

Table 2.2: Energy prices for the four convex hull models

Both primal and dual models have the same objective value, which is equal to $-52870$ €. For model (2.12), constraint (2.12d) must be respected and the values of (2.1a) and (2.14a) (each side of constraint (2.12d)) must be equal to the objective values of the primal and dual models. In both cases of maximisation and minimisation, both sides have the same value as the objective values of the primal and dual models.

All four models have thus the same costs, how could it thus be possible that there are three different pricing solutions ? In order to investigate this, let us look into the dual model (2.14). Only constraints (2.14b) and (2.14j) involve dual variable $\pi$, the price of energy. Energy prices for each time step can thus be bounded by the following equation :

$$
\eta_{c,t} + \zeta_{c,t} + \omega \leq \pi_t \leq c_g - \nu_{g,t} - \mu_{g,t} - \delta_{g,t} \qquad \forall c \in \mathcal{C}, g \in \mathcal{G}, t \in \mathcal{T} \tag{2.13}
$$

The only dual variable that is both in a bound of the energy prices or in the dual objective function is $\zeta$. Tables 2.3 and 2.4 show the value of each variable present in both the lower and upper bound of equation (2.13) (only for unit one for the variables of generator, the values of units two and three are available in table A.1).

For example, the values of $\nu$ in table 2.4 for the maximisation and minimisation of model (2.12), are not the same. The other difference for the same situation is variable $\phi^u$, which is different in order to keep the same minimal cost. Variables $\eta$, $\nu$ and $\phi^u$ differ between the primal and dual. Prices differ between those four results are different but the cost remains the same, because there are multiple optimal solutions to those problems.

Thanks to the results presented here, it can be concluded that convex hull prices are not unique.

| | $\eta$ | $\zeta$ |
|---|---|---|
| Primal | $[-240, -235, -210]$ | $0\,\forall t$ |
| Dual | $[-240, -240, -210]$ | $0\,\forall t$ |
| Maximisation of (2.12) | $0\,\forall t$ | $[-240, -210, -210]$ |
| Minimisation of (2.12) | $0\,\forall t$ | $[-240, -240, -210]$ |

Table 2.3: Value of dual variables of the first node present in the lower bound of 2.13.

| | | $\nu$ | $\mu$ | $\delta$ | $\psi^u$ | $\psi^v$ | $\psi^z$ |
|---|---|---|---|---|---|---|---|
| Unit one | Primal | $[0, -5, -30]$ | $0\,\forall t$ | $0\,\forall t$ | [0,0,-1200] | $0\,\forall t$ | $0\,\forall t$ |
| | Dual | $[0, 0, -30]$ | $0\,\forall t$ | $0\,\forall t$ | $[0, 0, -1000]$ | $0\,\forall t$ | $0\,\forall t$ |
| | Maximisation of (2.12) | $[0, -30, -30]$ | $0\,\forall t$ | $0\,\forall t$ | $[0, -1200, -1000]$ | $0\,\forall t$ | $0\,\forall t$ |
| | Minimisation of (2.12) | $[0, 0, -30]$ | $0\,\forall t$ | $0\,\forall t$ | $[0, 0, -1000]$ | $0\,\forall t$ | $0\,\forall t$ |

Table 2.4: Value of dual variables present in the upper bound of 2.13.

**Dual problem of the unit commitment (without minimum up and down times)**

$$\max_{\nu,\mu,\pi,\psi,\alpha,\chi} \sum_{t\in\mathcal{T}} \left( \sum_{g\in\mathcal{G}} \left(\psi^u_{g,t} + \psi^v_{g,t} + \psi^z_{g,t}\right) + \sum_{c\in\mathcal{C}} \left(\kappa_c D_t \zeta_{c,t}\right) \right) \tag{2.14a}$$

$$\nu_{g,t} + \mu_{g,t} + \delta_{g,t} + \pi_t \leq c_g \quad [x_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.14b}$$

$$-\overline{P}_g \nu_{g,t} - \underline{P}_g \mu_{g,t} + \psi^u_{g,t} + \alpha^u_{g,t} - \xi_{g,t} + \chi_{g,t+1} \leq K_g \quad [u_{g,t}] \quad \forall g \in \mathcal{G}, t = 1 \tag{2.14c}$$

$$-\overline{P}_g \nu_{g,t} - \underline{P}_g \mu_{g,t} + \psi^u_{g,t} + \alpha^u_{g,t} - \chi_{g,t} \leq K_g \quad [u_{g,t}] \quad \forall g \in \mathcal{G}, t = |\mathcal{T}| \tag{2.14d}$$

$$-\overline{P}_g \nu_{g,t} - \underline{P}_g \mu_{g,t} + \psi^u_{g,t} + \alpha^u_{g,t} - \chi_{g,t} + \chi_{g,t+1} \leq K_g \quad [u_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T}\backslash\{1, |\mathcal{T}|\} \tag{2.14e}$$

$$\psi^v_{g,t} + \alpha^v_{g,t} + \xi_{g,t} \leq S_g \quad [v_{g,t}] \quad \forall g \in \mathcal{G}, t = 1 \tag{2.14f}$$

$$\psi^v_{g,t} + \alpha^v_{g,t} + \chi_{g,t} \leq S_g \quad [v_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T}\backslash\{1\} \tag{2.14g}$$

$$\psi^z_{g,t} + \alpha^z_{g,t} - \xi_{g,t} \leq 0 \quad [z_{g,t}] \quad \forall g \in \mathcal{G}, t = 1 \tag{2.14h}$$

$$\psi^z_{g,t} + \alpha^z_{g,t} - \chi_{g,t} \leq 0 \quad [z_{g,t}] \quad \forall g \in \mathcal{G}, t \in \mathcal{T}\backslash\{1\} \tag{2.14i}$$

$$-\pi_t + \eta_{n,t} + \zeta_{n,t} \leq -\omega \quad [d_{c,t}] \quad \forall c \in \mathcal{C}, t \in \mathcal{T} \tag{2.14j}$$

$$\mu_{g,t}, \alpha^u_{g,t}, \alpha^v_{g,t}, \alpha^z_{g,t}, \delta_{g,t} \geq 0 \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.14k}$$

$$\nu_{g,t}, \psi^u_{g,t}, \psi^v_{g,t}, \psi^z_{g,t} \leq 0 \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.14l}$$

$$\eta_{c,t} \geq 0, \zeta_{c,t} \leq 0 \quad \forall g \in \mathcal{C}, t \in \mathcal{T} \tag{2.14m}$$

$$\pi_t \text{ free}, \chi_{g,t} \text{ free} \quad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{2.14n}$$

# Chapter 3

# The energy market with co-optimization of energy and reserve

Outages can happen to every agent that is trading in energy. Indeed, generators may fail and consequently a production plant may struggle to deliver as much energy as it is intended to, or worse, can be unexpectedly offline. The units that produce renewable energy are also a significant source of unexpected events on the grid. Indeed, in a wind farm for example, if the wind is not as strong as expected, the output of the wind farm would be smaller than the system was expected to receive. Or the other way around, the wind may be too strong and as a result some wind farms may be forced to shut down their production because the wind exceeds the technical specifications of the wind turbine.

As for consumers, they can experience diverse issues, like for example, a sudden drop of demand from a big factory that usually requires a significant amount of energy, or the other way around, when consumers need more energy than expected.

To cope with those problems, and to re-balance energy production and energy consumption, additional services can be traded. The services that we are interested in in this section, are reserves. Those are services that are used to cope with outages of the system, and bring balance back to the system. It is a capacity that is saved by generators to deal with temporary outages in the system, and to prevent over production or electricity shortage, which are consequences of decrease or increase of demand and decrease or increase of production. The management of reserve while meeting energy consumption is one of the most important task, since this balance process keeps the production and consumption on the same level.

The major consequence of the introduction of reserve to a system is an increase of the costs covered by the producers, since generation capacity that may have been used for energy is now used for reserve. This therefore incurs fixed costs, since generators may be turned online to meet reserve consumption, and it is important that everyone recovers those fixed costs. This is thus crucial to price this service correctly.

There are several types of reserves, but in this work the distinction will not be made, and only one type of reserve will be studied.

The first sections of this chapter explains the motivation of co-optimization of energy and reserve and the modeling choices. Section 3.3 provides a way to incorporate reserve into the unit commitment model, while the pricing models are presented in section 3.4. Finally, section 3.5 explains how to compute uplifts for each agent while co-optimizing energy and reserve.

## 3.1 Motivation of co-optimization of energy and reserve

The exchange of reserves, according to [DV13], has the potential to reduce costs.

The fact that renewable energy is not one hundred percent reliable can become expensive. For instance, as cited in [Far+12], according to predictions from the European Wind Energy association, the production capacities using wind as a resource in Europe could have been up to 210GW last year (2020), amongst which 40GW are offshore. In less than ten years from now, in 2030, 150GW could be operational offshore. Since offshore installations are not directly connected to a country, the increase of this number would significantly expand the challenges of transmission and production planning forecast, as well as increasing significantly the need of a qualitative optimization of reserve. In this paper, reserve procurement and activation are modeled for the Northern European System (the Nordic region as well as Germany and the Netherlands). This procurement is co-optimized with energy in the day-ahead market allowing the solution of the unit commitment model to be closer to optimum. The modeling of reserve in [Far+12] is a short-term procurement, which is helpful to cope with the increasing uncertainties in the system of renewables.

The restricted capacity of transmission abilities in the Northern European system, which prevents the Nordic system from providing 100% of the required reserves, is a concern mentioned in [Far+12]. However, by combining energy and reserve optimization, it is possible to make greater use of interconnections while minimizing socioeconomic losses in this market, which is based on a fixed amount of reserve.

## 3.2 Modeling choices

Several ways are available to incorporate reserve into the unit commitment model. Indeed, [CGG14] mentions that two ways are possible.

- Either to define beforehand the amount of reserve the system must meet. This will be considered as a fixed input, that can be defined for each time step. This amount can be based on predictions and historical data of outages. What is left out in this modelling of reserve is the consequences of reserve transmission on transmission constraints and limitations. Indeed, a higher and unscheduled flow of reserve can occur at a certain time step, and that may exceed the transmission abilities of the system. This amount of reserve can either be modeled as a system-wide reserve requirement or can be split across the transmission system (explained in chapter 4). The most common way to define this is to take a certain percentage of the total demand of the system, or either to take the maximum run capacity of the unit that can produce the most.

- The main matter of the article [CGG14] is to model the reserve requirement as a variable of the optimization models as well as to consider deliverability issues. A clever way to decrease zonal reserve requirements is also presented, which can ensure deliverability. Indeed, identifying all potential deliverability issues could be difficult and one other major challenge is that pre-defining the amount of zonal reserve requirement could lead to a solution of the unit commitment model that is either sub-optimal or infeasible. Indeed, the model using a pre-defined amount of zonal reserve requirement considers that this amount will be produced, regardless of what happens on the grid. The flows of reserve are considered to be shipped even if reserve is not produced.
  Therefore, [CGG14] introduces in the optimization model two variables of reserve (one that is solved based on contingencies scenarios and one that represents the actual production of reserve). These two variables still needs to be at least higher than some minimal reserve requirements inputs but these inputs are derived through research of reserve zone configurations and requirements. Transmission constraints depend on the reserve variable which is based on these scenarios, and can thus account for the potential activation and deliverability of reserve.

The most popular way of modeling reserve is the first one. Indeed, this method makes the implementation easier and also has a cheaper computational cost, since the reserve requirement is just a fixed amount in input. This would allow to compute reserve prices by computing the dual variable of reserve market clearing constraint(s). As the main matter of this work is how to price reserve correctly, this method of modeling will be used used.

## 3.3 Unit commitment model

The introduction of reserve into the unit commitment model is thus an additional product to trade on the market, still between producers and consumers.

- Consumers wish to consume energy but can also consume reserve[1]. They remain modeled as before, but they have an additional valuation for reserve $\omega^R$. Each consumer can consume up to a fixed fraction for each consumer $\kappa_c^R$ of the total required reserve of the system $R$. As an example, if two consumers are in a system, $\kappa_1^R = 1/3, \kappa_2^R = 2/3$ would represent that the maximum consumption of consumer one (two) is equal to one third (two third) of the global amount of reserve of the system.
- Producers now have the ability to produce reserve as well. They still are dependent

[1] The transmission system operator is the one who purchases reserve auctions in the day-ahead market on behalf of the public. It is indeed its role to ensure reliability of the system and to protect consumers. The fact that reserve is modeled to be consumed by consumers is thus an approximation.

on every constraint as in chapter 2, but now have additional constraints to respect:

1. the production of reserve is only possible if the unit is also producing energy.
2. the amount of reserve saved per time step of each generator does not exceed the maximum amount that can be saved per time step, i.e. the upward ramp rate $R_g$, which is specific to each generator.
3. each generator that produces energy and saves some of its remaining available production for reserve stays in its operating range.

Model (3.1) is the complete unit commitment model in a context of co-optimization of energy and reserve, which minimizes costs of generators. Of course, it is still possible to transform this model into a welfare maximisation problem, by taking the opposite maximisation problem.

**Unit commitment model co-optimizing energy and reserve**

$$\min_{\substack{x,r,u,v,z\\ d,\tilde{r}}} \sum_{t\in\mathcal{T}} \left( \sum_{g\in\mathcal{G}} (x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum_{c\in\mathcal{C}} \left(d_{c,t}\omega + \tilde{r}_{c,t}\omega^R\right) \right) \tag{3.1a}$$

$$\text{s.t. } (2.1b) - (2.1l) \tag{3.1b}$$

$$\sum_{g\in\mathcal{G}} r_{g,t} = \sum_{c\in\mathcal{C}} \tilde{r}_{c,t} \qquad [\pi_t^R] \quad \forall t\in\mathcal{T} \tag{3.1c}$$

$$0 \le \tilde{r}_{c,t} \le \kappa_c^R R \qquad \forall c\in\mathcal{C}, t\in\mathcal{T} \tag{3.1d}$$

$$x_{g,t} + r_{g,t} \le u_{g,t}\overline{P}_g \qquad \forall g\in\mathcal{G}, t\in\mathcal{T} \tag{3.1e}$$

$$0 \le r_{g,t} \le R_g \qquad \forall g\in\mathcal{G}, t\in\mathcal{T} \tag{3.1f}$$

Constraints (2.1b)-(2.1l) have the same definitions as in chapter 2, while constraint (3.1c) is the market clearing constraint of reserve, which balances reserve production and reserve consumption. Constraint (3.1d) sets the upper bound on the amount of reserve each consumer can consume and constraint (3.1e) limits the total production of energy and reserve for each generator. Finally, constraint (3.1f) sets a lower and an upper bound on reserve production, which must be non-negative and lower than the amount of reserve the generator can produce per time step.

