

Design, modelling and prototyping of an electrodynamic self-bearing axial-flux permanent magnet motor

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Abstract

Two topics have been the subject of numerous researches in the field of rotating machineries those past years. On the one hand, electrodynamic magnetic bearings (EDBs) ensure a contactless guidance of the rotor relying on the interaction between the permanent magnets (PMs) magnetic fields and the currents they induce in the conducting parts of the bearing structure. On the other hand, axial-flux permanent magnet (AFPM) machines allow to drive a PMs rotor disk by injecting three-phased currents in the windings. Topologies of electrodynamic thrust bearings (EDTBs) and AFPM machines present a significant number of similarities. Starting from this observation, this Master's thesis introduces a new machine topology combining both a bearing and motor behavior to obtain a self-bearing AFPM machine in a single winding.

First, a model describing the axial and rotational dynamics of such a machine is developed. This model directly relates the axial displacement to the force developed by the EDTB and the rotational motion to the driving torque generated by the AFPM machine and the drag torque arising from the EDTB. The obtained model is constituted of eight non-linear first-order differential equations with a set of twelve parameters. The calculations required to determine those parameters using static FEM simulations are also detailed. Furthermore, this model can be used to obtain several results regarding the quasi-static and dynamic behavior of the self-bearing AFPM machine. Based on the linearization of the obtained model, this work assesses the stability properties of the system under constant rotational speed, based on a root locus analysis. The evolution of the quasi-static equilibrium points as a function of the spin speed is also discussed for a fixed axial displacement or a given external force applied to the rotor. Moreover, this model allows to simultaneously obtain the axial and rotational dynamic behavior of the self-bearing AFPM machine in various situations, e.g. at start-up and when subjected to external force perturbations or variations of the reference spin speed. A last use of the model is to determine optimal dimensions and parameters for the machine, based on a set of criteria describing the EDTB and AFPM motor performance. In this work, this purpose is achieved through a parametric analysis, but this model can be used to conduct a conventional optimization on the machine parameters.

Finally, the prototype of a self-bearing AFPM motor and the associated testbench are developed as a proof-of-concept. The functioning of the EDTB and AFPM motor alone has been partially validated based on experimental measurements, but the fact that the prototype is not completely functional has kept us from determining the performance of the complete system. However, critical issues in the development of such a machine have been highlighted and solutions to avoid and solve them have been proposed. This Master's thesis thus paves the way for further works combining a fully passive magnetic guidance and a motor behavior within the same device.

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List of symbols

α	Angular position of the rotor	[rad]
δ	Phase angular positions	[rad]
δ_0	Angular position of the first phase	[rad]
δ_k	Angular position of the k-th phase	[rad]
Φ_0	Constant part of the magnetic flux linkage amplitude	[Wb]
Φ_L	Lower windings magnetic flux linkages	[Wb]
Φ_M	Motor flux linkages	[Wb]
Φ_S	Suspension flux linkages	[Wb]
Φ_U	Upper windings magnetic flux linkages	[Wb]
ω	Spin speed at equilibrium	[rad/s]
ω_α	Rotational damping frequency	[rad/s]
ω_m	Motor natural electrical frequency	[rad/s]
ω_s	EDTB natural electrical frequency	[rad/s]
ω_z	Axial damping frequency	[rad/s]
$\mathbf{A}$	Magnetic potential vector	[T/m]
C_α	Rotational damping	[kg·m ² /s ²]
$C_{\max}$	External axial damping required to stabilize the EDTB	[kg/s]
C_z	External axial damping	[kg/s]
F_D	Detent force	[N]
F_{ext}	External force	[N]
F_{ED}	Electrodynamic axial force	[N]
I_d	Direct-axis current	[A]
$\mathbf{I}_L$	Lower windings currents	[A]
$\mathbf{I}_M$	Motor currents	[A]
I_q	Quadrature-axis current	[A]
$\mathbf{I}_S$	Suspension currents	[A]
$\mathbf{I}_U$	Upper windings currents	[A]
$\mathbf{j}$	Current density vector	[A/m ²]
J	Inertia of the rotor	[kg·m ²]
k	Axial stiffness of the EDTB	[N/m]
k_∞	Axial stiffness of the EDTB at infinite spin speed	[N/m]
k_D	Detent force stiffness	[N/m]
k_{bearing}	Bearing performance criterion	[s]
k_{motor}	Motor performance criterion	[(Nm/s) ^{1/2}]
k_{tot}	Total axial stiffness of the EDTB	[N/m]
$k_{\text{tot},\infty}$	Total axial stiffness of the EDTB at infinite spin speed	[N/m]

K_Φ	Coefficient of proportionality between the magnetic flux linkage amplitude and the axial displacement	[Wb/m]
$\mathbf{L}$	Matrix of self and mutual inductance coefficients for the upper and lower windings	[H]
m	Mass of the rotor	[kg]
$\mathbf{M}$	Matrix of mutual inductance coefficients between upper and lower windings	[H]
N	Number of phases	[-]
p	Number of pole pairs	[-]
R	Winding resistance	[Ω]
T_{ext}	External torque	[Nm]
T_{ED}	Electrodynamic torque	[Nm]
T_M	Motor torque	[Nm]
T_D	Drag torque	[Nm]
$\mathbf{u}$	Phase voltages	[V]
u_d	Direct-axis voltage	[V]
u_k	Voltage of the k-th phase	[V]
u_k^{emf}	Emf induced on the k-th phase	[V]
u_q	Quadrature-axis voltage	[V]
z	Axial displacement	[m]

For some decades now, the transition from nuclear and fossil fuel energy to renewable energy sources, such as wind, solar and geothermal energy, has become a key concern of our society. The pursued objective is to reduce greenhouse gas emissions and to stop the global warming, but it also comes from an effort to produce clean energy, free of any waste damageable for our planet. Lately, this concern has been paired with intensive researches in many technical fields to design systems with high performance and reliability but low power consumption, both to enhance their durability and to reduce their energetical needs. Some researches even go further, by proposing self-powered systems relying on energy-harvesting, such as wireless nodes for the Internet-of-Things. As far as rotating machineries are concerned, the main trend aims at reducing the energy losses related to the bearings. Indeed, those mechanical components ensuring the guidance and the support of the rotor, i.e. the moving components of the machine, with respect to the stator, i.e. the stationary elements of the machine, come with significant losses mostly related to mechanical friction. Several types of mechanical bearings exist, among which the most common are the journal, ball and roller bearings.

Other types of bearings, such as hydrostatic and magnetic bearings, ensure quasi or full non-contact guidance and support of the rotor. One of their major advantages is that mechanical friction and wear are avoided, resulting in a longer lifetime for the machine, but also in the possibility to reach higher rotational velocities. However, in the case of magnetic bearings, the speed is limited by the mechanical resistance of the components constituting the machine, but also by eddy currents and hysteresis losses, and sometimes air losses, for very high rotational speed applications. The absence of mechanical friction also results in the fact that lubrication is no longer required. Furthermore, this allows using rotating machinery in domains such as food, semiconductor and biomedical industry, as contamination by particles resulting from the mechanical wear is no longer a problem [1].

Active and passive magnetic bearings

Among magnetic bearings, two main categories can be identified : the active ones (AMBs) and the passive ones (PMBs). A vast majority of current industrial applications using magnetic bearings, e.g. energy storage flywheels, high-speed rotors and rotating pumps, rely on AMBs. They are composed of a ferromagnetic rotor, maintained in levitation by electromagnets. The currents supplied to these electromagnets, usually through a power amplifier, depend on the position of the rotor, measured by gap sensors. A typical AMB structure is depicted in Fig. 1. Actively controlled magnetic bearings have the advantage of flexibility, as the values for the stiffness and the damping of the bearing can be varied widely within physical limits of the system and can thus be adapted

to various applications. Moreover, high stiffness and power densities can be obtained, compared to passive magnetic bearings [2]. Furthermore, AMBs allow to stabilize all the degrees of freedom of the system, something which cannot be achieved using only passive magnetic bearings. However, a drawback of AMBs is that they require additional components for the control system, e.g. sensors and actuators, as well as an external power supply. In addition, to reach a high precision positioning of the rotating elements, high resolution sensors are required, as the precision of the control is directly linked to that of the sensor.

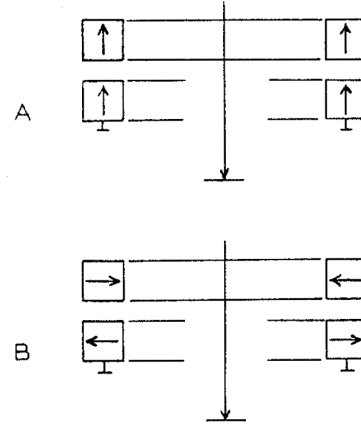
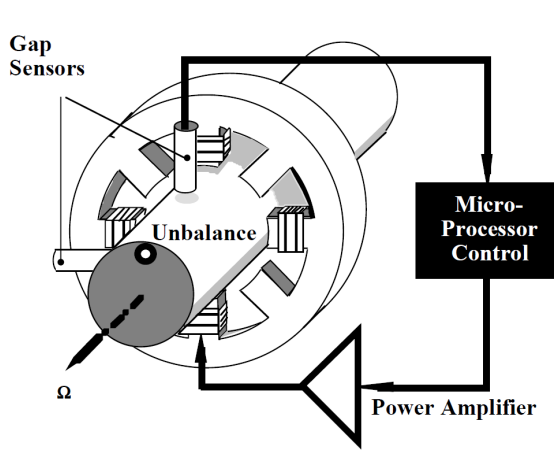


Fig. 1: Typical structure of a radial AMB, [1]. Fig. 2: Typical configurations of a radial PMB, [3].

Passive magnetic bearings (PMBs) constitute the second category of magnetic bearings. They do not require any control and usually rely on passive magnetic phenomena to stabilize the rotating elements of the machine, i.e. interactions between permanent magnets (PMs) magnetic fields or fields generated by DC currents. Electrodynamic bearings also fall into this category and are based on the interaction between PMs magnetic fields and currents induced by those fields. Fully passive magnetic bearings are only based on permanent magnets [3] [4], but are unable to stabilize all the degrees of freedom of the system, as stated by Earnshaw's theorem [5]. Indeed, stabilizing one degree of freedom and thus, having a positive stiffness in the stabilized direction, results in a negative stiffness in the other directions, corresponding to an unstable equilibrium. They typically rely on ring-shaped magnets, either radially or axially polarized, and are based on repulsion or attraction forces, as shown in Fig. 2. The main advantage of PMBs is that they can lead to energy savings, given that no sensing, control and actuation system is required, which also results in a higher reliability. However, the stiffness and damping of such devices are fixed, and the cost of the PMs can become an issue if the volume of magnets is too important. Among PMBs, two different types can be found: radial PMBs, stabilizing the transverse degrees of freedom [6] [7], and axial PMBs, stabilizing the axial degree of freedom [3] [8].

Researches on how to circumvent Earnshaw's theorem have led to two different fields of research. On the one hand, superconducting bearings have been used, evading Earnshaw's theorem due to their diamagnetic properties, but requiring the use of cryogenic system to reach their functioning temperature. On the other hand, electrodynamic magnetic bearings (EDBs), relying on electrodynamic force generation, have been developed [9] [10] [6] [7]. In [9], stabilization against unstable degrees of freedom is accomplished by using periodic Halbach arrays of PMs. These configurations represent optimally efficient ways to assemble PMs, creating a strong periodically varying magnetic field at one face of the array while nearly canceling the field on the back face of the array. The complete structure of this transverse stabilizer is depicted in Fig. 3. The periodic magnetic field generated by the rotating Halbach arrays then produces a time-varying flux in each inductively loaded circuit and, consequently, induces currents in those circuits. These

currents, interacting with the magnetic field of the array, exert a transverse restoring force on the rotating magnet array. As for classical PMBs, radial and axial EDB topologies have been developed [10].

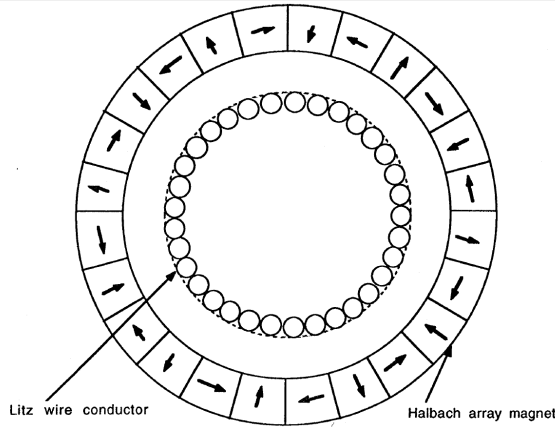


Fig. 3: Schematic drawing of an Halbach array stabilizer, [9].

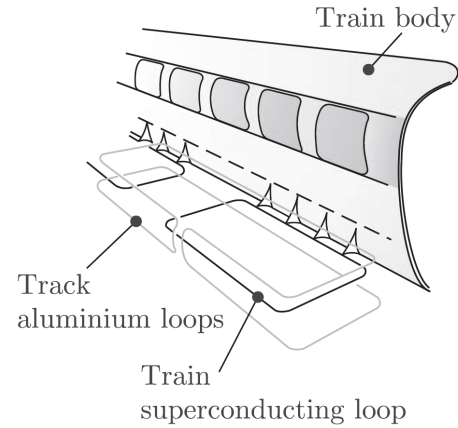


Fig. 4: Illustration of the null-flux principle for the levitation of ground vehicles, [10].

A drawback of such EDBs is that currents are induced in the coils by the time-varying magnetic flux of the PMs even when they are not needed, leading to significant Joule losses. A way to solve this problem is to rely on the null-flux principle, proposed by J. Powell and G. Danby in 1967 for the stabilization system of ground vehicles [11], more precisely high-speed trains. The proposed suspension is composed of an array of magnets with alternating polarity, attached to the train, and a set of stationary, vertically oriented coils. The magnetic flux linked by the coils varies linearly with the levitation height and passes through zero at the equilibrium position. A well-known application for the null-flux suspension is the Japanese MagLev trains [12]. As explained in [13] and [7], this concept has only been extended recently to rotating machineries and mostly relies on the symmetrization of the different topologies. In this particular case, when the rotor is at its equilibrium position - either axial or radial, depending on the type of bearing - the PMs magnetic flux linked by the coils is equal to zero. This means that no currents are induced in the coils, resulting in no restoring force being applied to the rotor. This case is depicted in Fig. 5. However, when the rotor is away from its equilibrium position, the magnetic flux linked by the coils is no longer equal to zero. This results in an electromotive force, inducing currents in the stabilizer coils. Those currents, by interaction with the magnetic field of the PMs, exert a restoring force on the rotating elements of the machine, as shown in Fig. 6.

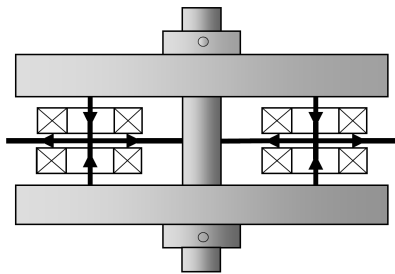


Fig. 5: Null-flux axial suspension at equilibrium position, [14].

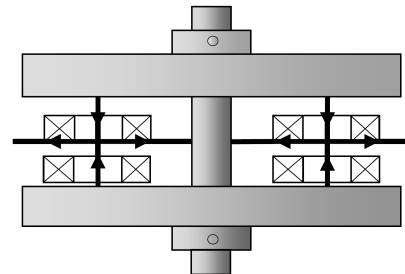


Fig. 6: Null-flux axial suspension away from its equilibrium position, [14].

Topologies for electrodynamic thrust bearings

As far as the electrodynamic thrust bearings (EDTBs) are concerned, the first topology of a null-flux EDTB was introduced in 1972 by G. Sacerdoti, A. Catitti and L.-L. Soglia in a patent [8]. The axial balancing system, depicted in Fig. 7, is composed of two identical static magnetic elements, composed of PMs (14a) and (14b) placed on back-irons (13a) and (13b). The PMs of the stationary elements are in even number, separated by non-magnetic material and arranged in a starlike pattern, as shown in Fig. 9. They are axially magnetized, but their polarity changes direction from one element to the adjacent ones, i.e. the PMs are arranged with an alternation of north and south poles. A rotating conductive disk (15) is placed between those stationary magnetic elements. A relative motion between the disk and the PM elements, forming a series of magnetic circuits by closure of the field across the back-irons (13a) and (13b), induces eddy currents in the disk. These currents in turn generate magnetic fluxes which oppose those produced by the magnets. Therefore, a reciprocal repulsion force is exerted between the rotating disk and each set of PMs. At the equilibrium position, those repulsive forces are balanced and the air gaps between the disk and the PMs are the same. This patent also introduces the dual topology for the proposed EDTB which, however, is not null-flux. Indeed, in Fig. 8, the rotating element is a PMs arrangement (21) and the stationary elements are conductive disks, (20a) and (20b) respectively. The principle remains the same, but an advantage of this topology is that the PMs volume and thus the cost is reduced while the same stabilization effect is achieved. The other degrees of freedom of the system are stabilized using classical PMBs.

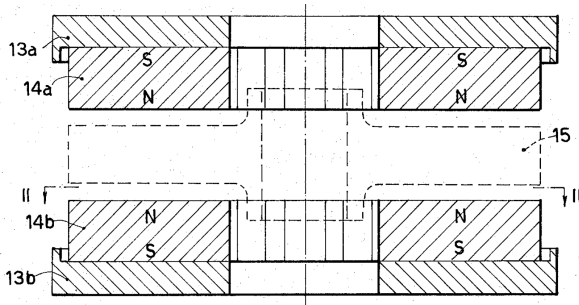


Fig. 7: Axial balancing system for a self-centering rotary magnetic suspension device, [8].

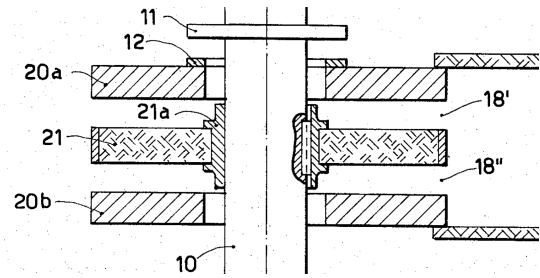


Fig. 8: Dual topology of the axial balancing system proposed in Fig. 7, [8].

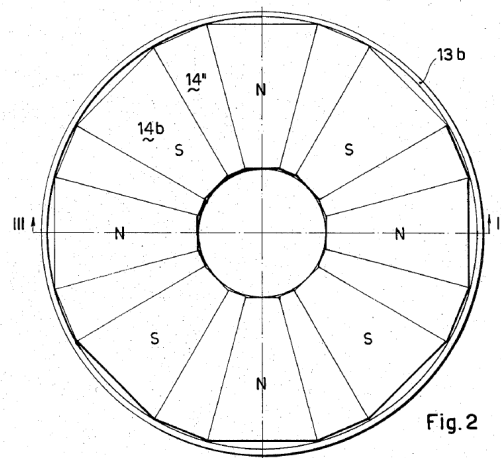


Fig. 9: Starlike arrangement of the PMs in the stationary elements of the EDTB, [8].

A second EDTB structure was proposed in 1996 by R. F. Post in [15]. As can be seen in Figs.

10 and 11, this topology is quite similar to the one proposed in [8] in terms of functioning, but relies on different technologies. The stationary element is a planar array of short-circuited Litz wire conductors, possibly inductively loaded, and sandwiched between two planar Halbach arrays, attached to the rotor and placed in opposition, such that their magnetic fields cancel each other midway between them, resulting in a null-flux magnetic suspension. In this case, the time-varying magnetic field due to the rotation of the Halbach arrays induces eddy currents in the conductors when the rotor is not centered axially. Those induced currents will then interact with the magnetic field of the rotating Halbach arrays, resulting in a high axial positive stiffness when the rotor is rotating fast enough. Litz wire, i.e. a conductor made up of a bundle of insulated fine wires, specially braided so that each wire in that conductor experiences the same average flux, is used to limit the eddy current losses, resulting from the fact that the conductors are immersed in a strong time-varying magnetic field. Furthermore, the radial and tilting degrees of freedom of the rotor are stabilized using classical PMBs. The axial stiffness of those elements is thus negative, according to Earnshaw's theorem, and must be compensated by the axial stiffness of the EDTB.

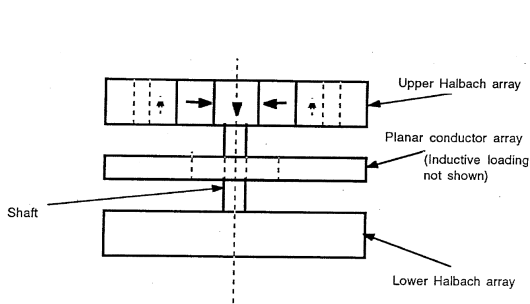


Fig. 10: Side view of the EDTB structure proposed in [15].

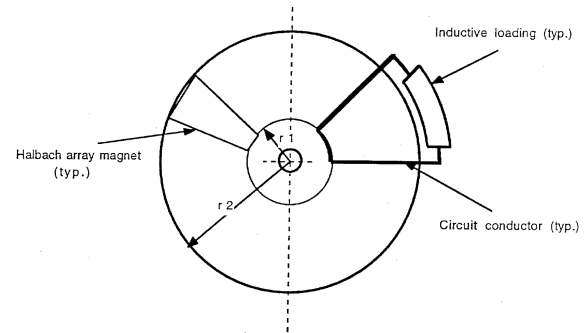


Fig. 11: Front view of the EDTB structure proposed in [15].

The same topology has been studied in more recent years by J. Sandtner and H. Bleuler in [14] and by F. Impinna and J. G. Detoni in [16], with the difference that two set of short-circuited coils are used instead of a single one. It has been successfully integrated in a complete passive magnetic suspension in [14] and models analyzing its dynamic and quasi-static behavior have been developed in [16]. In [17], a similar topology with two stationary stabilization coils and an angular shift of one fourth of a wavelength is used. Those coils have the same number of poles as the Halbach arrays and the addition of a second coil results in a reduction of the ripple on the restoring force.

Quite a different EDTB topology was introduced by J. Sandtner and H. Bleuler in [18] in 2006. It is depicted in Fig. 12. The stator, located at the center of bearing, is constituted by distributed windings, i.e. several layers of axial conductors, arranged to form a conductive ring around the laminated core. Those conductors are connected at both ends, in order to create rectangular loops. The rotor is external to the bearing structure and is composed of two PMs arrangements. They are radially magnetized, with alternating polarities from one PM to the adjacent ones. Those two structures are concentric and have an angular shift between each other, such that the magnetic field of the first one is maximal when the magnetic field of the second one is minimal. When the rotor is at its equilibrium position, the net magnetic flux of the PMs arrangements linked by the rectangular loops of the stator is null. Indeed, exactly half of the flux penetrates inward in and the other half of the flux outward of the stator copper structure, which means there are no induced currents in the stator coils and thus no restoring force applied to the rotor. Consequently, this EDTB topology respects the null-flux principle. However, when the rotor is not centered axially, the net magnetic flux across the loops is no longer equal to zero, as the proportion of the magnetic flux penetrating inward in the copper structure is either less or more important than the proportion of the flux going outward of it. This results in currents being induced in the stator copper structure and an axial restoring force being applied to the rotor. However, a significant advantage

of this topology is that, for a given speed and position, the eddy currents created within the copper structure rotate synchronously with the rotor and are consequently not pulsating. This also means that the restoring force is not pulsating. Furthermore, it should be noted that the copper structure length must be smaller than the PMs length, such that its ends remain in a region where the magnetic field is radial.

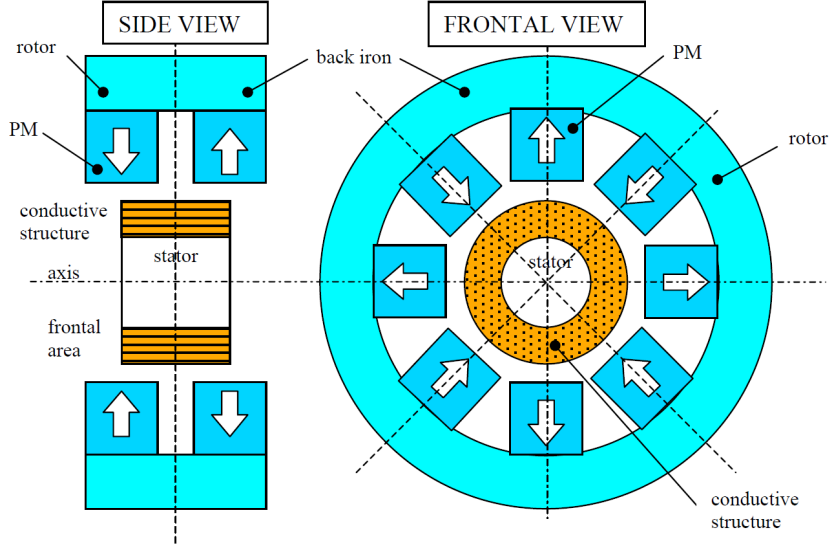


Fig. 12: Front and cross-sectional view of the EDTB proposed in [18].

A last EDTB topology, combining a bearing and a drive unit, was proposed by H. K. Asper in [19] in 2003. A front and side view of the EDTB structure are depicted in Fig. 13 and an exploded view in Fig. 14. The rotating element is made of two PMs arrays with opposite direction polarities from one PM to the adjacent ones, separated by a non-magnetic wall (6), as shown in Fig. 13. This element is located between two stationary set of coils (L), wound on coil holders (5) placed on soft iron rings (7). The coils of both stationary elements are connected in series. A similar topology has been used by J. Van Verdeghe in [20] in 2016, the innovation being in the use of PCB windings instead of conventional windings.

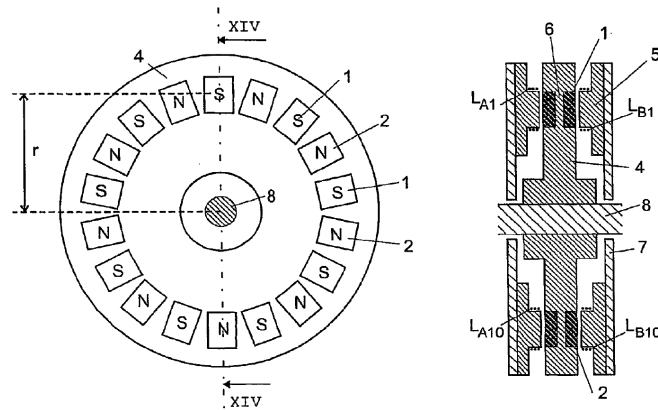


Fig. 13: Front and side view of the EDTB structure proposed in [19].

The principle of functioning of the EDTB is illustrated in Figs. 15 and 16. In Fig. 15, corresponding to the case where the rotor is axially centered, the PMs array is located at equal distances from the series connected stator coils L_A and L_B . This means that the electromotive forces U_A and U_B induced respectively in coils L_A and L_B have the same amplitude. As a result, the total

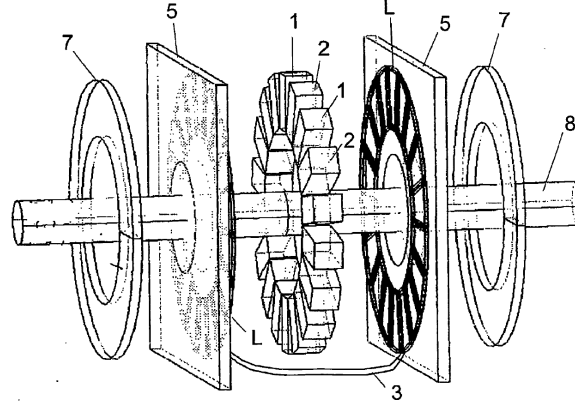


Fig. 14: Exploded view of the EDTB structure proposed in [19].

electromotive force on the series connected coils is equal to zero, which results in a null induced current. However, in the case of an axial excursion Δx of the rotor, as depicted in Fig. 16, the distance between the coils and the PMs array is different. It results in an unbalance of the emfs U_A and U_B and, consequently, in a non-zero emf on the series connected coils, resulting in an induced current. The interaction of this induced current with the magnetic field of the PMs array leads to a restoring force, stabilizing the axial degree of freedom of the rotor.

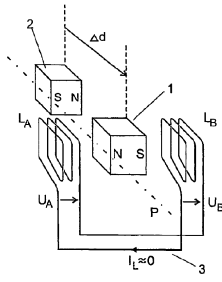


Fig. 15: Electromotive force induced in the coils A and B when the rotor is at equal distances of each coil, [19].

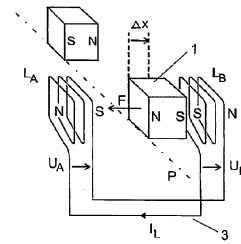
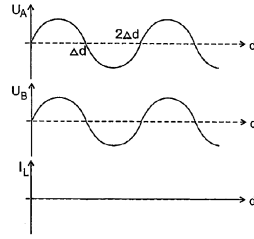
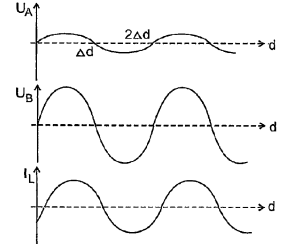


Fig. 16: Electromotive force induced in the coils A and B when the rotor is at unequal distances of each coil, [19].



A special feature of the system proposed in [19] is the possibility of using a first set of series connected coils to achieve the bearing function, and a second set of series connected coils linked to a source to achieve the drive function, as shown in Fig. 17. The advantage is that the stabilization of the axial degree of freedom relies on an EDTB using a rotor with a PMs array, and that this same array can also be used to rotate the device. This corresponds to an passive axial-flux self-bearing motor.

Topologies for axial-flux self-bearing motor

Another topic that has been the focus of numerous researches over the last decades is the axial flux self-bearing motor, consisting in the combination of an axial flux motor and a radial or axial AMB. As explained in [21], axial flux permanent magnet (AFPM) machines already are considered as a mature technology for a wide variety of applications, where extreme axial compactness, high torque density and high efficiency are required. Thus, it seems logical to attempt to combine this already successful technology with magnetic bearings in order to reduce the losses due to mechanical components.

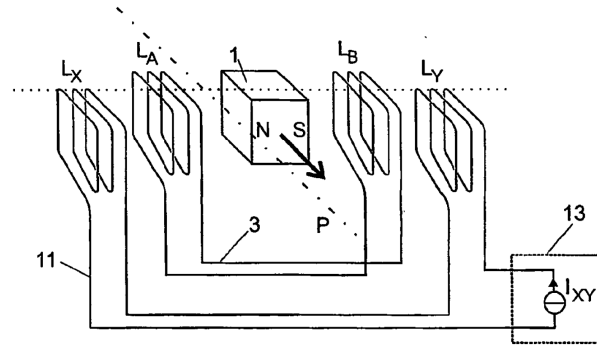


Fig. 17: Principle of the combined bearing and drive unit in [19].

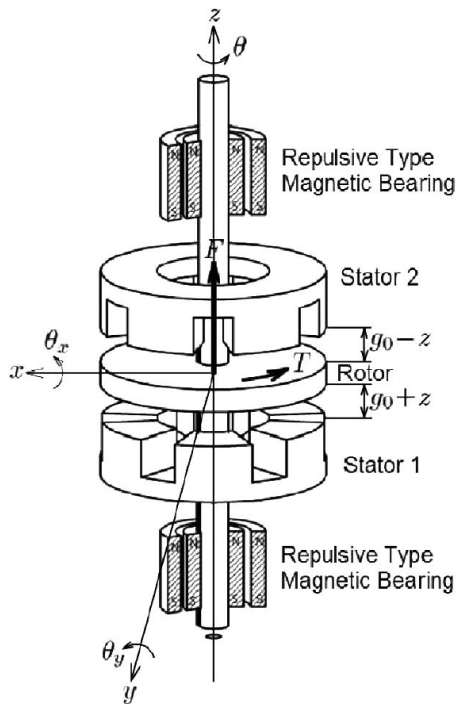


Fig. 18: Axial gap self-bearing motor, as proposed in [22], [23] and [24].

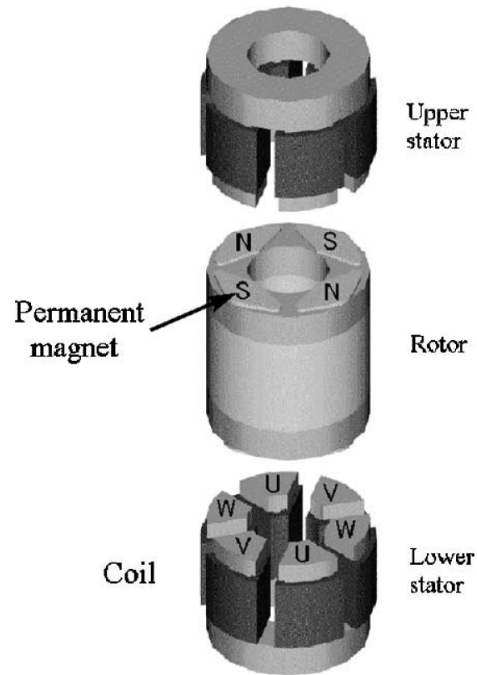


Fig. 19: Axial type self-bearing motor, as proposed in [25] and [26].

The first interesting topology was introduced in 2000 by S. Ueno and Y. Okada in [22]. It deals with a permanent magnet axial gap self-bearing motor (AGBM), which is an electrical combination of an axial thrust bearing and an axial flux motor. As shown in Fig. 18, the rotor consists in a flat disk with PMs attached on both faces to create either a salient- or nonsalient-pole rotor. The motor-bearing system relies on two stators, one on each rotor side. Three-phase windings that generate rotating magnetic fluxes in the air gap are attached to each stator. These flux linkages respectively produce motor torques T_1 and T_2 and generate attractive forces F_1 and F_2 between the rotor and the stators. The total motoring torque T corresponds to the sum of these torques and the axial force F is either the sum of these forces in the nonsalient case [23], or the difference of these forces in the salient case [24]. The control method relies on the vector control principle : the axial force is controlled through the direct-axis current I_d and the rotating torque, by the quadrature-axis current I_q . It should be noted that the axial position is actively controlled, meaning that a position feedback control is required. Typically, the attractive force is controlled by changing the magnitude and phase of the currents in the two stators. Furthermore, the radial and tilting motion rely on classical PMBs. A similar topology has been proposed in 2002 by K. Ohmori and Y. Okada in the framework of artificial heart pumps, in [25] and [26]. It is depicted in Fig. 19.

A second type of axial flux self-bearing motor was introduced by J. Asama and A. Chiba in [27] in 2013. It uses the structure of a single-drive axial flux bearingless motor, where field weakening and strengthening with d-axis current modulation produces the suspension force for the axial motion control. As far as the structure is concerned, the rotor consists in two disk-shaped rotor irons, with a PM ring attached on it at both ends. Those two disks are connected by a shaft. It corresponds to the central element in Figs 20(a) and (b). A three-phase winding used for both rotation and magnetic suspension is wound around the stator, on the left of Fig. 20. As said previously, the axial position is controlled actively through a modulation of the d-axis current. When the rotor is centered in the axial direction, the magnetic attraction forces are the same at both ends of the rotor. However, when the magnetic flux is strengthened in the wound stator air gap, the magnetic flux densities at both stators air gaps are unbalanced, resulting in a positive suspension force. This situation is depicted in Fig. 20(a). To generate a negative restoring force, the same principle is used, but the magnetic flux is weakened instead of strengthened. As far as the motor torque is concerned, it is proportional to the q-axis current. Moreover, the other degrees of freedom of the rotor are stabilized with passive magnetic suspensions.

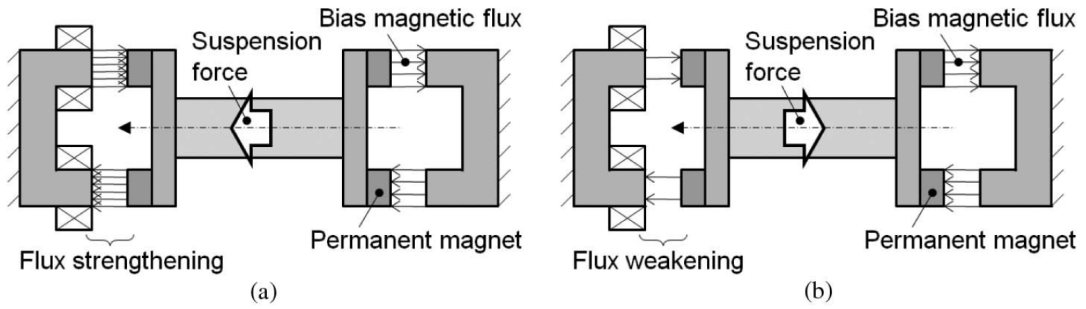


Fig. 20: Schematic diagram of the suspension force generation proposed in [27]. (a) Positive suspension force. (b) Negative suspension force.

A very similar topology, introduced by the same researchers in [28] in 2014, is shown in Fig. 21. The radial and tilting motions of the rotor are stabilized using a PM ring, attached to the stator, and a PM rod, attached to the rotor, both magnetized axially. This corresponds to the green elements in Fig. 21. Together, they form a repulsive type magnetic bearing, resulting in an unstable axial motion of the rotor. As far as the axial gap surface-mounted PM motor is concerned, it is composed of a rotor with axially magnetized PMs, used both for suspension force and torque generation, mounted on a back yoke. Furthermore, coreless coils are fixed to the stator. The active suspension force and the motor force are generated independently through the d- and q-axis currents respectively, based on Lorentz forces. Figure 22 illustrates how the axial suspension force is generated.

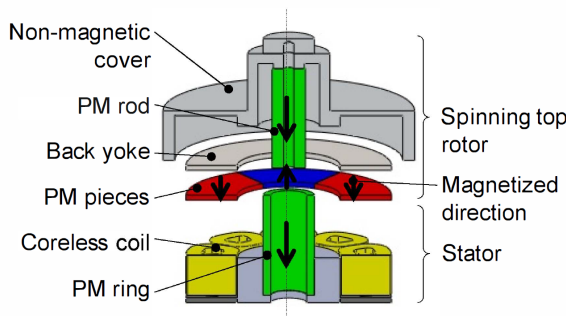


Fig. 21: Structure of the single-drive bearingless motor as proposed in [28].

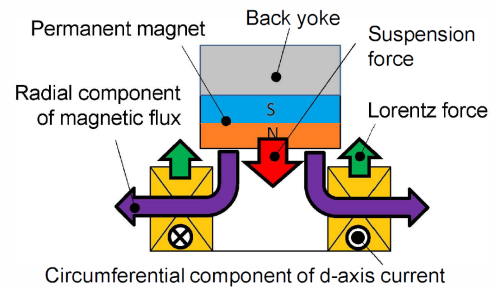


Fig. 22: Axial suspension force generation principle, [28].

Other types of axial flux self-bearing motors allow to simultaneously generate a motor torque and a radial transverse force. In [29] and [30], a third topology for axial flux self-bearing motors, in which the radial degree of freedom is actively controlled, is introduced. It relies on the use of the Lorentz forces for both torque and radial suspension force generation. The self-bearing motor is composed of an 8-pole simple [29] or double [30] disk rotor with PMs mounted on back-irons. The stator is constituted of 6 coreless concentrated windings, which can be supplied by motor or bearing currents, controlling the motor torque and radial force through the current amplitude and phase. When the coils are supplied with a motor current, as depicted in Fig. 23, the Lorentz forces generated by the interaction of those currents with the magnetic field of the PMs have opposite direction and thus result in a motor torque. However, when the stator coils are supplied with levitation currents, the generated Lorentz forces are in the same direction, resulting in a positive transverse force, as shown in Fig. 24.

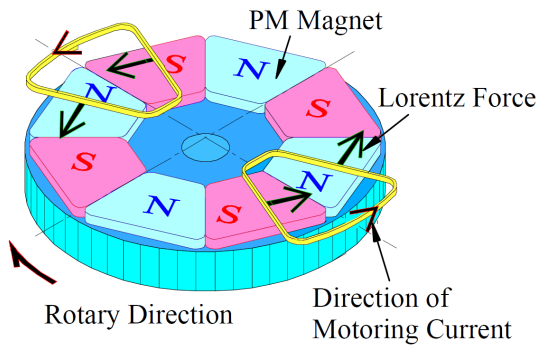


Fig. 23: Principle of motor torque generation, [29].

