Louvain School of Management

The impacts of Option onto a naïve portfolio.

Author(s): VilLLANCE Adrien
Supervisor(s): VRINS Frédéric
Academic year 2024-2025
Dissertation for the master of Business Engineering
Master subject and focus: Finance

UCLouvain

LOUVAIN School of Management

DECLARATION REGARDING AI TOOL USAGE IN MASTER'S THESIS

We recognize that AI tools might be valuable aids during the master's thesis work, but they are not infallible. Remember that transparency fosters trust, and acknowledging AI's role enhances the credibility of your work.

Therefore, when deciding to use such a tool, you need to adhere to the following principles of responsible use of AI.

1. **Critical Evaluation:**
    - We critically assessed the AI-generated output, ensuring its alignment with our research objectives.
    - Any modifications or corrections were made based on our expertise and domain knowledge.
2. **Transparency:**
    - We acknowledge the use of **ChatGPT** transparently, emphasizing that it contributed to our work but did not replace human judgment.
    - Our commitment to transparency ensures the integrity of this thesis.
3. **Ethical Considerations:**
    - We actively monitored for biases or unintended consequences introduced by the AI tool.
    - Our ethical responsibility guided our decisions throughout the research process.

Declaration **(This declaration is mandatory and must appear on the first page (after the title page) of the document.**

During the preparation of this master's thesis, the author(s) utilized **ChatGPT** for the following purpose:

1.[REASON]: Coding in VBA, Python and LaTex.

2. After using **ChatGPT**, the author(s) diligently reviewed and edited the content produced by the tool. We take full responsibility for the final content presented in this thesis.

By signing this declaration, we affirm that the content of this master's thesis reflects our original work, augmented by the responsible use of AI.

Feel free to customize the text by replacing "[NAME TOOL / SERVICE]" with the actual AI tool you used and providing a concise explanation of its purpose. Remember to sign and date the declaration appropriately.

04/08/2025, Adrien Villance

Acknowledgements

The writing of this thesis is the final step of my academic formation at UCLouvain. This final step would not have been possible without the help of various persons.

I would like first to express my gratitude to Mr. Vrins, for his patience, his constant availability and his useful pieces of advice. His expertise and professionalism allowed me to push this master thesis subject further and to provide results that would not have been possible without him.

I would also like to thank my parents, my brother and my friends, for the positive, constructive and encouraging mindset adopted for the past years, making this academic adventure a pleasure.

Finally, I want to thank my two supervisors at work, Annick and Philippe, who allowed me to adapt my schedule when necessary, and always proposed their help when circumstances were complicated.

Table des matières

1. Introduction *1*

1.1 Context 1

1.2 Objective of the thesis and Options potential 2

2. Literature review *4*

2.1 Portfolio optimization base 4

2.2 Options literature 8

2.3 Conclusion of the literature review 11

3 Methodology *12*

3.1 Benchmark creation and Options strategies 12

3.1.1Benchmark creation. 12

3.1.2 Options Strategies 15

3.2 Payoffs, Costs computation and ITS/OTS analysis 16

3.2.1 Payoffs 16

3.2.2 Costs computations 17

3.2.3 ITS and OTS analysis 19

4. Model for the analysis OTS *21*

4.1 Mathematical model 21

4.2 Explanation of the model 26

5. Results and analysis *28*

5.1 Stock and Portfolio descriptive analysis 28

5.1.1 Stocks analysis 28

5.1.2 Naïve Portfolio result: 30

5.2 Model Results: 31

6. Limitations and Conclusion *34*

6.1 Limitations 34

6.2 Conclusion 36

7. Appendices *37*

7.1 Illustrative Tables 37

7.2 Code 40

8. References *48*

1. Introduction

1.1 Context

Modern portfolio theory was introduced in 1952 by H. Markowitz. The objective of the Nobel prize winner was to create an optimized portfolio which maximized the expected return while maintaining a maximum level of risk. This risk adjusted portfolio is also called the mean-variance model, which in other words are the expected return divided by the variance of this same asset.

Markowitz claimed that, in order to minimize risk and maximize the expected return, the portfolio has to be composed of several non-correlated assets. This « Total risk calculation » approach allows the investor to take indirectly into his calculation both the systemic (risk of the entire market) and the specific risk (risk of the firm in which we invest).

Although Markowitz's theory might be the starting point of mean-variance optimal portfolio calculator, the model has faced criticism. One of main the criticism is that the portfolio is highly dependent of the input given. The main article describing this inconvenience is the one of Michaud (1989), which claims that the Markowitz optimizer might drastically change its allocation of assets due to very small changes in the input set. Another problem of the optimizer is the use of a Normal distribution in Markowitz's model, instead of using a Gaussian distribution, which better represents the reality. (Rachev & Mittnik, 2000). And lastly, the article published by Fama & French in 2004, which argues that the model overlooks many other important factors in the allocation of weight to each asset, such as liquidity, investors behavior and market dynamics.

Due to those critics and due also to the underperformance of this model during the financial crisis in 2008, there is a need to look for more recent optimizers, which would combine a more accurate view of portfolio allocation and annihilate some of the critics that the Markowitz's optimizer has faced. The book written by Bodie, Z., Kane, A., & Marcus, A. J. (2023) allows us to identify more precise models and the improvements they offer.

For example, there is the Treynor-Black model (1973), which compared to the Markowitz model, explicitly incorporates both active and passive components, allowing more clarity than Markowitz's single-focus approach. It introduces risk-adjusted performance metrics, such as the Treynor Ratio, for effective evaluation of active management.

Another model is the Black-Litterman (1992), which allows the investor to incorporate their specific view in the calculations. If the investors have information that the market doesn't know for example, this model allows to re-evaluate each weight of assets according to the available information and the impact the investor think they could have on their returns. The second advantage of the Black-Litterman is that it generates a more stable and diversified portfolio by mitigating the extreme asset weights that can arise from reliance solely on historical data.

Those models that are more efficient in representing the investor's view and the tendances of the market, also faced lots of critics and improvement since their creation, meaning that at this date, no optimizer have replaced a traditional investment strategy requiring several professional investors. The research of this thesis will focus on a subject that might outperform traditional optimizer, the main purpose: Measuring what are the impacts of integrating options into a portfolio and measure the positive/negative effect that it can have onto it.

To summarize, the objective is to measure how the expected returns could be improved through a dynamic integration of options and looking-forward method.

1.2 Objective of the thesis and Options potential

Most articles have described, analyzed and improved portfolio optimizers, but few have explored the impact that options can have on the returns and how they would enhance portfolio optimization.

According to Bodie, Kane and Marcus (2023): « *A **call option** gives its holder the right to purchase an asset for a specified price, called the **exercise** or **strike price,** on or before a specified expiration date.* » and « *A **put option** gives its holder the right to sell an asset for a specified exercise price on or before a specified expiration date.* ».

To clarify, profits on call options increase when the asset price increases, profits on put options increase when the asset price falls. The strike price (or exercise price) represents the pre-determined price at which the holder of an options contract can buy or sell the underlying asset. The date in the contract corresponds to the date up to which the seller/buyer can exercise the option, it is known as the expiration date or maturity, although there is a difference between European and American option: the first one can only be exercised at this precise maturity date and the second one can be exercised at any time up to the expiration date.
Another important aspect of option which must be emphasized is that: "*An options give its holder the right to do something g. The holder does not have to exercise this right. This is what distinguishes forwards and futures; where the holder is obligated to buy or sell the underlying asset. Whereas it costs nothing to enter into a forward or futures contract, there is a cost to acquiring an option*" (Hull, 2021).

There are two ways profits can be made from options:
- A direct profit from the premium, which is the cost to enter the option contract, represents an immediate profit for the option writer (seller), who receives the premium upfront in exchange for taking on the obligation of the contract. This profit might be transformed into losses because of the market fluctuations.
- An indirect profit, from the difference between the strike price and the market price of the underlying asset at expiration, depending on how the market fluctuates. For example, a call option becomes profitable when the market price exceeds the strike price plus the premium paid.

Options can be used whether for hedging or speculation. For hedging, if an investor feels that the actual share price will decline (S_t), he can protect himself by buying put options to a specified strike price (K) and exercise the option in period t+n where n respresents the maturity, this contract gives the holder the right to sell those shares at the specified strike price on the maturity date. Of course, he would have to pay upfront the premium (C) that this contract offers.
The investor benefits from this operation if the loss avoided by selling at the strike price exceeds the cost of the premium. In other word, for a K > S_t, This condition is satisfied if:

$$(K) - (S_{t+n}) > (C)$$

Concerning the speculation aspect, options benefit from a financial advantage called the "leverage effect". It allows investors to control a relatively large exposure to an underlying asset by investing a smaller amount of capital (the option premium). Of course, the losses can also exceed the losses in case of regular investment. this also reflects the risk that can arise from options management. In this analysis, only the speculation aspect of option will be used.

Those options can therefore excess market returns. If we predict correctly the market tendance, we can improve our portfolio's return. Another approach is to minimize the risk (or mitigating the potential losses) of the portfolio.
Our main goal will be to determine whether or not options would have a significant positive effect on the portfolio.

This work will be divided in two parts: the benchmark portfolio creation and the impact of integrating options in the same portfolio. The second part will be supported by code and algorithm in the following coding language: VBA and Python. We intend to extract all the data needed for those calculations from Bloomberg, which stays the more precise and complete database for such project. The VBA code will be used for largecomputations and calculating parameters beforehand, so that the code of the global portfolio can run in a smoother manner.

2. Literature review

2.1 Portfolio optimization base

The first step of this work will be to synthesize the literature review in order to establish the theoretical foundation of our work. Many articles have been published concerning optimizers and algorithms since the groundbreaking article published by Harry Markowitz in 1952. But too few have explored the potential advantages that options can offer. In the frame of this thesis, the purpose is to maximize the gain that the option portfolio can produce, the decision variable will be based on a ratio combining returns with respect to risk. The optimized portfolio's that base their allocations with respect to risk are called risk-adjusted portfolio, they take into account the returns with respect to the risk.

The first algorithm published by Harry Markowitz in 1952 is technically the first optimized portfolio based on the risk-adjusted returns. It implies to maximize the expected return while maintaining a desired level of risk through diversification of the assets. Being the pioneer of MV model, the first main critic of this method and more generally of this type of model is that an optimizer with a MV model may tend to usurp many of the integrative functions of the committee, and that those MV optimizers have serious financial deficiencies, which will often lead to financially meaningless "optimal" portfolios (Michaud, 1989). The other problem pointed by Michaud is that even the slightest change in the input given to the MV optimizer might lead to drastic change in the “optimal” portfolio given before those adjustments.

The Markovitz’s MV optimizer might not be the best algorithm to build a portfolio on. What about the several other methods based on Markovitz’s algorithm? One of the improvements made in 1973 was “*the exploration of the link between conventional subjective, judgmental, work of the security analyst, on one hand-rough cut and not very quantitative-and the essentially objective, statistical approach to portfolio selection of Markowitz and his successors, on the other*.” (Treynor & Black, 1973)
This method aims to construct a portfolio linked with market and therefore takes into account the macroeconomics risk and the individual-firm risk. Based on the Single-Index-Model (SIM), the Treynor-Black model is a practical application of the SIM, which use the model to calculate the active part of the portfolio.

According to Bodie & al. (2023), the Bêta supposed to estimate the macroeconomics risk in the formula impact both the risk premium and the risk of the portfolio itself. If a security has a negative alpha, it takes a short position or is excluded from the active portfolio if shorting is not allowed. A well-diversified active portfolio with more positive-alpha securities will reduce reliance on the passive index. The index portfolio is only efficient when all securities have zero alpha, as nonzero alphas indicate opportunities for active optimization, while zero-alpha securities only add uncompensated firm-specific risk.