There is actually no costs to produce reserve in the objective value. For this reason, it is preferable that cheaper units produce energy while units that are more expensive produce reserve.

**Example 3.1** (Solution of energy and reserve dispatch of example 1)**.**
*Picking up on example 1, reserve can be introduced in this model in order to illustrate the*

*unit commitment solution. The unique consumer can now also buy reserve. One additional characteristic must be described for each producer, which is $R_g$, the maximum of reserve that can be provided by each generator in a single time step. It will be equal to*

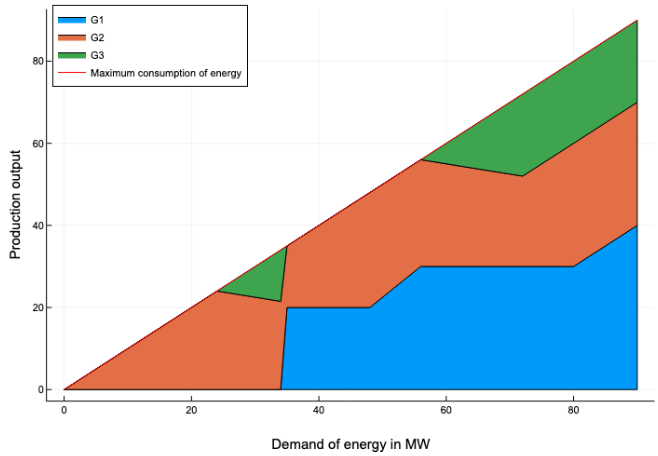
$$[R_1, R_2, R_3] = [10, 15, 0]$$

*Let us consider several demands of energy going from 1 to 90MW and a maximum consumption of reserve equal to one fourth of the energy demand (i.e. going from 0.25MW to 22.5MW). The consumers' willingness to pay for reserve will be taken to be equal to 200€/MWh while the valuation of energy is still the same as in example 1.*

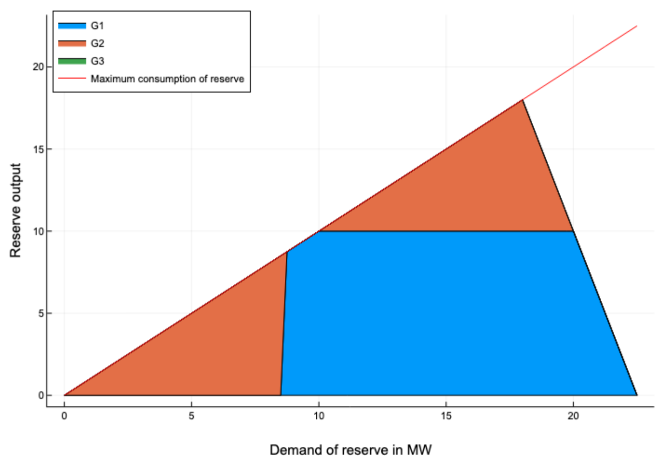
*Figure 3.1 shows the dispatch of both energy and reserve. The consumer always prefers to consume as much energy as possible. While reserve is also totally consumed up to a certain point, at a one point, there is a trade-off between reserve and energy. Let us look in detail into the dispatch in figure 3.1:*

- *The second producer is the first to provide energy and reserve, as it is the cheapest.*
- *Before turning* ON *unit one, since this producer has a start up cost, it is cheaper to first use unit three.*
- *Once unit one has been put* ON *(i.e. at a demand of energy of 35MW and a demand of reserve of 8.75MW), only this unit provides reserve. Indeed, this allows to keep generator one at its minimum run capacity for energy, while providing the required reserve, while also allowing unit two to have room for energy production, which is cheaper.*
- *Unit three is turned back* ON *at an energy demand of 57MW and a reserve demand of 14.25MW. The reason why it is turned* ON *is because with a demand of 56MW of energy and 14MW of reserve (i.e. total demand of 70MW) both generator one and two are producing at their maximum run capacities. This is when a trade-off between energy and reserve for units two and three takes place. Indeed, at this point, unit three begins to produce energy. Unit two, which was producing at its maximum run capacity, can now decrease its energy production in order to provide reserve, while keeping the demand fully satisfied.*
- *At some point, unit three is running at its maximum output and the maximum amount of reserve that can be consumed becomes so high, that units one and two must decide to trade reserve for energy. And as consumers have a higher valuation for energy, it is more beneficial for them to consume more energy. At the end, there would be a total of 90MW of energy consumed and 0MW of reserve.*

*One thing to be noticed is that when the system reaches a demand of energy of 35MW, unit one has been put* ON *and at this point, only this unit provides reserve since it is preferable to use more expensive units for reserve.*




(a) Dispatch of energy.




(b) Dispatch of reserve.

Figure 3.1: Dispatch of the solution of example 7 for several levels of demand (cumulative production).

The introduction of reserve did not change the fact that there are still non-convexities, since model (3.1) is still a mixed integer problem. As mentioned in section 2.1, it may not be possible to obtain uniform prices.

## 3.4 Pricing models

The additional product that is reserve creates a new continuous variable and adds constraints that use the already defined binary variables, but does not create any new binary variables. The unit commitment model is still a mixed-integer problem with non-convexities, therefore the pricing rules that are applied to the optimization of energy in section 2.2 can also be used here.

As before, the only thing that matters in the solutions of those pricing models are the prices. This time, besides the price of energy $\pi$ (dual variable of constraint (2.1d)), there is one additional price. This will be the price of reserve $\pi^R$, which is the dual variable of market clearing constraint for reserve (3.1c).

### Convex hull pricing

The continuous relaxation of the unit commitment model is still equivalent to the Lagrangian dual where the market clearing constraints are dualized. Indeed, the introduction of reserve did not modify the linearity of the objective function (3.1a), hence this function is already equal to its convex envelope.

As already mentioned, this new product did not influence the number of binary variables, hence to find the convex hull of the feasible set theorem 2.1 can be applied and constraint (2.1l) is still relaxed into:

$$u_{g,t}, v_{g,t}, z_{g,t} \geq 0 \qquad [\alpha^u_{g,t}, \alpha^v_{g,t}, \alpha^z_{g,t}] \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3.2a}$$

$$u_{g,t}, v_{g,t}, z_{g,t} \leq 1 \qquad [\psi^u_{g,t}, \psi^v_{g,t}, \psi^z_{g,t}] \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3.2b}$$

The other way to compute convex hull prices, which is model (2.3) in a context of energy-only, will be a bit different due to the additional reserve price. The Lagrangian dual of the unit commitment model (3.1), where both market clearing constraints have been dualized, is now the following:

**Lagrangian dual of unit commitment model 3.1**

$$\mathcal{P}(\pi, \pi^R) = \left\{ \begin{array}{ll} \min\limits_{\substack{x,r,u,v,z \\ d,\tilde{r}}} & \sum\limits_{t\in\mathcal{T}} \left( \sum\limits_{g\in\mathcal{G}} (x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum\limits_{c\in\mathcal{C}} \left( d_{c,t}\omega + \tilde{r}_{c,t}\omega^R \right) \right) \\ & - \sum\limits_{t\in\mathcal{T}} \left( \pi_t \left( \sum\limits_{g\in\mathcal{G}} x_{g,t} - \sum\limits_{c\in\mathcal{C}} d_{c,t} \right) + \pi^R_t \left( \sum\limits_{g\in\mathcal{G}} r_{g,t} - \sum\limits_{c\in\mathcal{C}} \tilde{r}_{c,t} \right) \right) \\ \text{s.t.} & (2.1b) - (2.1c) \text{ and } (2.1e) - (2.1l) \\ & (3.1d) - (3.1f) \end{array} \right\} \tag{3.3}$$

Based on this Lagrangian dual, convex hull prices are obtained by solving model (3.4).

$$\max_{\pi, \pi^R} \ \mathcal{P}(\pi, \pi^R) \tag{3.4}$$

The complete convex hull pricing model is model (3.1) with constraints (2.1l) relaxed into equations (3.2a)-(3.2b). If this model is solved again, now the dual variables of market clearing constraint (2.1d) can be obtained, which are the prices of energy as well as the dual variables of reserve market clearing constraint (3.1c), which are the prices of reserve.

## IP pricing

Similarly as in section 2.2, IP pricing will use the optimal solution of the unit commitment model and fix the non-convex part of this model to this optimal solution, fixing thus the non-convex part and dealing only with a convex model.

As for the model that only optimizes energy, constraint (2.1l) is relaxed into:

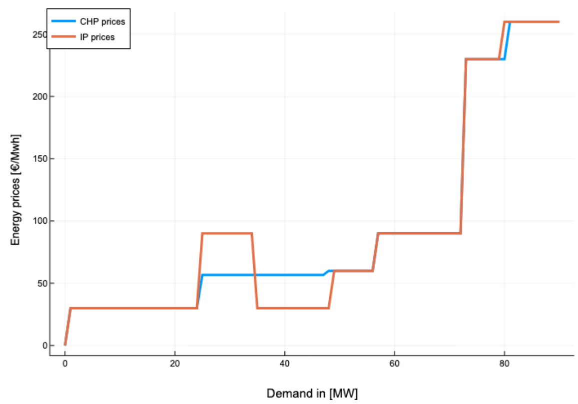
$$u_{g,t} = u^*_{g,t} \qquad [\beta^u_{g,t}] \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3.5a}$$
$$v_{g,t} = v^*_{g,t} \qquad [\beta^v_{g,t}] \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3.5b}$$
$$z_{g,t} = z^*_{g,t} \qquad [\beta^z_{g,t}] \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \tag{3.5c}$$

The full IP model that co-optimizes reserve and energy is thus model (3.1) with constraint (2.1l) relaxed into constraints (3.5a)-(3.5c), which allows the computation of the dual variable of the market clearing constraints of both energy (2.1d) and reserve (3.1c), which allows computing their respective prices $\pi$ and $\pi^R$.

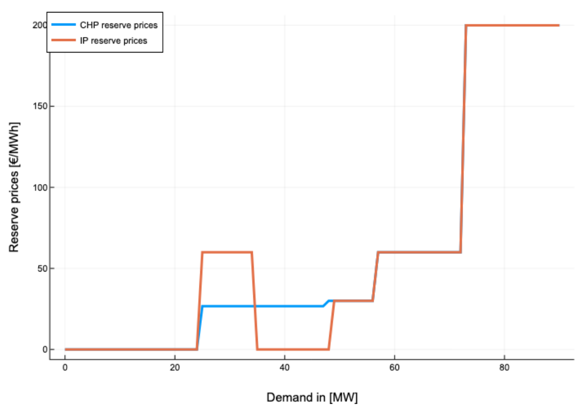
**Example 3.2** (Pricing solutions of example 3.1)**.**
*Results of applications of both convex hull pricing and IP pricing to example 3.1 can be seen on figure 3.2. Energy and reserve prices seem to follow the same pattern, except that reserve prices are sometimes equal to zero. To have a guess at how reserve prices behave, it can be analysed on several arbitrary intervals. Only IP prices are analysed since they are the marginal prices, where those can be computed "by hand" in a small system.*

- $D \in ]0, 24]$ *and* $R \in ]0, 6]$*: only unit two is producing. Reserve prices are equal to zero because there is no compromise between energy and reserve, while convex hull and IP energy prices are equal to 30€/MWh, which is the marginal cost of unit two. When* $D = 24$, $R = 6$*, the total demand is equal to 30MW, hence unit two will produce at its maximum run capacity.*
- $D \in [25, 34]$ *and* $R \in [6.25, 8.5]$*: this is where unit three is turned* ON*, since unit two does not have any available production left. IP prices are marginal, which is the reason why they are equal to 90€/MWh for energy (the cost of producing 1 additional MW of energy would be to use unit three) and to 60€/MWh for reserve. The cost of producing 1 additional MW of reserve would be to decrease the energy production of unit two by 1MW, in order that unit two can produce 1MW more reserve, while the 1MW hole would be produced by unit three, hence this would affect the total cost of the system to* $-30 + 90 = 60$*€.*
- $D \in [73, 79]$ *and* $R \in [18.25, 19.75]$*: the energy and reserve IP prices are respectively equal to 230€/MWh and 200€/MWh. In those intervals, unit two is trading reserve*

*for energy. Indeed, there is a higher valuation for energy, hence consumers benefit more from energy consumption than reserve. Energy prices are equal to this value since in order to produce an additional MW, the production of reserve needs to be decreased by 1MW and the production of energy has to be increased by the same amount. The cheapest way of doing this trade between the two products is by having unit two do it, hence an influence of cost of* $200 + 30 = 230$*. For the reserve price, the price is equal to 200€/MWh, because this is the only price that ensures that the consumer would agree to consume only a fraction of the maximum reserve consumption. Indeed, if the price was higher than its valuation, the consumer would not consume reserve, while if the price was lower than its valuation, he would want to consume as much reserve as he possibly can.*




(a) Energy prices.




(b) Reserve prices.

Figure 3.2: Results of the application of convex hull and IP pricing to the model of example 3.2.

