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How to measure interconnectedness in the financial system using Graph Theory? Review and application of the indicators of interconnectedness for the cross-border exposures between 2011 and 2020.

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This thesis represents the completion of my master’s degree in business engineering. I decided to combine the knowledge acquired in the Graph Theory course of my bachelor’s degree with my interest in International Finance.

This was a challenging task, however my continuous curiosity in this topic brought me here.

I would like to thank my relatives for their support and the Professor Olieslagers who gave me precious guidance all along the way.

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LIST OF ABBREVIATIONS

BCBS	Basel Committee on Banking Supervision
BIS	Bank for International Settlements
CBS	Consolidated Banking Statistics
CDS	Credit Default Swap
CGFS	Committee on the Global Financial System
CRD	Capital Requirements Directive
CRR	Capital Requirements Regulation
ECB	European Central Bank
EU	European Union
EVS	Espinosa-Vega and Solé
FSAP	Financial Sector Assessment Program
FSB	Financial Stability Board
GFC	Global Financial Crisis
G-SIBs	Global Systemically Important Banks
HHI	Herfindahl-Hirschman Index
IMF	International Monetary Fund
IMF FSIs	International Monetary Fund Financial Soundness Indicators
IRB	Immediate Risk Basis
LBS	Locational Banking Statistics
LCR	Liquidity Coverage Ratio
NSFR	Net Stable Funding Ratio
OTC	Over the Counter
RWA	Risk-Weighted Assets
SIFIs	Systemically Important Financial Institutions
URB	Ultimate Risk Basis
USD	United States Dollars

INTRODUCTION

The contraction of cross-border bank lending activities following the collapse of the investing bank Lehman Brothers has highlighted the multiple connections that exist between financial institutions (Haas & Horen, 2012). It has challenged the understanding of financial actors and banking supervisors of the risks arising from these connections. So far, the International Monetary Fund (2010) calls "the number of bilateral links between economies, as share of all possible bilateral links" *interconnectedness*. But creating interconnectedness with other financial institutions has always been a challenge. Indeed, there is a trade-off for financial institutions to create links with others. On one hand, it creates diversification of risk exposures (Glasserman & Young, 2016). On the other hand, higher connections facilitate the propagation of losses in case of a shock originating from an institution (Cabrales et al., 2017). To cope with this trade-off, banks have developed an increasingly complex and fragmented set of connections that complicates the monitoring of these interconnections.

To understand the complex formation of links between banks and the systemic risk it generates, the need for a global view of the financial system has become crucial for supervisors (International Monetary Fund, 2010). Consequently, the literature has investigated the opportunities of mapping the interconnections to improve our understanding of interconnectedness.

To uncover these links, network and graph theory appears as a solid tool. In fact, Allen and Babus (2008) argue that network representation captures the complex structure of interconnections between financial networks. Namely, it can identify the financial institutions that would be key drivers of propagation of a shock (Alves et al., 2013). Even banking supervisors try to include a network perspective, but the measures used to identify the Global systemically important banks, or G-SIBs, only account for the magnitude of the interconnections. The definition of interconnectedness lacks considering the key role of the structure of the network as a potential source of systemic risk. It is with the aim of reinforcing the benefits of graph theory for macro-economic supervision and to provide some thoughts on the assessment of interconnectedness that this thesis addresses the following question: **How to measure interconnectedness in the financial system using Graph Theory?**

This thesis reviews the measures of interconnectedness proposed in the literature and compares them to reveal the different dimensions of interconnectedness. To answer the question, the thesis will be conducted in four steps. The first part reviews how the literature on graph theory considers interconnectedness for a financial network. It also examines how the banking regulation defines interconnectedness as well as its usage in macro-economic supervision.

The second part justifies the methodology adopted to answer the research question. The main challenge was to find available granulated data on the interconnections between banks. To address this challenge, the research focusses on reliable data series. Therefore, the data sources are international banking statistics compiled by central banks and the national authorities. These data sources allow to create a financial network using graph theory. The financial network analysed accounts for the cross-border claims of 17 countries between 2011 and 2020. The second part also outlines the appropriate measures that will be assessed in the empirical research. Finally, it explains the contagion model simulated on the network and the statistical tests applied in the fourth part.

In the third part, the empirical research exploits the appropriate measures of graph theory to reveal features of the network and provide opportunities to identify the key players in a financial system. The research consists first of some summary statistics and a mapping of the network. Second, it assesses the measures derived from the structure of the graph. Third, it uses the contagion model to simulate shocks onto the network.

Eventually, independence tests are performed in the fourth part to compare the different measures and reveal similarities between them. The complete research redefines the concept of interconnectedness. Before proposing some recommendations to the policy makers, the thesis presents its own limits. Namely, the lack of granular data on interbank market reduces the interpretability of the results. The purpose of this thesis is primarily to illustrate and regroup the techniques of graph theory to model interconnectedness. Of course, it would be unrealistic to pretend to go beyond the research made by the banking supervisors. But I am confident that this work contributes to the understanding of interconnectedness in the financial system and help in the macro-prudential regulatory framework.

PART I - LITERATURE REVIEW

The literature review aims to gather information on the use of graph theory to measure the level of interconnectedness in the global financial system. To this end, the first chapter presents the graph theory and how this theory can be applied to a financial system. In the second chapter, the reader will find a review of the different measures of interconnectedness adapted to the financial network. Then, the chapter three states the approach of bank regulation to cope with interconnectedness between financial institutions.

Chapter 1 - Model interconnectedness with graph theory

The issue to be addresses is to capture the dimensions of interconnectedness. Let us look at what we already know from the primary definition. The Cambridge dictionary defines interconnectedness as “the state of having different parts or things connected or related to each other” (Cambridge University Press, n.d.). In our world, there are different systems presenting interconnectedness. For example, the brain is made up of tons of neurons connected by synapses, the friendship connects people together or the Web where the pages are connected through hyperlinks. For all these systems or networks, the graph theory enables to map the different connections.

1.1 Graph theory

In graph theory, the interconnections between participants can be represented by a mathematical object, a graph. In the course of Kunegis (2017), a graph G is defined as a triplet (V, E, Ψ) such that:

- $V(G)$ is a list of vertices (or nodes or participants)
- $E(G)$ is a list of edges (or links or connections)
- Ψ is an incidence function that associates each edge with a vertex or a pair of vertices.

The whole structure of a graph is represented thanks to these three elements. The Figure 1 gives a mapping of a simple graph with $V(G) = \{u, v, \dots, y\}$ and $E(G) = \{a, b, c, \dots, g\}$ and the incidence function is given by $\Psi\_G(a) = uv$ and many others.

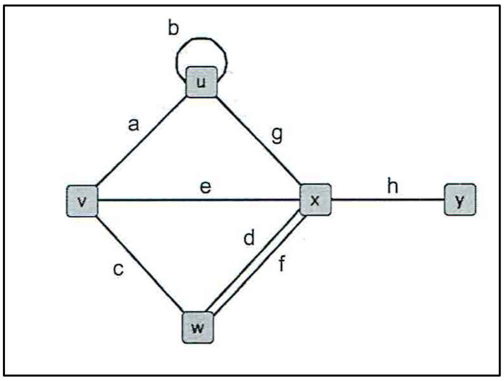

Figure 1: Representation of a simple graph (Kunegis, 2017).

Starting with this mathematical object, it is possible to characterise the network and establish some properties. The edges can have a direction or not. If applicable, we will say that the graph or network is directed. In the node list $V(G)$, each node can have attributes that describe it. Also, network analysis can characterize edges by their size or weight, introducing a weighted function. The weighted function of a graph (V, E, Ψ) is a function from $E(G)$ to Real which associates a real number to every edge. The weights can be retrieved as an attribute in the edge list $E(G)$ (Kunegis, 2017).

This mathematical object is widely studied in chemistry, social sciences, mathematics, and computer science. For instance, social sciences were concerned about the spread of a rumour, wondering how connections between people affect the transmission of information (Christakis & Fowler, 2009). The mapping of the graph of interconnections can illustrate how people learn, gather news and also how a disease spreads (Newman, 2010). For this purpose, social sciences researchers work on the identification of the more central or important nodes in a graph. To define the node centrality, Freeman (1978) distinguishes three common centrality measures for the vertices: degree, closeness and betweenness. These measures capture different aspects of the nodes. In his book, Newman (2010) gives a comprehensive definition and application for the principal measures. The next lines present these measures and the related formulas.

To define these measures, we need the concept of adjacency matrix used in the formulas. The adjacency matrix is a matrix where each element will indicate if there is a link between two nodes. In mathematical terms, the adjacency matrix of a graph G is the squared matrix A_{ij} of dimension the number of vertices times the number of vertices where the element ij is the number of edges (or links) between the vertex i and the vertex j.

1.1.1 Three principal measures of centrality in a graph (Newman, 2010)

- The degree of a vertex, k_i, is the number of edges incident to this vertex. In a directed graph, we can evaluate the in-degree and the out-degree. The in-degree stands for the number of edges ending on this vertex (or number of times the vertex is in second position in the incidence function). The out-degree stands for the number of edges starting from this vertex (or number of times the vertex is the first element in the incidence function).

$$k_i = \sum_j A_{ij}$$

The eigenvector centrality x_i^E extends the degree centrality by considering the importance of neighbours. A vertex is important in the eigenvector centrality sense if this vertex is connected to vertices with a high degree. Bonacich (1987) defines it as

$$x_i^E = \frac{1}{\lambda} \sum_j A_{ij} x_j$$

where λ is a constant (x_i is an eigenvector of A_{ij} and λ is its associated value)[1]. For directed graph, the PageRank measure x_i^P is more suitable than eigenvector to identify the most important vertices. The difference with eigenvector is that PageRank lowers the effect of having a lot of out-edges. The PageRank of a vertex is defined as

$$x_i^P = \sum_j \frac{A_{ij}}{k_j^{out}} x_j$$

where k_j^{out} is the out-degree of the vertex j.

- The betweenness of a vertex i, x_i^B , is the proportion of smallest paths (a path is a sequence of distinct vertices) between any vertices where the vertex i is included. A vertex is central if it tends to be close to others. The betweenness is defined as

[1] See https://textbooks.math.gatech.edu/ila/1553/eigenvectors.html for more details on eigenvector and eigenvalues.

$$x_i^B = \sum_{j \neq k \neq i} \frac{\sigma(j,k,i)}{\sigma(j,k)}$$

where $\sigma(j,k)$ is the number of smallest paths between j and k.

- The closeness x_i^C measures the mean distance (the number of vertices to pass to attain another vertex on the shortest path) between a vertex and all other vertices of the graph. The closeness is defined as

$$x_i^C = \frac{1}{N-1} \sum_{j \neq i} \frac{1}{d(i,j)}$$

where $d(i,j)$ is the distance between node i and node j.

1.2 Graph theory in finance

The graph theory applied to finance is simply the mapping of the interconnections between financial institutions (Allen & Babus, 2008). The mapping provides a mathematical object, a graph or network from which several features can emerge. Research explores various types of financial networks. In the following section, the participants and possible interconnections identified by the literature are detailed and reviewed.

1.2.1 Vertices

In a financial network, the vertices represent generally financial institutions. The European Regulation No 575/2013 of Capital Requirement Directive and Regulation defines a financial institution as “an undertaking of which the principal activity is to acquire holdings or pursue one or more activities” like lending and deposits, payment services, guarantees and commitments, investing and trading in money market instrument, foreign exchange, and others (Official Journal of the European Union, 2013). Under this definition, a financial institution could be a commercial bank, an investment bank, an insurance company, a hedge fund, and many more. To understand better the participants of financial network, we can decompose the balance sheet structure of a bank. Glasserman and Young (2016) decompose the asset side of a bank in in-network assets and outside assets as illustrated in Figure 2. Outside assets refer to common assets and claims on non-financial institutions. It could be mortgages or commercial loans. In-network assets are claims on other banks such as interbank loans and exposures through derivatives.

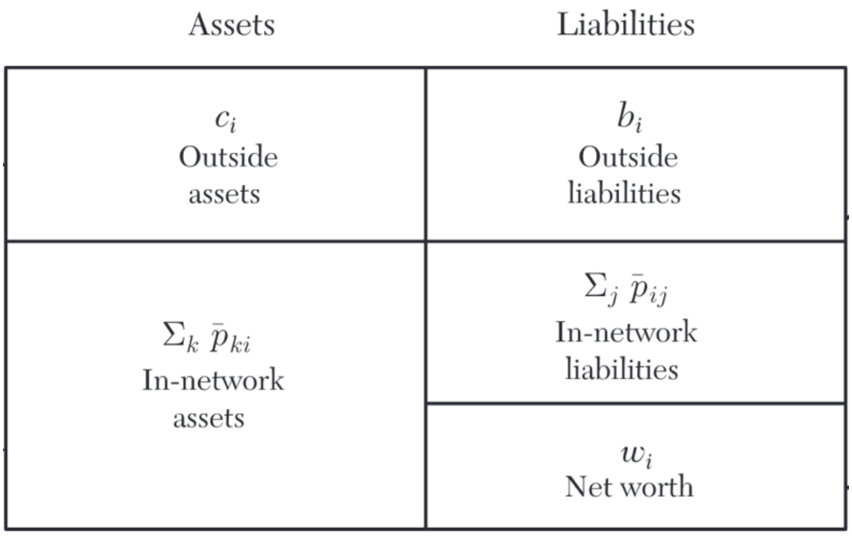

Figure 2: Balance sheet structure of a bank (Glasserman & Young, 2016).

1.2.2 Edges

Beside the vertices, the edges are the second component of a graph. The edges (or links) are determined by the incidence function that associates two financial institutions i and j in the financial network. Previous literature regroups network links in two types: direct linkages and indirect linkages. The direct linkage approach maps exposures on the balance sheet of the institutions like interbank loans and sometimes off-balance sheet exposures like derivatives. The indirect approach reconstructs network linkages from interconnections on the market (Kara et al., 2015).

For the direct links, Alves et al. (2013) identifies three different sources of interbank connections: credit claims which contains interbank deposits and repurchase agreements, debt securities and other securities. These connections are retrieved in the “in-network assets” of the balance sheet (see Figure 2). Besides the balance sheet, banks may have many links with other financial institutions: derivatives (like CDS), guarantees extended, credit commitments. Because of the confidentiality of the data, literature gives interest in approaches estimating the direct interconnections in the financial system (Anand et al., 2018). Researchers estimate direct bilateral links through maximum entropy using total asset and liabilities of balance sheet (Kara et al., 2015).

The confidentiality of the data also leads to the use of other data sources. The indirect links gains increasing interest as it may be potential channels of contagion (Roncoroni et al., 2019). These approaches reconstruct a network by leveraging market data. Billio et al. (2012) and Diebold and Yilmaz (2015) use equity returns to construct a financial network of interconnections. Bricco and Xu (2019) also identify similar portfolio holdings. To generate a link from these market data sets, researchers use "economic analytical techniques to estimate empirical connections" (Kara et al., 2015). Techniques can be Principal-Component Analysis, Variance Decomposition or Granger-causality for the returns on hedge funds or bank insurances. Starting with the portfolio holdings, Bricco and Xu (2019) identify three sources of information where indirect links can be created: information spill over, Mark-to-market losses (through fire sales) and market perception of counterparties credit worthiness. The Mind Map in Figure 3 illustrates the different types of interconnections described before.