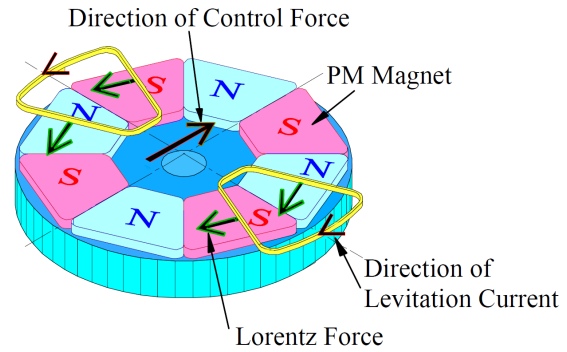


Fig. 24: Principle of radial suspension force generation, [29].

Other less conventional topologies of axial flux self-bearing motors are able to generate a suspension force, either axial or radial, and a motor torque. In [31], for example, the system is composed of a bell-shaped rotor with inner and outer back irons, and PMs blocks fixed to the outer back iron. The stator consists in air-core coils, whose end windings are able to produce non-zero levitation forces in the axial direction, provided that the rotor magnet system features a specific magnetization pattern. In [32], the device consists in a rotor with twin external Halbach arrays and a coreless stators with non-overlapping concentrated windings. Those coils are used for both torque and radial force generation and are energized by 2 separate symmetrical three-phase currents, one set for torque generation and the other one for suspension force generation. In that way, the rotation and levitation can be controlled independently.

Objectives of this Master's thesis

Based on the afore mentioned topologies, the perceptive reader should by this point have noticed the significant number of similarities between the structure of EDTBs and AFPM motors. In particular, the electrodynamic bearing and drive unit proposed by H. K. Asper in [19] and revisited by J. Van Verdegheem in [20] are very close in terms of structure to the axial gap self-bearing motor proposed by S. Ueno and Y. Okada in [22] and its dual topology proposed by W. Geng and Z. Zhang in [32]. Indeed, in each case, the system consists in a disk rotor with a PMs arrangement, where adjacent PMs have opposite polarities. This disk is comprised between two stators with a set of three-phase windings. In the patent of H. K. Asper, at each stator, one set of short-circuited windings is used to conduct the suspension currents of the EDTB and the second set is used to drive the axial flux motor. In the paper of S. Ueno and Y. Okada, only one set of three-phase windings is used at each stator, but it can generate an axial suspension force and a motor torque through the control of the d- and q-axis current respectively.

Consequently, the cornerstone of this Master's thesis is the idea that, at each stator, a single set of windings can be used to conduct both the suspension currents generated by the null-flux EDTB and the motor currents provided by a N-phased source, to drive the AFPM motor. The chosen topology is a double-sided AFPM machine with an internal rotor, as shown in Fig. 25. The stator can either be slotted, slotless or with salient poles. The main characteristics of this machine are its number of pole pairs p and its number of phases N . The stator is composed of $2pN$ coils. pN of them are placed above and pN under the PMs and are denoted respectively with a subscript U for "upper" and L for "lower". One winding is constituted by the series connection of the stator coils of one phase. Upper and lower windings are connected in parallel to a N-phased voltage source and constitute a phase winding.

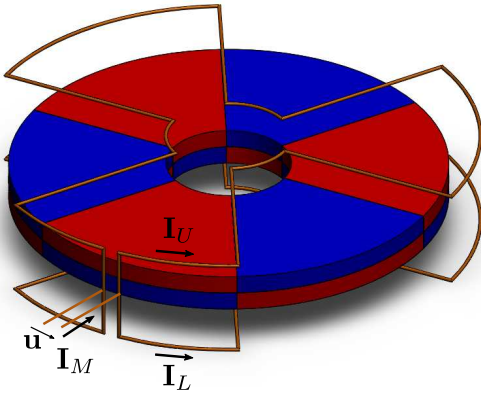


Fig. 25: 3D representation of a double-sided AFPM machine with three pole pairs ($p=3$) and three phases ($N=3$).

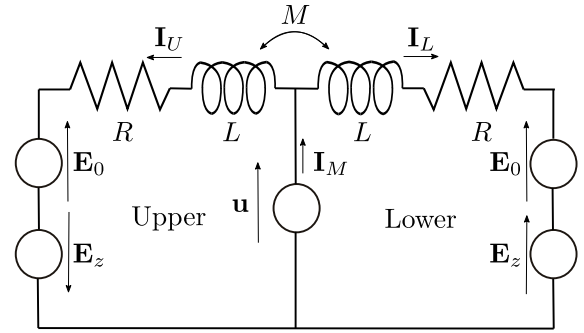


Fig. 26: Equivalent circuit of the double-sided AFPM machine.

From the point of view of the EDTB, the windings constituting a phase winding are seen in series, thus forming a short-circuited winding, as depicted in Figure 26. When the rotor is centered, the magnetic flux linked by the upper and lower windings have the same amplitude but are opposite in signs. The emf E_0 on the upper and lower coils thus exactly compensate for each other, what corresponds to a null-flux behavior. However, when the rotor is not centered, the axial displacement causes an imbalance E_z in the emf of the upper and lower coils, inducing suspension currents in the phase winding. These currents will generate an axial centering force as well as a drag torque which will impact the operation of the AFPM function. Other topologies based on upper and lower windings and exhibiting a null-flux behavior exist [33]. They can be obtained by rotation of the upper windings and PMs with respect to the lower ones, by an angle $\Delta\theta_a = \frac{k\pi}{p}$, or by placing the windings between the two PMs arrangements.

Thesis outline

This Master's thesis is divided into three chapters. In the first chapter, starting from the governing electrical equations for the upper and lower windings and the mechanical equations for the axial and rotational motion, a dynamic model describing the axial and rotational dynamics of the AFPM self-bearing motor is developed. Based on a linearization of this system of equations, the stability properties and equilibrium points at constant rotational speed can be discussed. Finally, a methodology to determine the model parameters using static FEM simulations is presented. The second chapter is dedicated to the application of the dynamic model to a study case. More precisely, a first section deals with the stability analysis of the bearing and drive system in quasi-static equilibrium. Furthermore, the influence of several model parameters on the stability properties is assessed, based on a root locus analysis. A second section is dedicated to the study of the system dynamic behavior, especially at start-up and when subject to an external force or torque perturbation. Then, a third section explains how it is possible to optimize the number of pole pairs and some geometric dimensions of the coils to reach an optimal performance for both the EDTB and AFPM motor. Finally, the third chapter presents the experimental setup and results, and compares the predictions provided by the dynamic model to the behavior of the real system.

In this chapter, the method used to obtain the dynamic model of the self-bearing AFPM machine is discussed, starting with the hypotheses that were made regarding the magnetic and electrical behavior of the system. The electrical equations, namely Faraday's law applied to the upper and lower windings, the electromechanical equations, relating the currents to the forces and torques, and the mechanical equations are then combined, after some changes of variable, to obtain the dynamic model of the machine. Finally, the method used to determine the model parameters based on static finite elements method (FEM) simulation is presented.

1.1 Hypotheses

In order to establish the dynamic model of the self-bearing AFPM machine, the following assumptions are made:

1. The materials are assumed to have linear magnetic characteristics, i.e. magnetic hysteresis and saturation are neglected;
2. Only rotational motion and small axial displacement of the rotor are considered. Consequently, the tilt and radial motions of the rotor are neglected;
3. Only the fundamental component of the PMs arrangements magnetic field is considered;
4. The inductance coefficients are independent of both the angular and axial position of the rotor;
5. The eddy currents are neglected except in the windings;
6. The skin effects in the conductors are neglected;

1.2 Magnetic flux linkages

According to assumptions 2 and 3, and as proven in [33], for small axial and radial displacements, the magnetic flux linkages Φ_U and Φ_L due to the PMs, respectively in the upper and the lower coils, are:

$$\Phi_{Uk} = (\Phi_0 - zK_\Phi) \cos(p(\delta_k - \alpha)) \quad (1.1)$$

$$\Phi_{Lk} = (\Phi_0 + zK_\Phi) \cos(p(\delta_k - \alpha)) \quad (1.2)$$

where $\delta_k = \frac{2\pi(k-1)}{pN} + \delta_0$ is the position of the k-th phase, δ_0 is the position of the first phase and α is the angular position of the rotor, Φ_0 is the constant part of the magnetic flux linkage amplitude and

K_Φ is the proportionality coefficient between the variation of the magnetic flux linkage amplitude and the axial displacement.

1.3 Electrical equations

Faraday's law can be written for the upper and lower windings yielding, under assumption 4, the following simplified equations:

$$\mathbf{u} = R\mathbf{I}_U + \mathbf{L} \frac{d\mathbf{I}_U}{dt} + \mathbf{M} \frac{d\mathbf{I}_L}{dt} + \frac{d\Phi_U}{dt} \quad (1.3)$$

$$\mathbf{u} = R\mathbf{I}_L + \mathbf{L} \frac{d\mathbf{I}_L}{dt} + \mathbf{M} \frac{d\mathbf{I}_U}{dt} + \frac{d\Phi_L}{dt} \quad (1.4)$$

where R is the resistance of a winding, $\mathbf{L}$ is the matrix of self and mutual inductance coefficients for both upper and lower windings, $\mathbf{M}$ is the matrix of mutual inductance coefficients between upper and lower windings, $\mathbf{I}_U = [I_{U1} \cdots I_{UN}]$ are the upper windings currents, $\mathbf{I}_L = [I_{L1} \cdots I_{LN}]$ are the lower windings currents, $\Phi_U = [\Phi_{U1} \cdots \Phi_{UN}]$ are the upper windings magnetic flux linkages and $\Phi_L = [\Phi_{L1} \cdots \Phi_{LN}]$ are the lower windings magnetic flux linkages. The electrical circuit corresponding to the k -th phase of these equations is represented in Fig. 1.1.

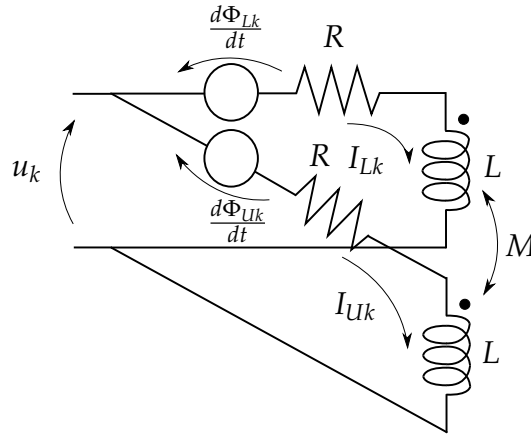


Fig. 1.1: Equivalent circuit of the k -th phase for the self-bearing motor electrical equations, where the coupling effect with the other phases through the matrices $\mathbf{L}$ and $\mathbf{M}$ is not represented.

Defining $\mathbf{I}_M = \mathbf{I}_L + \mathbf{I}_U$ as the motor currents supplied by the source, $\mathbf{I}_S = \frac{\mathbf{I}_L - \mathbf{I}_U}{2}$ as the suspension currents, $\Phi_M = \frac{\Phi_L + \Phi_U}{2}$ as the motor fluxes and $\Phi_S = \Phi_L - \Phi_U$ as the suspension fluxes, and combining (1.3) and (1.4), we can derive the two following electrical equations:

$$\mathbf{u} = \frac{R}{2}\mathbf{I}_M + \frac{(\mathbf{L} + \mathbf{M})}{2} \frac{d\mathbf{I}_M}{dt} + \frac{d\Phi_M}{dt} \quad (1.5)$$

$$0 = 2R\mathbf{I}_S + 2(\mathbf{L} - \mathbf{M}) \frac{d\mathbf{I}_S}{dt} + \frac{d\Phi_S}{dt} \quad (1.6)$$

Using the definition of Φ_{Uk} and Φ_{Lk} given by (1.1) and (1.2) respectively, the motor and suspension fluxes can be expressed as:

$$\Phi_{Mk} = \frac{\Phi_{Lk} + \Phi_{Uk}}{2} = \Phi_0 \cos(p(\delta_k - \alpha)) \Rightarrow \Phi_M = \Phi_0 \cos(p(\delta - \alpha)) \quad (1.7)$$

$$\Phi_{Sk} = \Phi_{Lk} - \Phi_{Uk} = 2zK_\Phi \cos(p(\delta_k - \alpha)) \Rightarrow \Phi_S = 2zK_\Phi \cos(p(\delta - \alpha)) \quad (1.8)$$

where $\delta = [\delta_1 \cdots \delta_N]$. Substituting in (1.5) and (1.6), it results that:

$$\mathbf{u} = \frac{R}{2} \mathbf{I}_M + \frac{(\mathbf{L} + \mathbf{M})}{2} \frac{d\mathbf{I}_M}{dt} + \Phi_0 \frac{d[\cos(p(\delta - \alpha))]}{dt} \quad (1.9)$$

$$0 = 2R\mathbf{I}_S + 2(\mathbf{L} - \mathbf{M}) \frac{d\mathbf{I}_S}{dt} + 2zK_\Phi \frac{d[\cos(p(\delta - \alpha))]}{dt} + 2\dot{z}K_\Phi \cos(p(\delta - \alpha)) \quad (1.10)$$

Equation (1.9) corresponds to that of a synchronous motor. It only involves the voltage source $\mathbf{u}$ and motor currents $\mathbf{I}_M$. Equation (1.10) intercepts the electrical behavior of the EDTB as it only links the suspension currents $\mathbf{I}_S$ to the axial dynamics ($z, \dot{z}$). Furthermore, this equation shows that the suspension currents $\mathbf{I}_S$ disappear if the rotor is centered and the axial velocity is zero. It is interesting to observe that these two electrical equations are perfectly decoupled as far as currents are concerned. Moreover, we can observe that the axial dynamics does not impact the motor currents $\mathbf{I}_M$ but that, on the contrary, the rotational dynamics impacts the suspension currents $\mathbf{I}_S$, meaning that the decoupling between them is not complete. Fig. 1.2 depicts the equivalent circuit corresponding to these two electrical equations. On the one hand, in the synchronous motor equivalent circuit, the resistance and inductance are divided by two, which highlights the fact that those elements are connected in parallel with respect to the source. On the other hand, in the EDTB equivalent circuit, the resistance and inductance are multiplied by two, as the elements are connected in series with respect to the thrust bearing.

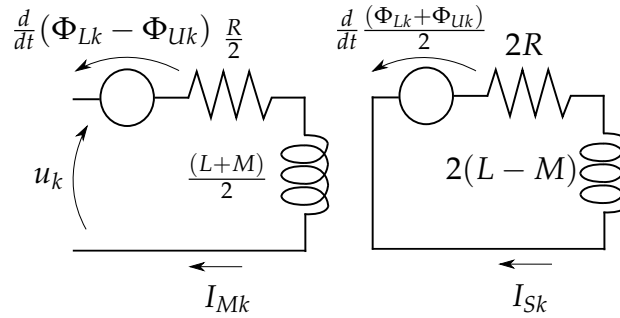


Fig. 1.2: Equivalent circuit of the k -th phase for the transformed self-bearing motor electrical equations, where the coupling effect with the other phases through the matrices $\mathbf{L}$ and $\mathbf{M}$ is not represented.

1.4 Electromagnetic forces and torques

Based on the assumptions, the only efforts generated on the rotor are the detent force and both electrodynamic force and torque along its rotating axis. According to the second assumption on the amplitude of the axial displacement, more specifically, the detent force can be expressed as:

$$F_D = -k_D z \quad (1.11)$$

where k_D , the stiffness along the axial direction, is negative, and therefore destabilizing.

The electrodynamic axial force is obtained as the sum of the forces on the upper and lower coils:

$$F_{ED} = \left[\frac{\partial \Phi_U}{\partial z} \right]^T \mathbf{I}_U^* + \left[\frac{\partial \Phi_L}{\partial z} \right]^T \mathbf{I}_L^* \quad (1.12)$$

From (1.1) and (1.2), we obtain that:

$$F_{ED} = 2K_{\Phi} [\cos(p(\delta - \alpha))]^T \mathbf{I}_S^* \quad (1.13)$$

The resulting force depends on the short-circuit suspension currents $\mathbf{I}_S$ but not on the motor currents $\mathbf{I}_M$ supplied by the voltage source $\mathbf{u}$.

The electrodynamic torque also results from the contribution of the upper and lower windings:

$$T_{ED} = \left[\frac{\partial \Phi_U}{\partial \alpha} \right]^T \mathbf{I}_U^* + \left[\frac{\partial \Phi_L}{\partial \alpha} \right]^T \mathbf{I}_L^* \quad (1.14)$$

$$= \underbrace{2pzK_{\Phi} [\sin(p(\delta - \alpha))]^T \mathbf{I}_S^*}_{\text{Drag Torque}} + \underbrace{p\Phi_0 [\sin(p(\delta - \alpha))]^T \mathbf{I}_M^*}_{\text{Motor Torque}} \quad (1.15)$$

The resulting torque has two components. The first is the drag torque T_D which is proportional to the axial displacement z and the suspension currents $\mathbf{I}_S$, themselves dependent on the axial dynamics. It is worth noticing that this drag torque only exists when the rotor is not at its equilibrium position $z = 0$. The second component is the driving torque T_M produced by the motor currents $\mathbf{I}_M$. Due to the drag torque, a coupling between the EDTB and synchronous motor functions exists regarding the rotational motion.

1.5 Dynamic model

In order to obtain a model able to describe the dynamics of the self-bearing motor, we first apply a change of variable in order to reduce the system, both for the suspension and motor currents, voltages and magnetic flux linkages, in the same way as in [20]:

$$\begin{cases} \mathbf{I} = \mathbf{U}\mathbf{I}^s \\ \Phi = \mathbf{U}\Phi^s \\ \mathbf{u} = \mathbf{U}\mathbf{u}^s \end{cases} \quad (1.16)$$

where the superscript s corresponds to the variables in the new coordinate system and the matrix $\mathbf{U}$ is defined as follows:

$$\begin{cases} U_{kl} = \frac{1}{\sqrt{N}} \exp\left(-j(k-1)(l-1)\frac{2\pi}{N}\right) \\ U_{kl}^{-1} = U_{kl}^* = \frac{1}{\sqrt{N}} \exp\left(j(k-1)(l-1)\frac{2\pi}{N}\right) \end{cases} \quad (1.17)$$

where $k, l = 1, \dots, N$. The motor and suspension flux linkages in the new coordinate system can be expressed as follows, this result being proved in [20]:

$$\Phi_M^s = \frac{\Phi_0 \sqrt{N}}{2} \begin{bmatrix} 0 \\ \exp(jp(\alpha - \delta_0)) \\ 0 \\ \vdots \\ 0 \\ \exp(-jp(\alpha - \delta_0)) \end{bmatrix} \quad (1.18)$$

$$\Phi_S^s = zK_{\Phi} \sqrt{N} \begin{bmatrix} 0 \\ \exp(jp(\alpha - \delta_0)) \\ 0 \\ \vdots \\ 0 \\ \exp(-jp(\alpha - \delta_0)) \end{bmatrix} \quad (1.19)$$

Based on (1.18) and provided that the source $\mathbf{u}$ is symmetrical, we observe that the only non-homogeneous electrical equations in the new coordinate system for the motor currents are the second and N -th ones. Similarly, based on (1.19), the only non-homogeneous electrical equations in the new coordinate system for the suspension currents are the second and N -th ones. It is then possible to reduce the initial $2N$ electrical equations to a set of four equations:

$$\begin{cases} \mathbf{u}^r = \frac{R}{2} \mathbf{I}_M^r + L_{Mc} \frac{d\mathbf{I}_M^r}{dt} + \frac{d\Phi_M^r}{dt} \\ 0 = 2R\mathbf{I}_S^r + L_{Sc} \frac{d\mathbf{I}_M^r}{dt} + \frac{d\Phi_S^r}{dt} \end{cases} \quad (1.20)$$

where L_{Mc} is the motor cyclic inductance and L_{Sc} is the suspension cyclic inductance, defined as:

$$L_{Mc} = \sum_{k=1}^N \frac{(L_{1k} + M_{1k})}{2} \cos(p\delta_k) \quad (1.21)$$

$$L_{Sc} = \sum_{k=1}^N 2(L_{1k} - M_{1k}) \cos(p\delta_k) \quad (1.22)$$

and with:

$$\begin{aligned} \mathbf{u}^r &= [u_2^s \quad u_N^s]^T \\ \mathbf{I}_M^r &= [I_{M2}^s \quad I_{MN}^s]^T \\ \mathbf{I}_S^r &= [I_{S2}^s \quad I_{SN}^s]^T \\ \Phi_M^r &= \begin{bmatrix} \Phi_{M2}^s \\ \Phi_{MN}^s \end{bmatrix} = \frac{\Phi_0 \sqrt{N}}{2} \begin{bmatrix} \exp(jp(\alpha - \delta_0)) \\ \exp(-jp(\alpha - \delta_0)) \end{bmatrix} \\ \Phi_S^r &= \begin{bmatrix} \Phi_{S2}^s \\ \Phi_{SN}^s \end{bmatrix} = zK_\Phi \sqrt{N} \begin{bmatrix} \exp(jp(\alpha - \delta_0)) \\ \exp(-jp(\alpha - \delta_0)) \end{bmatrix} \end{aligned}$$

1.5.1 Motor electrical equations

In this new coordinate system, the motor torque can be expressed as:

$$T_M = \left[\frac{\partial \Phi_M^r}{\partial \alpha} \right]^T \mathbf{I}_M^{r*} = jp \frac{\Phi_0 \sqrt{N}}{2} [I_{M1}^{r*} \exp(-jp(\alpha - \delta_0)) - I_{M2}^r \exp(-jp(\alpha - \delta_0))]$$

As the drag torque can only have a real value, it results that the two components of the reduced motor currents $\mathbf{I}_M^r$ are complex conjugates, i.e. $I_{M2}^r = I_{M1}^{r*}$. This property directly results from the change of variable and is proved in Appendix A. This yields that the torque can be expressed as:

$$T_M = p\Phi_0 \sqrt{N} \Im \{ I_{M1}^r \exp(-jp(\alpha - \delta_0)) \} \quad (1.23)$$

in which we notice that $\Im \{ I_{M1}^r \exp(-jp(\alpha - \delta_0)) \}$ corresponds to the q-axis current of a synchronous machine. Consequently, we can define the d- and q-axis motor currents and voltages:

$$\begin{aligned} I_d &= \Re \{ I_{M1}^r \exp(-jp(\alpha - \delta_0)) \} \\ I_q &= \Im \{ I_{M1}^r \exp(-jp(\alpha - \delta_0)) \} \\ u_d &= \Re \{ u_1^r \exp(-jp(\alpha - \delta_0)) \} \\ u_q &= \Im \{ u_1^r \exp(-jp(\alpha - \delta_0)) \} \end{aligned}$$

Substituting in (1.23), the synchronous motor torque corresponds to:

$$T_M = p\Phi_0 \sqrt{N} I_q \quad (1.24)$$

The observation that $I_{M2}^r = I_{M1}^{r*}$ also yields that the motor electrical equations reduce to:

$$u_1^r = \frac{R}{2} I_{M1}^r + L_{Mc} \frac{dI_{M1}^r}{dt} + \frac{d\Phi_{M1}^r}{dt} \quad (1.25)$$

Multiplying by $\exp(-jp(\alpha - \delta_0))$ and separating into two equations corresponding to the real and imaginary parts allows us to express the d- and q-axis electrical equations of the synchronous motor.

$$\begin{cases} u_d = \frac{R}{2} I_d + L_{Mc} \left(\frac{dI_d}{dt} - p\dot{\alpha} I_q \right) \\ u_q = \frac{R}{2} I_q + L_{Mc} \left(\frac{dI_q}{dt} + p\dot{\alpha} I_d \right) + \frac{p\Phi_0\sqrt{N}}{2} \dot{\alpha} \end{cases} \quad (1.26)$$

1.5.2 Electrodynamic thrust bearing electrical equations

As far as the EDTB is concerned, the electrodynamic axial force F_{ED} and the drag torque T_D can be related to the reduced suspension currents $\mathbf{I}_S^r$. It can also be observed that $I_{S2}^r = I_{S1}^{r*}$, by the same proof as the one done for the motor currents in Appendix A. The electrodynamic axial force is expressed as:

$$\begin{aligned} F_{ED} &= \left[\frac{\partial \Phi_S^r}{\partial z} \right]^T \mathbf{I}_S^{r*} \\ &= K_\Phi \sqrt{N} [I_{S1}^{r*} \exp(-jp(\alpha - \delta_0)) + I_{S1}^r \exp(-jp(\alpha - \delta_0))] \end{aligned} \quad (1.27)$$

and the drag torque is given by:

$$\begin{aligned} T_D &= \left[\frac{\partial \Phi_S^r}{\partial \alpha} \right]^T \mathbf{I}_S^{r*} \\ &= jpzK_\Phi \sqrt{N} [I_{S1}^{r*} \exp(-jp(\alpha - \delta_0)) - I_{S1}^r \exp(-jp(\alpha - \delta_0))] \end{aligned} \quad (1.28)$$

Defining $Y_S = I_{S1}^r \exp(-jp(\alpha - \delta_0))$ allows us to rewrite the axial force as:

$$\begin{aligned} F_{ED} &= K_\Phi \sqrt{N} [Y_S^* + Y_S] \\ &= 2K_\Phi \sqrt{N} \Re \{Y_S\} \end{aligned} \quad (1.29)$$

and the drag torque as:

$$\begin{aligned} T_D &= jpzK_\Phi \sqrt{N} [Y_S^* - Y_S] \\ &= 2pzK_\Phi \sqrt{N} \Im \{Y_S\} \end{aligned} \quad (1.30)$$

Combining (1.29) and (1.30), Y_S can be expressed as:

$$Y_S = \frac{1}{2K_\Phi \sqrt{N}} \left(F_{ED} + j \frac{T_D}{pz} \right) \quad (1.31)$$

To find the instantaneous current in the k-th phase, the variable change (1.16) is applied reversely, remembering that only two components of $\mathbf{I}_S^s$ are non-zero, namely the second and the N-th ones. This yields

$$\begin{aligned} I_{Sk} &= \frac{2}{\sqrt{N}} \Re \left\{ Y_S \exp(jp(\alpha - \delta_0)) \exp \left(-j(k-1) \frac{2\pi}{N} \right) \right\} \\ &= \frac{1}{K_\Phi N} \left(F_{ED} \cos \left[p(\alpha - \delta_0) - (k-1) \frac{2\pi}{N} \right] - \frac{T_D}{pz} \sin \left[p(\alpha - \delta_0) - (k-1) \frac{2\pi}{N} \right] \right) \end{aligned} \quad (1.32)$$

The RMS phase current can also be obtained as:

$$I_{rms} = \frac{1}{\sqrt{2}K_{\Phi}N} \sqrt{F_{ED}^2 + \left(\frac{T_D}{pz}\right)^2} \quad (1.33)$$

The observation that $I'_{S2} = I'_{S1}^*$ also yields that the EDTB electrical equations reduce to:

$$0 = 2RI'_{S1} + L_{Mc} \frac{dI'_{S1}}{dt} + \frac{d\Phi'_{S1}}{dt} \quad (1.34)$$

Multiplying by $\exp(-jp(\alpha - \delta_0))$, separating into two equations corresponding to the real and imaginary parts and substituting Y_S allows us to express the electrical equations of the EDTB as:

$$\begin{cases} 0 = 2R\Re\{Y_S\} + L_{Sc} \left(\frac{d\Re\{Y_S\}}{dt} - p\dot{\alpha}\Im\{Y_S\} \right) + K_{\Phi}\sqrt{N}\dot{z} \\ 0 = 2R\Im\{Y_S\} + L_{Sc} \left(\frac{d\Im\{Y_S\}}{dt} + p\dot{\alpha}\Re\{Y_S\} \right) + pK_{\Phi}\sqrt{N}\dot{\alpha}z \end{cases} \quad (1.35)$$

Substituting (1.29) and (1.30) in (1.35), the EDTB electrical equations are expressed in terms of the electrodynamic axial force F_{ED} and the ratio of the drag torque and the axial displacement $\left(\frac{T_D}{z}\right)$:

$$\begin{cases} 0 = 2RF_{ED} + L_{Sc} \left(\dot{F}_{ED} - \dot{\alpha} \left(\frac{T_D}{z} \right) \right) + 2K_{\Phi}^2 N \dot{z} \\ 0 = 2R \left(\frac{T_D}{z} \right) + L_{Sc} \left(\left(\frac{\dot{T}_D}{z} \right) + p^2 \dot{\alpha} F_{ED} \right) + 2(pK_{\Phi})^2 N \dot{\alpha} z \end{cases} \quad (1.36)$$

1.5.3 Mechanical motion equations

Two mechanical equations are used to describe the motion of the system. First, the axial acceleration, speed and displacement can be related to the axial forces using Newton's law:

$$m\ddot{z} + C_z\dot{z} = F_{ED} + F_D + F_{ext} \quad (1.37)$$

where m is the mass of the rotor, C_z is the axial damping coefficient, and F_{ext} is the external axial force. Similarly, the rotational dynamics can be described using Euler's equation for the rotation of rigid bodies.

$$J\ddot{\alpha} + C_{\alpha}\dot{\alpha} = T_M + T_D + T_{ext} \quad (1.38)$$

where J is the inertia of the rotor, C_{α} is the rotational damping coefficient and T_{ext} is the external torque.

1.5.4 Final set of dynamic equations

Finally, combining the electrical equations with the motion equations, we obtain the following dynamic equations:

$$\dot{F}_{ED} = -\frac{2R}{L_{Sc}}F_{ED} + \dot{\alpha} \left(\frac{T_D}{z} \right) - \frac{2K_\Phi^2 N}{L_{Sc}}\dot{z} \quad (1.39)$$

$$\left(\frac{\dot{T}_D}{z} \right) = -\frac{2R}{L_{Sc}} \left(\frac{T_D}{z} \right) - p^2 \dot{\alpha} F_{ED} - \frac{2p^2 K_\Phi^2 N}{L_{Sc}} z \dot{\alpha} \quad (1.40)$$

$$\dot{I}_d = \frac{1}{L_{Mc}} u_d - \frac{R}{2L_{Mc}} I_d + p \dot{\alpha} I_q \quad (1.41)$$

$$\dot{I}_q = \frac{1}{L_{Mc}} u_q - \frac{R}{2L_{Mc}} I_q - p \dot{\alpha} I_d - \frac{p \Phi_0 \sqrt{N}}{2L_{Mc}} \dot{\alpha} \quad (1.42)$$

$$\ddot{z} = \frac{F_{ED}}{m} + \frac{F_{ext}}{m} - \frac{k_D}{m} z - \frac{C_z}{m} \dot{z} \quad (1.43)$$

$$\ddot{\alpha} = \frac{p \Phi_0 \sqrt{N}}{J} I_q + \frac{1}{J} \left(\frac{T_D}{z} \right) z + \frac{T_{ext}}{J} - \frac{C_\alpha}{J} \dot{\alpha} \quad (1.44)$$

Equations (1.39) and (1.40) model the dynamic behavior of the EDTB function. Equations (1.41) and (1.42) are the equations of the AFPM motor function in the d-q coordinate space. The current I_d and I_q have been chosen as state variables, as I_q is linearly linked to the torque generated by the AFPM motor. Equations (1.43) and (1.44) correspond to the mechanical equations of the rotor axial and rotational motions respectively.

1.6 Model parameters determination

The proposed dynamic model comprises 12 parameters. For a specific machine, whose dimensions and material are completely defined, some of them can be calculated through analytical formulae, like the winding resistance R , the rotor mass m and the inertia J . The others, like the cyclic inductances L_{Sc} and L_{Mc} , or the parameters Φ_0 and K_Φ , are more complex to compute analytically, but can be obtained through static FEM simulations. In this work, COMSOL Multiphysics has been used to perform those simulations. The way to derive these parameters is detailed hereafter.

1.6.1 Inductances

First, in order to calculate the suspension and motor cyclic inductances, respectively L_{Sc} and L_{Mc} , the coefficients of the inductance matrices $\mathbf{L}$ and $\mathbf{M}$ must be determined. When calculating the elements of the matrix $\mathbf{L}$, we first compute the self-inductance coefficient L_{11} of a winding. To do so, a current I is injected in the winding, considered as a volume conductor, and the magnetic flux going through it is calculated according to the following definition of the flux, derived in Appendix B:

$$\Phi(I) = \frac{1}{I} \int_V \mathbf{j} \cdot \mathbf{A} dV \quad (1.45)$$

where V is the volume of the winding, I is the injected current, $\mathbf{j}$ is the current density vector and $\mathbf{A}$ is the magnetic vector potential. The self-inductance is then found as:

$$L_{11} = \frac{\Phi_{11}(I)}{I} \quad (1.46)$$

To determine the value of the mutual inductances between the windings in the same plane, the same current I is injected in the same direction in both windings and the flux is computed in one of them. The number of calculations required is $\frac{N}{2}$ if N is even and $\frac{N-1}{2}$ if N is odd. In the case of

a three-phase motor, a single simulation is required because the mutual is the same between each winding. Thus, the mutual is computed as:

$$L_{12} = L_{13} = L_{23} = \frac{\Phi_{12}(I)}{I} - L_{11} \quad (1.47)$$

The mutual inductance matrix $\mathbf{M}$ can be found in the exact same way by changing the geometry of the problem. However, in the case of a double-sided AFPD machine, the elements of this matrix are negligible if the rotor, located between the upper and lower windings, is constituted of iron.

1.6.2 Magnetic flux linkages

Then, the constant part of the magnetic flux linkage amplitude, denoted Φ_0 , and the proportionality coefficient between the variation of the magnetic flux linkage amplitude and the axial displacement, denoted K_Φ , must be determined. To do so, a current I is injected in the winding and the magnetic flux is calculated for an axial displacement $z = 0$. This simulation must be performed with the coil positioned exactly above one of the PMs. The amplitude of the magnetic flux linkage is obtained as:

$$\Phi_0 = \Phi_{mag}(z = 0) - L_{11}I \quad (1.48)$$

where $\Phi_{mag}(z = 0)$ is calculated using (1.45), which is the reason why injecting a current in the winding is required. Then, to determine K_Φ , a second simulation is done by moving the winding further away from the PMs by a distance Δz in the axial direction. K_Φ is then calculated as:

$$K_\Phi = \frac{\Phi_{mag}(z = 0) - \Phi_{mag}(z = \Delta z)}{\Delta z} \quad (1.49)$$

In the case where an iron stator yoke is considered, the detent force stiffness k_D can be obtained with a single simulation. The force on the back-iron is computed at a distance Δz from the point at which the rotor is perfectly centered, corresponding to a detent force equal to zero. According to the definition of the detent force given in (1.11), k_D is found as:

$$k_D = \frac{-F_z}{\Delta z} \quad (1.50)$$

Uses of the Model

This chapter is dedicated to the possible uses of the model developed in the previous chapter. First, section 2.1 presents a case study that is used as a basis for the sections that follow, providing numerical values for the model parameters. Then, the properties of the system in quasi-static equilibrium are discussed in section 2.2, based on the linearization of the dynamic equations describing the self-bearing AFPM machine. Moreover, a stability analysis based on the root locus is conducted, for the AFPM motor, the EDTB and the self-bearing AFPM motor. Section 2.3 deals with the dynamic behavior of the machine at start-up and when subjected to external force perturbations or variation of the reference spin speed. Finally, a parametric analysis on several geometrical and topological parameters of the machine is undertaken in section 2.4 to assess the influence of those dimensions on the performance of the EDTB and the AFPM motor, in cases with or without iron stator yoke.

2.1 Case study

Let us consider the double-sided AFPM machine shown in Fig. 2.1. This machine number p of pole pairs is six and the PMs remanent magnetization is 1.42 [T]. The armature winding is composed of three phase concentrated windings, as shown in Fig. 2.1. The geometrical parameters are given in Table 2.1. The remaining parameters are obtained through static FEM simulations, as explained previously. All these parameters are given in Table 2.2. The mass is chosen to be $m = 0.5$ [kg] and the rotor inertia is $J = 0.4$ [g·m²]. The detent force stiffness k_D is taken to be zero, corresponding to the fact that the stator yokes are made out of non-magnetic material. The axial damping coefficient C_z can also be chosen and is set to ensure stability of the system for every angular speed. The rotational damping coefficient C_α corresponds to a non-ideality of the machine and is here chosen to be equal to zero.

R_{in}	R_{out}	e	h_{PM}	h_{iron}	t_{coil}	w_{coil}
20	50	1	6	5	4	4

Table 2.1: Geometrical parameters
[mm].

R [mΩ]	L_{Mc} [nH]	L_{Sc} [nH]	Φ_0 [mWb]
0.507	143	143.11	0.546
K_Φ [Wb/m]	k_D [N/m]	C_z [kg/s]	C_α [kg·m ² /s ²]
0.0796	0	5	0

Table 2.2: Model parameters.

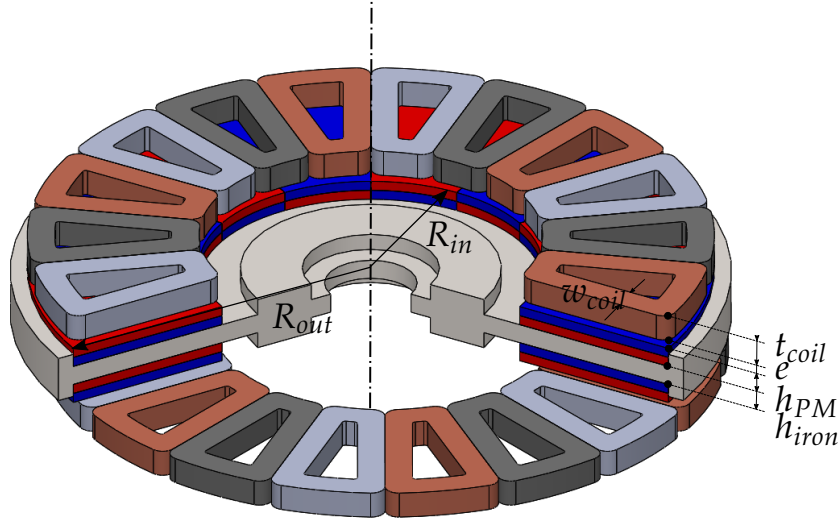


Fig. 2.1: 3D representation of the double-sided AFPM machine with six pole pairs and three phases. The three phase concentrated windings are represented respectively in grey, dark grey and brown.

2.2 Stability analysis

This section is divided into three parts. First, the linearization of the dynamic model equations around a quasi-static equilibrium point is discussed. Then, the evolution of this equilibrium as a function of the spin speed, with a given external force applied to the rotor or a fixed axial displacement, is considered. Finally, the root loci for the AFPM motor, the EDTB and the self-bearing AFPM motor are analyzed.