*One can thus conclude that IP reserve prices are non-zero only when one unit needs to decrease its energy production in order to produce reserve. Convex hull prices, the same way as in example 2.3, are monotone, in both cases of energy and reserve.*

## 3.5 Uplifts

The same methodology as in section 2.3 will be used, i.e. first solve the selfish maximisation problem of each agent, and then subtract from this what the unit is earning at the unit commitment optimal solution.

The only difference with section 2.3 is that, on top of market clearing constraint (2.1d), there is an additional constraint that takes several agents into account, which is the market clearing constraint of reserve (3.1c). Before obtaining the selfish maximisation of each agent, this constraint also needs to be dualized and is exactly model (3.3).

This model can be transformed into a maximisation problem by taking the objective function to be the opposite sign, which is model (3.6).

$$\begin{aligned}
\max_{x,r,u,v,z,d,\tilde{r}} \sum_{t\in\mathcal{T}} & \left(\pi_t\left(\sum_{g\in\mathcal{G}} x_{g,t} - \sum_{c\in\mathcal{C}} d_{c,t}\right) + \pi_t^R\left(\sum_{g\in\mathcal{G}} r_{g,t} - \sum_{c\in\mathcal{C}} \tilde{r}_{c,t}\right)\right) \\
& - \sum_{t\in\mathcal{T}}\left(\sum_{g\in\mathcal{G}}(x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum_{c\in\mathcal{C}}\left(d_{c,t}\omega + \tilde{r}_{c,t}\omega^R\right)\right) && (3.6a) \\
\text{s.t. } & (2.1b) - (2.1c) \text{ and } (2.1e) - (2.1l) && (3.6b) \\
& (3.1d) - (3.1f) && (3.6c)
\end{aligned}$$

### 3.5.1 Producers

Given prices $\pi$ and $\pi^R$, and by keeping only constraints and objective terms related to a specific generator $g$, the selfish maximisation problem of each generator $g \in \mathcal{G}$ is the following:

$$\begin{aligned}
\max_{x,r,u,v,z} & \sum_{t\in\mathcal{T}}(\pi_t - C_g)x_{g,t} + \pi_t^R r_{g,t} - u_{g,t}K_g - v_{g,t}S_g && && (3.7a) \\
\text{s.t. } & (2.8b) - (2.8i) && && (3.7b) \\
& x_{g,t} + r_{g,t} \leq u_{g,t}\overline{P}_g && \forall t \in \mathcal{T} && (3.7c) \\
& 0 \leq r_{g,t} && \forall t \in \mathcal{T} && (3.7d) \\
& r_{g,t} \leq R_g && \forall t \in \mathcal{T} && (3.7e)
\end{aligned}$$

Given the solution of this selfish maximisation problem $\overline{x}, \overline{r}, \overline{u}, \overline{v}, \overline{z}$, and the unit commitment solution $x^*, r^*, u^*, v^*, z^*$, and similarly as in section 2.3.2, the uplift is equal to:

**Uplift computation for producer $g$ with reserve**

$$\begin{aligned}\text{uplift}_g = & \underbrace{\sum_{t\in\mathcal{T}} \left( (\pi_t - C_g)\, \overline{x}_{g,t} + \pi_t^R \overline{r}_{g,t} - K_g \overline{u}_{g,t} - S_g \overline{v}_{g,t} \right)}_{\text{what the producer wants to do selfishly}} \\ & - \underbrace{\sum_{t\in\mathcal{T}} \left( \left( (\pi_t - C_g)\, x^*_{g,t} + \pi_t^R r^*_{g,t} - K_g u^*_{g,t} - S_g v^*_{g,t} \right) \right)}_{\substack{\text{profit/loss producer is having} \\ \text{at the unit commitment solution}}}\end{aligned} \tag{3.8}$$

### 3.5.2 Consumers

Following the same methodology as for producers, given prices, the selfish maximisation problem of each consumer $c \in \mathcal{C}$ is:

$$\max_{d,\tilde{r}} \sum_{t\in\mathcal{T}} \left( d_{c,t}\, (\omega - \pi_t) + \tilde{r}_{c,t} \left( \omega - \pi_t^R \right) \right) \tag{3.9a}$$

$$\text{s.t. } 0 \le d_{c,t} \qquad \forall t \in \mathcal{T} \tag{3.9b}$$

$$d_{c,t} \le \kappa_c D_t \qquad \forall t \in \mathcal{T} \tag{3.9c}$$

$$0 \le \tilde{r}_{c,t} \qquad \forall t \in \mathcal{T} \tag{3.9d}$$

$$\tilde{r}_{c,t} \le \kappa_c^R R \qquad \forall t \in \mathcal{T} \tag{3.9e}$$

Once the solution of the selfish maximisation problem $\overline{d}, \overline{\tilde{r}}$ is found, and given the unit commitment solution $d^*, \tilde{r}^*$, the uplift computation for each consumer $c \in \mathcal{C}$:

**Uplift computation for consumer $c$ with reserve**

$$\text{uplift}_c = \underbrace{\sum_{t\in\mathcal{T}} \left( \overline{d}_{c,t}\, (\omega - \pi_t) + \overline{\tilde{r}}_{c,t} \left( \omega^R - \pi_t^R \right) \right)}_{\text{what the consumer wants to do selfishly}} - \underbrace{\sum_{t\in\mathcal{T}} \left( d^*_{c,t}\, (\omega - \pi_t) + \tilde{r}^*_{c,t} \left( \omega^R - \pi_t^R \right) \right)}_{\substack{\text{profit/loss consumer is having} \\ \text{at the unit commitment solution}}} \tag{3.10}$$

IP pricing still don't give any uplifts to consumers, for the same reason as stated in section 2.3.3.

# Chapter 4

# The co-optimization of energy and reserve with a transmission system

This work focused on models that considered all producers and consumers in one single place. This chapter aims at incorporating transmission capabilities into those models. Indeed, producers and consumers are not often located in the same place, and the transmission of energy and reserve are dictated by physical laws. This significantly increases the complexity of the energy and reserve trading. This additional feature will allow us to conduct more realistic simulations in chapter 5.

The introduction of the transmission system adds a new agent to the models, which is the transmission system operator. This adds a new traded product: transmission. Combined with energy and ancillary services, those are the three major products traded in electricity markets.

As cited in [Far+12], the transmission system could be used to build an offshore grid, as explained in section 3.1. They conclude in this paper that the current transmission system could not support the proposed offshore grid, due to the lack of capacities of the transmission system. Indeed, the use of a transmission system in a system that co-optimizes energy and reserve means that not only energy will be shipped, but reserve could also be shipped, in order to re-balance the system. And since the moment when reserve could be actually shipped (and not just saved) is not planned, nor the actual amount, this can make the transmission system full. This significantly increases the complexity of the dispatch, because:

- by allowing shipments of energy and reserve, this is an opportunity for cheaper units to produce more energy in order to ship it to places where there are only more expensive generators, that could be kept offline,
- the maximum capacity of the lines could be more rapidly reached.

Section 4.1 presents the introduction of the transmission system to the unit commitment model. The pricing models are discussed in section 4.2, while the computations of uplifts are explained in section 4.3.

## 4.1 Unit commitment model

The transmission network will be modeled using nodes and edges, as a graph. The nodes will represent several locations having energy and/or reserve demand or not and having generators or not (i.e. the generation capacities of those generators can provide energy and reserve to this node). The edges will represent the lines connecting the nodes together. Those lines have minimum and maximum capacities. The set of nodes (lines) is noted $\mathcal{N}$ ($\mathcal{L}$) and its index is $n$ ($l$).

### 4.1.1 Description of the transmission system

To incorporate the transmission system in the unit commitment model, several new parameters as well as new variables needs to be defined:

- Link matrix $L$: this matrix will indicate which generators are connected to which node. $L_{g,n} = 1$ $(= 0)$ if generator $g$ is (not) connected to node $n$.
- Incidence matrix $B$: to tell which line connects which node together and in which direction. $B_{l,n} = -1$ $(= 1)$ if line $l$ goes out (in) of node $n$.
- Minimum and maximum capacities of lines $\overline{f}, \underline{f}$: each line has its specific minimum and maximum capacity in MW, which defines the shippable amount of energy or reserve.
- Variable $I_{n,t}$: this new variable will represent the injection of energy in node $n$ at time step $t$. This will thus be the amount of energy in $MW$ that is injected in a specific node. This variable will be positive if there is energy shipped from one node (import), or negative if there is energy shipped to one node (export).
- Variable $f_{l,t}$: the shipment of energy on each line is defined by this variable, which will indicate the amount of energy in MW that is being shipped on line $l$ at time step $t$.

Note also that from now on, each consumer is attached to one node, and can consume energy and reserve. In other words, there is no difference between $d_{c,t} = d_{n,t}$ and $\tilde{r}_{c,t} = \tilde{r}_{n,t}$. The balance constraint should ensure that the consumption of each consumer is met, by production in the node itself and/or production shipped to this node but produced in another node.

### 4.1.2 Reserve market design

From now on, this work will now focus on two different ways to co-optimize energy and reserve in a system with a transmission network. From [VBD18], the two possible options to optimize reserve that are studied in this work are either *approach one (uncoordinated* *procurement)*, which implies that reserve must be provided to each node only by the generators connected to the node, or *approach three (coordinated procurement with network constraints)*, which implies that reserve can also be shipped.

### Approach one, uncoordinated procurement

$$\sum_{g \in \mathcal{G}} L_{n,g} r_{g,t} = \tilde{r}_{n,t} \qquad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{4.1}$$

Since reserve can only be provided to the consumer located in one node by generators connected to this specific node, only constraint (4.1) is needed. This will result in a nodal price for reserve. A consequence of this market design is that consumption of reserve in nodes that have zero generation capacity is not possible, hence only energy can be consumed in those nodes.

### Approach three, coordinated procurement with network constraints

From [VBD18], approach three introduces three additional constraints:

$$\sum_{g \in \mathcal{G}} r_{g,t} = \sum_{n \in \mathcal{N}} \tilde{r}_{n,t} \qquad \forall t \in \mathcal{T} \tag{4.2a}$$

$$\sum_{g \in \mathcal{G}} L_{n,g} r_{g,t} + \sum_{l \in \mathcal{L}} B_{l,n} \Delta f_{l,t} = \tilde{r}_{n,t} \qquad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{4.2b}$$

$$\underline{f_l} \leq f_{l,t} + \Delta f_{l,t} \leq \overline{f_l} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{4.2c}$$

One can see that constraint (4.2a) is a consequence of constraint (4.2b). Indeed, if the sum on every node of constraint (4.2b) is computed, we find back constraint (4.2a). The three terms are equal to:

- $\sum_{n \in \mathcal{N}} \sum_{g \in \mathcal{G}} L_{n,g} r_{g,t} = \sum_{g \in \mathcal{G}} r_{g,t}$, the sum on every node of all generation capacity is equal to the sum of all generation capacity
- $\sum_{n \in \mathcal{N}} \sum_{l \in \mathcal{L}} B_{l,n} \Delta f_{l,t} = 0$, the sum on every node of all reserve flows is equal to zero, since what goes out from one place goes into another place
- $\sum_{n \in \mathcal{N}} \tilde{r}_{n,t}$

Hence, constraint (4.2a) is not needed and can be dropped. Approach three has only one reserve market clearing constraint, which is constraint (4.2b), its dual variable is the price of reserve, and as for approach three, is therefore a nodal reserve price.

**Example 4.1** (Introduction of transmission system to example 3.1)**.**
*This example builds on top of example 3.1 by introducing a transmission network to the system. Figure 4.1 shows the transmission network, using two nodes A and B, connected by one bi-directional line $l_1$. The incidence matrix is taken to be equal to $B = [-11]$. This line*

*therefore has a minimum (maximum) run capacity equal to -15MW (15MW). A negative flow represents a flow in the opposite direction, i.e. in this case from B to A. Generator one, the one that has a start-up cost, is connected to node A, while the other two generators ($G_2, G_3$) are connected to node $B$. Each node is connected to a specific consumer that can consume energy and reserve ($D_A, R_A$ and $D_B, R_B$).*
*Approach one will only allow line $l_1$ to ship energy, while approach three will allow this line to also ship reserve. In both approaches, there is one price of energy and one price of reserve per node, hence for this system four prices $\pi_A, \pi_A^R, \pi_B, \pi_B^R$.*

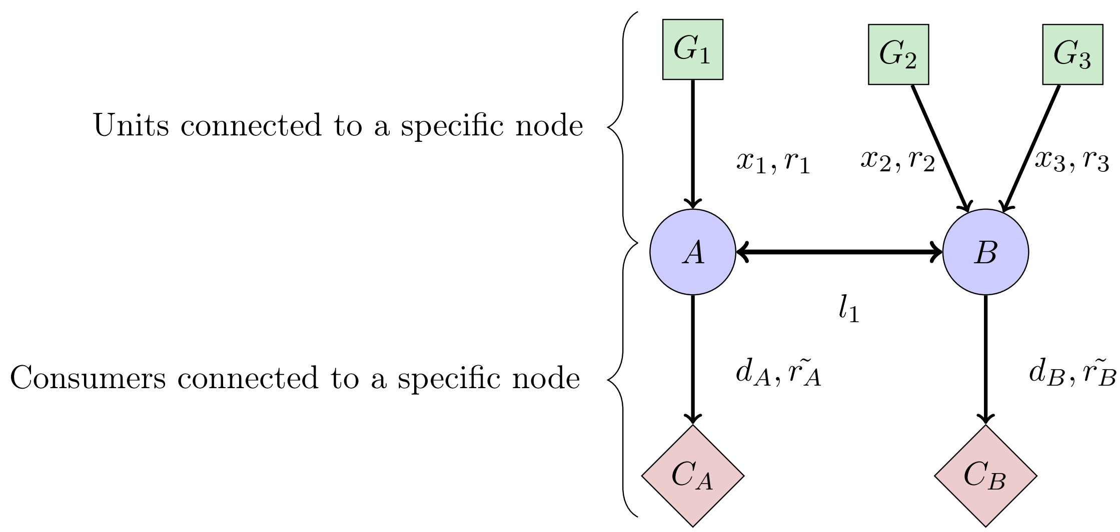



Figure 4.1: Transmission system of example 9.