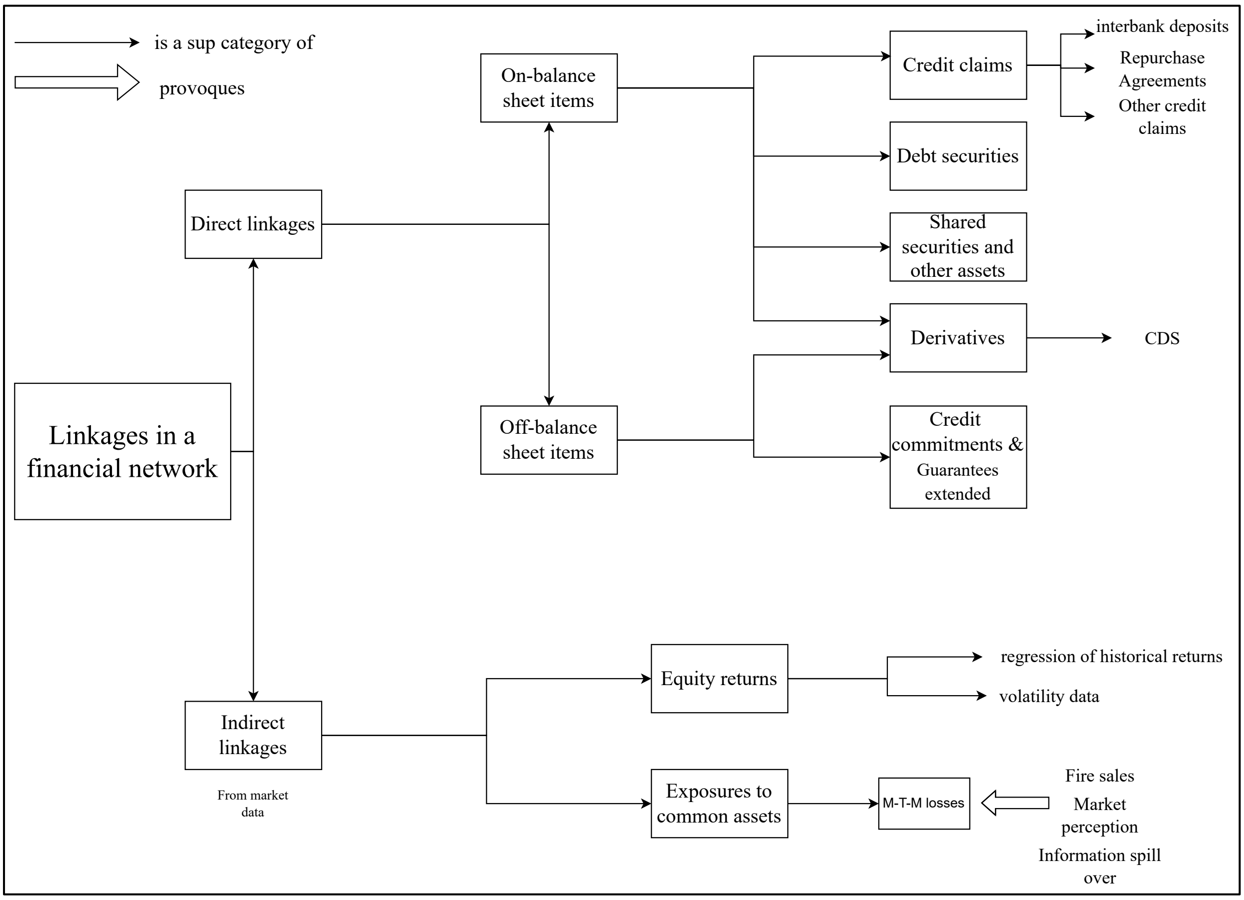

Figure 3: Mind Map of the different links between financial institutions.

Thus, the range of linkages (or edges) of the financial network is disparate. Recently, researchers develop a comprehensive structure of links between financial institutions. Aldasoro and Alves (2018) use multiplex interbank networks to model financial interconnectedness. They consider the network as a set of multiple types of instruments: credit claims, debt securities, derivatives, off-balance sheet, and other links.

Moreover, literature investigates principally three levels of interconnections for the financial institutions (Bricco & Xu, 2019). First, at the level of the country, for example, Degryse and Nguyen (2004) analyse bilateral interbank exposures in the Belgium banking system. Second, Peltonen et al. (2019) show that it is also relevant to look at the cross-sector interconnections. In that perspective, the Bank for International Settlements provides statistics on financial institutions divided in different sectors: the private banks, investment banks but also hedge funds as part of non-bank financial sector (BIS Monetary and Economic Department, 2015). Third, financial interconnections can be analysed at the international level highlighting the exposures consolidated for countries. For instance, Minoiu and Reyes (2013) investigate the "global banking network using data on cross-border banking flows". Besides, Gorton and Metrick (2010) analyse cross-border links between banks and non-bank financial institutions. They demonstrate that the total amount of money interchanged on this market increased considerably in the 2010's, and even more than the total cross-border claims market. Also, central counterparties became a new central player in the network of financial transactions during the global financial crisis (Gorton & Metrick, 2010) (Claessens et al., 2012).

To wrap it all up, we have seen that graph theory models the possible interconnections between participants. In finance, there are various types of interconnections between financial institutions that constitute interconnectedness. From these interconnections such as interbank market exposures, it is possible to build a graph. The next chapter reviews the features that can emerge from such graph.

Chapter 2 – Measures of interconnectedness

Based on the network mapping, a detailed analysis of interconnectedness can emerge. Literature investigates two types of analysis. First, static analysis describes the structural properties of a graph (Alves et al., 2013). Second, dynamic analysis introduces a disruptive shock in the network to understand the mechanisms of transmission (Alves et al., 2013). The financial network analysed in this thesis is the cross-border direct exposures network consolidated at the level of countries. In this network, the vertices represent countries, and a link (or edge) is the connection between two countries. The connections get a value determined by the amount of outstanding claims from one country on another. The data series supporting the graph are described in the methodology part. This chapter provides the reader with the most reliable measures of interconnectedness derived from the graph theory.

2.1 Static analysis

Looking at the structure of the graph, static analysis applied to financial network aims to assess the robustness of the graph and to identify the central edges. As it considers the architecture of the graph, the static analysis is suitable for macro-prudential analysis and especially systemic risk analysis (Ecial, 2012). First, the analysis determinates the extent to which the system is resilient to contagion. Cerutti et al. (2017) define the resilience or robustness of a financial network as its capacity to connect all vertices after the deletion of edges or vertices. Second, network analysis identifies the central players or major triggers of contagion effects (European Central Bank, 2010). von Peter (2007) identifies central nodes or "international banking centres" in the cross-border financial network based on several centrality measures. The next lines review the properties that assess first the robustness of the graph, and second the centrality role of the nodes or vertices. The purpose is to integrate these properties in the concept of interconnectedness.

2.1.1 Robustness measures for the financial network

A first indicator of the robustness of the graph is the degree distribution of the vertices. We recall that the degree of a financial institution is the number of financial links incident to this financial institution. When analysing the distribution of all the degrees, some features can emerge. According to Ecial (2012), the degree helps to analyse the network's activity. Further, Sachs (2014) found that the distribution of claims between financial institutions also matters in the stability of the financial network. While Allen and Gale (2000) argue that a complete network is more robust, Sachs (2014) points out that an incomplete network but with equally distributed claims would be more robust. Literature largely establishes that the degree distribution of the graph of interconnections between financial institutions presents a power-law distribution (Haldane, 2013) (Kranakis, 2013). A small number of institutions participates massively in the total of the interconnections. More, Haldane (2013) presents the long tail distribution resulting from this. For him, the distribution is an indicator of robustness of the financial system. The system is more robust to random attacks. On the contrary, if the shock is targeted to an important institution, it might have tremendous impact. Boss et al. (2004) also demonstrate for the Australian interbank network that the degree distribution follows a power law distribution indicating the existence of hubs in the network.

Besides the degree distribution, the reciprocity in the graph can be measured. Haldane (2013) defines the reciprocity in the financial market as "the number of exposures that have a corresponding counter-exposure". The claims represented as out-degree are compensated by debt or, in-degree. Ecial (2012) advances that a high level of reciprocity would conduct to a low probability of systemic risk. Indeed, the unbalanced exposures are partly offset with reciprocal exposures. Considering the reciprocal effect of a financial interconnection, concentration of the network is also an indicator of its robustness. Sachs (2014) assesses a concentration ratio for core banks, i.e., "the share of total bank assets that core banks hold". The other main measure of concentration that can be applied is the density. It is the proportion of potential links that are actually present in the network (Newman, 2010). Freixas et al. (2000) argue that a denser network, a network that tends to be complete, is more resilient in case of distress. A loss will spread among more institutions and thus will be attenuated. Nevertheless, in the case of a rise of exposures faster than density, a higher network density increases contagion risk (Gai & Kapadia, 2010). A classic measure of concentration is the Herfindahl-Hirschman Index (HHI). This index is calculated by summing the squared of market shares of each institution in the market or network. It indicates the level of competitivity in the market (Rhoades, 1993). In our cross-border network, the market shares are represented by the total amount of claims a country owns to the rest of the world.

2.1.2 Principal node-centrality measures and its application in a financial network

Besides the robustness of the network, the static analysis aims to identify the most central or core banks in the network. To that end, literature uses network/graph metrics on nodes to evaluate the so-called node centrality. In the particular case of cross-border financial network, von Peter (2007) establishes a comprehensive methodology to identify core nodes in the international banking network. Based on his work, three categories of centrality can be observed for a node:

- well-connected overall
- well positioned on the flows of interconnections
- connected to important nodes.

Well-connected overall

To designate a node or vertex well connected overall, the first principal measure is the degree of a particular node. It demonstrates its ability to interact with other financial institutions by means of liquidity distribution (von Peter, 2007). A country with a high degree plays the role of shock absorber in the network because its degree facilitates risk-shifting: "exposures are diversified across a wider set of institutions" (Gai & Kapadia, 2010). However, a high degree represents the ability of a vertex to accentuate the spread of a shock (Ecial, 2012). In brief, the degree is a pertinent candidate to evaluate the trade-off between risk sharing and amplification of shock.

In the case of directed graph, in and out-degree can also be determined. The in-degree represents the "choices of entities abroad to place funds with a certain financial institution" (von Peter, 2007). In the case of cross-border network, the graph is directed. In that way, the International Monetary Fund evaluates the in-degree on direct bilateral exposures of interbank market as a measure of the diversification in a financial network (International Monetary Fund, 2011). von Peter (2007) results reveal that the central countries tend to have a higher in-degree than out-degree. In a weighted graph, the degree of a node can be complemented with its strength. The strength summarizes the total weight of all links incidents to the vertex (Cerutti et al., 2017). Furthermore, Minoiu and Reyes (2013) measure the Herfindahl-Hirschman Index not at the level of the total market but rather at the level of a node. The node-HHI assesses the level of concentration or diversification of a country in its lending and borrowing activities. For Minoiu and Reyes (2013), it "reflects the country's vulnerability to negative shocks due to excessive dependence to its counterparties".

Well positioned

Freeman (1978) points out that a vertex can have a low degree but be in a central position in the network structure. So, the position of a vertex is also a category of centrality. In the graph theory, we saw that closeness and betweenness consider the existence of paths between vertices. Therefore, closeness and betweenness are additional measures of centrality. Closeness measures the ability of a vertex to attain every other vertex quickly. For Von Peter (2007), it helps to identify international centres with the broadest reach to smaller and remote countries. For Ecial (2012), a high level of closeness for a bank could also indicate a proximity to "healthy big institutions" and thus could protect the bank. The betweenness measures the number of times a vertex is on the shortest path between two other vertices (Alves et al., 2013). For Ecial (2012), it describes the power of control of a vertex in the network activity. A high level of betweenness would indicate a high fragility in case of failure of other banks. In contrast, a low level of betweenness would define the country as a systemically important centre. If the institution faces difficulties, it would have more impact on the network (Ecial, 2012).

However, the papers of Cerutti et al. (2017) and Soramäki et al. (2007) demonstrate that closeness and betweenness are not applicable for graphs like the cross-border network. Betweenness and closeness measures search for the geodesic shortest path. However, the structure of the graph is special. As the cross-border network stands with direct exposures, the edges are directed from one country or financial institution to another. The connections are not constructed with the view to attain a certain vertex as quickly as possible. For Cerutti et al. (2017), "bank flows do not prefer to traverse the network via the shortest path".

Alternatively, Hauton and Héam (2016) use another perspective of closeness that does not consider the geodesic path. For them, two financial institutions can be close in term of "substitutability" and "integration" (Hauton & Héam, 2016). Both measures are components of the level of interconnectedness of a financial institution. Substitutability looks for similarities between two institutions in the distribution of their bilateral exposures to the entire network. Integration tries to spot the institutions that tend to be more interconnected to the rest of the network.

Connected to important neighbours

Furthermore, the importance of the neighbours can also highlight central nodes or vertices. The eigenvector centrality includes the influence or importance of the financial institution (see Chapter 1 for the formal definition). One drawback is that the eigenvector centrality does not consider directed links and thus does not differentiate incoming edges and outcoming edges. A similar method that handles directed networks is the Katz centrality (Katz, 1953) (equivalent to the "Prestige" measure of von Peter (2007)). The Katz centrality measure integrates the importance of external vertices for eigenvector centrality with a small bias term (Newman, 2010). For Ecial (2012) alpha-centrality, equivalent to Katz centrality, includes the power of external sources of influence in the measure. "A nil value of alpha denotes that only external influence matters, whereas a large value denotes that only the innate characteristic of the bank matters" (Ecial, 2012). According to Cerutti et al. (2017), Katz centrality turns out to be an indicator of the fragility of countries in the transmission of shocks. Indeed, a high level of Katz centrality indicates that the country is strongly connected to important countries (Cerutti et al., 2017). If a shock occurs in one of the systemic important countries, countries with high Katz centrality will be more impacted than countries with low Katz centrality. Heipertz et al. (2019) study balance-sheet contagion mechanisms and show that Katz centrality is correlated to systemic risk. Even further, Puhr et al. (2012) demonstrate on the Australian interbank market that Katz centrality is the best candidate to determine the contagiousness of a node in the transmission of shock, and more particularly to predict default cascades. These concepts will be developed in the next section on dynamic analysis.

As an extension of Katz centrality, PageRank attenuates the direct centrality of important nodes (Rovira Kaltwasser & Spelta, 2019). The PageRank algorithm ranks the most popular pages on the web. The PageRank centrality of a node i corresponds to the "probability of being at the node in the steady state of a teleporting random walk" (Korniyenko et al., 2018). The algorithm simulates the path of a random entry in the pages and then chooses a random link to another page. At the end, it measures the popularity of the pages, i.e., the probability to reach a certain page. It considers that if a page has a lot of links, the importance of each link should be diminished compared to a page that have a few links. Rovira Kaltwasser and Spelta (2019) accommodate it in finance to determine systemically important institutions. They point out the benefits of the PageRank algorithm for financial regulation. For them, the algorithm accounts not only for the systemic risk of each individual institutions but also for "how the exposures of all the banks in the system affect each other and drive the rankings of each individual institution" (Rovira Kaltwasser & Spelta, 2019).

In 2012, Battiston et al. (2012) introduce the DebtRank, particularly adapted to a financial network. It is a measure, like PageRank, which excludes repeated links in the walks. This measure helps to find financial institutions that would cause the widest spread of economic distress on the network in case of contagion (Battiston et al., 2012).

Similarly to Katz centrality, hub and authority scores are derived from the relative importance of neighbours to assess the centrality of a vertex (Cerutti et al., 2017). León-Rincón and Pérez-Villalobos (2013) clarify that the hub and authority scores include the "mutually reinforcing centrality arising from nodes pointing to other nodes (i.e., *hubs*) and from nodes being pointed-to by other nodes (i.e., *authorities*)". Their result shows that this measure helps to identify the Systemic Important Financial Institutions (SIFIs). The concept of systemic important financial institutions is developed in the next chapter.

To sum up, the Table 1 regroups some of the static measures that can be applied to a graph. Alves et al. (2013) reflect that the static network measures "illustrate that credit and funding events can be expected to be widespread in the interbank market, because institutions are close to each other and well connected". With all these metrics, we end up with the conclusion that important vertices would reduce the impact of a shock but facilitate the spread of a loss. In line with the challenge of risk-sharing versus propagation of shock, network analysis can thus reveal insightful specificities of the interconnectedness in the financial network (Cabrales et al., 2017). Centrality measures like the Katz centrality can strongly be interpreted as a measurement of systemic risk (Puhr et al., 2012). For indirect links, Diebold and Yilmaz (2015) demonstrate that centrality measures can be used to determine the level of connectedness among financial institutions. Puhr et al. (2012) even argues that Katz centrality is an accurate predictor of default cascades. It is therefore interesting to process a dynamic analysis and compare the results.

<table>
<tr><td rowspan="8">Node centrality measures</td><td rowspan="2">Well connected</td><td>Degree</td></tr>
<tr><td>Strength</td></tr>
<tr><td rowspan="2">Well positioned</td><td>Closeness</td></tr>
<tr><td>Betweenness</td></tr>
<tr><td rowspan="4">Connected to important vertices</td><td>Eigenvector centrality</td></tr>
<tr><td>Katz centrality (or alpha-centrality or Prestige)</td></tr>
<tr><td>PageRank and DebtRank</td></tr>
<tr><td>Hub and authority scores</td></tr>
<tr><td rowspan="4">Robustness measures</td><td>-</td><td>Degree distribution</td></tr>
<tr><td rowspan="3">Reciprocity</td><td>Reciprocity ratio</td></tr>
<tr><td>Concentration ratio</td></tr>
<tr><td>HHI</td></tr>
</table>

Table 1: Static measures from graph theory.

