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Forecasting market-based inflation expectations volatility: the role of energy variables

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Declaration regarding AI tools usage in Master's thesis

During the preparation of this master's thesis, the author(s) utilized AI tools (e.g. ChatGPT) and Deepl for the following purposes:

1. [Spelling an grammar checks]: Deepl was used primarily and ChatGPT to do a review of the potential mistakes (grammar and orthography)

2. [Rephrasing]: Deepl was used to reformulate some sentences and see if there was a clearer and better way to express my thoughts. ChatGPT was used at the end of my work to clarify and reduce some of my arguments, remove unnecessary sentences, and avoid redundancy.

After using Deepl and ChatGPT, the author(s) diligently reviewed and edited the content produced by the tools. The author(s) take full responsibility for the final content presented in this thesis.

By signing this declaration, the author(s) affirm that the content of this thesis reflects his original work, augmented by a responsible use of AI tools.

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Foreword & Acknowledgments

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1. Introduction

Market-based inflation expectations have become pivotal indicators for central banks and market participants in shaping forward-looking monetary policy and understanding inflation dynamics (European Central Bank, 2024; Grølund et al., 2024). Derived from instruments like inflation swaps, they provide real-time insight into how financial markets forecast future price developments. However, their volatility remains limitedly understood especially regarding the influence of certain macroeconomic factors, among which energy variables such as crude oil and natural gas that play increasingly influent roles.

Crude oil and natural gas prices are known to influence inflation through direct and indirect transmission channels (Alessandrini & Gazzani; 2025). It's also reported that crude oil prices affect transportation and production costs globally, while natural gas impacts electricity and heating costs, especially in the Euro Area (ACER, 2024). Looking beyond realized inflation, makes appear that changes in these energy variables may affect the volatility of market-based inflation expectations as well.

In the last 15 years, the world has witnessed multiple energy-related shocks, including the oil price crashes of 2014, and the energy crisis related to the Russia-Ukraine conflict starting in 2022 (IEEFA, 2024). These events have underscored the need of establishing the impact of how energy prices volatility like crude oil and natural gas may transmit into inflation expectations uncertainty, especially across different time horizons, short, long and very-long term, and distinct regions that could be compared.

In that sense, this thesis, based on a quantitative research approach including data collection, regression models and statistical analysis, aims to investigate the impact of crude oil and natural gas price volatilities on the forecast of market-based inflation expectations volatilities in the Euro Area and the United States in the last decades. It uses EGARCH(1,1) models shaped with local energy variables as external regressors, respectively Brent and Dutch TTF for the Euro Area and Western Texas Intermediate (WTI) and Henry Hub for the United States of America, to isolate the impact of each variable on the volatility of 1y1y, 5y5y and 10y10 zero-coupon inflation swap rates. Crude oil spot and natural gas spot prices were all collected from Bloomberg for the period January 2008 to December 2023, representing a sample of 15 years

monthly data. Inflation Linked Swap Zero Coupon for the same periods and for each mentioned time horizon were also retrieved on Bloomberg.
This thesis aims to answer the following research question: “How do crude oil and natural gas prices, in the Euro Area and in the United States, impact the forecast of market-based inflation expectations volatilities between 2008 and 2023?”.

This thesis’ methodology employs an empirical strategy using ARMA-EGARCH(1,1) models incorporating exogeneous regressors for regional energy prices, model selection procedures based on statistical criteria (AIC, BIC and Log-Likelihood) and robustness diagnostics of the final selected model (Ljung-Box, Nyblow, Sign Bias and Pearson Goodness-of-fit).The final work provides a comparative, time-horizon-sensitive view of energy variables’ roles in shaping the volatility of forward-looking inflation metrics and enhance how energy shocks propagate through financial markets.

This research starts with a review of the existing theoretical and empirical literature and presents the tested hypotheses. The third section of this paper highlights the detailed methodology used to obtain the statistical findings outlined in the fourth section. Last, this paper discusses the potential limitations and implications of the study and ends with a general conclusion.

2. Literature review

This literature review summarizes empirical and theoretical contributions on the relationship between energy price dynamics, with a more specific focus on crude oil and natural gas prices, and the volatility of market-based inflation expectations.

2.1. Energy prices and inflation expectations

The European Central Bank (ECB) underlines that the foundational rationale linking energy prices with inflation expectations lies in the structure self of modern economies, where energy plays a central role as one of the major inputs in production, transportation and household consumption (European Central Bank, 2024, Grølund et al., 2024). Therefore, fluctuations in crude oil and natural gas prices can impact both realized inflation and the anticipated future inflation prices into financial instruments like inflation-linked bonds and inflation swaps. This transmission mechanisms finds itself relevant when speaking of inflation forecasts directly derived from market instruments, as opposed to the others often used transmission mechanisms using survey data or macroeconomic models. Furthermore, the data retrieved from market-based measures are available at a high frequency and ensure up-to-date information, despite likely incorporating time-varying inflation risk premia. (Casiraghi & Miccoli, 2018; Gurkayanak et al., 2010; Burban et al., 2021; Boeckx et al., 2025). According to the ECB (2014), the inflation premia reflect the abilities of nominal bonds and inflation-linked securities, such as bonds and swaps, to provide protection against economic shocks. Those hedging characteristics strongly depend on the type of economic noise investors expect. This premium is influenced by how nominal bonds perform in a deflationary environment compared to inflation-linked instruments (European Central Bank, 2014). So, if inflation drops, investors may be less willing to pay for inflation protection, meaning that today's low inflation swap rates likely reflect both weak inflation expectations but also low demand for insurance against rising inflation. (ECB, 2024; Casiraghi & Miccoli, 2018; ECB, 2007). Casiraghi and Miccoli (2018), Burban et al. (2021) and Boeckx et al. (2025) build on this by demonstrating that inflation swap rates incorporate time-varying risk premia and therefore require adjustment to extract pure inflation expectations. Their work shows that these premia vary systematically with business cycle indicators such as the Purchasing Managers' Index (PMI) and financial market volatility as represented by the Volatility Index (VIX). The PMI serves as leading indicator to signal future economic activity and that is closely observed by central banks, investors and analyst to define growth periods (S&P Global, 2025). The VIX, a real-time market-based indicator published by the Chicago Board Options Exchange (CBOE), measures the expected volatility in the US stock market over the next 30 days and is specifically based on S&P500 option prices. A higher VIX occurs often during crisis as markets expect more volatility while a lower VIX means that markets expect calm and stability (CBOE, 2025). Casiraghi and Miccoli (2018) find also out that estimated inflation risk premia are higher in the Euro Area than in the US on average and that they increase with the maturity of the contract. Finally, their research underscores the fact that methodological rigor is required when isolating true expectations from market-based measures. Inflation-linked swaps are considered as market-based measures as they are not yet realized, like ICP are, and forward-looking which is important when assessing market-based inflation expectations volatilities.

2.2. Insights from the Euro Area

Crude oil, in particular, has long been at the heart of discussions when shaping how inflation is understood and discussed. Historically, oil price shocks tend to be followed by increases in headline inflation (Hammoudeh & Reboredo, 2018; Lee & Kim, 2024). While the pass-through from energy prices to core inflation is generally limited, it remains statistically significant in many economies (Tanaka & Yoon, 2025), with expectations often adjusting in anticipation of monetary policy responses. Alessandrini and Gazzani (2025) also report how the nature of the shock can substantially influence the inflationary outcome, whether it is supply- or demand-driven, underscoring the importance of economic context when understanding crude oil's impact on inflation expectations.

In Europe especially, natural gas has gained rising prominence in the recent years, mainly due to the structural shifts operated within the continent to stabilize energy markets and address geopolitical developments challenges (European Commission, 2022; European Council, 2024). Unlike crude oil, whose market is global and historically anchored, natural gas markets are more regional, with supply often dependent of off- and on-land infrastructure constraints like effective pipeline networks and liquefied natural gas (LNG) terminals (ACER, 2024). As a result, gas price volatility can vary across regions and lead distinct inflationary implications (ACER, 2024).

In the Euro Area, recent empirical contributions emphasize the relevance of energy variables in modelling inflation expectations. Boeck and Zörner (2024) provide evidence that natural gas prices have had substantial effects on inflation expectations. Especially in the last years with the wake of geopolitical conflicts like the Russia-Ukraine conflicts that started in February 2022. Their semi-structural Baeysian VAR model analysis, retrieving data from 2004 to 2022, underscores that gas price volatility has not only increased but has also been a pivotal driver of uncertainty surrounding inflation expectations. Boeck and Zörner (2024) findings state that a 10% gas price shock raises inflation by 8 basis points[1] (bps) and inflation expectations by 5 bps on impact while the peak effects of such shocks generally appear after around 1 year. The study reveals that long-term expectations remain stable, implying no evidence of de-anchoring and at the same time, that second-round effects are substantial on the short-term inflation expectations. Those findings align with structural vulnerabilities in the Euro Area's energy supply chains, which have made inflation expectations more sensitive to gas price shocks. Similarly, Morao (2025) uses structural vector autoregressions (SVAR) to show how short-term increases in crude oil prices lead to strong but transitory peaks in short-term market-based inflation expectations. The same study also reports that while inflation expectations are reactive to energy shocks, the duration of these effects depends on the nature and duration of the price change. Longer-term inflation expectations (5y5y) show limited responsiveness, indicating that long-term expectations are relatively well-anchored and suggest that aside from energy prices, demand-driven pressures might be stronger drivers of inflation. Short-term expectations are more volatile and less well-anchored than the longer ones as agents expects the gas and oil shocks to be temporary. That divergence in responses may emphasize agents' forward-looking behaviour, aligning with rational expectations theories (Boeck & Zörner, 2024; Morao, 2025). Morao (2025) and Alessandri and Gazzani (2025), with their analysis over asymmetries between supply and demand, also point potential inefficiencies or frictions in expectation formation due to the existence of a temporal lag between energy price spikes and the adjustments in market-based inflation indicators.