2.2.1 Linearization around quasi-static equilibrium points

The objective of the dynamic model obtained in the previous chapter is on the one hand, to predict the dynamic behavior of the motor and suspension functions, and on the other hand, to determine under which conditions the system will be stable in quasi-static equilibrium. To assess those stability properties, the system is linearized around a quasi-static equilibrium point, characterized by a constant angular speed and a given external force and torque. The choices made to compute this equilibrium point are detailed later in this section. The objective is thus to write a state model for our system, under the form:

$$\dot{\mathbf{q}} = \mathbf{A} \cdot \mathbf{q} + \mathbf{B} \cdot \mathbf{x} + \mathbf{D} \cdot \mathbf{v} = \mathbf{f}(\mathbf{q}, \mathbf{x}, \mathbf{v}) \quad (2.1)$$

where $\mathbf{q} = (F_{ED} \quad \frac{T_D}{z} \quad I_d \quad I_q \quad z \quad \dot{z} \quad \dot{\alpha})^T$ is the state variable vector, $\mathbf{x} = (u_d \quad u_q)^T$ is the input vector, composed of the d- and q- axis voltages, $\mathbf{v} = (F_{ext} \quad T_{ext})^T$ is the perturbation vector, composed of the external force and torque. This system can be linearized around its quasi-static equilibrium point, where $\mathbf{q}^*$, $\mathbf{x}^*$ and $\mathbf{v}^*$ denote the equilibrium values, using a first order Taylor series:

$$\mathbf{f}(\mathbf{q}, \mathbf{x}, \mathbf{v}) = \mathbf{f}(\mathbf{q}^*, \mathbf{x}^*, \mathbf{v}^*) + \underbrace{\frac{\partial \mathbf{f}}{\partial \mathbf{q}}}_{\mathbf{A}} \cdot (\mathbf{q} - \mathbf{q}^*) + \underbrace{\frac{\partial \mathbf{f}}{\partial \mathbf{x}}}_{\mathbf{B}} \cdot (\mathbf{x} - \mathbf{x}^*) + \underbrace{\frac{\partial \mathbf{f}}{\partial \mathbf{v}}}_{\mathbf{D}} \cdot (\mathbf{v} - \mathbf{v}^*) \quad (2.2)$$

By definition of the equilibrium point, $\mathbf{f}(\mathbf{q}^*, \mathbf{x}^*, \mathbf{v}^*)$ is equal to zero. The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{D}$ are respectively the state, input and perturbation matrices, and are found as derivatives of $\mathbf{f}(\mathbf{q}, \mathbf{x}, \mathbf{v})$ evaluated at the quasi-static equilibrium point. The matrix $\mathbf{A}$, which will be of interest for the root locus analysis, is defined by (2.3). The matrices $\mathbf{B}$ and $\mathbf{D}$ are defined by (2.4) and (2.5) respectively.

$$\mathbf{A} = \begin{pmatrix} -\frac{2R}{L_{Sc}} & \omega & 0 & 0 & 0 & -\frac{2K_{\Phi}^2 N}{L_{Sc}} & \left(\frac{T_D}{z}\right)^* \\ -p^2 \omega & -\frac{2R}{L_{Sc}} & 0 & 0 & -\frac{2(pK_{\Phi})^2 N \omega}{L_{Sc}} & 0 & -p^2 \left(F_{ED}^* + \frac{2K_{\Phi}^2 N}{L_{Sc}} z^*\right) \\ 0 & 0 & -\frac{R}{2L_{Mc}} & p\omega & 0 & 0 & pI_q^* \\ 0 & 0 & -p\omega & -\frac{R}{2L_{Mc}} & 0 & 0 & -p \left(I_d^* + \frac{\Phi_0 \sqrt{N}}{2L_{Mc}}\right) \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{m} & 0 & 0 & 0 & -\frac{k_D}{m} & -\frac{C_z}{m} & 0 \\ 0 & \frac{z^*}{J} & 0 & \frac{p\Phi_0 \sqrt{N}}{J} & \frac{1}{J} \left(\frac{T_D}{z}\right)^* & 0 & -\frac{C_{\alpha}}{J} \end{pmatrix} \quad (2.3)$$

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & \frac{1}{L_{Mc}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_{Mc}} & 0 & 0 & 0 \end{pmatrix}^T \quad (2.4)$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{J} \end{pmatrix}^T \quad (2.5)$$

2.2.2 Quasi-static equilibrium

In order to compute the equilibrium point under quasi-static conditions, some choices must be made regarding the values of the state variables, inputs and perturbations. First, as the AFPM self-bearing motor is assumed to be in quasi-static equilibrium, the speed in the axial direction must be zero, i.e. $\dot{z} = 0$. This leaves us with six non-trivial equations, and ten variables. Consequently, four of those variables must be given a value. The following choices are made:

1. The spin speed is constant and equal to a value ω , i.e. $\dot{\alpha}^* = \omega$.
2. The controller on the d-axis current is designed in such a way that it enforces $I_d^* = 0$ at equilibrium.
3. The external torque is equal to zero at equilibrium, i.e. $T_{ext}^* = 0$, which corresponds to the case of a motor with no load. In other words, a flywheel for energy storage.
4. The last choice to be made is related to the EDTB and can be done in two ways. It is possible to either impose the external force applied to the rotor or to impose the axial excursion of the rotor. One should select the approach according to the problem to be solved.

Equilibrium point of the AFPM motor equations

As far as the AFPM motor equations are concerned, the quasi-static equilibrium values of the d-axis voltage and the q-axis current and voltage can be expressed as:

$$u_d^* = -p\omega L_{Mc} I_q^* \quad (2.6)$$

$$u_q^* = \frac{R}{2} I_q^* + \frac{p\Phi_0 \sqrt{N}}{2} \omega z^* \quad (2.7)$$

$$I_q^* = \frac{C_\alpha \omega - T_D^* - T_{ext}^*}{p\Phi_0 \sqrt{N}} \quad (2.8)$$

In this work, as the parasitic rotational damping coefficient C_α is considered to be equal to zero and as the external torque is, by our third choice, also equal to zero, (2.8) reduces to:

$$I_q^* = -\frac{T_D^*}{p\Phi_0 \sqrt{N}} \quad (2.9)$$

Imposing the external force F_{ext}

The first possibility, which also seems the most natural, is to impose the value of the external force at equilibrium. Thus, all the quasi-static equilibrium values related to the EDTB, namely z^* , F_{ED}^* and T_D^* , can be expressed in terms of this external force F_{ext} and of the spin speed ω of the motor:

$$z^*(\omega, F_{ext}) = \frac{\left[\left(\frac{2R}{L_{Sc}} \right)^2 + (p\omega)^2 \right]}{\left[\left(\frac{2R}{L_{Sc}} \right)^2 + (p\omega)^2 \left(1 + \frac{k_\infty}{k_D} \right) \right]} \cdot \frac{F_{ext}}{k_D} \quad (2.10)$$

$$F_{ED}^*(\omega, F_{ext}) = -\frac{k_\infty}{k_D} \cdot \frac{(p\omega)^2}{\left[\left(\frac{2R}{L_{Sc}} \right)^2 + (p\omega)^2 \left(1 + \frac{k_\infty}{k_D} \right) \right]} \cdot F_{ext} \quad (2.11)$$

$$T_D^*(\omega, F_{ext}) = -\frac{2R}{L_{Sc}} \cdot \frac{k_\infty}{k_D} \cdot \frac{(pL_{Sc})^2 \left[\left(\frac{2R}{L_{Sc}} \right)^2 + (p\omega)^2 \right]}{\left[\left(\frac{2R}{L_{Sc}} \right)^2 + (p\omega)^2 \left(1 + \frac{k_\infty}{k_D} \right) \right]^2} \cdot \omega F_{ext}^2 \quad (2.12)$$

where k_∞ is the stiffness associated to the electrodynamic axial force F_{ED} generated by the EDTB at infinite rotational speed, defined as:

$$k_\infty = \lim_{\omega \rightarrow \infty} -\frac{F_{ED}(\omega)}{z} = \frac{2K_\Phi^2 N}{L_{Sc}} \quad (2.13)$$

Based on (2.10) to (2.12), it is possible to observe the EDTB behavior in two extreme cases regarding the spin speed. First, when it is close to zero, meaning that the terms involving the spin speed ω become negligible compared to the constant ones, those expressions reduce to:

$$z_0^* = \frac{F_{ext}}{k_D} \quad (2.14)$$

$$F_{ED,0}^* = -\frac{k_\infty}{k_D} \cdot \left(\frac{pL_{Sc}}{2R} \right)^2 \cdot \omega^2 F_{ext} \quad (2.15)$$

$$T_{D,0}^* = -\frac{L_{Sc}}{2R} \cdot \frac{k_\infty}{k_D} \cdot (pL_{Sc})^2 \cdot \omega F_{ext}^2 \quad (2.16)$$

Secondly, when the spin speed is very large, meaning that the constant terms become negligible compared to the ones involving the spin speed ω , those expressions reduce to:

$$z_{\infty}^* = \frac{F_{ext}}{k_{\infty} + k_D} \quad (2.17)$$

$$F_{ED,\infty}^* = -\frac{k_{\infty}}{k_{\infty} + k_D} \cdot F_{ext} \quad (2.18)$$

$$T_{D,\infty}^* = -\frac{2R}{L_{Sc}} \cdot \frac{k_{\infty}}{k_D} \cdot \left(\frac{L_{Sc}}{1 + \frac{k_{\infty}}{k_D}} \right)^2 \cdot \frac{F_{ext}^2}{\omega} \quad (2.19)$$

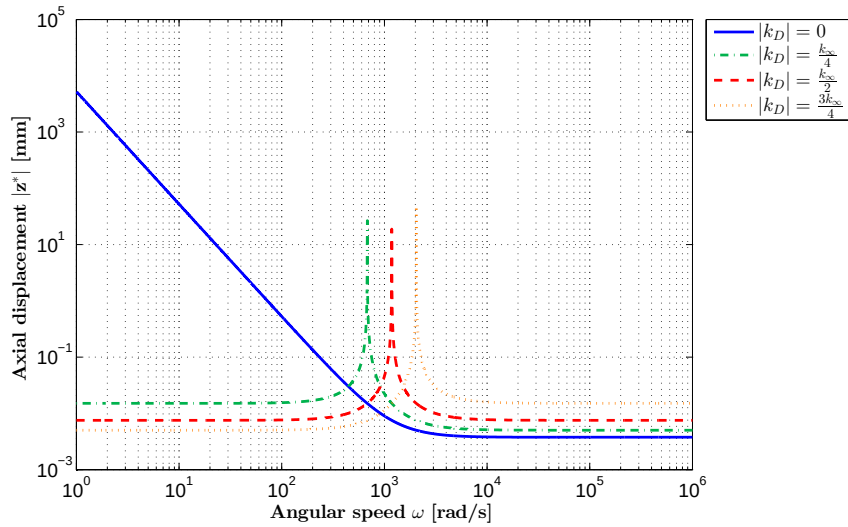


Fig. 2.2: Absolute value of the axial displacement at equilibrium as a function of the spin speed ω , for values of k_D ranging from 0 to $\frac{3k_{\infty}}{4}$.

Figure 2.2 depicts the absolute value of the axial displacement at equilibrium, as a function of the spin speed ω , when the rotor is subjected to an external force $F_{ext} = 1$ [N]. First, let us consider the case where no detent force is applied to the rotor, i.e. $k_D = 0$. At very low spin speed, the EDTB fails to stabilize the rotor as the axial displacement at equilibrium is larger than the airgap of the machine, namely $e = 1$ [mm]. However, as the spin speed increases, the axial excursion becomes smaller than the airgap, around approximately 100 [rad/s]. Finally, at very high spin speed, the axial displacement levels off around $z_{\infty}^* = \frac{F_{ext}}{k_{\infty}} = 3.76$ [μm], as predicted by (2.17)

Let us now discuss the case where a detent force is applied to the rotor, i.e. $k_D \neq 0$. When the spin speed is very low, the rotor stabilizes around a negative axial position, given by $z_0^* = \frac{F_{ext}}{k_D}$. This axial displacement is negative as $k_D < 0$ by definition. However, when the spin speed increases, the axial displacement remains negative but increases in absolute value until a frequency ω_0 at which the axial displacement goes to infinity. This frequency can be determined by identification of the denominator of (2.10) with a canonical second order polynomial $(j\omega)^2 + 2\zeta\omega_0(j\omega) + \omega_0^2$, where ζ is the damping coefficient and ω_0 is the natural frequency. In this case, $\zeta = 0$ meaning that the system is not damped. The natural frequency is expressed as:

$$\omega_0 = \frac{1}{p} \left(\frac{2R}{L_{Sc}} \right) \sqrt{\frac{-k_D}{k_D + k_{\infty}}} \quad (2.20)$$

It will be seen later in this section that this spin speed corresponds to the case where the total stiffness is equal to zero, thus yielding an infinite axial displacement for a finite external force.

Finally, at very high spin speed, the rotor stabilizes around a positive axial position $z_\infty^* = \frac{F_{ext}}{k_\infty + k_D}$. Assuming that $k_D < k_\infty$, as $|k_D|$ increases, therefore strengthening the destabilizing detent force, the value of the axial excursion becomes larger, as can be seen in Fig. 2.2.

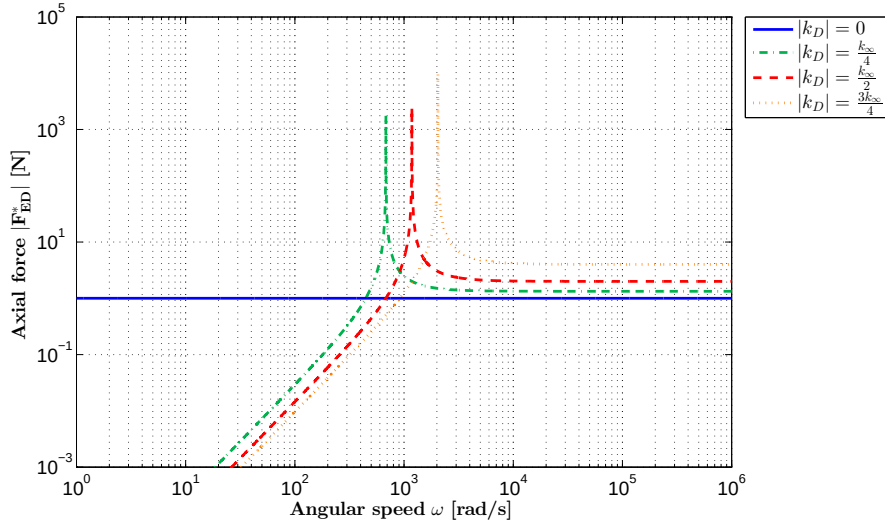


Fig. 2.3: Absolute value of the electrodynamic axial force at equilibrium as a function of the spin speed ω , for values of k_D ranging from $\frac{3k_\infty}{4}$.

As far as the electrodynamic axial force is concerned, Fig 2.3 shows that, when the rotor is not subjected to a detent force, the axial force is exactly the opposite of the external force, i.e. $F_{ED}^* = -F_{ext}$. On the contrary, when the rotor is subject to a detent force, at low spin speed, the axial force is no longer restoring but destabilizing. Its amplitude increases like ω^2 , as stated by the simplified expression (2.15). However, above the frequency ω_0 , the electrodynamic axial force becomes restoring and stabilizes at a value $F_{ED,\infty}^* = -\frac{k_\infty}{k_\infty + k_D} \cdot F_{ext}$ when the spin speed goes to infinity, as stated by (2.18).

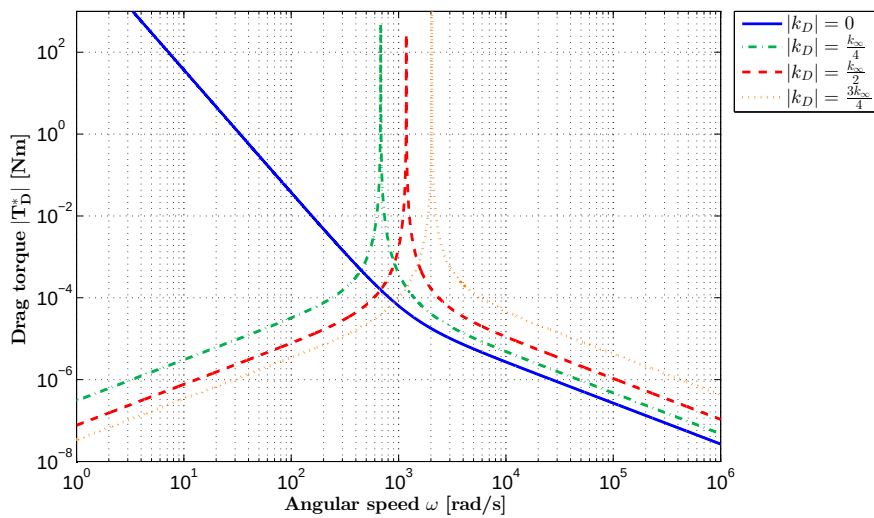


Fig. 2.4: Absolute value of the drag torque at equilibrium as a function of the spin speed ω , for values of k_D ranging from 0 to $\frac{3k_\infty}{4}$.

Figure 2.4 presents the drag torque at equilibrium as a function of the spin speed. When no detent force is applied to the rotor, this drag torque goes to infinity when the spin speed goes to

zero. At low spin speed, it decreases like $\frac{1}{\omega^3}$, as can be found from (2.12). However, at higher spin speed, the drag torque still decreases, but like $\frac{1}{\omega}$, going to zero when the angular speed goes to infinity, as stated by (2.19). In the case where a detent force is applied to the rotor, the drag torque initially contributes positively to the rotation of the machine and increases like ω . However, this case must be avoided as the electrodynamic axial force is destabilizing as long as the spin speed is lower than ω_0 . Above this frequency, the drag torque contributes negatively to the acceleration of the rotor and decreases like $\frac{1}{\omega}$. Just as in the case without detent force, the drag torque goes to zero for an infinite spin speed.

Another important quantity related to the EDTB behavior is the value of the RMS phase current, given by (1.33) and expressed in this case as:

$$I_{rms}^* = \frac{1}{\sqrt{2}K_\Phi N} \cdot \sqrt{(F_{ED}^*)^2 + \left(\frac{T_D^*}{pz^*}\right)^2} \quad (2.21)$$

Combining (2.18) and (2.19) with (2.21), the RMS phase current at infinite spin speed is expressed as:

$$I_{rms,\infty}^* = \frac{1}{\sqrt{2}K_\Phi N} \cdot \frac{k_\infty}{k_\infty + k_D} \cdot F_{ext} \quad (2.22)$$

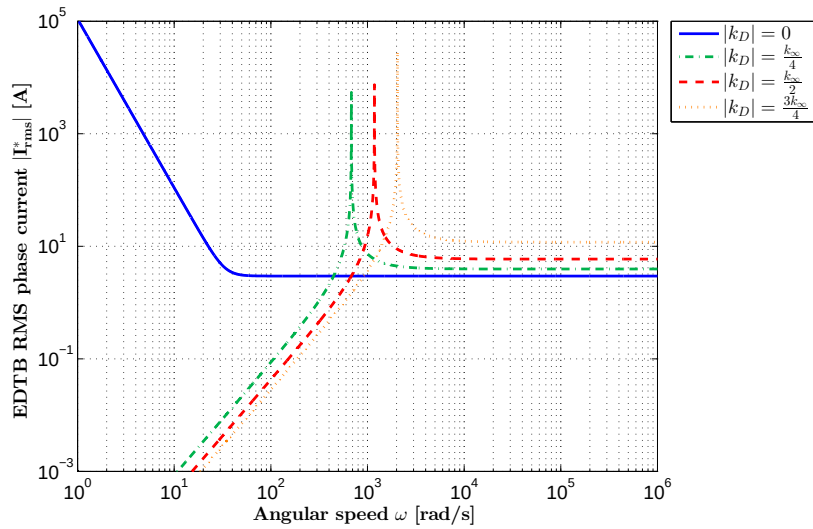


Fig. 2.5: RMS phase current in the EDTB at equilibrium, as a function of the spin speed ω , for values of k_D ranging from 0 to $\frac{3k_\infty}{4}$.

Figure 2.5 represents the RMS phase current as a function of the spin speed, for different values of k_D . In the case without detent force, for a given external force, the required RMS value for the phase current goes to infinity at low spin speed and levels off around a value $I_{rms,\infty}^* = 2.96$ [A] at high spin speed, as predicted by (2.22). When a detent force is applied to the rotor, this value increases, reaching $I_{rms,\infty}^* = 11.84$ [A] in the case where $k_D = \frac{3k_\infty}{4}$. The presence of a detent force thus increases the power consumption of the EDTB at high spin speed, as the detent force must be compensated besides the external force.

As far as the AFPM motor is concerned, the equilibrium value of the quadrature-axis current I_q^* is not represented, as it has the same graph as the drag torque, scaled by a factor $\frac{-1}{p\Phi_0\sqrt{N}}$. However, it is interesting to observe the evolution of the direct- and quadrature- axis voltages at equilibrium as a function of the spin speed. The d-axis voltage can be expressed as:

$$u_d^* = \frac{\omega L_{Mc} T_D^*}{\Phi_0 \sqrt{N}} \quad (2.23)$$

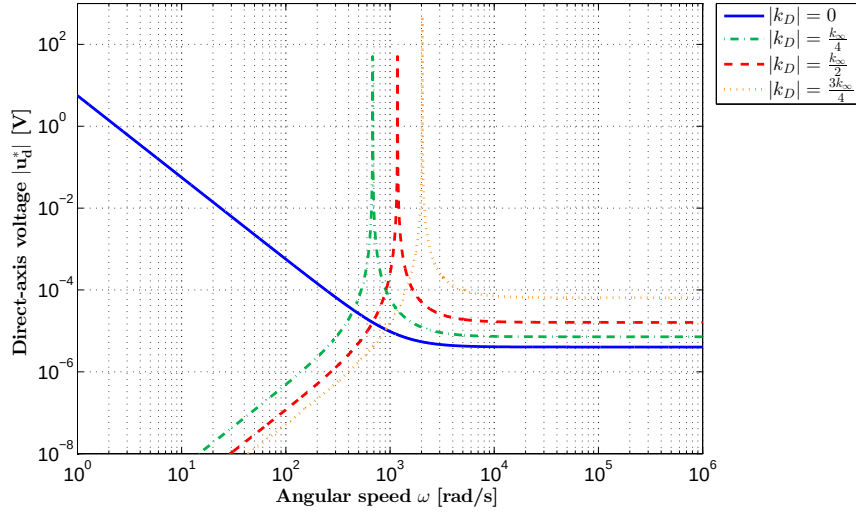


Fig. 2.6: Direct-axis voltage at equilibrium as a function of the spin speed ω , for values of k_D ranging from 0 to $\frac{3k_\infty}{4}$.

According to the observations on the behavior of the drag torque as a function of the spin speed, the evolution of the d-axis voltage can be deduced as it corresponds to T_D^* multiplied by ω . It can be seen in Fig. 2.6 that, at low spin speed, the d-axis voltage decreases like $\frac{1}{\omega^2}$ when $k_D = 0$ and increases like ω^2 when $k_D \neq 0$. However, when a detent force is applied to the rotor, u_d^* is positive below the spin speed ω_0 and goes to infinity at ω_0 . Regardless of the presence of a detent force, at high spin speed, the direct-axis voltage reaches a plateau whose value is given by:

$$u_{d,\infty}^* = -\frac{2R}{\Phi_0\sqrt{N}} \cdot \frac{L_{Mc}}{L_{Sc}} \cdot \frac{k_\infty}{k_D} \cdot \left(\frac{L_{Sc}F_{ext}}{1 + \frac{k_\infty}{k_D}} \right)^2 \quad (2.24)$$

The absolute value of the d-axis voltage $|u_{d,\infty}^*|$ increases by roughly one order of magnitude when $|k_D|$ goes from zero to $\frac{3k_\infty}{4}$. However, this value remains small, as it is below 10^{-4} [V].

As far as the quadrature-axis voltage is concerned, its analytical expression is slightly more complicated, so its evolution with the spin speed is only discussed based on Fig. 2.7. On the one hand, in the case where $k_D = 0$, the q-axis voltage u_q^* goes to infinity when the spin speed goes to zero. It decreases like $\frac{1}{\omega^3}$ at low spin speed and increases like ω at high spin speed. On the other hand, when $k_D \neq 0$, the q-axis voltage always increases like ω , but presents a positive peak at the frequency ω_0 and a negative peak at a slightly lower frequency.

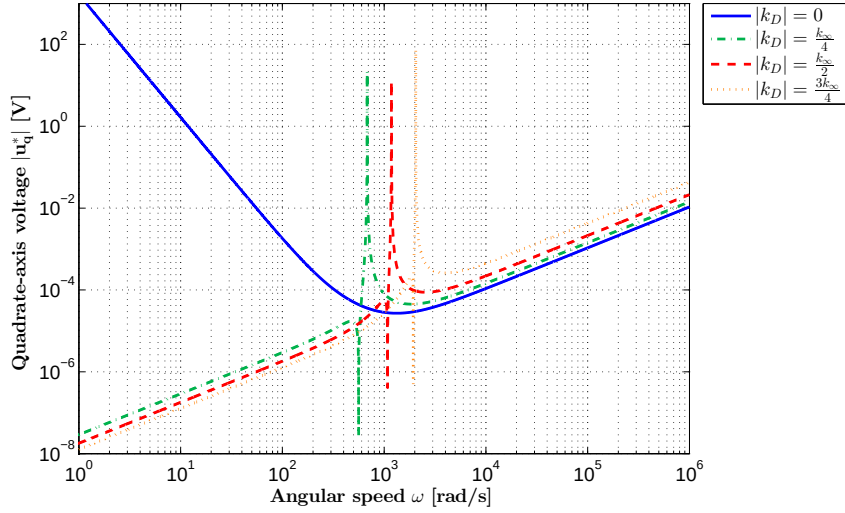


Fig. 2.7: Quadrature-axis voltage at equilibrium as a function of the spin speed ω , for values of k_D ranging from 0 to $\frac{3k_\infty}{4}$.

Imposing the axial displacement z

The second possibility to determine the quasi-static equilibrium point consists in imposing the axial displacement z and to express the other quantities in terms of this axial excursion. A possible motivation for using this method could be to determine the maximum axial load that the EDTB can sustain for a given axial displacement and spin speed. This method yields the following expressions for F_{ext}^* , F_{ED}^* and T_D^* :

$$F_{ext}^*(\omega, z) = \left(k_\infty \frac{(p\omega)^2}{(p\omega)^2 + \left(\frac{2R}{L_{Sc}}\right)^2} + k_D \right) z \quad (2.25)$$

$$F_{ED}^*(\omega, z) = -k_\infty \cdot \frac{(p\omega)^2}{(p\omega)^2 + \left(\frac{2R}{L_{Sc}}\right)^2} \cdot z \quad (2.26)$$

$$T_D^*(\omega, z) = -k_\infty \cdot \frac{2R}{L_{Sc}} \cdot \frac{p^2\omega}{(p\omega)^2 + \left(\frac{2R}{L_{Sc}}\right)^2} \cdot z^2 \quad (2.27)$$

As in the situation where the external force is imposed, two extreme cases regarding the spin speed are considered, the first one corresponding to a spin speed close to zero. In this case, (2.25) to (2.27) reduce to:

$$F_{ext,0}^* = k_D \cdot z \quad (2.28)$$

$$F_{ED,0}^* = -k_\infty \cdot \left(\frac{pL_{Sc}}{2R} \right)^2 \cdot \omega^2 z \quad (2.29)$$

$$T_{D,0}^* = -k_\infty \cdot \left(\frac{p^2 L_{Sc}}{2R} \right) \cdot \omega z^2 \quad (2.30)$$

Again, when the spin speed becomes very large, those expressions reduce to:

$$F_{ext,\infty}^* = (k_\infty + k_D) \cdot z \quad (2.31)$$

$$F_{ED,\infty}^* = -k_\infty \cdot z \quad (2.32)$$

$$T_{D,\infty}^* = -k_\infty \cdot \frac{2R}{L_{Sc}} \cdot \frac{z^2}{\omega} \quad (2.33)$$

Figure 2.8 depicts the absolute value of the external force that can be sustained by the EDTB at equilibrium, for a given axial displacement, here $z = 0.1$ [mm], different values of the detent force stiffness k_D , and as a given spin speed. In the case where no detent force is applied to the rotor, the axial load that can be sustained is always positive and increases like ω^2 at low frequency, according to (2.25), until reaching a plateau whose value is given by (2.31), i.e. $F_{ext,\infty}^* = k_\infty \cdot z = 26.6$ [N], at high spin speed. When a detent force is applied to the rotor, at low spin speed, the external force that can be sustained is negative and equal to $F_{ext,0}^* = k_D \cdot z$, as stated by 2.28. However, as the spin speed increases, the sustainable axial load increases until it reaches zero at the frequency ω_0 , given by (2.20). This comes from the fact that the total stiffness is equal to zero at this particular frequency, as mentioned earlier. The external force then becomes positive and reaches a plateau at $F_{ext,\infty}^* = (k_\infty + k_D) \cdot z$, which is slightly lower than the value found when considering the case with no detent force. Concretely, this means that the presence of a detent force decreases the axial load that can be sustained by the EDTB.

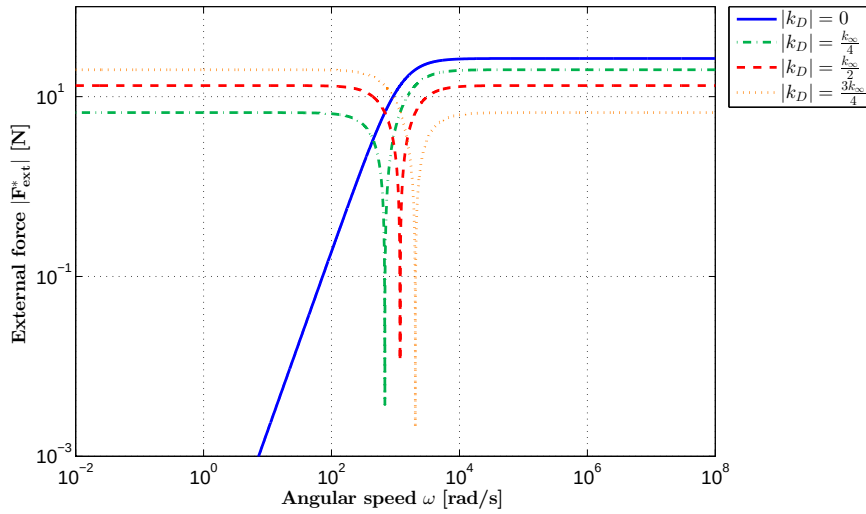


Fig. 2.8: Absolute value of the external force at equilibrium as a function of the spin speed ω , for values of k_D ranging from 0 to $\frac{3k_\infty}{4}$.

The absolute value of the electrodynamic axial force F_{ED}^* and of the drag torque T_D^* at equilibrium is shown in Fig. 2.9, as a function of the spin speed, for three different axial displacements. As far as the axial force is concerned, its value is close to zero at low spin speed, but increases like ω^2 , as can be seen from (2.26). However, at high spin speed, the axial force levels off around $F_{ED,\infty}^* = -k_\infty \cdot z$, as stated by (2.32). It is interesting to notice that this plateau depends linearly on the axial displacement z . The drag torque, however, depends quadratically on the axial displacement z . This torque goes to zero at very low and very high spin speed, as it increases like ω at low spin speed and decreases like $\frac{1}{\omega}$ at high spin speed. Furthermore, it reaches a maximum at a frequency $\omega_{\max}$ given by:

$$\omega_{\max} = \frac{1}{p} \left(\frac{2R}{L_{Sc}} \right) \quad (2.34)$$

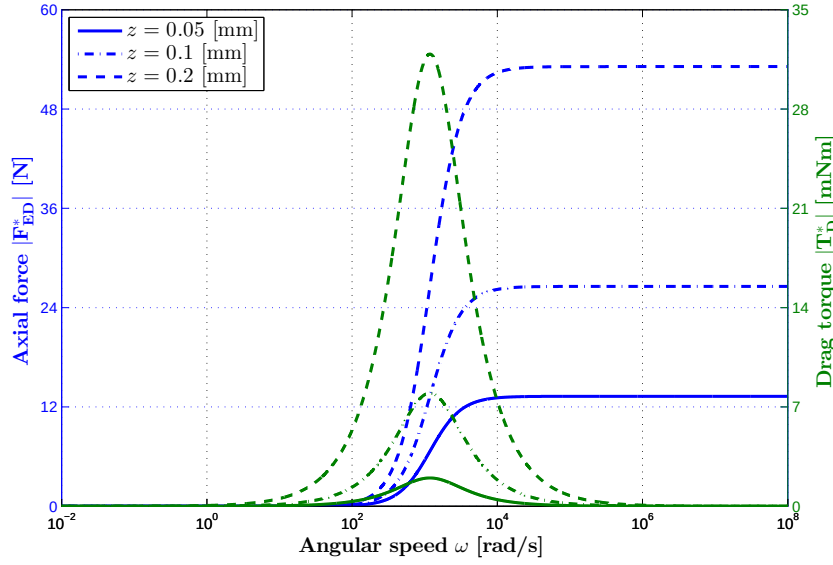


Fig. 2.9: Absolute value of the electrodynamic axial force and the drag torque at equilibrium, as functions of the spin speed ω , for values of z ranging from 0.05 to 0.2 [mm].

Another interesting result is the expression of the total stiffness developed by the EDTB as a function of the spin speed and the axial displacement in quasi-static conditions. First, let us consider the expression of the EDTB stiffness:

$$k_{\text{tot}}(\omega) = -\frac{F_{ED}(\omega) + F_D}{z} = k_{\infty} \cdot \frac{(p\omega)^2}{(p\omega)^2 + \left(\frac{2R}{L_{Sc}}\right)^2} + k_D \quad (2.35)$$

At infinite spin speed, the stiffness is maximal and has the following value:

$$k_{\text{tot},\infty} = \lim_{\omega \rightarrow \infty} k_{\text{tot}}(\omega) = k_{\infty} + k_D \quad (2.36)$$

and at zero spin speed, the stiffness is minimal and has a negative value:

$$k_{\text{tot},0} = \lim_{\omega \rightarrow 0} k_{\text{tot}}(\omega) = k_D \quad (2.37)$$

The evolution of the total stiffness of the EDTB with the rotational speed is depicted in Fig. 2.10. It can be seen that the stiffness starts to increase around roughly 100 [rad/s], and reaches a maximum value of 265 [N/mm] above 10 [krad/s] when $k_D = 0$. Two other cases are depicted in this figure, for $|k_D| = \frac{k_{\infty}}{4}$ and $|k_D| = \frac{k_{\infty}}{2}$ respectively. The addition of a detent force shifts the curve downwards, thus creating a range of low spin speeds where the stiffness is negative. This explains the instability of the EDTB at low spin speed. The transition from negative to positive stiffness is done at a spin speed ω_0 given by 2.20.

As far as the RMS value of the phase current in the EDTB is concerned, combining (2.26) and (2.27) with (1.33) allows to write I_{rms}^* as:

$$I_{rms}^* = \sqrt{2} \frac{p\omega K_{\Phi} z}{\sqrt{(p\omega L_{Sc})^2 + (2R)^2}} \quad (2.38)$$

When the rotational speed goes to infinity, this expression reduces to:

$$I_{rms,\infty}^* = \frac{\sqrt{2} K_{\Phi} z}{L_{Sc}} \quad (2.39)$$

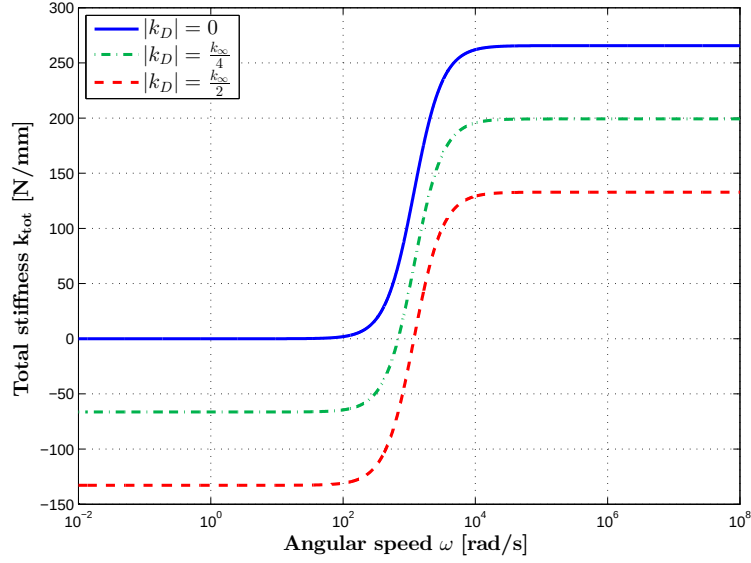


Fig. 2.10: Total stiffness of the EDTB as a function of the spin speed of the motor.

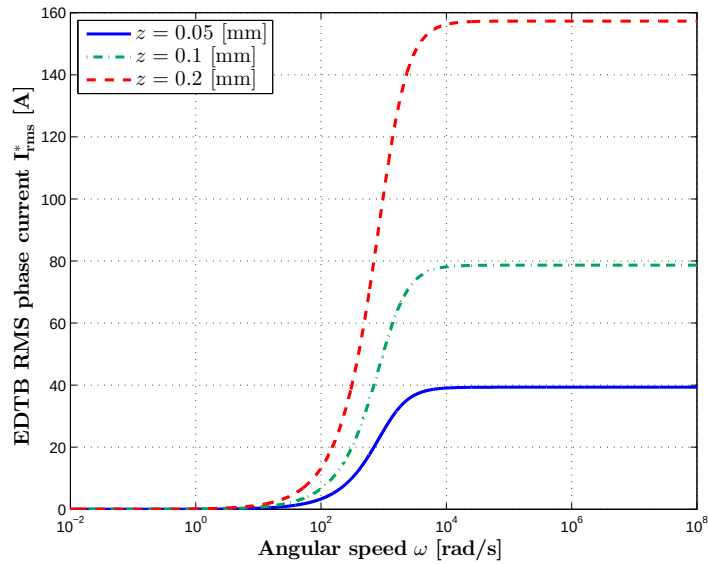


Fig. 2.11: RMS value of the phase current in the EDTB, as a function of the spin speed ω , for values of z ranging from 0.05 to 0.2 [mm].

Figure 2.11 presents the evolution of the RMS phase current in the EDTB, as described by (2.38), for three different axial excursions. It should be noticed that the RMS phase current depends linearly on the axial displacement. Another important quantity that has to be considered is the RMS current density $J_{rms} = \frac{I_{rms}}{S}$, where S is the cross section of the conductors constituting the coils. Based on the geometrical parameters of Table 2.1, the cross section of the conductors is $S = 16 \text{ [mm}^2\text{]}$. The current density for the different axial displacements can be found in Table 2.3. For axial displacements under $z = 0.1 \text{ [mm]}$, the current density remains below the admissible $5 \text{ [A/mm}^2\text{]}$ usually considered to be a good limit in the literature.

As far as the AFPM motor is concerned, let us first examine the evolution of the q-axis current I_q^* at equilibrium. Equation (2.8) states that this current is proportional to the drag torque, in the absence of viscous friction and external torque. Therefore, Fig. 2.12 is similar to the graph of the

z [mm]	I_{rms}^* [A]	J_{rms}^* [A/mm ²]
0.05	39.3	2.46
0.1	78.7	4.92
0.2	157.3	9.83

Table 2.3: RMS phase current and current density for values of z ranging from 0.05 to 0.2 [mm].

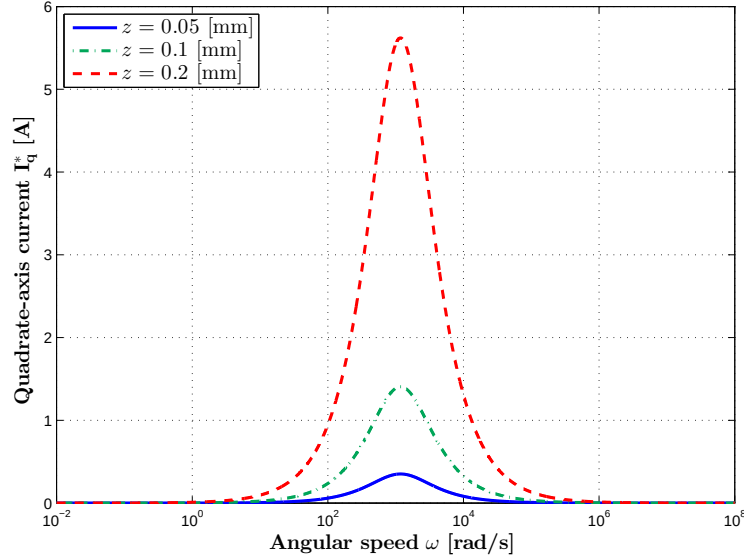


Fig. 2.12: Quadrature-axis current as a function of the spin speed ω , for values of z ranging from 0.05 to 0.2 [mm].

drag torque shown in Fig. 2.9 and also highlights a quadratic dependency of the q-axis current on the axial displacement z . However, it is still interesting to obtain numerical values for the q-axis current. Its maximum value, reached at a spin speed $\omega_{\max}$, is given in Table 2.4, together with the corresponding current density. One should note that the current values are much lower compared to the EDTB RMS phase current and that the admissible limit of 5 [A/mm²] is never reached.

z [mm]	I_q^* [A]	J_q^* [mA/mm ²]
0.05	0.35	21.9
0.1	1.41	88.1
0.2	5.62	351.2

Table 2.4: Maximum q-axis current and current density for values of z ranging from 0.05 to 0.2 [mm].