2.2 Dynamic analysis

Literature not only looks at the static properties of the financial network. An important part of the research investigates the transmission mechanisms triggered by a shock on one part of the system. This refers to the dynamic analysis of a financial network. For a bank, a shock could be the sudden high demand of liquidity from the consumers (Allen & Gale, 2000). Similarly, a shock could be the arbitrary failure of an institution (Espinosa-Vega & Solé, 2010). The dynamic analysis examines how the different financial institutions will be impacted and will react from such shock (Alves et al., 2013). Therefore, dynamic analysis complements robustness measures of the static analysis. This section presents what is a contagion model, which one to use, and how it measures interconnectedness.

2.2.1 Definition of contagion

The work of Allen and Gale (2000) is a pioneer in contagion analysis. For them, the transmission of a shock (or spillover effects) can be defined as the event "when one region suffers a bank crisis, the other regions suffer a loss because their claims on the troubled region fall in value" (Allen & Gale, 2000). Smaga (2014) covers more largely "the instability of the given institution" as the trigger of transmission mechanisms leading to a system-wide crisis. Literature on contagion aims to evaluate how a shock could impact the whole system. Smaga (2014) proposes to define systemic risk as "the risk that a shock will result in such a significant materialisation of imbalances that it will spread on the scale impairing the functioning of financial system and to the extent that it adversely affects the real economy". Literature explores various paths of contagion in interbank market and with its own methodology to assess the potential systemic risk of such spillover effects (Upper, 2011) (Alves et al., 2013).

2.2.2 Core model

We will start here with the Eisenberg and Noé (2001) model. It states that shock comes either from liquidity shocks (asset fires sales) or from information contagion. For direct links, Babus (2016) extends the contagion process identifying liquidity shocks that could arise from fire sales of funding trouble and solvency shocks that emerge from a loss in value of total asset. From these different contagion mechanisms, the level of interconnectedness in the interbank or cross-border exposures network can be evaluated. Some static metrics already include a shock transmission. PageRank and DebtRank suppose a random path in the network. In the next sub-section, we review some practical measures of interconnectedness in the dynamic analysis of a network.

2.2.3 Measures of interconnectedness in the contagion model

For Alves et al. (2013), running a default contagion on the financial system enables identifying the systemic role of the financial institutions, i.e., systemic importance and systemic fragility. Upper (2011) runs the contagion process of Eisenberg and Noé by starting to generate the failure of a particular financial institution and then recording the systemic role of each institution. Systemic importance of a financial institution X measures the number of other institutions in default when we initially default the institution X. Systemic fragility is measured by the number of times the financial institution X defaults when we run scenario of default starting with other institutions (Alves et al., 2013). In his survey, Upper (2011) tests the reliability of different simulation models of contagion in the interbank market. He argues that simulation models are suitable candidates to predict “whether or not, contagion could be an issue and, possibly, also identify banks whose failure could give rise to contagion” (Upper, 2011).

In conclusion, we have classified various centrality measures that can be applied to the financial network. The centrality measures feature how well a financial institution is connected, how it is positioned and how it influences others. Results of static analysis potentially reveal financial institutions at risk in case of a crisis. More notably, some measures can be interpreted as indicator of the systemic risk, namely the Katz centrality. We have also shown that the dynamic analysis gives information about the spillover effects in a scenario of default. It could identify the systemic role of the financial institutions constituting the network. Some approaches even regroup direct links and transmission mechanisms to model the interconnectedness. For instance, Bicu and Candelon (2013) complement the direct cross-border links with indirect exposures. Indirect exposures are defined as the connections between two institutions caused by the intermediate external counterparties (Bicu & Candelon, 2013). They develop a transmission mechanism to identify the effects of the intermediate counterparties for the two institutions in case of a shock.

Chapter 3 – Interconnectedness in regulatory policies

As the financial crisis of 2008 highlighted the effects of interconnections in the propagation of a shock, the financial regulation evolved to adopt a network perspective. This chapter goes through some of the safeguards put in place by the financial supervisors to face the systemic risk that comes from interconnections.

3.1 Systemic risk

The emergence of network-related policies comes from the recognition that the network structure plays a key role in systemic risk. In their report to G20 Finance ministers and governors in 2009, the International Monetary Fund (IMF), the Financial Stability Board (FSB) and the Bank for International Settlements (BIS) defined the systemic risk as a "risk of disruption to financial services that is caused by an impairment of all or parts of the financial system and has the potential to have serious negative consequences for the real economy" (Financial Stability Board, 2009). In the aftermath of the Global Financial Crisis (GFC), the Dodd-Frank Reform strengthened the financial regulation to tackle systemic risk. In October 2009, the European Central Bank organised the workshop "recent advances in modelling systemic risk using network analysis" (European Central Bank, 2010). The aim was to raise awareness of network modelling as a tool for managing systemic risk. It emerges that network theory provides good techniques for identifying systemically important market participants and for assessing the resilience of the financial system to contagion. More, this workshop suggests the need of more granular data. Over the past few years, the ECB has produced numerous working papers on network theory. These papers show that network analysis is well accepted as a monitoring tool, but not yet as a tool for implementing the rules. To face systemic risk, the ECB also develops a "Risk Dashboard", i.e., "a set of quantitative and qualitative indicators of systemic risk in the EU financial system" (European Systemic Risk Board, 2022). Alongside indicators of macro risk, credit risk, market risk and others, there are indicators that express the "risk related to central counterparties" (European Systemic Risk Board, 2022). Simultaneously, Basel III, through the identification of the G-SIBs, has incorporated a network perspective to address systemic risk.

3.2 Basel III reforms

One significant lesson learned from the global financial crisis is that some financial institutions played a bigger role in the chain of losses. It appears that it is crucial to identify these institutions and apply specific measures to them (Mesnard et al., 2016).

The Basel Committee on Banking Supervision (BCBS) "is the primary global standard setter for the prudential regulation of banks" (*Basel Committee Charter*, 2018). To strengthen the regulation, supervision, and management of banks, the BCBS establishes the Basel Framework (Basel Committee on Banking Supervision, 2022). It is a set of standards to apply to the internationally active banks (Basel Committee on Banking Supervision, 2022). In the Basel Framework, the 14 standards can be regrouped in three pillars. Concerning the Pillar 1 of Basel III, all banks should meet minimum capital requirements. It is decomposed in higher quality and level of capital with minimum common equity capital, capital loss absorption capacity, capital conservation buffer and countercyclical buffer. Notably, countercyclical buffer acts as a macro-economic buffer. It can be adapted by the jurisdiction to face the level of risk arising from "periods of excessive aggregate credit growth" (Basel Committee on Banking Supervision, 2019). Along capital in their balance sheet, banks should be able to quickly release cash in scenario of stress within the economy. The Liquidity Coverage Ratio has been introduced for the short term and the Net Stable Funding Ratio for the long term.

In addition to risk-based capital and liquidity ratio requirements, BCBS introduces a set of reforms to explicitly account for interconnectedness. The BCBS identifies core financial institutions and banks that pose higher risk to the financial system, named G-SIBs (Basel Committee on Banking Supervision, 2013). The following sub-section describes the methodology used to identify the systemic important banks.

3.2.1 G-SIBs

The G-SIBs are the Global Banks that are Systemically Important, i.e., "their failure would cause significant disruption to the wider financial system and economy activity" (Financial Stability Board, 2014).

The Financial Stability Board reports each year the list of global systemically important banks (G-SIBs). For each bank, a score of risk is calculated according to the assessment methodology from the BCBS. There are 12 indicators regrouped in five categories: size (the total exposures defined in the leverage ratio worksheet of Basel III Dec 2012), jurisdictional activity, interconnectedness (intra-financial system assets, intra-financial system liabilities and securities outstanding), substitutability and complexity. For interconnectedness, the intra-financial system assets (liabilities) are calculated as the sum of:

- lending (deposits) to financial institutions (including undrawn committed lines for both)
- holdings of (all marketable) securities issued by other financial institutions (by the bank)
- net mark to market reverse repurchase agreements with other financial institutions
- net mark to market securities lending (borrowing from) to financial institutions
- net mark to market OTC derivatives with financial institutions.

Then, the score is calculated as the amount of the category divided by the total sum of the category for all the banks. Each category represents 20 percent of the total score, which determines the set of policy measures put in place to prevent systemic risk. The total score of a bank indicates on which bucket the bank at risk is allocated. Based on its bucket, the bank is notably required to hold a specific level of additional capital buffers (currently from 1% to 2.5%) in addition to the Total Loss-Absorbing Capacity (TLAC) requirements (Financial Stability Board, 2021). That is the first stated introduction of interconnectedness in the regulation framework with a direct measure of the level of asset and liabilities for banks.

At the European level, the Capital Requirement Directive and Capital Requirement Regulation (CRD IV and CRR) implement the co-reported G-SIBs framework from the Basel Committee and the Financial Stability Board. It develops all the rules that are applied in all the member countries of the European Union. One of the key objectives is to “improve the EU banking sector’s ability to withstand economic shocks” (*Banking Union*, 2022).

3.3 Financial Sector Assessment Program

Another learning from the global financial crisis was the worldwide implication of the "health and functioning of a country's financial sector" (*Financial Sector Assessment Program (FSAP)*, 2021). The IMF introduces the Financial Sector Assessment program (i.e., an in-depth analysis of the financial sector of countries) in the broader IMF country surveillance Article IV consultations. The main goal is to assess the stability of the financial sectors of each systemic country by the mean of stress tests, examination of the resilience of financial and non-financial sectors, analysis of systemic risk from cross-border spillover effects. For example, the FSAP of Spain introduces interconnectedness analysis in a technical note in 2017 (International Monetary Fund, 2017).

3.4 Relevance of network theory for the regulation

As a result, a greater understanding of the network of interconnections between banks can help regulators in their macro-prudential framework for financial supervision. The use of network theory in financial regulation has become an evidence after the 2008 financial crisis (Haldane, 2013).

On the one hand, static analysis, and more specifically the analysis of the centrality of financial institutions can provide information to policy makers with a view to identifying "the financial institutions potentially causing greater systemic harm" (Enriques et al., 2019). For instance, Haldane (2013) demonstrates that the degree distribution is an indicator of robustness of the network. He shows that it exhibits a power-law long-tail distribution. Thus, a small number of financial institutions owns most of the links. So, it suggests that it is relevant to manage specific important institutions.

On the other hand, dynamic analysis allows to determine the exact amounts exposed to a default. For example, the DebtRank estimates the potential economic loss of the default of one financial institution (Battiston et al., 2012). The Eisenberg and Noé (2001) model gives a reference pattern of the events following a bank failure. "Policy makers should identify the most appropriate measure of systemic relevance by looking at the specific characteristics of the regulation to be applied, its objectives, and the information available to both the regulator and the regulated" (Enriques et al., 2019). So, the challenge is to choose the right tools to measure interconnectedness and manage systemic risk.

3.5 Identified gap and contribution

The measure of interconnectedness is also subject to discussion. Where the established measure from G-SIBs seems the most direct measure, others try to precise the value of the network structure in the measurement of interconnectedness. The purpose of this thesis is to compare and review the different graph measures that can be applied to a financial network and develop the concept of interconnectedness from these measures. This could help the financial supervisors in their macro-prudential regulatory framework. This research will give valuable insights to answer to the following question: **how to measure interconnectedness in the financial system using Graph Theory?**

PART II – RESEARCH METHODOLOGY

To answer the research question, we will undertake a research synthesis. The Handbook of Research Synthesis (1994) defines the research synthesis as “the conjunction of a particular set of literature review characteristics”. In other words, it is the combination of papers from the literature with the aim of “increasing the generality and applicability of, and the access to those findings” (Wyborn et al., 2018). Form the research synthesis, this thesis brings an application of the concepts of graph theory for a financial network.

In the literature, there are a variety of articles that develop the measures of graph theory and their application in a financial network. Unfortunately, the literature lacks to compare the scope of these measures. Therefore, this research presents the most appropriate measures of interconnectedness explored in the literature for the cross-border financial network. As we examine the results of mathematical measures, the quantitative research is suitable. It also assesses the independence between the network measures selected. The purpose is to find patterns or similarities in the measures. This allows to generalise and illustrate a practical measurement of the level of interconnectedness. This part motivates the approach used to answer the research question. In the first chapter, the collection of the data as well as the choice of the data series is discussed. The second chapter outlines the analysis steps followed and why they allow a clear definition of the concept of interconnectedness.

Chapter 1 – Data description

This chapter aims to present and justify the pertinence of the data selected to construct a financial network. The first section outlines the data collection process and the quality of the sources of data. Then, the second section describes the data base built to carry out the analysis of the graph.

1.1 Data collection

To map the cross-border interconnectedness between financial institutions, data series of the links and the vertices are needed. Presently, the publicly available data bases from the Bank for International Settlements, the ECB Statistical Data Warehouse, and the IMF Financial

Soundness Indicators are the most complete and credible data sources on banking statistics. The following lines give a brief description of the three different sources of data.

First, the Bank for International Settlements (BIS) publishes international banking statistics that "cover the balance sheets of internationally active banks" (Bank for International Settlements, n.d.). Its mission is to "support central banks in the pursuit of monetary and financial stability through international cooperation" (Bank for International Settlements, n.d.). The Committee of the Global Financial System (CGFS) is responsible for the collection of the data from each country. National authorities like central banks collect data from "internationally active banks" in the jurisdiction of their country every month or quarter (Bank for International Settlements, 2019). The authorities of the reporting country aggregate the data at national level. The CGFS then validates the data and reports the aggregated data to the Bank for International Settlements at country-level.

Second, the ECB Statistical Data Warehouse provides statistics and indicators on "Euro area monetary policy, financial stability and other topics" (*European Central Bank - Statistical Data Warehouse*, n.d.). National central banks and credit institutions provide data to the ECB. Then, the ECB aggregates it and make it available on the internet via the ECB Statistical Data Warehouse.

Third, the International Monetary Fund offers Financial Soundness Indicators "of the current financial health and soundness of the financial institutions in a country" (*Financial Soundness Indicators Compilation Guide 2019*, 2019).

1.2 Data description

The three sources of data previously developed allow to construct a graph object G consisting of a vertices table $V(G)$ and an edge table $E(G)$. The financial network uses cross-border claim exposures of 17 countries from 2011 to 2020. The graph is mapped in the first chapter of the empirical analysis.

The International Monetary Fund (IMF) recalls that cross-border claims are part of the main sources of systemic interconnectedness risks (International Monetary Fund, 2017). In 2017, the International Monetary Fund (IMF) suggests that cross-border, with cross-sectoral linkages should become part of regular systemic risk surveillance of a country. In the Financial Sector Assessment Program of countries (FSAP), the IMF examines the interconnectedness based on BIS Consolidated Banking Statistics and regulatory capital provided by IMF Financial Soundness indicators. In a working paper for the European Central Bank, Peltonen et al. (2019) confirm the value of cross-border exposures for the policy-making via an early-warning model. Their model suggests that the risk coming from lending activities "predicts more accurately bank crisis" (Peltonen et al., 2019).

1.2.1 Vertices table, V

For the modelling of the network, vertices represent countries. The Bank for International Settlements (BIS) publishes data series of the exposures between countries. It differentiates reporting countries, i.e., the countries reporting the holding of claims and the counterparty country. A reporting country regroups all the "internationally active banking groups"[1] headquartered in the country, excluding intragroup positions. The 17 reporting countries from BIS analysed are: Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Altogether, it counts for 80 percent of the total reported claims from BIS. Fourteen of the seventeen countries are part of the European Union. Alike Hattori and Suda (2007), this empirical analysis collects data only on reporting countries from Bank for International Settlements since a non-reporting country will only counts the exposures of others on this country and not inversely (e.g. Luxembourg or Turkey).

The model of contagion described in the next chapter requires the capital level of each country. This data is compiled from two sources of data: the IMF FSIs and the ECB statistical data warehouse. To merge the data series, the scope of the institutions covered in a country must be consistent across data sources. For the ECB, the data series cover domestic credit institutions. It regroups large, medium-size, and small domestic banks as well as foreign EU and non-EU controlled subsidiaries and branches. The level of capital defined by ECB is the "total equity" (Official Journal of the European Union, 2017). The IMF accounts for the

[1] See (Bank for International Settlements, 2019) for the complete definition.

"financial institutions of a country" (*Financial Soundness Indicators Compilation Guide 2019*, 2019). IMF FSIs provides the data series "regulatory capital RWA" for several countries (*Financial Soundness Indicators Compilation Guide 2019*, 2019). As BIS statistics exclude activities of non-internationally active banks, IMF FSIs may overestimate the level of capital. It should be noted that the United Kingdom is no longer part of the European union and thus, the ECB data warehouse does report the level of capital for the fourth quarter of 2020. To cope with that, we give the value of the fourth quarter of 2019.