### 4.1.3 Complete model

The complete unit commitment model that co-optimizes energy and reserve with transmission abilities is model (4.3). It reproduces almost every constraint from previous chapters, except for energy market clearing constraint (2.1d) and reserve market clearing constraint (3.1c) which need to be modified in order to add transmission abilities to the system.

Constraint (4.3d) is the new energy market clearing constraint, which ensures that the consumption in each node is equal to the addition of the injection to this node and the production of the generators connected to this node. The injection variable only indicates if a node is exporting energy (negative injection) or importing energy (positive injection), but nothing in the energy balance constraint indicates the values of flows of energy. This is the goal of constraint (4.3e), which ensures that an injection in a node is equal to in-going flows minus out-going flows. Constraint (4.3f) ensures that each line stays in its operating range, which depends on the minimum (maximum) capacity $\underline{f_l}$ ($\overline{f_l}$).

**Unit commitment model with transmission system co-optimizing energy and reserve**

$$\min_{\substack{x,r,u,v,z\\ d,\tilde{r},I,f,\Delta f}} \sum_{t\in\mathcal{T}} \left( \sum_{g\in\mathcal{G}} (x_{g,t}c_g + u_{g,t}K_g + v_{g,t}S_g) - \sum_{n\in\mathcal{N}} \left(d_{n,t}\omega + \tilde{r}_{n,t}\omega^R\right) \right) \tag{4.3a}$$

$$\text{s.t. } (2.1b) - (2.1c) \text{ and } (2.1e) - (2.1l) \tag{4.3b}$$

$$(3.1d) - (3.1f) \tag{4.3c}$$

$$d_{n,t} - I_{n,t} - \sum_{g\in\mathcal{G}} L_{g,n}x_{g,t} = 0 \qquad [\pi_{n,t}] \quad \forall n\in\mathcal{N}, t\in\mathcal{T} \tag{4.3d}$$

$$I_{n,t} - \sum_{l\in\mathcal{L}} B_{l,n}f_{l,t} = 0 \qquad \forall n\in\mathcal{N}, t\in\mathcal{T} \tag{4.3e}$$

$$\underline{f_l} \leq f_{l,t} \leq \overline{f_l} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{4.3f}$$

$$\mathbf{A1}: \sum_{g\in\mathcal{G}} L_{n,g}r_{g,t} = \tilde{r}_{n,t} \qquad [\pi^R_{n,t}] \quad \forall n\in\mathcal{N}, t\in\mathcal{T} \tag{4.3g}$$

$$\mathbf{A3}: \sum_{g\in\mathcal{G}} L_{n,g}r_{g,t} + \sum_{l\in\mathcal{L}} B_{l,n}\Delta f_{l,t} = \tilde{r}_{n,t} \qquad [\pi^R_{n,t}] \quad \forall n\in\mathcal{N}, t\in\mathcal{T} \tag{4.3h}$$

$$\mathbf{A3}: \underline{f_l} \leq f_{l,t} + \Delta f_{l,t} \leq \overline{f_l} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{4.3i}$$

Finally, while approach one only has constraint (4.3g) which balances reserve production and consumption in each node individually, approach three introduces two constraints, the first one being the market clearing constraints ensuring that consumption in each node is equal to the sum of production in this node and the difference between in-going flows and out-going reserve flows. The second ensures that each line that ships energy and reserve stays in its operating range.

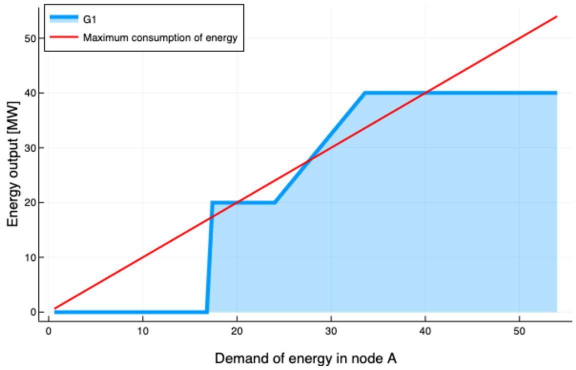
**Example 4.2** (Solution of example 4.1)**.**
*First, it should be remembered that generators used in this example are the same as in example 2.1. Model (3.1) can be used to solve example 4.1. The system has been solved 90 times, with a global energy demand of the system going from 1MW to 90MW. This demand is split across nodes A and B, and the consumer of node A can consume up to 60% of the global energy demand (*$\kappa_A = 0.6$*), and consumer of node B up to 40% (*$\kappa_A = 0.4$*). The total demand of reserve was fixed to be equal to one fourth of the global demand of energy,*

*and only the consumer in node B can consume reserve (i.e.* $\kappa_A^R = 0, \kappa_B^R = 1$*).*

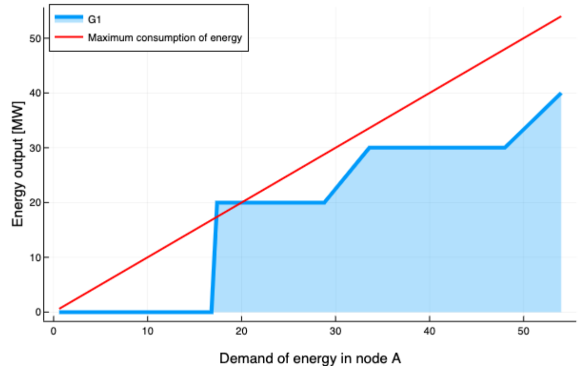
*Both approaches one and three are solved. In figures 4.2a, 4.2b, 4.2c and 4.2d, it can be seen that the demand of energy is not influenced by those different approaches, thanks to the transmission system which allows energy to be shipped from one node to the other. The maximum run capacity of node A (i.e. generator one) becomes lower than the maximum demand at a certain point, but the maximum run capacity of node B (i.e. generator two and three) is always bigger than the demand in this node. Hence, energy can be dispatched the cheapest way possible.*

*On the other hand, the dispatch of reserve is totally different as it is displayed in figures 4.2e and 4.2f. Indeed, while with approach one the consumption of reserve in node B can only be produced by generator two, approach three allows shipments of reserve, which enables generator one to also produce reserve and to ship it to node B. Those two figures also show that the production of reserve by generator* 2 *is a lot different between the two approaches. Indeed, generator two produces up to 15MW of reserve (its maximum run capacity of reserve) and then its production begins to decrease with approach one, in order to produce more energy, With approach three, the first demands of energy and reserve are met by generator two, but when generator one must be turned* ON *in order to provide the additional demand of energy and reserve, only generator one provides reserve and ships it to node B.*

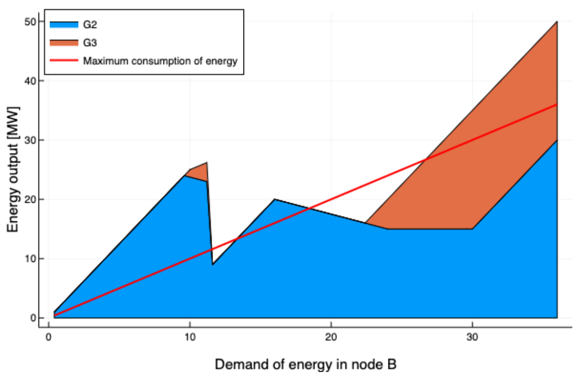
*Finally, figures 4.3a and 4.3b show the flows between the two nodes, where it can be seen that since generator one is shipping reserve to node B, it is node B that ships back energy to node A, ensuring a lot of consumption of both products. In figure 4.3b, for a demand of energy in node A equal to 45MW, the flow of energy from B to A is equal to 15MW, which is the maximum capacity of the line. There is also a flow of reserve from A to B equal to 10MW. The line is not binding because it would result in a net flow of 5MW from B to A should reserve be produced.*



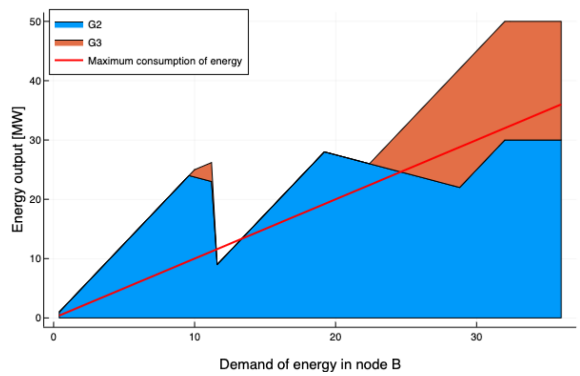

(a) Energy production in node A with approach one.



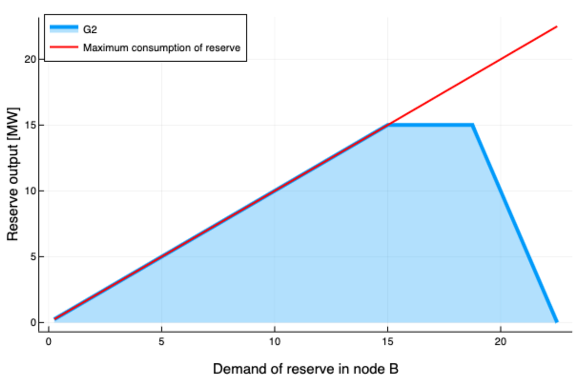

(b) Energy production in node A with approach three.



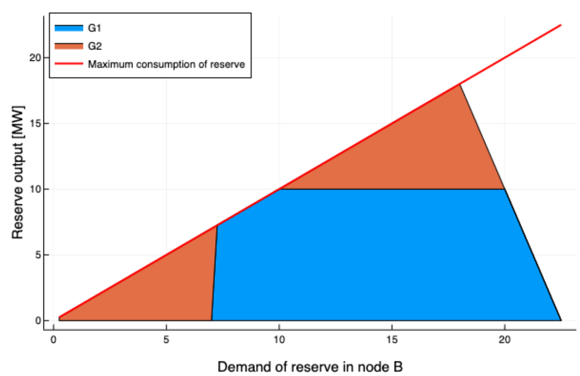

(c) Cumulative energy production in node B with approach one.




(d) Cumulative energy production in node B with approach three.

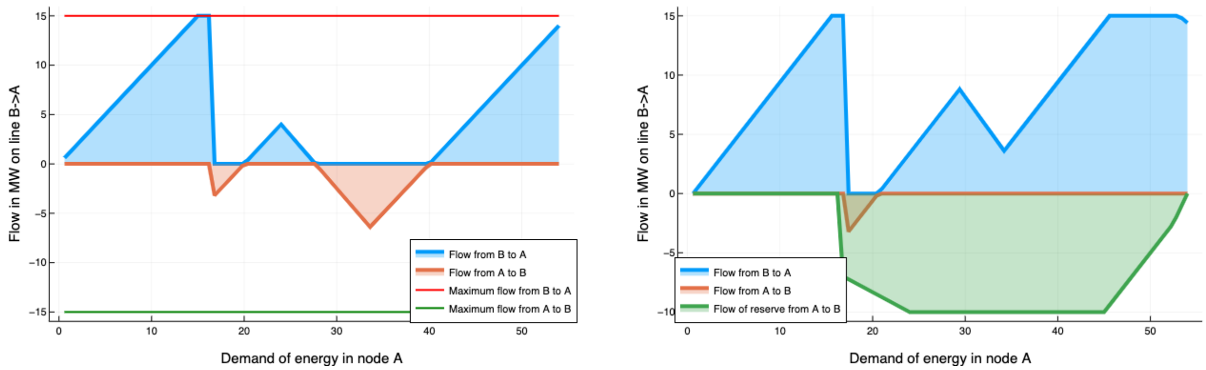



(e) Reserve production in node B by generator 2 with approach one.




(f) Cumulative reserve production in node B with generator 2 and import of reserve by generator 1 in node A with approach three.

Figure 4.2: Energy and reserve dispatch of example 4.2.




(a) Flows between nodes A and B with approach one.

(b) Flows between nodes A and B with approach three.

Figure 4.3: Flows of energy and reserve of example 4.2.

Example 4.2 enlightened that model (4.3) considered that, with approach three, the flow of reserve is modeled as if reserve would definitely be activated. This is a weakness of this model, since reserve may or may not be produced. The decision of production depends on real-times contingencies and outages that could happen on the grid.
[CGG14] discusses the modeling of reserve with two variables: minimum zonal reserve requirements and zonal cleared reserve production. The advantage of these variables is that deliverability issues are taken care of. Indeed, there is a transmission constraint that ensures that, if reserve is deployed due to a certain contingency, the impact by outages of generation units and the consequences of the production of this reserve due to this event on the transmission systems and constraints are considered.

One can see that, similarly to the introduction of reserve, the introduction of the transmission system did not influence the non-convex part of the unit commitment model. Indeed, it still is a mixed integer problem. Section 2.1 mentioned that there may not exist uniform prices for such a model.

## 4.2 Pricing models

The establishment of a transmission system adds two (three in the case of approach three) continuous variables, and several new constraints. None of these constraints uses binary variables, hence this new feature to the unit commitment model does not influence the non-convex part of the unit commitment model.

Since the model is still non-convex, the same pricing models as in the previous chapters will be used in order to obtain near-equilibrium prices.

### Convex hull pricing

It is again possible to compute these prices with the continuous relaxation of the unit commitment model. The solution of this model is indeed still equivalent to the solution of the Lagrangian dual of the unit commitment model where the balance constraints are dualized, for two reasons.

- The objective function is still linear, since the introduction of the transmission system did not modify this function. The convex envelope of the objective function is exactly equal to this function.
- The added constraints to the feasible set are all convex since they do not involve any binary variables. The continuous relaxation of this set will thus remains the same as before.