The d-axis voltage is depicted in Fig. 2.13. It can be seen that, below the spin speed $\omega_{\max}$, corresponding here to the point where the slope is maximum, the u_d^* voltage is approximately zero. As the spin speed increases, the voltage increases in absolute value and then levels off at roughly 10 [krad/s]. The values obtained are relatively small, as the maximum voltage equals -11.5 [mV] in the case where $z = 0.2$ [mm]. It is also interesting to note that the d-axis voltage depends quadratically on the axial displacement, just as the drag torque and the q-axis current.

As far as the q-axis voltage is concerned, it always increases like ω , except around $\omega_{\max}$ where it flattens a bit due to the contribution of the current I_q^* which decreases like $\frac{1}{\omega}$ above $\omega_{\max}$. Also, the larger the axial displacement, the larger the value of the q-axis voltage. However, compared to the d-axis voltage, the relationship with the axial displacement is not that straightforward as u_q^* is composed of a term including I_q^* , which depends quadratically on z , and a term in ωz , which

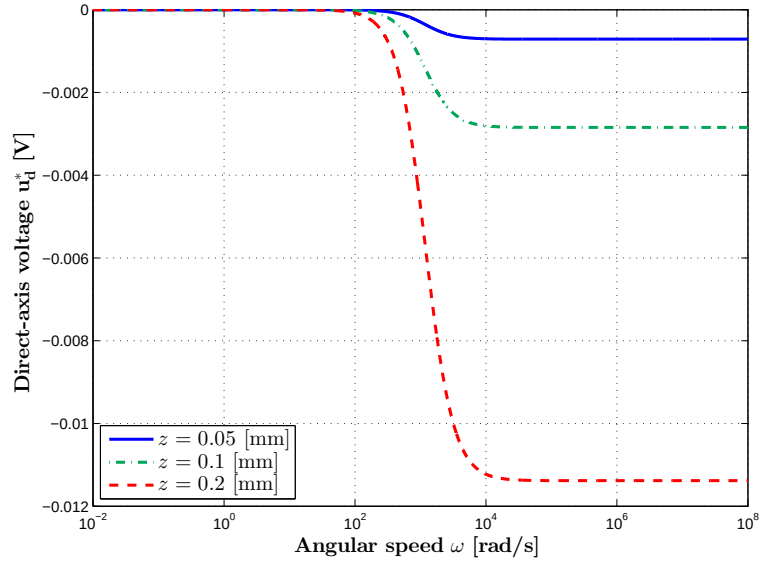


Fig. 2.13: Direct-axis voltage as a function of the spin speed ω , for values of z ranging from 0.05 to 0.2 [mm].

depends linearly on z .

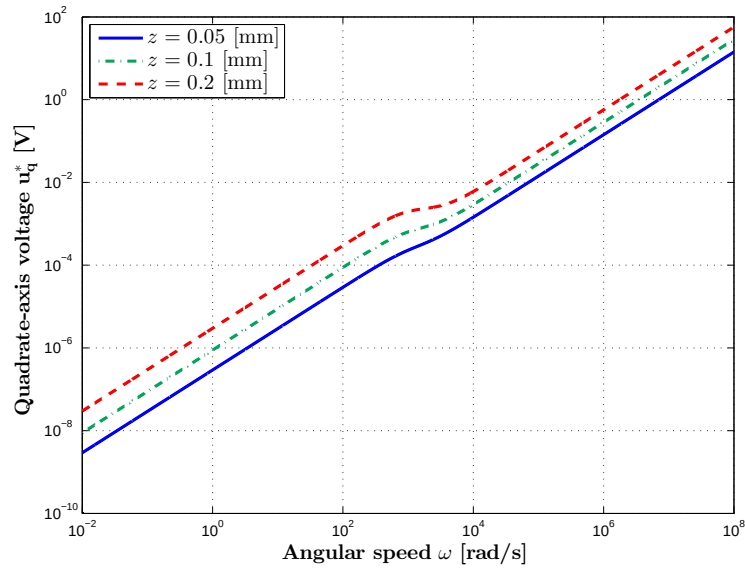


Fig. 2.14: Quadrature-axis voltage as a function of the spin speed ω , for values of z ranging from 0.05 to 0.2 [mm].

2.2.3 Root locus analysis

Provided that the matrix $\mathbf{A}$ resulting from the linearization of the dynamic model equations is known, it is possible to discuss the stability properties of the system based on the eigenvalues of this matrix. In particular, a detailed analysis of the root locus is required to determine under which conditions the system is asymptotically stable. To do so, let us first discuss the stability of the AFPM motor and the EDTB separately, before considering the stability of the self-bearing AFPM motor.

AFPM motor stability

In the case of the AFPM motor, only equations (1.41), (1.42) and (1.44) of the dynamic model are considered. The state variables are respectively the direct-axis current I_d , the quadrature-axis current I_q and the spin speed $\dot{\alpha}$. Based on those equations, the corresponding matrix $\mathbf{A}$ is given by:

$$\mathbf{A} = \begin{pmatrix} -\frac{R}{2L_{Mc}} & p\omega & pI_q^* \\ -p\omega & -\frac{R}{2L_{Mc}} & -p\left(I_d^* + \frac{\Phi_0\sqrt{N}}{2L_{Mc}}\right) \\ 0 & \frac{p\Phi_0\sqrt{N}}{J} & -\frac{C_\alpha}{J} \end{pmatrix} \quad (2.40)$$

Using the fact that the d-axis current controller enforces $I_d^* = 0$ at equilibrium, the characteristic polynomial of the matrix can be calculated as:

$$\begin{aligned} \det(s\mathbf{I} - \mathbf{A}) &= \begin{vmatrix} (s + \omega_m) & -p\omega & -pI_q^* \\ p\omega & (s + \omega_m) & \frac{p\Phi_0\sqrt{N}}{2L_{Mc}} \\ 0 & -\frac{p\Phi_0\sqrt{N}}{J} & (s + \omega_\alpha) \end{vmatrix} \\ &= s^3 + (2\omega_m + \omega_\alpha)s^2 + \left(\omega_m^2 + 2\omega_m\omega_\alpha + (p\omega)^2 + \frac{(p\Phi_0)^2 N}{2JL_{Mc}}\right)s \\ &\quad + \left(\omega_\alpha[\omega_m^2 + (p\omega)^2] + \frac{p^2\Phi_0\sqrt{N}}{J} \left[p\omega I_q^* + \omega_m \frac{\Phi_0\sqrt{N}}{2L_{Mc}}\right]\right) \end{aligned} \quad (2.41)$$

where $\omega_m = \frac{R}{2L_{Mc}}$ is the natural electrical frequency of the motor and $\omega_\alpha = \frac{C_\alpha}{J}$ is its natural damping frequency. Using the Routh table and the fact that, at equilibrium, $I_q^* = \frac{C_\alpha\omega - T_{ext}}{p\Phi_0\sqrt{N}}$, the conditions guaranteeing that the system is asymptotically stable are the following:

1. This first condition is trivially satisfied, as both ω_m and ω_α are strictly positive in the case of a physical system.

$$2\omega_m + \omega_\alpha > 0 \quad (2.42)$$

2. To discuss the second stability condition, two cases are considered:

- When $C_\alpha = 0$, the spin speed ω must satisfy the following inequality:

$$\omega^2 + \frac{T_{ext}}{2J\omega_m} \cdot \omega + \left(\left(\frac{\omega_m}{p}\right)^2 + \frac{\Phi_0^2 N}{4JL_{Mc}}\right) > 0 \quad (2.43)$$

This inequality holds for any positive value of ω .

- When $C_\alpha \neq 0$, the spin speed ω must satisfy the following inequality:

$$\left(1 - \frac{\omega_\alpha}{2\omega_m}\right) \omega^2 + \frac{T_{ext}}{2J\omega_m} \cdot \omega + \left(\left(\frac{\omega_m + \omega_\alpha}{p} \right)^2 + \frac{\Phi_0^2 N}{2JL_{Mc}} \frac{\omega_m + \omega_\alpha}{2\omega_m} \right) > 0 \quad (2.44)$$

This equation has real solutions if $\Delta \geq 0$. Assuming that $\omega_\alpha < \omega_m$, only one of those solutions is positive. It is expressed as:

$$\omega_{m,1} = \frac{T_{ext}}{2J(\omega_\alpha - 2\omega_m)} + \frac{1}{2 \left(1 - \frac{\omega_\alpha}{2\omega_m}\right)} \sqrt{\Delta} \quad (2.45)$$

$$\text{where } \Delta = \left(\frac{T_{ext}}{2J\omega_m} \right)^2 - 4 \cdot \left(1 - \frac{\omega_\alpha}{2\omega_m}\right) \cdot \left(\left(\frac{\omega_m + \omega_\alpha}{p} \right)^2 + \frac{\Phi_0^2 N}{2JL_{Mc}} \frac{\omega_m + \omega_\alpha}{2\omega_m} \right)$$

This also means that if $\Delta < 0$ and thus, if the rotational damping coefficient C_α is small enough, the AFPM motor is asymptotically stable, as there are no intersections between the root locus and the imaginary axis. To find the maximum admissible value for C_α , the roots of the following third order polynomial in ω_α are computed numerically:

$$\Delta(\omega_\alpha) = \left(\frac{2}{p^2\omega_m} \right) \omega_\alpha^3 + \left(\frac{\Phi_0^2 N}{2JL_{Mc}\omega_m^2} \right) \omega_\alpha^2 - \left(\frac{6\omega_m}{p^2} + \frac{\Phi_0^2 N}{2JL_{Mc}\omega_m} \right) \omega_\alpha \quad (2.46)$$

$$+ \left(\left(\frac{T_{ext}}{2J\omega_m} \right)^2 - \left(\frac{2\omega_m}{p} \right)^2 - \frac{\Phi_0^2 N}{JL_{Mc}} \right) \quad (2.47)$$

The maximum value that the rotational damping C_α can take, to ensure that the motor is asymptotically stable, is depicted in Fig 2.15. It only depends on the external torque applied to the motor. Nevertheless, its variation with T_{ext} is very small. For the parameters provided in the framework of our case study, the maximum value is around 1.4 [kg·m²/s²].

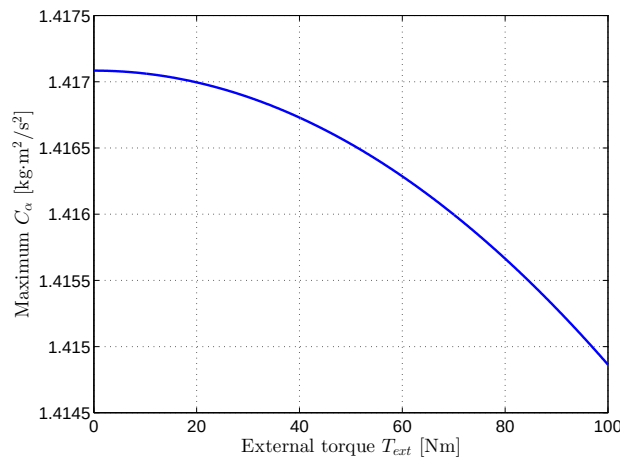


Fig. 2.15: Evolution of the maximum rotational damping coefficient C_α .

3. To discuss the third stability condition, two cases are also considered:

- When $C_\alpha = 0$, the spin speed ω must satisfy the following inequality:

$$\omega < \omega_{m,2} = \frac{\Phi_0^2 N \omega_m}{2L_{Mc} T_{ext}} \quad (2.48)$$

This means that, without rotational damping, the synchronous motor is intrinsically unstable at high spin speed.

- When $C_\alpha \neq 0$, the spin speed ω must satisfy the following inequality:

$$\omega^2 - \frac{T_{ext}}{2J\omega_\alpha} \cdot \omega + \left(\frac{1}{2} \left(\frac{\omega_m}{p} \right)^2 + \frac{\Phi_0^2 N}{4JL_{Mc}} \frac{\omega_m}{\omega_\alpha} \right) > 0 \quad (2.49)$$

This equation has one real positive solution, given by:

$$\omega_{m,2} = \frac{T_{ext}}{4J\omega_\alpha} + \frac{1}{2} \sqrt{\left(\frac{T_{ext}}{2J\omega_\alpha} \right)^2 - 4 \left(\frac{1}{2} \left(\frac{\omega_m}{p} \right)^2 + \frac{\Phi_0^2 N}{4JL_{Mc}} \frac{\omega_m}{\omega_\alpha} \right)} \quad (2.50)$$

if the rotational damping C_α is contained in the following interval:

$$C_\alpha \in \left[-\frac{(p\Phi_0)^2 N}{4\omega_m L_{Mc}} \pm \frac{p^2}{4\omega_m} \sqrt{\left(\frac{\Phi_0^2 N}{L_{Mc}} \right)^2 + 2 \left(\frac{T_{ext}}{p} \right)^2} \right] \quad (2.51)$$

In practice, a solution can only be found for negative values for C_α , which are not realistic in the case of a physical system as the rotational damping must necessarily be positive. Thus, this condition is always satisfied.

First, let us discuss the general shape of the root locus. In Fig. 2.16, the root locus corresponding to the case where $T_{ext} = 1$ [Nm] and $C_\alpha = 0$ is depicted. It can be seen that the two eigenvalues on the left of the graph, corresponding to the electrical ones, always remain in the stable region of the complex plane. However, the eigenvalue on the right, denoted s_1 , has a positive real part when the spin speed exceeds $\omega_{m,2}$, given by (2.48). The synchronous motor without rotational damping is thus, as said previously, intrinsically unstable at high spin speed. A more detailed view of the eigenvalues on the left of Fig. 2.16, denoted s_2 and s_3 , is shown in Fig. 2.17. At low spin speed, both eigenvalues are real, but they become complex conjugates as the spin speed increases.

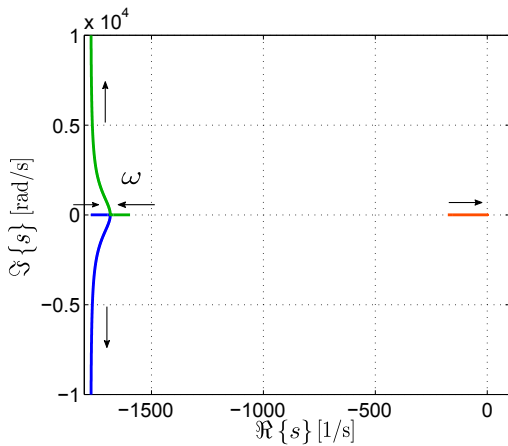


Fig. 2.16: Root locus for $T_{ext} = 1$ [Nm] and $C_\alpha = 0$.

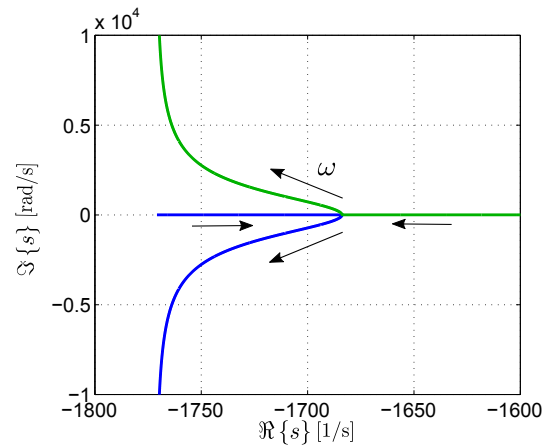


Fig. 2.17: Detail of the root locus of Fig. 2.16 for $T_{ext} = 1$ [Nm] and $C_\alpha = 0$.

Let us now analyze the influence of the rotational damping C_α and the external torque T_{ext} on the eigenvalues of the linearized system. As far as the rotational damping is concerned, Fig. 2.18 shows that, when increasing C_α , the shape of the root locus changes and the two complex conjugates eigenvalues move closer to the imaginary axis. For a value of C_α larger than the maximum one found in (2.47), the root locus intersects with the imaginary axis at a spin speed $\omega_{m,1}$ given by (2.45). It is also interesting to note that, below the maximum value for C_α , the eigenvalue s_1 no longer intersects with the imaginary axis. This means that, for a rotational damping such that $C_\alpha \in [0, C_{\alpha,max}]$, the synchronous motor is asymptotically stable for any spin speed.

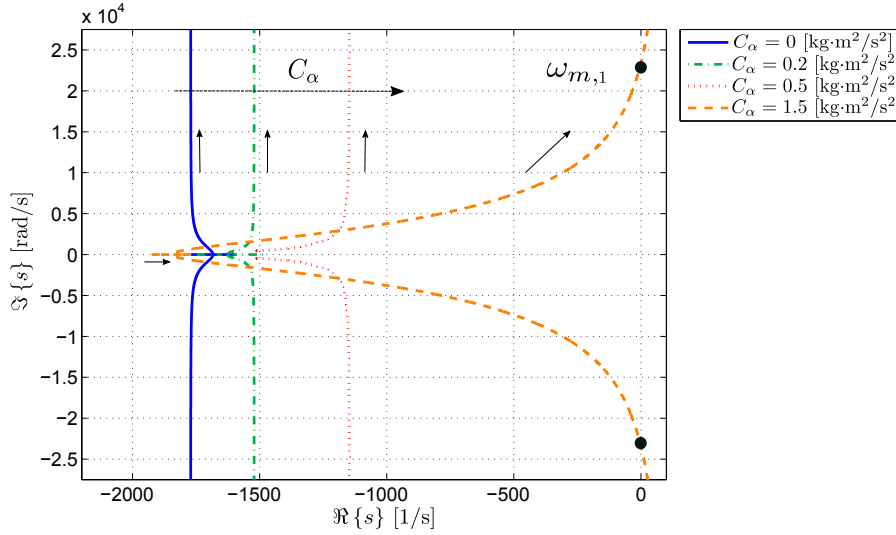


Fig. 2.18: Detail of the root locus for $T_{ext} = 1$ [Nm] and $C_\alpha = 0$ to 1.5 [$\text{kg}\cdot\text{m}^2/\text{s}^2$].

Figure 2.19 depicts the evolution of the root locus for different values of the external torque. When this torque increases, the eigenvalues s_2 and s_3 progressively shift to the left of the complex plane, becoming even more stable. However, as far as the eigenvalue s_1 is concerned, its real part becomes positive above the spin speed $\omega_{m,2}$. The larger the external torque applied to the motor, the lower the spin speed above which the system becomes unstable, as $\omega_{m,2}$ is proportional to $\frac{1}{T_{ext}}$. This behavior is shown in Fig. 2.20. For an external torque of 1 [Nm], $\omega_{m,2}$ is approximately 5.5 [krad/s], against 1 [krad/s] for a torque of 5 [Nm].

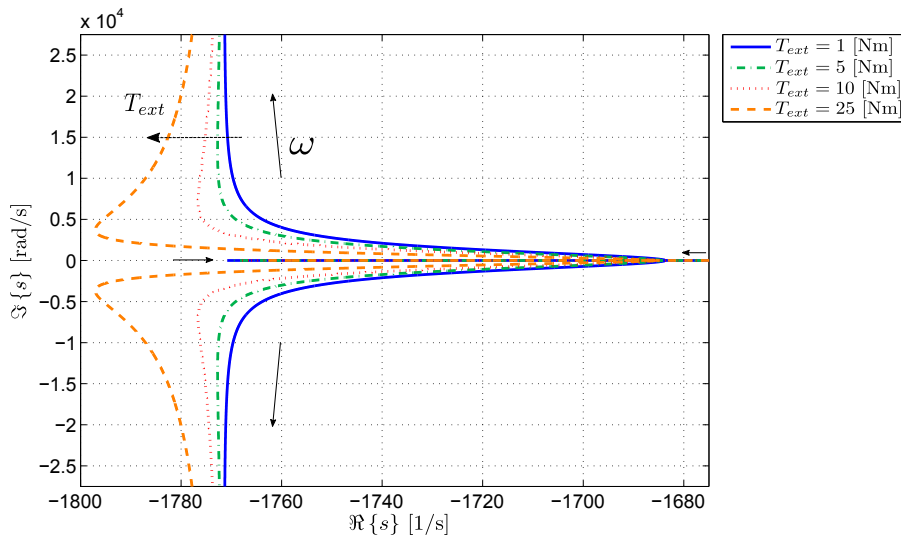


Fig. 2.19: Detail of the root locus for $C_\alpha = 0$ and $T_{ext} = 1$ to 25 [Nm].

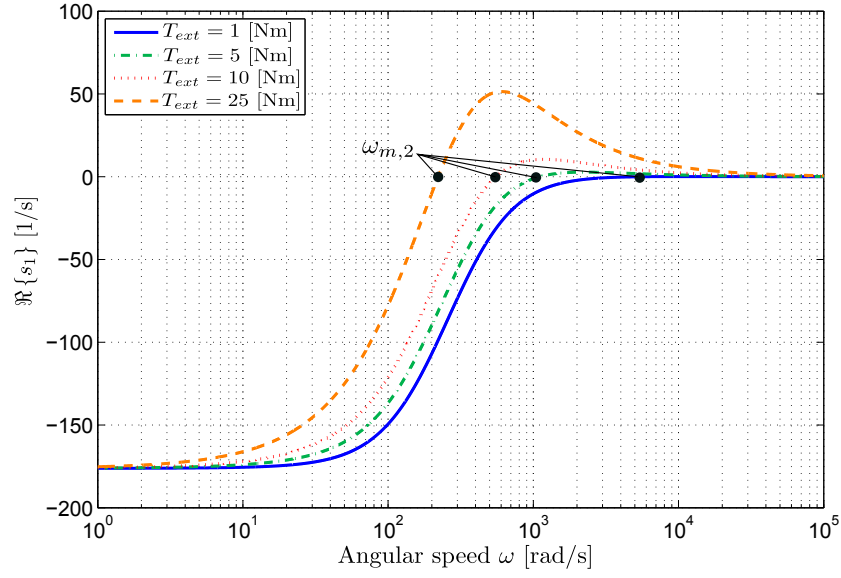


Fig. 2.20: Real part of the eigenvalue s_1 as a function of the spin speed ω , for $C_\alpha = 0$ and $T_{ext} = 1$ to 25 [Nm].

Electrodynamic thrust bearing stability

In the case of the EDTB, equations (1.39), (1.40) and (1.43) of the dynamic model are considered. The trivial equation $\frac{dz}{dt} = \dot{z}$ is also used, as a linear system must be obtained. The state variables are the electrodynamic axial force F_{ED} , the ratio of the drag torque and the axial position $\frac{T_D}{z}$, the axial position z and the axial velocity $\dot{z}$. First, the stability conditions are expressed without any assumption. Then, the hypotheses introduced in [34] are used to simplify the obtained expressions and derive analytical formulae for interesting features of the root locus. Those hypotheses are the following:

- The square of the EDTB natural electrical frequency is larger than the square of the maximal natural frequency of the spring-mass system constituted of the rotor and the EDTB:

$$\omega_s^2 \gg \frac{k_\infty}{m} \quad (2.52)$$

- The EDTB natural electrical frequency is larger than its natural damping frequency:

$$\omega_s \gg \omega_z \quad (2.53)$$

where $\omega_s = \frac{2R}{L_{Sc}}$ is the natural electrical frequency of the EDTB and $\omega_z = \frac{C_z}{m}$ is its natural damping frequency. The matrix of the linearized system constituted of the EDTB equation is defined as:

$$\mathbf{A} = \begin{pmatrix} -\frac{2R}{L_{Sc}} & \omega & 0 & -k_\infty \\ -p^2\omega & -\frac{2R}{L_{Sc}} & -p^2\omega k_\infty & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{m} & 0 & -\frac{k_D}{m} & -\frac{C_z}{m} \end{pmatrix} \quad (2.54)$$

To calculate the eigenvalues of this matrix, let us determine its characteristic polynomial as:

$$\begin{aligned} \det(s\mathbf{I} - \mathbf{A}) &= \begin{vmatrix} (s + \omega_s) & -\omega & 0 & k_\infty \\ p^2\omega & (s + \omega_s) & p^2\omega k_\infty & 0 \\ 0 & 0 & s & -1 \\ -\frac{1}{m} & 0 & \frac{k_D}{m} & (s + \omega_z) \end{vmatrix} \\ &= s^4 + (2\omega_s + \omega_z)s^3 + \left(\frac{k_\infty + k_D}{m} + 2\omega_s\omega_z + \omega_s^2 + (p\omega)^2 \right) s^2 \\ &\quad + \left(\frac{k_\infty + 2k_D}{m} \cdot \omega_s + [\omega_s^2 + (p\omega)^2]\omega_z \right) s + \left(\frac{k_\infty + k_D}{m} \cdot (p\omega)^2 + \frac{k_D}{m} \cdot \omega_s^2 \right) \end{aligned} \quad (2.55)$$

Using the Routh table, the conditions guaranteeing that the system is asymptotically stable are the following:

1. The first condition is, as in the case of the AFPM motor, trivially satisfied as both ω_s and ω_z are strictly positive in the case of a physical system.

$$2\omega_s + \omega_z > 0 \quad (2.56)$$

2. The second condition on the stability of the EDTB is the following:

$$2\omega_s ((\omega_s + \omega_z)^2 + (p\omega)^2) + \frac{k_\infty}{m} \cdot (\omega_s + \omega_z) + \frac{k_D}{m} \cdot \omega_z > 0 \quad (2.57)$$

Translating this condition in terms of the the spin speed ω yields:

$$\omega > \pm \frac{1}{p} \sqrt{-\left(\frac{1}{2m} \left[k_{\infty} \left(1 + \frac{\omega_z}{\omega_s} \right) + k_D \frac{\omega_z}{\omega_s} \right] + (\omega_s + \omega_z)^2 \right)} \quad (2.58)$$

This solution only makes sense if the argument of the square root is positive, otherwise the stability condition is always satisfied. This means that the following polynomial in ω_z must be negative:

$$\omega_z^2 + \left(2\omega_s + \frac{k_{\infty} + k_D}{2m\omega_s} \right) \omega_z + \left(\omega_s^2 + \frac{k_{\infty}}{2m} \right) \leq 0 \quad (2.59)$$

which is only true if the natural damping frequency ω_z is such that:

$$\omega_z \in \left[-\left(\omega_s + \frac{k_{\infty} + k_D}{4m\omega_s} \right) \pm \frac{1}{2} \sqrt{\Delta_{\omega_z}} \right] \quad (2.60)$$

where $\Delta_{\omega_z} = \left(\frac{k_{\infty} + k_D}{2m} \right)^2 \frac{1}{\omega_s^2} + \frac{2k_D}{m}$. Again, this solution only makes sense if $\Delta_{\omega_z} \geq 0$, yielding a condition of the following polynomial in k_D :

$$k_D^2 + (2k_{\infty} + 8\omega_s^2 m)k_D + k_{\infty}^2 > 0 \quad (2.61)$$

This equation always has two real solutions as $\Delta_{k_D} = 64\omega_s^4 m^2 \left(1 + \frac{k_{\infty}}{2m} \frac{1}{\omega_s^2} \right)$ is strictly positive. Thus, the values of k_D that satisfy to this condition are:

$$k_D \notin \left[-k_{\infty} - 4\omega_s^2 m \left(1 \pm \sqrt{1 + \frac{k_{\infty}}{2m} \frac{1}{\omega_s^2}} \right) \right] \quad (2.62)$$

Using assumption (2.52), this condition further simplifies to:

$$k_D \notin [-k_{\infty} - 8\omega_s^2 m; -k_{\infty}] \quad (2.63)$$

The value of k_D should be comprised between $-k_{\infty}$ and 0, otherwise the detent force is always larger than the restoring force generated by the EDTB. If it is the case, it means that $\Delta_{\omega_z} > 0$ and thus, that it is always mathematically possible to find values of ω_z verifying condition (2.59). However, in a physical system, the damping is necessarily positive, but the values given by (2.60) are necessarily negative, considering assumption (2.52). Finally, this means that this stability condition is always verified for a physical system as $|k_D| < k_{\infty}$ and $C_z \geq 0$.

3. The third condition to ensure that the EDTB is asymptotically stable is the following:

$$K_1(p\omega)^4 + K_2(p\omega)^2 + K_3 > 0 \quad (2.64)$$

where the coefficients K_1 , K_2 and K_3 are given by:

$$K_1 = 2\omega_s \omega_z \quad (2.65)$$

$$K_2 = 2\omega_s \omega_z (\omega_s^2 + (\omega_s + \omega_z)^2) - \frac{k_{\infty}}{m} \cdot \omega_s (2\omega_s + 3\omega_z) - \frac{4k_D}{m} \cdot \omega_s \omega_z \quad (2.66)$$

$$K_3 = 2\omega_s^3 \omega_z (\omega_s + \omega_z)^2 + \frac{k_{\infty}}{m} \cdot \omega_s^2 (2\omega_s^2 + 5\omega_s \omega_z + 3\omega_z^2) + \frac{4k_D}{m} \cdot \omega_s^2 (\omega_s \omega_z + \omega_z^2) + \frac{1}{m^2} (k_{\infty}^2 \omega_s (\omega_s + \omega_z) + k_{\infty} k_D \omega_s (2\omega_s + 3\omega_z) + 2k_D^2 \omega_s \omega_z) \quad (2.67)$$

Two cases are considered regarding the axial damping coefficient C_z and the detent force stiffness k_D . First, let us derive the exact stability solution for $C_z = 0$ and $k_D \neq 0$. In this case, we obtain that:

$$\omega < \omega_{s,1} = \frac{1}{p} \sqrt{\omega_s^2 + \frac{k_\infty + 2k_D}{2m}} \quad (2.68)$$

Secondly, let us consider the general case, with $C_z \neq 0$ and $k_D \neq 0$, but using the hypotheses of [34]. The coefficients simplify to:

$$K_1 = 2\omega_s\omega_z \quad (2.69)$$

$$K_2 = 2\omega_s^2 \left(2\omega_s\omega_z - \frac{k_\infty}{m} \right) \quad (2.70)$$

$$K_3 = 2\omega_s^4 \left(\omega_s\omega_z + \frac{k_\infty}{m} \right) \quad (2.71)$$

Those coefficients are identical to the ones obtained in [34] while using the simplifying assumptions directly on the characteristic polynomial. This confirms that no additional term has been forgotten. The stability condition is then $\omega \in -\infty; \omega_{s,1}] \cup [\omega_{s,2}; +\infty$, where:

$$\omega_{s,1-2} = \frac{1}{p} \sqrt{\frac{\omega_s}{2\omega_z} \left[\left(\frac{k_\infty}{m} - 2\omega_s\omega_z \right) \mp \sqrt{\frac{k_\infty}{m} \left(\frac{k_\infty}{m} - 8\omega_s\omega_z \right)} \right]} \quad (2.72)$$

4. The last stability condition on the spin speed is the following:

$$\omega > \omega_0 = \frac{\omega_s}{p} \sqrt{\frac{-k_D}{k_D + k_\infty}} \quad (2.73)$$

This condition tells us that, when a detent force is applied to the rotor, the EDTB is unstable at low spin speed due to this detent force. This conclusion has already been drawn hereabove, dealing with the quasi-static equilibrium. In particular, one should note that the frequency ω_0 introduced in (2.20) corresponds to the lower limit on the spin speed ensuring stability.

Let us now analyze the shape of the root locus in a general case, i.e. $C_z \neq 0$ and $k_D \neq 0$. The eigenvalues related to the electrical behavior of the system, shown in Fig. 2.21, always remain in the stable part of the complex plane. The case of the mechanical eigenvalues, depicted in Fig. 2.22, is more complex. If the spin speed is lower than ω_0 , one of the eigenvalues has a positive real part, leading to an unstable behavior of the EDTB. This corresponds to the case where the detent force stiffness k_D is larger in absolute value than the electrodynamic axial force stiffness $k(\omega)$ due to the EDTB. As the spin speed increases, those eigenvalues become complex conjugates but with a negative real part. Then, when the spin speed further increases and is comprised between $\omega_{s,1}$ and $\omega_{s,2}$, given by (2.72), the EDTB is mechanically unstable. It becomes stable again above $\omega_{s,2}$.

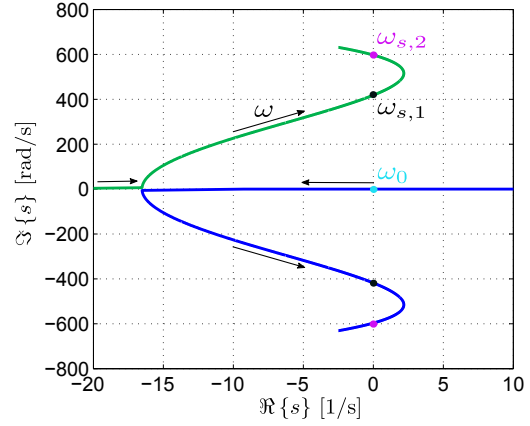
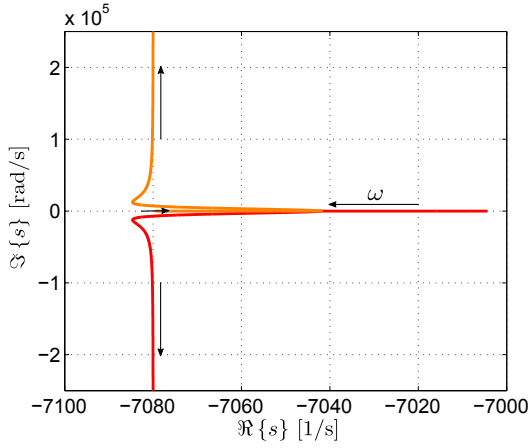


Fig. 2.21: Electrical eigenvalues for $C_z = 2.5$ [kg/s] and $|k_D| = \frac{k_\infty}{4}$. Fig. 2.22: Mechanical eigenvalues for $C_z = 2.5$ [kg/s] and $|k_D| = \frac{k_\infty}{4}$.

The changes in the root locus shape related to the detent force stiffness k_D and the axial damping C_z are now considered. In Fig. 2.23, the root locus is depicted for $|k_D|$ ranging from 0 to $\frac{3k_\infty}{4}$. The first observation that can be done is that, when $k_D = 0$, the EDTB is only unstable at high spin speed. However, when $k_D \neq 0$, the EDTB is also unstable when the spin speed is lower than ω_0 . It can also be seen that the location at which the mechanical eigenvalues are equal shifts progressively to the right of the complex plane, eventually crossing the imaginary axis when $|k_D| > \frac{k_\infty}{2}$. The EDTB is also unstable at any spin speed for these values of k_D . Moreover, the imaginary part of these eigenvalues when intersecting the imaginary axis decreases when $|k_D|$ increases. This comes from the fact that the total stiffness $k_{\text{tot}} = k(\omega) + k_D$ decreases, resulting in a reduction of the natural mechanical frequency $\sqrt{\frac{k_{\text{tot}}}{m}}$.

As far as the axial damping is concerned, when $C_z = 0$ and $k_D = 0$, the root locus only intersects the imaginary axis at the spin speed $\omega_{s,1}$ and becomes unstable above this speed. The effect of adding axial damping to the EDTB is to shift the root locus to the left of the complex plane, as can be seen in Fig. 2.24. For a given range of axial damping, the root locus intersects with the imaginary axis at two given spin speeds, namely $\omega_{s,1}$ and $\omega_{s,2}$, and the system is unstable between those two frequencies. This is for example the case for $C_z = 2.5$ [kg·m²/s²]. Nevertheless, if the axial damping becomes large enough, the EDTB becomes stable at any spin speed. Therefore, external axial damping should be added in order to ensure the stability of the EDTB.

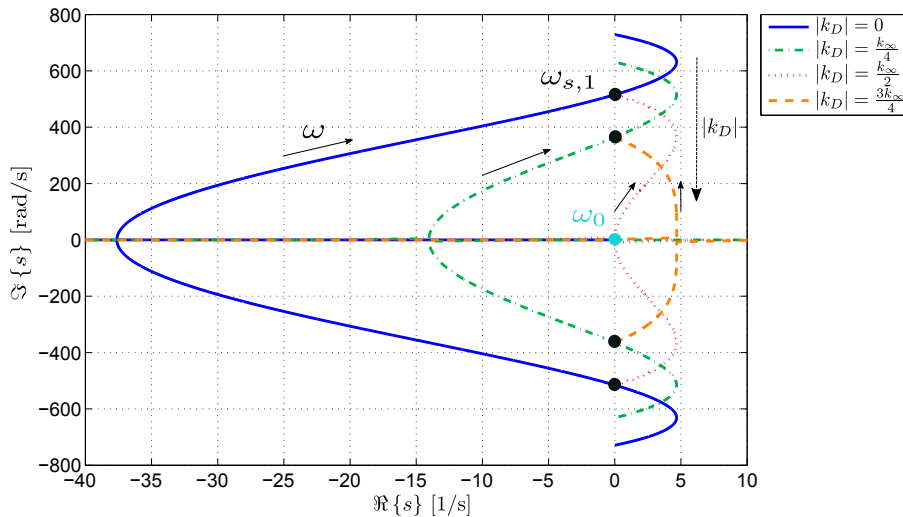


Fig. 2.23: Detail of the root locus for $C_z = 0$ and $|k_D| = 0$ to $\frac{3k_\infty}{4}$.

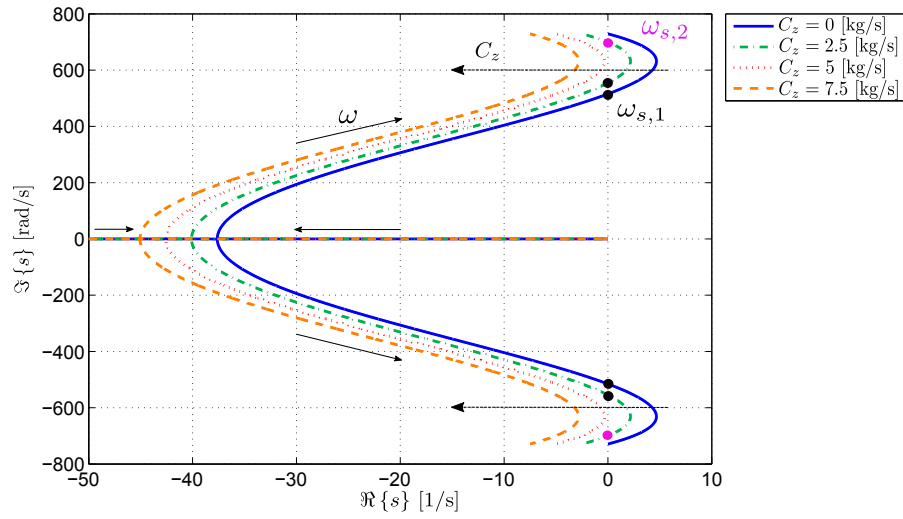


Fig. 2.24: Detail of the root locus for $k_D = 0$ and $C_z = 0$ to 7.5 [kg/s].