Nevertheless, the guidelines for the BIS statistics "are aligned with standards for prudential reporting" of the Basel Committee on Banking Regulation (Bank for International Settlements, 2019). BIS aims to promote the use of "common transparent methodology" (Bank for International Settlements, 2019). Therefore, the datasets can be aligned with datasets of other official institutions like the International Monetary Funds and the European Central Bank data warehouse. Moreover, the values are similar for countries represented in both data sets. We recall that this research has only illustrative purpose by reviewing the possible network measures applied to the financial network.

1.2.2 Edges table, E

The other part constituting the graph is the edges table, $E(G)$. The BIS international statistics are composed of two main data sets: the Locational Banking Statistics (LBS) and the Consolidated Banking Statistics (CBS). In 2017, The BIS estimates that the Locational Banking Statistics cover 94% of the total cross-border claims of all banks worldwide (Bank for International Settlements, 2020). The following research uses rather a similar data set, the Consolidated Banking Statistics, as it measures "bank's country risk exposures on a worldwide basis" (BIS Monetary and Economic Department, 2015). The advantage of Consolidated Banking Statistics (CBS) is that the data is compiled with a nationality perspective. The statistics focus on the country where the parent of the banking group is headquartered whereas Locational Banking Statistics are based on the residence of the banking office and may thus overstate the importance of intermediate vertices (Cerutti et al., 2017). Moreover, CBS exclude intra-group positions similarly to the consolidation approach followed by banking supervisors (Bank for International Settlements, 2019). The aim of this research is to assess the level of interconnectedness of banks from one country to another.

Therefore, data series focussing on nationality of the country are more suitable for the purpose of this research. In the CBS data, the Bank for International Settlements presents two different formats of cross-border exposures: exposures from Immediate Risk Basis (IRB) and from Ultimate Risk Basis (URB). In scenario of contagion, the transmission of loss in asset value is not stopped at intermediate nodes so the network structure needs to reflect the ultimate receiver of risk. URB adjusts from IRB inward and outward risk transfers across countries (Bank for International Settlements, 2019). From URB, we select the instrument "cross-border claims excluding derivatives" (Bank for International Settlements, 2019). It is composed of loans and deposits including reverse repos, debt securities holdings and equity instruments (Bank for International Settlements, 2019). The links can be classified as direct links as part of credit claims and debt securities of the Mind Map in Figure 3 of the literature review. Thus, the edges $E(G)$ represent the total amount outstanding claims from BIS CBS in USD between two countries in $V(G)$. The three data sources are compared in
Table 2.

	ECB Data warehouse	IMF	BIS
Type of data	Supervisory and prudential statistics	Financial Soundness Indicators	Consolidated Banking Statistics
Data series	Total equity	Total regulatory capital	Total claims on guarantor basis
Series code	LE000	FS_ODX_CRT_FSKRC	Q:S:5A:4R:U:C:A:A:T01
Currency	Euro	Country currency	US dollars
Frequency	Quarterly	Quarterly	Quarterly
Scope	Country = large, medium-size, and small domestic banks as well as foreign EU and non-EU controlled subsidiaries and branches	Country = all financial institutions	Country = internationally active banking groups
Coverage	European Union countries	Countries publishing Financial Stability Reports	The list of CBS-reporting countries counts 25 countries
Network parameter	Capital as vertices attribute	Capital as vertices attribute	Edges weights

Table 2: Comparison of the ECB, IMF, and BIS data series.

Chapter 2 – Research procedures

After setting the scope of study, we can now discuss the research process carried out. To comprehensively capture interconnectedness in the network, the research is conducted in three phases (Bricco and Xu, 2019):

- the mapping and descriptive analysis of the financial network
- the analysis of the interconnectedness and contagion
- the policy discussion.

The two first steps are detailed in this chapter.

2.1 Descriptive analysis

The first obvious description of the network is the mapping. The mapping is considered as descriptive preliminary statistics that help to balance the results and have a global picture of the financial system (Alves et al., 2013). After the modelling of the graph, descriptive analysis contextualises the data series and help to understand the scope of the network. It shows the main destinations and origins of exposures. The research is restricted to direct links focussing the work on the understanding of balance sheet interconnectedness. The evolution of the cross-border claims can also reveal a trend. The empirical research is conducted for 10 periods from the fourth quarter of 2011 to the fourth quarter of 2020. It seeks to demonstrate that static measures are good proxies of dynamic measures.

2.2 Measures of interconnectedness

From the literature review, we can confirm the benefit of using static analysis to measure the level of interconnectedness. Ecial (2012) shows that the static analysis is valuable in the macroprudential analysis because it summarises "stylised facts of the network architecture as a whole". For instance, stylised facts extracted from the static network include the identification of central or systemic groups of institutions (or nodes) in the network. The literature review identifies robustness and centrality measures suitable for the cross-border network of exposures. All the measures in Table 3 capture different dimensions of interconnectedness within the network studied. The next lines present the formal definition used to calculate each measure.

Measures of robustness	Measures of node-centrality	Measures of systemic role
Degree distribution	Degree	Systemic importance
Reciprocity	Strength	Systemic fragility
Herfindahl-Hirschman Index	Node-HHI	
	Katz centrality	
	Hub and Authority scores	

Table 3: The different measures of interconnectedness applied to the financial network.

2.2.1 Robustness measures

The degree distribution is defined as the distribution $P_{deg}(k)$ = fraction of nodes in the graph with degree k (Nykamp DQ, n.d.). The formal definition of degree is given in the literature review.

The reciprocity defines the proportion of mutual connections in a directed graph: $\frac{\sum_{i,j}(A.*A')_{ij}}{\sum_{i,j}A_{ij}}$ where $A.* A'$ is the element-wise product of the adjacency matrix A and its transpose (Nepusz & Csardi, n.d.).

The Herfindahl-Hirschman Index is defined as: $HHI = \sum_{i=1}^{N} s_i^2$ where s_i is the market share of institution i and N the total number of institutions (Rhoades, 1993).

2.2.1 Centrality measures

Martinez-Jaramillo et al. (2014) gives the formal definition for the strength of a node i, $s_i = \sum_{j\, \in N(i)} wij$ where $N(i)$ is the set of vertices linked to i.

The formal definition of out-HHI (or lending HHI) from Martinez-Jaramillo et al. (2014) is: $HHI^L(i) = \sum_{j\, \in N(i)}(\frac{w_{ij}^L}{\sum_j w_{ij}^L})^2$.

The Katz centrality a node i is defined as: $x_i = \alpha * \sum_j A_{ij}x_j + \beta$ "where the adjacency matrix A has eigenvalues Λ. The parameter β controls the initial centrality and $\alpha < \frac{1}{\Lambda_{max}}$"*(Jalili, n.d.).*

Kleinberg (1999) defines the hub and authority scores.

The authority score of the vertices is defined as the principal eigenvector of $A^T A$, where A is the adjacency matrix of the graph and A^T the transpose of this matrix while the hub score of the vertices is defined as the principal eigenvector of AA^T[1].

2.3 Contagion model

The dynamic analysis follows the suggestions of Bricco and Xu (2019). They promote the International Monetary Fund largely using the Espinoza-Vega and Solé model (EVS) in its Financial Sector Assessment Program. This model presents similarities to the one of Eisenberg-Noé but is directly constructed around BIS data. The model illustrates how network analysis could make a significant contribution in the financial surveillance of cross-border exposures. Basically, the EVS model simulates an idiosyncratic credit shock triggered by the default of one arbitrary country (Espinosa-Vega & Solé, 2010). Then, a sequential algorithm will measure the total asset losses. The algorithm assumes the arbitrary failure of a country, let's denoted country h. Therefore, the country h will not be able to repay its entire debts to other countries.

The balance sheet of the other countries will determine their ability to face the credit event. For these countries, the shock appears as a loss in their asset side of the balance sheet. Let's take the example of the country i that suffers from the failure of country h. The Figure 4 shows the effects of the shock on a bank i that holds assets in country h. On the asset side, the EVS model identifies the loans from bank i to all countries j: $\sum x_{ij}$ with country h included in j. The rest of the asset side is defined as other assets. On the liabilities side, the EVS model distinguishes the capital of country i as k_i, the long-term and short-term borrowings of country i as b_i, the deposits of country i as d_i, and the borrowings from all countries j as $\sum x_{ij}$. Then, the algorithm checks for each country of j if the country can absorb the loss. For the country i, the country will fail if the capital of the country, k_i is lower than the loss on the asset side x_{hi} times the loss given default from the failed country. For each new failed country, the steps are repeated until there is no more failure.

[1] More details on the algorithm that rank the vertices in a network can be found in (Kleinberg, 1999).

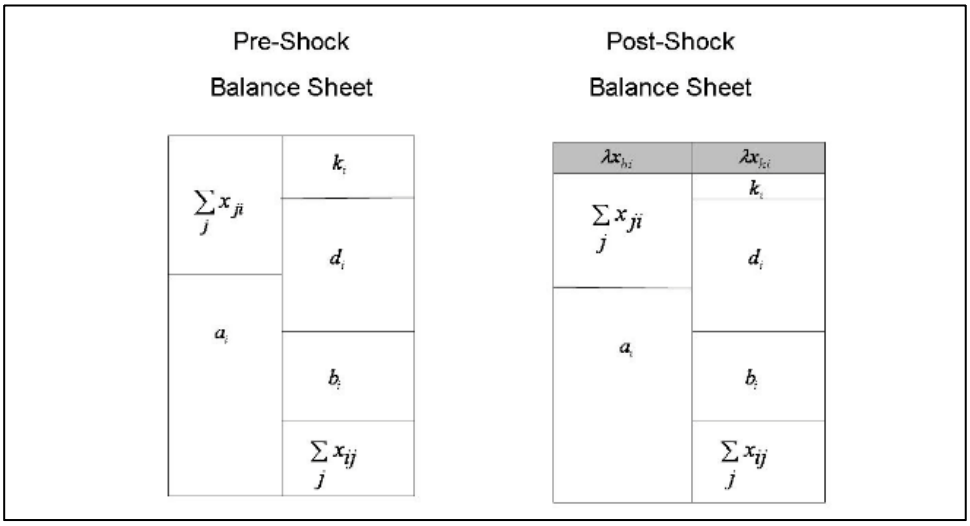

Figure 4: Effect of a credit shock on a bank's balance sheet (Espinosa-Vega and Solé, 2010).

This model allows to construct a classification of the systemic role of each country. As seen in the literature review, Aldasoro and Alves (2018) defines the systemic role from the number of failures generated in a contagion model. For Hauton and Héam (2016), it handles the contagion risk of the institutions.

2.4 Test of independence

After the static measures, the contagion model gives a dynamic measure: the systemic role. The purpose is to assess the relation between the measures. In line with Hauton and Héam (2016), exact Fisher tests of independence are performed. The exact Fisher test reveals the independence between two measures. To do so, the values of the measures allow to split the countries into groups in a contingency matrix. The groups formed in the static analysis will be crossmatched with the groups of the dynamic analysis. A comparison of these measures through statistical treatment will help to determine the relevance of the static measures.

The null hypothesis of exact Fisher test is that the two measures are independent. The alternative hypothesis suggests the dependence between the two measures. When the test gives a p-value lower than 1%, we can reject the null hypothesis of independence. The exact Fisher test is more robust than chi-square test because our sample contains only 17 countries.

PART III – EMPIRICAL ANALYSIS

The literature review highlights the appropriate measures of graph theory to apply to a financial network. After, the research methodology motivates the methods used to measure interconnectedness. As a result, the empirical research can construct a financial network from graph theory. The financial network analysed is the graph of cross-border gross claims between internationally active banking groups of 17 countries[1] for 10 annual periods (from the fourth quarter of 2011 to the fourth quarter of 2020). For each year, a graph G is constructed. The graph is composed of vertices $V(G)$ and edges $E(G)$ determined by the incidence function Ψ. The vertices $V(G)$ represent the 17 countries reporting claims. The edges $V(G)$ represent the holding of claims from one country to another. The amount of claims is an attribute of the edges and is named weight. In the network, an amount of claims against a country is considered as an exposure for the reporting country and a debt for the counterparty country. This part develops the concept of interconnectedness by a mapping and summary statistics of the network and by performing static and dynamic measures from graph theory outlined in the literature review.

Chapter 1 – Summary statistics

The summary statistics contextualise the financial network analysed and give a global view of the complex interconnections between countries. It is a first measure of the interconnectedness among financial participants. This chapter presents and illustrates summary statistics of the graph constructed.

1.1 Evolution of the interconnections over years

By tracking the evolution of the total of interconnections over years, we can appreciate the magnitude of this financial network in the entire financial system. From 2012 to 2017, there is a plunge of the amount of cross-border dollars interchanged in the global financial system.

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[1] Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Ireland, Italy, The Netherlands, Portugal, Spain, Sweden, Switzerland, the United Kingdom, the United States. It should be noted that China, Russia, and Luxembourg, among others, do not report Consolidated Banking Statistics.

The cross-border claims tend to increase from 2017 to stand at 14.169,659,248 trillion USD at the end of 2020. The Figure 5 illustrates the evolution of the total amount of claims held in the financial network. The rise of cross-border exposures has benefits, but it also poses risks to the financial system. On one hand, it reduces the cost of funding and thus it allows countries to allocate more efficiently their capital through risk-sharing (Korniyenko et al., 2018). On the other hand, it increases external vulnerabilities of countries that are highly dependent of central countries (Korniyenko et al., 2018).

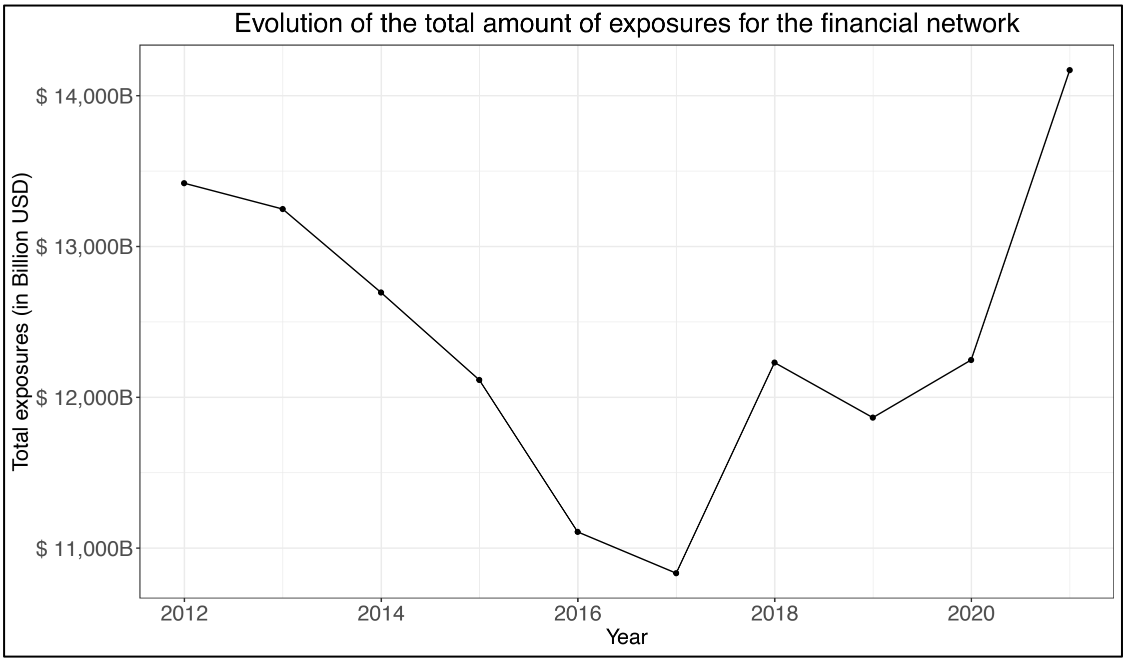

Figure 5: Evolution of the total cross-border exposures between 17 countries from December 2011 to December 2020.