Therefore, theorem 2.1 can be used again to find the convex hull of the feasible set which is described by its continuous relaxation, hence:

$$u_{g,t} \in \{0,1\} \longrightarrow 0 \leq u_{g,t} \leq 1 \tag{4.4a}$$

$$v_{g,t} \in \{0,1\} \longrightarrow 0 \leq v_{g,t} \leq 1 \tag{4.4b}$$

$$z_{g,t} \in \{0,1\} \longrightarrow 0 \leq z_{g,t} \leq 1 \tag{4.4c}$$

To compute convex hull prices, one can also use the Lagrangian dual of the unit commitment model where market clearing constraints have been dualized. This time, the Lagrangian dual is

**Lagrangian dual of unit commitment model 4.3**

$$\mathcal{Q}(\pi,\pi^R) = \left\{ \begin{array}{ll} & \min\limits_{\substack{x,r,u,v,z\\ d,\tilde{r},I,f,\Delta f}} \sum\limits_{t\in\mathcal{T}} \left( \sum\limits_{g\in\mathcal{G}} (x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum\limits_{c\in\mathcal{C}} \left(d_{c,t}\omega + \tilde{r}_{c,t}\omega^R\right) \right) \\ & \quad - \sum\limits_{t\in\mathcal{T}} \sum\limits_{n\in\mathcal{N}} \left( \pi_{n,t} \left( \sum\limits_{g\in\mathcal{G}} L_{g,n} x_{g,t} + I_{n,t} - d_{n,t} \right) \right) \\ & \quad \textbf{(A1)} \; - \sum\limits_{t\in\mathcal{T}} \sum\limits_{n\in\mathcal{N}} \left( \pi^R_{n,t} \left( \sum\limits_{g\in\mathcal{G}} L_{n,g} r_{g,t} - \tilde{r}_{n,t} \right) \right) \\ & \quad \textbf{(A3)} \; - \sum\limits_{t\in\mathcal{T}} \sum\limits_{n\in\mathcal{N}} \left( \pi^R_{n,t} \left( \sum\limits_{g\in\mathcal{G}} L_{n,g} r_{g,t} + \sum\limits_{l\in\mathcal{L}} B_{l,n} \Delta f_{l,t} - \tilde{r}_{n,t} \right) \right) \\ \text{s.t.} & (2.1b) - (2.1c) \text{ and } (2.1e) - (2.1l) \\ & (3.1d) - (3.1f) \\ & (4.3e) - (4.3f) \text{ and } \textbf{(A3)} \; (4.3i) \end{array} \right\} \tag{4.5}$$

Convex hull prices are obtained by solving the following optimization problem:

$$\max_{\pi,\pi^R} \; \mathcal{Q}(\pi, \pi^R) \tag{4.6}$$

To obtain convex hull prices, it is thus needed to solve either (4.6) or the unit commitment model (4.1) with constraint (2.1l) relaxed into constraints (4.4).

## IP pricing

Just like for convex hull prices, the method used for the model without a transmission system can be adopted here. The only thing to modify is constraint (2.1l), which is now fixed to the optimal unit commitment solution.

$$u_{g,t} \in \{0,1\} \quad \longrightarrow \quad u_{g,t} = u^*_{g,t} \tag{4.7a}$$

$$v_{g,t} \in \{0,1\} \quad \longrightarrow \quad v_{g,t} = v^*_{g,t} \tag{4.7b}$$

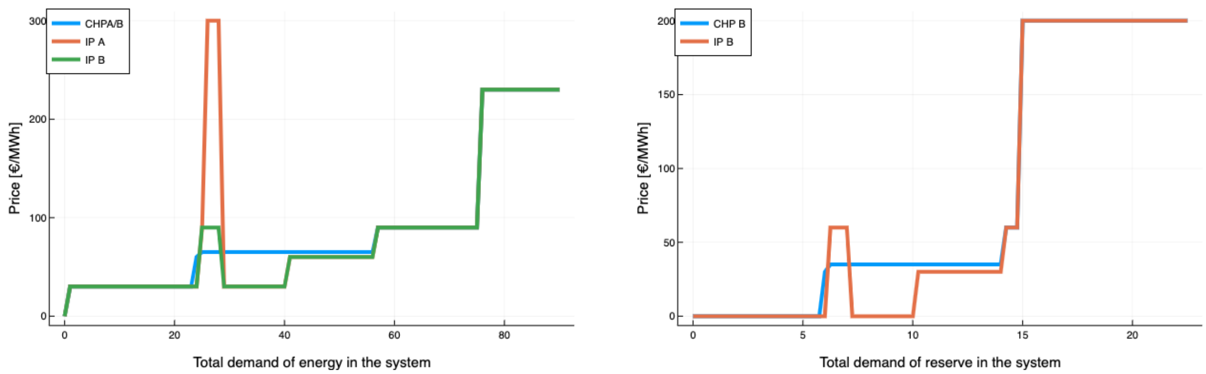
$$z_{g,t} \in \{0,1\} \quad \longrightarrow \quad z_{g,t} = z^*_{g,t} \tag{4.7c}$$

The complete IP pricing model is unit commitment model (4.1) with constraint (2.1l) relaxed into constraints (4.7). As a reminder, when solving the IP pricing model, the non-convex part is now fixed.

**Example 4.3** (Application of pricing models to example 4.2)**.**
*This example builds on top of the previous example to show the influence of the transmission system on prices.*

*With approach one, figures 4.4a and 4.4b respectively show energy and reserve prices of the solution in the previous example using approach one. The higher volatility of IP prices can directly be seen. One consequence of the transmission system is that it can incur price differentials between zones, which is the case for a global demand of 26MW and 27MW with IP pricing. The cause of this price differential is that the line connecting A to B is binding, i.e. it is shipping energy at its full capacity.*

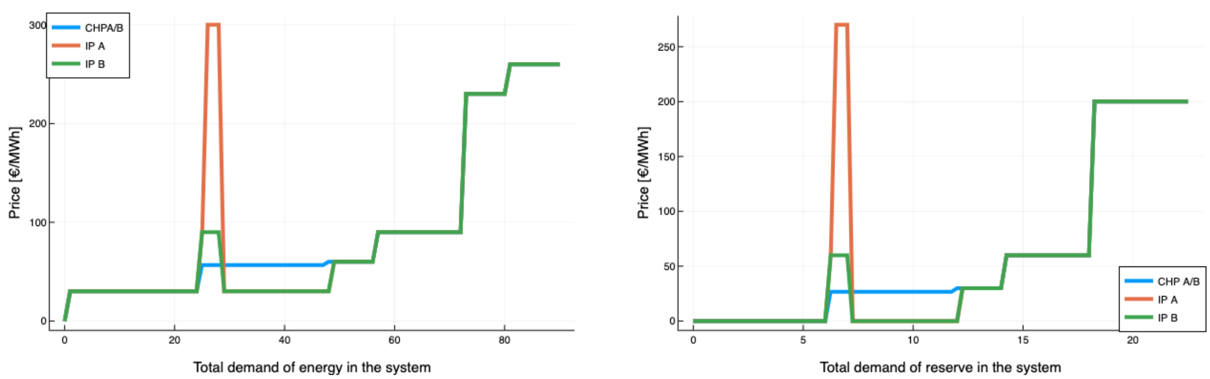
*The consumer of node A can only consume energy, since reserve can not be imported, reserve prices in this node are equal to zero for approach one. For this reason, only prices of reserve in node B are displayed in figure 4.4b. This is not the case for approach three, since now node A can ship reserve to node B and vice-versa, hence prices of reserve in node A can be positive for this approach.*




(a) Energy prices with approach one.

(b) Reserve prices with approach one.

Figure 4.4: Energy and reserve prices of example 4.3 with approach one.




(a) Energy prices with approach three.

(b) Reserve prices with approach three.

Figure 4.5: Energy and reserve prices of example 4.3 with approach three.

*With approach three, the higher volatility can also be seen. The number of price differentials is the same as with approach one, because the dispatch does not result in more frequent binding lines (which can be seen on figure 4.3b). On the other hand, for reserve, node A is shipping reserve to node B, which creates positive prices of reserve in node A.*

Example 4.3 enlightened the fact that prices differentials occur when lines were binding. In fact, with the transmission constraints that are used for approach three, a line might not be binding. Indeed, as mentioned in example 4.2, if a line is shipping energy in one direction and reserve in the other, even if constraint (4.3f) is tight, constraint (4.3i) might not be tight, since the two flows would not have the same sign.

On top of that, as mentioned in [Sch+15], a price differential can occur even if the line is not binding in the optimal dispatch of the unit commitment model. Indeed, when there is a price differential, the difference between the two prices is pocketed by the TSO, i.e. represents a congestion price. This can happen for example when the unit commitment model does not dispatch lines to be binding, but the convex hull pricing model has a dispatch that does make the lines binding.

## 4.3 Uplifts

The pricing models are the same as before and give near-equilibrium prices, and they can cause loss or opportunity costs. This is why uplifts are still needed. This time, in addition to uplifts of producers and consumers, since there is an additional agent in the system, uplift must be computed for this agent as well.

By taking the Lagrangian dual of unit commitment model (4.3), this can be transformed into a maximisation problem, in order to find the selfish maximisation sub-problem for each agent. The welfare maximisation problem of the Lagrangian dual is the following:

$$
\begin{aligned}
\max_{\substack{x,r,u,v,z\\ d,\tilde{r},I,f,\Delta f}} & \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}}\left(\pi_{n,t}\left(\sum_{g\in\mathcal{G}} L_{g,n}x_{g,t} + I_{n,t} - d_{n,t}\right)\right)\\
&\textbf{(A1)} \quad + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}}\left(\pi^R_{n,t}\left(\sum_{g\in\mathcal{G}} L_{n,g}r_{g,t} - \tilde{r}_{n,t}\right)\right)\\
&\textbf{(A3)} \quad + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}}\left(\pi^R_{n,t}\left(\sum_{g\in\mathcal{G}} L_{n,g}r_{g,t} + \sum_{l\in\mathcal{L}} B_{l,n}\Delta f_{l,t} - \tilde{r}_{n,t}\right)\right)\\
&- \left(\sum_{g\in\mathcal{G}}(x_{g,t}C_g + u_{g,t}K_g + v_{g,t}S_g) - \sum_{c\in\mathcal{C}}\left(d_{c,t}\omega + \tilde{r}_{c,t}\omega^R\right)\right)
\end{aligned}
\tag{4.8a}
$$

$$
\begin{aligned}
\text{s.t. } & (2.1b) - (2.1c) \text{ and } (2.1e) - (2.1l) && (4.8b)\\
& (3.1d) - (3.1f) && (4.8c)\\
& (4.3e) - (4.3f) \text{ and } \textbf{(A3)}\ (4.3i) && (4.8d)
\end{aligned}
$$

### 4.3.1 Producers

The selfish maximisation problem of one producer is obtained by taking only terms and constraints related to this unit. One can see that in model (4.8), the introduction of the transmission system did not influence the terms of producers, except that they are producing in a given node and that the resulting prices of energy and reserve are also nodal. As the uplift is computed for each producer separately, the focus is put only on prices of the node a producer $g$ is connected to. If $\tilde{n}$ denotes this node, we can only keep prices of this node, and hence note $\pi_{\tilde{n},t} = \pi_t, \pi^R_{\tilde{n},t} = \pi^R_t$. Thanks to this notation, the exact same selfish maximisation problem of each producer as the one described in section 3.5.1 can be found again.

As the computation of uplifts takes only the solution of the selfish maximisation problem of a given producer, and what profit/loss this producer is making at the unit commitment solution, the uplift of each producer $g$ is computed with the exact same formula (3.8).

### 4.3.2 Consumers

As with producers, the only thing that affects consumers is that now they are consuming energy and reserve in a given node, and that prices are nodal. If prices are denoted the same way as in the previous sub-section, the exact same selfish maximisation problem of each consumer as in section 3.5.2 is found.

And again, as in previous subsection, the uplift for each consumer is computed with the same formula (3.10).

The same statement as in section 2.3.3 is still applied here, hence consumers do not have any uplifts with IP pricing.

### 4.3.3 Transmission system operator

Only variables $I$ and $f$, and in the special case of approach three for reserve $\Delta f$, are related to the transmission system operator. The same methodology as in previous sub-sections and chapters will be used.

The selfish maximisation problem of the transmission system operator is obtained by only keeping terms of the objective function and constraints related to the previously mentioned variables. Since this agent has either two variables with approach one or three with approach three, this sub-section will be divided in two paragraphs which explain the uplifts computation for each approach separately.

**Approach one** Given prices $\pi$, the selfish maximisation problem will be model (4.9).

$$\max_{I,f} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} \pi_{n,t} I_{n,t} \tag{4.9a}$$

$$(4.3e) - (4.3f) \tag{4.9b}$$

Following the same method as in previous sub-sections and chapters, given prices $\pi$ and based on the solution $\overline{I}, \overline{f}$ of the model up-above, and the solution $I^*, f^*$ of the unit commitment model, the uplift of the TSO is computed with (4.10)

**Uplift computation for TSO with reserve and transmission system with approach one**

$$\text{uplift}_{TSO} = \underbrace{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \pi_{n,t}\overline{I}_{n,t}}_{\text{what the TSO wants to do selfishly}} - \underbrace{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \pi_{n,t}I^*_{n,t}}_{\substack{\text{profit/loss TSO is having}\\ \text{at the unit commitment solution}}} \tag{4.10}$$

**Approach three** Following the same method as the previous paragraph, the selfish maximisation problem of the TSO will now be model (4.11).

$$\max_{I,f,\Delta f} \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \pi_{n,t}I_{n,t} + \pi^R_{n,t}\left(\sum_{l\in\mathcal{L}} B_{l,n}\Delta f_{l,t}\right) \tag{4.11a}$$

$$(4.3e) - (4.3f) \text{ and } (4.3i) \tag{4.11b}$$

And finally, given prices $\pi, \pi^R$, with the solution of the selfish maximisation problem (4.11) $\overline{I}, \overline{f}, \overline{\Delta f}$, and with the solution of the unit commitment model $I^*, f^*, \Delta f^*$, the uplift of the TSO is computed with formula (4.12) with approach three.