Alessandri and Gazzani (2025) and Hoyinck and Rossi (2023) further enrich those views by clarifying the effects of demand- and supply-driven natural gas shocks. They deepen the previous analysis by adding that supply shocks coming from disruptions in global supply chains have a more marked and lasting impact on Euro Area inflation expectations than demand shocks. This new layer of information suggests us that the asymmetry of supply and demand is critical when it comes to understand the regional sensitivity to energy variables, particularly when we know that the Euro Area relies on a greater dependence on imported natural gas compared to the United States (IEEFA, 2024; ACER, 2024). Alessandri and Gazzani (2025) findings also highlight that crude oil and natural gas supply shocks have different intensities and inflation transmission channels. Crude oil supply shocks impact inflation quickly, mainly

[1] A basis point is a financial unit of measure used to express the variations in percentage of interest rates, inflation rates, etc. A basis point of 1 accounts for 0.01%.

through transportation fuel costs while natural gas supply shocks are stagflationary by simultaneously reducing output and increasing inflation. The natural gas shocks act via electricity prices which rise faster and more than after oil shocks and are three times more responsive to gas shocks than oil shocks, meaning their effects are gradual, stronger and last longer than oil shocks (Alessandri & Gazzani, 2025).

2.3. Insights from the United States of America

United States-focused studies often relate crude oil as a major driver or a dominant energy variable impacting market-based inflation expectations. In that sense, Hammoudeh and Reboredo (2018), using a Gaussian affine term structure model, find out that the crude oil prices changes affect inflation expectation nonlinearly. Their findings also relates that these oil prices variations seem to have much stronger effects when prices are above 67$/barrel. A 1% increase in oil prices raises 5-year inflation expectations by 0.0061percentage points when above the 67$/barrel threshold, but only by 0.0036 when below it. Hammoudeh and Reboredo (2018) also relate that a similar pattern is observable for 10-year inflation expectations but it remains smaller. More importantly, Hammoudeh and Reboredo (2018) highlight the importance of decomposing breakeven inflation rates to understand the relationship between crude oil prices and inflation expectations. For instance, when oil prices are relatively high, the inflation risk premia is accountable for 70% of the total breakeven response at the 10-year horizon, showing the inflation risk premia explain a significant part of crude oil effect on inflation expectations. Furthermore, their study suggests that the comovement between these variables intensifies during period of financial stress. This last relayed fact is also emphasizing the need for more flexible models capable of accommodating structural change and volatility clustering. Another study from Lee and Kim (2024), which draw a parallel between the Korean crude oil shocks and the one observed in the US, reveals as well asymmetrical effects aligning with the US evidence. The inflationary impact of positive oil shocks is significantly greater than the one of negative ones which clearly highlights behavioural asymmetries in inflation expectation formation. Lee and Kim (2024) complete their findings by adding that energy shocks may exacerbate inflation expectation volatility under certain macroeconomic regimes, highlighting the existence of regional asymmetries. The systemic nature of oil shocks means it can have an impact on both inflation levels and uncertainty, especially when it's accompanied by changes in monetary policies (Hammoudeh & Reboredo, 2018; Lee & Kim, 2024).

2.4. Regional asymmetries

Many studies are mutually reinforcing when highlighting regional asymmetries. Boeck and Zörner (2024), Morao (2025), and Alessandri and Gazzani (2025) all report the growing impact of natural gas in shaping Euro Area inflation expectation, and especially during supply shocks such as the ones known in February 2022 with the effective start of the Russia-Ukraine war. Meanwhile, most of the literature of the US remains focus on the impact of WTI crude oil, it still underscores regional asymmetries. These asymmetries come not only from energy market structure but also from different levels of energy dependence throughout the regions/countries. Whang and Zhou (2024) and Alessandri and Gazzani (2025) agree on the conclusion that the nature of the energy shocks is essential in determining its inflationary impact whether it comes from a supply- or demand-side.

2.5. Volatility modelling frameworks

Engle (1982), who first introduced the ARCH-models (Autoregressive Conditional Heteroskedasticity) modelled the log returns Y_T of the analysed time series evaluated at all time t, as a white noise ϵ_t multiplied by the volatility σ_t. Based on Engle's model assumptions, Bollerslev (1986) propose an observation-driven model to frame financial time series that are not independently realized by extending the ARCH-model to allow the conditional variance σ_t^2 to have an additional autoregressive structure within itself. From their findings appears the GARCH-model (Generalized Autoregressive Conditional Heteroskedasticity) which allows to highlight the high and low volatility clusters as well as autocorrelations in absolute returns. The creation of this observation-driven model allows financial time series to be more economically interpretable (Dufays, 2011). Peng et al. (2024) also points that, nowadays, this model is renowned when it comes to analysing volatility clusters and has numerous variants that are still widely used for economical interpretations. The basic GARCH model is written as follows in its simplest form, if a time series is defined by $Y_T = \{y_1, y_2, \dots, y_T\}$:

$$\epsilon_t = \sigma_t z_t \quad \text{with} \quad E(z_t) = 0,\ VAR(z_t) = 1.$$
$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

where z_t is an unobservable random variable part of an i.i.d. process and where E(.) and VAR(.) respectively represent the expectation and the variance operators. $\{\omega, \alpha, \beta\}$ are the GARCH-model parameters respectively representing a constant term, the ARCH-term (how large the previous shock was) and the GARCH-term (how large the previous period's volatility was).

The parameters α and β are restricted to be non-negative to ensure model stationarity, meaning that the sum of both coefficients is less than 1: $\alpha + \beta < 1$. By meeting this condition, the model's conditional variance σ_t^2 has a finite expected value and always remains non-negative t. Agnolucci (2009) also add that GARCH models are not accurate models to produce forecasts as their performance quickly deteriorates but provide a good record when estimating the volatility of returns from financial assets.

2.6. Hypotheses

Based on the already existing literature, a few hypotheses have been formulated to guide the answer to the research question.

H1: Crude oil price volatilities negatively affect the forecasting of market-based inflation expectations volatilities.

H2: Natural gas prices volatilities positively affect the forecasting of market-based inflation expectations volatilities.

H3: Natural gas prices have a stronger impact on the Euro Area's volatility inflation expectations forecasts than on US ones.

H4: The impact of energy variables differs across time horizon (1y1y, 5y5y, 10y10y).

3. Methodology

This thesis section is dedicated to explaining all the assumptions made to model how the selected energy variables may have an impact on the forecast of market-based inflation expectations volatilities. To compute the model, a R program using the rugarch and the rmgarch packages has been used.

3.1. Research objective

This research paper aims at determining the impact of crude oil and natural gas variables on the forecast of market-based inflation expectations volatility. Moreover, this research also put an emphasis on two distinct regions, the Euro Area (EA) and the United States of America (US). This paper deeps dive into the previously mention objective by using a quantitative approach. The aim was to gather a sample of 15 years data on each defined variable to ensure strong data consistency and a large period overview.

3.2. Data collection

In order to attain the research objective, the data for the determined dependant (energy variables: Brent, Western Texas Intermediate, Dutch TTF and Henry Hub) and independent variables (Inflation Linked Swaps Zero Coupon for the EA and the US as they are market-based measures of inflation expectations) have been retrieved on Bloomberg platform. Bloomberg is a powerful financial tool gathering worldwide financial data and often used in the financial world as a database platform.

It was determined to take into account in the research the crude oil variables as the crude oil serves as a reference on the energy market and the natural gas variables as this energy sees its market exposure soaring and is more often used as main energy source worldwide, particularly in the Euro Area, that puts itself as one of the major natural gas importers around the world (IEEFA, 2024), making its integration into the analysis even more relevant. Additionally, it has been decided to retrieve the short- (1y1y), long- (5y5y) and very-long-term (10y10y) horizons data to describe separately how energy markets impact market-based inflation expectations at different horizons.

3.3. Data description

The final dataset consists of 1152 observations, representing 192 observations for each variable, ranging from the analysed period of January 2008 to December 2023, representing 192 months. In fact, a monthly frequence has been chosen to get a more detailed view on the volatilities throughout the time. To remain consistent in the analysis, the starting period of January 2008 has been selected as data for Dutch TTF were only available starting at this period. This sample also allows to highlight the past 15 years international events outcomes on the inflation expectations volatility potentially driven by events like the Russian Ukrainian War. This specific tab has been formatted to compute the month-over-month (MoM) difference to highlight volatility month by month and enabling to work with more stationary data, which is a requirement to use effectively the econometric models selected in the following pages of this research paper. The difference of the log returns of the different energy variables have been computed to ensure consistency in the analysis. Collection source and transformation types have been summarized in Table 1.

Table 1. – Source collection and transformation

Variable	**Source**	**Index**	**Transformation**
Inflation Linked Swaps			
EA ILS Zero Coupon bond price (1y)	*Bloomberg*	*EUSWI1*	*Difference*
EA ILS Zero Coupon bond price (5y)	*Bloomberg*	*EUSWI5*	*Difference*
EA ILS Zero Coupon bond price (10y)	*Bloomberg*	*EUSWI10*	*Difference*
US ILS Zero Coupon bond price (1y)	*Bloomberg*	*USSWIT1*	*Difference*
US ILS Zero Coupon bond price (5y)	*Bloomberg*	*USSWIT5*	*Difference*
US ILS Zero Coupon bond price (10y)	*Bloomberg*	*USSWIT10*	*Difference*
Crude oil			
Brent Crude spot price	*Bloomberg*	*CO1:COM*	*Diff. of Log returns*
WTI Crude Oil spot price	*Bloomberg*	*CL1:COM*	*Diff. of Log returns*
Natural gas			
Dutch TTF spot price	*Bloomberg*	*TTFG1:COM*	*Diff. of Log returns*
Henry Hub spot price	*Bloomberg*	*NG1:COM*	*Diff. of Log returns*

3.3.1. Summary statistics

Table 2 is a summary statistic of the data used through this thesis.