Self-bearing AFPM motor stability

The case of the self-bearing AFPM motor is now considered. Studying its stability requires to use the complete set of equations constituting the dynamic model. The matrix $\mathbf{A}$ defined by (2.3) is rewritten here using the notations defined when studying the stability of the motor and the bearing equations:

$$\mathbf{A} = \begin{pmatrix} -\omega_s & \omega & 0 & 0 & 0 & -k_\infty & \left(\frac{T_D}{z}\right)^* \\ -p^2\omega & -\omega_s & 0 & 0 & -p^2k_\infty\omega & 0 & -p^2(F_{ED}^* + k_\infty z^*) \\ 0 & 0 & -\omega_m & p\omega & 0 & 0 & pI_q^* \\ 0 & 0 & -p\omega & -\omega_m & 0 & 0 & -p\left(I_d^* + \frac{\Phi_0\sqrt{N}}{2L_{Mc}}\right) \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{m} & 0 & 0 & 0 & -\frac{k_D}{m} & -\omega_z & 0 \\ 0 & \frac{z^*}{J} & 0 & \frac{p\Phi_0\sqrt{N}}{J} & \frac{1}{J}\left(\frac{T_D}{z}\right)^* & 0 & -\omega_\alpha \end{pmatrix} \quad (2.74)$$

The calculation of the characteristic polynomial being extremely complex, only the independent term of the polynomial will be computed. This term will be useful to determine at which spin speed one of the eigenvalues is equal to zero. Let us reuse the assumption that the d-axis current of the motor is equal to zero at equilibrium ($I_d^* = 0$) and let us consider that the detent force stiffness and the rotational damping are equal to zero ($k_D = 0$ and $C_\alpha = 0$). This yields the following polynomial in ω for the independent term:

$$K_1\omega^3 + K_2\omega^2 + K_3\omega + K_4 \quad (2.75)$$

where

$$K_1 = p^4 \left[\frac{F_{ED}^* + 2k_\infty z^*}{m} \cdot \frac{1}{J} \left(\frac{T_D}{z}\right)^* + \frac{k_\infty}{m} \cdot \frac{p\Phi_0\sqrt{N}I_q^*}{J} \right] \quad (2.76)$$

$$K_2 = p^2 \left[\omega_m \cdot \frac{k_\infty}{m} \cdot \frac{(p\Phi_0)^2 N}{2JL_{Mc}} - \omega_s \cdot \frac{1}{Jm} \cdot \left(\frac{T_D}{z}\right)^{*2} \right] \quad (2.77)$$

$$K_3 = (p\omega_m)^2 \cdot \frac{F_{ED}^* + 2k_\infty z^*}{m} \cdot \frac{1}{J} \left(\frac{T_D}{z}\right)^* \quad (2.78)$$

$$K_4 = -\frac{\omega_m^2 \omega_s}{Jm} \cdot \left(\frac{T_D}{z}\right)^{*2} \quad (2.79)$$

Those coefficients do not depend on the axial damping C_z , but are related to the quasi-static equilibrium point and therefore, to the external force and torque applied to the rotor. Nevertheless, it remains tedious to determine the spin speed at which $s = 0$ and thus, to find the roots of this polynomial in terms of ω , as F_{ED}^* , z^* , $\left(\frac{T_D}{z}\right)^*$ and I_q^* are themselves functions of the spin speed.

Let us first examine the root locus depicted in Fig. 2.25 and corresponding to the case where $|k_D| = 0$, $C_z = 2.5$ [kg/s], $C_\alpha = 0$ and $F_{ext} = 1$ [N]. One should first notice that it is not obtained by simple superposition of the root locus for the AFPM motor and the EDTB. This comes from the fact that the drag torque introduces a coupling between the motor and the bearing function. Moreover, the matrix $\mathbf{A}$ clearly depends on the equilibrium point of the EDTB, which was not the case when only the bearing equations were considered. As far as the electrical eigenvalues are concerned, they all remain in the stable part of the complex plane at any spin speed. The two eigenvalues on the left (orange and red) are the electrical eigenvalues of the EDTB, while the ones in the center (light blue and light green) are the electrical eigenvalues of the AFPM motor. The locus of those electrical eigenvalues is quite complex, especially around $\Re\{s\} = -2000$, but the

important observation is that they always remain stable. The mechanical values of the self-bearing AFPM motor are depicted on the right side of Fig. 2.25. The purple one is the mechanical eigenvalue of the AFPM motor, and the two other ones (dark blue and dark green) are the mechanical eigenvalues of the EDTB. As far as the stability is concerned, those eigenvalues cross the imaginary axis and become unstable at high spin speed, but the dark blue one is also unstable at low spin speed. The rotational speed at which $s = 0$ is found by determining the roots of (2.75).

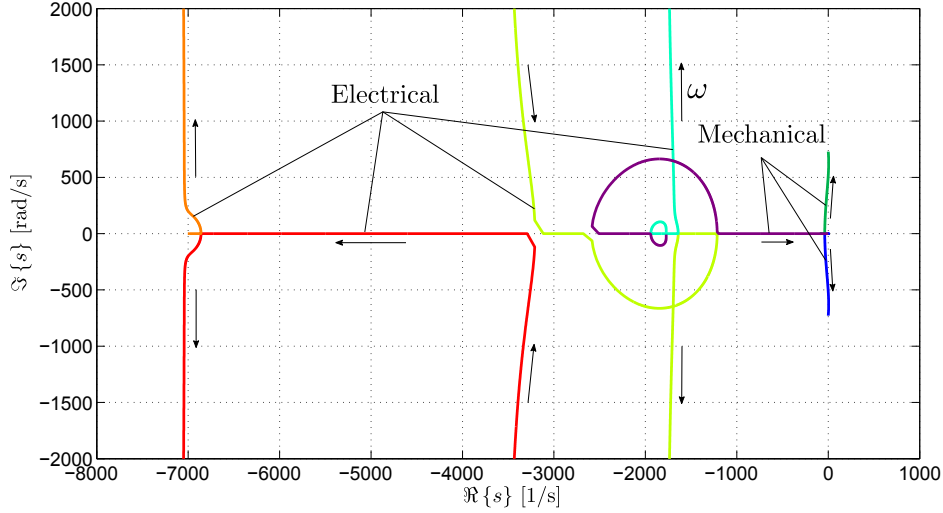


Fig. 2.25: Root locus for $|k_D| = 0$, $C_z = 2.5$ [kg/s], $C_\alpha = 0$ and $F_{ext} = 1$ [N].

Figures 2.26 to 2.28 show the root locus in the case where $|k_D| = \frac{k_\infty}{4}$, $C_z = 2.5$ [kg/s], $C_\alpha = 0$ and $F_{ext} = 1$ [N]. Obvious differences are visible compared to the case where $k_D = 0$. In Fig. 2.26, we observe that the mechanical eigenvalue of the AFPM motor (purple) is initially unstable. As the spin speed increases, it moves closer to the imaginary axis, eventually becoming stable. At very high spin speed, this eigenvalue remains stable and tends to $s = 0$. As far as the mechanical eigenvalues of the EDTB are concerned, they initially are both stable. They become equal at a spin speed ω_1 , then move to the left of the complex plane in a curved pattern. They move back towards the imaginary axis at higher spin speed, eventually crossing it and becoming unstable. The electrical eigenvalues are shown in Fig. 2.27, for the ones of the EDTB and in Fig. 2.28, for the ones of the AFPM motor. Again, the pattern is relatively complex, but they both remain stable at any spin speed. They are shown here as an information.

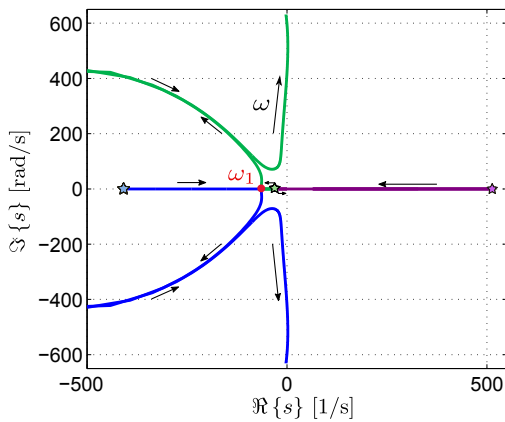


Fig. 2.26: Mechanical eigenvalues of the EDTB (blue and green) and of the motor (purple).

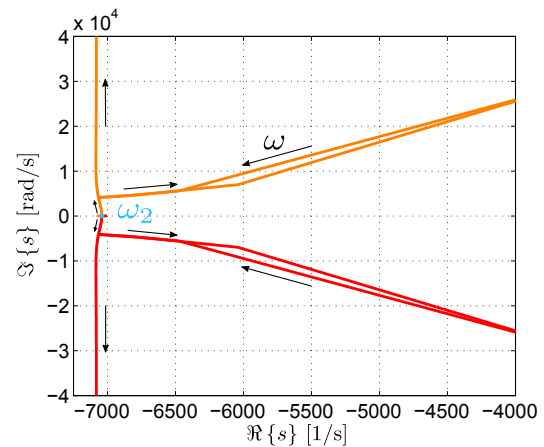


Fig. 2.27: Electrical eigenvalues of the EDTB.

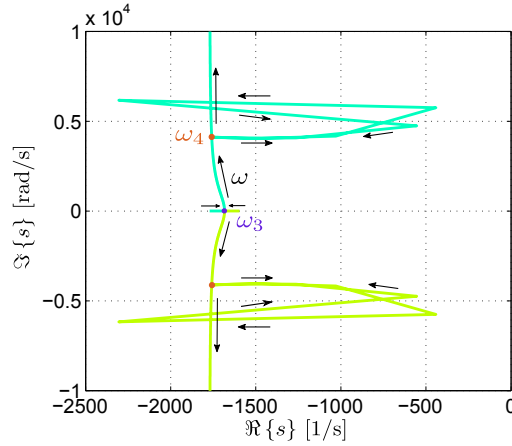
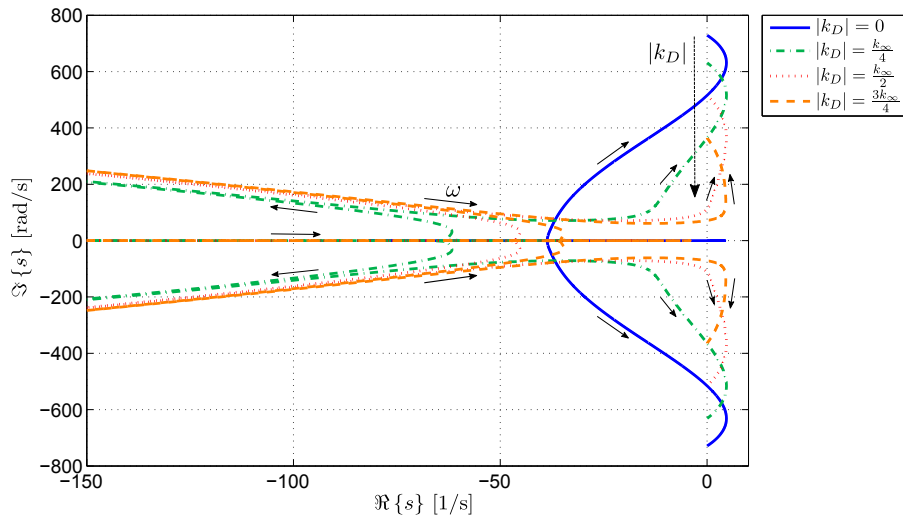


Fig. 2.28: Electrical eigenvalues of the AFPM motor.

Then, the influence of the different parameters on the root locus shape is discussed, starting with the detent force stiffness k_D . To do so, all the other parameters are kept equal to the ones used in Fig. 2.25. The effect of k_D on the mechanical eigenvalues of the system is depicted in Fig. 2.29. When $k_D = 0$, the usual shape of the locus for the EDTB mechanical eigenvalues can be observed. However, when $k_D \neq 0$, the behavior already observed in Fig. 2.26 is repeated. As $|k_D|$ increases, the imaginary part of the mechanical eigenvalues is reduced, due to the fact that the total stiffness and thus, the natural mechanical frequency of the EDTB is reduced. This behavior is similar to the one observed when discussing the stability of the EDTB equations.

Fig. 2.29: Detail of the root locus for $|k_D| = 0$ to $\frac{3k_\infty}{4}$.

A variation of the axial damping coefficient C_z gives a figure which looks exactly the same as Fig. 2.24. The effect of the axial damping is thus to shift the mechanical eigenvalues of the EDTB to the left of the complex plane. However, it does not influence the mechanical eigenvalue of the AFPM motor. A variation of the rotational damping coefficient C_α , on the contrary, has an important effect on the shape of the root locus, particularly on the electrical eigenvalues of the AFPM motor, as shown in Fig. 2.30. The peaks on the left, corresponding to the eigenvalues at low spin speed, are shifted to the left, and the peaks on the right, corresponding to the eigenvalues at high spin speed, are shifted to the right towards the imaginary axis, eventually crossing it if the value of C_α is large enough. This makes the AFPM motor unstable. This effect is similar to the one observed in the case of the AFPM motor equations, except that it also has more complex effects on the root locus shape, as the one depicted in Fig. 2.31.

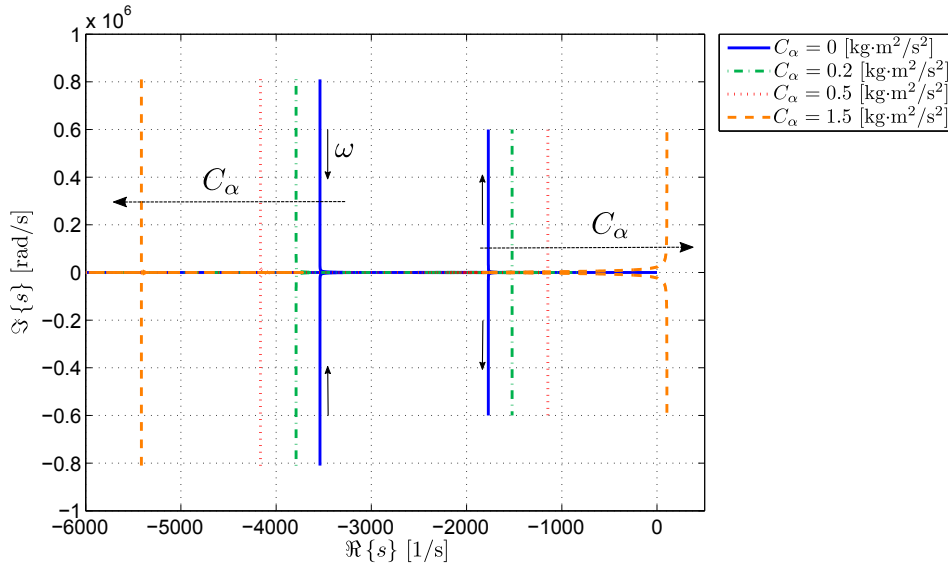


Fig. 2.30: Electrical eigenvalues of the AFPM motor for $C_\alpha = 0$ to $1.5 \text{ [kg}\cdot\text{m}^2/\text{s}^2]$.

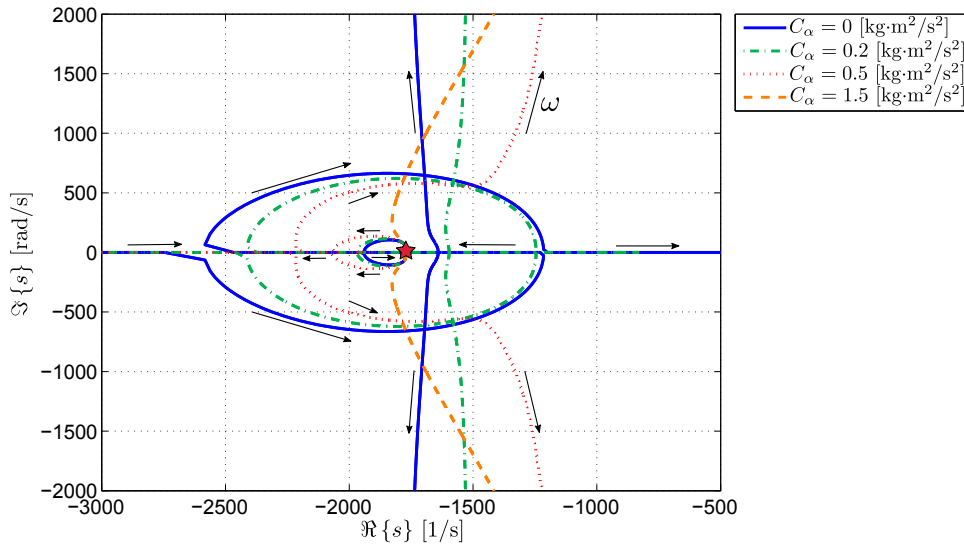


Fig. 2.31: Detail of Fig. 2.30.

Finally, let us analyze the effect of the external force perturbation on the root locus shape. Figure 2.32 presents the mechanical eigenvalues of the EDTB for different values of external forces, ranging from 0 to 25 [N]. The change of the root locus shape is quite similar to the one obtained by increasing the detent force stiffness. However, at very high spin speed and in the cases where the external force is equal to 10 and 25 [N] respectively, the mechanical eigenvalues cross the imaginary axis a second time and become stable when the spin speed goes to infinity. As far as the electrical eigenvalues are concerned, the effect on the ones related to the EDTB is not really significant, but the effect on the ones related to the AFPM motor is depicted in Fig. 2.33. Again, the shape is quite complex, but the important conclusion is that the eigenvalues remain in stable part of the complex plane.

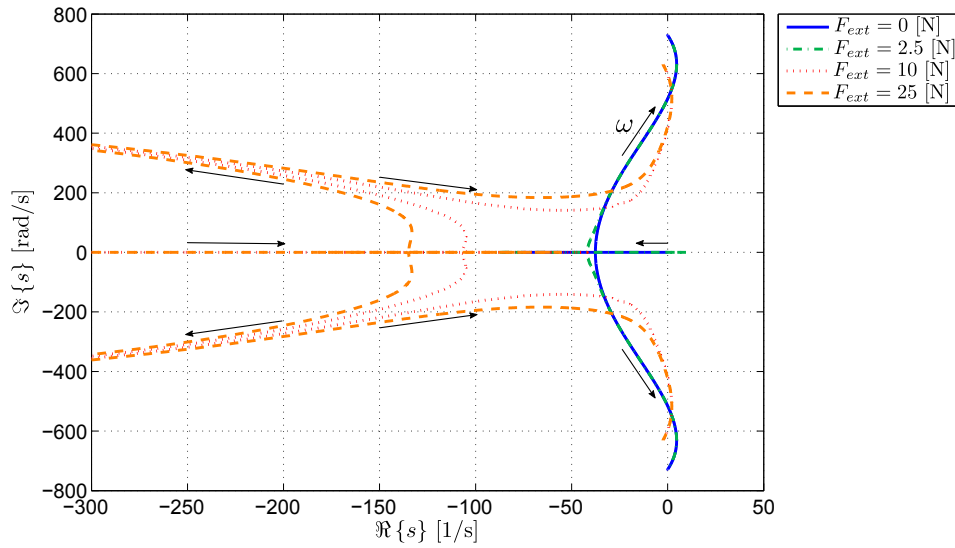


Fig. 2.32: Mechanical eigenvalues for $F_{ext} = 0$ to 25 [N].

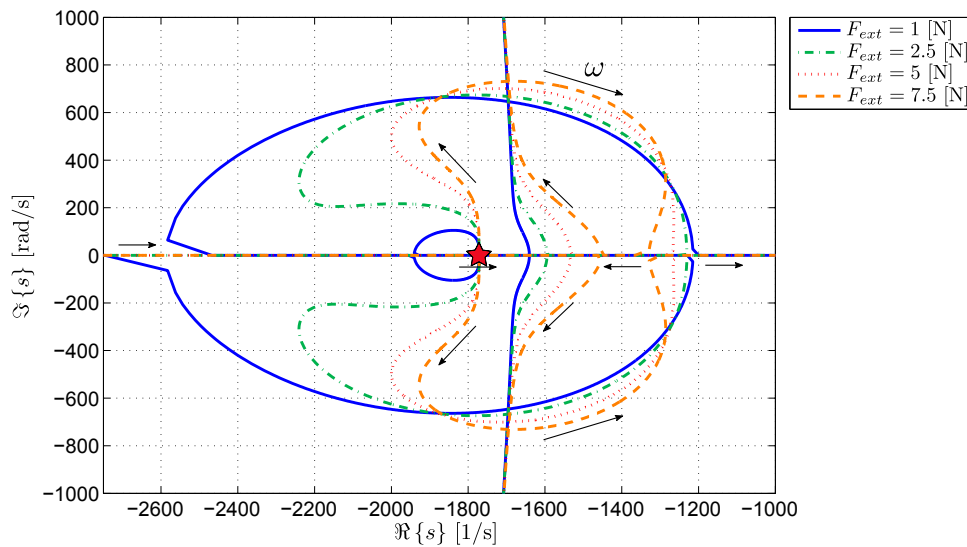


Fig. 2.33: Electrical eigenvalues for $F_{ext} = 0$ to 25 [N].

2.3 Dynamic behavior

In this section, the dynamic behavior of the self-bearing AFPM motor is discussed. First, the control of the AFPM motor is presented. Then, the dynamic behavior of the system is studied, during the start-up of the motor, when subjected to external force and torque perturbations and finally, when subjected to variations of the reference speed.

2.3.1 Motor control

As far as the motor control is concerned, it is performed in terms of u_d and u_q voltages. Two levels of controllers are used. On the low level, two internal PI controllers with a time response of 0.75 [s] are used to control the d- and q-axis currents I_d and I_q . On the top level, the motor is controlled in speed, also by a PI controller, but with a response time of 10 [s]. This response time is at least one order of magnitude larger than the one of the current controllers, thus ensuring that the low level controllers stabilize faster than the high level one. The control scheme is presented in Fig. 2.34. It is the classical controller for a synchronous machine in the d-q domain, as presented in [35]. The dimensioning of those controllers can be found in Appendix C. As stated earlier, the axial dynamics of the rotor is not controlled.

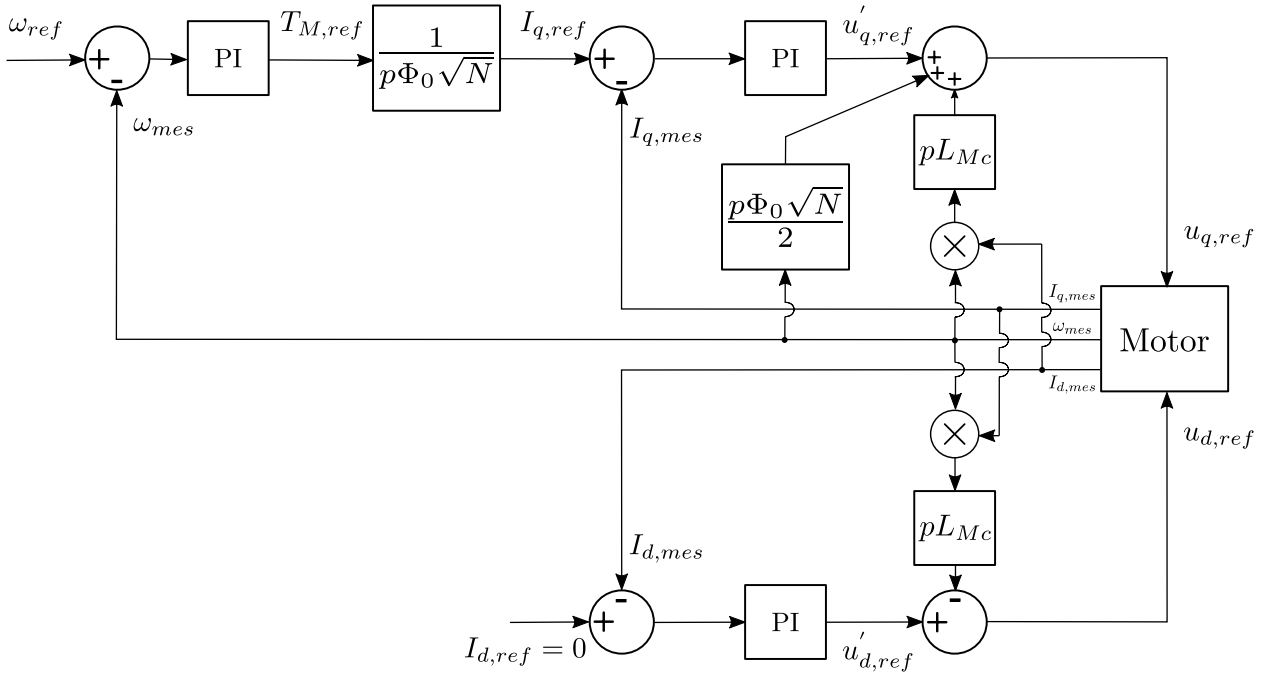


Fig. 2.34: Control scheme of the AFPM motor with the dynamic compensation terms on $u_{d,ref}$ and $u_{q,ref}$.

The model used to simulate the EDTB and motor function is the non-linear one presented in the previous chapter, based on (1.39) to (1.44). The simulations are performed using the SIMULINK tool in MATLAB.

2.3.2 Start-up

The start-up of the machine is shown in Fig. 2.35, with three different setpoints for the spin speed at 250, 500 and 750 [rad/s]. The response time of the speed controller, chosen to be 10 [s], is respected. Nevertheless, this proportional-integral controller may be a bit violent, as it might require too much torque to accelerate the self-bearing AFPM motor, as the torque peak reaches up to 200 [mNm] in the case of a setpoint at 750 [rad/s].

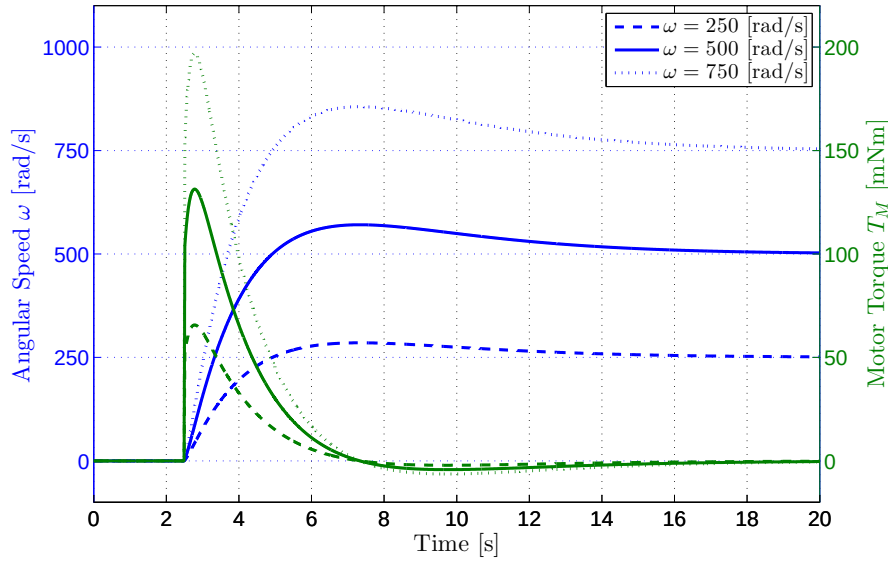


Fig. 2.35: Start-up of the AFPM motor for different speed setpoints, namely 250, 500 and 750 [rad/s].

2.3.3 Behavior under an external force perturbation

To assess the coupling between the motor and bearing behavior, an external force perturbation, i.e. a constant value of 5 [N], which could correspond to the gravity, is applied to the rotor for a duration of 25 [s]. This force perturbation is applied when the rotor is rotating at a constant spin speed. Three spin speeds are considered here, 500, 750 and 1000 [rad/s] respectively. In Fig. 2.36, the axial displacement stabilizes around a non-zero value when the rotor is subject to an external force. This axial excursion decreases as the spin speed increases. Indeed, the axial displacement stabilizes around 0.12 [mm] at 500 [rad/s] but around 0.045 [mm] at 1000 [rad/s]. This behavior is mathematically confirmed by (2.10). It is also interesting to note that an overshoot is present in the case of a spin speed of 500 [rad/s]. Moreover, it can be observed in Fig. 2.37 that the axial displacement presents oscillations just after the external force perturbation. The oscillation frequency is related to the natural mechanical frequency $\sqrt{\frac{k_{tot}}{m}}$ and therefore increases with the spin speed, as the axial stiffness developed by the EDTB increases. One should also note that those oscillations last longer in the 1000 [rad/s] case.

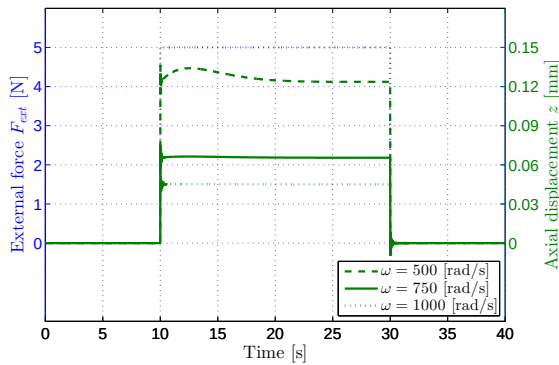


Fig. 2.36: Axial displacement of the rotor under an external force perturbation, for spin speeds of 250, 500 and 750 [rad/s], respectively in dashed, solid and dotted.

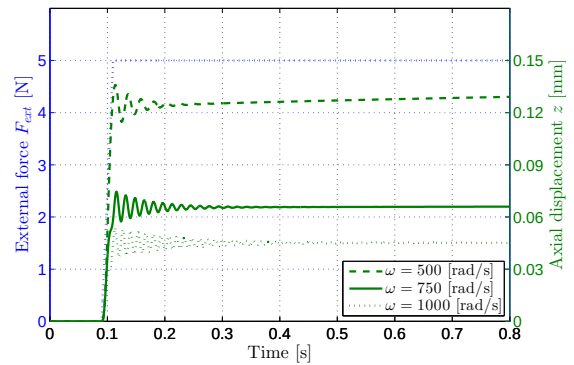


Fig. 2.37: Detail of Fig. 2.36 around the external force perturbation.

In Fig. 2.38, the drag torque immediately increases when the external force perturbation is applied to the system. This comes from the fact that it is proportional to the square of the axial displacement. In Fig. 2.39, depicting the difference between the current spin speed and the reference spin speed, it can be seen that the presence of this drag torque decreases the angular velocity of the rotor. The speed controller thus increases the reference motor torque to compensate for this drag torque. The torque overshoot in Fig. 2.38 comes from the fact that the rotor must be accelerated to come back to its initial spin speed. After that, the motor torque stabilizes at a value which is exactly the opposite of the drag torque and compensates for it. The axial force developed by the EDTB levels off around the external force. When the perturbation is removed from the rotor, the axial displacement and thus the drag torque go to zero. However, as the motor torque cannot change instantaneously, a positive speed overshoot appears. The speed controller then forces the motor torque to take a negative value to slow down the rotor. After a few seconds, the force, torques and axial displacement return to zero.

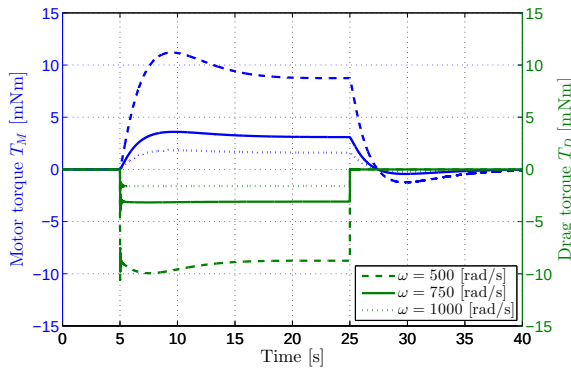


Fig. 2.38: Drag and motor torque developed by the self-bearing AFPM machine under an external force perturbation, for spin speeds ranging from 250 to 750 [rad/s].

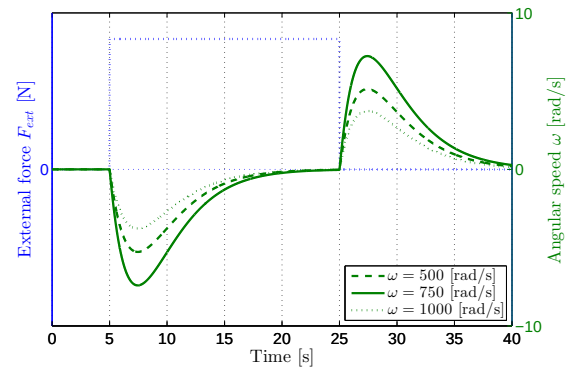


Fig. 2.39: Difference between measured and reference spin speed under an external force perturbation, force perturbation, for spin speeds ranging from 250 to 750 [rad/s].

2.3.4 Behavior under an external force perturbation with a detent force

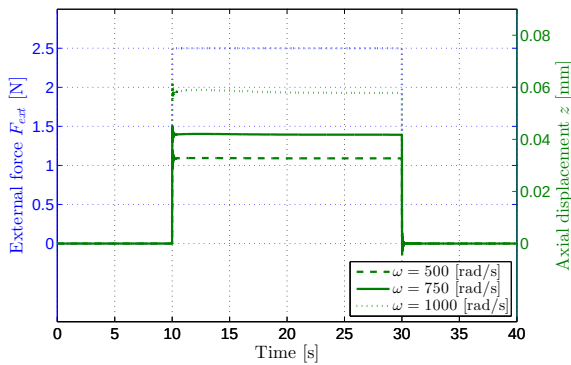


Fig. 2.40: Variation of the axial displacement under an external force perturbation, for detent force stiffnesses of 0, $\frac{k_\infty}{16}$ and $\frac{k_\infty}{8}$, respectively in dashed, solid and dotted.

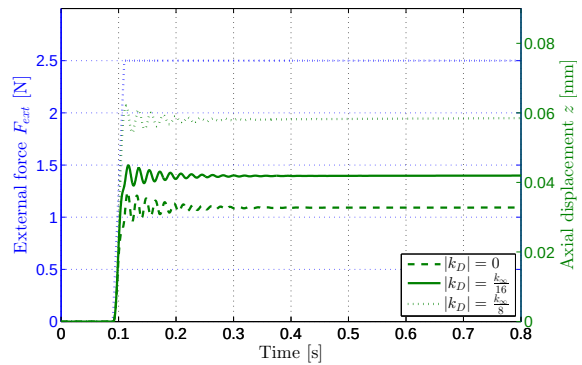


Fig. 2.41: Detail of Fig. 2.40 around the external force perturbation.

Another case that can be simulated using the model developed in the previous chapter is the response to an external force perturbation, but with the addition of a detent force regarding the axial dynamics of the rotor. In Fig. 2.40, an external force of 2.5 [N] is applied to the system, resulting in an axial displacement of roughly 0.03 [mm] when the detent force stiffness $k_D = 0$.

When a small detent force is added, e.g. $|k_D| = \frac{k_\infty}{16}$, the axial excursion increases to 0.04 [mm]. If it is further increased, a displacement of 0.06 [mm] is reached, i.e. twice the initial axial excursion. The value of k_D can be increased up to the point where the stiffness developed by the EDTB is equal in absolute value to that of the detent force. The detail provided in Fig. 2.41 shows that the frequency of oscillation of the axial displacement decreases when k_D increases. This comes from the fact that the total stiffness $k_{\text{tot}} = k(\omega) + k_D$ decreases as $k_D < 0$.

Figures 2.42 and 2.43 present the axial force, and the motor and drag torque developed by the self-bearing motor. As far as the force is concerned, it stabilizes around a value which is no longer the opposite of the external force when $k_D \neq 0$, but around the sum of the external and detent force. In other words, the axial force compensates for the detent force besides the external force. In Fig. 2.43, it can be seen that the drag torque and consequently the motor torque increase due to the increment in the axial displacement, but the behavior is otherwise similar to the one observed previously.

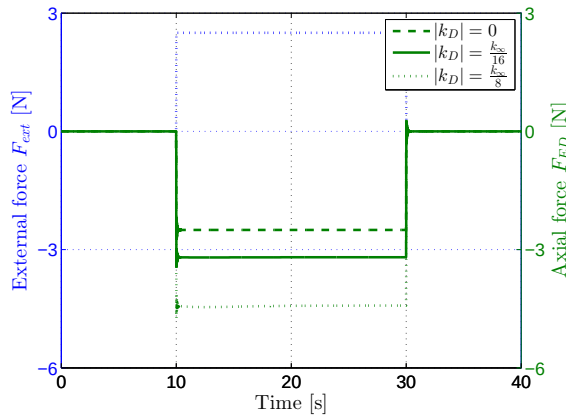


Fig. 2.42: Variation of the electrodynamic axial force under an external force perturbation, for a detent force ranging from 0 to $\frac{k_\infty}{8}$.

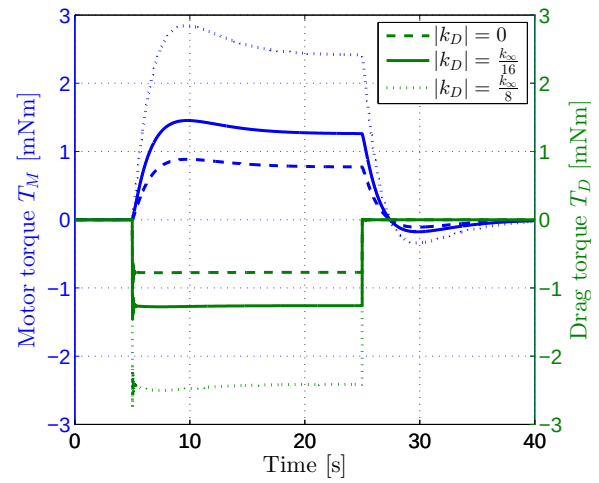


Fig. 2.43: Variation of the drag and motor torque under an external force perturbation, for a detent force ranging from 0 to $\frac{k_\infty}{8}$.

2.3.5 Behavior under a variation of the reference spin speed

In Fig. 2.44 to 2.46, the angular speed changes linearly from 750 [rad/s] to a new setpoint at 900 [rad/s], under different external forces applied to the system. Fig. 2.44 depicts the variation of the axial displacement under those conditions. Based on this graph, we conclude that the higher the angular speed, the higher the axial stiffness, as the axial displacement reduces with an increment of the spin speed. It can also be noticed that increasing the external force applied to the system results in a larger axial displacement. This increase is linear with the external force. As far as the drag torque due to the EDTB is concerned, a quadratic dependence to the axial displacement is clearly visible in Fig. 2.45, thus corresponding to what was stated earlier in (2.27). In Fig. 2.46, the motor torque is initially stable around a value which compensates for the drag torque. Then, as an increase in spin speed is required, the motor torque grows to accelerate the rotor. When the required spin speed is reached, the motor torque reduces and levels off around the new value of the drag torque, which in this case is lower than the previous one. Note that this is not always the case, as the drag torque first increases at low spin speed and decreases at high spin speed, as discussed previously.

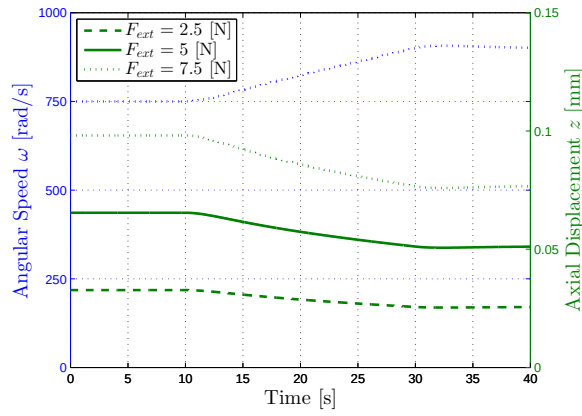


Fig. 2.44: Variation of the axial displacement for a variation of the spin speed, under external forces ranging from 2.5 to 7.5 [N].

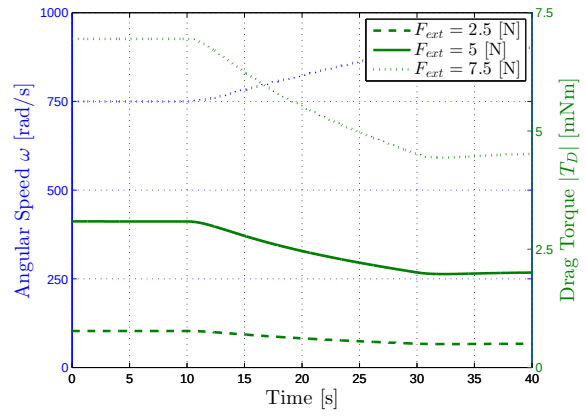


Fig. 2.45: Variation of the drag torque for a variation of the spin speed, under external forces ranging from 2.5 to 7.5 [N].