**Uplift computation for TSO with reserve and transmission system with approach three**

$$\text{uplift}_{TSO} = \underbrace{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \pi_{n,t}\overline{I}_{n,t} + \pi^R_{n,t}\left(\sum_{l\in\mathcal{L}} B_{l,n}\overline{\Delta f}_{l,t}\right)}_{\text{what the TSO wants to do selfishly}} - \underbrace{\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \pi_{n,t}I^*_{n,t} + \pi^R_{n,t}\left(\sum_{l\in\mathcal{L}} B_{l,n}\Delta f^*_{l,t}\right)}_{\substack{\text{profit/loss TSO is having}\\ \text{at the unit commitment solution}}} \tag{4.12}$$

The transmission system operator is a convex agent. For this reason, as with consumers, IP pricing will not give any uplifts to this agent, since this pricing scheme fixes the non-convex part of the model, and ensures that the dispatch is the same as the one of the unit commitment model. Hence, prices obtained in a convex model are guaranteed to maximize the utilities of convex agents. Those prices will thus maximize the utility of the transmission system operator, it will thus not receive any uplift.

One can notice that the transmission system operator would enjoy a lot of price differentials, because this would earn him money as it is shipping energy (and reserve with approach 3) from one cheap place to another place which is more expensive.

**Example 4.4** (Uplift of TSO of example 4.3)**.**
*This example uses results of example 4.3 to compute the uplift of the transmission system operator.*

*One can notice that with convex hull prices, prices are always equal in both nodes. This would consequently make the objective value be equal to 0, hence whatever the transmission system operator would do, it would not lose or earn money.*

*There are price differentials with IP pricing, but since those prices ensure that convex agents are maximizing their utilities, it don't have any uplift with those prices.*

*To illustrate the uplift of the TSO with approach one, one can set arbitrary convex hull energy prices* $[\pi_{A,1}, \pi_{B,1}] = [300, 240]$ *and unit commitment dispatch equal to*

$$[I^*_{A,1}, I^*_{B,1}] = [10, -10]$$

*The selfish maximisation problem would be equal to:*

$$\begin{aligned} \max_{I,f}\ & 300 I_{A,1} + 240 I_{B,1} \\ s.t.\ & I_{A,1} = f_{1,1} \quad I_{B,1} = -f_{1,1} \\ & -15 \leq f_{1,1} \leq 15 \end{aligned}$$

*The optimal value of the problem above is*

$$[\bar{I}_{A,1}, \bar{I}_{B,1}] = [15, -15]$$

*with optimal objective value equal to 900. Hence, the uplift would be equal to:*

$$\mathit{uplift}_{TSO} = 300 \times 15 + 240 \times -15 + 300 \times 10 + 240 \times -10 = 300$$

*With approach three, with reserve convex hull prices equal to*

$$[\pi^R_{A,1}, \pi^R_{B,1}] = [150, 90]$$

*and the same energy prices above. The unit commitment dispatch is equal to*

$$[I^*_{A,1}, I^*_{B,1}] = [-10, 10], \Delta f^*_{1,1} = 5$$

*which represents a shipment of energy (reserve) from A to B equal to 10MW (2MW). The selfish maximisation problem would be equal to:*

$$\begin{aligned} \max_{I,f,\Delta f}\ & 300 I_{A,1} + 240 I_{B,1} + 60 \Delta f_{1,1} \\ s.t.\ & I_{A,1} = f_{1,1} \quad I_{B,1} = -f_{1,1} \\ & -15 \leq f_{1,1} \leq 15 \quad -15 \leq f_{1,1} + \Delta f_{1,1} \leq 15 \end{aligned}$$

*The optimal solution of this selfish maximisation problem is equal to*

$$[\bar{I}_{A,1}, \bar{I}_{B,1}] = [15, -15], \Delta f^*_{1,1} = 30$$

*which gives an optimal objective value equal to 900. The uplift would thus be equal to:*

$$\mathit{uplift}_{TSO} = 300 \times -15 + 240 \times 15 + 60 \times 30 - (300 \times -10 + 240 \times 10 + 60 \times 5) = 1200$$

# Chapter 5

# Application to Belgium

The results have been computed with the system of Belgium (which is displayed in figure 5.1) having twenty-four Belgian nodes (red and blue) of which twelve (in red) have an energy demand and ten have no generator connected to them (circled in black). This system also has six import nodes of which three are in France, one in Luxembourg and two in the Netherlands. One could notice that there are eight nodes which have energy demand and no generators connected to them, hence a lot of the energy demand will need to be transported. The red nodes with numbers {2,7,9,12} have respectively one, eleven, five and two generators connected to them. For them, one can think there will be fewer imports.

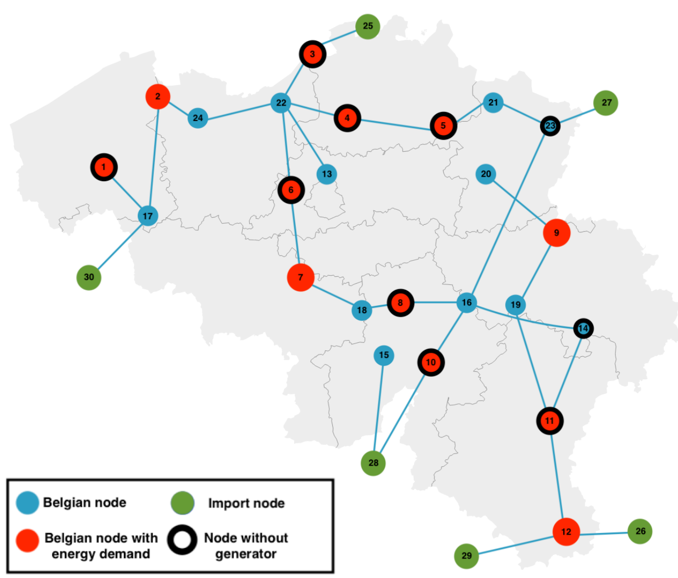



Figure 5.1: Belgium's system used in the simulations.

### Consumers description

On the one hand, energy consumption only occurs in red nodes, each of which has one consumer. The maximum energy one can consume is based on the inelastic demand of our data set, multiplied by a certain percentage specific to each node. For example, the consumer of node 6 just near Brussels can consume up to 21.3 % of the inelastic demand.

On the other hand, consumption of reserve only takes place in nodes which have generators connected to them, i.e. generators without a black circle around them. The maximum reserve one can consume is equal to the global reserve demand multiplied by the sum of maximum output of generators of this node divided by the sum of maximum outputs of all generators of the system, i.e.:

$$R_n = \frac{\sum_{g \in \mathcal{G}_n} \overline{P}_g}{\sum_{g \in \mathcal{G}} \overline{P}_g} R$$

Having those locations for reserve is interesting in that we can see approach three as a relaxation of approach one, because approach one is approach three with a zero reserve shipments requirement. From [PS17], Belgium has 55MW of primary upward and downward reserve, 140MW of secondary upward and downward reserve and 350MW of tertiary reserve. Since types of reserve are not split, the system is considered to have a total reserve requirement $R$ equal to 545MW.

Consumers have a fixed willingness to pay for energy equal to 8300 €/MWh and have a curve of willingnesses to pay for reserve (i.e. a reserve demand curve) that goes from 8000 €/MWh to 0 €/MWh split over 100 intervals.

### Description of the simulation settings

Sixty-eight generators are located in Belgium and the six import nodes are modeled as a linear supply function. This supply function is linear and goes from 0 to 200 €/MWh (spread across ten blocks), with a capacity equal to the capacity of the corresponding cross-border transmission line[1].

Reserve can only be produced in Belgium, hence green import nodes are not able produce reserve. There is thus no reserve shipments on cross-border lines, which forces node 15 to produce itself its reserve.

This system has been simulated over eight days (96 periods of 15 minutes for each day), one weekday and one weekend day for each season. Therefore, annual values can be computed by taking, for each computed values, a weighted sum over season, weekday and weekend day. The volatility and average of energy and reserve prices are computed over a matrix having five weekdays and two weekend days per season for all four seasons.

First, some insights into the solution will be displayed. Then, for each agent, results will be displayed and analysed in six situations which are a combination of convex hull pricing

[1]For the green node being connected to Belgium by two lines, the capacity of this node is equal to the sum of those two lines.

and IP pricing and a system without reserve, one with approach one and finally one with approach three. Let us note that the uplifts displayed in these results are considered to be paid by an external agency and not consumers. Finally, a small summary assesses the results.

## 5.1 Solution analysis

The main goal of this section is to analyse the solution, first with the average of energy (reserve) prices $\mu$ ($\mu^R$) and their volatility $\sigma$ ($\sigma^R$) and then with global remarks about the optimal dispatch.

### 5.1.1 Analysis of prices

#### Energy prices

On the one hand, it can be seen in table 5.1 that prices are higher with IP pricing than with convex hull pricing for a system without reserve. Once reserve has been introduced, with approach one, it results in an opposite trend, with prices being higher with convex hull pricing. Once approach one has been relaxed into approach three, it ends up being IP prices again that are higher.

In [MMO21], it is observed that when switching from an IP pricing model to a convex hull pricing model, two things can occur.

1. A generator producing at its minimum run capacity sets a higher price with convex hull pricing, instead of a cheap generator with headroom setting a low price with IP pricing.
2. An offline generator sets a lower price with convex hull pricing instead of an expensive but price-responsive consumer setting a high price with IP pricing. The reason to do that is to avoid high prices which lead to higher loss of opportunity costs with convex hull pricing, whereas IP pricing is paying a lot of uplifts in order to keep generators offline.

Either the first effect dominates or the other. Here, it is thus the second effect that dominates for the systems without reserve and approach three, while the first effect dominates for the system with approach one.

On the other hand, one can conclude that IP prices are always more volatile than convex hull prices. This is due to the fact that IP prices are determined by the generator that is ON but does not produce at full capacity (marginal). Thus if there is a change in the load across time, generators would produce more/less than the previous time step, which results in a change of the unit which is marginal.

| | | Without reserve | Approach one | Approach three |
|---|---|---|---|---|
| $\mu$ | CHP | 43.2 | 43.4 | 42.7 |
| | IP | 45.7 | 40 | 44.2 |
| $\sigma$ | CHP | 8.3 | 7.5 | 8.9 |
| | IP | 10.6 | 9.4 | 12.6 |

Table 5.1: Average and volatility of energy prices.

### Reserve prices

One can also look into the average of reserve prices $\mu^R$ and their volatility $\sigma^R$. In the case of approach three, the network prices and their volatility are also analysed. In table 5.2, reserve prices are on average almost respectively five times bigger with IP pricing for the system using approach one, while being on average equal to the reserve prices of the system using approach three. IP reserve prices are more volatile with approach one while being less volatile with approach three.

Due to the no shipments of reserve requirement of approach one and because it is not necessarily the cheapest dispatch, one may think that this would lead to higher prices than the ones with approach three. To contradict this, nodes 9 and 20 and the line binding those two nodes can be isolated from the system. In total, there are seven generators connected to those two nodes. If this system is solved over one time period and that the maximum consumption of energy and reserve is equal to 200MW for each node (i.e. 200MW of reserve demand and 200MW of energy demand in each node), the average of prices are displayed on table 5.3, which are relatively equal.

| | | Approach one | Approach three |
|---|---|---|---|
| $\mu^R$ | CHP | 17.8 | 5.5 |
| | IP | 97.1 | 5.5 |
| $\sigma^R$ | CHP | 31.1 | 18.2 |
| | IP | 493.2 | 9.2 |

Table 5.2: Average and volatility of reserve prices.

| | | Approach one | Approach three |
|---|---|---|---|
| $\mu^R$ | CHP | 85.5 | 82.4 |
| | IP | 50 | 50 |

Table 5.3: Average reserve prices of the isolated system.

Overall, the introduction of reserve did not have any significant influence on energy prices and volatility except that convex hull prices have a higher average than IP when approach one is used.

If the neighbouring countries of Belgium could provide reserve, the prices of reserve would drop to zero. In this case, the entire reserve production would come from importations.

### 5.1.2 Dispatch analysis

Incorporating reserve can also change the dispatch significantly. Indeed, in order to provide reserve, a generator must be producing energy, i.e. only a production of reserve is forbidden. This has a great impact on the dispatch since for approach one, reserve has a no shipments restriction and must be provided by a local production in the node. This may force some expensive generators to produce a lot of reserve or to cut energy production in order to provide reserve, while other cheaper generators with a lot of room for reserve production may not be used for reserve.

In order to illustrate this statement, the amount of energy and reserve each node produces and consumes with each approach can be investigated, and displayed in table B.4. The number of the nodes are displayed in figure 5.1. Several things can be seen in this table.

- Approach one has an effect on energy production. Indeed, as reserve needs to be produced locally, generators that are not necessarily ON with the system without reserve, are forced to be ON to produce reserve, hence some nodes produce more energy with approach one than they do with any other systems.
- Some nodes, which are already producing energy, need to import energy in order to be able to produce local reserve with approach one.
- The introduction of reserve has no influence on energy consumption.
- When reserve is introduced, one can see that the dispatch differs completely between approach one and three. Since reserve can be shipped with approach three, the system will prefer to import it from other cheaper places, hence some nodes are producing reserve with approach one but no reserve with approach three.

Table B.1 shows the amount of energy and reserve traded throughout the year, and we can see that each time the system consumes as much energy as it can, approach three allows more reserve consumption than approach one, due to the fact that generators with less room for production are not forced to produce reserve since this product can be shipped with approach three.

It can also be noted that with the system without reserve 21.6% of energy consumed is imported from other countries, while only 18.4% and 19.3% are respectively imported with the system using approach one and three. Approach one imports less since some generators are forced to be ON to produce reserve.