1.2 Map

The mapping of the graph allows to view the distribution of the exposures between countries. It is a visual representation of all the claims held in the financial network. The graph constructed represents the different countries as vertices and the claims as edges from one country to another country. In the Figure 6, the orange nodes represent the countries, and the grey edges represent the exposure of one country to another. The graph bears some properties that specify the scope of study. It is a weighted graph. Therefore, each edge in $E(G)$ has a specific weight as attribute (the amount outstanding claims from one country to the other). Moreover, the graph is directed. The exposure is considered as a flow from the lender to the borrower country. In the network, the direction is pictured by the arrow from the reporting country to the counterparty country. The graph is not complete because some countries do not report or have no claims on other countries. The amount of capital of each country is represented by the size of the orange circle. Also, the sizes of the arrows are proportional to the amount of the claims attached to the edges. The mapping clearly points out the United States as a central participant in the cross-border interconnections. The United States has considerable amounts of claims with other countries. It also has the highest level of capital. The mapping of the cross-border claims network for the year 2011 and 2016 can be retrieved in Appendix 1 and 2.

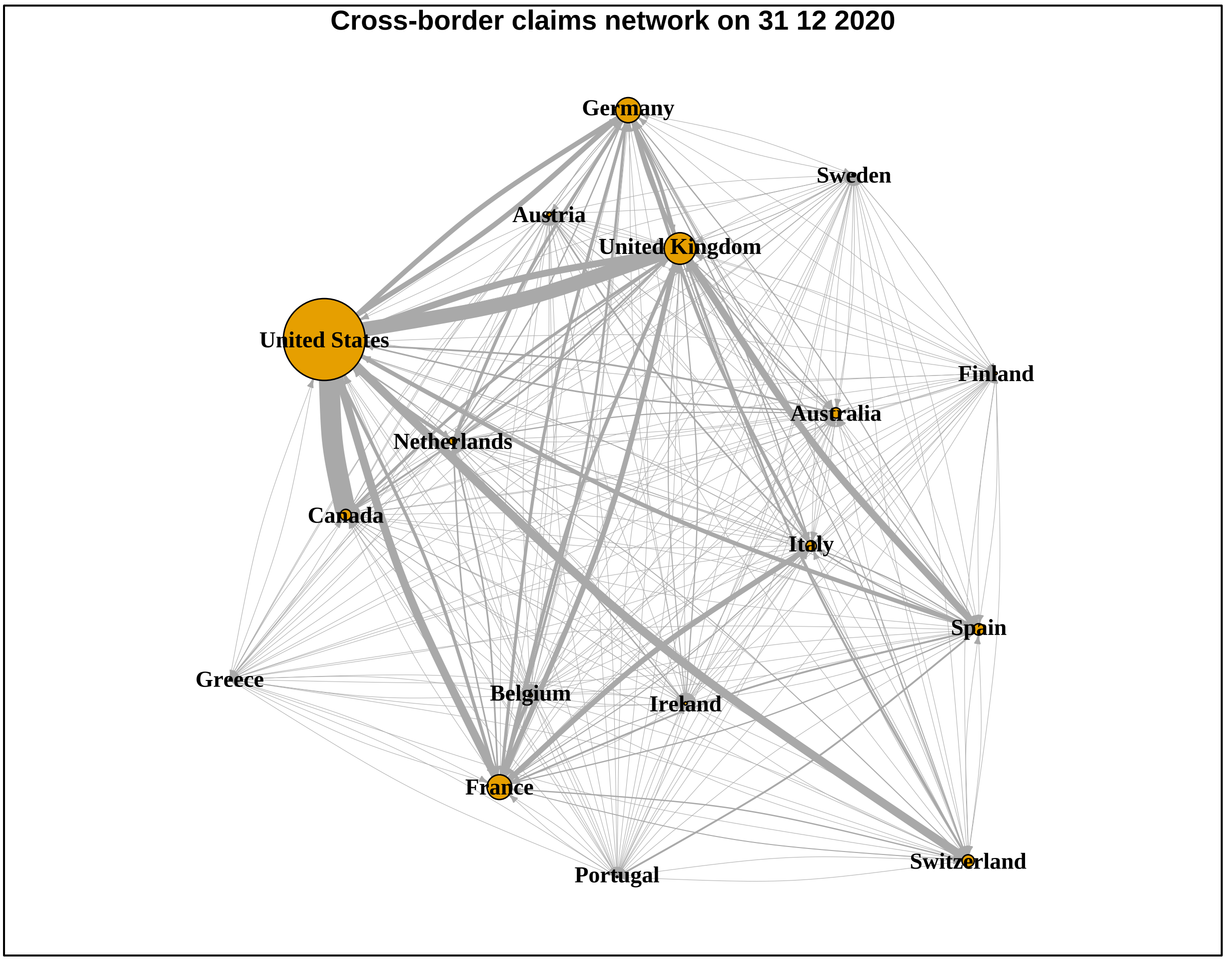

Figure 6: Cross-border claims network of the 17 countries on 31 December 2020.

1.3 Heatmap

The heatmap complements the mapping with a different representation of the links. The heatmap allows to quickly visualize the countries responsible for most of the interconnections. With the heatmap, in-degree and out-degree can be rapidly detected. In the network represented in the Figure 7, the highest amount of claims is the exposure of Canada on the United States. It states at 1.45 trillion USD. It shows the high-risk exposure that Canada has on the United States. Moreover, the United States appears as the first destination of lending activities in front of the United Kingdom. France, the United Kingdom, and the United States seems to be the three first holders of claims on all the countries. At first glance, the amounts of exposures are condensed in few countries.

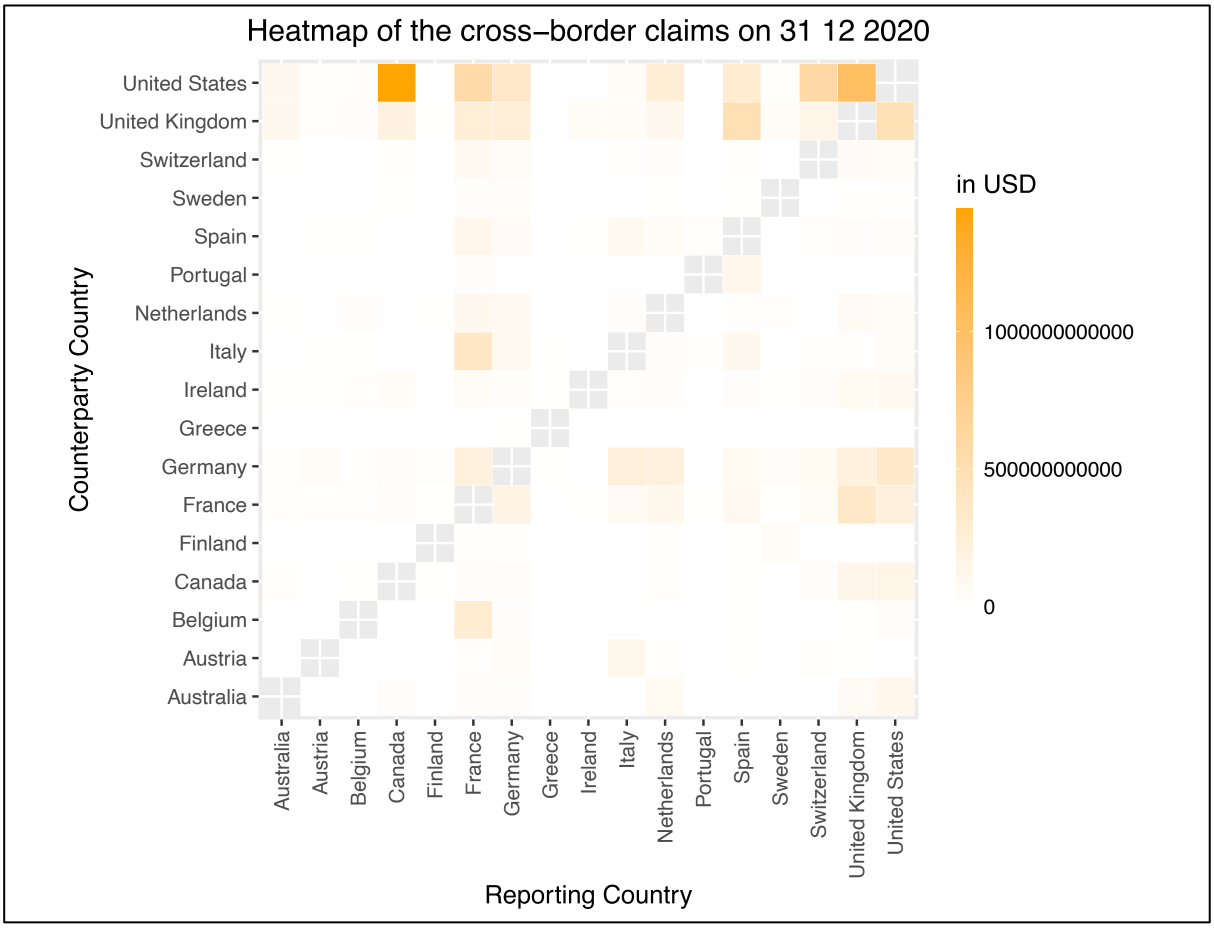

Figure 7: Heatmap of the cross-border claims on 31 December 2020. Each square represents the number of claims owned from the reporting country (x-axis) to the counterparty country (y-axis).

1.4 Size of capital

One attribute of the constructed graph is the capital of each country. Let's remember that the capital analysed is the total equity of countries measured by the European Central Bank and the International Monetary Fund. In terms of capital, the United States clearly stands out from the other countries with a level of capital that reaches over 2 trillion USD by the end of 2020. The United States is followed by the United Kingdom. With a lower amount of capital, Germany and France present a similar evolution of their capital since 2011. The other countries admit a capital below 500 billion USD. In the Figure 8, the evolution of capital is represented for the top countries with a colour line and a grey line for the other countries.

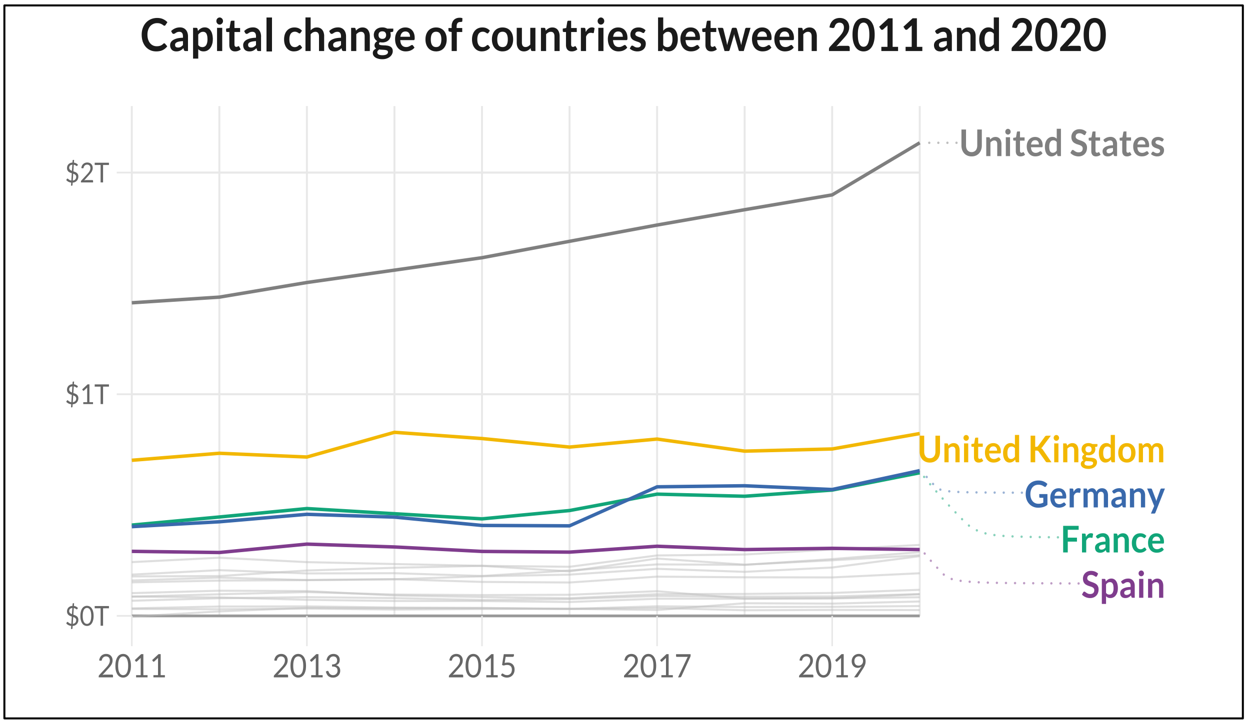

Figure 8: Evolution of capital (in trillion USD) for the 17 countries between December 2011 and December 2020. Top countries are represented with a colour line while other countries are represented by a grey line.

Chapter 2 – Static analysis of the robustness of the financial network over years

After mapping the graph, we can apply graph theory and more precisely robustness and centrality measures to the network. This chapter regroups the results of the measures of robustness: degree distribution, reciprocity, and concentration. The next chapter focusses on the node centrality measures. The measures developed in the literature review highlight the dimensions of the concept of interconnectedness. The purpose is to identify the countries that cause systemic risk (Alves et al., 2013). The static and dynamic analysis consider only amounts of claims above 100 billion USD. It facilitates the identification of central countries and minor the effect of small interconnections.

2.1 Degree distribution

The first indicator in graph theory is the degree, i.e., the number of links incident to the vertex (Kunegis, 2017). For the network constructed, the degree of a country counts the number of times this country has a claim against or has a debt to another country. The degree distribution shows nearly a power long tail distribution in Figure 9. There are a few countries (the United States, the United Kingdom, and France) with a high degree while many others have only a few links above 100 billion USD. According to Boss (2014), it suggests the existence of hubs. The financial system would be exposed to systemic risk in case of the failure of US, UK, or France banks.

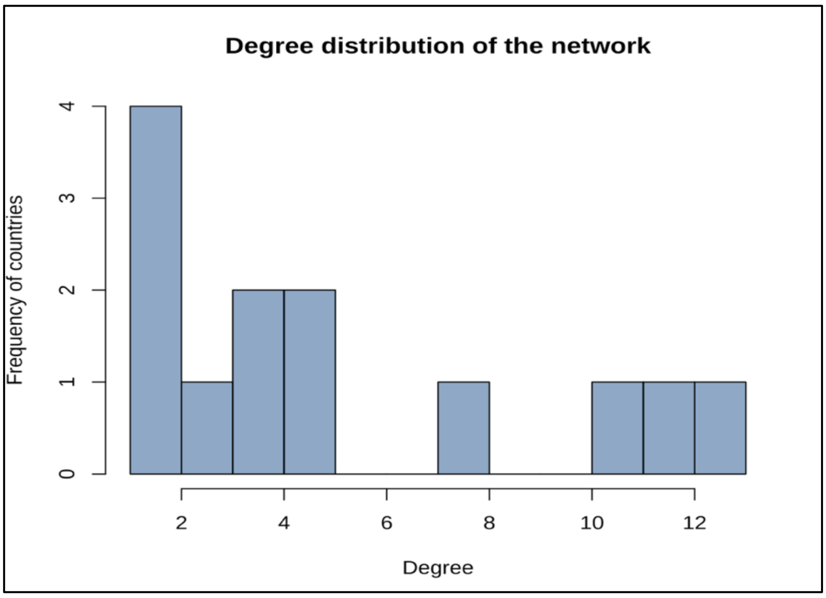

Figure 9: Degree distribution of the financial network of 2020 for countries with claims above 100 000 000 000 USD

2.2 Reciprocity

The reciprocity measure counts the number of links that have a corresponding counter-exposure (Alves et al., 2013). It stands at 59 percent for the financial network in 2011. It stays slightly above 50 percent over the years. This means that, in general, the exposure of one country to another is often counterbalanced by the inverse exposure. A higher reciprocity in the network could lower the systemic risk it causes (Alves et al., 2013).

2.3 Concentration

The Herfindahl-Hirschman Index (HHI) is an indicator of market concentration (Rhoades, 1993). It sums the squared market shares of each institution. In our cross-border network, the number of institutions is reflected by the number of countries. The market share is expressed by the total amount of claims a country owns to the rest of the world. It captures both the number of linkages and the size (or weight) of the linkages. The network constructed presents a HHI around 1050 points for all the periods. According to Lapid et al. (2021), an index over 2500 points indicates a “high concentration market”. A value between 1500 and 2500 indicates a moderate level of concentration and below 1500 a low concentration. This allows us to advance that the cross-border claims network remains with a low level of concentration. This indicator demonstrates that the total amount of claims is well distributed among the countries.

Chapter 3 – Static analysis of the centrality of countries over years

The static analysis of centrality measures aims to identify the central or core countries. Two dimensions of interconnectedness are measured in this chapter: how well the nodes are connected and the influence of the central countries. We recall from the literature review that measures of position in the graph, namely betweenness and closeness, are inappropriate for the cross-border network.