Table 2 – Summary Statistics: ILS and Energy Variables

	μ	σ	s	k
ILS EA				
1y1y	0.00300	0.46145	-1.57431	12.93383
5y5y	0.00132	0.19041	-0.93223	9.02716
10y10y	0.00104	0.13183	-0.56976	7.85544
ILS US				
1y1y	0.00068	0.56454	-2.45491	20.81199
5y5y	0.00003	0.22967	-1.20332	8.20716
10y10y	-0.00056	0.16091	-0.40089	4.57583
Crude oil				
Brent Crude spot price	-0.00018	0.11167	2.12069	16.65684
WTI Crude Oil spot price	0.00006	0.12126	0.95543	14.96919
Natural gas				
Dutch TTF spot price	-0.00104	0.18646	0.13997	5.50834
Henry Hub spot price	0.00539	0.14412	0.23349	5.12274

Notes: Table 2 reports the summary statistics of EA and US ILS at 1-, 5- and 10-year horizons, Crude oil (Brent and WTI) and Natural gas (Dutch TTF and Henry Hub) spot prices after transformation (difference). Symbols: μ = average; σ = standard deviation; s = skewness; k = kurtosis.

3.4. Statistical model

In order to compare different regions across the world, this section first proposes the application of two EGARCH(1,1)-models used for the observed regions, respectively the Euro Area and the United States. Then, it elaborates on the process of building those two distinct models and the assumptions used behind it.

3.4.1. EGARCH-models: Application

Euro Area model

In the Euro Area (EA)-model, the Euro Area Inflation Linked Swap Zero Coupon (ILS_{EA}), is the dependent variable, European crude oil (Brent) and natural gas (Dutch TTF) spot prices variables are set as the independent ones. The regression model is represented as follows:

$$\Delta ILS_{EA,t} = \alpha_0 + \alpha_1 \Delta\log(Brent_t) + \alpha_2 \Delta\log(DTTF_t) + \alpha_3(\Delta\log(Brent_t) * \Delta\log(DTTF_t)) + \epsilon_t \tag{1a}$$

- $ILS_{EA,t}$: Inflation Linked Swap Zero Coupons of Euro Area at time *t*.
- α_0: a constant coefficient
- $\alpha_1, \alpha_2, \alpha_3$: the coefficients of the energy variables
- $Brent_t$: Brent price at time *t*
- $DTTF_t$: Dutch TTF price at time *t*
- ϵ_t: residuals

where ϵ_t follows a EGARCH(1,1)-process for modelling the volatility of the Euro Area market-based inflation expectations. An EGARCH(1,1)-model to frame the volatility of inflation expectations can is:

$$\epsilon_t = \sigma_t z_t \tag{1b}$$

where $\epsilon_t$is the residual term at time *t*, σ_t: the conditional standard deviation (volatility) at time *t* and z_t is the white noise process.

The conditional variance σ_t^2 of the EA EGARCH(1,1)-model is represented as:

$$\ln(\sigma_t^2) = \omega + \alpha \frac{|\epsilon_{t-1}|}{\sigma_{t-1}} + \alpha\gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta \ln(\sigma_{t-1}^2) + \delta_1 \Delta\log(Brent_t) + \delta_2 \Delta\log(DTTF_t) + \delta_3(\Delta\log(Brent_t) * \Delta\log(DTTF_t)) \tag{1c}$$

where $\ln(\sigma_t^2)$ is log of the conditional variance at time *t*, ensuring positivity, ω is a constant term, $\alpha \frac{|\epsilon_{t-1}|}{\sigma_{t-1}}$ captures the magnitude of the standardized shock, $\alpha\gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}}$ captures asymmetry (leverage effect) and if < 0 negative shocks increase more volatility than positive ones, $\beta\ln(\sigma_{t-1}^2)$ is the volatility persistence term and $\delta_1, \delta_2, \delta_3$ are the coefficients of the exogeneous energy variables and the interaction term.

United States of America model

The US-model is structurally similar to the EA-model but varies with the selection of different dependent and independent variables. US Inflation Linked Swap Zero Coupon (ILS_{US}), is the dependent variable while crude oil (WTI) and natural gas (Henry Hub – HH) spot prices, the independent variables.

$$\Delta ILS_{US,t} = \alpha_0 + \alpha_1 \Delta\log(WTI_t) + \alpha_2 \Delta\log(HH_t) + \alpha_3(\Delta\log(WTI_t) * \Delta\log(HH_t)) + \epsilon_t \quad \text{(2a)}$$

- $ILS_{US,t}$: Inflation Linked Swap Zero Coupons of Euro Area at time *t*.
- α_0: a constant coefficient
- $\alpha_1, \alpha_2, \alpha_3$: the coefficients of the energy variables
- WTI_t: Brent price at time *t*
- HH_t: Dutch TTF price at time *t*
- ϵ_t: residuals

where ϵ_t follows a EGARCH(1,1)-process for modelling the volatility of the United States market-based inflation expectations. An EGARCH(1,1)-model to frame the volatility of inflation expectations can is:

$$\epsilon_t = \sigma_t z_t \quad \text{(2b)}$$

where $\epsilon_t$is the residual term at time *t*, σ_t: the conditional standard deviation (volatility) at time *t* and z_t is the white noise process.

The conditional variance σ_t^2 of the US EGARCH(1,1)-model is represented as:

$$\ln(\sigma_t^2) = \omega + \alpha \frac{|\epsilon_{t-1}|}{\sigma_{t-1}} + \alpha\gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta\ln(\sigma_{t-1}^2) + \delta_1 \Delta\log(WTI_t) + \delta_2 \Delta\log(HH_t) + \delta_3(\Delta\log(WTI_t) * \Delta\log(HH_t)) \quad \text{(2c)}$$

where $\ln(\sigma_t^2)$ is log of the conditional variance at time *t*, ensuring positivity, ω is a constant term, $\alpha \frac{|\epsilon_{t-1}|}{\sigma_{t-1}}$ captures the magnitude of the standardized shock, $\alpha\gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}}$ captures asymmetry (leverage effect) and if < 0 negative shocks increase more volatility than positive ones, $\beta\ln(\sigma_{t-1}^2)$ is the volatility persistence term and $\delta_1, \delta_2, \delta_3$ are the coefficients of the exogeneous energy variables and the interaction term.

3.4.2. Statistical model: Framework

This section explain how the final EGARCH(1,1)- model used to compute the results over time series volatilities in this thesis has been obtained. The (1,1) combination choice is explained by different studies. Wang et al. (2009) and Liu and Hung (2010) have demonstrated that the degree of prediction and explanation is the lowest p and q combination. One of the main challenges particular to the GARCH-model's field is the presence of heteroskedastic effects, meaning there is inconsistency in the volatility of the considered process. In our theoretical research, volatility is considered to be the squared root of conditional variance. Applied to our application case, we define the volatility σ_t as the square root of the conditional variance σ_t^2 of the log return process given its previous value. In our case, if $X_T = \{x_1, x_2, \dots, x_T\}$ is the time series of energy variables evaluated at time t, we define the log returns as:

$$\Delta Y_t = \log(X_t) - \log(X_{t-1}) \tag{3}$$

and the volatility σ_t, where:

$$\sigma_t^2 = VAR(Y_t^2 \mid \mathcal{F}_{t-1}) \tag{4}$$

where $\mathcal{F}_{t-1}$ is the σ-algebra generated by $X_0, \dots, X_{t-1}$. Intuitively, we can expect the volatility of such processes to change over time, merely influenced by external factors like economics and politics. Inflation Linked Swaps were computed as differences as they are already given in returns.

Then, and as mentioned in the literature review, we use the ARCH-model set up by Engle (1982) as a basis to build or GARCH-model. The ARCH-model is structured as follows:

$$\epsilon_t = \sigma_t z_t \tag{5}$$

where ϵ_t is the residual term that is split into a time dependent standard deviation σ_t and a stochastic part z_t, also called white noise that is i.i.d.

The conditional variance given by the ARCH-model (Engle, 1982) is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \tag{6}$$

As the conditional mean and variance must be positive, $\omega > 0$ and $\alpha_i \geq 0, i \in [1, q]$. To ensure process covariance stationarity, $\sum_{i=1}^{q} \alpha_i < 1$ is necessary.

A few years later, Bollerslev (1986) contributes to Engle's approach by proposing a conditional variance model where the past conditional variances (p) of the series are added to the initial ARCH-model, resulting in a GARCH-model (Generalized ARCH).

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{7}$$

where $\omega > 0$ and $\alpha_i \geq 0, i \in [1, q]$; $\beta_i \geq 0, i \in [1, p]$. to ensure a positive conditional variance. As mentioned in the literature review, $\sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 < 1$ to guarantee a stationary covariance process. The model now offers more flexibility and allows to capture both volatility clustering and dense tail returns.