Treynor-Black algorithm (1973) highlights the relevance of information ratios in effective security assessments. A positive alpha security benefits the active portfolio by adding a nonmarket risk premium, but it also increases portfolio variation due to the firm-specific risk. The macroeconomics risk component (beta) is neutral since it impacts both risk and return in

a balanced manner for all assets with the same beta, resulting in neither an intrinsic benefit nor disadvantage.
Securities with negative alpha take short positions in the active portfolio; however, if shorting is not permitted, their weight drop to 0, and limit diversification. Concerning the zero alpha securities, they do not contribute to the portfolio's attractiveness because they introduce firm-specific risk without compensating in terms of returns. Furthermore, the market index is only efficient if all equities have zero alpha, a rare occurrence. If a security is identified through the security analysis with a non-zero alpha and included into the portfolio, the portfolio will be found less attractive. The addition of a non-zero alpha security will just add the specific risk to the systematic risk that is supported by beta.

While the Treynor algorithm provides a structured way to merging active and passive portfolio elements, its efficacy drops to 0 if any non-zero alpha security is found. (Bodie & al, 2023). The Treynor-Black model might be more performant than the traditional Markovitz model in real practical cases due to the many errors that can occurs when calculating the covariance matrix of the Markovitz one. However, from a theoretical point of view, the Markovitz model might still be the better model. (Bodie & al, 2023)

We therefore must look for other algorithms that would provide us with an efficient portfolio. The main problem that arises when we investigate others optimization problems is the error estimation. The Black-Litterman method (Black & Litterman, 1992) might seem more accurate and representative because it allows the user to implement the investor vision in the calculations. Problem is that MV optimizers, including the Black-Litterman one, exclude completely the effect of measurement error in global (Jorion, 2007). Implementing the investor view can lead to negative effect if the accuracy is questionable. Given the complexity of that kind of algorithm and the accuracy they require to produce good results, the Black-Litterman method might not be the solution.

Another criticism of the Black-Litterman method is their definition of the mean-variance/tracking error (MVTE) which is defined as circular: “A criticism of this entire analysis is that the logic used in justification of the MVTE utility function is circular. If the benchmark represents the actual portfolio that the investor currently holds, then the optimization exercise is given the solution as part of its inputs. This approach is valid if the investor chooses the benchmark, but it does not apply if the investor selects a different portfolio" (Chow, 1995).

Given all the limitations or complexity of the models analyzed, maximizing the expected returns might be too complicate to implement accurately in the frame of this thesis. What about minimizing the risks?

The candidate for this method would be the risk parity model (RPM), developed in the late 1990 and improved until 2017 (Dalio, 2017) Although Dalio popularized this model, he is not the first to explore this field. Lörtscher (1990) and Kessler and Schwarz (1996) also documented scientifically the risk parity model, although they were speaking about an "equal risk benchmark". The RPM does not consider the expected returns and only focus on the respective risk estimated for each asset. Thanks to the absence of calculations that require the estimations of the expected return, this model gives more reliable results in term of risk and asset allocation. The main goal of the model is to equalize all assets' risk contribution between them, ensuring that no asset dominate the portfolio's risk profile. This model became popular in 2008 after the crisis stroke the USA. Investor being frustrated by the losses endeavored; they were seeking for safer portfolio.
In the paper detailing the performance of parity risk portfolio (Anderson et al., 2012) it becomes very clear that the low-risk portfolio can outperform market or value-weighted onto very long periods (85 years). But for long sub periods (37 years) it is not true anymore. Given the relatively short time window planned for this thesis, the parity risk portfolio model will also not be accurate to use as a benchmark. The market conditions being a very important factor, this method will not be classified as "better" than simpler naïve diversification.

Therefore, finding article that proves statistic evidence that MPT optimizers outperforms naïve diversification is quite complicated. As demonstrated with a tangible example for the geographic parameter and the property-type (real estate) (Cheng, 2000), it was noticed that there is no evidence of a better strategy than the naïve one for those examples. Extrapolate those results to the financial field might be touchy and would require caution. But the purpose of this thesis is to measure the impact of options based on a reliable benchmark. According to

DeMiguel & al. (2009), naïve diversification beats the MVP in most cases and produce more reliable result in terms of Sharpe Ratio.

The naïve diversification (1/N) follows a simple formula which is also called the equally weighed technic. Under a naive diversification approach, each of the N assets receives an allocation of 1/N of the total portfolio.

$$w_i = \frac{1}{N} \text{ for each i} = 1,2, \dots, \text{N}$$

Where:

w_i is the weight of the i-th asset in the portfolio.

N is the total number of assets in the portfolio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

Where:

R_p is the return of the portfolio or investment

R_f is the risk-free rate of return

σ_p is the standard deviation of the portfolio's excess return.

The marginal benefits that MVP optimizers give is not worth the work that their calculations require. Choosing naïve diversification as the technic to build the benchmark will be the more reliable (Cheng & Liang, 2000) and minimize all potential estimation errors that make optimized portfolios underperform.

2.2 Options literature

Now that the solution of a reliable benchmark has been found, our research will focus onto the impact of options, and how could those options impact the profitability of the portfolio analyzed.

One of the main works done on options is the one provided by Black & Scholes (1973). It allows to estimate the effective price of European options. The main difference between American and European options being that "*American option" is one that can be exercised at any time up to the date the option expires. A "European option" is one that can be exercised only on a specified future date*" (Black & Scholes, 1973). This work argues that the underlying option asset follows a Brownian motion process which is a stochastic process that models continuous time random motion. The BS model "only" allows to evaluate European option prices, but as explained by Merton (1973) American option pricing are more difficult to value than European options. The main problem being that American options have a positive probability at any time to be exercised before the expiration date, requiring therefore very intensive computations.

Since the publication of the groundbreaking article of Black & Scholes, several scientists have tried to mathematically simulate how algorithms could take into account the specifications of American-style options. In this special financial derivative, investors compare the direct payoff with the expected future return of the option, if the direct payoff is higher, they will sell. Thus, the optimal exercise strategy is fundamentally determined by the conditional expectation of the payoff from continuing to keep the option alive. (Longstaff et al., 2001). Longstaff & al have analyzed the problem by developing the Least Square Monte Carlo (LSM) method, which consists in minimizing the sum of the squared differences between observed values and predicted values. These differences are also known as residuals. The main goal of this strategy is to maximize American Option payoff correctly by identifying optimal exercise strategies under the risk-neutral framework. The LSM approach offers more scalability, is computationally less demanding and more flexible than other method such as the binomial method also used for the American option pricing (Cox et al., 1979).

Black and Scholes also shows that " *The actual standard deviation which would result over the life of an option would be a better input into the model if it were known in advance*" (Latané & Rendleman, 1976). Latané and Rendleman therefore develop this concept, now called implied volatility. Their study explains several new conclusions for the volatility parameter. By inputting the actual market price option into the BS model, and solve it for the volatility, they obtain an "implied standard deviation" which transforms the traditional historical volatility into a forward-looking measure. This new parameter serves as a reliable indicator on future variability in the underlying asset's price, since these values aggregate forward-looking opinions and assessments of risk that the market embed in option prices.

A tangible case and test of the BS model including stochastic volatility is the one provided by Yuan and Rieger (2020), who classes options investment strategies into two categories: "*There are two strategies to diversify portfolios consisting of options: one is to combine options on single underlying stocks, and the other one is to buy an option based on the index of these stocks*."

The authors find that index-based call options tend to perform better for moderately risk-averse investors particularly when the proportion of wealth allocated to calls is low (1% to 4%). For higher allocations (e.g., 5%), individual stock-based call options may slightly outperform for highly risk-averse investors. On the other hand, stock-based put options consistently outperform index-based puts across all levels of risk aversion and parameter settings. The study also highlights the sensitivity of the results to market parameters. For example, higher correlations among stocks reduces the diversification benefits of index-based options. Similarly, stock-based options perform better in turbulent markets with higher volatility ($\sigma \geq 35\%$), while index-based puts become more attractive in markets with negative expected returns. The optimal choice of strategy also depends on the relationship between strike prices and current asset prices.

Therefore, Yuan and Rieger proves that integrating options enhances portfolio performances, enhances expected utility and diversification potential. The choice between individual stock-based and index-based derivatives depends on market conditions, the investor's risk aversion, and the proportion of derivatives in the portfolio.

Another tangible case of the impact of options and mainly call options in that case is the case provided by Isakov and Morard (2001). The empirical analysis is performed on the Swiss Exchange from 1989 to 1996. In this study, Isakov and Morard state, based on the work of Bookstaber and Clarke (1985), that "*The introduction of options in stock portfolios can considerably alter the distribution of returns, and notably that the covered call strategy has* *the effect of reducing variance and inducing negative skewness. Comparisons based solely on the mean and the variance of the distribution could therefore lead to wrong conclusions*".

Another work developing the potential positive aspect options can have onto a portfolio is the one provided by Jones (2013). The author argues that financial derivatives can be implemented to improve portfolio efficiency both in terms of implementing investment strategies and in broadening the range of investment opportunities. The methodology used is a quantitative one in the form of principal components. This in short means that the first principal components explain the most variance, the second principal explains the second most variance… According to Jones (2013) "*Principal components analysis (PCA) is a procedure to generate a set of linearly uncorrelated factors (eigenvectors) that represent a set of observations of possibly correlated variables (e.g. stocks in an equity index or asset classes in a multi-asset portfolio)*". The paper mainly focuses on global market exposures rather than investment into specific assets. The conclusion of the author is that those derivatives, including the options, offer significantly more diversification than traditional approaches, and the approaches chosen in the paper illustrates better risk measurement and ongoing risk monitoring. This being the proof that option can indeed boost portfolio performances in terms of risks.

2.3 Conclusion of the literature review

Thanks to this complete literature review, insights over past and recent studies are given concerning global portfolio optimization and the integration of options. It will allow us to build a reliable benchmark, and to integrate properly options given the potential they offer.

The past optimizers that have been developed concerning modern portfolio theory require very intensive computations, and don't offer a 100% reliable benchmark. Each algorithms present its limitations, and the potential errors (small or consequent) can lead to completely unreal results. However, as we've seen, diversification is a key factor to protect the portfolio from the systemic risk (Markowitz, 1952). Therefore, building a reliable benchmark based on the naïve diversification (Demiguel et al., 2009)can include a correlation parameter, by ensuring to integrate manually assets that are uncorrelated between themselves. This will allow to make sure that the systemic risk of the global portfolio will be diminished, without the need of a mean-variance optimizer. This method is also more in line with reality, given the fact that professionals don't use optimizers for several practical reasons cited in the literature.

Concerning the options part, the use of a traditional Black-Scholes model will be the model used for the pricing of options. The model is not yet perfect, but it is the more reliable model known as of todayThanks to those researches we will be able to accurately estimate the pricing of European call and put options.

3 Methodology

3.1 Benchmark creation and Options strategies

3.1.1Benchmark creation.

The analysis of options impacts onto a portfolio will be divided in two parts. The first part will be to build a reliable benchmark based on the naïve diversification technic (1/N). This benchmark will regroup 28 randoms assets (**Table 1** – Section 7.1) from the EUROSTOXX 50 and will allow to estimate the efficiency of a benchmark. The second part will be dedicated to the options and their impact on the portfolio performance, which will include the same underlying assets as the benchmark with naïve diversification, but with the major difference that options with several strategies and maturities can be exercised. The timeframe chosen to evaluate those portfolios' expected returns across time will be of 6 years (January 2019 – December 2024). The data before 2019 will be used in the in-the-sample analysis, where a computation of the most profitable options with respect to their costs will be conducted.

The model integrating option will be tested against its equivalent with naïve diversification, the evaluation criteria is therefore crucial in our analysis. The main parameters that will allow us to judge which portfolio is the more performant will be the Sharpe Ratio (SR), the volatility (σ_p^2) and the returns (R_p) estimated for both portfolios.