## 5.2 Consumers

Consumers make more profits when energy or reserve prices are low, because that is what they pay. Note that consumers do not have any uplifts with IP pricing since this pricing rule fixes the non-convex part of the model to their unit commitment optimal values, hence the model is convex and has the same dispatch as the unit commitment model.

Results of consumers are available in table 5.4. Uplifts displayed in this table are rather small compared to the profits they make, and this is due to the fact that consumers consume as much energy as they can during the whole year, while consumers never consume as much reserve as they can with approach one, and consume as much as they can during 357 days of the year with approach three.

### Reserve preferences

Note that results of the system without reserve must not be compared to the results of the other two systems that use reserve since reserve creates a lot of welfare for consumers, hence only the comparison between approach one and three is made here:

- **System with convex hull pricing:** before receiving uplifts, approach three is the preferred modeling of reserve, thanks to lower energy and reserve prices. After receiving uplifts, it is still approach three that is the most preferable. Indeed, although uplifts are higher with approach one, it does not succeed to fill the gap made by profits of approach three.
- **System with IP pricing:** before receiving uplifts approach one is the most desirable for consumers. Although reserve prices are higher with this approach, the lower energy prices are more profitable to consumers. As explained in section 4.3, consumers do not receive any uplifts with IP pricing.

### Pricing preferences

This time, it is the use of reserve that is fixed, and preferences of pricing rules are analysed for each system:

- **System without reserve:** convex hull pricing is the most beneficial to consumers thanks to lower energy prices. In this case, they do not receive any uplifts because they consume as much as they can, hence there is no loss to compensate or lost opportunity costs to pay.
- **System with approach one:** this time, IP pricing is the most preferable. Indeed, although reserve prices are higher with this pricing rule than with convex hull pricing, the energy prices are lower and this pricing scheme provides twice as much profits from energy as the convex hull pricing. After receiving uplifts, even though IP pricing does not give any, it is still the same rule that gives more advantages, because the amount of given uplifts with convex hull pricing is not big enough to cancel out the difference.

- **System with approach three:** before uplifts, the most preferable pricing rule is this time convex hull pricing. Indeed, the two pricing rules produce equal reserve prices on average, but energy prices are lower with convex hull pricing. Because IP pricing does not give any uplifts to consumers, the conclusion is the same after uplifts.

| | | | CHP | IP |
|---|---|---|---|---|
| Without reserve | Profits | Energy | A+1 250 470.7 | A−1 061 854.7 |
| | | Reserve | \ | \ |
| | | Total Profits | A+B−209 010 676.8 | A+B−211 323 002.2 |
| | Uplifts | | C | C |
| | Total (uplifts + profits) | | A+B+C−209 010 676.8 | A+B+C−211 323 002.2 |
| Approach one | Profits | Energy | A+1 276 856.1 | A+4 746 804.9 |
| | | Reserve | B−1 552 355.1 | B−3 015 269.8 |
| | | Total Profits | A+B−275 499 | A+B+1 731 535.1 |
| | Uplifts | | C+477 051.7 | C |
| | Total (uplifts + profits) | | A+B+C+201 552.7 | A+B+C+1 731 535.1 |
| Approach three | Profits | Energy | A+1 709 035.5 | A=8 127 170 680.9 |
| | | Reserve | B+45 542.4 | B=210 261 147.5 |
| | | Total Profits | A+B +1 754 577.9 | A+B=8 337 431 828.4 |
| | Uplifts | | C+809.7 | C=0 |
| | Total (uplifts + profits) | | A+B+C+1 755 387.6 | A+B+C=8 337 431 828.4 |

Table 5.4: Annual profits and uplifts in euros for consumers.

## 5.3 Producers

Of course, producers enjoy higher prices since they resell the energy and reserve they produce. They also have costs due to their production, start up costs and minimum load costs. They have uplifts since they are not necessarily producing at maximum run capacity, or producing at a loss. Indeed, one generator that has a marginal cost lower than the price of energy would want to produce as much as he can, while one generator that has a marginal cost higher would not want to produce at all.

Following the same structure as in the consumers section, attention will be paid to which use of reserve is preferable for producers with a fixed pricing method, and then which pricing method is better for each use of reserve. The results used in these analysis are displayed in table 5.5.

### Reserve preferences

With a fixed pricing rule, we can analyse which use of reserve producers prefer.

- **System with convex hull pricing:** before receiving uplifts, the system without reserve would be preferable for producers, thanks to costs being the lowest. After receiving them, it is now slightly more desirable for generators to be in a system using approach one, thanks to uplifts being the highest with this use of reserve. Those are the highest due to the no shipment requirement, which does not necessarily allows the cheapest units to produce reserve, while the cheapest unit would prefer to produce more reserve since they would make a bigger profit.
- **System with IP pricing:** before receiving uplifts, the system without reserve is again the most profitable, thanks to the lowest costs and prices being higher than with the other two systems. After receiving uplifts, the one using approach one is clearly preferable, thanks to reserve prices being almost twenty times bigger with approach one than with approach three, significantly increasing lost opportunity costs for cheapest units that are not dispatched.

### Pricing preferences

The next analysis was made separately for each use of reserve (no reserve, approach one and three) in order to find out which pricing method gives more advantages to producers.

- **System without reserve:** before receiving uplifts, IP pricing is more profitable since those prices are 2.5 € higher than convex hull prices. After uplifts it is still IP pricing that producers prefer. Indeed, if IP pricing is more profitable before uplifts, it would hardly be possible that convex hull pricing becomes more profitable, since by construction, those prices minimize uplifts.
- **System with approach one:** although for reserve it is IP pricing which yields higher prices, for energy it is convex hull prices that are higher, leading to more profits for convex hull pricing. After earning uplifts, IP pricing this time is more profitable thanks to uplifts being more than ten times more expensive than with convex hull pricing. Indeed, reserve prices being almost five times higher with IP, the cheaper units would prefer to produce more reserve. This is not necessarily possible since reserve is only meant to be produced locally with this approach.
- **System with approach three:** IP pricing is the most desirable pricing rule for this system, thanks to higher energy prices. On average, IP and convex hull pricing produce equal reserve prices, so the difference between reserve profits is very small (around ten thousands of euros). After uplifts being paid, IP pricing is still profitable, which is not a surprise thanks to the uplifts minimizing characteristic of convex hull prices.

| | | | CHP | IP |
|---|---|---|---|---|
| Without reserve | Profits | Energy | 25 962 320.4 | 27 718 235.6 |
| | | Reserve | \ | \ |
| | | Costs | −3 112 144.6 | −3 112 144.6 |
| | | Total Profits | 22 850 175.8 | 24 606 091 |
| | Uplifts | | 69 527.1 | 694 200.3 |
| | Total (uplifts + profits) | | 22 919 702.9 | 25 300 291.3 |
| Approach one | Profits | Energy | 25 034 781.8 | 21 236 612.4 |
| | | Reserve | 1 398 968.9 | 2 861 883.7 |
| | | Costs | −7 993 705.8 | −7 993 705.8 |
| | | Total Profits | 18 440 044.9 | 16 104 790.3 |
| | Uplifts | | 4 568 841.3 | 47 032 766.4 |
| | Total (uplifts + profits) | | 23 008 886.2 | 63 137 556.7 |
| Approach three | Profits | Energy | 25 116 430.2 | 25 448 280.5 |
| | | Reserve | 286 791.5 | 296 812.5 |
| | | Costs | −4 831 923.7 | −4 831 923.7 |
| | | Total Profits | 20 571 298 | 20 913 169.2 |
| | Uplifts | | 1 882 612.6 | 3 044 499.2 |
| | Total (uplifts + profits) | | 22 453 910.6 | 23 957 668.4 |

Table 5.5: Annual profits and uplifts in euros for producers.

## 5.4 Transmission system operator

The transmission system operator only ships energy from one place to another if there is a positive price differential between those two places. It will make money from energy trading with a system without reserve and with approach one, but also from reserve trading with approach three.

For the TSO, it is not necessary needed to divide preferences by pricing and systems, since no matter what the pricing rule is, it is always the system using approach three that is the most preferred. Indeed, although when using approach three with convex hull pricing the TSO does not make any revenue off reserve, and even loses money, the amount of money it makes with energy is the highest with this system. If IP pricing is used in the system, approach three allows the TSO to earn a lot more money with reserve than with any other system, and on top of that, it has an additional stream of revenue from reserve, which comforts his earnings.

If all systems are compared using different pricing rules, we can note that each time IP pricing is preferable before and after receiving uplifts. Indeed, the TSO wants to ship more energy when there are price differentials between nodes, and thanks to the volatility being higher for IP pricing, there may be more price differentials, and thus more shipping of energy. It is worth noting that IP prices are prices that guarantee the welfare maximization of convex agents.

| | | | CHP | IP |
|---|---|---|---|---|
| Without reserve | Profits | Energy | 1 103 715.8 | 1 660 125.9 |
| | | Reserve | \ | \ |
| | | Total Profits | 1 103 715.8 | 1 660 125.9 |
| | Uplifts | | 897.5 | 0 |
| | Total (uplifts + profits) | | 1 104 613.3 | 1 660 125.9 |
| Approach one | Profits | Energy | 1 026 163.1 | 1 354 383.7 |
| | | Reserve | \ | \ |
| | | Total Profits | 1 026 163.1 | 1 354 383.7 |
| | Uplifts | | 1748.4 | 0 |
| | Total (uplifts + profits) | | 1 027 911.5 | 1 354 383.7 |
| Approach three | Profits | Energy | 1 171 777 | 2 548 962.3 |
| | | Reserve | −5575.1 | 29 946.3 |
| | | Total Profits | 1 166 201.9 | 2 578 908.6 |
| | Uplifts | | 6326.2 | 0 |
| | Total (uplifts + profits) | | 1 172 528.1 | 2 578 908.6 |

Table 5.6: Annual profits and uplifts in euros for the transmission system operator.

## 5.5 Summary

In this section, a brief summary of previously found results will be described. This summary will be based on those previous results as well as table 5.9, which describes the amount of money consumers pay for energy and reserve, the amount of money generators receive, and the difference between what consumers pay and generators receive which are the pre-uplift profits of the transmission system operator.

The preferences of pricing rules for each agent are displayed in table 5.7. Globally 7 times out of 9, IP pricing is preferable.

Consumers pay less money with convex hull prices with the system without reserve and the one using approach three as it can be seen in table 5.9. After receiving uplifts, they prefer convex hull prices with the system without reserve and with approach three, thanks to lower prices with this pricing rules. With the system approach one, the trend in prices is the other way around, i.e. convex hull prices being higher, which leads to IP pricing being more beneficial to consumers, since they pay less.

Producers have a higher pre-uplift profit two times out of three with IP pricing, which can be seen on table 5.9. They always prefer IP pricing after receiving uplifts. Indeed, IP prices are higher for the system without reserve, leading to higher energy profits in the end. The system using approach one has higher convex hull prices, but IP reserve prices are five times higher creating a lot more lost opportunity costs in uplifts. With approach three IP pricing is the most desirable thanks to energy prices being higher with this pricing rule.

The TSO always prefers IP pricing, thanks to the highest volatility of prices.

| | Without reserve | Approach one | Approach three |
|---|---|---|---|
| Consumers | CHP | IP | CHP |
| Producers | IP | IP | IP |
| TSO | IP | IP | IP |

Table 5.7: Preferences of pricing rules for each agent after receiving uplifts.

In table 5.8, which shows preferences after receiving uplifts, the system without reserve is never the preferred one. The pre-uplifts profits of consumers are in advantage of approach three with convex hull prices and approach one with IP pricing. Indeed, in table 5.9, it can be seen that the least amount of money they pay with convex hull prices is with approach three, and with approach one with IP pricing, thanks to the dynamic of prices. After receiving uplifts, it is still approach one that is the most beneficial with IP pricing and approach three is financially more interesting for convex hull prices.

Producers always prefer approach one, due to the high amount of uplifts. Even though, it can be seen in table 5.9, they are receiving more money before uplifts with the system using approach one with convex hull prices, and with the system which does not use reserve with IP pricing, the huge amount of uplifts for approach one forces them to prefer approach one after receiving them. The TSO also prefers approach three, no matter what the pricing rule is, thanks to the additional revenues they earn with reserve flows.

| | CHP | IP |
|---|---|---|
| Consumers | Approach three | Approach one |
| Producers | Approach one | Approach one |
| TSO | Approach three | Approach three |

Table 5.8: Preferences of systems for each agent after receiving uplifts.

The added value of approach three, i.e. being able to ship reserve, can be seen directly in the costs of producers in table 5.5 which are 3M€ lower with approach three than with approach one, but also the TSO has an additional source of revenue ending up making more money.

| | Without reserve | | Approach one | | Approach three | |
|---|---|---|---|---|---|---|
| | CHP | IP | CHP | IP | CHP | IP |
| 1. What consumers pay for energy | 44 212 977 | 46 525 302.4 | 44 186 591.5 | 40 716 642.7 | 43 754 412.1 | 45 463 447.6 |
| 2. What consumers pay for reserve | \ | \ | 1 398 968.9 | 2 861 883.7 | 281 216.4 | 326 758.8 |
| 3. What generators receive | 43 109 261.2 | 44 865 176.4 | 44 559 397.3 | 42 224 142.7 | 42 869 426.7 | 43 211 297.8 |
| 4. What goes in TSO's pocket (1.-2.) | 1 103 715.8 | 1 660 125.9 | 1 026 163.1 | 1 354 383.7 | 1 166 201.9 | 2 578 908.6 |

Table 5.9: Annual values of what consumers pay, what generators receive and what the TSO is earning.