3.1 well connected

3.1.1 degree and strength

After looking at the degree distribution of the whole graph, it is interesting to examine the node degree of each country. The node degree is the most common measure of centrality in graph theory (León-Rincón & Pérez-Villalobos, 2013). In the cross-border network of 2020, the United States appears as the more central node with a degree of 13. It means that the United States has 13 connections (claims or debts) above 100 billion USD with other countries. A high degree for the United States expresses its ability to accentuate the spread of a shock. On the contrary, it allows risk sharing among other countries. The Table 4 ranks the countries by degree over the years.

As it is a directed graph, in and out-degree can reveal other features. In our network, a high degree is mainly explained by the in-degree. The United States presents an in-degree of eight and an out-degree of five while the United Kingdom presents an in-degree of seven and an out-degree of five. On the opposite, France presents a higher out-degree than in-degree. It means that France is more likely to hold claims. von Peter (2007) suggests that a high in-degree demonstrates more the centrality of the country. Thus, even if France has a high degree, it is less central than the United States and the United Kingdom.

This result is in line with the heatmap presented before. The in-degree is distributed like the total degree while the out-degree is more disparate between the countries. Over the years, the ranking of the most central countries in term of degree remains stable with the United States, the United Kingdom, Germany, and France in the top 4. Although the degree measure is simple, it has two major limitations. First, it does not consider the size of the connections.

Second, it does not capture the influence of the other nodes. Thus, other measures could reveal new features of the graph.

Ranking of degree	2011	2016	2020
1	United States (13) United Kingdom (13)	United States (12)	United States (13)
2		France (10)	United Kingdom (12)
3	Germany (11)	United Kingdom (9)	France (11)
4	France (10)	Germany (8)	Germany (8)
5	Netherlands (6)	Spain (3)	Spain (5)
6	Spain (4)	Netherlands (3)	Netherlands (5)
7	Italy (4)	Canada (3)	Italy (4)
8	Ireland (2)	Italy (2)	Canada (4)
9	Sweden (2)	Switzerland (2)	Australia (3)
10	Australia (2)	Belgium (1)	Switzerland (2)
11	Switzerland (2)	Finland (1)	Austria (1)
12	Canada (2)	Sweden (1)	Belgium (1)
13	Austria (1)	Australia (1)	Portugal (1)
14	Belgium (1)		
15	Finland (1)		

Table 4: Rankings of the degree of countries from the network constructed over the years.

Because it is a weighted graph, the analysis can complement the degree indicator with strength measure. The strength exposes the amount of claims (i.e., size or weight) incident to a country. In the network constructed, degree measure and strength measure are correlated. Basically, countries with a high degree presents also a high weight related to other countries. There is no digression in the ability to connect to other countries and the amounts engaged in these connections. The evolution of the country strength from 2011 to 2020 illustrates which countries stand out in Figure 10. The United States exhibits a much greater strength than other countries, following by the United Kingdom. We can see that post-crisis dynamics shows a shrinkage in the strength of the overall amount of cross-border lending until 2016. Since 2016, the strength of the linkages increases for most of the countries.

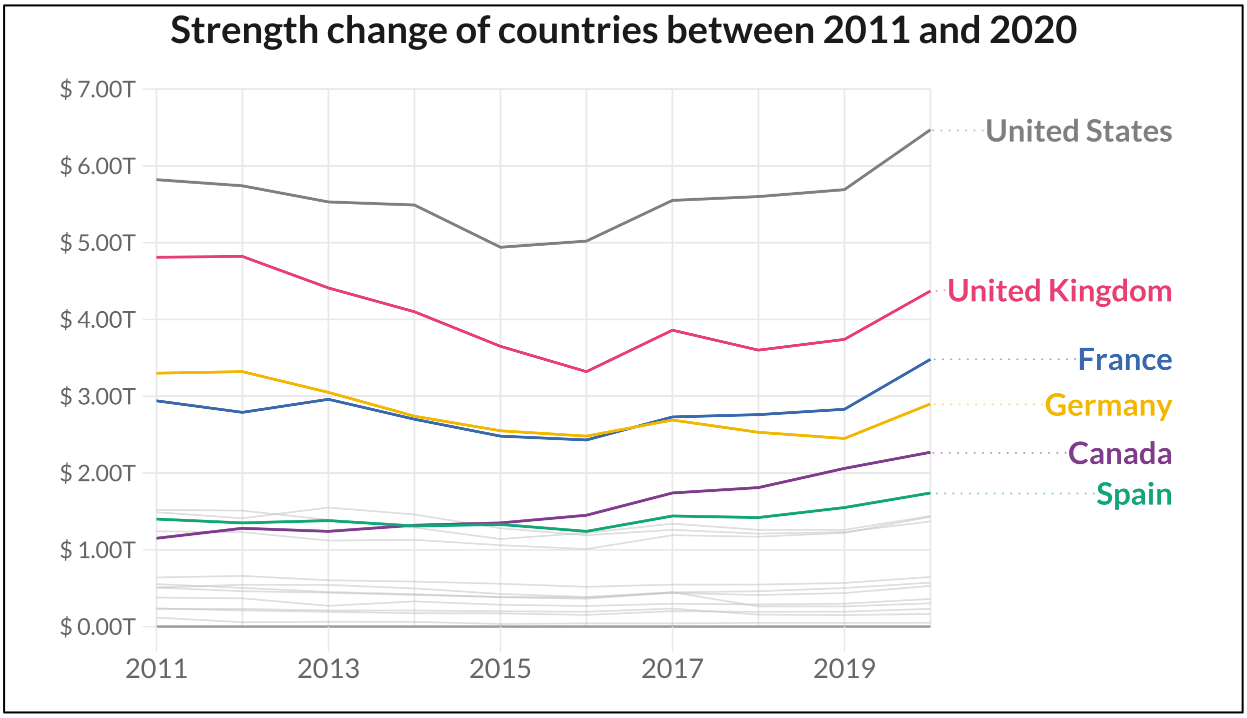

Figure 10: Strength change of countries from the networks constructed from 2011 to 2020. Only the top 6 countries are represented with a color line, the other countries are represented by a grey line.

3.1.2 Node-HHI

At the level of the node, the Herfindahl-Hirschman Index (HHI) measures the concentration of the borrowing and lending activities of countries (Minoiu & Reyes, 2013). For the lending activities (out-HHI), the index sums up the squared shares of the borrowers in the total outflows of each lender (out-strength) and inversely for the in-HHI (Minoiu & Reyes, 2013).

The Figure 11 highlights the ranking of out-HHI calculated for all the countries of the network in 2020. In 2020, Canada, Ireland and Switzerland appear to have the highest levels of concentration in their lending activities. The three countries seem highly dependent to their counterparties and thus, more vulnerable to a negative shock on the financial system (Martinez-Jaramillo et al., n.d.). On the contrary, France and Germany, two countries with a high strength, present a low out-HHI. This difference accounts for the divergent behaviour of countries in response to a shock on a shared counterparty. For example, Canada has a higher dependence than Germany and France on the United States concerning the lending activities. In the scenario of failure of the United States, Canada will suffer larger losses than Germany and France. Thus, it can potentially lead to transmission of failure. According to Martinez-Jaramillo et al. (2014), node-HHI is a relevant index to measure contagion transmission. Therefore, the node-HHI measure reveals another dimension of interconnectedness. Interconnectedness is not only defined by the magnitude of interconnections, but also by the choice of counterparties in the distribution of the interconnections.

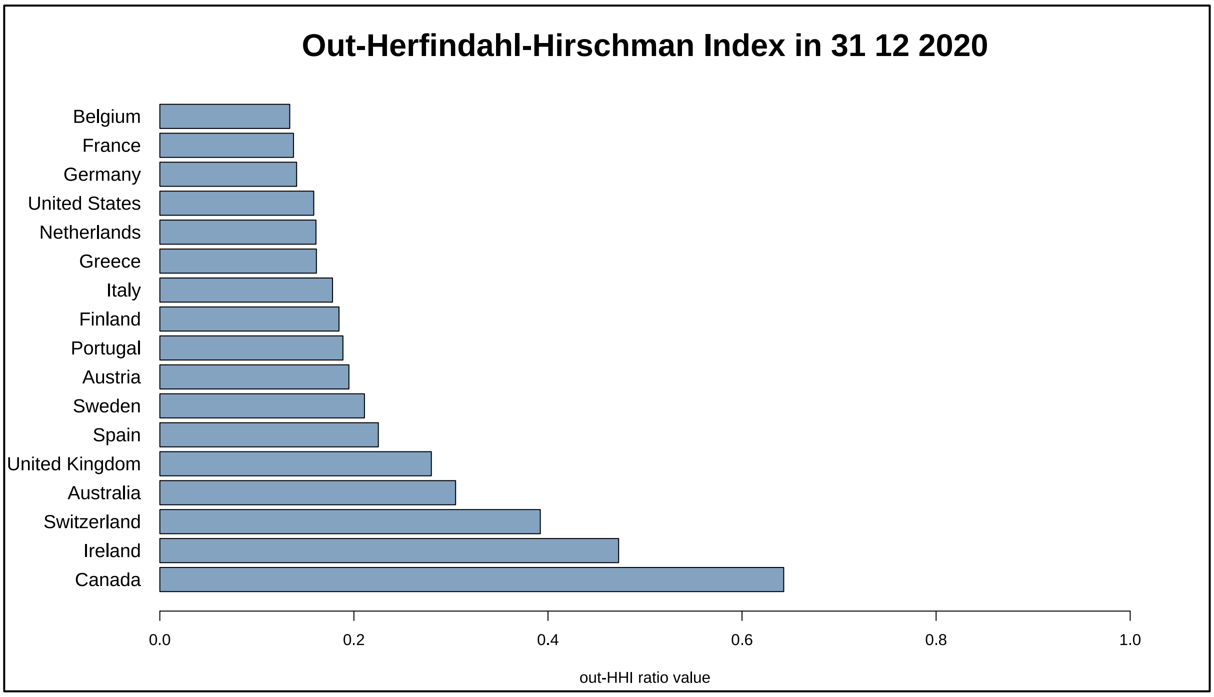

Figure 11: Out-HHI ratio of the 17 countries from the network constructed in 2020.

3.2 Connected to important countries

Now that we have presented the central countries and the importance of the choice of counterparties, we can assess the influence of central countries using the measure of Katz centrality and authority and hub scores discussed in the literature review.

3.2.1 Katz centrality

Katz centrality turns out to be an indicator of the fragility of countries in the transmission of shocks (Cerutti et al., 2017). Indeed, a high level of Katz centrality indicates that the country is strongly connected to important countries. If a shock occurs in one of the "systemic important" countries, countries with high Katz centrality will be more impacted than countries with low Katz centrality (Cerutti et al., 2017). The impact consists of a loss of assets for the financial institution. In our network, the United Kingdom, the United States and Germany count for the top three levels of Katz centrality. The countries with high degree present the highest values of Katz centrality. It reflects the tendency of the central countries to connect to other central countries. Over the years, the ranking of the countries as well as the values does not significantly change.

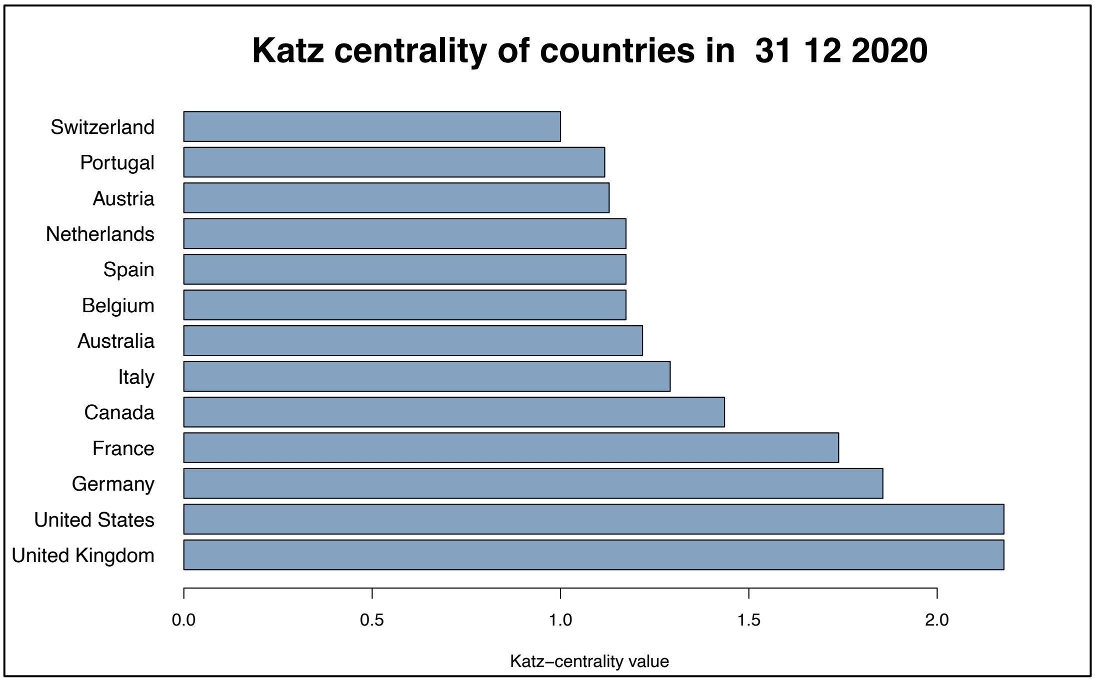

Figure 12: Katz centrality of the countries from the network constructed in December 2020.

3.2.2 Hub and authority scores

Similarly to Katz centrality, the hub and authority scores consider the role of the counterparties, but with a mutually reinforcing relationship between countries (Cerutti et al., 2017). Basically, each country is assigned an authority and hub centrality score "proportional to the sum of the hub and authority scores of its neighbours" (León-Rincón & Pérez-Villalobos, 2013). For Cerutti et al. (2017), a country is a hub if it is the "principal creditor to a large number of borrowers”, or it is an authority if it is ‘heavily dependent on its corresponding hub for cross-border funding”. The authority and hub score can be calculated for the countries in our network, the ranking of hub and authority scores is displayed in Table 5. In 2020, the United States appears as the first authority but not as a hub. It reflects the large amounts it borrows from various countries. Also, Canada appears as the main hub. It seems that the large amount of claims it holds on to the United States, 1 446 570 769 000 USD, explains the ranking. Canada becomes a hub because it is highly connected to the most central countries. In a nutshell, most of the systemic important banks seem to be located in the United States and the United Kingdom.

Ranking of the countries	Hub score	Authority score
1	Canada	United States
2	United Kingdom	United Kingdom
3	France	France
4	Switzerland	Germany
5	Germany	Italy

Table 5: Ranking of the countries based on their hub and authority scores from the network constructed in 2020.

As a result, the concept of interconnectedness cannot be limited at the size of the interconnections. The size or strength of the cross-border claims would designate the United States as the core country followed by the United Kingdom, France, Germany, and Canada (Figure 10). The Katz-centrality ranking is in line with the ranking of strength of the countries. However, some measures point out the important role played by other countries. Proof of this is the highest hub score and out-HHI ratio of Canada. The measures demonstrate that Canada has highly concentrated lending activities and is highly influenced by other countries. To sum up, the graph measures applied on the financial network reveal new dimensions of interconnectedness: the influence that countries have on others, the role they play in transmission of a shock and the central position they have in the network. All the results of the centrality measures can be retrieved in Appendix 3 for the network of 2011, Appendix 4 for the network of 2016, and Appendix 5 for the network of 2020.

Chapter 4 – Dynamic analysis of the systemic role of countries over the years

Some static measures determine the role countries play in transmission of a shock. It would therefore be relevant to simulate the transmission of a shock and compare the results. The chapter four presents the results of contagion simulations. The model of contagion is developed in the research methodology. Starting with the arbitrary failure of a country, it simulates the propagation of asset losses between the countries. The model enables to identify the systemic role of each country as defined by Alves et al. (2013). The systemic role is composed of the systemic fragility and systemic importance of each country. The systemic fragility counts the number of times the country fails when we run the contagion process starting with the failure of other countries. The systemic importance of a country counts, when you shock this country, the number of countries that it causes to fail.

4.1 Systemic role in 2011

In 2011, the United States, the United Kingdom and Germany are the three systemic important countries. If one of these countries fails, it generates the failure of fifteen other countries. Nevertheless, the propagation of losses starting with the failure of Germany differs from the United Kingdom and the United States. While the two English countries cause the failure of several countries in the first iteration, Germany only causes the failure of Greece and Netherlands. Then, the failure of the Netherlands causes the failure of Italy. The process repeats itself to end up with fifteen failed countries. Thus, the ultimate failure of every country is more a result of the cascade default from the contagion process of Germany than the initial failure of Germany. This distinguishes the influence of the systemic important countries.