The volatility of the stock portfolio will be calculated thanks to the data gathered on Bloomberg for each stock included in the portfolio. The formula for the volatility, or more commonly the global risk of the portfolio, can be calculated as the sum of the multiplication of each asset i and j weight multiplied by their respective covariance. This gives the following formula describing the variance, which is the total risk (σ_p, Standard deviation) squared:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j} \qquad (1)$$

Where the parameter $\rho_{i,j}$ is the correlation between the asset i and j.

Therefore, the calculation of the covariance matrix between the expected returns of the assets can be formulated as a matrix of size N*N (28*28 in our case).

The covariance matrix is the following:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{1,2} & \cdots & \sigma_1\sigma_N\rho_{1,N} \\ \sigma_2\sigma_1\rho_{2,1} & \sigma_2^2 & \cdots & \sigma_2\sigma_N\rho_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_N\sigma_1\rho_{N,1} & \sigma_N\sigma_2\rho_{N,2} & \cdots & \sigma_N^2 \end{pmatrix}$$

Associated with the weight vector:

$$w = \begin{pmatrix} \frac{1}{N} \\ \frac{1}{N} \\ \vdots \\ \frac{1}{N} \end{pmatrix}_{N\times 1}$$

Given that all weights within the portfolio are equal in terms of value at t = 0, the variance σ_p^2 can be summarize as:

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_i \sigma_j \rho_{i,j}$$

The expected returns ($E[R_p]$) of the naïve portfolio strategy will simply be the sum of all underlying returns (μ_i) divided by N.

$$E[R_p] = \sum_{i=1}^{N} w_i \ \mu_i = \sum_{i=1}^{N} \frac{1}{N} \ \mu_i = \frac{1}{N} \sum_{i=1}^{N} \mu_i \qquad (2)$$

The Sharpe ratio for this naïve strategy will be:

$$\mathrm{SR} = \frac{E[R_p] - R_f}{\sigma_p}$$

Where R_f is the risk-free rate. By inputting the previous equation into this simplified SR, we obtain:

$$\mathrm{SR} = \frac{\frac{1}{N} \sum_{i=1}^{N} \mu_i \ - \ R_f}{\sqrt{\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j \rho_{i,j}}} \qquad (3)$$

For the risk-free rate of European options, several rates can be chosen. In this work, we will equal the risk free rate to 0, which is common for simulations.

3.1.2 Options Strategies

« An option is said to be **in the money (ITM)** when its exercise would produce a positive cash flow. Therefore, a call option is in the money when the asset price is greater than the exercise price, and a put option is in the money when the asset price is less than the exercise price. Conversely, a call is **out of the money (OTM)** when the asset price is less than the exercise price; no one would exercise the right to pay the strike price for a stock when its market value is less than that amount. A put option is out of the money when the exercise price is less than the asset price. Options are **at the money (ATM)** when the exercise price and asset price are equal. » (Bodie & al, 2023)

The three strategies have their advantages. From a buyer point of view, the ITM ones are less risky than the OTM because they already have intrinsic value, if they could be exercised immediately they would give a positive payoff. Even if the market doesn't move much, ITM options are more likely to retain some value. The OTM have a lower probability to be profitable, they are entirely speculative, relying on significant market movement to gain intrinsic value but also benefit from more leverage effect. The ATM strategy is the middle ground between the two other strategies. From a seller point of view this relation is inverted (almondztrade.com).

The data extracted from Bloomberg allows to calculate the price of options for maturities of 30 days, 3 months, 6 months and 1 year. For each of these maturities, Bloomberg had available data, allowing to collect the implied volatility at 80%, 90%, 95%, 100%, 105% 110% and 120%. This results in a total of 28 different option prices per day for each stock. To ensure correct pricing of the option, it was decided to choose the implied volatility instead of the historical one. These implied volatility figures are available from 2005 to December 2024 (unless for the 0.8 and the 1.2 strategies, for which data was available starting from 2008).

This implied volatility ensures a more precise approach of the traditional historical volatility. This parameter considers the stock's future volatility, derived from the price of its options. It represents the market's expectations of the stock's fluctuation. This measure is forward looking and captures market sentiment, the supply/demand aspect of the options and the upcoming events that might influence the stock/option price.

3.2 Payoffs, Costs computation and ITS/OTS analysis

3.2.1 Payoffs

The payoffs of call and put options are simple. As explained earlier, holding a call option gives you the right to purchase a stock at the exercise price K (which was set previously when buying the option). If the value of the stock at maturity t (S_t) is greater than the exercise price this difference is the gain.

$$\text{Payoff of a call holder} = \begin{cases} S_t - K, if\ S_t > K \\ 0, if\ S_t \leq K \end{cases}$$

$$\text{On the other hand, payoff of a put holder is } = \begin{cases} 0, if\ S_t \geq K \\ K - S_t, if\ S_t < K \end{cases}$$

But the payoffs do not guarantee that the holder has made profit, due to the prime paid for the option. Profit can be generated when S_t > K + the premium paid (C) to hold the options (for call options).

Hereunder a visual representation of the differences between profit and payoff can be found for both put and call options.

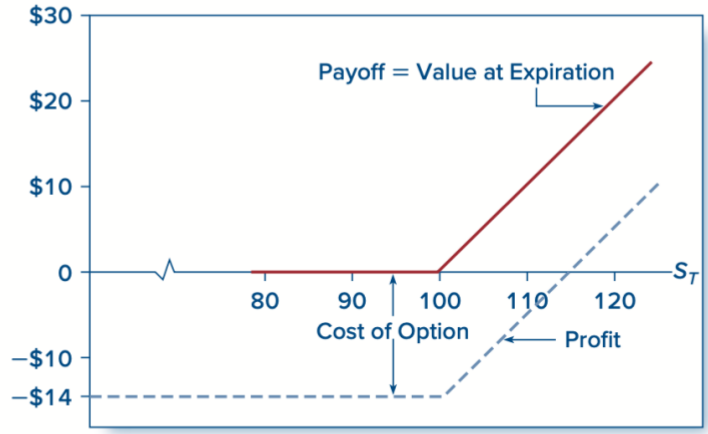

Figure 1: Payoff of a call

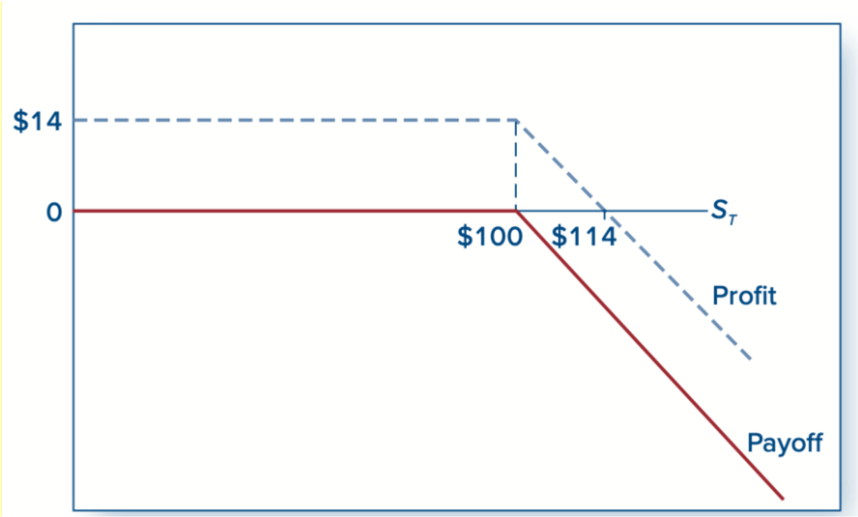

Figure 2: Payoff of a put

These payoffs will be one of the main components of our analysis concerning options. Their relative prime paid in time t-m, where t is the period at which the option is acquired and m the maturity of the options, implies a double temporality in the global expected returns calculations.

This double temporality complexifies the profit computation for options since simulations have to associate the correct payoff at t = s+m where s is the buying period t of the option (Section 7.2 Appendix, Code 1). To compute the profit, it is still needed to deduct from those payoffs the cost encountered at t = s. For options with a payoff = 0, the profit is the cost multiplied by -1.

3.2.2 Costs computations

The second main components of the option modelization are the costs. The premium paid upfront for acquiring an option are mandatory to know how much cash we invest in those options.

Given that the payoffs of the assets will be counted in the future, options buyers still must invest money at time $t_{(i)}$, investment onto which they can hope to recollect some profit only at time $t_{(i+m)}$. In the model used for this analysis, the timeframe of each i will be 1 month. Given the fact that the options available are minimum equal to 1 month, the model will be constrained to buy options only the 1st of each month.

To model these costs, the Black-Scholes formula allows to estimate the premium for puts and calls (See section 7.2 - Appendix – code 2)

Premium for a call:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \qquad (4)$$

And for a put:

$$P = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \qquad (5)$$

Where d_1 is the sensitivity of the option to the stock price and d_2 is the risk neutral probability that the option will expire ITM.

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

And

$$d_2 = d_1 - \sigma\sqrt{T}$$

Where all parameters can be understood as

S_0 = price of the underlying stock

K = Strike price

r = Risk free rate (0 in our simulation)

T = time to maturity (in year)

σ_t = Implied volatility at time t, historical one if not available

$N(x)$ = cumulative distribution function of the standard normal distribution

This model makes the assumptions that:

1) No dividends are paid during the option's life
2) No transaction costs or taxes
3) Risk-free rate and volatility are constant (The value of the option will be calculated with an implied volatility equals to the Ivol at purchase time t)
4) Markets are efficient

Thanks to this subdivision of the problem, the costs of the options can be calculated as a parameter for the next steps, reducing therefore computation needs for the final model.

3.2.3 ITS and OTS analysis

Based on historical return, the purpose of this section is to evaluate the past returns of the of options across a large horizon of time (ITS), allowing to reduce in the out of sample analysis (OTS) the number of options the model can acquire, focusing on the ones that are more profitable historically.

For this, since the historical data gathered from Bloomberg covers the period from May 2005 to December 2024, we decided to split the ITS and OTS with an approximate factor of 2/3 and 1/3, covering respectively the period from 2005 to December 2018 for the ITS and the period from 2019 to December 2024 for the OTS.

To evaluate the past historical returns of options, the first step was to compute the cost of each options, and assesses the profits generated from them for the past 13,5 years.
As explained in the previous section, to evaluate accurately these costs, the Black and Scholes formula was used, which evaluates the premium paid for an option based on the expected return on the asset and the probability under the risk neutral assumption that the option will finish in the money.

Since the payoffs and the costs could be computed for the ITS options, the analysis of the past historical returns will determine the options included in the OTS analysis, restraining only the available options in the OTS to the ones most profitable historically.

In **Table 2** and **Table 3** (Section7.1 – Appendix) are the results of the profits made from May 2005 to December 2018. The options that will be available in the OTS analysis are the ones that have a <u>**median**</u> <u>**profit**</u> positive (**Table 4** – Section 7.1 – Appendix). As displayed in the tables, the median profits for put option are consistently negative, leading to focus the strategy of the OTS analysis to only call options.

This aspect can be explained by the fact that the benchmark chosen are 28 random companies from the EUROSTOXX50 in 2025, leading to a bias because those enterprises being in the top 50 European capitalizations in 2025, ensures a certain profitability for the analysis starting in 2005. Since those past 19 years can be summed up as a “rising market”, it is therefore logic that betting on calls that would allow to buy the underlying to a lower price are profitable.

4. Model for the analysis OTS

4.1 Mathematical model

Let's first design a model that will define the constraints wanted. The total allowed budget to buy options will be 10% of the total portfolio value. Given the fact that the model will handle 14 types of options (strategy and maturity allowed in the ITS analysis) for 28 different stocks, it results that at each time t, the model has 392 potential options available, it will therefore be forced to pick the top 6 options with the best ratio based on the historical returns and the standard deviation of the same returns for the previous 12 months.