In 1991, Nelson finally proposed an exponential GARCH-model (EGARCH) to capture both the sign and the size of past residuals. Nelson and Cao (1992) also argue that linear GARCH-model are too restrictive as they have non-negativity constraints. This EGARCH-model finally accommodates for leverage effects, allows to work in absolute variations and is nowadays typically used for interest rates, swaps and inflation trends. As it is the case in a general GARCH-model, the EGARCH-model also needs to guarantee a stationary covariance process but the stationarity is not expressed as $\sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 < 1$, but as $|\beta| < 1$. Additionally, the model allows now to work directly with the delta between to data instead of using log returns. The EGARCH (Nelson, 1991) representation is:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \frac{|\epsilon_{t-i}|}{\sigma_{t-i}} + \sum_{i=1}^{q} \alpha_i \gamma_i \frac{\epsilon_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^{p} \beta_i \ln(\sigma_{t-j}^2) \tag{8}$$

The leverage effect shows that, with γ_i< 0, bad events will increase volatility more than positive shocks. The log-conditional variance $\ln(\sigma_t^2)$ ensures there is no restriction in the other parameters for a positive conditional variance.

For this thesis, the model, set up in the rugarch package, follows an ARMA(p',q')-EGARCH(p,q). The ARMA representation is mathematically written as follow:

$$Y_t = \mu + \sum_{i=1}^{q} \phi_i Y_{t-i} + \sum_{j=1}^{p} \theta_i \epsilon_{t-j} + \epsilon_t \tag{9}$$

The EGARCH(p,q) keeps the same form as developed in the equation 6. In the ARMA(p',q') term, the dependant variable is, in this thesis case, the month over month difference of Inflation Linked Swap rates Zero Coupon for the EA and the US. μ corresponds to the mean of the time series, ϕ_i the autoregressive coefficient and θ_i is the moving average coefficient. An autoregressive coefficient ϕ_i is valid if it stays within the stationary bounds -1 < ϕ_i < 1. For instance, if ϕ_i > 0, the series tends to move in the same direction as the previous value, which is called positive persistence, else we face a case of mean-reverting with oscillation.

In order to see if the selected energy variables among the regions a role have to play in the forecast of market-based inflation expectation volatilities, additional external regressors have been added to the original GARCH-model. The approach involves adding $\sum_{j=1}^{m} \delta_i v_{jt}$, where m stands for the possible number of external regressors v_j, to the volatility of our model. The ARMA part remains unchanged while the EGARCH-model is now represented as:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \frac{|\epsilon_{t-i}|}{\sigma_{t-i}} + \sum_{i=1}^{q} \alpha_i \gamma_i \frac{\epsilon_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^{p} \beta_i \ln(\sigma_{t-j}^2) + \sum_{j=1}^{m} \delta_i v_{jt} \tag{10}$$

The external regressors used are the logarithmic differences of crude oil and natural gas spot prices time and are specific for each region. The Euro Area and the United States models include respectively Brent, Dutch TTF and Western Texas Intermediate, Henry Hub.

3.5. Heteroskedasticity test and best model definition

As mentioned in the Section 3.4., a prerequisite to justify the usage of a GARCH-model is to have a presence of heteroskedasticity in the conditional variance (Bollerslev, 1987). Therefore, this thesis uses the ARCH LM Test to evaluate whether heteroskedasticity is observable into the different financial time series. The residuals of the linear regression are reported in the ARCH LM Test, for which a FinTS package on R is used. The ARCH LM Test evaluates whether the null hypotheses: no ARCH effects can be rejected. In this case, lags are equal to 12, corresponding to 12 months, which allows the model to account for a full year of past shocks, helping to report whether the conditional variance is more volatile on the short- or long-term. Rejecting the null hypotheses means that the conditional variance is not homoscedastic. To visualize potential volatility clustering (ARCH effects), ACF (Autocorrelation Function) of squared residuals can be plotted as the ARCH LM test checks whether the past squared residuals help explain current squared residuals. On the other hand, failing to reject the null hypotheses would not justify the use of a GARCH-model. However, to have empirical validation and analyse the robustness of the results, 3 evaluation measures have been chosen: the AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion) for in-sample and the Log-Likelihood. The common point of both this measures it that the better the model is, the lower the value is. AIC (Akaike, 1974) and BIC (Schwarz, 1978) representations are written as follow:

$$AIC = 2\kappa - 2\ln(\hat{L}) \tag{11}$$

$$BIC = \kappa \ln(n) - 2\ln(\hat{L}) \tag{12}$$

where κ represents the number of parameters estimated, $\hat{L}$ the maximised value of the likelihood function of the estimated model and n the sample size.

Additionally, and to evaluate the accurate distribution method, the Jarque-Bera test (1987) has been performed on each sample of collected time series to know under which distribution the modelled EGARCH(1,1)-models should work. The Jarque-Bera is represented as:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{13}$$

where n is the number of observations, S the Skewness and K the Kurtosis. The null hypotheses is: Normal distribution, therefore a p-value lower than 0.05 indicates strong statistically evidence to reject the null hypotheses and so the conclusion is that month over month difference for all inflation expectation is not normal distributed.

3.5.1. Euro Area

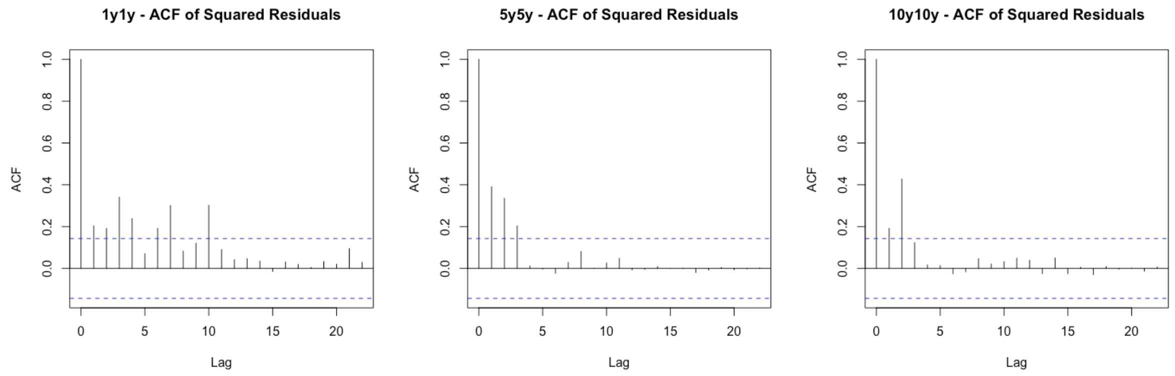
The results clearly highlight that 1y1y, 5y5y and 10y10y inflation expectation time series reject the ARCH LM Test's null hypotheses: no ARCH effect, meaning conditional heteroskedasticity is present in the series and justify a GARCH-model use. Then, to see whether a model with external regressors is better than another model without them, the Log Likelihood, AIC and BIC measures were retrieved, see Table 3. Globally, the EGARCH(1,1)-model with external regressors added on is better and has thus been used to build on the results analysed in the Section 4 of this thesis.

Table 3. Euro Area – Tests Outcomes and Fit Analysis

Test	**1y1y**	**5y5y**	**10y10y**
ARCH LM Test (p-value, df = 12)	1.469e-10	0.0003	0.0051
Jarque-Bera (p-value)	0.0000	0.0000	0.0000
EGARCH(1,1) without external regressors			
Log Likelihood	-61.034	87.567	144.368
AIC	0.7238	-0.8571	-1.4614
BIC	0.8443	-0.7366	-1.3409
EGARCH(1,1) with external regressors			
Log Likelihood	-12.672	135.892	182.889
AIC	0.2518	-1.3286	-1.8286
BIC	0.4412	-1.1393	-1.6392

Complementarily to empirical test, the ARCH effects have been modelled through an ACF plot of squared residuals per time horizon, see Figure 1. Volatility clustering is observed in each time series as bar pikes above the 95% confidence interval (delimited by the blue dotted lines on the charts below) which suggests the presence of conditional heteroskedasticity in the residuals of the EA mean model, justifying GARCH(1,1)-model use to capture time-varying volatility structure in the data.

Figure 1. Autocorrelation function (ACF) of Squared residuals on Euro Area 1y1y, 5y5y and 10y10y



Source: Graphs prepared by the author

3.5.2. United States of America

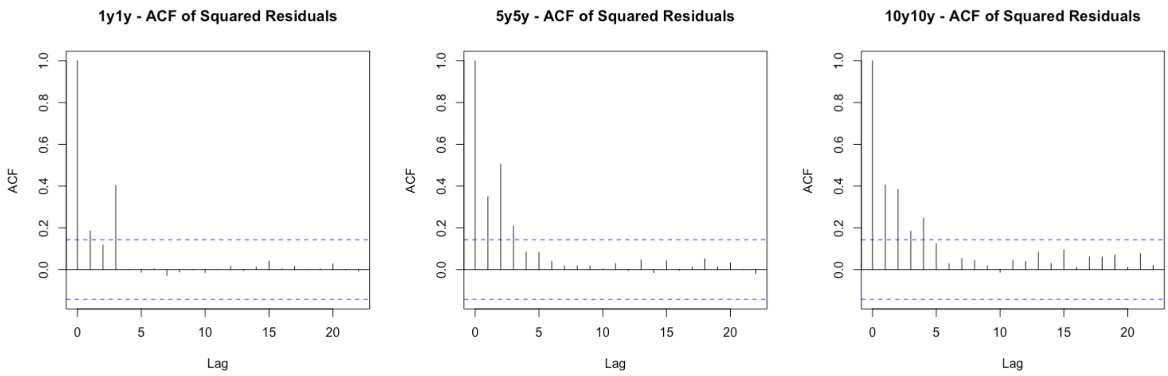
As performed for the Euro Area and theoretically outlined in the Section 3.5., ARCH effects tests and comparative model have been run for the United States as well. The Table 4 summarizes the different results given by those processes. It has been concluded that ARCH LM null hypotheses can be generally rejected excepted for the 1y1y horizon. Furthermore, EGARCH-modelling including external regressors appears to be better than the models without them and have thus been used in this thesis next Section.