Another result is the fact that Finland is the only country that never fails in every simulation on the financial network. This reflects the low level of claims Finland holds on all countries, but with a similar level of capital as Belgium, Ireland, Portugal, or Sweden. The contagion process also identifies Greece as a systemic fragile country. Basically, it is due to the negative level of capital it faces in 2011. The systemic role of the countries based on the network of 2011 can be retrieved in Figure 13.

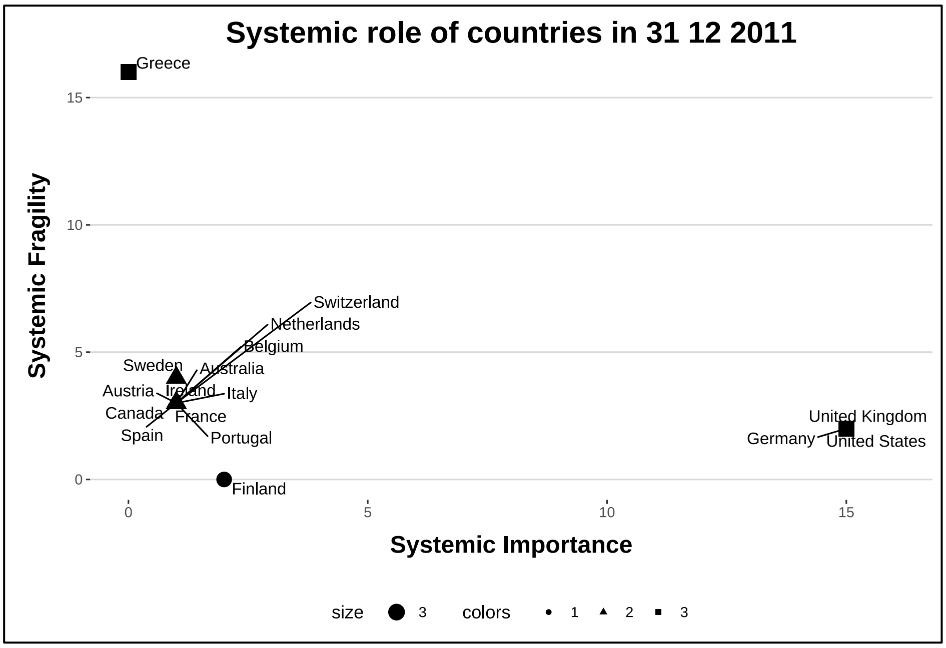

Figure 13: Systemic role of countries in 2011 from the network constructed.

4.2 Systemic role in 2012

In 2012, Greece returns to a positive level of capital and thus became less systemic fragile. The United Kingdom and the United States are still systemic important countries. They both causes the failure of fifteen countries. However, Germany is no longer a systemic important country. Although it still causes the failure of the Netherlands, it does not generate the failure of Italy, which increased its capital from 2011 to 2012. The systemic role of the countries based on the network of 2012 can be retrieved in Figure 14.

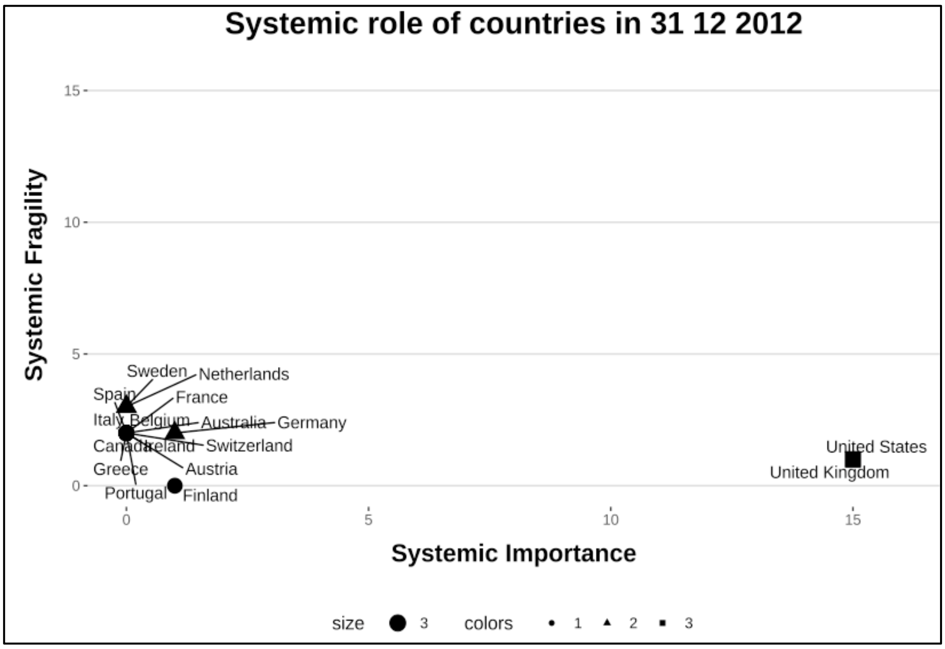

Figure 14: Systemic role of countries in 2012 from the network constructed.

4.3 Systemic role in 2020

In 2020, the United States remains a systemic important country. Over the years, the United States plays a central role in the contagion processes. No other country causes the failure of several countries. It is in line with the evolution of strength. The increased number of connections for all the countries in 2019 and 2020 was covered by the increase of capital for all countries. The systemic role of the countries based on the network of 2020 can be retrieved in Figure 15.

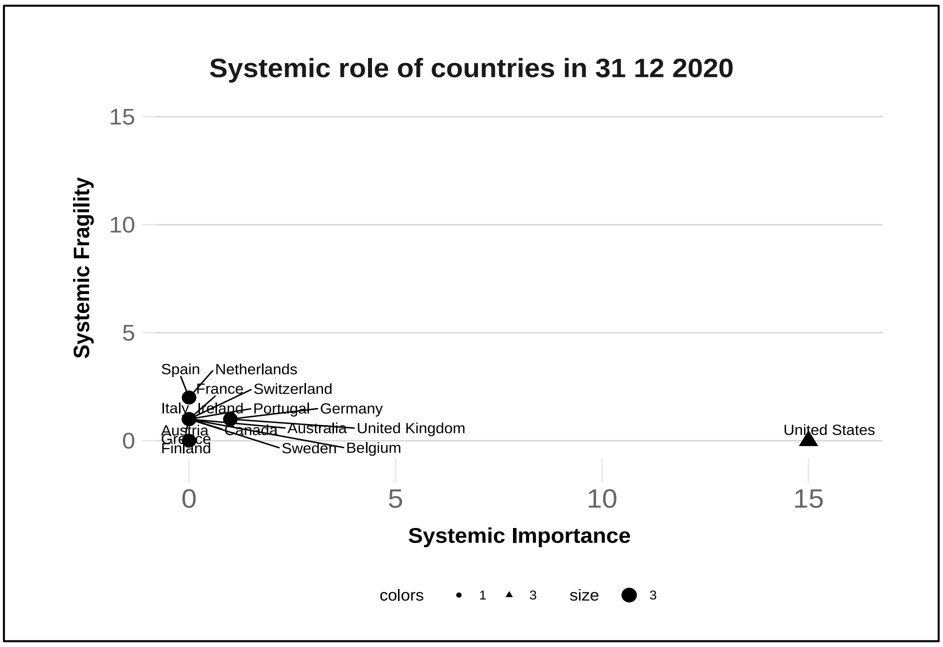

Figure 15: Systemic role of countries in 2020 from the network constructed.

To conclude this empirical research, let's review the definition of interconnectedness from the banking regulation. The Basel Committee on Banking Supervision (BCBS) measures interconnectedness as the amount of intra-financial system assets, intra-financial system liabilities and securities outstanding of each bank (Basel Committee on Banking Supervision, 2022). Similarly, our network considers the cross-border claims of the internationally active banks of each country. The results of the measures presented in the empirical research demonstrate that the measure of BCBS is not sufficient to measure interconnectedness. The state of having links with other participants and their magnitude does not cover all the dimensions of interconnectedness. The empirical research supports the view that the interconnections cannot be dissociated from the graph structure they build. The measures assessed include the structure of the graph. Firstly, the connections can be mapped via a network or a heatmap to highlight the layout of the interconnections. In supplement of the magnitude of the links, it is crucial to account for the capital of the financial institutions. Secondly, we can assess the robustness of the graph through the degree distribution, the measure of reciprocity and the concentration of lending and borrowing activities. Third, we can identify the central players of the graph based on their degree, strength, and node-HHI level. A country can also be central in term of influence it generates on other countries. It is measured by the Katz centrality and the hub and authority scores. Due to the structure of their interconnections, countries could play a systemic role in the financial system.

PART IV – RESULTS AND RECOMMENDATIONS

The static and dynamic analysis have revealed the dimensions of interconnectedness on the financial network. To go further, it would be insightful to understand the relevance of the measures. For this purpose, the first chapter of the results section reports tests of independence between the static and dynamic measures. Independence determines how a static measure can support the results of the dynamic measure (systemic role). The second chapter aims to confront the results of this research with the current economic climate.

Chapter 1 – Independence between measures of interconnectedness

On the static analysis, the measures identify groups of countries that are more central or highly dependent of their neighbours. On the dynamic analysis, the contagion model samples the countries in systemic important and systemic fragile countries. The purpose here is to assess how the contagion-based measure of interconnectedness is related to static measures. According to Hauton and Héam (2016), Exact Fisher tests enable to assess the independence between two measures of interconnectedness. This part presents the results of the exact Fisher tests for the financial network constructed.

1.1 Results of the exact Fisher tests

For each measure of interconnectedness, the countries are split into two groups. The countries that present a value equal or above the mean value of all the countries are separated from the countries with a value lower than the mean value. The exact Fisher test evaluates the independence between each static measure and the systemic role measure. The methodology of the exact Fisher test is explained in the research methodology part. The Table 6 reports the results of the exact Fisher tests for the cross-border network of 2011.

	Systemic important	Systemic fragile
Degree	++	+
Strength	+++	++
Katz centrality	++	+
Hub score	.	.
Authority score	+++	++

Table 6: Results of the exact Fisher test between a measure of the first column and the two measures of systemic role in the network constructed of 2011. If the exact Fisher test rejects the null hypothesis of independence for the two measures, a "+" is displayed. A p-value of the exact Fisher test lower than 1% is illustrated by "+++", "++" when the p-value is lower than 5%, "+" for a p-value lower than 10%, and "." if we cannot reject the null hypothesis.

The exact Fisher tests indicates that we can reject the null hypothesis of independence between groups of countries for the authority score and systemic importance for the financial network of 2011. Still, we have not proven that the two measures are dependent. We can only state that there is no evidence that the measures are independent. We end up to the same conclusions for the strength measure. For the periods of 2016 and 2020, the exact Fisher tests indicate independence between static measures and the systemic role measure (see Appendix 6 and Appendix 7).

To sum up, the authority measure and the strength measure seem good predictors of the systemic role of countries in the network of 2011. This result highlights the contribution of statics measures based on the graph theory for the identification of systemic important financial institutions. Nevertheless, the result is not consistent over the years. Thus, we cannot completely interpret the measures as a reliable estimator of systemic risk in the network.

Chapter 2 – The G-SIBs

In this chapter, the ranking of countries identified as systemic in this empirical research is compared to the list of Global Systemically Important Banks (G-SIBS) reported by the Financial Stability Board (FSB).

The 2021 list of G-SIBs identifies, based on end-2020 data, the banks that are required to meet additional capital buffers and other requirements (Financial Stability Board, 2021). They are assigned to a bucket that determine the corresponding level of additional requirements. As seen in the literature review, the G-SIBs-methodology measures, among four other indicators, the level of interconnectedness of each bank. The measure of interconnectedness consists of the total intra-financial system assets, intra-financial system liabilities and securities outstanding of the bank. In contrast, the empirical research uses graph theory measures and contagion model to assess the level of interconnectedness.

2.1 G-SIBs versus systemic important countries

In 2021, the list of G-SIBs identifies JP Morgan Chase as systemically important on the bucket of 2.5% additional capital buffer (Financial Stability Board, 2021). BNP Paribas, Citigroup, and HSBC are required 2.0% of additional capital buffers (Financial Stability Board, 2021). JP Morgan Chase and Citigroup are American financial institutions. BNP Paribas is French, and HSBC is British. In this list, a slight majority of firms are American. The Table 7 indicates that the United States is the most systemic country in terms of the number of G-SIBs it regroups and in terms of the systemic role identified in the research.

Buckets	2021 List of G-SIBs by their nationality		Ranking of the systemic important countries in 2020	
3.5%				
2.5%	1 American		The United States	1
2.0%	1 American 1 French 1 British		Germany, The United Kingdom	2
1.5%	2 Americans 3 Chinese 1 British 1 German 1 Japanese			3

Table 7: G-SIBs as of November 2021 and their allocated buckets (from FSB, 2021) versus the ranking of systemic important countries from the network constructed of 2020.

2.2 G-SIBs versus authority score

The authority score designates the United States and the United Kingdom as the most interconnected countries. The 2021 list of G-SIBs also identifies these countries as home of G-SIBs (Financial Stability Board, 2021) in Table 8.

Buckets	2021 List of G-SIBs by their nationality		Ranking of the Authority countries in 2020	
3.5%				
2.5%	1 American		The United States (1)	1
2.0%	1 American 1 French 1 British		The United Kingdom (0.23)	2
1.5%	2 Americans 3 Chinese 1 British 1 German 1 Japanese		France (0.13)	3
1.0%			Germany (0.12)	4
			…	

Table 8: G-SIBs as of November 2021 and their allocated buckets (from FSB, 2021) versus the ranking of authority scores from the network constructed of 2020. The value of the authority score is in parenthesis.

Our dynamic measure of systemic importance as well as authority score indicate a large difference between the United States and all other countries. At the same time, there is a large presence of American financial institutions in the list of G-SIBs.

Chapter 3 – Limitations

We should recall that the results of the empirical research are only illustrative of the opportunities of graph theory to assess the level of interconnectedness for a financial network. It does not pretend to identify the countries generating systemic risk. This chapter presents a non-exhaustive list of the limitations of this research.

3.1 Data confidentiality

For confidentiality reasons, internationally active banking groups do not report their bilateral exposures, or at least it is only accessible to supervisors (Cerutti et al., 2012). Data on bank-bilateral linkages can help to precise the losses a shock generates to other institutions. This would enable to construct a solid tool for policy decision in time of crisis (Cerutti et al., 2012). Even more, the regulation consolidates the data at sector-level of different countries. The lack of granular data influences the research in some ways. For instance, it induces interpretation bias as we cannot testify the ultimate financial institution responsible of the loan. The results are therefore interpreted at the country level. This may underestimate the systemic risk generated by banks, as their contribution is counterbalanced by a multitude of other banks and regrouped at country level. To cope with that, the G20 Data Gaps initiative helps to increase the global coverage of locational banking statistics from the BIS to 94 percent in 2017 (International Monetary Fund, n.d.) (International Monetary Fund, n.d.). Literature also investigates other methods to face the lack of data on direct links. For instance, Diebold and Yılmaz (2015) deal with equity returns to measure the financial network connectedness. As depicted in Figure 3, these are indirect links extracted from market data.

3.2 Network measures

The main limitation encountered in this research is the lack of evidence to identify systemic fragilities in the network. Literature identifies Katz centrality as an indicator of the fragility of countries when a shock occurs to the system (Cerutti et al., 2017). However, the interpretation of systemic fragility is solely determined by this graph measure. We still need to determine the interpretative power of centrality measures on the financial system fragility.

To easily interpret the centrality measures, we focus on claims above 100 billion USD. Unfortunately, it diverges the results from the real network.

3.3 Graph and contagion model

The graph constructed does not account for all the direct links that connect financial institutions. An interesting strand of the literature investigates the multiplex network. Aldasoro and Alves (2018) consider different types of exposures and propose measures of the systemic importance. This enables to decompose the contribution of each type of instrument in the measure of interconnectedness.

The contagion model constructed easily communicates on the cascade default process. This leads to simplifications and the omission of some features. Namely, it assumes a loss given default parameter of 100 percent. It also assumes that the banks have no direct response to face the default of another institution. However, the network topology changes when facing crisis (European Central Bank, 2010). In the ideal model, the defaulted loans should be revaluated before impacting another institution (Espinosa-Vega & Solé, 2010).