This constraint of allowing to invest in only the best options type based on the historical returns ensure to have possible investment, not having very small quantities of the 392 options available in time t.

The model will invest the same amount into those 6 options. For example, if an option is valued at 50€ and another one at 100€, the model will buy 2 times more options of the option one compared to the option 2.

The indices are therefore:

- $t \in \{0, 1, 2, ..., T\}$ be the time index (months).
- $j \in \{1, ..., 28\}$ be the stock ID.
- $i \in \{1, ..., 7\}$ represents the strike strategy index (e.g., 0.8, 0.9, …, 1.2).
- $m \in \{1, 2, \dots 12\}$ be the time to maturity in months.
- $\tau \in \{call, put\}$ be the option type.
- $s \subseteq t$, where s is the period at which option has been bought and a subset of t
- $O_t \subset i \times j \times m \times \tau$: available option types at time t
- $S_t \subset O_t$: Subset of available strategies with the top 6 ratios

Parameters:

$\mathrm{Cost}_t^{ijm\tau}$ = Cost to purchase one unit of the option at time t.

$\mathrm{Payoff}_t^{i,j,m,\tau}$ = Payoff generated by option bought at time s with maturity m and where t = s+m

Price_t^{j} = Price of stock j at time t

$\mathrm{I}_t^{\mathrm{target}} = 0{,}1 * P_t$

$\mathrm{I}_t^{\mathrm{current}} = \sum_{i,j,m,\tau} q_t^{ijm\tau} * Value_t^{ijm\tau}$ for t with s <= t < s+m (current investment in options)

$\mathrm{I}_t^{\mathrm{req}} = \mathrm{I}_t^{\mathrm{target}} - \mathrm{I}_t^{\mathrm{current}}$ (Investment required to reach correct balance)

$\mathrm{I}_t^{\mathrm{stock}} = \mathrm{I}_t^{\mathrm{req}} - \sum_{\mathrm{i,j,m,\tau}} \mathrm{Payoff}_t^{\mathrm{i,j,m,\tau}}$ (investment from stock to reach correct balance)

$Value_t^{ijm\tau}$ = Value of option at time t

P_t = Portfolio Value at time t

$\mathrm{K}_t^{ijm\tau}$ = Strike price of option

$\mathrm{R}_t^{ijm\tau}$ = Ratio between the returns of the past **12 available months** of the same options type and the volatility of those returns.

Variables

h_t^{j} = quantity of stock j at time t.

$\mathrm{q}_t^{ijm\tau}$= Quantity of options in possession at time t

k_t^{j} = quantity of stock j sold at time t.

SB_t^{j} = Stock bought with payoff at time t

$\mathrm{OB}_t^{i,j,m,\tau}$ = Option bought with payoff at time t

Calibration of the parameters:

Portfolio's value

$$\mathrm{P_t} = \sum_{i,\mathrm{j,m,\tau}} \mathrm{h_t^j} * \mathrm{Price_t^j} + \sum_{s=0}^{t} \sum_{i,\mathrm{j,m,\tau}} 1_{\mathrm{t<s+m}} * \mathrm{q_t^{ijm\tau}} * Value_{\mathrm{t}}^{\mathrm{ijm\tau}} + \sum_{\mathrm{i,j,m,\tau}} \mathrm{Payoff_t^{i,j,m,\tau}} \qquad (6)$$

Payoffs

$$\mathrm{Payoff_t^{i,j,m,\tau}} = \mathrm{Max}\left(\mathrm{K_t^{ijm\tau}} - \mathrm{Price_t^j}; 0\right) \text{ for a put with } \mathrm{t = s + m}$$
$$\mathrm{Payoff_t^{i,j,m,\tau}} = \mathrm{Max}\left(\mathrm{Price_t^j} - \mathrm{K_t^{ijm\tau}};\ 0\right) \text{ for a call with } \mathrm{t = s + m}$$

Ratio based onto which we will pick the 6 best options:

$$\mathrm{R_t^{ijm\tau}} = \sum_{\substack{\mathrm{s} \in [\mathrm{t-12,t-1}] \\ \mathrm{t'' = t'+m}}} 1_{t'' \in [t-12, t-1]} * \mathrm{Profit_{t''}^{i,j,m,\tau}} / \sigma_{\mathrm{Profit_{t''}^{i,j,m,\tau}}} \qquad (6)$$

$$\mathcal{S}_t = \underset{\substack{\mathcal{A} \subseteq \mathcal{O}_t \\ |\mathcal{A}|=6}}{\arg\max} \sum_{(i,j,m,\tau) \in \mathcal{A}} R_t^{i,j,m,\tau}$$

Where for all t:

$$\mathrm{OB_t^{i,j,m,\tau}} > 0 \Rightarrow (i, j, m, \tau) \in S_t$$

Reinvestment of Payoffs

If (in scope range)

$$8\% \leq \sum_{s=0}^{t} \sum_{i,\mathrm{j,m,\tau}} 1_{\mathrm{t<s+m}} * \mathrm{q_t^{ijm\tau}} * Value_{\mathrm{t}}^{\mathrm{ijm\tau}} / \mathrm{P_t} \leq 12\% \qquad (7)$$

Then

$$\sum_{j} \mathrm{SB}_{\mathrm{t}}^{\mathrm{j}} * \mathrm{Price}_{\mathrm{t}}^{\mathrm{j}} / \sum_{\mathrm{i,j,m,\tau}} \mathrm{Payoff}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}} = 0{,}9 \qquad (8)$$

$$\sum_{i,j,m,\tau} \mathrm{OB}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}} * \mathrm{Cost}_{\mathrm{t}}^{\mathrm{ijm\tau}} / \sum_{\mathrm{i,j,m,\tau}} \mathrm{Payoff}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}} = 0{,}1 \qquad (9)$$

And

$$h_{t+1}^{j} = \sum_{j} \mathrm{h}_{\mathrm{t}}^{\mathrm{j}} + \sum_{j} \mathrm{SB}_{\mathrm{t}}^{\mathrm{j}} \qquad (10)$$

$$\mathrm{q}_{\mathrm{t+1}}^{\mathrm{ijm\tau}} = \sum_{i,j,m,\tau} \mathrm{OB}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}} + \sum_{i,j,m,\tau} q_{t}^{ijm\tau} \qquad (11)$$

<u>If below scope range</u>

$$\sum_{s=0}^{t} \sum_{i,\mathrm{j,m,\tau}} 1_{\mathrm{t<s+m}} * \mathrm{q}_{\mathrm{s}}^{\mathrm{ijm\tau}} * Value_{\mathrm{s}}^{\mathrm{ijm\tau}} / P_{t} < 8\% \qquad (12)$$

Then

$$\sum_{i,j,m,\tau} \mathrm{OB}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}} * \mathrm{Cost}_{\mathrm{t}}^{\mathrm{ijm\tau}} = min(\mathrm{I}_{\mathrm{t}}^{\mathrm{req}}, \sum_{\mathrm{i,j,m,\tau}} \mathrm{Payoff}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}}) \qquad (13)$$

<u>if i req < payoff</u>

$$\sum_{i,j,m,\tau} \mathrm{SB}_{\mathrm{t}}^{\mathrm{j}} * \mathrm{Price}_{\mathrm{t}}^{\mathrm{j}} = \sum_{\mathrm{i,j,m,\tau}} \mathrm{Payoff}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}} - \sum_{i,j,m,\tau} \mathrm{OB}_{\mathrm{t}}^{\mathrm{i,j,m,\tau}} * \mathrm{Cost}_{\mathrm{t}}^{\mathrm{ijm\tau}} \qquad (14)$$

And

$$\sum_{i,j,m,\tau} \mathrm{OB}_t^{i,j,m,\tau} * \mathrm{Cost}_t^{ijm\tau} = \mathrm{I}_t^{req} \qquad (15)$$

And

$$h_{t+1}^{j} = \sum_{j} \mathrm{SB}_t^{j} + \sum_{j} \mathrm{h}_t^{j} \qquad (16)$$

$$\mathrm{q}_{t+1}^{ijm\tau} = \sum_{i,j,m,\tau} \mathrm{OB}_t^{i,j,m,\tau} + \sum_{i,j,m,\tau} q_t^{ijm\tau} \qquad (17)$$

<u>if I req > profit</u>

$$\sum_{j} \mathrm{k}_t^{j} * \mathrm{Price}_t^{j} = \mathrm{I}_t^{stock} \qquad (18)$$

And

$$\sum_{i,j,m,\tau} \mathrm{OB}_t^{i,j,m,\tau} * \mathrm{Cost}_t^{ijm\tau} = \sum_{j} \mathrm{k}_t^{j} * \mathrm{Price}_t^{j} + \sum_{\mathrm{i,j,m,}\tau} \mathrm{Payoff}_t^{i,j,m,\tau} \qquad (19)$$

and

$$h_{t+1}^{j} = \sum_{j} \mathrm{h}_t^{j} - \sum_{j} \mathrm{k}_t^{j} \qquad (20)$$

$$\mathrm{q}_{t+1}^{ijm\tau} = \sum_{i,j,m,\tau} \mathrm{OB}_t^{i,j,m,\tau} + \sum_{i,j,m,\tau} q_t^{ijm\tau} \qquad (21)$$

If exceeds range

$$\sum_{s=0}^{t}\sum_{i,j,m,\tau} 1_{t<s+m} * q_s^{ijm\tau} * Value_s^{ijm\tau}/P_t > 12\% \qquad (22)$$

Then

$$\sum_{i,j,m,\tau} OB_t^{i,j,m,\tau} * Cost_t^{ijm\tau} = 0 \qquad (23)$$

$$\sum_{i,j,m,\tau} SB_t^{j} * Price_t^{j} = \sum_{i,j,m,\tau} Payoff_t^{i,j,m,\tau} \qquad (24)$$

And

$$h_{t+1}^{j} = \sum_{j} h_t^{j} + \sum_{j} SB_t^{j} \qquad (25)$$

$$q_{t+1}^{ijm\tau} = \sum_{i,j,m,\tau} q_t^{i,j,m,\tau} \qquad (26)$$

4.2 Explanation of the model

The objective is to observe the behavior of the most profitable historical strategies, through picking only the top 6 options type of the past 12 periods t. The model ensures to come as close as possible to the 10% target of investment in options at all time.

At each time t, the model forms a subset $S_t \subset O_t$ of exactly 6 option types, where O_t represents the universe of admissible options available at time t. This selection is driven by a profitability ratio $R_t^{ijm\tau}$ which aggregates the realized profits from previous purchases of that option type (same maturity, same underlying, same strike price in %) in a rolling window.

The portfolio is managed with the purpose of maintaining a target allocation of 10% in options. But to avoid unrealistic rebalancing from very small transactions, the model authorized the actual option weight to oscillate between 8% to 12%. If the current allocation lies within this band, the reinvestment into options is 10% of the Payoff made at time t, the money left is invested into stocks. When the allocation drops below 8%, the model uses a mix of available payoff and proceeds from stock sales to increase the exposure toward 10%. And lastly, if the allocation exceeds 12%, no new options are bought and all the payoffs are invested in stocks.

The balance of the portfolio is at t = 0, 900 000 in stock (equally allocated in terms of cash) and 100 000 in options. This balance is updated dynamically through purchases and liquidations of stocks, based on the needs of the options allocation mechanism.

Each option held in the portfolio is dynamically revalued at every time t using the Black-Scholes pricing formula. The value is computed using:

- the current underlying price (i.e., updated moneyness).
- the remaining time to maturity T expressed in years.
- a constant implied volatility σ, fixed at the level observed at the time of purchase s.

The model thus reflects realistic time decay and market evolution, while maintaining a stable volatility assumption in line with the original market pricing context. This continuous re-evaluation ensures that the portfolio captures the actual mark-to-market value of the options held, rather than relying on historical cost or simplified payoffs. The corresponding code can be visualized in (Section 7.2 – Code 4)

The value of the total portfolio is also computed at each period t, summing the value of stocks held, options alive and payoffs made from the options where s+m = t
This model forms a dynamic adaptive and self-consistent one, ensuring to be the closest to a 10% allocation in options at every time t.