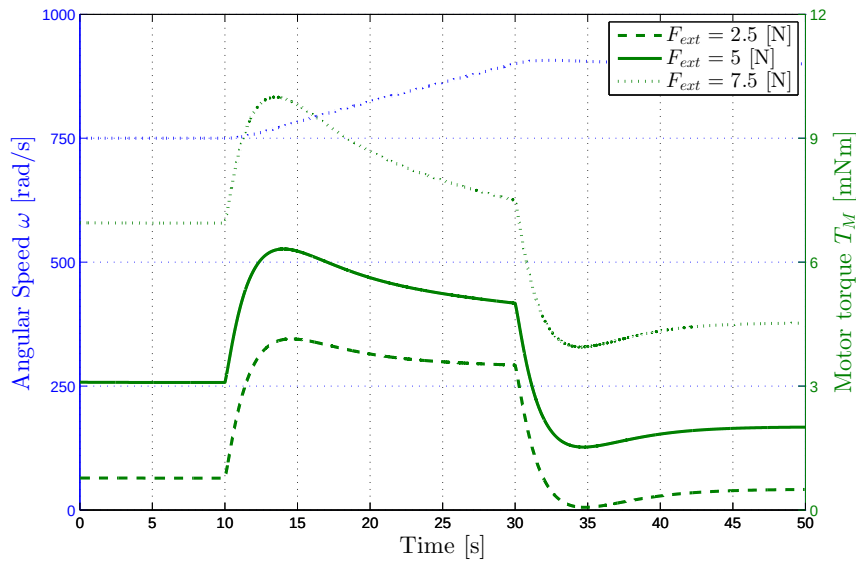


Fig. 2.46: Variation of the motor torque for a variation of the spin speed, under external forces ranging from 2.5 to 7.5 [N].

2.4 Parametric analysis

This section is dedicated to the parametric analysis of two geometrical parameters of the self-bearing AFPM motor, namely the width and the thickness of the coils constituting the windings, denoted w_{coil} and t_{coil} respectively, and of the number p of pole pairs. First, the performance criteria of the EDTB and the AFPM motor are presented. Then, the results obtained through static FEM simulations for various self-bearing AFPM motors are discussed.

2.4.1 Performance criteria of the self-bearing AFPM motor

Four criteria are used to characterize the performance of a self-bearing motor. This first one characterizes the motor performance, while the other ones, taken from [34], are related to the EDTB characteristics:

1. The first criterion is related to the motor performance and is given by the ratio between the torque T_M developed by the motor and the square root of the Joule losses. This ratio must be maximized.

$$k_{\text{motor}} = \frac{T_M}{\sqrt{P_{\text{Joule}}}} = \frac{p\Phi_0\sqrt{N}I_q}{\sqrt{\frac{R}{2}I_q^2}} = p\Phi_0\sqrt{\frac{2N}{R}} \quad (2.80)$$

It is also possible to find the Joule losses related to the q-axis current using the following relationship:

$$P_{\text{Joule}} = \frac{T_M^2}{k_{\text{motor}}^2} \quad (2.81)$$

2. The second criterion is related to the bearing performance and is given by the ratio between the electrodynamic axial force F_{ED} developed by the EDTB and the power dissipated by the drag torque, denoted $P_{\text{drag}} = T_D\omega$, divided by the axial displacement z . This division makes the criterion independent of z . This ratio defining the bearing performance must be maximized.

$$k_{\text{bearing}} = \frac{F_{ED}}{\frac{T_D}{z}\omega} = \frac{L_{Sc}}{2R} \quad (2.82)$$

As in the case of the motor, the dissipated power can be retrieved as:

$$P_{\text{drag}} = \frac{F_{ED} \cdot z}{k_{\text{bearing}}} \quad (2.83)$$

3. The third criterion corresponds to the axial damping that must be introduced in the system to ensure that the EDTB remains stable at any spin speed. It corresponds to the value of C_z which causes (2.72) to give only one solution. This value must be minimized.

$$C_{\text{max}} = \frac{K_{\Phi}^2 N}{8R} \quad (2.84)$$

4. The fourth criterion is the total axial force stiffness developed by the EDTB at infinite velocity. It is given by (2.36):

$$k_{\text{tot},\infty} = \frac{2K_{\Phi}^2 N}{L_{Sc}} + k_D \quad (2.85)$$

2.4.2 Simulation results

To evaluate the variation of the self-bearing AFPM motor performance with respect to the geometrical dimensions of the coils and the number of pole pairs, and to determine which values optimize this performance, the model parameters of several machines are evaluated by performing static FEM simulations in COMSOL Multiphysics. As far as the geometrical parameters are concerned, the coil thickness is comprised between 1 and 20 [mm], with steps of 1 [mm], while the coil width is comprised between 0.5 and 3 [mm], with steps of 0.5 [mm]. Furthermore, the number of pole pairs is comprised between 2 and 6. One should also note that two cases are considered, with or without iron stator yoke. The results are obtained for an airgap of 1 [mm]. For the electrodynamic axial force and the power consumption, the values are found for an axial excursion of $z = 0.5$ [mm], corresponding to half the airgap, and a spin speed $\omega = 1$ [krad/s].

The effect of the coil width is always to increase each criterion, except for the total stiffness, where the effect is to decrease it monotonically. Therefore, the evolution of the four criteria with this geometrical dimension are not depicted in this work. In the simulation results that follow, only the minimum and maximum values obtained for each number of pole pairs are presented. The minimum value corresponds to $w_{\text{coil}} = 0.5$ [mm] and the maximum value, to $w_{\text{coil}} = 3$ [mm], and the other way round for the total stiffness.

AFPM motor performance

Let us first consider the motor performance. By comparison of Figs. 2.47 and 2.48, the AFPM motor with an iron stator yoke is more effective, with a performance increase of roughly 50% compared to the case without iron yoke. Increasing the number of pole pairs also has a positive effect on the motor performance, improving the criterion by 50% when going from $p = 2$ to $p = 6$ with an iron yoke and by 60% without iron yoke. However, as the number of pole pairs increases, the performance enhancement becomes smaller. The performance criterion is even expected to decrease for larger values of p . This comes from the fact that, even though k_{motor} depends linearly on p , the magnetic flux Φ_0 is reduced for large p values due the edge effects. The resistance also increases as the length of a winding increases. Another important element is that, in both cases, a maximum is reached for a given value of the coil thickness. The optimal thickness value decreases with p , and is larger without iron yoke than with one.

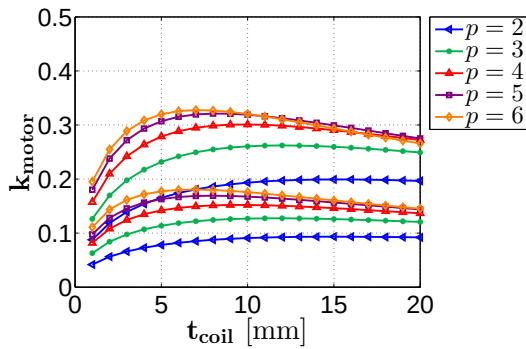


Fig. 2.47: Motor performance criterion as a function of the coil thickness, without iron stator yoke.

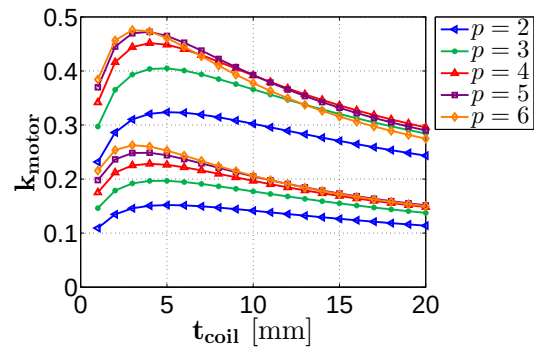


Fig. 2.48: Motor performance criterion as a function of the coil thickness, with an iron stator yoke.

As far as the power dissipated by the motor is concerned, the case without iron yoke consumes less, especially for small coil thicknesses between 1 and 5 [mm], according to Figs. 2.49 and 2.50. The power increase with an iron yoke is of 40 to 50%. This result is obtained by considering that the only torque that the motor needs to produce is used to counteract the drag torque. Our motor

thus operates as a flywheel. Nevertheless, Figs. 2.51 and 2.52 show that for the same axial displacement and spin speed, the drag torque, and thus the motor torque, is more important in the case with an iron yoke, thus explaining the difference in terms of power consumption.

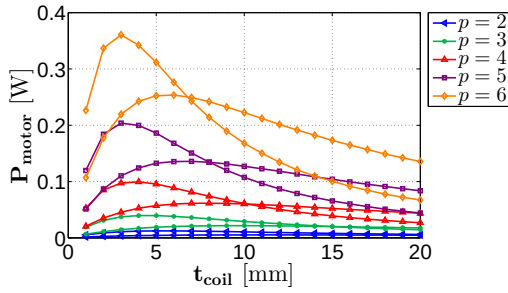


Fig. 2.49: Power dissipated by the motor as a function of the coil thickness, without iron stator yoke.

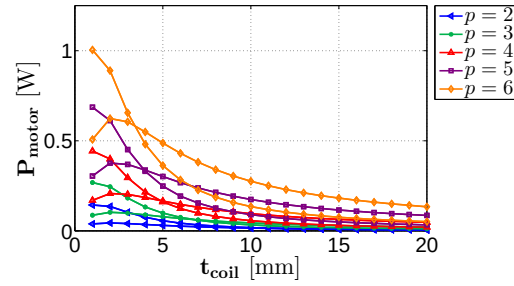


Fig. 2.50: Power dissipated by the motor as a function of the coil thickness, with an iron stator yoke.

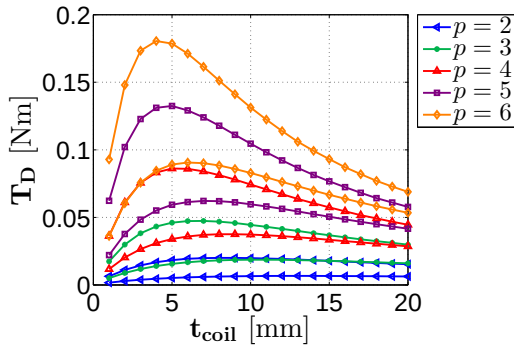


Fig. 2.51: Drag torque as a function of the coil thickness, without iron stator yoke.

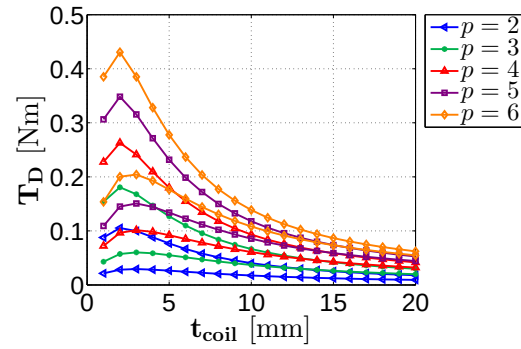


Fig. 2.52: Drag torque as a function of the coil thickness, with an iron stator yoke.

EDTB performance

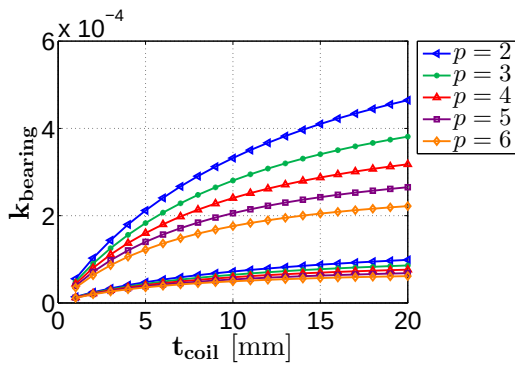


Fig. 2.53: Bearing performance criterion as a function of the coil thickness, without iron stator yoke.

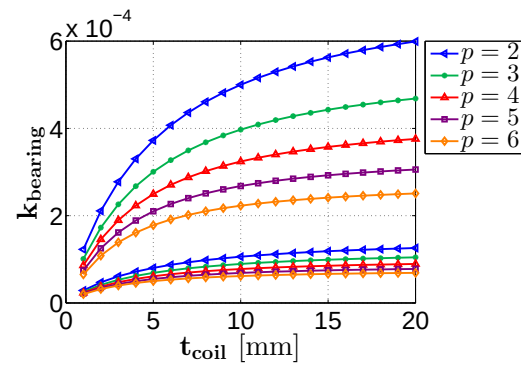


Fig. 2.54: Bearing performance criterion as a function of the coil thickness, with an iron stator yoke.

The performance of the electrodynamic thrust bearing is now discussed, starting with the criterion comparing the axial force generated by the EDTB, multiplied by the axial excursion, with the power dissipated by the drag torque. The case with an iron yoke performs 20 to 30% better than without iron yoke. This means that, for the same dissipated power and axial displacement, a machine with an iron stator yoke can produce a larger restoring force. Moreover, contrary to the

case of the motor, increasing the number of pole pairs decreases the bearing performance by 50 to 60% when going from $p = 2$ to $p = 6$.

Figures 2.55 and 2.56 depict the power dissipated by the bearing, and Figs. 2.57 and 2.58 represent the restoring force generated by the EDTB. An important observation is that the maximum power consumption resulting from the drag torque nearly corresponds to the maximum restoring force, explaining why the performance criterion does not present a maximum. Moreover, the power consumption of a machine with an iron yoke is slightly less than two times more important than without iron yoke, but the axial restoring force, however, is more than two times larger. This explains that an EDTB with an iron yoke is more efficient regarding this criterion.

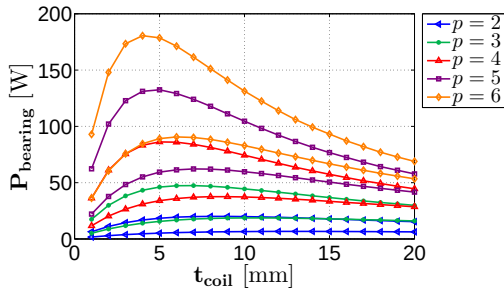


Fig. 2.55: Power dissipated by the bearing as a function of the coil thickness, without iron stator yoke.

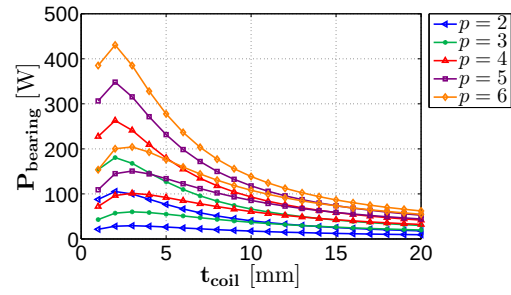


Fig. 2.56: Power dissipated by the bearing as a function of the coil thickness, with an iron stator yoke.

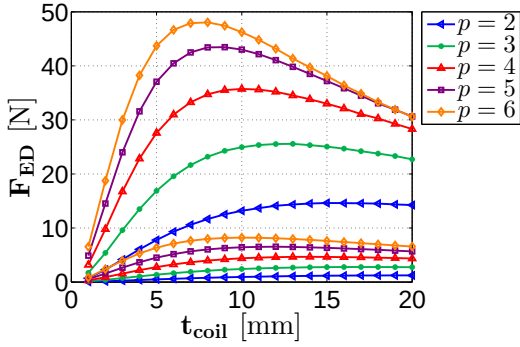


Fig. 2.57: Electrodynamic axial force developed by the bearing as a function of the coil thickness, without iron stator yoke.

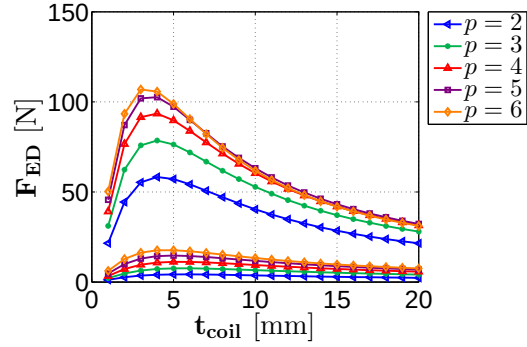


Fig. 2.58: Electrodynamic axial force developed by the bearing as a function of the coil thickness, with an iron stator yoke.

Let us now consider the amount of axial damping that must be introduced in the system to ensure that it is stable at any spin speed. It is represented in Figs. 2.59 and 2.60. For a machine without iron yoke, this quantity remains below 5 [kg/s], which is a reasonable value. It can be observed that it first increases with the number of pole pairs, reaches a maximum for $p = 4$ and then reduces as p becomes larger. A maximum is reached for each curve, at $t_{\text{coil}} = 14$ [mm] for $p = 2$ and $t_{\text{coil}} = 7$ [mm] for $p = 6$. The thickness at which the criterion is maximum decreases with the number of pole pairs. Significant differences are observed in the case with an iron yoke. First, the axial damping is three to four times the value of the case without iron yoke. Then, it only decreases with the number of pole pairs. Finally, it presents a maximum for $t_{\text{coil}} = 3$ [mm], which is not influenced by the number of pole pairs. According to this criterion, a machine without stator yoke should be preferred as it requires a lower axial damping to make it stable.

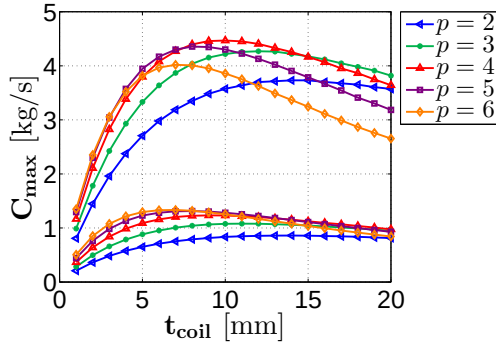


Fig. 2.59: Required axial damping as a function of the coil thickness, without iron stator yoke.

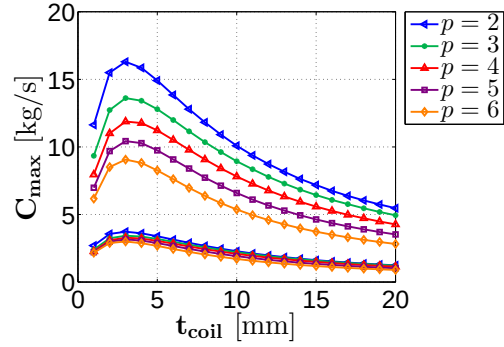


Fig. 2.60: Required axial damping as a function of the coil thickness, with an iron stator yoke.

The last criterion to discuss is the axial force stiffness at infinite spin speed, corresponding to the maximum stiffness that the EDTB can generate. When no iron stator yoke is used, the total stiffness is maximum for the lowest possible coil thickness. It then decreases monotonically with the thickness. As far as the number of pole pairs is concerned, increasing p enhances the stiffness, as this increases K_{Φ} and the inductance L_{Sc} . Nevertheless, the increase of K_{Φ} has more impact as the stiffness is proportional to its square. However, when an iron stator yoke is used, a detent force exists between the PM rotor and the yokes, resulting in a negative detent stiffness k_D which degrades the total stiffness. For small coil thicknesses, below 3 [mm], the total stiffness is strongly negative. As the coil thickness increases, the stiffness becomes positive and reaches a maximum before monotonically decreasing. The coil thickness giving the maximum stiffness decreases with p and goes from $t_{\text{coil}} = 13$ [mm] for $p = 2$ to $t_{\text{coil}} = 7$ [mm] for $p = 6$. The maximum stiffness obtained without iron yoke goes from 125 [N/mm] for $p = 2$ to 360 [N/mm] for $p = 6$, which is more than two times larger than the 60 [N/mm] for $p = 2$ and 195 [N/mm] for $p = 6$ obtained with an iron yoke. Also, the presence of a detent force makes the EDTB unstable at low frequency. This problem cannot be fixed by the addition of external axial damping.

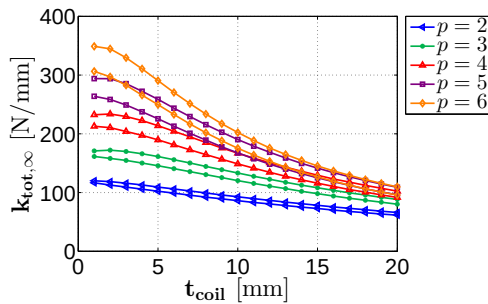


Fig. 2.61: Total stiffness as a function of the coil thickness, without iron stator yoke.

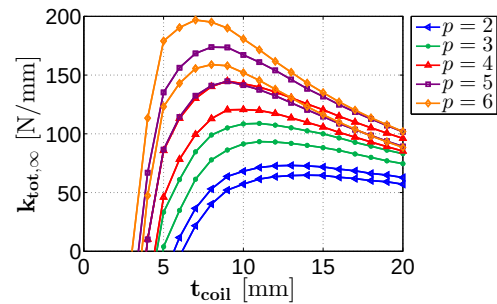


Fig. 2.62: Total stiffness as a function of the coil thickness, with an iron stator yoke.

Figure 2.63 depicts the maximum value for the total stiffness, obtained for a coil width $w_{\text{coil}} = 0.5$ [mm], in the cases with and without iron stator yoke. It is interesting to observe that the total stiffness with an iron yoke is always lower than the one without iron yoke, but tends to the total stiffness without iron yoke when the coil thickness is large enough, i.e. when the back-iron is far for the PM rotor and the detent force stiffness becomes negligible.

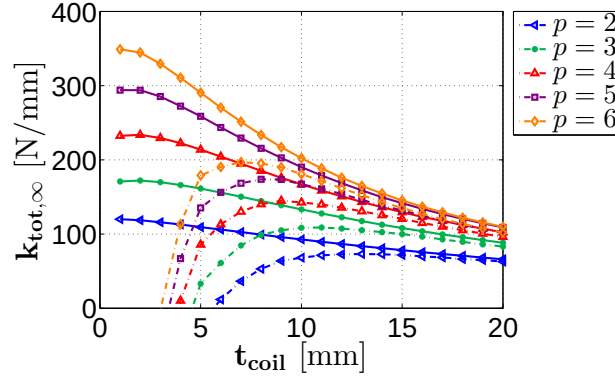


Fig. 2.63: Comparison between the maximum total stiffness without iron stator yoke (solid line) and with iron stator yoke (dashed-dotted line).

2.4.3 Assumption on the bearing electrical and mechanical frequencies

In section 2.2, while considering the root locus of the EDTB, an assumption (2.52) was made regarding the natural mechanical and electrical frequencies of the bearing. The parametric analysis can be used to confirm that this assumption holds for our machine topology. Figure 2.64 represents the ratio between the mechanical and electrical frequency of the bearing in percent. This ratio always remains below 3%. Even better, with 6 pole pairs, the maximum value is roughly 1%. In the case with an iron yoke, shown in Fig. 2.65, this percentage remains below 4%. The assumption that was made earlier is thus respected in our case.

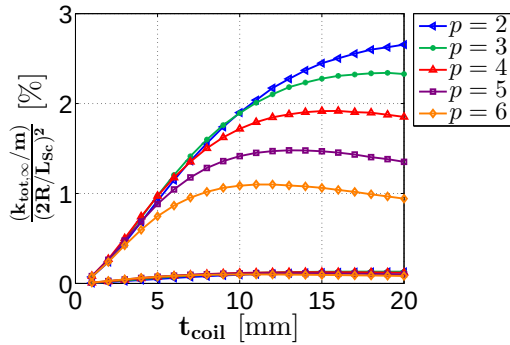


Fig. 2.64: Ratio of the mechanical and electrical bearing frequencies as a function of the coil thickness, without iron stator yoke.

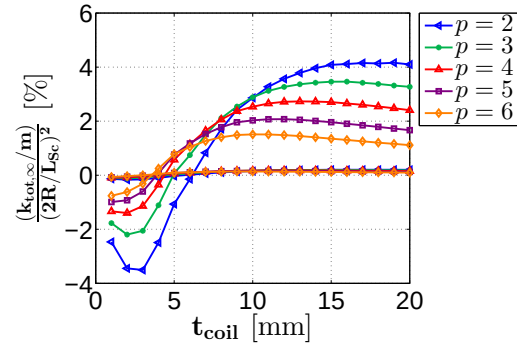


Fig. 2.65: Ratio of the mechanical and electrical bearing frequencies as a function of the coil thickness, with an iron stator yoke.

2.4.4 Conclusion

Those simulation results highlight the fact that a self-bearing motor with an iron stator yoke is more efficient in terms of motor torque and restoring force generation, compared to the power required to produce them. However, the external damping required to stabilize the system at any spin speed is lower in the case of a machine without iron stator yoke, and the total stiffness at infinite velocity is also larger in this case. Moreover, the fact that the EDTB is stable a low frequency without iron yoke is an incentive for using this topology instead of the other one. Therefore, the topology without iron stator yoke is preferred, even though the motor and bearing performance are lower. Indeed, stability issues are more critical for the functioning of the system than performance.

Experimental Validation

This chapter intends to present the prototype and the testbench developed in the framework of this Master's thesis, starting from the work done in [20]. Another objective is to measure the performance of the self-bearing AFPM motor to compare it with the theoretical results obtained from the model developed in Chapter 1. Section 3.1 presents the global topology of the self-bearing AFPM motor, and details the rotor and the stator structure. The two types of phase windings manufactured for the armature are also detailed. Section 3.2 is dedicated to the presentation of the testbench for the EDTB and the self-bearing AFPM motor and to the explanation of its different possible configurations. In section 3.3, the 12 parameters required for using the model are determined or evaluated based on measurements performed on the prototype. Those parameters are injected in the model in section 3.4 to determine the expected behavior of the self-bearing AFPM motor. Finally, section 3.5 discusses the experimental results, namely the emf measurements on the open circuit windings and phase windings and the characterization of the EDTB and self-bearing AFPM motor performance.

3.1 Prototype

This section comes back on the general topology chosen for our prototype, and describes into more details the rotor and stator structure.

3.1.1 Topology

As already explained in the introduction, the topology chosen for our prototype is a double-sided AFPM motor with an internal rotor. In this case, the number of pole pairs p is six and the number of phases N is three, just as for the case study presented in Chapter 2. As far as the rotor is concerned, it is constituted of two PMs arrangements placed on an iron disk. These arrangements are positioned with an angular shift $\Delta\theta_{PM} = \frac{180^\circ}{p} = 30^\circ$ between them, meaning that the upper and lower PMs attract each other, thus reinforcing the magnetic field. This rotor is placed between the two stator windings, with an air gap of 1 [mm] on each side when the rotor is perfectly centered. The maximum axial excursion is thus limited by the value of the air gap, even though the self-bearing AFPM motor is theoretically designed to generate a restoring force able to maintain the axial displacement well below this value, assuming that the spin speed is large enough.

As far as the stator is concerned, the windings are connected in series with an angular shift between them equal to zero. The armature windings are fixed to two stator yokes made of non-magnetic material. Indeed, the parametric analysis conducted in Chapter 2 has highlighted the fact that the use of iron stator yokes has a negative impact on the total stiffness generated by the EDTB and causes stability issues at low spin speed, due to the detent force between the PMs rotor and the yokes. Two different types of stator windings are developed in this work. On the one

hand, the first type relies on 18 coils per stator, six per phase, made of solid copper cut in a spiral shape with four turns. On the other hand, the second winding type is based on three concentrated windings per stator. Those are made out of Litz wire. The particularities of this type of wire and the reason why it was used are discussed later in this section.

3.1.2 Rotor

Structure

For this prototype, three configurations are possible for the rotor. Let us first discuss the common elements shared by those different configurations. They are all composed of a ferromagnetic yoke (4), on which the PMs arrangements facing the upper windings (3a) and the lower windings (3b) are fixed. The PMs are attached to this yoke by their attraction force with the ferromagnetic material but, for safety reasons, an epoxy glue has been used to permanently fix the magnets to the yoke. Thus, the possibility of rearranging them to make, for example, a two or three pole pairs rotor is lost. One should also note that a rim exceeding half of the magnets height blocks their radial motion. The shaft is composed of two parts screwed to each other through the ferromagnetic yoke. Part (5) always remains the same and its flat surface on the left of Figs. 3.1 to 3.3 is used to measure the axial displacement of the shaft with a laser position sensor. Parts (2) and (7), however, are interchangeable and depend on the chosen configuration for the prototype. The different possibilities are the following:

1. Coupling with a DC motor: In this configuration, depicted in Fig. 3.1, a pin (1) is locked into the shaft (2) end. A compliant mechanical coupling can then be fixed to this pin and coupled to a DC motor shaft. This configuration is used to characterize the EDTB performance, as the drive function is ensured by the DC motor. Moreover, if the coupling is stiff enough, measurements at a given axial displacement can be performed.

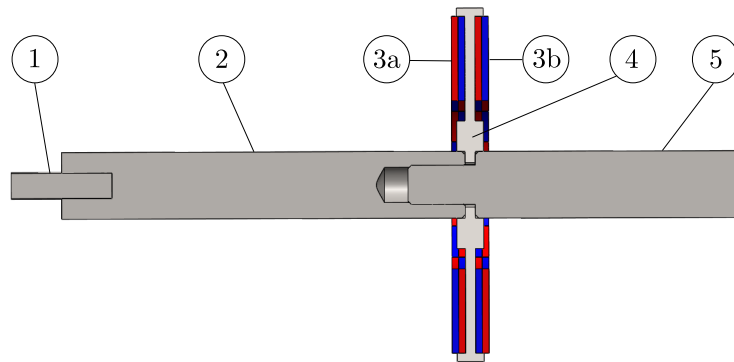


Fig. 3.1: Cross-sectional view of the rotor configuration for a coupling with a DC motor.

2. Compressed air turbine: In Fig. 3.2, the shaft (7) is used. A turbine (6) can be placed at its end to drive the motor using compressed air. The turbine is maintained axially by the bolt screwed to the threaded shaft end. Even though this configuration has not been used in our experiments, it is still a possibility to drive the EDTB without using the motor function of the self-bearing AFPM motor.
3. Self-bearing AFPM motor: This configuration, shown in Fig. 3.3, is the one used when the prototype is operated as a self-bearing AFPM rotor. A small permanent magnet (8) is fixed to the threaded end of shaft (7) by screwing a 3D printed element (9). This PM is used by a AMS Hall sensor to determine the angular position of the motor. As a reminder, the angular position of a synchronous motor must be known to be able to control it properly. In this case, a MAXON EPOS2 24/5 electronic controller is used to drive the motor.

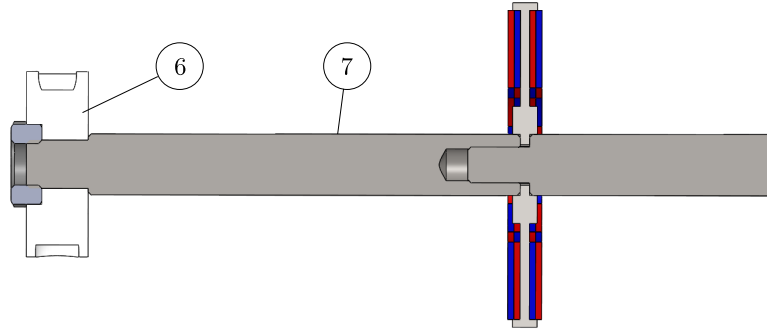


Fig. 3.2: Cross-sectional view of the rotor configuration with a turbine.

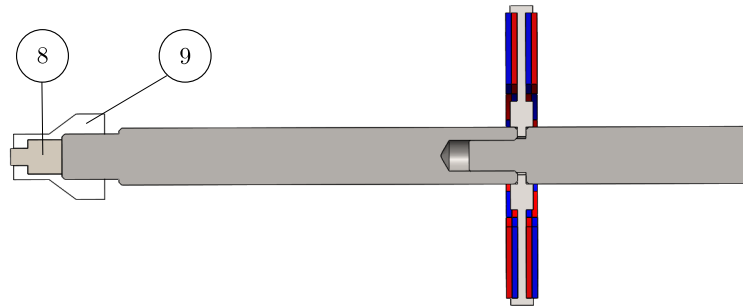


Fig. 3.3: Cross-sectional view of the rotor configuration for the self-bearing AFPM motor operation.

PMs arrangements

Each permanent magnets arrangements consists of 12 axially polarized segment magnets with an angular aperture of 30° , as depicted in Fig. 3.4. The exact same arrangement is placed on both sides of the ferromagnetic yoke, with an angular shift $\Delta\theta_{PM} = 30^\circ$ between them, meaning that the upper and lower PMs arrangements attract each other. This reinforces the magnetic field produced by the rotor, as their magnetic fields are in the same direction. Moreover, the PMs are surface-mounted and glued to the ferromagnetic yoke with an epoxy glue to avoid any motion during the machine operation. Furthermore, they are arranged with alternating polarities. However, a better result could be obtained using an Halbach array configuration, producing a magnetic flux density closer to a pure sine, thus limiting the relative amplitude of the higher order harmonics. The reason why this configuration has not been chosen is essentially technical, as it is difficult and expensive to find azimuthally polarized segment magnets on the market. The assembly of periodic Halbach arrays for an axial-flux six pole pairs rotor also constitutes a technical challenge. The characteristics of the PMs are found in Table 3.1.

Characteristics	Value
Inner radius R_{in}	25 [mm]
Outer radius R_{out}	50 [mm]
Thickness h_{PM}	4 [mm]
Angular aperture α_{PM}	30°
Remanent magnetization B_r	1.38 [T] (N48SH)
Maximum temperature	150°C
Material	NdFeB
Mass	14.65 [g]

Table 3.1: Characteristics of the chosen permanent magnets.



Fig. 3.4: SOLIDWORKS view of the rotor with the PMs arrangements.



Fig. 3.5: Picture of the rotor with the PMs arrangements.

3.1.3 Stator

The upper and lower stators are composed of a non-magnetic yoke, on which armature windings are fixed. The two different types of windings tested in this work are presented into more details.

Solid copper windings

The first type of windings consists in a coil with four turns, cut directly into a solid copper plate, using a wire electro-erosion cutting technique. The desired coil shape is shown in Fig. 3.6 and the achieved result is depicted in Fig. 3.7, with a one euro coin to give an idea of the size. Two housings are cut into the coils to allow soldering copper rods for the electrical connections. The objective behind this particular coil type is to use the geometrical dimensions found to give a satisfying trade-off between the AFPM motor and EDTB performance, based on the parametric analysis conducted in section 2.4. The resulting coil dimensions are $t_{\text{coil}} = 4$ [mm] and $w_{\text{coil}} = 4$ [mm]. Moreover, several constraints have motivated the use of this particular coil type. Relying on the experience acquired during the construction of the prototype presented in [20], eddy currents due to the axial component of the PMs magnetic induction field are known to be critical. For this reason, the surface of the coils normal to the the axial magnetic induction field component is reduced to 0.75 [mm]. Also, to be able to control the motor within a reasonable range of voltages, the number of turns has to be limited, as the magnetic flux linked by the windings is proportional to the square of the number of turns. Obtaining the required coil thickness, width and radius of curvature with classical wire is not possible without reaching a significant number of turns, which is the reason why a solid copper coil is preferred.

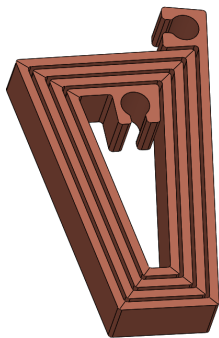


Fig. 3.6: SOLIDWORKS view of a solid copper coil.

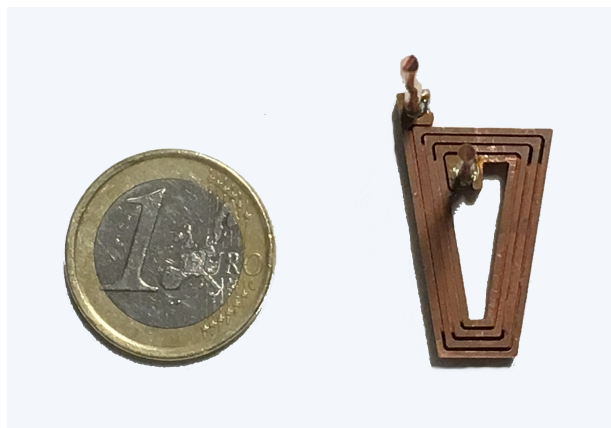


Fig. 3.7: Picture of a solid copper coil, with a one euro coin for the scale.

Figure 3.8 depicts the expected result after placing the 18 coils constituting a stator on the non-magnetic yoke, made out of bakelite. The motivations for using this particular type of material are that it is non-magnetic, non-conducting, it has a satisfying mechanical strength and it can resist to temperatures over 150 [°C]. This last characteristic is required to be able to solder the coils on the yoke using stain, without any risk of damaging the yoke itself. As far as the topology of the windings is concerned, each winding is constituted of 6 coils connected in series. The coils are placed on the stator by alternating the three phases, meaning that the first coil belongs to phase A, the second to phase B, the third to phase C, and so on. The connections are realized by connecting the rods soldered into the coil housings using shaped pieces of a 2 [mm] copper sheet, as depicted in Fig. 3.10. The stators obtained using this topology are shown in Figs. 3.9 and 3.11.

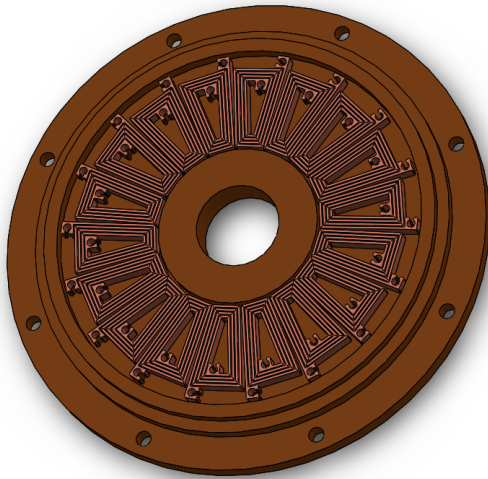


Fig. 3.8: SOLIDWORKS view of the 18 coils constituting the stator windings.



Fig. 3.9: Picture of the 18 coils constituting the stator windings.

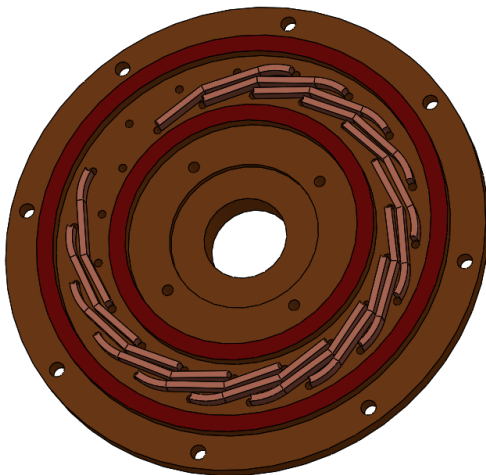


Fig. 3.10: SOLIDWORKS view of the coils connections.

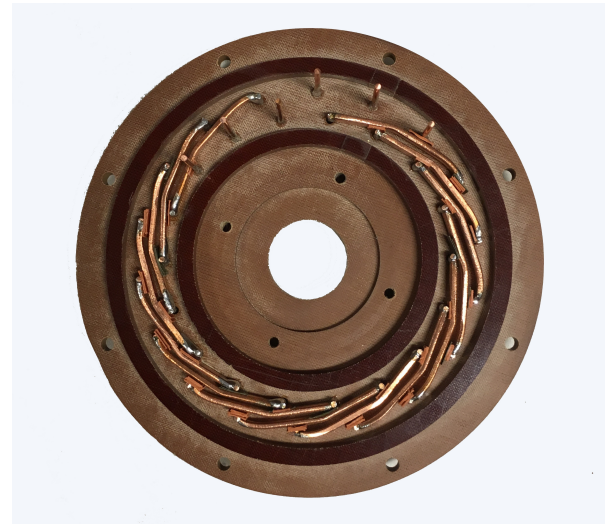


Fig. 3.11: Picture of the coils connections.

The tests performed with those stators have shown poor results as they develop a braking torque when the rotor is put into motion. This braking torque is due to the fact that the azimuthal component of the magnetic induction field, normal to the lateral section of the coils, induces strong eddy currents into the coils, given that the coil thickness is 4 [mm]. This explanation is supported by a COMSOL Multiphysics simulation performed by Dr. Virginie Kluyskens. This non-desired effect keeps us from using those stators for the self-bearing AFPM motor.