3.4 Scope of study

We have seen in the chapter three of the literature review that the regulation framework has changed during the period analysed. This research would benefit from including the impact of additional capital requirements in the analysis. It could identify the extent to which this additional capital was involved in determining the values of the centrality measures. And therefore, to what extent it has potentially reduced systemic risk.

Chapter 4 – Policy recommendations

These results demonstrate that there are opportunities for the regulation to better understand the dimensions of financial interconnectedness thanks to graph theory.

The Basel Committee on Banking Supervision has introduced interconnectedness in the regulatory framework via the measure of G-SIBs. However, this measure only accounts for the size of the direct links between financial institutions. It does not consider the influence, the central position, and the role of financial institutions in transmission of a shock.

Our results show that graph theory helps identifying banks potentially causing systemic risk due to the structure of the network. The centrality metrics of countries can be used to complement the risk assessment of countries. In the Financial Sector Assessment Program (FSAP) of Spain in 2018, the IMF already suggests to “incorporate cross-sectoral and cross-border dimensions in monitoring financial stability risks and systemic risks. Interconnectedness analysis could also be incorporated as part of the Financial Stability Review on a regular basis. Different quantitative methodologies could be considered to enhance the monitoring of interconnectedness and systemic risks”. This thesis recommends the reinforcement of the risk assessment by considering the dimensions of interconnectedness found in this research.

CONCLUSION

So far, we have been able to synthesize the measures of interconnectedness in three phases. First, we have defined the concept of interconnectedness thanks to graph theory. We have identified the different types of interconnections between internationally active banks consolidated at country-level. The approach of bank regulation on interconnectedness was also outlined.

Second, we have motivated the approach used to assess interconnectedness. The different data sources available were discussed as well as the appropriate measures to apply on the financial network constructed in the third part.

Third, we have assessed the robustness of the network and the centrality of nodes by an application of graph theory measures. It has emerged that the measures capture different dimensions of interconnectedness. We have seen that interconnectedness cannot be reduced to the magnitude of the interconnections between financial institutions. Interconnectedness can be measured by four types of information: summary statistics, robustness measures, centrality measures, and systemic role of the financial institutions.

With the summary statistics, interconnectedness can be evaluated by a mapping of the interconnections and by the evolution over years of the total amount of these links. Noting that the evolution of the capital of each country must be considered. With the robustness analysis, interconnectedness can be evaluated by the degree distribution of the network, the reciprocity, and the concentration of the network. With the centrality measures, interconnectedness can be measured by identifying specific important countries. On one hand, there are countries with a high centrality in the network, i.e., countries with a high degree, with a high strength and a high level of node-HHI. On the other hand, there are countries that highly influence or are highly influenced by others. It was assessed with the Katz centrality measure, and the hub and authority scores. This demonstrates that the measures of interconnectedness must consider the influence of the financial institutions on each other. With the contagion model, interconnectedness can be measured via the systemic role of each financial institution. Either the financial institutions of one country can cause the failure of other countries, or they can be subject to failure when a shock occurs in another country.

In addition, we have compared the measures of interconnectedness to better understand the scope they cover. We have compared the measures of centrality and systemic role thanks to exact Fisher tests. It indicates that Katz centrality and authority measures could predict the systemic role of countries in 2011 but not in 2016 and 2020.

Therefore, this empirical research enables to synthesize some of the work presented in the literature about interconnectedness analysis. This thesis facilitates the understanding of the opportunities that graph theory generates for financial supervision. In particular, with the current context of war in Ukraine, the financial stability in Europe is threatened by the impacts on energy prices and inflation, without overlooking the direct impact on the population. The war in Ukraine raises the question of how banks integrate the geopolitical context in their distribution of cross-border claims with other countries.

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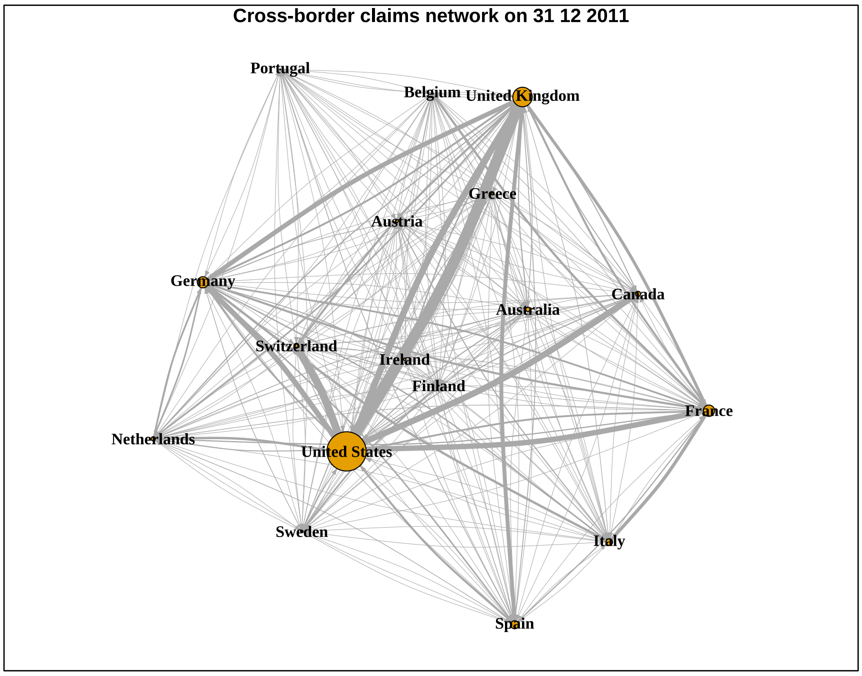
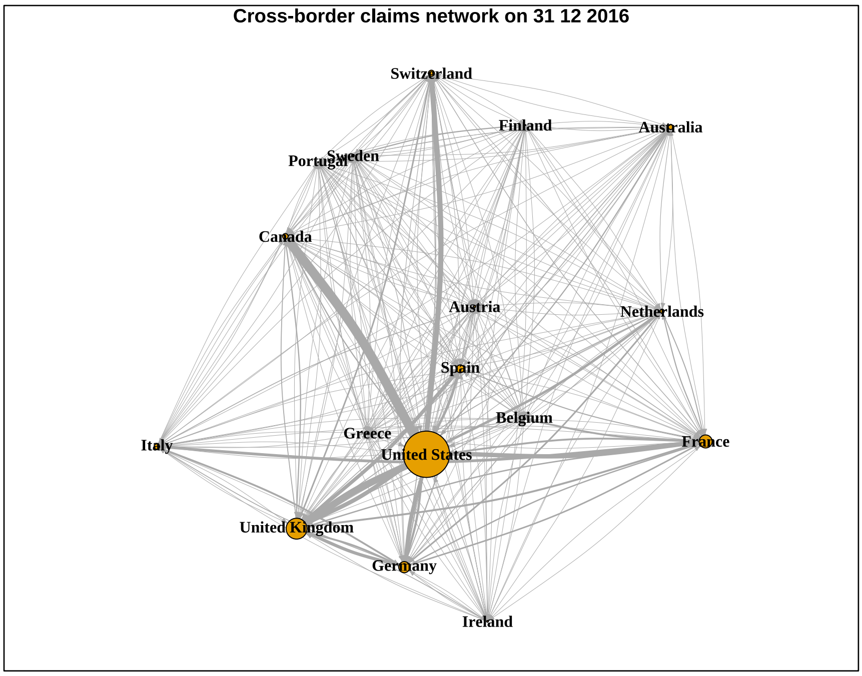
APPENDICES

Appendix 1: Mapping of the cross-border claims network on 31 December 2011.



Appendix 2: Mapping of the cross-border claims network on 31 December 2016.



Appendix 3: Table of results for all centrality measures for the 2011 network.

	degree	strength	in.HHI	out.HHI	alpha.centrality	Katz.centrality	Page.Rank	Hub.score	Authority.score
Austria	1	100840000000	0.287071958299279	0.163893229480202	−100617118913.409	1.13524012731106	0.0231382719264712	0	0.00108687068369329
Belgium	1	223928000000	0.545851319218261	0.151255454845685	1.65129341203571	1.16290259266358	0.0237039558507907	0	0.0484868560455913
Germany	11	2576985000000	0.126302343556132	0.150649304181592	−0.0000000000022130347405504	1.89498680447026	0.110032788195494	0.618356232837695	0.147401074053245
Spain	4	857573000000	0.161275099191176	0.29166090240528	−1.8995083325848	1.35240127311061	0.0274637591627712	0.2731935957659	0.0582931520195434
Finland	1	162637000000	0.600931710956993	0.124777317104209	162637000001	1.1	0.0211138505156801	0	0.00514937115773248
France	10	2395663000000	0.175157736050736	0.147977558643569	0.0000000000290849474847141	1.6290259266358	0.0901201462122056	0.584688121289424	0.164842506668778
United Kingdom	13	4119758000000	0.145950314490008	0.260060228910083	0.0000000000225874378054824	2.20767109957833	0.243509551813107	1	0.306249877789742
Ireland	2	265226000000	0.200418670340028	0.658194839022522	3.88856899697069	1.22076710995783	0.0281121755577824	0.0353131506169325	0.0473595497314238
Italy	4	801573000000	0.291461151242756	0.23266251012456	−0.997789755200409	1.35240127311061	0.0363281723433637	0.029104067217557	0.102637342014971
Netherlands	6	942330000000	0.17094404897847	0.154402753365925	1.34171447604712	1.57316838306844	0.0449440196199814	0.248209628324371	0.116734497050679
Sweden	2	263632000000	0.189841321938953	0.259371703259596	1	1	0.0138508444366	0.0854955456787398	0.000000000000000243678711676664
Australia	2	219382000000	0.206760564137899	0.268259035584075	−1.28307551377289	1.21876013623094	0.033193372514956	0.0288283683091923	0.00847517235357625
United States	13	5288448000000	0.156923754068232	0.209495399848264	−0.0000000000212852342768843	2.18760136230944	0.257414398889832	0.213361150456931	1
Switzerland	2	969667000000	0.158004935866449	0.455587050520289	1	1	0.0138508444366	0.693749983518717	0.000000000000000243678711676664
Canada	2	722116000000	0.203551296327043	0.596778596212787	−1.28667271836568	1.21876013623094	0.0332238485243652	0.516072238251338	0.0084885258010339

Appendix 4: Table of results for all centrality measures for the 2016 network.

	degree	strength	in.HHI	out.HHI	alpha.centrality	Katz.centrality	Page.Rank	Hub.score	Authority.score
Belgium	1	184626000000	0.503164277229543	0.12590213911035	−0.177229296491788	1.1574055146042	0.0270697564359873	0.000000000000000524740381370873	0.044145755572295
Germany	8	1726295954000	0.131056465919967	0.167176050535735	−0.509353019539117	1.78449251233369	0.154175607739973	0.509075873229604	0.132559230566618
Spain	3	709839304000	0.165659567797126	0.257315882751724	0.35297214026081	1.1574055146042	0.0216967030654695	0.371743344650584	0.0242633561954603
Finland	1	114332237000	0.450130065802527	0.143232594920548	114332237001	1.1	0.028008505096967	0.000000000000000524740381370873	0.000000000000000912006797638179
France	10	1895465000000	0.144460264173456	0.141625323805305	−0.0000000000063762920525375	1.57405514604201	0.110058680762948	0.56674340256252	0.084601589833994
United Kingdom	9	2552519211000	0.138658549719091	0.267450989523976	0.496301433628956	1.87996558701577	0.20937907982095	0.76433342340685	0.285151599044055
Italy	2	459363128000	0.309690561474305	0.205942293246714	−0.773960964520562	1.1574055146042	0.0331170261677228	0.0251993604977752	0.0665230192349904
Netherlands	3	517115351000	0.167588685043149	0.171899198290539	0.306093641090554	1.1574055146042	0.0221717690696552	0.284622033982644	0.0260212862538893
Sweden	1	114332237000	0.156314863799119	0.248262032253464	1	1	0.015139732484847	0.000000000000000377810889395006	0.00000000000000006607155522159
Australia	1	112339928000	0.204022838665705	0.305912366848788	1	1	0.015139732484847	0.117888826187215	0.00000000000000006607155522159
United States	12	4486968845000	0.152331787708261	0.159408774829974	−0.0000000000179946905012981	2.07609336107069	0.302577995863317	0.170889903544054	1
Switzerland	2	667474677000	0.162040348209849	0.379442766797388	1	1	0.015139732484847	0.585427350329565	0.000000000000000013214311044318
Canada	3	1147956622000	0.225668060208986	0.668214159069985	−1.02218933446437	1.20760933610707	0.046325678522468	1	0.00810220932448263

Appendix 5: Table of results for all centrality measures for the 2020 network.

	degree	strength	in.HHI	out.HHI	alpha.centrality	Katz.centrality	Page.Rank	Hub.score	Authority.score
Austria	1	114686801000	0.312034018773331	0.194881419003513	−17755506824.0684	1.12912249907437	0.0274952537509602	0	0.000734740097761568
Belgium	1	296029000000	0.602532803486775	0.13390282303105	−1.29810740432544	1.17384090067606	0.0322244537308028	0	0.0463002637748836
Germany	8	2019767568000	0.138958344288066	0.140973119897966	−0.106246249125386	1.85579309682816	0.13764404108522	0.293611488950791	0.120754870073977
Spain	5	1145367955000	0.166760252111599	0.225186582147288	0.00000000001719319247356	1.17384090067606	0.0234546276488769	0.280139368181873	0.0201477131597004
France	11	2833024427000	0.167397949455883	0.137744807124592	−0.00000000000776311579043081	1.73840900676056	0.120342395499267	0.463720607197358	0.126414740099547
United Kingdom	12	3761268187000	0.136489359218703	0.279840697885151	0.00000000000559420238190792	2.17722803505067	0.189231255590164	0.736908078643887	0.23040485627314
Italy	4	832086851000	0.317526212476349	0.177976651819372	−0.154817351868315	1.29122499074366	0.0386295255992707	0.0189944836224089	0.0691473256285555
Netherlands	5	810063596000	0.151317050488711	0.16091812078712	0.163292627624556	1.17384090067606	0.0223511812790948	0.220853870350201	0.016857119165329
Portugal	1	127420412000	0.533066545956669	0.188723578230531	3.19076366857631	1.11738409006761	0.0191973757137395	0	0.0120394309729434
Australia	3	333379135000	0.214939039570006	0.304820887827882	−0.673324416014737	1.21772280350507	0.0385592848224985	0.0886244931951817	0.00518760111584158
United States	13	5977421374000	0.180739011240193	0.158633076713805	−0.0000000000136319578887511	2.17722803505067	0.277803421354429	0.125300343892265	1
Switzerland	2	741416299000	0.15501250924291	0.392107290669833	1	1	0.0166984246704752	0.418969636718099	0.00000000000000035733685608082
Canada	4	1932805307000	0.226059300714565	0.642962247307395	−0.361233355169162	1.43544560701013	0.0563687592552008	1	0.0393395459537912

Appendix 6: Results of the exact Fisher test between a measure of the first column and the two measures of systemic role in the network constructed of 2016. If the exact Fisher test rejects the null hypothesis of independence for the two measures, a "+" is displayed. A p-value of the exact Fisher test lower than 1% is illustrated by "+++", "++" when the p-value is lower than 5%, "+" for a p-value lower than 10%, and "." if we cannot reject the null hypothesis.

	Systemic important	Systemic fragile
Degree	.	.
Strength	.	.
Katz centrality	.	.
Hub score	.	.
Authority score	.	.

Appendix 7: Results of the exact Fisher test between a measure of the first column and the two measures of systemic role in the network constructed of 2020. If the exact Fisher test rejects the null hypothesis of independence for the two measures, a "+" is displayed. A p-value of the exact Fisher test lower than 1% is illustrated by "+++", "++" when the p-value is lower than 5%, "+" for a p-value lower than 10%, and "." if we cannot reject the null hypothesis.

	Systemic important	Systemic fragile
Degree	.	.
Strength	.	.
Katz centrality	.	.
Hub score	.	.
Authority score	.	.

Abstract: The events following the collapse of Lehman Brothers in 2008 highlighted the multiple connections that exist between financial institutions. It has challenged the understanding of financial actors and banking supervisors of the risks arising from these connections. Interconnectedness became a subject of interest in the financial literature. In particular, the Graph Theory allows to map the interconnections between financial institutions. After a review of the different measures applied in Graph Theory, we will assess the relevant measures for the cross-border claims network. Then, we will simulate the transmission of a shock to the network between 2011 and 2020. The measures will also be compared to assess their relevance. This will lead to the conclusion that interconnectedness should not be only defined by the size of the exposures but also by the fragility of the structure that these interconnections generate.

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