5. Results and analysis

5.1 Stock and Portfolio descriptive analysis

5.1.1 Stocks analysis

This section aims at describing a statistical analysis of the 28 stocks included in our investment universe. This analysis focused on average monthly returns, volatility (standard deviation and variance), asymmetry (skewness), tail risk (kurtosis), and return normality using the Shapiro-Wilk test. The goal of this section is to better understand the returns distributions of individual stocks and to provide an initial overview of the risks associated with each, prior to their integration into the final optimization model. (Section 7.2 - Code 4)

Stock	Mean	Median	Std Dev	Variance	Skewness	Kurtosis	Shapiro_W	p_value
MC FP Equity	0,01285	0,014957	0,070839	0,005018	0,181829	0,040728	0,98508	0,563690488
NDA FH Equity	0,010449	0,016624	0,078973	0,006237	-1,536302	5,756871	0,897517	6,45928E-05
AI FP Equity	0,010933	0,018253	0,049285	0,002429	-0,369539	0,031682	0,982661	0,433792622
BN FP Equity	0,003523	0,011571	0,051976	0,002702	-0,38297	1,0203	0,967665	0,063879723
SAP GR Equity	0,015767	0,028156	0,085906	0,00738	-1,481721	4,72981	0,90728	0,000152259
SU FP Equity	0,021775	0,024036	0,075537	0,005706	-0,529262	0,439366	0,975769	0,185368128
SAN FP Equity	0,006024	0,006885	0,052953	0,002804	-0,438148	1,147619	0,967688	0,064072029
IBE SM Equity	0,012737	0,0177	0,059952	0,003594	-0,677017	1,654211	0,963939	0,043897059
SGO FP Equity	0,017481	0,03455	0,09419	0,008872	-1,324126	3,654118	0,919132	0,000217373
BAYN GR Equity	-0,020632	-0,012654	0,109109	0,011905	-0,111973	-0,408872	0,986142	0,691740043
ABI BB Equity	-0,000637	0,008144	0,085176	0,007255	-0,634281	2,132949	0,953599	0,010470134
BAS GR Equity	-0,002545	-0,008408	0,090335	0,00816	-0,28956	1,098671	0,976158	0,250439793
INGA NA Equity	-0,002545	-0,008408	0,090335	0,00816	-0,28956	1,098671	0,976158	0,250439793
AIR FP Equity	0,008782	0,017172	0,124296	0,01545	-2,629877	16,36923	0,756876	1,67265E-09
DG FP Equity	0,007226	0,012049	0,074854	0,005603	-0,512328	4,284996	0,931869	0,0008303
DHL GR Equity	0,008912	0,004699	0,081339	0,006616	-0,280392	0,283106	0,982001	0,474152722
AD NA Equity	0,008714	0,006834	0,047213	0,002229	-0,226309	-0,857345	0,972315	0,117985473
ENI IM Equity	0,004322	0,004442	0,086226	0,007435	0,547509	3,155041	0,936494	0,002613869
ADS GR Equity	0,003161	0,012063	0,096025	0,009221	-0,215048	-0,075874	0,989553	0,866966266
NOKIA FH Equity	-0,007833	-0,003845	0,0935	0,008742	-0,616459	0,87576	0,973828	0,190825147
WKL NA Equity	0,01716	0,022939	0,051091	0,00261	-0,328966	0,674715	0,974973	0,167150709
IFX GR Equity	0,007735	0,025145	0,129602	0,016797	-0,450521	0,829678	0,979064	0,347557642
RI FP Equity	-0,002256	0,003641	0,05572	0,003105	-0,309528	0,590561	0,979067	0,282064691
SAF FP Equity	0,011007	0,020346	0,108641	0,011803	-2,423935	14,63716	0,781389	6,64109E-09
DB1 GR Equity	0,009609	0,005623	0,060397	0,003648	-0,186331	0,972182	0,977348	0,286946635
KER FP Equity	-0,006835	0,001576	0,089038	0,007928	0,089757	-0,813839	0,979906	0,312889727
RMS FP Equity	0,021024	0,024819	0,074315	0,005523	0,373759	-0,104422	0,98379	0,491919799
EL FP Equity	0,011582	0,015052	0,073242	0,005364	-0,545023	2,115329	0,965447	0,047716497

Figure 3 – Descriptive analysis of the stocks from 2019 to Dec. 2024

Among the stocks analyzed, SU FP Equity (Schneider Electric) and RMS FP Equity (Hermès) delivered the highest average monthly returns, with 2.18% and 2.10%, respectively, signaling strong performance for the time series chosen (2019-2024). On the opposite end, INGA NA Equity and BAS GR Equity recorded average monthly returns of approximately -0.25%, suggesting underperformance relative to the rest of the universe.
In most cases, the mean and median returns are relatively close, indicating symmetric return distributions without major skew from outliers. However, some stocks show notable discrepancies, BN FP Equity, for example, has a mean of 0.35% but a median of 1.52%, suggesting positively skewed episodes that were less frequent but had a strong impact.
Volatility varied widely across the sample. AIR FP Equity (Airbus) and IFX GR Equity (Infineon Technologies) were among the most volatile stocks, with monthly standard deviations of 12.42% and 12.96%, respectively. In contrast, AI FP Equity (L'Air Liquide) and BN FP Equity reported relatively low volatilities (below 5.5%), providing more stable return profiles. Stocks like SAF FP Equity and SAP strike a more balanced risk-return profile, combining moderate volatility with solid average returns.

From a skewness perspective, most stocks are negatively skewed, suggesting a tendency for downside risk. Notably, AIR FP Equity and SAF FP Equity reported strong negative skewness, at -2.63 and -2.42, respectively indicating more frequent extreme losses than gains. A few exceptions exist: ENI IM Equity and MC FP Equity exhibit positive skewness, suggesting they had more occasional large gains than losses.

The kurtosis statistics reveal how "fat-tailed" the return distributions are. Stocks such as AIR FP Equity (16.37) and SAF FP Equity (14.64) show extremely high kurtosis values, meaning they are more exposed to tail events or sharp jumps/drops. On the other hand, AD NA Equity and RMS FP Equity exhibited negative or near-zero kurtosis, indicating more normal-like return patterns with limited extreme moves.

The Shapiro-Wilk test results indicate that most return series deviate significantly from normality. Most p-values fall below the 0.05 threshold, meaning the null hypothesis of normality is rejected for those stocks. Exceptions include ADS GR Equity, DHL GR Equity, and RMS FP Equity, which pass the normality test with high p-values (>0.47), suggesting more normally distributed return behavior.

This statistical exploration underscores the variety in return characteristics across the equity universe. Some stocks like SU FP, SAP, and RMS FP offer strong returns with relatively moderate volatility, making them suitable for a core position in a portfolio. Others, such as AIR FP and IFX GR, while offering higher returns, come with elevated volatility and non-normality, and thus may be better suited for tactical or satellite exposure.
Together, these observations provide valuable inputs for portfolio construction and optimization, where return expectations must be balanced against risk and tail-event exposure.

5.1.2 Naïve Portfolio result:

As introduced in the previous section, the benchmark portfolios was constructed to assess performance if no option trading were effectuated.

For this benchmark, an equally-weighted investment strategy was adopted. This means that at the initial investment date (t = 0), an equal amount of capital was allocated to each stock (1 000 000/28). The performance evaluation period extends from January 1st, 2019, to December 1st, 2024, a period which includes exceptional events, notably the significant drop due to the COVID-19 crisis, allowing to measure later the positive/negative impact of this with the option included portfolio.

The equally weighted Portfolio produced the following performance results over the 6 years analyzed:

- Annualized Return: **13.26%**
- Annual Volatility (Standard Deviation): **18.11%**
- Sharpe Ratio: **0.7324**

These results indicate that for every unit of risk undertaken, the investor earned a return of 0.7324. While the average return of 13.26% annually is considered significant, a Sharpe Ratio being below 1 implies that the portfolio is relatively inefficient from a risk-adjusted return perspective. In general, a Sharpe Ratio below 1 is considered suboptimal (www.investopedia.com), particularly for diversified equity portfolios, as it suggests a less favorable balance between reward and volatility. (Bodie&al, 2023)

One crucial aspect of this analysis that can't be ignored, is the bias this benchmark has. The companies in which money is invested, are companies that are included today in the EUROSTOXX 50. It means that based on their historical good performances, it is nearly sure to benefit from investing in it, because on the 01/01/2005 they were not necessarily included in this benchmark.

5.2 Model Results:

The results of the model with integration of options gave the following results:

- Annualized Return: **21.30%**
- Annual Volatility (Standard Deviation): **30.69%**
- Sharpe Ratio: **0.6939**

The investment being 900 000 euros in stocks at the beginning of the simulation, and 100 000 in option, resulting in a total portfolio value of 1 000 000 euros at t = 0.
At the last period of our analysis (December 2024), the value of stocks is equal to 1 967 093 and the final payoff of the options is equal to 282 785. The final value of the portfolio is therefore 2 249 878 euros, compared to 1 967 093 euros for the benchmark.

The first results that stands out of this model is the significantly higher annualized return (>< 13,26% for the benchmark). This portfolio higher returns than stocks, but at what cost ? As we can see the standard deviation of the total portfolio value is also higher (>< 18,11%), therefore implying a lot more risk when investing in option.

These higher returns in options are also due to the ITM aspect of those same options. ITM options typically follow closely the price movements of the underlying stock, but still offer the asymmetric payoff structure of options limiting downside to the premium paid, while maintaining significant upside potential. Over time, and especially in rising markets, these characteristics result in amplified returns

In the figure below, transactions and results from 2019 to 2024:

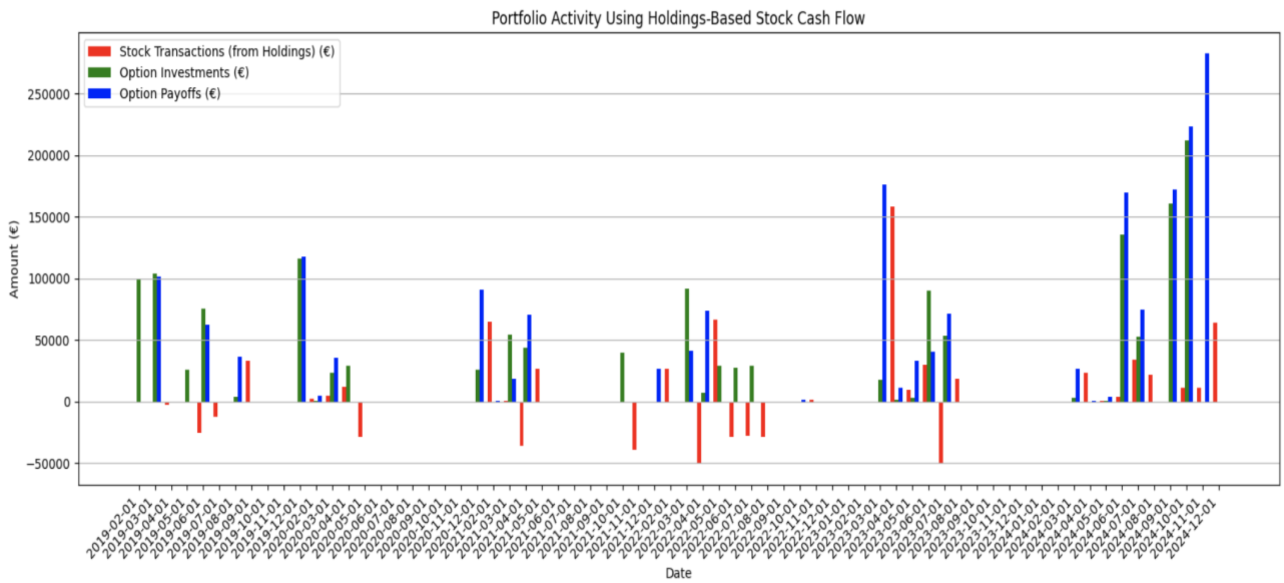

Figure 4: Transaction log of optimization model

As it can clearly be seen, the high annualized returns were driven by the huge payoff made from options in the last periods, therefore also driving up the volatility of the portfolio. On the last 3 periods of the analysis, options have generated payoffs equal to 678 000 euros, for an initial cost of 565 000 euros.