Table 4. United States – Tests Outcomes and Fit Analysis

Test	**1y1y**	**5y5y**	**10y10y**
ARCH LM Test (p-value, df = 12)	0.2222	0.0038	0.0030
Jarque-Bera (p-value)	0.0000	0.0000	0.0000
EGARCH without external regressors			
Log Likelihood	-96.317	52.8414	104.8405
AIC	1.0991	-0.4877	-1.0409
BIC	1.2196	-0.3672	-0.9204
EGARCH with external regressors			
Log Likelihood	-54.859	101.877	141.782
AIC	0.8900	-0.9668	-1.3913
BIC	0.6943	-0.7774	-1.2019

The ACF plots of squared residuals per time horizon, see Figure 2, also reveal volatility clustering but spread on differently on the lags as we compare it to the Euro Area. The 1y1y inflation expectations data displays notable significant positive autocorrelation in the first lag before starting decreasing around lag 5. This pattern suggests the presence of conditional heteroskedasticity in the residuals of the US mean equation, justifying the usage of a GARCH(1,1)-model as it was the case back for the EA.

Figure 2. Autocorrelation function (ACF) of Squared residuals on United States 1y1y, 5y5y and 10y10y



Source: Graphs prepared by the author

3.6. GARCH Robustness Diagnostics methods

The rugarch package on R offers a series of diagnosis tools to interpret the outcomes of the model and its goodness-of-fit. Four of those diagnostic tools, have been used in this thesis rejects or fail to reject their null hypotheses under the conditions specified below and assess the model robustness First, Ljung-Box Tests on Standardized Residuals and on Standardized Squared Residuals results are expressed in p-values with a 5% confidence level. The null hypotheses is expressed as: H0: no serial autocorrelation. A p-value higher than 0.05 would fail to reject the null hypotheses and confirm that the final model does not include autocorrelation, which improve model's robustness. Then, Sign Bias Test results are expressed in p-values with a 5% confidence level. The null hypotheses is expressed as: H0: no asymmetry, a rejected null hypothesis would show to our model models well asymmetry, which is important in the case of market-based inflation expectation volatility analysis. Thirdly, the Nyblom Test results are expressed in critical value with a 1% confidence level for Asymptotic Critical Value. The null hypothesis is expressed as: H0: no parameter instability. Having parameter instability may cause instability within the model itself, instable parameters are then removed if the significantly affect the model robustness. Last, Chi-Squared Pearson Goodness-of-fit Tests are expressed in p-values with a 5% confidence level. The null hypothesis is expressed as: H0: no significant difference between the observed and the expected value. As the p-value is higher than 0.05, we fail to reject the null hypotheses, meaning that our model doesn't include significant differences between the expected values and the observed one, meaning a good model fit. The diagnostic results are available in Appendix I.

4. Results analysis

The goal of the thesis is to evaluate the role of crude oil and natural gas in the forecast of the volatility for short-, long- and very-long-terms samples of market-based inflation expectations, respectively 1y1y, 5y5y and 10y10y, between January 2008 and December 2023 for the Euro Area and the United States. Therefore, the methodology given in the previous section has been applied to obtain statistical results. This section investigates first the results given by the ARCH LM Test, which is determinant in evaluating the presence of heteroskedasticity in the time series, as they are mandatory to justify an EGARCH(1,1)-model use. Then, the selected EGARCH(1,1)-model outcomes are analysed.

4.1. EGARCH analysis

In order to assess the influence of energy prices dynamics on inflation expectations volatility in the Euro Area and in United States distinctly, an EGARCH(1,1) model has been estimated using a short-, long- and very-long-term Zero-Coupon inflation swap rate as the dependant variable. Those various models includes both crude oil and natural gas spot prices as well as their interaction term as external regressors in both the conditional mean and variance equations. This dual inclusion allows for an evaluation of how level changes and interaction effects in the chosen EA and US energy markets translate both into the mean and volatility of market-based inflation expectations. As Jarque-Bera Tests null hypotheses has been rejected for each time series, the EGARCH(1,1)-models all follow Student-t distribution, allowing also to account for fat tails. As discussed in the Section 4.1., there are empirical justifications to apply EGARCH(1,1)-models to all 6 “Zero Coupon Inflation Swap Rate” time series as their ARCH LM Tests prove the null hypotheses: no ARCH effect can be rejected and demonstrate conditional heteroskedasticity presence within the time series sample.

4.1.1. Euro Area

Tables 4, 5 and 6 respectively present parameter estimates for the conditional mean and variance equations of an EGARCH(1,1)-model fitted to the 1y1y, 5y5y and 10y10y inflation expectation time series for the Euro Area. The model results only include Brent crude oil, Dutch TTF natural gas coefficients as the interaction term, originally included in both the mean and the variance equations (Brent x Dutch TTF) has been removed as it doesn’t provide any added information on the energy dynamics, see Equation 9. The stationarity condition is respected for each EGARCH(1,1)-model as $|\beta| < 1$.

Table 5 results report that only Brent and Dutch TTF are both statistically significant at a 5% confidence level in the mean equation, meaning that Brent crude oil prices and Dutch TTF natural gas prices are drivers of 1y1y horizon inflation expectations. The autoregressive term AR(1) is significant at a 10% confidence level suggesting predictability in the short-term evolution of the mean. Its negative sign implies a mean-reverting process. In the variance equation Brent is also positively significant at a 5% confidence level while Dutch TTF is statistically significant and negative, suggesting that Brent crude oil prices may not help anchor volatility inflation expectations, while Dutch TTF may well. The volatility dynamics parameters (α, γ) are significant at a 5% confidence level and positive. α suggests that recent return shocks meaningfully increase future volatility of 1y1y market-based inflation expectations and a negative γ indicates asymmetry where negative shocks lead to greater increases in uncertainty than positive ones. β is insignificant and does not confirm strong persistence in inflation expectations volatility despite its value close to 1 (0.999). The shape parameter (10.09) of the t-Student distribution is estimated significant at a 5% confidence level and supports the presence of fat tails in the distribution of standardized residuals. All diagnostic tests support model adequacy, see Appendix I.

Table 5. Euro Area – 1y1y EGARCH(1,1) Results

Parameter	**Estimate**	**Robust Std. Error**	**t-value**	**p-value**
Panel A. Mean Equation				
β_0	-0.000011	0.001506	-0.01	0.994
AR(1)	-0.122006	0.072748	-1.68	0.094
Brent (β_1)	-1.714786	0.387326	-4.43	0.000
Dutch TTF (β_2)	-0.365324	0.087510	-4.17	0.000
Panel B. Variance Equation				
ω	-0.017892	0.000596	-30.03	0.000
ARCH-term (α)	0.169923	0.035652	4.77	0.000
GARCH-term (β)	0.999999	0.902882	1.11	0.268
Leverage effect (γ)	-0.163297	0.037343	-4.37	0.000
Brent (δ_1)	1.859507	0.096225	19.32	0.000
Dutch TTF (δ_2)	-1.349553	0.073901	-18.26	0.000
Panel C. Distribution parameter				
Shape - t Student	10.092946	2.410408	4.19	0.000

Results summarized in the Table 6 indicates that for the 5y5y EGARCH(1,1)-model's mean equation, Brent and Dutch TTF are statistically significant, at a 5% confidence level, resulting in Brent crude oil and Dutch TFF natural gas prices being consistent drivers of 5y5y inflation expectations. Moreover, Brent is also statistically significant at a 5% confidence level in the variance equation while Dutch TTF is significant at a 10% confidence level. The core volatility dynamics defined by α, β and γ remains well captured and are all significant at a 5% confidence level. The ARCH-term is not significant at a 5% confidence level, the GARCH-term is large (0.969) which suggests strong volatility persistence in the 5y5y inflation expectations time series, and the leverage effect indicates asymmetry where positive shocks lead to greater increases in uncertainty than negative ones. In contrast to the 1y1y EGARCH(1,1)-model results, the shape parameter of the Student-t distribution is insignificant (0.346) at a 5% confidence level, suggesting that a model with a normal distribution may suit. However, keeping this Student-t distribution allows to account for fat tails. Finally, all diagnostics applied on the model support its robustness, see Appendix I.

Table 6. Euro Area – 5y5y EGARCH(1,1) Results

Parameter	**Estimate**	**Robust Std. Error**	**t-value**	**p-value**
Panel A. Mean Equation				
β_0	-0.005648	0.005338	-1.06	0.290
AR(1)	-0.092207	0.098928	-0.93	0.351
Brent (β_1)	-0.814155	0.088516	-9.20	0.000
Dutch TTF (β_2)	-0.118458	0.040220	-2.95	0.003
Panel B. Variance Equation				
ω	-0.137292	0.114346	-1.20	0.229
ARCH-term (α)	0.060608	0.069444	0.87	0.383
GARCH-term (β)	0.969854	0.024774	39.15	0.000
Leverage effect (γ)	0.251500	0.107966	2.33	0.019
Brent (δ_1)	2.577812	0.853239	3.02	0.003
Dutch TTF (δ_2)	-1.235129	0.706594	-1.75	0.080
Panel C. Distribution parameter				
Shape - t Student	14.606158	15.504505	0.94	0.346

Table 7 results highlight that Brent and Dutch TTF are statistically significant at a 5% confidence level in the mean equation, meaning that both variable prices are consistent drivers of inflation expectations. However, none of Brent or Dutch TTF are significant at a 5% confidence level in the 10y10y variance equation. Expect for the ARCH-term, the volatility dynamics parameters (β, γ) are significant at a 5% confidence level and positive. β confirms strong persistence in inflation expectations volatility with a value close to 1 (0.925), and a positive γ indicates asymmetry where positive shocks lead to greater increases in uncertainty than negative ones. The t-Student distribution is estimated significant through its shape parameter equal to 6.29 and supports the presence of fat tails in the distribution of standardized residuals. All diagnostic tests support model adequacy and robustness, see Appendix I.