Finally, one can wonder if these results would draw the same conclusions if uplifts were not of the responsibility of an external agency but must be paid by consumers. Indeed, as IP pricing may be the most frequently interesting, it raises significantly huge amounts of uplifts. The transmission system operators as well as producers will not be affected by this change of policy. The conclusions are still the same for these agents. However, consumers preferences might be modified, because they need to pay for their uplifts, but also the uplifts of every other agent. The sum of uplifts and the final profits of consumers are displayed in table 5.10.

| | | CHP | IP |
|---|---|---|---|
| Without reserve | Total uplifts | 70 424.6 | 694 200.3 |
| | Total Profits | A-206 036 602.2 | A-208 972 703.3 |
| Approach one | Total uplifts | 5 047 641.4 | 47 032 766.4 |
| | Total Profits | A-1 801 589.5 | A−42 256 732.1 |
| Approach three | Total uplifts | 1 889 748.5 | 3 044 499.2 |
| | Total Profits | A+2 910 138.3 | A=8 334 387 329.2 |

Table 5.10: Total uplifts of each system with each pricing rule and the corresponding final profits of consumers in the case where they need to pay uplifts.

One can see that the conclusions of which system is preferable did not change, except for the fact that consumers prefer approach three in asystem using IP pricing now. The preferences of pricing rules are the same for the system without reserve and the one using approach three, i.e. convex hull pricing is the more beneficial. For the system which uses approach one, the result is the opposite. Indeed, when uplifts are not the responsibility of consumers, IP pricing is more beneficial for them, which gives around 1.5M€ more than convex hull pricing. When these agents must pay uplifts, this is the other way around, i.e. convex hull pricing is the most financially interesting. This is due to the fact that the amount of uplifts for this system is almost ten times bigger with IP pricing than with convex hull pricing, which must now be paid by them. These 47M€ of uplifts also adds up to what consumers are already paying for energy and reserve (see table 5.9) with IP pricing, more than doubling the amount of money they are spending.

# Chapter 6

# Conclusion

Chapter 2 showed the optimization of energy with non-convexities but without any trading of reserve, while also showing that convex hull pricing and IP pricing schemes must be supported by uplifts in a non-convex market, how to compute them, but also proving that convex hull prices were not unique. Chapter 3 set out why reserve was crucial in an energy market and how to introduce it to the models of the previous chapter, explained how to apply the pricing schemes to it and how uplifts were affected by this incorporation of the new product. Chapter 4 highlighted what the added value of the establishment of a transmission network is and how to build it, while proposing two different market designs for reserve and their consequences on reserve prices, as well as indicating the application of pricing schemes to this new model and explaining everything about the new agent, the transmission network operator. Finally, chapter 5 applied everything from the previous chapters to a data set of Belgium.

This last chapter allowed to prove some theories and led to some findings.

- If uplifts are paid by an external agency, after receiving them, IP pricing is beneficial more frequently than convex hull prices. With all three kinds of agent and the three different systems (without reserve, with reserve and approach one, with reserve and approach three), this pricing scheme is seven times out of nine more beneficial.
- If uplifts are paid by consumers, convex hull pricing is always more beneficial than IP pricing for consumers. However, IP pricing is the one that is more financially interesting for the transmission system operator and the producers. As it is their responsibility, they would prefer on a theoretical level convex hull pricing since they are the minimizer of uplifts, which means that they will pay less uplifts to other agents than with IP pricing.
- While the higher volatility of energy prices is indeed with IP pricing, with this exact simulation of Belgium, reserve prices are two times more volatile with convex hull prices with approach three.
- If one must choose a system with reserve between approach one and three, it would

depend on the pricing scheme. Indeed, with convex hull prices, it is approach three that is more financially interesting for consumers and the transmission system operator, while approach one is only beneficial for producers. With IP pricing, it is approach one that is the most beneficial to both producers and consumers, while the TSO prefers approach three.

- In the particular case of this system of Belgium, approach three allows respectively almost four times lower reserve prices and twenty times lower with convex hull prices and IP prices. But as demonstrated, this is not always true.

As said many times throughout this work, a lot of causes make the trading of reserve crucial to have in the process of optimizing energy. Looking thus at table 5.7, which shows the benefits after receiving uplifts when consumers do not need to pay them, five times out of six IP pricing is more beneficial if the system is co-optimizing energy and reserve. **This is why, when not considering the amount of side payments, and that consumers are not responsible for uplifts, this would be the first choice of pricing rule if the goal is to satisfy most of the agents**. Indeed, in a system using approach one, IP pricing is the rule that is preferred by everyone, while in a system using approach three, this rule is the more beneficial for producers and the transmission system operator while convex hull pricing gives around 1.5M€ more for consumers. Then looking at table 5.8, with the IP pricing scheme, it is approach one that is two times out of three more beneficial. **The choice that would suit most of the agents is therefore IP prices with approach one.**

We need to consider the total amount of uplifts given with a system using this pricing rule with this market design of reserve, which is the significant amount of 47M€ per year. This method gives this money exclusively to generators, because IP pricing does not give uplifts to convex agents (i.e. consumers and the transmission system operator), but also does not allow the optimization to procure reserve in every node, which can yield potential outages in some nodes that could not be solved.

If consumers need to pay uplifts in a system that co-optimizes energy and reserve, convex hull pricing would always be better for consumers, while IP is the most preferred pricing rule for the TSO and the producers. **Again, the pricing rule that would suit most of the agents is IP pricing.**

It is important to remember that uplifts are required to cope with non-convexities in an optimization problem of energy (and reserve), but they are not desirable. **Therefore, the most significant benefit of convex hull pricing is that it minimizes such uplifts. Considering this, as well as the need of reserve co-optimization and the ability to obtain reserve anywhere, a system based on approach three and convex hull pricing would be the most prudent in this regard, and allows shipments of reserve.**

This work could be improved by inspiring ourselves from [CGG14] and by considering actual contingencies in the optimization process. This would allow the models to be closer to reality, by considering the maximum amount of reserve that can be consumed as a variable that depends on events that happen based on probabilities. By doing so, reserve is only produced based on some contingencies events, and the consequence of this activation of reserve on transmission constraints is directly taken into account, instead of considering that reserve is produced and shipped no matter what. Finally, these models could be scaled to a bigger model of, for example, Europe.

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# Appendix A

# Additional tables concerning the uniqueness of convex hull prices

| | | $\nu$ | $\mu$ | $\delta$ | $\psi^u$ | $\psi^v$ | $\psi^z$ |
|---|---|---|---|---|---|---|---|
| Unit two | Primal | $[-30, -35, -60]$ | $0\,\forall t$ | $0\,\forall t$ | $[-900, -1050, -1800]$ | $0\,\forall t$ | $0\,\forall t$ |
| | Dual | $[-30, -30, -60]$ | $0\,\forall t$ | $0\,\forall t$ | $[0, -900, -1800]$ | $[-900, 0, 0]$ | $0\,\forall t$ |
| | Maximisation of (2.12) | $[-30, -30, -60]$ | $0\,\forall t$ | $0\,\forall t$ | $[0, -1800, -1800]$ | $[-900, 0, 0]$ | $0\,\forall t$ |
| | Minimisation of (2.12) | $[-30, -30, -60]$ | $0\,\forall t$ | $0\,\forall t$ | $[0, -900, -1800]$ | $[-900, 0, 0]$ | $0\,\forall t$ |
| Unit three | Primal | $0\,\forall t$ | $0\,\forall t$ | $[30, 25, 0]$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ |
| | Dual | $0\,\forall t$ | $0\,\forall t$ | $[30, 30, 0]$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ |
| | Maximisation of (2.12) | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ |
| | Minimisation of (2.12) | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ | $0\,\forall t$ |

Table A.1: Value of dual variables present in the upper bound of 2.13

# Appendix B

# Additional solution analysis

| | Without reserve | Approach one | Approach three |
|---|---|---|---|
| Energy | 27 570 332 | 27 570 332 | 27 570 332 |
| Reserve | \ | 1 454 242.7 | 1 464 907.5 |

Table B.1: Amount of energy and reserve traded throughout the year in MW

| | Without reserve | Approach one | Approach three |
|---|---|---|---|
| France | 10.2% | 8.5% | 9.2% |
| Netherlands | 10.1% | 8.7% | 8.8% |
| Luxembourg | 1.3% | 1.2% | 1.3% |

Table B.2: Fraction of the total energy consumed that is imported from France, Netherlands and Luxembourg

One can also compute the percentage of the energy consumption the system imports from other countries, and also to look into how much of the consumed energy is imported from other nodes, i.e. not produced locally. In order to do so, the production of each node $n$ that has energy demand (in red on figure 5.1) is divided by how much it imports from other nodes (i.e. value of variable $I$). The calculation is therefore based on the yearly injection in node $n$, $\hat{I}_n$, but also on the yearly consumption in node $n$, $\hat{d}_n$:

$$\text{percentage shipped to node } n = \frac{\hat{I}_n}{\hat{d}_n}$$

In table B.2, approach one imports less than any other system. This is due to the fact that, since reserve is forced to be provided locally, generators which are not the cheapest ones are forced to be online in order to provide reserve, hence they produce energy and take the place of generators that are the cheapest when not enforcing approach one.

One can already see that in node $\{1, 3, 4, 5, 6, 8, 10, 11\}$ this percentage is equal to 100% since those nodes don't have generators connected to them, hence no nodal production and only energy imported from other nodes. The other nodes $\{2, 7, 9, 12\}$, import:

| | Without reserve | Approach one | Approach three |
|---|---|---|---|
| Node 2 | 98.7% | 96.3% | 98.8% |
| Node 7 | 80.2% | 90.5% | 82.9% |
| Node 9 | 99.7% | 99.4% | 99.5% |
| Node 12 | 90.5% | 78% | 86.3% |

Table B.3: Percentage of the total energy consumed in four nodes that is imported from other nodes

| | Energy production | | | Energy consumption | | | Reserve production | | Reserve consumption | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Without reserve | Approach one | Approach three | Without reserve | Approach one | Approach three | Approach one | Approach three | Approach one | Approach three |
| Node 1 | 0 | 0 | 0 | 3 680 499 | 3 680 499 | 3 680 499 | 0 | 0 | 0 | 0 |
| Node 2 | 43 800 | 114 568.7 | 38 125.7 | 3 151 919.9 | 3 151 919.9 | 3 151 919.9 | 4580.5 | 4874.3 | 4580.5 | 4580.4 |
| Node 3 | 0 | 0 | 0 | 132 135.2 | 132 135.2 | 132 135.2 | 0 | 0 | 0 | 0 |
| Node 4 | 0 | 0 | 0 | 1 416 742 | 1 416 742 | 1 416 742 | 0 | 0 | 0 | 0 |
| Node 5 | 0 | 0 | 0 | 1 686 615.8 | 1 686 615.8 | 1 686 615.8 | 0 | 0 | 0 | 0 |
| Node 6 | 0 | 0 | 0 | 5 878 731.4 | 5 878 731.4 | 5 878 731.4 | 0 | 0 | 0 | 0 |
| Node 7 | 825 115.6 | 394 381.3 | 679 488.2 | 3 924 433.6 | 3 924 433.6 | 3 924 433.6 | 201 589.9 | 819 684.6 | 201 589.9 | 201 589.9 |
| Node 8 | 0 | 0 | 0 | 682 651 | 682 651 | 682 651 | 0 | 0 | 0 | 0 |
| Node 9 | 14 600.2 | 26 059.1 | 23 348.3 | 4 326 042.4 | 4 326 042.4 | 4 326 042.4 | 18 554.4 | 4961.5 | 18 554.4 | 28 208.1 |
| Node 10 | 0 | 0 | 0 | 1 063 638.1 | 1 063 638.1 | 1 063 638.1 | 0 | 0 | 0 | 0 |
| Node 11 | 0 | 0 | 0 | 381 311.1 | 381 311.1 | 381 311.1 | 0 | 0 | 0 | 0 |
| Node 12 | 128 122.9 | 279 699.6 | 165 630 | 1 245 612.3 | 1 245 612.3 | 1 245 612.3 | 37 194 | 175 566.5 | 37 194 | 37 183.4 |
| Node 13 | 7 835 520 | 7 568 506.3 | 7 823 709.9 | 0 | 0 | 0 | 267 013.66 | 11 810.1 | 267 013.7 | 267 045.9 |
| Node 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Node 15 | 0 | 661 168.1 | 672 523.7 | 0 | 0 | 0 | 40 034 | 40 034 | 40 034 | 40 034 |
| Node 16 | 8 120 448 | 7 843 729.8 | 8 120 448 | 0 | 0 | 0 | 276 718.2 | 0 | 276 718.2 | 276 756.7 |
| Node 17 | 0 | 621 830.7 | 0 | 0 | 0 | 0 | 32 063.8 | 0 | 32 063.8 | 32 062.6 |
| Node 18 | 80 370 | 801 348.1 | 68 150 | 0 | 0 | 0 | 46 171.9 | 0 | 46 171.9 | 46 171.9 |
| Node 19 | 0 | 63 419.4 | 0 | 0 | 0 | 0 | 40 400.4 | 0 | 40 400.4 | 40 388.8 |
| Node 20 | 889 945.5 | 671 096.8 | 787 197.4 | 0 | 0 | 0 | 46 721.3 | 366.3 | 46 721.3 | 46 735.6 |
| Node 21 | 91 630 | 218 767.3 | 78 182 | 0 | 0 | 0 | 73 197.2 | 1290 | 73 197.2 | 73 197.2 |
| Node 22 | 997 544.9 | 1 010 688 | 958 499.1 | 0 | 0 | 0 | 172 032 | 12 947.3 | 172 032 | 172 989 |
| Node 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Node 24 | 2 575 836.2 | 2 183 918.1 | 2 656 695.9 | 0 | 0 | 0 | 197 971.3 | 393 372.8 | 197 971.3 | 197 964 |
| Node 25 | 1 468 890 | 1 321 424.9 | 1 312 248.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Node 26 | 371 478.8 | 334 383.1 | 376 259.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Node 27 | 1 308 566.5 | 1 103 894.5 | 1 159 342.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Node 28 | 895 879 | 770 712.4 | 797 424.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Node 29 | 395 644.4 | 356 346.7 | 403 243.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Node 30 | 1 526 940 | 1 224 388.9 | 1 449 815.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table B.4: Production and consumption of each node in MW with the three systems (without reserve, with approach one and with approach three

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