The total value invested in option equals 1 554 000 euros, and the total payoffs are 1 964 000. Resulting in a positive P&L of around 410 000 euros and a global return of 26%.

69,8% of the options bought were profitable (meaning the **profit** > costs), which underlines the rising market since the model was only allowed to buy options ITM or ATM. 59% of the options bought among the approved set formed based on historical returns, where option with maturity of 1 year.

This implies that due to their long-term profile, the model spots them as high quality option type since their ratio profit/volatility were among the top 6, and boosted considerably the returns of options.

One of the main point that needed to be verified was that the model tried as much as possible to keep 10% of total portfolio value in options.

Hereunder the graph of the percentage of the portfolio's value allocated to options.

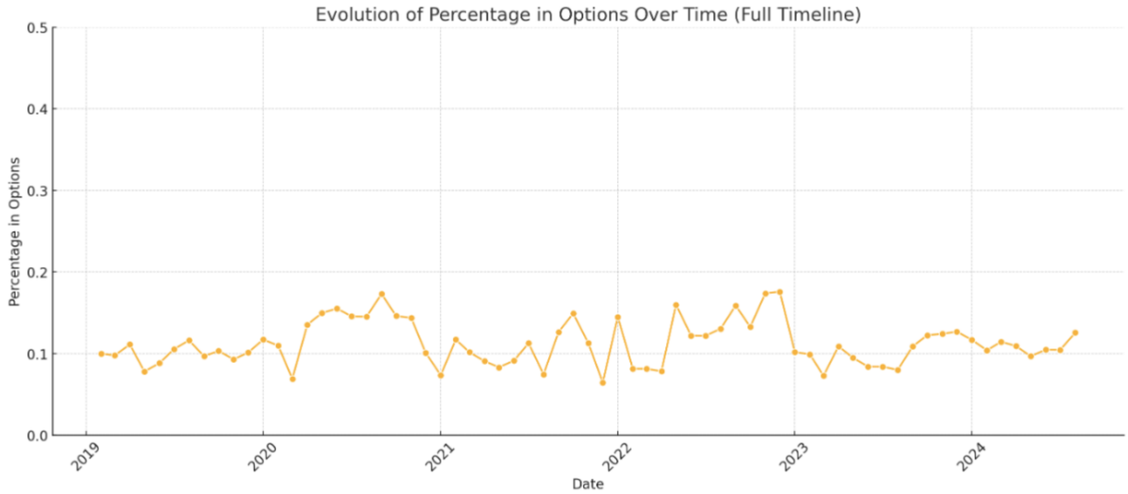

Figure 5: Evolution of the total options value held at each t in percentage

As displayed in the **Figure 5**, the total value of options oscillates around 10% of the global portfolio value (the graph includes the payoff made from option at each time t, since this money has not yet been spent neither on stock or option and will be invested in options directly if needed). Since the value of options is dynamic across each period t, several periods excess this 10% target. This is due to the option value increasing with the dynamic BS formula computation with the new maturity and the new moneyness.

For those periods, the model only re-invest in stocks as displayed in **Figure 4** (graph transaction), the model is forced to wait for some active options to arrive at maturity or for stocks to gain significant value since it wasn't allowed to sell options.

In conclusion, the main purpose of the model was to maximize profit, based on a decision variable that takes into account the historical (up to 1 year before) return of those same options type, in a market that follows the same trend as the years before, the returns were indeed enhanced through the integration of options, at the cost of volatility.

6. Limitations and Conclusion

6.1 Limitations

Despite the realistic investment constraints, and the clear logic of the simulation, several limitations emerge concerning the results obtained and the methodology used.

First, the metric used to select options (The best returns/volatility for the past year) does not clearly evaluate the same asset. It is indeed a relatable metric, because those assets do have lots of parameters in common such as the maturity, the underlying and the strike price and the option type, but this is not clearly the same asset.
Therefore the link that can be made between those two assets is limited, even if still profitable in our analysis. Maximizing the real Sharpe Ratio of the option (same asset) would have been a better alternative in terms of option picking.

The second important point of our analysis is the value estimation of the option across time. The model acknowledges a fixed implied volatility over the whole life of the option, which is not correct. When maturity and moneyness change, the option moves on the volatility surface and the model should have found the new implied volatility corresponding to this new moneyness and maturity. This would have given the correct implied volatility at time t for this specific option. However, allowing to re-evaluate the option with a fixed volatility is a fine compromise, that "only" under-estimates the option value with the delta between accurate Ivol and the fixed one.

Third point is the bias of the selected stocks, where a bias can be detected. Since the initial stocks choices for the whole analysis (2005 to 2024) were made based on the stocks present in the EUROSTOXX50 in 2024, it is logic that those stocks will gain value in the end, ensuring profit and strong performances beforehand. The ITS analysis conducted advising for call option was therefore more likely to be profitable, since those same stocks were still ranked in the top 50 european stocks, acknowledging a significant capitalization 6 years later.

Lastly, the rising market into which the analysis has been conducted doesn’t ensure the model to be profitable in period of important crisis. The timeframe did not allow to test the model in period of recession.

In sum, the model offers a clean and structured framework to simulate a 10% option strategy, but omit some aspect of option modelization, limiting therefore accuracy of their pricing across time. Some structural aspects of the period analyzed also contributed to boost the results. But with those limitations taken into account, the model still showed that moderate investment in option is profitable, in terms of returns.

6.2 Conclusion

The main purpose of this thesis was to measure the impact of options into a naive portfolio. The model developed through this idea was based on extensive computations, simulating more than 350 000 option costs and profits, allowing to spot a precise strategy from the ITS analysis and replicated in an OTS analysis through a mathematical model.

From this point, the research question can be answered: With the limitations mentioned, options have a positive impact on a naïve portfolio, and can be used to boost portfolio performances in terms of returns.

7. Appendices

7.1 Illustrative Tables

Table 1 – Stocks of the benchmark

Ticker	Company Name
MC FP Equity	LVMH Moët Hennessy Louis Vuitton SE
NDA FH Equity	Nordea Bank Abp
AI FP Equity	Air Liquide SA
BN FP Equity	Danone SA
SAP GR Equity	SAP SE
SU FP Equity	Schneider Electric SE
SAN FP Equity	Sanofi
IBE SM Equity	Iberdrola SA
SGO FP Equity	Compagnie de Saint-Gobain SA
BAYN GR Equity	Bayer AG
ABI BB Equity	Anheuser-Busch InBev SA/NV
BAS GR Equity	BASF SE
INGA NA Equity	ING Groep N.V.
AIR FP Equity	Airbus SE
DG FP Equity	Vinci SA
DHL GR Equity	Deutsche Post AG
AD NA Equity	Koninklijke Ahold Delhaize N.V.
ENI IM Equity	Eni S.p.A.
ADS GR Equity	adidas AG
NOKIA FH Equity	Nokia Oyj
WKL NA Equity	Wolters Kluwer N.V.
IFX GR Equity	Infineon Technologies AG
RI FP Equity	Pernod Ricard SA
SAF FP Equity	Safran SA
DB1 GR Equity	Deutsche Börse AG
KER FP Equity	Kering SA
RMS FP Equity	Hermès International SCA

Table 2 – Call historical profit from 2005 to 2018

Sheet	Strategy	Mean	Median	Std Dev
Profit_call_12mth	0,8	7,206437	2,468202	19,50551
Profit_call_12mth	0,9	5,107508	1,239923	16,76436
Profit_call_6mth	0,8	3,323014	1,115193	13,34036
Profit_call_12mth	0,95	4,63133	0,877376	15,97462
Profit_call_6mth	0,9	2,366212	0,591516	11,20591
Profit_call_3mth	0,8	1,572701	0,504042	9,729841
Profit_call_12mth	1	4,074289	0,478488	14,98755
Profit_call_6mth	0,95	2,043304	0,309195	10,45028
Profit_call_3mth	0,9	1,115034	0,272529	8,213804
Profit_call_30d	0,8	0,512768	0,18411	5,468871
Profit_call_30d	0,9	0,367304	0,118648	4,637914
Profit_call_3mth	0,95	0,921942	0,075921	7,538441
Profit_call_12mth	1,05	3,452157	0,071765	13,83117
Profit_call_30d	0,95	0,268955	0,00936	4,144553
Profit_call_30d	1,2	-0,02538	-0,01657	0,245319
Profit_call_6mth	1	1,65978	-0,03144	9,436643
Profit_call_30d	1,1	-0,03034	-0,07557	1,122862
Profit_call_3mth	1,2	0,065087	-0,09591	2,581331
Profit_call_3mth	1	0,695233	-0,17419	6,561757
Profit_call_30d	1	0,133565	-0,17719	3,191393
Profit_call_30d	1,05	0,004894	-0,2091	2,052647
Profit_call_12mth	1,1	2,845916	-0,2602	12,57909
Profit_call_3mth	1,05	0,457999	-0,27661	5,399368
Profit_call_6mth	1,05	1,259154	-0,27949	8,2132
Profit_call_3mth	1,1	0,248016	-0,29411	4,245639
Profit_call_6mth	1,2	0,473462	-0,29879	4,765298
Profit_call_6mth	1,1	0,913808	-0,35623	6,882265
Profit_call_12mth	1,2	2,234536	-0,374	11,05856

Table 3 – Put historical profit from 2005 to 2018

Sheet	Strategy	Mean	Median	Std Dev
Profit_put_30d	0,8	-0,053238	-0,028195	0,1479524
Profit_put_30d	0,9	-0,129828	-0,112454	0,7136669
Profit_put_30d	0,95	-0,228177	-0,28669	1,5575887
Profit_put_30d	1	-0,363567	-0,462279	2,6461282
Profit_put_30d	1,05	-0,492238	-0,340849	3,7526118
Profit_put_30d	1,1	-0,527474	-0,260869	4,4386999
Profit_put_30d	1,2	-0,591387	-0,227482	5,4267283
Profit_put_3mth	0,8	-0,214568	-0,159141	0,617882
Profit_put_3mth	0,9	-0,433779	-0,439854	1,5437305
Profit_put_3mth	0,95	-0,626871	-0,719911	2,5302107
Profit_put_3mth	1	-0,85358	-0,928845	3,7429874
Profit_put_3mth	1,05	-1,090814	-0,866619	4,9985248
Profit_put_3mth	1,1	-1,300797	-0,763136	6,1415467
Profit_put_3mth	1,2	-1,722182	-0,695183	8,5093345
Profit_put_6mth	0,8	-0,529825	-0,474672	1,2194455
Profit_put_6mth	0,9	-0,879029	-0,890234	2,5492149
Profit_put_6mth	0,95	-1,201937	-1,221449	3,5619014
Profit_put_6mth	1	-1,585461	-1,527494	4,7564689
Profit_put_6mth	1,05	-1,986087	-1,658732	6,0764036
Profit_put_6mth	1,1	-2,331433	-1,646872	7,4183364
Profit_put_6mth	1,2	-3,379377	-1,812395	10,849315
Profit_put_12mth	0,8	-1,256736	-1,080128	1,825449
Profit_put_12mth	0,9	-1,653727	-1,558132	3,7922812
Profit_put_12mth	0,95	-2,129905	-1,983476	4,8351389
Profit_put_12mth	1	-2,686946	-2,409928	6,0523355
Profit_put_12mth	1,05	-3,309078	-2,673758	7,4606549
Profit_put_12mth	1,1	-3,91532	-2,808249	9,0307338
Profit_put_12mth	1,2	-6,228636	-3,834596	12,616404