Table 7. Euro Area – 10y10y EGARCH(1,1) Results

Parameter	**Estimate**	**Robust Std. Error**	**t-value**	**p-value**
Panel A. Mean Equation				
β_0	-0.006165	0.005210	-1.18	0.237
AR(1)	-0.035106	0.092698	-0.38	0.705
Brent (β_1)	-0.563803	0.082657	-6.82	0.000
Dutch TTF (β_2)	-0.085751	0.039433	-2.17	0.029
Panel B. Variance Equation				
ω	-0.361344	0.208183	-1.74	0.083
ARCH-term (α)	0.081845	0.070069	1.17	0.243
GARCH-term (β)	0.925777	0.042421	21.82	0.000
Leverage effect (γ)	0.384411	0.158726	2.42	0.015
Brent (δ_1)	1.863243	1.462251	1.27	0.203
Dutch TTF (δ_2)	-0.545819	0.666880	-0.82	0.413
Panel C. Distribution parameter				
Shape - t Student	6.298891	2.792752	2.25	0.024

4.1.2. United States of America

In this section, the United States-related EGARCH(1,1)-models results are analysed. Compared to the initial model, see Equation 10, the interaction term (WTI x Henry Hub) in the mean and the variance equations has been removed as it doesn't provide any added information on the energy dynamics. The stationarity condition of an EGARCH-model is respected in each case as $|\beta| < 1$.

In the 1y1y horizon, the mean equation results, reported in Table 8, indicates that only WTI is significant at a 5% confidence level and has a negative effect on the level of market-based inflation expectations. The autoregressive term does not appear to be significant, implying a short/limited memory in the conditional mean of the 1y1y time series. WTI is associated with a statistically significant rise (2.364) in conditional variance while Henry Hub is insignificant at a 5% confidence level. Those findings may indicate that WTI crude oil prices may contribute to the unanchoring of inflation expectations, whereas Henry Hub natural gas spot prices have a limited impact on uncertainty. The ARCH-term is insignificant at a confidence level of 5%, suggesting that past shocks have a limited impact on current volatility. The GARCH-term (β = 0.88) is highly significant and indicates that volatility is extremely persistent over time as its coefficient value is close to 1. Last, the leverage term (γ = 0.38) is also highly significant at a 5% confidence level and positive, confirming the presence of an asymmetric effect. This implies that positive shocks have a stronger impact on future volatility than the negative shocks of equal magnitude. The distribution parameter analysis reveals that the t-Student distribution is statistically significant, validating the use of a Student-t distribution to capture fat tails. Additionally, the different diagnostic tests support the robustness of the model, see Appendix I.

In the 5y5y horizon, the autoregressive AR(1) is statistically significant in the mean equation, Table 9 reports that both external regressors (WTI and Henry Hub) are significant at a 5% confidence level and negatively associated with the dependent variable. In the variance equation, the GARCH-term is highly significant which is an indicator of strong volatility persistence. On the contrary, the ARCH-term is not significant at a 5% confidence level. The leverage effect is positive and significant at a 5% confidence level, supporting the presence of asymmetry in volatility. WTI is significantly positive, indicating a amplifying effect on volatility and Henry Hub is insignificant at a 5% confidence level. The Student-t distribution shape parameter is not significant, suggesting that deviations from normality might be modest. At last, the diagnostic tests indicate no specification issues, see Appendix I.

Table 8. United States – 1y1y EGARCH(1,1) Results

Parameter	Estimate	Robust Std. Error	t-value	p-value
Panel A. Mean Equation				
β_0	0.013379	0.038835	0.34	0.730
AR(1)	0.033344	0.108688	0.31	0.759
WTI (β_1)	-2.116260	0.322007	-6.57	0.000
Henry Hub (β_2)	-0.112616	0.414883	-0.27	0.786
Panel B. Variance Equation				
ω	-0.235606	0.106427	-2.21	0.027
ARCH-term (α)	0.108612	0.133302	0.81	0.415
GARCH-term (β)	0.889216	0.045814	19.41	0.000
Leverage effect (γ)	0.387376	0.118562	3.27	0.001
WTI (δ_1)	2.364530	1.041660	2.27	0.023
Henry Hub (δ_2)	0.637378	0.942495	0.68	0.499
Panel C. Distribution parameter				
Shape - t Student	3.805966	1.096374	3.47	0.001

Table 9. United States – 5y5y EGARCH(1,1) Results

Parameter	Estimate	Robust Std. Error	t-value	p-value
Panel A. Mean Equation				
β_0	-0.003085	0.000543	-5.68	0.000
AR(1)	-0.001445	0.000426	-3.39	0.001
WTI (β_1)	-0.884138	0.064490	-13.71	0.000
Henry Hub (β_2)	-0.172721	0.022057	-7.83	0.000
Panel B. Variance Equation				
ω	-0.202063	0.105198	-1.92	0.055
ARCH-term (α)	-0.039055	0.120146	-0.33	0.745
GARCH-term (β)	0.949859	0.026057	36.45	0.000
Leverage effect (γ)	0.389996	0.159746	2.44	0.015
WTI (δ_1)	1.577676	0.592671	2.66	0.008
Henry Hub (δ_2)	-0.663826	0.697554	-0.95	0.341
Panel C. Distribution parameter				
Shape - t Student	99.984864	63.395508	1.58	0.115

The Table 10 indicates that both WTI and Henry Hub are significant at a 5% confidence level in the mean equation and have a negative effect on the level of market-based inflation expectations. The autoregressive term is not significant. In the variance equation, WTI is associated with a statistically significant increase (1.23) of the conditional variance but insignificant at a 5% confidence level. Henry Hub is significant at a 5% confidence level and associated with a reduction (-1.26) of the conditional variance. Those findings may indicate that that Henry Hub natural gas spot prices may anchor the inflation expectations, while WTI has a limited impact on these expectations. The ARCH-term is insignificant at a confidence level of 5%, suggesting that past shocks have limited impact on current volatility. The GARCH-term is highly significant and indicates that volatility is extremely persistent over time. Last, the leverage effect is also insignificant at a 5% confidence level, refuting the presence of an asymmetric effect. The distribution parameter analysis reveals that the t-Student distribution is not statistically very different from infinity (99.99), indicating that the residual distribution is close to normal. Additionally, the different diagnostic tests support the robustness of the model, see Appendix I.

Table 10. United States – 10y10y EGARCH(1,1) Results

Parameter	**Estimate**	**Robust Std. Error**	**t-value**	**p-value**
Panel A. Mean Equation				
β_0	-0.012695	0.049152	-0.26	0.796
AR(1)	-0.038240	0.153445	-0.25	0.803
WTI (β_1)	-0.588918	0.080933	-7.28	0.000
Henry Hub (β_2)	-0.127952	0.061732	-2.07	0.039
Panel B. Variance Equation				
ω	-0.065913	0.644723	-0.10	0.919
ARCH-term (α)	-0.264144	0.564448	-0.47	0.640
GARCH-term (β)	0.981890	0.179637	5.47	0.000
Leverage effect (γ)	0.196133	0.223572	0.88	0.380
WTI (δ_1)	1.232116	1.330198	0.93	0.354
Henry Hub (δ_2)	-1.260991	0.586317	-2.15	0.032
Panel C. Distribution parameter				
Shape - t Student	99.999710	752.137744	0.13	0.894

4.2. Summary of the results

In the 1y1y horizon, both regions display volatility persistence and leverage effects, but Brent and Dutch TTF drive inflation expectation more strongly in the Euro Area than WTI and Henry Hub actually do in the US. While we observe that Brent, Dutch TTF and WTI significantly impact volatility through the results obtained by the variance equation, Henry Hub appears to be insignificant at a 5% confidence level. ARCH effects (α) are significant in the EA while they aren't in the US. Volatility persistence (β) is relatively high and asymmetry (γ) is present in both regions. The Student-t distribution is significant in both cases highlighting a leptokurtic behaviour.

In the 5y5y horizon, Brent and WTI appear to be significant at a 5% confidence level in the mean equation as well as natural gas variables. In the variance equation, Brent and WTI both positively affects volatility. Dutch TTF is marginally significant if a 10% confidence level is taken, and Henry Hub is insignificant. Core volatility dynamic parameters (β, γ) are significant but ARCH effects are insignificant in both regions The Student-t distribution is not significant in both cases, suggesting a nearly normal distribution.

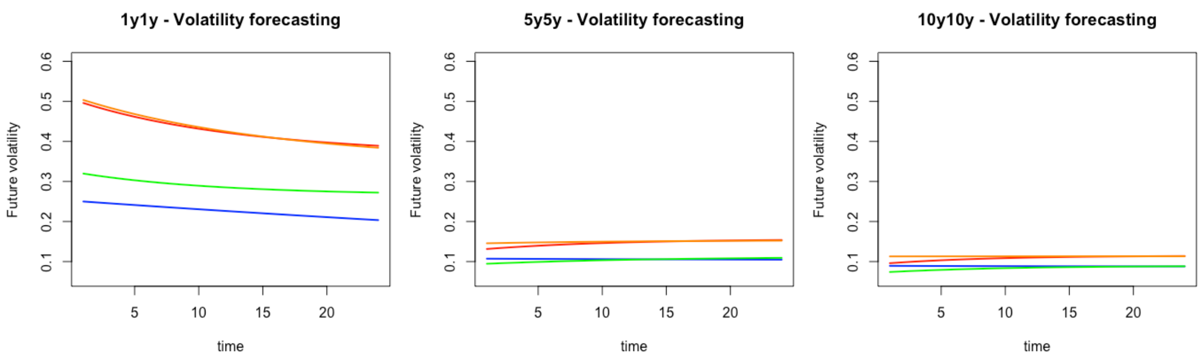
In the 10y10y horizon, the US model shows that WTI is insignifican while Henry Hub is negatively significant, decreasing volatility of inflation expectations. The EA model reflects Brent and Dutch TTF minor role in driving volatility of inflation expectation, both being insignificant at a 5% confidence level. The ARCH-term is insignificant in both cases, while the GARCH-term remains large and significant in both models. The leverage effect is only significant at a 5% confidence level in the EA model, suggesting the presence of asymmetric effects. In the EA, a leptokurtic behaviour is still observed so the Student-t distribution remains significant, on the contrary, the US distribution observe a behaviour closer to a normal distribution.