Litz wire concentrated windings

The second type of windings developed in this work is based on concentrated windings made out of Litz wire. The shape of a winding is depicted in Fig. 3.12 and realized in Fig. 3.13. One should note that two turns are present in the final version. The objective of this winding type is to deal with the problem encountered in the design of the first type of windings, namely the induced eddy currents. To do so, a particular type of electrical wire, called "Litz" wire, is used. It is composed of several thin copper wires, electrically insulated from each other and connected in parallel. The use of thin wires completely solves the problem of the eddy currents, as the surface normal to the magnetic induction field is negligible in all directions.

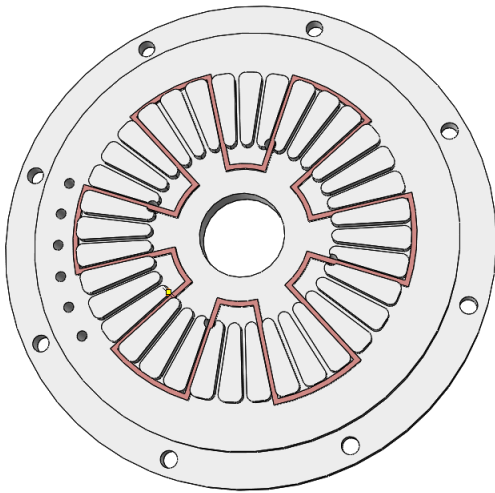


Fig. 3.12: SOLIDWORKS view of one stator with one of the three windings.

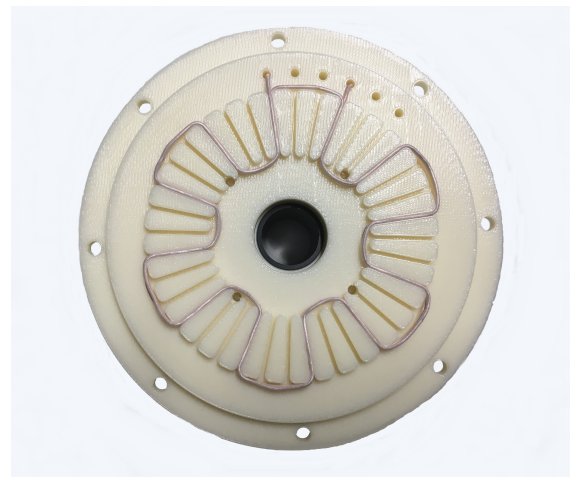


Fig. 3.13: Picture of one stator with one of the three windings.

Three windings as the one depicted in Fig. 3.12 are used, with a mechanical angular shift of 10° between them. This corresponds to 60° from an electrical point of view, which means that the second phase must be connected in the opposite direction with respect to the first and third phase to obtain a three-phased system. The three windings are represented in Fig. 3.14 and realized in Fig. 3.15. One should note that the stator yoke is manufactured by 3D printing and is also composed of a non-magnetic, non-conducting material.

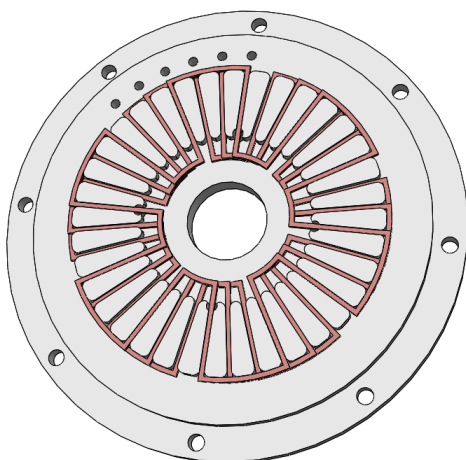


Fig. 3.14: SOLIDWORKS view of one stator with the three windings.

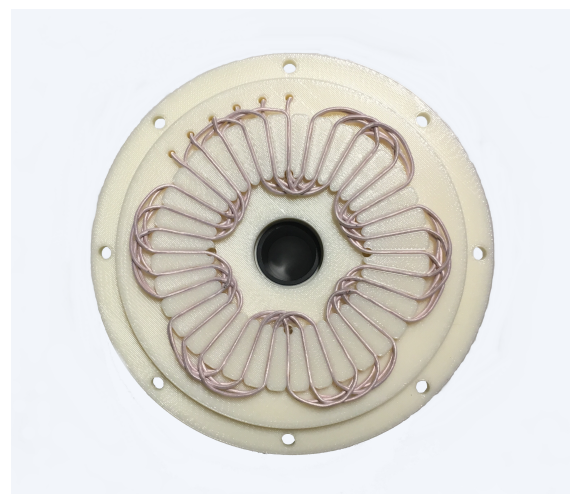


Fig. 3.15: Picture of one stator with the three windings.

3.2 Testbench

This section is dedicated to the description of the testbench implemented to validate the EDTB and AFPM motor behavior. The objective of this testbench is both to ensure the radial guiding of the rotor and to measure its axial displacement. In the case of the self-bearing AFPM motor, an angular position measurement is also required to be able to control the motor.

3.2.1 Configurations

Three configurations are possible for the testbench. In all three cases, a laser position sensor (3) is fixed to the right stator. Then, it is possible to switch from one configuration to the other by changing the rotor structure and by choosing the suitable 3D printed part to be placed on the left stator. The two first testbench configurations are used to characterize the EDTB behavior. The most simple one, which is not used in this work but is still left as a possibility, uses a turbine powered by compressed air to drive the rotor. It is depicted in Fig. 3.16. A second possibility is to drive the rotor using a DC motor, as explained earlier. In this case, the 3D printed part is used to maintain the DC motor attached to the testbench.

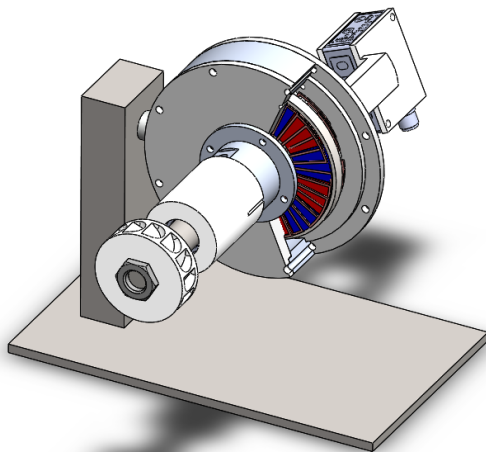


Fig. 3.16: Configuration of the testbench with a turbine.

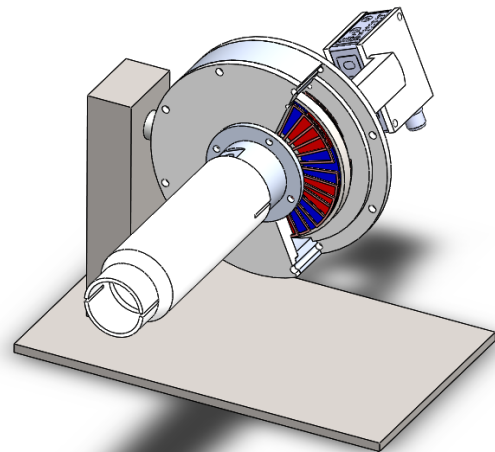


Fig. 3.17: Configuration of the testbench with a DC motor.

A third configuration can be used to drive the self-bearing motor using a MAXON EPOS2 24/5 electronic controller. This configuration is depicted in Fig. 3.18. The EDTB in itself, including the armature windings and the PMs rotor disk (1) is located between the two stator yokes (5). Those two yokes are fixed to a protective shield (2) also used, as its name indicates, as a protection against machine parts projection in case of a complete crash of the rotor. Moreover, a threaded hole in this protective shield is used to connect the testbench to the support (4). The sleeves of the radial air bearings (6) are fixed to the stator yokes to ensure the radial guiding of the rotor. Finally, a 3D printed part (7) is used both to support the axial damper, whose goal is to introduce the external axial damping required to make the EDTB stable at any spin speed, and the Hall sensor placed in (8), using the small permanent magnet attached to the rotor shaft to determine the angular position.

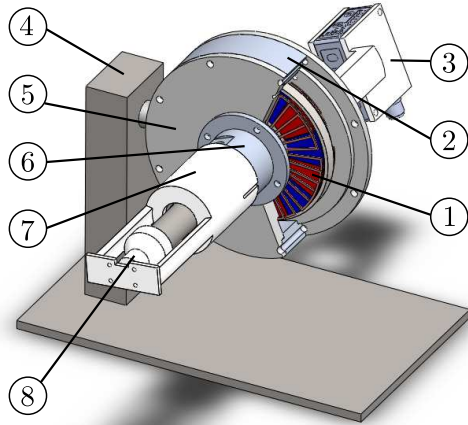


Fig. 3.18: Configuration of the testbench for the self-bearing AFPM motor operation.

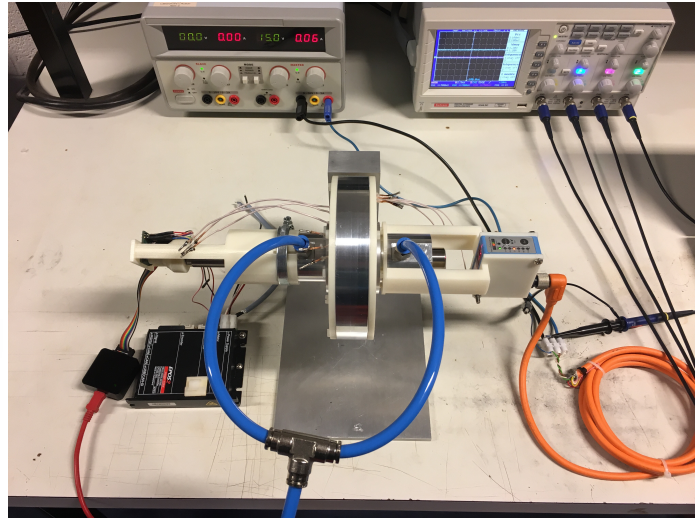


Fig. 3.19: Picture of the testbench for the self-bearing AFPM motor operation.

3.2.2 Radial air bearings

As explained hereabove, the radial guiding of the rotor is ensured by two air bearings fixed to the stator yokes, and supporting the radial load due to the rotor weight. Those bearings are composed of two elements: the air bushings and the sleeves. The air bushings are inserted in the sleeves and compressed air is injected into the bearing through the hole on the top of the sleeve. The advantage of using air bearings is that they are not subject to mechanical friction as a thin film of air is present between the inner surface of the air bushing and the rotor surface. Further works could focus on the integration of the self-bearing AFPM motor into a fully passive suspension, thus replacing those air bearings by passive magnetic bearings.

3.2.3 Axial damper

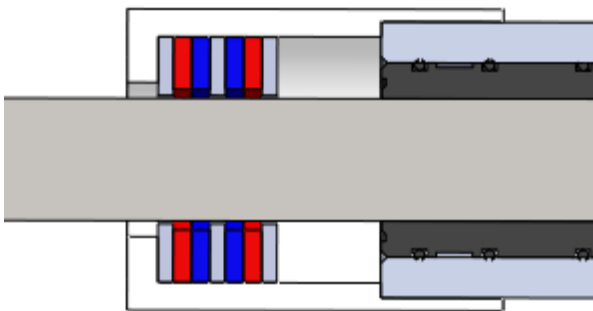


Fig. 3.20: Cross-sectional view of the magnetic damper used to introduce external axial damping in the system.

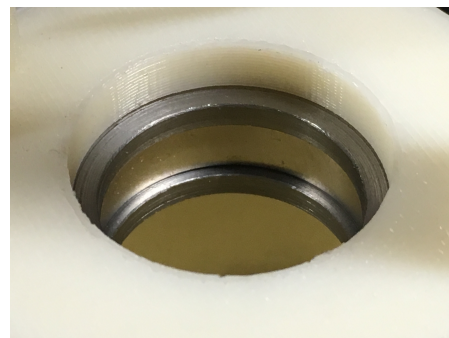


Fig. 3.21: Picture of the magnetic damper.

The purpose of the damper is to introduce an external axial damping in the system in order to make the EDTB stable at any spin speed, based on the results obtained in section 2.2. The damper is composed of permanent magnets with opposite polarity, placed between several iron rings, as

can be seen in Figs. 3.20 and 3.21. The characteristics of the PMs are shown in Table 3.2. When the rotor moves with respect to those dampers, eddy currents are induced in the iron rotor. Those currents then interact with the PMs magnetic field and generate a force which opposes the motion of the rotor and is proportional to its axial velocity. This principle has been validated with COM-SOL Multiphysics. However, due to the compliance of the 3D printed part, the iron rings drags along the rotor shaft, hindering the good functioning of the prototype. Therefore, the damper has not been used during the tests performed on the prototype.

Characteristics	Value
Inner diameter D_{in}	23 [mm]
Outer diameter D_{out}	40 [mm]
Thickness	6 [mm]
Remanent magnetization B_r	1.25 [T] (N38)
Maximum temperature	120°C
Material	NdFeB
Mass	37.66 [g]

Table 3.2: Characteristics of the damper permanent magnets.

3.2.4 Sensors

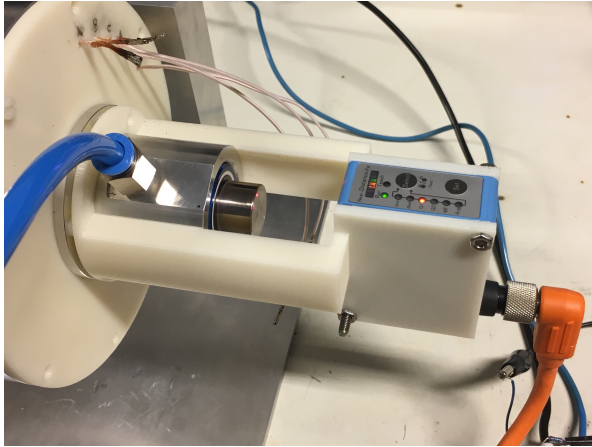


Fig. 3.22: Laser sensor used for the axial position measurement.

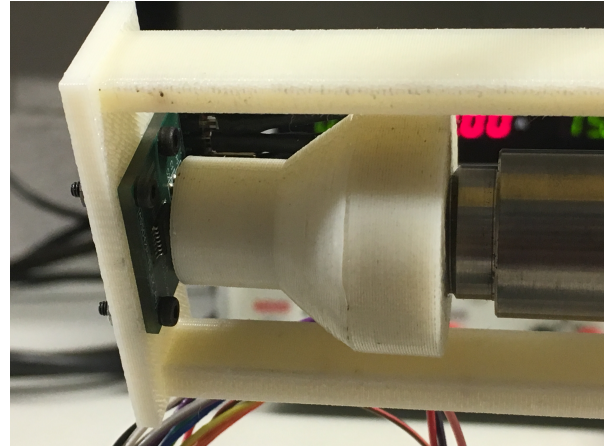


Fig. 3.23: Hall sensor used for the angular position measurement.

Position sensor

To measure the axial displacement of the rotor, a SICK OD2-P30W04I0 laser position sensor is used. It can measure distances within a range of 26 to 34 [mm], with a resolution of 2 [μ m], which thus corresponds to our needs. Figure 3.22 shows the laser sensor fixed to the stator. It is oriented towards the EDTB as the reference taken for the measurements is the right end of the rotor shaft. With the configuration used in the laboratory, the output of this sensor is comprised between 0.35 and 4.36 [V], corresponding to 26 and 34 [mm] respectively. Moreover, in the position corresponding to $z = 0$, found by using the DC motor drive and moving it axially until the amplitude of the generated emfs of the same phase is equal for the upper and lower coils, the output voltage of the laser sensor is 2.87 [V]. The axial displacement z is thus given by the following formula:

$$z(U_{laser}) = 2.005 \cdot (U_{laser} - 2.87) \text{ [mm]} \quad (3.1)$$

Angular position sensor

In the case of a synchronous motor, a measurement of the angular position is required to be able to control the motor. This measurement is done by means of a Hall sensor, composed of two elements. On the one hand, a small permanent magnet is attached to the left end of the rotor shaft using a 3D printed part, as can be seen in Fig. 3.23. On the other hand, a AMS5047P board with the sensor itself is fixed to another 3D printed part attached to the stator yoke. This board can be directly connected to the MAXON controller and generates signals indicating the position with respect to the three phases. A reference that can be chosen to define the angular position corresponding to zero is to align the rotor with the magnetic axis of phase A, by supplying it with a DC current.

3.2.5 Drive systems

Three distinct systems are used to drive the rotor, the first one being a turbine powered by compressed air, as depicted in Fig. 3.24. This possibility has not been used in our experiments as it is very difficult to achieve a precise spin speed using this turbine, but also because it exerts axial and radial forces on the rotor when compressed air is used. A second possibility, used during our measurements to drive the rotor at a given spin speed, is to connect the rotor shaft to a MAXON DC Motor through a flexible bellows coupling, which can be seen in Fig. 3.25. This particular type of coupling has been selected as its stiffness is relatively low and consequently does not completely constraints the axial degree of freedom of the EDTB. As far as the motor is concerned, it is maintained in the axial direction by a 3D printed part attached to the stator yoke using a hose clamp. Furthermore, it can be supplied with a voltage up to 48 [V] and can sustain a maximum power of 90 [W]. Finally, the last possibility to drive the motor is to use the Hall sensor together with the MAXON EPOS2 24/5 electronic controller to supply the self-bearing AFPM motor with a three-phase voltage system. However, even though this controller manages to drive the motor, it fails to determine proper gains for its internal speed controller, thus keeping us from being able to control the motor.

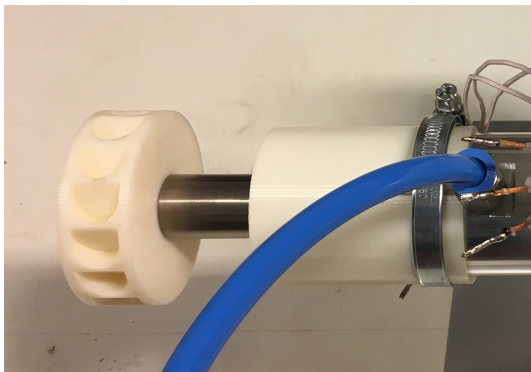


Fig. 3.24: Picture of the turbine drive system.

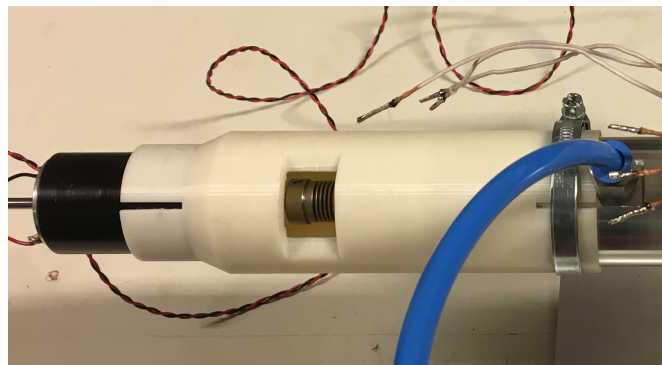


Fig. 3.25: Picture of the DC motor drive system.

3.3 Model parameters

Among the 12 parameters of the model developed in Chapter 1, several are already known, such as the number of phases ($N = 3$) and the number of pole pairs ($p = 6$). The axial and rotational damping coefficients, respectively C_z and C_a , are supposed to be equal to zero, as on the one hand, the magnetic axial damper is not used due to the mechanical issues mentioned hereabove, and as on the other hand, no rotational damping is introduced in the system. Moreover, no ferromagnetic parts are used in the construction of this prototype, except the ferromagnetic rotor yoke, which means that no detent force is applied to the rotor resulting in $k_D = 0$. This leaves us with seven parameters to determine. This section is divided into three categories of parameters to determine, namely the electrical, magnetic and mechanical ones.

3.3.1 Electrical parameters

As only the second type of windings is actually used for the prototype, the electrical parameters, namely the winding resistance and the motor and suspension cyclic inductances, are determined for these windings.

Winding resistance

The winding resistance is determined experimentally using a RLC-meter. The measurements are taken at 120 [Hz] and 1 [kHz], but thanks to the use of Litz wire, the resistance is not expected to vary with the electrical frequency. Indeed, the skin effect is negligible due to the small section of the electrical wires constituting the Litz wire. The measurements are reported in Table 3.3. Based on those results, the mean value of a winding resistance is $R = 104.5$ [mΩ].

120 [Hz]	Phase A	Phase B	Phase C
Upper windings	0.112	0.113	0.102
Lower windings	0.101	0.099	0.111
1 [kHz]	Phase A	Phase B	Phase C
Upper windings	0.110	0.104	0.101
Lower windings	0.098	0.097	0.106

Table 3.3: Resistance measurements at 120 [Hz] and 1 [kHz], in [Ω].

Cyclic inductances

The motor and suspension cyclic inductances can be calculated using (1.21) and (1.22) respectively. In the case of a three-phase system, i.e. for $N = 3$, those expressions reduce to:

$$L_{Mc} = \frac{1}{4} [(2L_{11} - L_{12} - L_{13}) + (2M_{11} - M_{12} - M_{13})]$$

$$L_{Sc} = (2L_{11} - L_{12} - L_{13}) - (2M_{11} - M_{12} - M_{13})$$

As our rotor is composed of a ferromagnetic part, the elements of the matrix $\mathbf{M}$, namely the mutual inductances between the upper and lower windings, are negligible. Moreover, in the case of a three-phase system, the mutual inductance between two windings is always the same. Using those simplifications, the cyclic inductance can finally be expressed as:

$$L_{Mc} = \frac{L_{11} - L_{1k}}{2}$$

$$L_{Sc} = 2(L_{11} - L_{1k})$$

The self-inductance L_{11} of a winding can be measured directly on the prototype using a RLC-meter. The mutual inductance L_{1k} between two windings 1 and k is determined by measuring the self-inductance of those two windings connected in series. The measured value, denoted $L_{s,1k}$ then corresponds to:

$$L_{s,1k} = 2(L_{11} + L_{1k}) \quad (3.2)$$

if the current circulates in the same direction in both windings. The self-inductance and the mutual inductance of the three phases are measured, both for the upper and lower coils, in order to verify that the system is well-balanced. Nevertheless, the inductance values being very low, the measurements are close to the lower bound of the RLC-meter measurement range. This means that the precision of our results is quite poor. Our measurements are presented in Table 3.4. Based on those results, the self-inductance L_{11} and the mutual inductance L_{1k} are obtained by calculating the mean value. The motor and suspension cyclic inductance values are given in Table 3.5.

L_{11}	Phase A	Phase B	Phase C
Upper windings	0.85	0.85	0.9
Lower windings	0.9	0.9	0.8
$L_{s,1k}$	Phases A-B	Phases B-C	Phases A-C
Upper windings	2.6	2.6	2.1
Lower windings	2.3	2.2	2
L_{1k}	Phases A-B	Phases B-C	Phases A-C
Upper windings	0.45	0.45	0.15
Lower windings	0.25	0.2	0.2

Table 3.4: Inductance measurements at 1 [kHz], in [μ H].

L_{11}	L_{1k}	L_{Mc}	L_{Sc}
867	317	275	1100

Table 3.5: Mean inductance values in [nH].

3.3.2 Magnetic parameters

To determine the magnetic parameters of our model, the emf induced in an open circuit winding is measured at different spin speeds and for various axial displacements. An analysis of the spectral content of these emfs is then used to extract the amplitude of the fundamental, which is directly proportional to the magnetic flux linkage amplitude. Indeed, the magnetic flux linkage of the k -th phase, given by (1.1) and (1.2) for the upper and lower windings respectively, is related to the induced emf by the relationship:

$$u_k^{\text{emf}}(t) = \frac{d\Phi_k(t)}{dt} \quad (3.3)$$

Thus, for the upper and lower windings, this expression corresponds to:

$$u_{k,U}^{\text{emf}}(t) = p\omega(\Phi_0 - zK_\Phi) \sin(p(\delta_k - \alpha)) - \dot{z}K_\Phi \cos(p(\delta_k - \alpha)) \quad (3.4)$$

$$u_{k,L}^{\text{emf}}(t) = p\omega(\Phi_0 + zK_\Phi) \sin(p(\delta_k - \alpha)) + \dot{z}K_\Phi \cos(p(\delta_k - \alpha)) \quad (3.5)$$

where $\omega = \dot{\alpha}$ is the spin speed of the self-bearing AFPM motor. If the axial speed $\dot{z} = 0$, those

expressions reduce to:

$$u_{k,U}^{\text{emf}}(t) = p\omega(\Phi_0 - zK_\Phi) \sin(p(\delta_k - \alpha)) \quad (3.6)$$

$$u_{k,L}^{\text{emf}}(t) = p\omega(\Phi_0 + zK_\Phi) \sin(p(\delta_k - \alpha)) \quad (3.7)$$

Magnetic flux linkage amplitude Φ_0

To determine the constant part of the magnetic flux linkage amplitude, the emf of the k -th phase of either an upper or lower winding is measured for an axial displacement $z = 0$ and the amplitude of the fundamental is determined based on the spectral content. In Fig. 3.26, the module of the Fourier transform of the measured emf is represented, for a mechanical spin speed ranging from 600 to 1500 [rpm]. As six pole pairs are used, the electrical frequency is, of course, six times larger than the mechanical one. The amplitude of the peaks corresponding to the fundamental increases linearly with the spin speed, as expected. One should be aware that the amplitude of the peaks in the Fourier domain corresponds to half the amplitude of the sine or cosine in the time domain. Also, one should note that a third and fifth harmonics are present, but their impact is discussed in section 3.5.

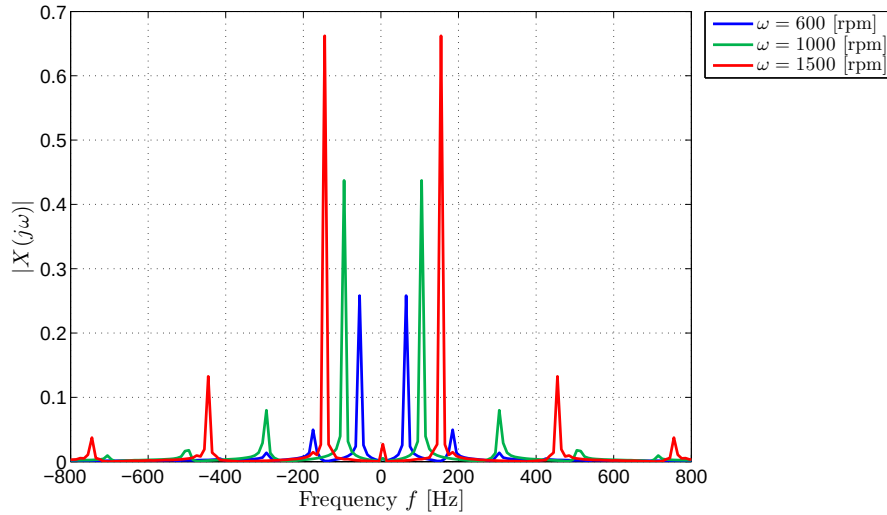


Fig. 3.26: Fourier transform of the emf of an open circuit winding, for mechanical spin speeds $\omega = 600, 1000$ and 1500 [rpm].

Considering the amplitude of the fundamental component of the emf at different spin speeds, it is possible to compute a linear regression between this amplitude and the electrical frequency $p\omega$. The slope of this regression corresponds to Φ_0 . Figure 3.27 presents the experimental data, measured at five different spin speeds, and the linear regression computed from this data. This gives a value of $\Phi_0 = 1.363$ [mWb].

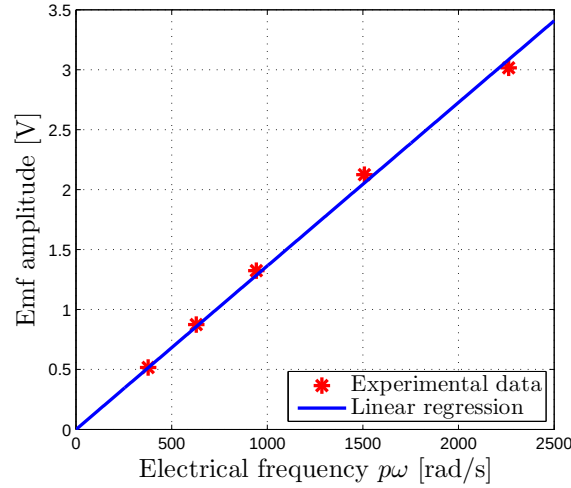


Fig. 3.27: Linear regression between the emf amplitude and the electrical frequency.

Coefficient K_Φ

To compute the coefficient of proportionality between the magnetic flux linkage amplitude and the axial displacement, the same technique is used as for computing Φ_0 . However, in this case, the emf is measured for a non-zero axial displacement. In Fig. 3.28, the Fourier transform of the emf induced in the upper and lower windings constituting the same phase is represented, in the case of a positive axial displacement. It can be seen that the amplitude of the emf in the lower windings, corresponding to the solid lines, is larger than the emf on the upper windings, corresponding to the dashed-dotted lines. This difference in the emf amplitude corresponds to $2(p\omega)K_\Phi z$.

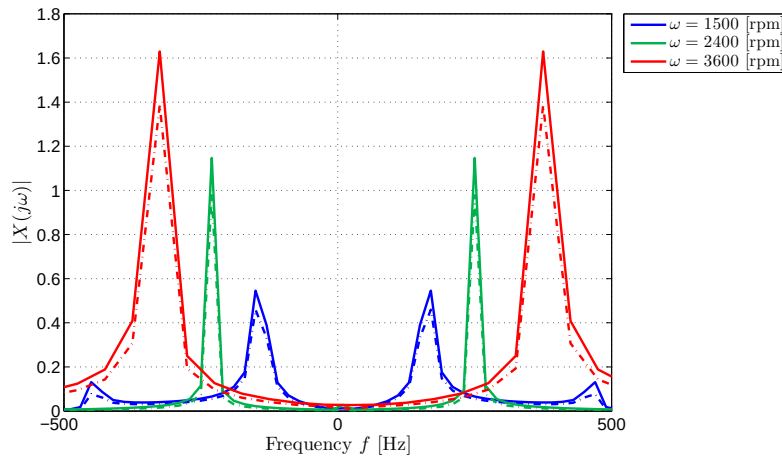


Fig. 3.28: Fourier transform of the emf, respectively for the lower windings (solid line) and upper windings (dashed-dotted line) of the same phase, for a positive axial displacement and spin speeds $\omega = 1500, 2400$ and 3600 [rpm].

Measuring the difference in terms of emf amplitude and dividing this result by the electrical frequency $p\omega$ gives the variation of the magnetic flux linkage with the axial displacement, corresponding to $2K_\Phi z$. A linear regression can thus be computed based on the experimental data. The slope is equal to $2K_\Phi$. This technique is depicted in Fig. 3.29, which presents the result of the linear regression, based on the magnetic flux linkage at three different mechanical spin speeds, for various axial displacements. The resulting value is $K_\Phi = 140.7$ [mWb/m].

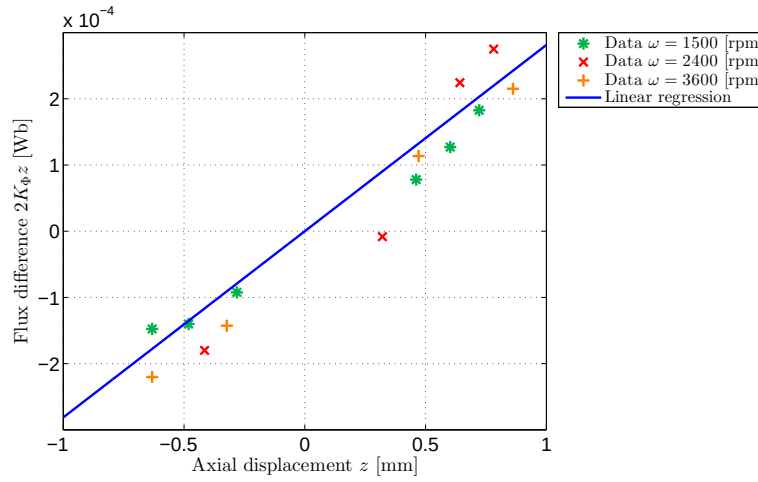


Fig. 3.29: Linear regression between the difference of magnetic flux linkages in the lower and upper windings and the axial displacement.

3.3.3 Mechanical parameters

Rotor mass and inertia

The rotor mass and inertia are determined by using the tools of the CAD software SOLIDWORKS, for the two possible rotor configurations. The obtained results are shown in Table 3.6.

Configuration	Mass [kg]	Inertia [$\text{g} \cdot \text{m}^2$]
Motor	1.097	0.916
Turbine	1.222	0.940
Sensor	1.197	0.922

Table 3.6: Mass and inertia of the rotor for the configuration to be used with a DC motor, or with a turbine/Hall sensor.

Stiffness of the mechanical coupling

To determine the stiffness of the mechanical coupling between the DC motor and the self-bearing AFPM motor, the rotor is attached to this coupling and put into vertical position. The rotor configuration used in this case is the motor one. The force exerted on the coupling equals the weight of the rotor. This results in an axial displacement $z = 0.461$ [mm], measured with the position sensor. Therefore, the coupling stiffness is given by:

$$k_{\text{coupling}} = \frac{m \cdot g}{z} = 19.2 \text{ [N/mm]} \quad (3.8)$$

3.4 Model predictions

Having identified the 12 parameters of the model, this section discusses some important model predictions, namely the axial stiffness generated by the EDTB and the ratio between the drag torque and the square of the axial displacement, based on the parameters identified in section 3.3.

In Fig. 3.30, let us first concentrate on the stiffness developed by the EDTB, represented as a function of the spin speed. A first interesting results is that no significant stiffness is generated below 10 [krpm]. Moreover, given that the stiffness of the mechanical coupling between the DC motor and the rotor shaft has a stiffness of roughly 20 [N/mm], it is impossible to observe the EDTB stabilizing effect below roughly 100 [krpm]. As the maximum spin speed used during our experiments was 4.8 [krpm], this explains why, indeed, no stabilizing effect has been observed. Furthermore, computing the stiffness of the EDTB at this maximum frequency gives a value of 27.2 [N/m], which is three orders of magnitude smaller than the one of the mechanical coupling. Nevertheless, when the spin speed becomes much higher, the stiffness becomes significant, reaching up to 108 [N/mm] at infinite spin speed. As far as the ratio of the drag torque and the square of the axial displacement is concerned, it tells us that this quantity reaches a maximum around 200 [krpm] and has significant values between roughly 10 [krpm] and 1 [Mrpm]. Finally, the instability frequency is $\omega = 302$ [krpm], a value that is impossible to reach from the mechanical point of view of our prototype.

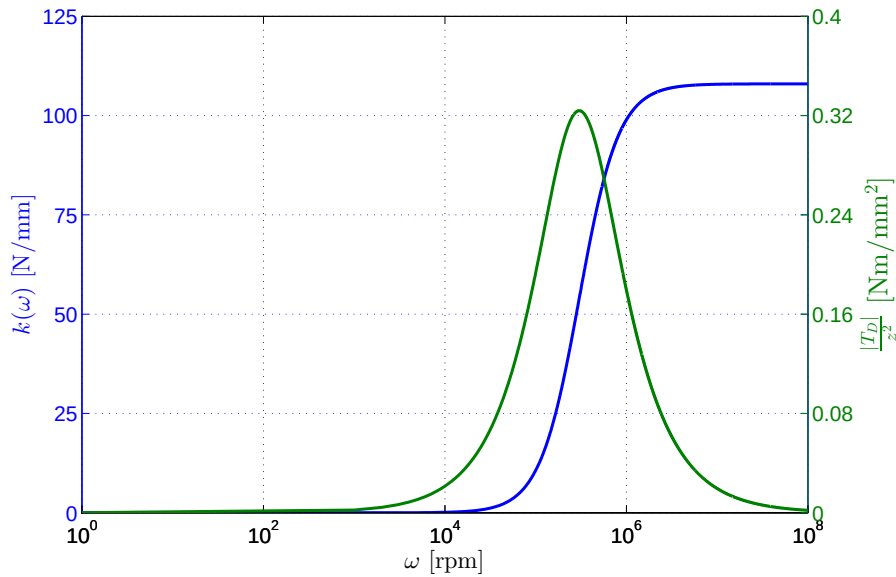


Fig. 3.30: Stiffness $k(\omega)$ and ratio between of the drag torque T_D and the square of the axial displacement z^2 as a function of the spin speed.

3.5 Experimental results

This final section presents the experimental results obtained from the self-bearing AFPM motor using the testbench. The electromotive forces induced on the open circuit windings and phase windings are measured and discussed. More precisely, the spectral content of the emf and the influence of the axial displacement on its amplitude are discussed. Finally, a brief word is said about the EDTB and AFPM motor performance.

3.5.1 Open circuit windings

First, the electromotive force induced on the upper and lower windings is observed. Figures 3.31 and 3.32 show the emfs of the three phases for the upper windings, respectively at 1000 and 1500 [rpm]. Note that the phase shift between them is 60° , due to the fact that the emf was taken in the same direction for each phase. However, it should have been taken in the opposite direction for phase B to obtain three emfs with a phase shift of 120° between them. Nevertheless, it can be observed that the three-phased system is well-balanced and that the phase shift corresponds to what is expected.

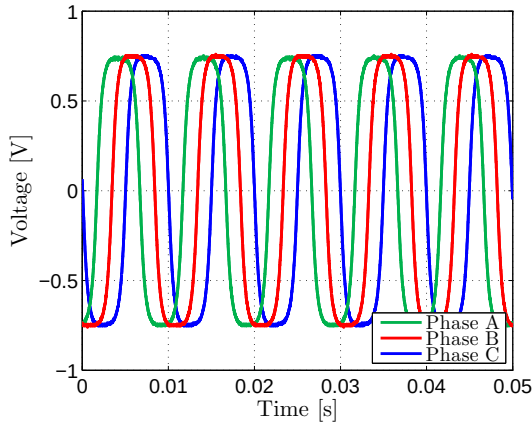


Fig. 3.31: Three-phased system of the upper windings at a spin speed $\omega = 1000$ [rpm].

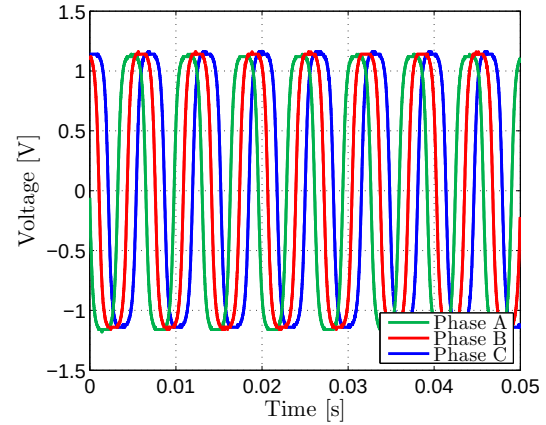


Fig. 3.32: Three-phased system of the upper windings at a spin speed $\omega = 1500$ [rpm].

Due to the quasi square shape of the concentrated windings, the observed emf is not a pure sine as it is assumed in our model. This statement is verified by Fig. 3.33, depicting the spectral content of the emf measured on a winding for a spin speed of 1000 [rpm]. Indeed, a third and fifth order harmonics are present, giving the emf a shape closer to a square than a pure sine or cosine. The ratio between the third harmonics and the fundamental is 18%, and the ratio between the fifth harmonics and the fundamental is 4%. Further works should focus on developing windings shape giving a purer sine than the one obtained in this work. A direct consequence of this non-ideality is that our assumption on the expression of the magnetic flux density is not entirely correct.

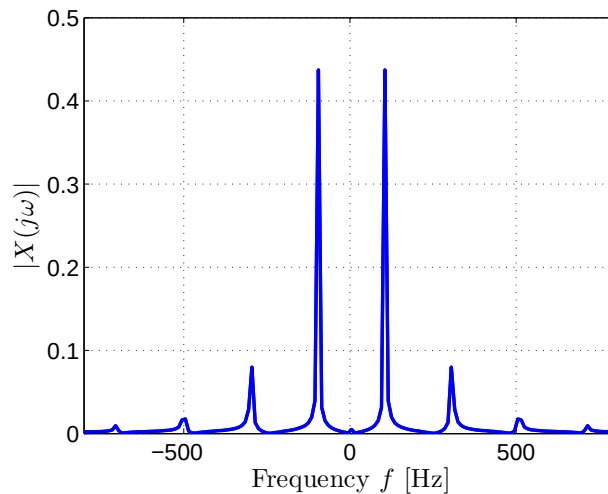


Fig. 3.33: Fourier transform of the emf at a spin speed $\omega = 1000$ [rpm].