Table 4 – Options available for the OTS model

Sheet	Strategy	Mean	Median	Std Dev
Profit_call_12mth	0,8	7,206437	2,4682019	19,505507
Profit_call_12mth	0,9	5,1075081	1,2399228	16,764359
Profit_call_6mth	0,8	3,3230138	1,1151926	13,340361
Profit_call_12mth	0,95	4,6313301	0,8773758	15,974625
Profit_call_6mth	0,9	2,3662116	0,5915165	11,205907
Profit_call_3mth	0,8	1,5727006	0,5040417	9,7298414
Profit_call_12mth	1	4,0742889	0,478488	14,987555
Profit_call_6mth	0,95	2,0433042	0,3091952	10,45028
Profit_call_3mth	0,9	1,1150345	0,272529	8,2138035
Profit_call_30d	0,8	0,5127684	0,1841101	5,4688711
Profit_call_30d	0,9	0,3673039	0,1186483	4,6379144
Profit_call_3mth	0,95	0,9219421	0,0759208	7,5384412
Profit_call_12mth	1,05	3,4521572	0,0717647	13,831167
Profit_call_30d	0,95	0,2689548	0,0093603	4,1445532

7.2 Code

Code 1: Payoff computation

```
Sub ComputeRealizedOptionPayoffs_WithNAHandling()

    Dim wb As Workbook
    Set wb = ThisWorkbook

    Dim maturities As Variant
    maturities = Array( _
        Array("30d", 1), _
        Array("3mth", 3), _
        Array("6mth", 6), _
        Array("12mth", 12) _
    )

    Dim arrFactors As Variant
    arrFactors = Array(0.8, 0.9, 0.95, 1, 1.05, 1.1, 1.2)

    Dim iMat As Integer, iStock As Integer, iRow As Long, j As Integer
    Dim offset As Integer, colPrice As Integer, colStart As Integer
    Dim S0 As Double, ST As Double, K As Double
    Dim payoffCall As Double, payoffPut As Double
    Dim valStart As Variant, valEnd As Variant

    Dim priceSheet As Worksheet
    Set priceSheet = wb.Sheets("Prices")

    Application.DisplayAlerts = False

    For iMat = LBound(maturities) To UBound(maturities)

        Dim matLabel As String
        matLabel = maturities(iMat)(0)
        offset = maturities(iMat)(1)

        ' Remove old output sheets
```

```
        On Error Resume Next
        wb.Sheets("Realized_Call_" & matLabel).Delete
        wb.Sheets("Realized_Put_" & matLabel).Delete
        On Error GoTo 0

        ' Create new result sheets
        Dim wsCall As Worksheet, wsPut As Worksheet
        Set wsCall = wb.Sheets.Add
        Set wsPut = wb.Sheets.Add
        wsCall.Name = "Realized_Call_" & matLabel
        wsPut.Name = "Realized_Put_" & matLabel

        ' Loop over 29 stocks
        For iStock = 0 To 28

            colPrice = 4 + iStock ' Column D = 4 in VBA
            colStart = 5 + iStock * 10 ' Column E = 5 for stock 1

            ' Write column headers
            For j = 0 To 6
                wsCall.Cells(3, colStart + j).Value = "Strike " & arrFactors(j)
                wsPut.Cells(3, colStart + j).Value = "Strike " & arrFactors(j)
            Next j

            ' Loop over each row until data runs out
            For iRow = 4 To 237

                valStart = priceSheet.Cells(iRow, colPrice).Value
                valEnd = priceSheet.Cells(iRow + offset, colPrice).Value

                If IsError(valStart) Or IsError(valEnd) Or _
                   Not IsNumeric(valStart) Or Not IsNumeric(valEnd) Then

                    ' Write #N/A if data is invalid
                    For j = 0 To 6
                        wsCall.Cells(iRow, colStart + j).Value = CVErr(xlErrNA)
                        wsPut.Cells(iRow, colStart + j).Value = CVErr(xlErrNA)
                    Next j

                Else
                    S0 = CDbl(valStart)
                    ST = CDbl(valEnd)

                    For j = 0 To 6
                        K = arrFactors(j) * S0
                        payoffCall = Application.WorksheetFunction.Max(ST - K, 0)
                        payoffPut = Application.WorksheetFunction.Max(K - ST, 0)

                        wsCall.Cells(iRow, colStart + j).Value = payoffCall
                        wsPut.Cells(iRow, colStart + j).Value = payoffPut
                    Next j
                End If

            Next iRow

        Next iStock

    Next iMat

    Application.DisplayAlerts = True
    MsgBox "Realized payoffs with #N/A handling completed for all maturities."

End Sub


```

Code 2 Option Premium Computation

```
Sub BlackScholesPremiums_AllMaturities()

    Dim sheetData As Variant
    sheetData = Array( _
        Array("Ivol per month 30d", 1 / 12, "30d"), _
        Array("Ivol per month 3mth", 1 / 4, "3mth"), _
        Array("Ivol per month 6mth", 1 / 2, "6mth"), _
        Array("Ivol per month 12 mth", 1, "12 mth") _
    )

    Dim wsInput As Worksheet, wsCall As Worksheet, wsPut As Worksheet
    Dim iRow As Long, j As Integer, idx As Integer
    Dim S As Double, sigma As Double, K As Double
    Dim T As Double, r As Double
    Dim d1 As Double, d2 As Double
    Dim callPrice As Double, putPrice As Double
    Dim arrFactors As Variant
    arrFactors = Array(0.8, 0.9, 0.95, 1, 1.05, 1.1, 1.2)

    r = 0 ' Assuming risk-free rate is 0

    Application.DisplayAlerts = False

    For idx = LBound(sheetData) To UBound(sheetData)

        Dim sheetName As String: sheetName = sheetData(idx)(0)
        T = sheetData(idx)(1)
        Dim suffix As String: suffix = sheetData(idx)(2)

        Set wsInput = Worksheets(sheetName)

        ' Delete old output sheets
        On Error Resume Next
        Worksheets("call_" & suffix).Delete
        Worksheets("put_" & suffix).Delete
        On Error GoTo 0

        ' Create output sheets
        Set wsCall = Worksheets.Add
        wsCall.Name = "call_" & suffix

        Set wsPut = Worksheets.Add
        wsPut.Name = "put_" & suffix

        ' Copy headers (optional)
        For headerCol = 5 To 12
            wsCall.Cells(3, headerCol).Value = wsInput.Cells(3, headerCol).Value
            wsPut.Cells(3, headerCol).Value = wsInput.Cells(3, headerCol).Value
        Next headerCol

        Dim colVol As Integer, colUnderlying As Integer
        colVol = 5

        Do While colVol <= 285
            colUnderlying = colVol + 7

            For iRow = 4 To 237
                If IsNumeric(wsInput.Cells(iRow, colUnderlying).Value) Then
                    S = CDbl(wsInput.Cells(iRow, colUnderlying).Value)

                    For j = 0 To 6
                        If IsNumeric(wsInput.Cells(iRow, colVol + j).Value) Then
                            sigma = CDbl(wsInput.Cells(iRow, colVol + j).Value) * 0.01
```

```
                            K = S * arrFactors(j)

                            If sigma > 0 And S > 0 And K > 0 Then
                                d1 = (Log(S / K) + (r + 0.5 * sigma ^ 2) * T) / (sigma * Sqr(T))
                                d2 = d1 - sigma * Sqr(T)

                                callPrice = S * WorksheetFunction.NormSDist(d1) - K * Exp(-r * T) * WorksheetFunction.NormSDist(d2)
                                putPrice = K * Exp(-r * T) * WorksheetFunction.NormSDist(-d2) - S * WorksheetFunction.NormSDist(-d1)

                                wsCall.Cells(iRow, colVol + j).Value = callPrice
                                wsPut.Cells(iRow, colVol + j).Value = putPrice
                            Else
                                wsCall.Cells(iRow, colVol + j).Value = CVErr(xlErrNA)
                                wsPut.Cells(iRow, colVol + j).Value = CVErr(xlErrNA)
                            End If
                        Else
                            wsCall.Cells(iRow, colVol + j).Value = CVErr(xlErrNA)
                            wsPut.Cells(iRow, colVol + j).Value = CVErr(xlErrNA)
                        End If
                    Next j
                Else
                    For j = 0 To 6
                        wsCall.Cells(iRow, colVol + j).Value = CVErr(xlErrNA)
                        wsPut.Cells(iRow, colVol + j).Value = CVErr(xlErrNA)
                    Next j
                End If
            Next iRow

            colVol = colUnderlying + 3
        Loop

    Next idx

    Application.DisplayAlerts = True
    MsgBox "Option premiums calculated for all maturities (calls and puts)."

End Sub