The forecasts of the market-based inflation expectations volatilities - with and without external regressors – are shown in Figures 3, 4, 5, and 6 to highlight their evolution across different horizons (1y1y, 5y5y and 10y10y). These graphs are plotted using a common Y-axis scale to allow a consistent comparison of forecasted volatility levels. The Y-axis represents the conditional standard deviation of the forecast residuals from the inflation swap rate series (expressed in absolute terms), while the X-axis represents the forecast length in months (here, up to 24). For each region, four model specifications are shown: "Green" is for only crude oil included, "Orange" is for only natural gas included, "Blue" stands for both external regressors included, and "Red" represents a model without any external regressors included.

4.2.1. Euro Area

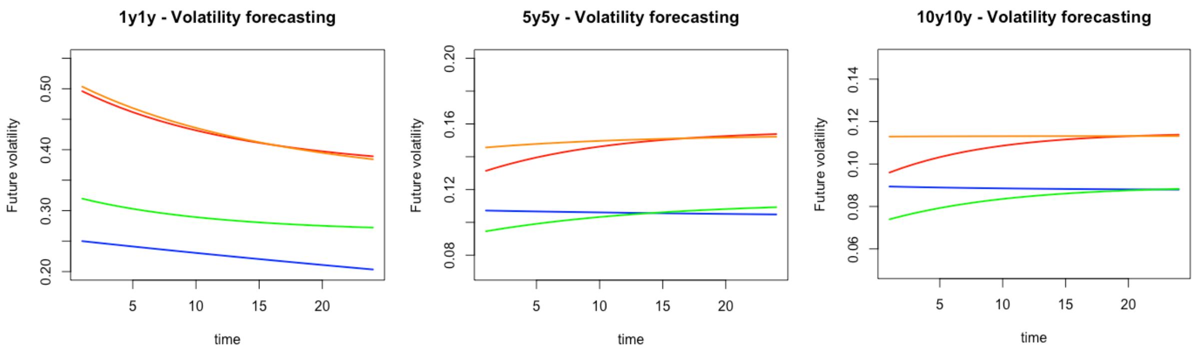
For the Euro Area, Figures 3 and 4 reveal that the 1y1y horizon exhibits the highest levels and variation in forecasted volatility. The model without external regressors (in red) predicts a conditional volatility that decrease from around 0.5 to 0.4, whereas the Brent model (in green) forecasts intermediary volatility, from around 0.32 to 0.28. This observation reflects Brent's significant (p-value < 0.05) role on volatility, confirming a stabilizing and explanatory role on short-term inflation expectations. On the contrary, Dutch TTF-model (in orange) leads to approximatively equal forecasted volatility or above, suggesting that gas price fluctuations are associated with higher perceived uncertainty, confirming hypotheses 2 (H2). The combined model with both external regressors (in blue) reflects a lower dynamic, where the Dutch TTF and Brent's combination highlight lower inflation expectation volatility on a shorter term, the destabilizing influence of Dutch TTF does not seem to outweigh the stabilizer role of Brent. This suggests a short-term sensitivity to gas shocks outweighed by Brent's impact. On the longer horizons (5y5y and 10y10y), Figures 3 and 4 highlight lower volatility levels (mostly increasing ones, from 0.09 to 0.1 for Brent and combined models, and from 0.13 to 0.15 for baseline and Dutch TTF models), with a notable increased effect from Dutch TTF (in orange) before stationarity. Brent continues to act as a stabilizer, though the difference between the models has severely decreased, validating the hypotheses H1 and H4. The Euro Area findings suggest that energy price volatility's impact fades over time, as market appears to treat their shocks as transitory in shaping longer-term market-based inflation expectations. This statement confirms hypotheses 4 (H4), stating that energy variables impact varies across horizons.

Figure 3. Euro Area – 24-month volatility forecasting for 1y1y, 5y5y and 10y10y horizons



Source: Graphs prepared by the author

Figure 4. Euro Area – Zoom on 24-month volatility forecasting for 1y1y, 5y5y and 10y10y horizons



Source: Graphs prepared by the author

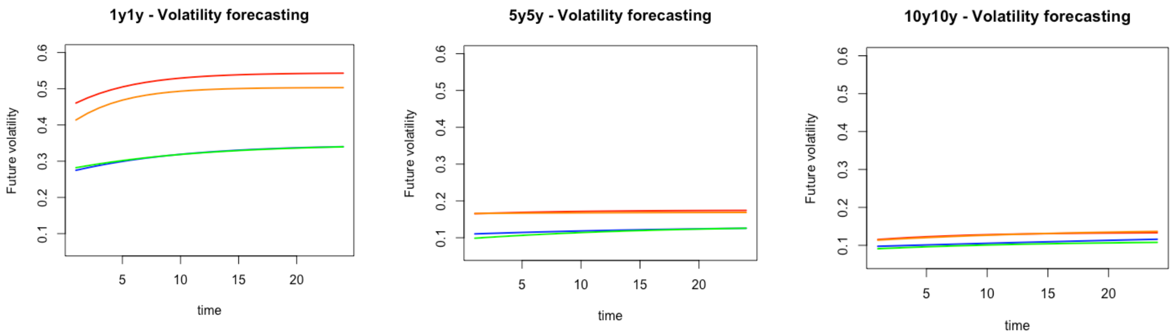
Notes Figures 3 and 4: Y-axis: Future volatility, representing the standard deviation of the inflation swap forecast residuals, is expressed in percentage points (e.g. 0.6 corresponds to a conditional volatility of 0.6 percentage points in the inflation swap rate). X-axis: Time (expressed in months). Legend: "With both regressors" = blue; "With no regressors" = red; "With Brent" = green; "With Dutch TTF" = orange.

4.2.2. United States of America

For the United States, Figures 5 and 6 relate that 1y1y horizon show the highest levels and greatest dispersion in forecasted volatility, as it was already the case for the Euro Area. But, in this case, the baseline model (in red) forecasts a growth in volatility from approximately 0.45 to 0.55 in the first 10 months, before reaching a stationary evolution. The WTI model (in green) exhibits the same forecasting curve but with lower predicted percentage points, starting at around 0.27 and ending at 0.32. As in the Euro Area, this reflects again the significant effect ($p < 0.05$) of WTI on the forecasted market-based inflation expectations volatilities, confirming its stabilizing role on a short horizon and validating hypotheses 1 (H1), stating that crude oil has a positive impact on the forecast of market-based inflation expectations volatility. Oppositely, Henry Hub inclusion (in orange), despite being statistically insignificant (p-value > 0.05), results in higher forecasted volatility and is closely shaped to the baseline model on a 1y1y horizon, suggesting that gas price fluctuation are again associated with higher perceived uncertainty. The combined model (in blue) exhibits a WTI-like volatility forecasting, where the dominance of WTI does not seem outweighed by the Henry Hub price fluctuations, outlining the stabilization role of the WTI.

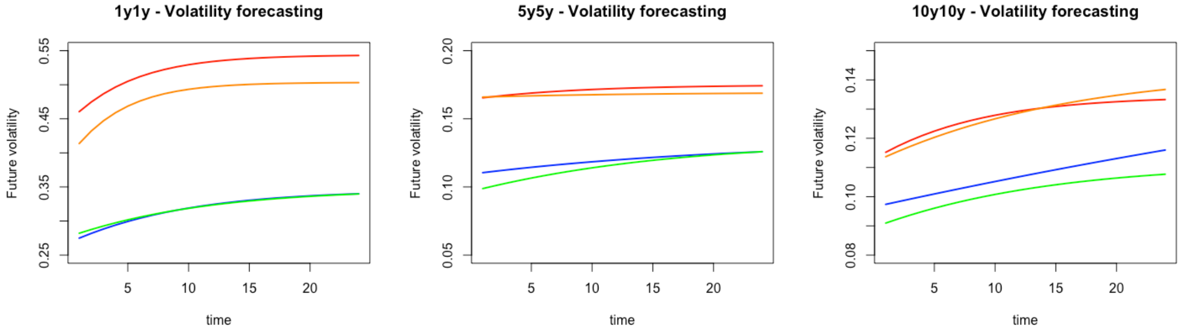
At 5y5y, Figure 6 exhibits that the Henry Hub-model (in orange), despite being insignificant, leads to lower volatility forecast than the baseline model (in red) which may highlight its marginal impact on medium-term inflation expectations volatility. Interestingly, the combined model (in blue) results in an intermediary volatility forecast plot, raising from 0.12 to 0.13, suggesting that the stabilized effect of WTI dominates the Henry Hub effect on a 5y5y horizon, and that the combined model proves its ability to better capture volatility dynamics than any other model. The model without any regressors (in red) performs almost similarly to the Henry Hub model, outlining the key stabilization role of WTI for market-base inflation expectations. Last, the 10y10y horizon provides us with different outputs than the previous ones. Henry Hub is now negatively statistically significant while WTI is not significant anymore. The Brent and the Henry Hub models, respectively in green and orange, have lower volatility forecasts than the baseline model, invalidating hypotheses 2 (H2), stating that natural gas as a positive impact on inflation expectations volatility. But in period 14, Henry Hub's curve is higher than the baseline model which, on the contrary, validates hypotheses 2 (H2). As we consider the combined model, we note that considering both variables account for lowering uncertainty on the long-term, despite Henry Hub being increasing uncertainty, suggesting that the market treats those shocks as increasingly transitory on long horizons, confirming hypotheses 4 (H4), stating that the impact of energy variables varies over horizons. Considering Euro Area results on natural gas impact on the forecasting of market-based inflation expectations volatility, we cannot confirm the hypotheses 3 (H3), stating that natural gas price fluctuations have more impact in the Euro Area than in the US, as it shows impact on different time horizons.