Furthermore, for an axial displacement $z = 0$, the upper and lower windings emfs are mea-

sured. They are compared in Figs. 3.34, 3.35 and 3.36 for each phase. It can be seen that those emfs have the same amplitude and phase and consequently, that both stators present well-balanced three-phased systems.

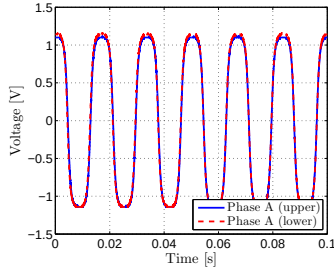


Fig. 3.34: Upper and lower emfs of phase A at $\omega = 1500$ [rpm].

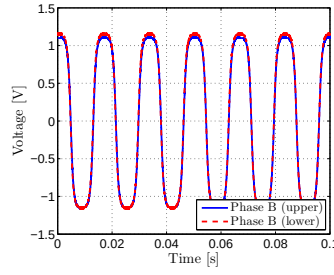


Fig. 3.35: Upper and lower emfs of phase B at $\omega = 1500$ [rpm].

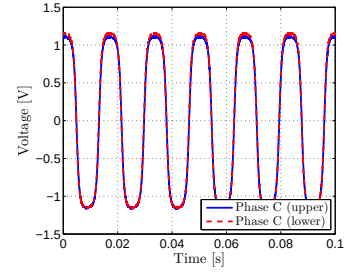


Fig. 3.36: Upper and lower emfs of phase C at $\omega = 1500$ [rpm].

Then, the electromotive forces on the upper and lower windings of the same phase are measured at a spin speed of 2400 [rpm] in Figs. 3.37 and 3.38 and of 3600 [rpm] in Figs. 3.39 and 3.40, for different axial displacements. It can be seen that a positive displacement results in a larger amplitude on the lower winding than on the upper one, and the other way round for a negative displacement.

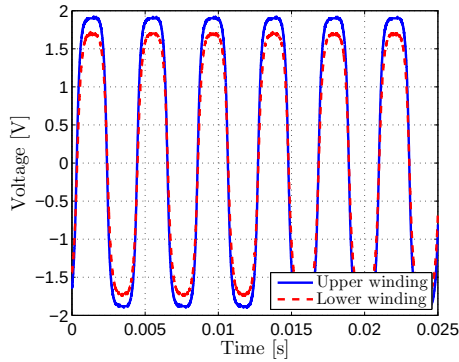


Fig. 3.37: Difference in emf amplitude at a spin speed $\omega = 2400$ [rpm] and an axial displacement $z = -0.42$ [mm].

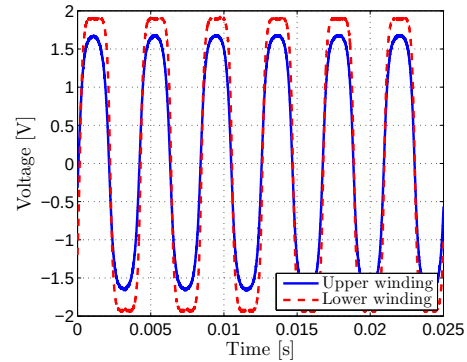


Fig. 3.38: Difference in emf amplitude at a spin speed $\omega = 2400$ [rpm] and an axial displacement $z = 0.78$ [mm].

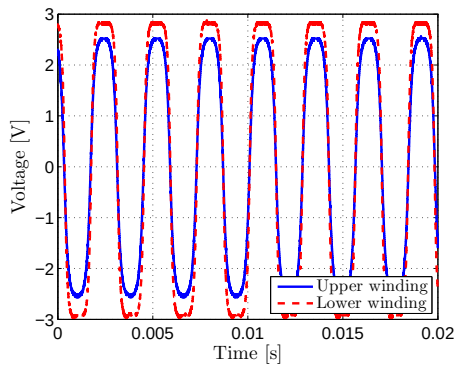


Fig. 3.39: Difference in emf amplitude at a spin speed $\omega = 3600$ [rpm] and an axial displacement $z = 0.86$ [mm].

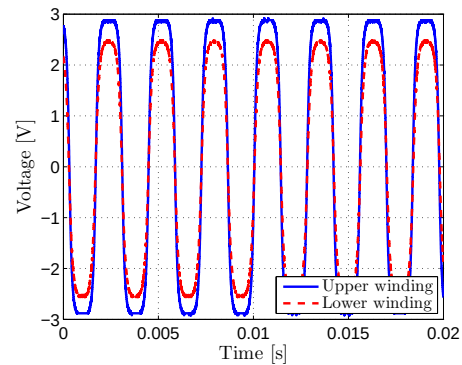


Fig. 3.40: Difference in emf amplitude at a spin speed $\omega = 3600$ [rpm] and an axial displacement $z = -0.63$ [mm].

3.5.2 Open circuit phase windings

Figures 3.41 to 3.44 depict the emf induced on a phase winding, resulting from the series connection of the upper and lower windings of the same phase. First of all, one should notice that the shape of the emf is not a pure sine, due to the presence of a third and fifth harmonics. Nevertheless, when the axial displacement is the same, which is the case for Figs. 3.41 to 3.43, the emf amplitude increases with the spin speed, going from 0.15 [V] at 1200 [rpm] to 0.65 [V] at 3600 [rpm].

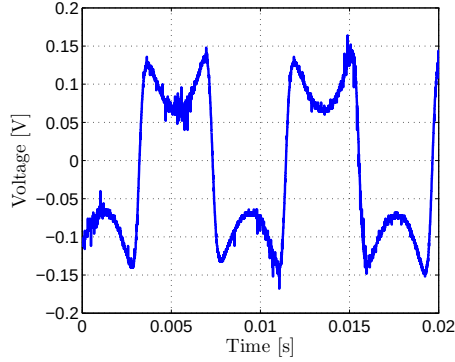


Fig. 3.41: Emf on a phase winding at a spin speed $\omega = 1200$ [rpm] and an axial displacement $z = 0.7$ [mm].

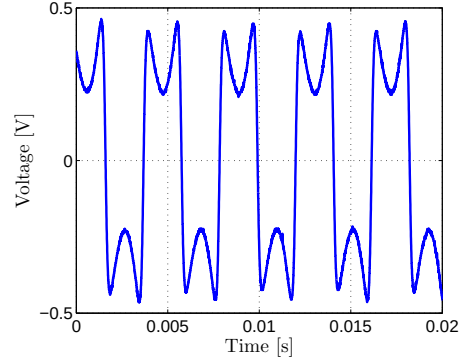


Fig. 3.42: Emf on a phase winding at a spin speed $\omega = 2400$ [rpm] and an axial displacement $z = 0.78$ [mm].

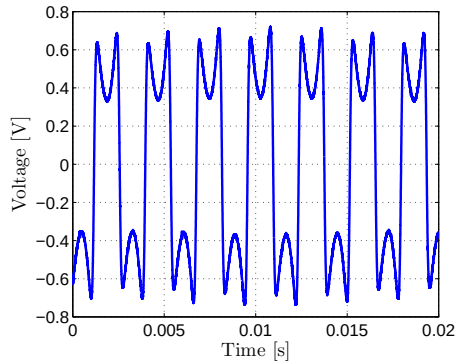


Fig. 3.43: Emf on a phase winding at a spin speed $\omega = 3600$ [rpm] and an axial displacement $z = 0.75$ [mm].

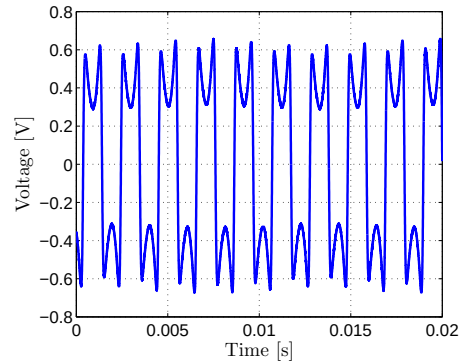


Fig. 3.44: Emf on a phase winding at a spin speed $\omega = 4800$ [rpm] and an axial displacement $z = 0.42$ [mm].

3.5.3 EDTB and AFPM motor performance

The EDTB and AFPM motor performance has not been characterized for two main reasons. In the case of the EDTB, as already explained in section 3.4, the stiffness it develops at the maximum frequency used during our experiments is negligible compared to the one of the mechanical coupling. Therefore, it is not possible to observe the stabilizing effect of the bearing without reaching a very high spin speed. For safety reasons, the prototype was not tested above 4800 [rpm], while according to the model predictions, interesting results are only observable above 100 [krpm]. A possibility to characterize the EDTB would be to use the drive function of the AFPM motor, as in this particular case, no mechanical coupling would be constraining the rotor axial dynamics. As far as the AFPM motor is concerned, the objective was to use the MAXON EPOS2 24/5 controller to supply the three-phased motor load. However, after several attempts, the motor successfully starts up, but the tuning of the velocity controller of the EPOS2 fails for a reason which remains unidentified at this point. This keeps us from characterizing the performance of the AFPM motor.

Conclusion

As a reminder, the starting point of this Master's thesis is the observation that the structure of EDTBs and AFPM motors presents a significant number of similarities, as discussed in the introduction. From there, a new type of bearingless machine combining the concepts of null-flux electrodynamic thrust bearings and double-sided AFPM machines is introduced. Three main objectives are defined regarding this electrodynamic self-bearing AFPM motor. First, developing a model describing the axial and rotational dynamics of such a machine. Secondly, using this model to study the quasi-static equilibrium and stability, as well as the dynamic behavior. Finally, designing a prototype and the associated testbench as a proof-of-concept and validating our model based on the experimental results. Those three objectives underline the structure of this Master's thesis, as each chapter is dedicated to one of them.

Chapter 1 is dedicated to the derivation of a dynamic model describing both the axial and rotational dynamics of the machine, under the assumption of small axial displacements. Compared to the work presented in [20], it is not possible to obtain a state-space representation as the resulting differential equations are non-linear. Nevertheless, a set of eight first-order non-linear differential equations completely describes the dynamics of the self-bearing AFPM motor. This model involves twelve parameters, among which some can be chosen and others can be computed either using analytical formulae or static FEM calculations. The procedure to determine those parameters is also detailed. The originality of this new machine concept and the lack of investigations regarding the combination of a bearing and a motor function within a single winding in the literature have motivated us to publish a conference paper describing our work at the 2017 *IEEE International Electrical Machines and Drives Conference* [36].

In Chapter 2, the afore mentioned model is first linearized around a quasi-static equilibrium point, corresponding to a constant spin speed and a given external force and torque. This allows to obtain a state-space representation of the system around this equilibrium point, and consequently, to assess the stability properties of the system based on a root locus analysis. Considering only the stability of the motor highlights the fact that, when subjected to an external torque, it becomes unstable at high spin speed without rotational damping. However, introducing external damping to the motor only stabilizes it if the damping remains below a given value, above which other high speed instabilities appear. As far as the bearing is concerned, it is unstable at low spin speed if a detent force is applied to the rotor, and at high spin speed regardless of the presence of a detent force. Nevertheless, introducing a sufficient amount of external axial damping can solve the issue of the instability at high spin speed. Then, combining the motor and bearing behavior gives a relatively complex root locus due to the coupling effects between both functions. However, the same tendencies than for the separate motor and bearing are observed, but with slight variations. The root locus also becomes dependent on the equilibrium point. Moreover, analyzing the quasi-static

equilibrium points shows that the electrodynamic axial force generated by the EDTB is proportional to the axial displacement, but that the drag torque, however, is proportional to its square. Finally, the performance criteria for the EDTB and AFPM motor are assessed for different coil dimensions and numbers of pole pairs. It results that using an iron stator yoke gives better results in terms of motor and bearing performance, but has a negative impact on the EDTB stiffness and causes instabilities at low spin speed that cannot be solved by introducing axial damping in the system. Therefore, a topology with a non-magnetic stator yoke is preferred for our prototype.

Chapter 3 presents the structure of the prototype and the testbench. As this Master's thesis is in the continuity of the work performed by Joachim Van Verdeghe in [20], the same testbench structure than the one developed last year has been used. New parts have been added, among others 3D printed parts for the fixation of a magnetic axial damper, a DC motor for the drive system and a Hall sensor for the angular position measurement. A completely new prototype has been manufactured, including a rotor and two different types of stator windings. The first one is rather prospective and is constituted of coils cut into a solid copper block using an electro-erosion cutting technique. The second one is constituted of concentrated windings made out of Litz wire, whose particularity is to avoid eddy currents. Let us now discuss the measurements performed on those two winding types. In the case of the first one, a braking torque due to eddy currents in the windings is observed, thus hindering the good functioning of the whole system. The problem comes from the fact that the lateral surfaces of the coils, normal to the time-varying azimuthal component of the magnetic field, are rather large, resulting in the generation of significant eddy currents. This explanation was confirmed by a numerical simulation on COMSOL Multiphysics, performed by Dr. Virginie Kluyskens. Therefore, no further measurements have been made on this winding type. As far as the Litz wire windings are concerned, they solve the problem of the eddy currents but their disadvantage is their cost. Regarding the EDTB function, the emfs induced in the open windings and phase windings have been measured, and their variations with the spin speed and the axial position have been assessed. The presence of a third and fifth harmonics, due to the shape of the windings, was observed. Moreover, the magnetic parameters have been identified based on those measurements. However, we have been unable to observe a stabilizing effect of the EDTB at the spin speeds considered during our experiments, which is normal according to the predictions of our model. As far as the AFPM motor is concerned, we have been able to drive the motor but not to characterize its performance yet. Finally, the complete system has not been validated due to the failing behavior of the EDTB.

As this Master's thesis is only the second document in the literature covering the topic of electrodynamic self-bearing AFPM motors, the first one being our paper presented at the IEEE IEMDC conference, major opportunities are still to come regarding this technology. Further works could for example focus on optimizing this new machine type for given applications, with defined specifications regarding the motor and the thrust bearing performance. Based on the literature, applications at very high spin speed, such as compressors, turbines and flywheels can be considered. On another scale, small heart pumps could be developed, as AFPM motors already exist for this particular application [22]. Based on the experience acquired with our prototype, new designs should either take into account the effect of eddy currents while dimensioning the windings, or opt for a solution using Litz wire but with a winding topology designed in such a way that the relative amplitude of the high order harmonics is limited and that the performance is carefully evaluated before moving to the prototyping phase. Furthermore, the development of an electrodynamic self-bearing radial-flux PM motor or the integration of the self-bearing AFPM motor in a fully passive magnetic suspension could be the topics of future works.

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Appendices

Property of the Change of Variable

In this appendix, we prove that the change of variable results in the two components of the reduced current vector to be complex conjugates. First, the expression of a component of the current vector after the change of variable is given:

$$I_k^s = \sum_{l=1}^N \frac{I_l}{\sqrt{N}} \exp \left(j(k-1)(l-1) \frac{2\pi}{N} \right) \text{ for } k = 1, \dots, N \quad (\text{A.1})$$

It results that the components of the reduced current vectors $I_1^r = I_2^s$ and $I_2^r = I_N^s$ are expressed as:

$$\begin{aligned} I_1^r &= \sum_{l=1}^N \frac{I_l}{\sqrt{N}} \exp \left(j(l-1) \frac{2\pi}{N} \right) \\ &= \sum_{l=1}^N \frac{I_l}{\sqrt{N}} \left[\cos \left((l-1) \frac{2\pi}{N} \right) + j \sin \left((l-1) \frac{2\pi}{N} \right) \right] \\ &= \frac{1}{\sqrt{N}} \left[\sum_{l=1}^N I_l \cos \left((l-1) \frac{2\pi}{N} \right) + j \sum_{l=1}^N I_l \sin \left((l-1) \frac{2\pi}{N} \right) \right] \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} I_2^r &= \sum_{l=1}^N \frac{I_l}{\sqrt{N}} \exp \left(j(N-1)(l-1) \frac{2\pi}{N} \right) \\ &= \sum_{l=1}^N \frac{I_l}{\sqrt{N}} \left[\cos \left((N-1)(l-1) \frac{2\pi}{N} \right) + j \sin \left((N-1)(l-1) \frac{2\pi}{N} \right) \right] \\ &= \frac{1}{\sqrt{N}} \left[\sum_{l=1}^N I_l \cos \left((N-1)(l-1) \frac{2\pi}{N} \right) + j \sum_{l=1}^N I_l \sin \left((N-1)(l-1) \frac{2\pi}{N} \right) \right] \end{aligned} \quad (\text{A.3})$$

To prove that $I_2^r = I_1^{r*}$, it must be demonstrated that:

$$\begin{cases} \Re \{I_1^r\} = \Re \{I_2^r\} \\ \Im \{I_1^r\} = -\Im \{I_2^r\} \end{cases}$$

Thus, based on (A.2) and (A.3), it must be proved that:

$$\begin{cases} \cos \left((l-1) \frac{2\pi}{N} \right) = \cos \left((N-1)(l-1) \frac{2\pi}{N} \right) \\ \sin \left((l-1) \frac{2\pi}{N} \right) = -\sin \left((N-1)(l-1) \frac{2\pi}{N} \right) \end{cases} \quad (\text{A.4})$$

The right-hand side of the first equation in (A.4) can be rewritten as:

$$\begin{aligned}\cos\left((N-1)(l-1)\frac{2\pi}{N}\right) &= \cos\left([(l-1)2\pi] - \left[(l-1)\frac{2\pi}{N}\right]\right) \\ &= \cos\left(-(l-1)\frac{2\pi}{N}\right) \\ &= \cos\left((l-1)\frac{2\pi}{N}\right)\end{aligned}$$

which proves that $\Re\{I_1^r\}$ and $\Re\{I_2^r\}$ are equal.

As far as the right-hand side of the second equation in (A.4) is concerned, it can be rewritten as:

$$\begin{aligned}\sin\left((N-1)(l-1)\frac{2\pi}{N}\right) &= \sin\left([(l-1)2\pi] - \left[(l-1)\frac{2\pi}{N}\right]\right) \\ &= \sin\left(-(l-1)\frac{2\pi}{N}\right) \\ &= -\sin\left((l-1)\frac{2\pi}{N}\right)\end{aligned}$$

which proves that $\Im\{I_1^r\}$ and $\Im\{I_2^r\}$ have opposite values.

Magnetic Flux Linkage Calculation

This appendix proves the formula used to compute the magnetic flux linked by a coil to determine the model parameters.

The definition of the magnetic flux is the following:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S} \quad (\text{B.1})$$

Using the relationship $\mathbf{B} = \nabla \times \mathbf{A}$, where $\mathbf{A}$ is the magnetic potential vector, this definition can be rewritten as:

$$\Phi = \int_S \nabla \times \mathbf{A} \cdot d\mathbf{S} \quad (\text{B.2})$$

Applying Stokes' theorem, this surface integral can be reduced to a line integral:

$$\Phi = \int_{\partial S} \mathbf{A} \cdot d\mathbf{l} \quad (\text{B.3})$$

One should note that this results corresponds to the integral of a single contour. However, in the case of a coil, the contribution of each contour must be taken into account and weighted by a factor $\frac{j dS'}{I}$ where j is the norm of the local current density vector, dS' is an infinitesimal surface element, and I is the total current circulating into the coil. The total flux is thus obtained as:

$$\Phi_{\text{tot}} = \int_{S'} \int_{\partial S} \mathbf{A} \cdot \mathbf{j} dl dS' \quad (\text{B.4})$$

$$= \frac{1}{I} \int_V \mathbf{j} \cdot \mathbf{A} dV \quad (\text{B.5})$$

Speed and Current Controllers

This appendix deals with the dimensioning of the PI controllers for the current and speed control of the AFPM motor. The control is assumed to be performed in the d-q domain.

Current control

Let us first dimension the low level controllers for the d- and q-axis currents. Figure C.1 depicts the control scheme for the q-axis current. The dynamic compensations are not represented, but it is assumed that they are added to the output of the current controller before it is applied to the motor. One should also note that this figure would be identical for the d-axis current, as long as the dynamic compensations are not represented. This means that both controllers can be dimensioned in the same way.

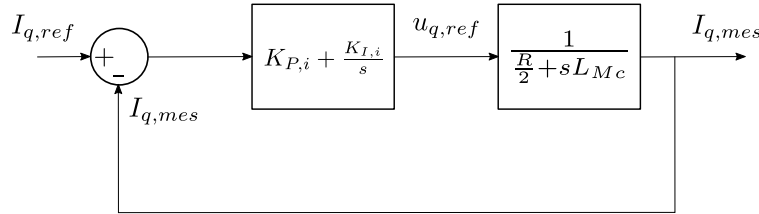


Fig. C.1: Block diagram for the q-axis current controller, where the compensation terms on $u_{q,ref}$ are not represented.

The analysis of the block diagram in Fig. C.1 gives that:

$$(I_{q,ref} - I_{q,mes}) \left(K_{P,i} + \frac{K_{I,i}}{s} \right) \frac{1}{\frac{R}{2} + sL_{Sc}} = I_{q,mes}$$

Simplifying this expression gives the transfer function between the measured current $I_{q,mes}$ and the reference current $I_{q,ref}$ in the Laplace domain as:

$$\frac{I_{q,mes}}{I_{q,ref}}(s) = \frac{K_{P,i} s + K_{I,i}}{L_{Sc} s^2 + \left(\frac{R}{2} + K_{P,i} \right) s + K_{I,i}} \quad (C.1)$$

By identification with the canonical second order polynomial, namely $s^2 + 2\zeta\omega_0 s + \omega_0^2$, it is possible to determine a value for the proportional and integral gains. Here, we choose to use the overdamped case ($\zeta > 1$). The response time at 98% is thus given by:

$$t_r = \frac{4}{\omega_0 (\zeta - \sqrt{\zeta^2 - 1})}$$

Choosing the response time t_r to be equal to 0.75 [s], the following gains are obtained:

$$K_{P,i} = \frac{8\zeta L_{Sc}}{t_r (\zeta - \sqrt{\zeta^2 - 1})} - \frac{R}{2} = 4.32 \cdot 10^{-4} \quad (C.2)$$

$$K_{I,i} = \frac{16L_{Sc}}{t_r^2 (\zeta - \sqrt{\zeta^2 - 1})^2} = 3.7 \cdot 10^{-3} \quad (C.3)$$

As far as the references of these controllers are concerned, they are either $I_{d,ref} = 0$ in the case of the d-axis and a non-zero $I_{q,ref}$ related to the output of the speed controller, in the case of the q-axis.

Speed control

Let us now dimension the high level speed controller, producing the motor torque reference $T_{M,ref}$ and, consequently, the reference q-axis current. The control scheme is depicted in Fig. C.2. An important detail is that the inertia J does not only correspond to the inertia of the self-bearing AFPM motor, but also to that of the load. However, as we consider that our motor acts as a flywheel, the speed controller is designed based on the motor inertia.

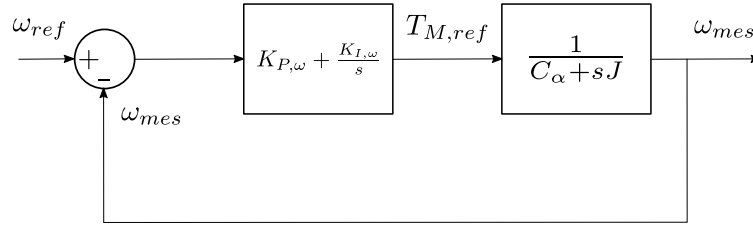


Fig. C.2: Block diagram for the speed controller.

The analysis of the block diagram in Fig. C.2 gives, similarly to the case of the current controllers, the transfer function between the measured speed ω_{mes} and the reference speed ω_{ref} in the Laplace domain as:

$$\frac{\omega_{mes}}{\omega_{ref}}(s) = \frac{K_{P,\omega} s + K_{I,\omega}}{J s^2 + (C_\alpha + K_{P,\omega}) s + K_{I,\omega}} \quad (C.4)$$

For this controller, the underdamped case ($\zeta \leq 1$) is used. This means that the response time at 98% is given by:

$$t_r = \frac{4}{\zeta \omega_0}$$

Choosing a response time t_r of 10 [s] for the speed controllers, which is at least one order of magnitude larger than the response time of the low level current controllers, and a damping factor $\zeta = 1$, gives us:

$$K_{P,\omega} = \frac{8J}{t_r} - C_\alpha = 3.2 \cdot 10^{-4} \quad (C.5)$$

$$K_{I,\omega} = \frac{16J}{t_r^2 \zeta^2} = 6.4 \cdot 10^{-5} \quad (C.6)$$



Technical Drawings

This appendix contains the technical drawings of the parts that were manufactured by Axel Jottard at the LAFAB laboratory. It includes:

1. The iron rings for the magnetic damper.
2. The stator aluminium protective shield.
3. The two stator bakelite yokes.
4. The rotor ferromagnetic disk.

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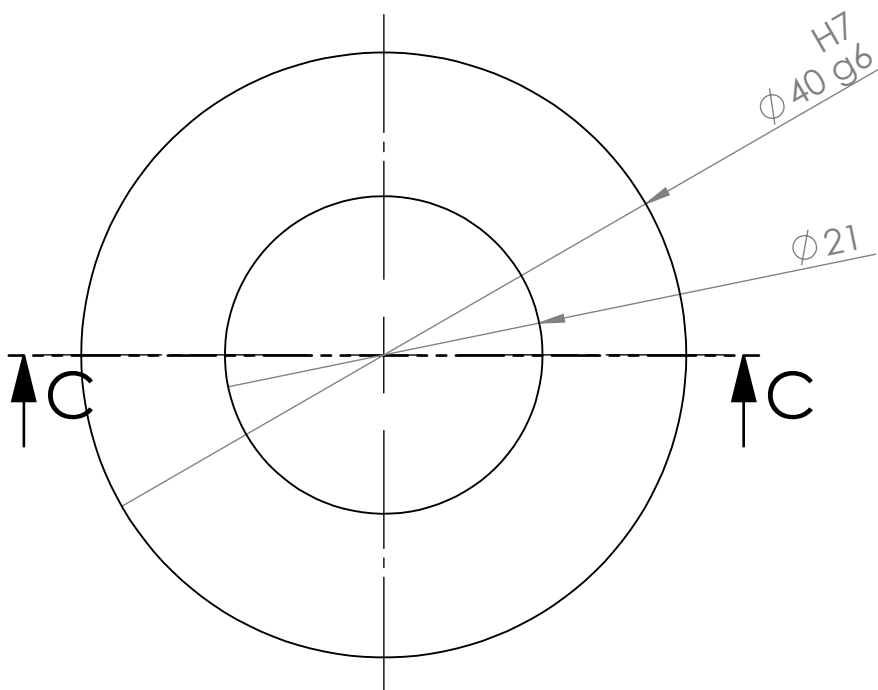
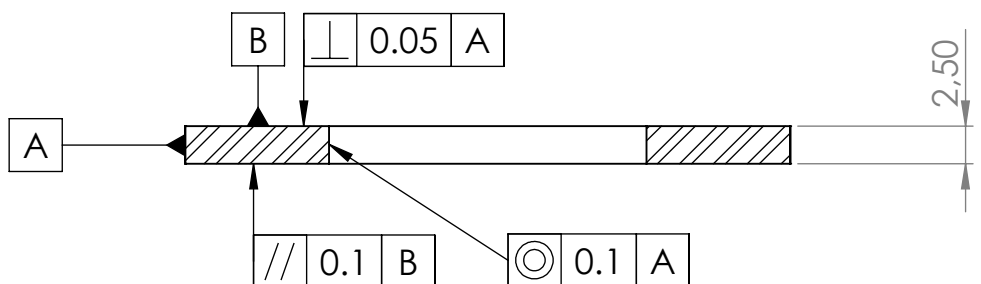
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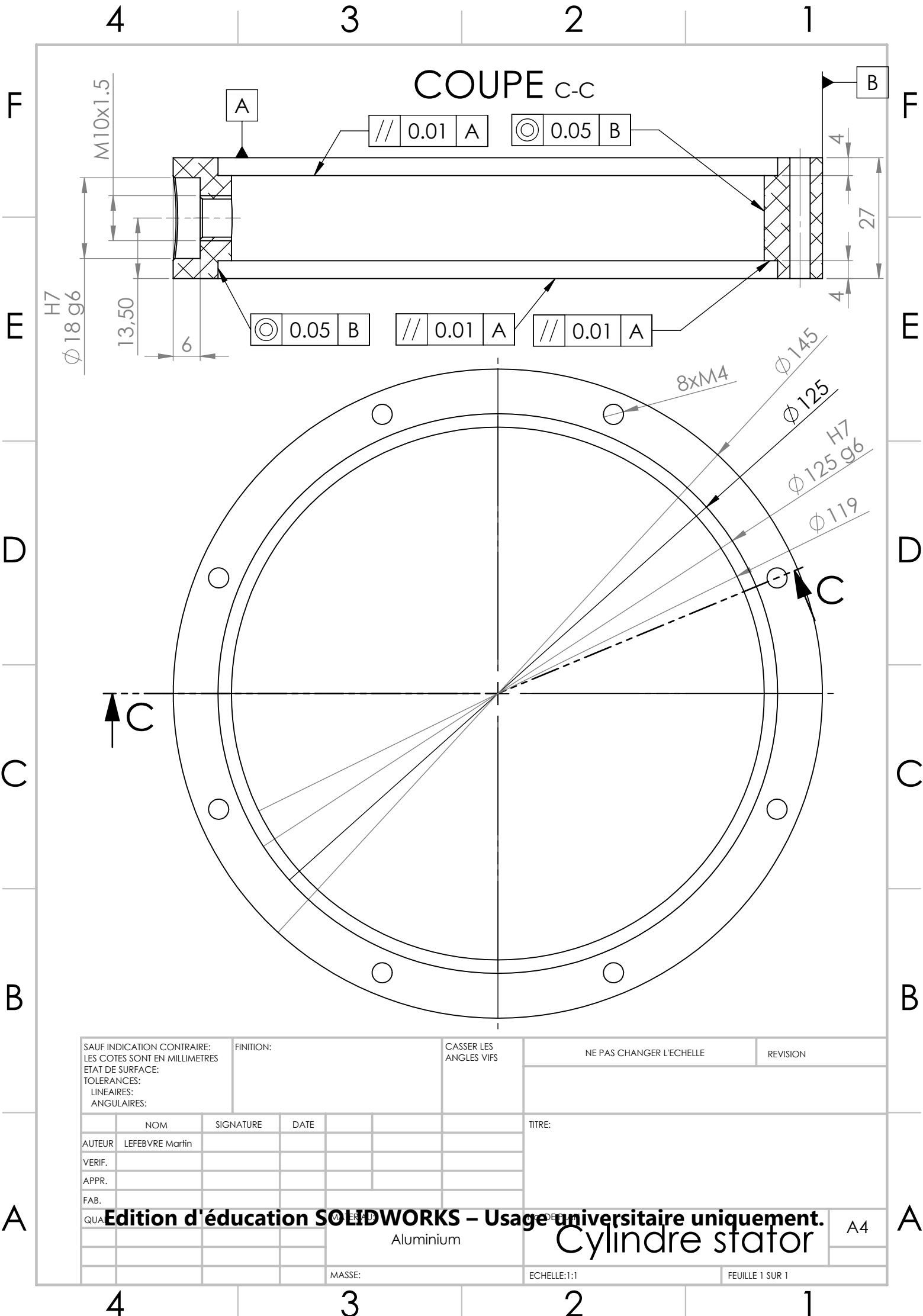
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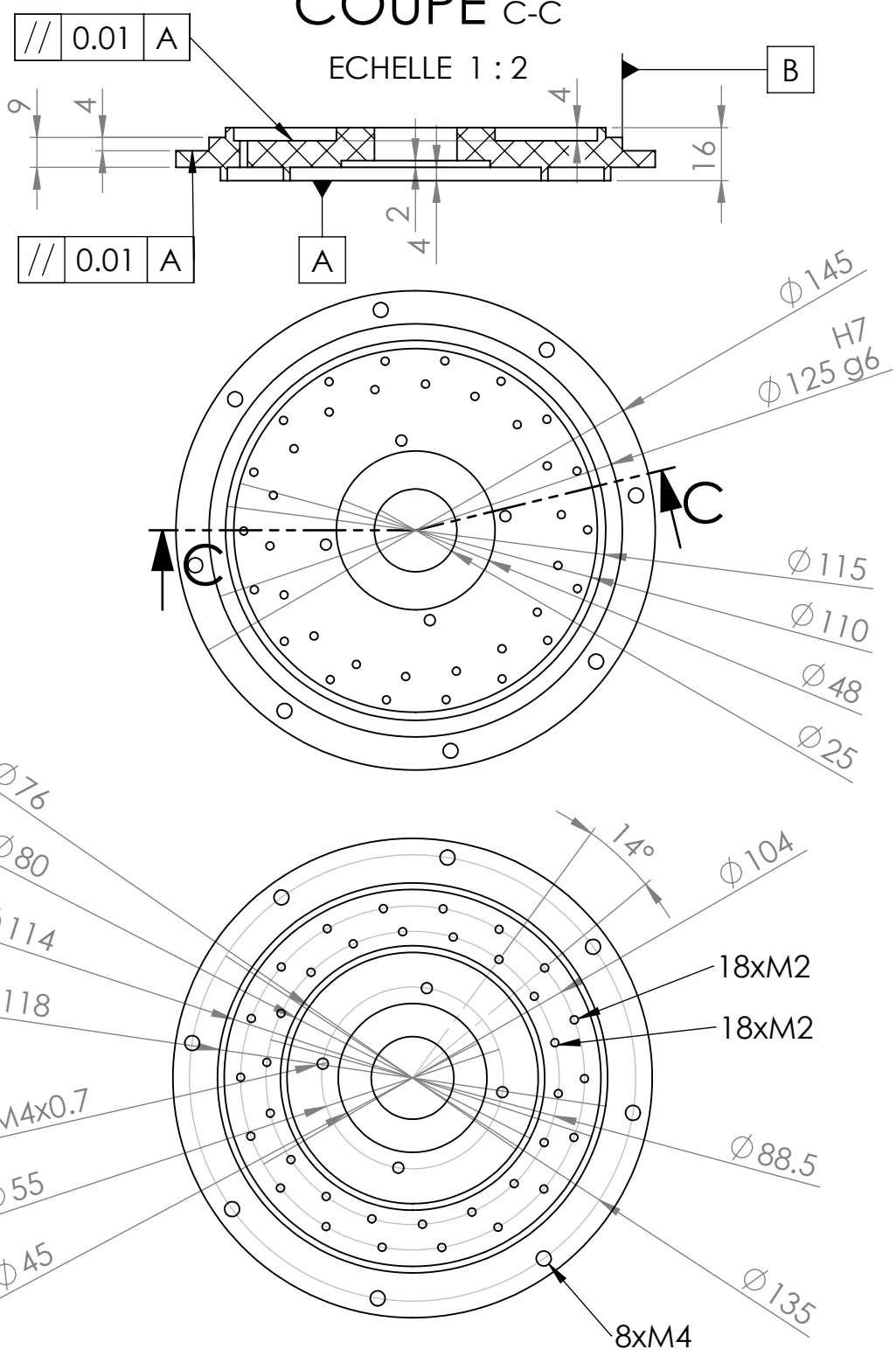
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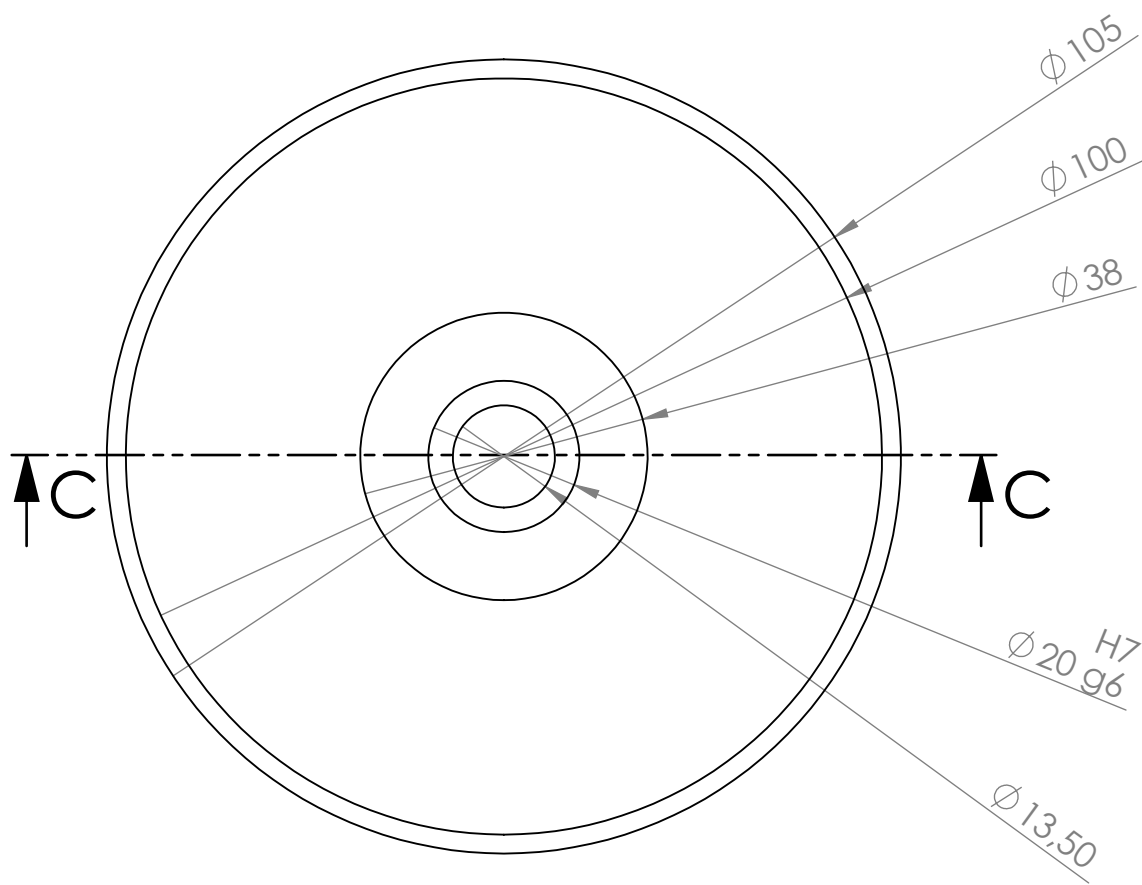
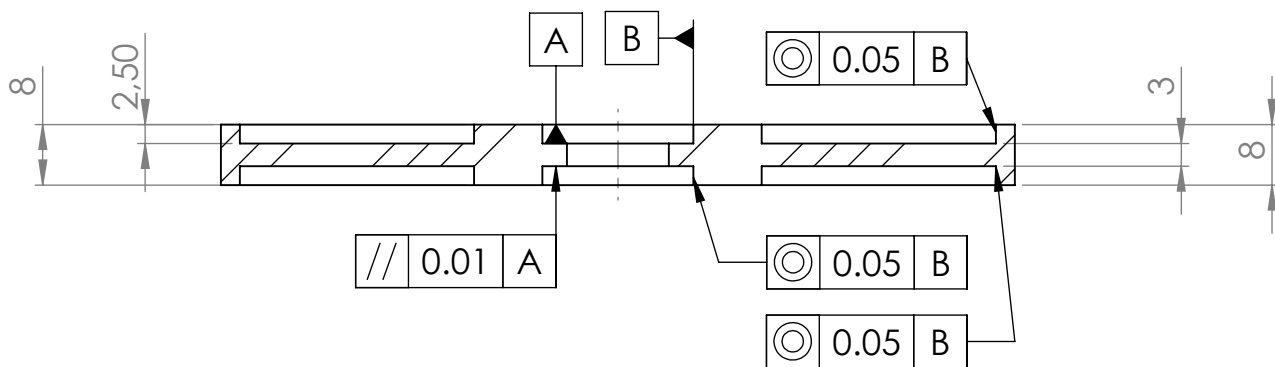
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