```

Code 3: Model with 10% target of investment in options

```
import pandas as pd
import numpy as np
from scipy.stats import norm

# === Load and preprocess data ===
options_file = "Option-Final file.xlsx"
underlying_file = "Prix sous jacent.xlsx"

# Load options
options_df = pd.read_excel(options_file, header=0)
options_df.columns = options_df.columns.str.strip().str.lower()
options_df['date'] = pd.to_datetime(options_df['date'], dayfirst=True)
options_df['option_type'] = options_df['option_type'].str.lower()
options_df['stock_id'] = options_df['stock_id'].astype(int)

# Load underlying prices
underlying_df = pd.read_excel(underlying_file, header=2)
underlying_df.rename(columns={underlying_df.columns[0]: 'date'}, inplace=True)
```

```python
underlying_df['date'] = pd.to_datetime(underlying_df['date'], dayfirst=True)
underlying_df.set_index('date', inplace=True)
underlying_df = underlying_df.apply(pd.to_numeric, errors='coerce').dropna()
underlying_df.sort_index(inplace=True)

# Create stock_id to ticker mapping
stock_id_to_ticker = list(underlying_df.columns)

# Filter to simulation dates only
start_date = pd.Timestamp("2019-01-01")
if (underlying_df.index >= start_date).any():
    underlying_df = underlying_df[underlying_df.index >= start_date]
else:
    start_date = underlying_df.index[0]

options_df = options_df[options_df['date'].isin(underlying_df.index)]

# Approved options
approved_options = [
    ('call', 1, 0.95), ('call', 12, 1.05), ('call', 3, 0.95),
    ('call', 1, 0.9), ('call', 1, 0.8), ('call', 3, 0.9),
    ('call', 6, 0.95), ('call', 12, 1.0), ('call', 3, 0.8),
    ('call', 6, 0.9), ('call', 12, 0.95), ('call', 12, 0.9),
    ('call', 6, 0.8), ('call', 12, 0.8)
]
approved_set = set(approved_options)

# === BS Formula ===
def black_scholes_call_price(S, K, T, sigma):
    if T <= 0:
        return max(S - K, 0)
    d1 = (np.log(S / K) + 0.5 * sigma ** 2 * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    return S * norm.cdf(d1) - K * norm.cdf(d2)

# === Initialize portfolio ===
initial_portfolio_value = 1_000_000
initial_date = underlying_df.index[0]
initial_prices = underlying_df.loc[initial_date]
stock_ids = underlying_df.columns.tolist()
stock_holdings = (0.90 * initial_portfolio_value / len(stock_ids)) / initial_prices.values

option_positions = []
portfolio_values = []
transactions = []
stock_holdings_log = []
option_valuations_log = []
live_options_tracking_log = []

# === Initial Option Purchase ===
option_budget = 0.10 * initial_portfolio_value
available_options = options_df[options_df['date'] == initial_date]
eligible = available_options[
    (available_options['option_type'] == 'call') &
    (available_options[['option_type', 'maturity', 'strategy']].apply(
        lambda row: tuple(row) in approved_set, axis=1)) &
    (available_options['sharpe'].notna())
]
top_options = eligible.sort_values('sharpe', ascending=False).head(6)

if not top_options.empty:
    per_investment = option_budget / len(top_options)
    for _, row in top_options.iterrows():
        stock_id = int(row['stock_id'])
        ticker = stock_id_to_ticker[stock_id - 1]
        S = initial_prices[ticker]
        strike = row['strategy'] * S
        quantity = per_investment / row['cost']
```

```
        expiry_date = row['date'] + pd.DateOffset(months=row['maturity'])
        option_positions.append({
            **row,
            'quantity': quantity,
            'expiry': expiry_date,
            'strike': strike,
            'ivol': row['ivol'],
            'date_bought': row['date']
        })
        transactions.append({
            'date': row['date'],
            'action': 'buy_option',
            'stock_id': stock_id,
            'strategy': row['strategy'],
            'maturity': row['maturity'],
            'option_type': row['option_type'],
            'cost': row['cost'],
            'quantity': quantity,
            'realised_profit': row['profit'] * quantity,
            'expiry': expiry_date
        })

# === Simulation Loop ===
for current_date in underlying_df.index:
    current_prices = underlying_df.loc[current_date]
    stock_value = np.sum(stock_holdings * current_prices.values)

    for sid, qty in enumerate(stock_holdings):
        stock_holdings_log.append({
            'date': current_date,
            'stock_id': sid + 1,
            'quantity': qty,
            'price': current_prices.iloc[sid],
            'value': qty * current_prices.iloc[sid]
        })

    realised_option_pnl = 0
    new_positions = []
    for pos in option_positions:
        if pos['expiry'] == current_date:
            realised_option_pnl += pos['profit'] * pos['quantity']
        else:
            new_positions.append(pos)
    option_positions = new_positions

    active_option_cost = 0
    updated_positions = []
    for pos in option_positions:
        if pos['date'] <= current_date < pos['expiry']:
            stock_id = int(pos['stock_id'])
            ticker = stock_id_to_ticker[stock_id - 1]
            S = current_prices[ticker]
            K = pos['strike']
            sigma = pos['ivol'] / 100

            T_months = (pos['expiry'].to_period("M") - current_date.to_period("M")).n
            T_years = T_months / 12

            if T_years <= 0:
                continue

            option_value = black_scholes_call_price(S, K, T_years, sigma)
            position_value = option_value * pos['quantity']
            active_option_cost += position_value

            option_valuations_log.append({
                'date': current_date,
                'stock_id': stock_id,
```

```
                'strike': K,
                'ivol': sigma,
                'underlying_price': S,
                'time_to_maturity': T_years,
                'bs_value': option_value,
                'quantity': pos['quantity'],
                'total_value': position_value,
                'expiry': pos['expiry'],
                'strategy': pos['strategy'],
                'option_type': pos['option_type']
            })

            live_options_tracking_log.append({
                'date_bought': pos['date_bought'],
                'eval_date': current_date,
                'stock_id': stock_id,
                'strategy': pos['strategy'],
                'strike': K,
                'ivol': sigma,
                'maturity': T_months,
                'cost': pos['cost'],
                'current_value': option_value,
                'underlying_price': S
            })

            updated_positions.append(pos)

    option_positions = updated_positions
    portfolio_value = stock_value + active_option_cost + realised_option_pnl

    option_allocation = active_option_cost / portfolio_value
    target_option_value = 0.10 * portfolio_value
    option_investment_needed = target_option_value - active_option_cost

    stock_sale_record = {}
    option_buy_records = []
    stock_buy_record = []

    if option_allocation > 0.12:
        option_cash = 0
        stock_cash = realised_option_pnl
    elif 0.08 <= option_allocation <= 0.12:
        option_cash = 0.10 * realised_option_pnl
        stock_cash = realised_option_pnl - option_cash
    else:
        option_cash = min(option_investment_needed, realised_option_pnl)
        remaining_needed = option_investment_needed - option_cash
        stock_cash = realised_option_pnl - option_cash

        if remaining_needed > 0:
            sell_fraction = remaining_needed / stock_value
            for idx in range(len(stock_holdings)):
                amount_sold = stock_holdings[idx] * sell_fraction
                stock_sale_record[idx + 1] = amount_sold
                stock_holdings[idx] -= amount_sold
            option_cash += remaining_needed

    if option_cash > 0:
        available_options = options_df[options_df['date'] == current_date]
        eligible = available_options[
            (available_options['option_type'] == 'call') &
            (available_options[['option_type', 'maturity', 'strategy']].apply(
                lambda row: tuple(row) in approved_set, axis=1)) &
            (available_options['sharpe'].notna())
        ]
        top_options = eligible.sort_values('sharpe', ascending=False).head(6)

        if not top_options.empty:
```

```
            per_investment = option_cash / len(top_options)
            for _, row in top_options.iterrows():
                stock_id = int(row['stock_id'])
                ticker = stock_id_to_ticker[stock_id - 1]
                S = current_prices[ticker]
                strike = row['strategy'] * S
                quantity = per_investment / row['cost']
                expiry_date = row['date'] + pd.DateOffset(months=row['maturity'])
                option_positions.append({
                    **row,
                    'quantity': quantity,
                    'expiry': expiry_date,
                    'strike': strike,
                    'ivol': row['ivol'],
                    'date_bought': row['date']
                })
                option_buy_records.append({
                    'stock_id': stock_id,
                    'strategy': row['strategy'],
                    'maturity': row['maturity'],
                    'option_type': row['option_type'],
                    'cost': row['cost'],
                    'quantity': quantity,
                    'realised_profit': row['profit'] * quantity,
                    'expiry': expiry_date
                })

    if stock_cash > 0:
        per_stock_cash = stock_cash / len(stock_ids)
        for i in range(len(stock_holdings)):
            additional_qty = per_stock_cash / current_prices[i]
            stock_holdings[i] += additional_qty
            stock_buy_record.append({
                'date': current_date,
                'action': 'buy_stock',
                'stock_id': i + 1,
                'price': current_prices[i],
                'quantity': additional_qty,
                'amount': per_stock_cash
            })

    stock_value = np.sum(stock_holdings * current_prices.values)
    portfolio_value = stock_value + active_option_cost + realised_option_pnl
    portfolio_values.append({
        'date': current_date,
        'portfolio_value': portfolio_value,
        'stock_value': stock_value,
        'options_value': active_option_cost,
        'realised_pnl': realised_option_pnl
    })

    for sid, amount in stock_sale_record.items():
        transactions.append({
            'date': current_date,
            'action': 'sell_stock',
            'stock_id': sid,
            'amount': amount
        })
    for opt in option_buy_records:
        transactions.append({
            'date': current_date,
            'action': 'buy_option',
            **opt
        })
    for sb in stock_buy_record:
        transactions.append(sb)

# === Export ===
```

```python
portfolio_df = pd.DataFrame(portfolio_values)
transactions_df = pd.DataFrame(transactions)
stock_holdings_df = pd.DataFrame(stock_holdings_log)
option_valuations_df = pd.DataFrame(option_valuations_log)
live_tracking_df = pd.DataFrame(live_options_tracking_log)

with pd.ExcelWriter("Results.xlsx", engine="openpyxl") as writer:
    portfolio_df.to_excel(writer, sheet_name="Portfolio Values", index=False)
    transactions_df.to_excel(writer, sheet_name="Transactions Log", index=False)
    stock_holdings_df.to_excel(writer, sheet_name="Stock Holdings", index=False)
    option_valuations_df.to_excel(writer, sheet_name="Option Valuations", index=False)
    live_tracking_df.to_excel(writer, sheet_name="Live Option Tracking", index=False)

print("Portfolio simulation complete. Results saved to 'portfolio_call_top6sharpe_valued.xlsx'")
```

8. References

- Anderson, R. M., Bianchi, S. W., & Goldberg, L. R. (2012). Will my risk parity strategy outperform? *Financial Analysts Journal, 68*(6), 75–93. https://www.jstor.org/stable/41714296

- Black, F., & Litterman, R. (1992). Global Portfolio Optimization. *Financial Analysts Journal, 48*(5), 28–43. https://www.jstor.org/stable/4479577

- Bodie, Z., Kane, A., & Marcus, A. J. (2023). *Investments* (13th ed.). McGraw-Hill Education.

- Bookstaber, R. M., & Clarke, R. G. (1985). Problems in evaluating the performance of portfolio models. *Financial Analysts Journal, 41*(1), 48–55. https://doi.org/10.2469/coxfaj.v41.n1.48

- Cheng, P. (2000). Cheng, P., & Liang, Y. (2000). Optimal diversification: Is it really worthwhile? *The Journal of Real Estate Portfolio Management, 6*(1), 7–16. https://www.jstor.org/stable/44154425

- Chow, G. (1995). Portfolio selection based on return, risk, and relative performance. *Financial Analysts Journal, 51*(2), 54–60. https://www.jstor.org/stable/4479831

- Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics, 7*(3), 229–263. https://doi.org/10.1016/0304-405X(79)90015-1

- DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *The Review of Financial Studies, 22*(5), 1915–1953. https://www.jstor.org/stable/30226017

- Fama, E. F., & French, K. R. (2004). The Capital Asset Pricing Model: Theory and evidence. *Journal of Economic Perspectives, 18*(3), 25–46. https://www.jstor.org/stable/3216805

- Isakov, D., & Morard, B. (2001). Improving Portfolio Performance with Option Strategies : Evidence from Switzerland. *European Financial Management, 7 (1),* 73-91. https://doi.org/10.1111/1468-036x.00145

- Jorion, P. (1992). Portfolio optimization in practice. *Financial Analysts Journal, 48*(1), 68–74. https://www.jstor.org/stable/4479507

- Longstaff, F. A., & Schwartz, E. S. (2001). Valuing American options by simulation: A simple least-squares approach. *The Review of Financial Studies, 14*(1), 113–147. https://doi.org/10.1093/rfs/14.1.113

- Markowitz, H. (1952). Portfolio selection. *The Journal of Finance, 7*(1), 77–91. https://doi.org/10.2307/2975974

- Merton, R. C. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science, 4*(1), 141–183. https://doi.org/10.2307/3003143

- Michaud, R. O. (1989). The Markowitz Optimization Enigma: Is Optimized Optimal? *Financial Analysts Journal, 45*(1), 31–42. https://www.jstor.org/stable/4479185

- Rachev, S. T., & Mittnik, S. (2000). *Financial Modelling with Lévy Processes*. John Wiley & Sons.

- Treynor, J., & Black, F. (1973). How to Use Security Analysis to Improve Portfolio Selection. *The Journal of Business, 46*(1), 66–86. https://www.jstor.org/stable/2351280

- Yuan, S., & Rieger, M. O. (2020). Diversification with options and structured products. *Review of Derivatives Research, 24*, 55–77. https://doi.org/10.1007/s11147-020-09169-x

Abstract:

Modern portfolio theory was introduced in 1952 by H. Markowitz. The objective of the Nobel prize winner was to create an optimized portfolio which maximized the expected return while maintaining a maximum level of risk.
Although Markowitz's theory might be the starting point of mean-variance optimal portfolio calculator, the model has faced criticism. With the rising popularity of derivatives, the Markovitz Optimizer does not reflect investor strategies. Therefore, forcing portfolio optimizer to become more and more complex, this work aims at measuring the impact of options, and in what way(s) can the returns be improved through them.

UNIVERSITÉ CATHOLIQUE DE LOUVAIN
Louvain School of Management
Place des Doyens, 1 bte L2.01.01, 1348 Louvain-la-Neuve
Boulevard Emile Devreux 6, 6000 Charleroi, Belgique
Chaussée de Binche 151, 7000 Mons, Belgique
www.uclouvain.be/lsm