Figure 5. United States – 24-month volatility forecasting for 1y1y, 5y5y and 10y10y horizons



Source: Graphs prepared by the author

Figure 6. United States – Zoom on 24 -month volatility forecasting for 1y1y, 5y5y and 10y10y horizons



Source: Graphs prepared by the author

Notes Figures 5 and 6: Y-axis: Future volatility, representing the standard deviation of the inflation swap forecast residuals, is expressed in percentage points (e.g. 0.6 corresponds to a conditional volatility of 0.6 percentage points in the inflation swap rate). X-axis: Time (expressed in months). Legend: “With both regressors” = blue; “With no regressors” = red; “With WTI” = green; “With Henry Hub” = orange.

5. Discussion

The results from the EGARCH(1,1)-models confirm that energy prices have a non-negligible role in shaping market-based inflation expectations volatility confirming Boeck and Zörner (2024) findings. Crude oil variables, Brent for the Euro Area and WTI for the United States, are consistently significant in variance equation for 1y- and 5y-horizon. On the other hand, natural gas variables, Dutch TTF for the Euro Area and Henry Hub for the United States are negatively significant respectively for short- and medium-term horizon in the EA while the 10y10y horizon is the only US exception. Table 11 summarizes the estimated energy coefficients in the variance equation for each selected horizon.

Table 11. Impact of energy variables on market-based inflation expectations volatilities

Parameter	**1y1y**	**5y5y**	**10y10y**
EA Variance Equation - Energy coefficients			
Brent	1.859507 *	2.577812 *	1.863243
Dutch TTF	-1.349553*	-1.235129*	-0.545819
US Variance Equation – Energy coefficients			
WTI	2.364530*	1.577676*	1.232116
Henry Hub	0.637378	-0.663826	-1.260991*

Notes: *are significant at a 5% confidence level.

5.1. Euro Area

In the Euro Area, except for a 10y-horizon, Brent is consistently positively significant, suggesting that an increase in Brent prices is consistently associated with an increase in the forecasting of market-based inflation expectation volatility across all horizons. Economically, this can be interpreted that higher Brent prices may increase the macroeconomic incertitude, possibly as agents anticipate less accurately the inflationary consequences of Brent price fluctuations. Economic agents are then more surprised which indicates a worse integration of Brent price signal in the inflation expectations, resulting in increased inflation expectations volatility. Alternatively, Dutch TTF prices show statistically negative significant effects on Euro Area inflation expectation volatilities on the same horizon periods. While this may seem surprizing considering recent geopolitical events and energy shocks in shocks in Europe, it could broader reflect the higher short-term volatility and lower global integration of the Dutch TTF market. Furthermore, only monthly data were considered to format this analysis, and extreme peaks on short periods of time might not have been fully covered by this data frequence, thereby underestimating the full impact of natural gas shocks on market-based inflation expectations volatility.

5.2. United States of America

In the US, WTI exhibits a similar role as Brent does in the Euro Area. Across short- and medium-term horizons, higher WTI prices are significantly associated with higher inflation expectation volatility, suggesting that US economic agents may also have a less well-anchored understanding of WTI dynamics effects on shorter term than on longer term, increasing as they are more surprised by its WTI inflation impact. However, the effect of Henry Hub natural gas presents a contrasted behaviour. While it is not significant in the 1y1y- and 5y5y-horizons, the coefficient become negative and significant on the 10y10y-horizon. This suggests that an increase in natural gas prices is associated with lower volatility in 10y10y market-based inflation expectations as well and reflects less incertitude towards natural gas-related energy shocks. This could come from the fact that natural gas remains a regional and less-predictable market than crude oil in the US. In addition, Henry Hub prices shocks may be perceived as more transitory in the US by the economic agents, contributing to less uncertainty about longer-term inflation risks driven by natural gas price fluctuations.

5.3. Thesis limitations

This thesis also presents some limitations. First, the monthly frequency of the data may smooth high-frequency volatility spikes. To capture more granularity in the trends, a daily or weekly analysis may be more adapted, especially around major geopolitical events like Russia-Ukraine war as we know changes arise quiet quickly in a very uncertain and disturbed geopolitical world. Moreover, monthly data sample may appear inaccurate when trying to assess impact on 1y1y short horizon as the data frequence is less dense than a weekly or daily one. Secondly, this thesis models each region independently. However, energy markets and especially crude oil market are globally integrated. Ignoring cross-market volatility transmission mechanisms limits a broader interpretability of the results obtained. In that sense, including multivariate volatility modelling could reveal spillovers between the regions and provide further data on how energy market interaction can have an impact on domestic market-based inflation expectation volatilities. Thirdly, other energy commodities could have been considered to assess the impact of energy variables in market-based inflation expectations volatility. Therefore, including alternative variables (e.g. coal, electricity futures) could refine the understanding of the energy impact on inflation expectations volatilities from another standpoint. Last, market-based expectations contain risk premia. The models formulated do not explicitly decompose this component, potentially conflating expectation volatility with risk aversion change.

6. Conclusion

Market-based inflation expectations have become pivotal indicators for central banks and economic agents in shaping forward-looking monetary policy and understanding inflation dynamics (European Central Bank, 2024, Grølund et al., 2024). This thesis the role of crude oil and natural gas price on market-based inflation expectations volatility in two key economic areas, the Euro Area and the United States, over three maturity horizons, respectively 1y1y, 5y5y and 10y10y. Using a 15-year monthly dataset (2008-2023), this study applies EGACRH(1,1)-models with region-specific external regressors: Brent and Dutch TTF for the Euro Area and WTI and Henry Hub for the United States. Inflation Linked Swaps Zero Coupon serve as the dependent variable as they are market-based measures of inflation expectations. To ensure model robustness and goodness-of-fit, diagnostics tools such as Ljung-Bow, Sign Bias Test, Pearson Goodness-of-fit and ARCH LM Tests have been used. The study reveals that crude oil prices (Brent and WTI) play a positive significant role on the forecast of market-based inflation expectations volatilities, suggesting that rising crude oil prices tends to increase volatility forecasts and act as inflation expectation driver. Crude oil being a globalized energy market, economic agents may be more uncertain towards crude oil price fluctuations than towards more regional ones from natural gas, enhancing more predictable price behaviour. In that sense, the findings also reveal that on a long horizon (10y10y) in the US, Henry Hub is statistically significant and negative, meaning that a higher natural gas price would lead to a decrease of the volatilities of market-based inflation expectations. The study also points out that on a longer term, market tends to treat energy price shocks transitory when shaping longer-term market-based inflation expectations volatility. Furthermore, the study also state that energy variables have different impact across different horizons. Nevertheless, this research paper has some limitations. Firstly, the monthly frequence may smooth out high-frequency volatility spikes, considering possible extensions to daily or weekly data samples. Then, the thesis focuses on independent regional models, meaning that cross-market dependencies and spillovers has been ignored. Thirdly, the focus on crude oil and natural gas has been arbitrarily chosen arbitrarily, extending the study to other energy commodities may add an additional layer of understanding. Lastly, inflation linked swaps, selected as market-based inflation expectation measures, contain time-varying risk premia, potentially conflicting volatility expectations with risk aversion change. By its findings, this thesis contributes new empirical insights into the link between energy variables and market-based inflation expectations. It provides valuable insights to the existing literature and can be used as a robust base for further research exploring inflation dynamics and energy financial links.

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Appendices

Appendix I – Diagnostics results

Diagnostics	EA 1y1y	EA 5y5y	EA 10y10y	US 1y1y	US 5y5y	US 10y10y
Ljung-Box tests on Standardized Residuals						
Lag 1	0.50	0.77	0.47	0.95	0.99	0.23
Lag 2	0.45	0.47	0.26	0.64	0.34	0.48
Lag 5	0.27	0.49	0.45	0.03	0.80	0.50
Ljung-Box tests on Standardized Squared Residuals						
Lag 1	0.71	0.91	0.43	0.35	0.55	0.62
Lag 5	0.35	0.91	0.89	0.04	0.22	0.47
Lag 9	0.50	0.97	0.98	0.03	0.23	0.41
Sign bias Test						
Sign Bias	0.82	0.67	0.95	0.33	0.91	0.96
Joint effect	0.99	0.49	0.89	0.64	0.73	0.93
Negative Sign Bias	0.96	0.17	0.78	0.27	0.94	0.70
Positive Sign Bias	0.85	0.66	0.50	0.94	0.41	0.60
Nyblom Stability test						
Joint Statistic	2.30	1.87	2.36	1.61	2.22	2.24
1% Asymptotic Critical Value	3.27	3.27	3.27	3.27	3.27	3.27
Chi-Squared Pearson Goodness-of-fit						
Group of 50	0.64	0.74	0.74	0.76	0.57	0.98

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