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Heterogeneity as Transmission

Common Shocks, Wage Regimes, Inflation Divergence, and Policy Stance in a Two-Country Currency Union

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Contents

Abstract

This study examines how differences in wage-setting institutions can transform a common shock into asymmetric national outcomes within a currency union. The mechanism operates through labour-market tightness, which determines whether new-hire wages remain anchored to the existing-wage norm or shift to flexible wage pressure once a country-specific threshold is crossed. Because the selected wage enters marginal cost, this threshold generates nonlinear inflation transmission. A reduced-form two-country Dynare model with an occasionally binding constraint (OccBin) compares Belgium and Germany under the same cost-push shock and common ECB interest rate while allowing their wage regimes to differ. The simulations generate distinct wage-regime paths, inflation responses, activity dynamics, and country-specific shadow-rate gaps. Reduced-form evidence from local projections and estimated tightness thresholds show corresponding differences in national inflation and real-rate responses, together with a stronger nonlinear wage–tightness relationship in Belgium. The findings identify institutional heterogeneity as an active transmission channel through which a shock that is common at impact becomes country-specific in its consequences.

Keywords: optimum currency areas; wage-setting institutions; labour-market tightness; nonlinear Phillips curve; inflation divergence; OccBin.

1 Introduction

A common shock is not necessarily common in its consequences. Once it enters an economy, it is filtered through the institutions, labour markets, wage rules, and price-setting structures of each country, so what looks symmetric at the point of impact can become asymmetric through transmission. This study starts from that observation. In a currency union, heterogeneity is not only a background condition that makes common monetary policy difficult; it can itself be the channel through which a common disturbance becomes a country-specific inflation and policy-stance outcome.

This argument is rooted in the theory of Optimum Currency Areas (OCA) (Kekre, 2022). A currency union removes country-specific monetary policy, so members share one nominal rate even when their economies differ, and the traditional OCA literature asks when that is costly: when shocks are asymmetric, relative prices adjust slowly, or risk sharing is incomplete. This study shifts the emphasis from asymmetric disturbances to asymmetric propagation, asking how the same common cost-push shock becomes asymmetric as it passes through different wage regimes.

The starting point of the mechanism, and the source of its nonlinearity, is labour-market tightness $\theta_t^i = V_t^i/U_t^i$, vacancies over unemployment. A high θ_t^i means a tight market where hiring is hard and competition bids wages up to a flexible wage that can exceed what incumbents are paid; a low θ_t^i means a slack market where hiring is easy, bargaining power is weak, and wages stay anchored to the existing-wage norm. Since firms hire at the margin, the new hire's wage drives marginal cost, and the economy selects it by comparison: the flexible wage when tight, the existing wage when slack. This selection is the nonlinearity. It connects to Benigno and Eggertsson (2023, 2024), who show that the Phillips curve is flat when labour markets are slack and steep once they tighten: while the existing-wage norm binds, the shock is absorbed, but once θ_t^i crosses the threshold, the flexible wage takes over and the shock passes into marginal cost and inflation, thus forming the inverse-L Phillips curve. That threshold is set by national wage institutions, which makes θ_t^i the variable on which the rest of the study turns.

Belgium and Germany make this concrete. The same shock selects different wage-regime paths in Belgium, with its indexation-style wage norm (Bijnens et al., 2023), and Germany, with its more staggered collective bargaining (Jäger et al., 2022), generating a marginal-cost and inflation gap. Since the shock is common, this cross-country difference comes from the transmission mechanism rather than from imposed asymmetric disturbances; and under one currency-union interest rate, the inflation divergence then maps into different country-level monetary conditions, measured through country-specific shadow-rate diagnostics.

The study develops this argument through a reduced-form two-country Dynare/OccBin mechanism exercise and a reduced-form empirical exercise. In the model, OccBin solves the occasionally binding new-hire wage threshold and traces the regime-dependent transmission of a common shock into inflation and country-specific policy gaps. In the data, local projections of inflation, real-rate, and wage–tightness responses check whether observable counterparts of the same transmission chain appear empirically. The empirical exercise does not estimate the structural model; it provides external reduced-form support for the mechanism.

The main claim is not simply that wage institutions differ across countries. It is that these differences shape the route through which a common shock moves across the economy, turning a symmetric disturbance into country-specific inflation and policy-stance outcomes.

The rest of the study is organised as follows. Section 2 positions the study within the related literature. Sections 3–5 develop the theoretical mechanism, its reduced-form Dynare/OccBin implementation, and the baseline calibration. Sections 6–7 present the simulation results and robustness checks. Sections 8–9 provide the data, empirical strategy, and reduced-form empirical evidence. Section 10 concludes. The appendices contain the theoretical derivations, computational diagnostics, robustness screen, data construction details, and empirical tables.

2 Related Literature

This study sits at the intersection of three literatures: the theory of optimum currency areas, the recent work on state-dependent inflation dynamics, and the comparative study of wage-setting institutions. Each is well developed on its own, but they are rarely connected, and it is their intersection that the study exploits.

The optimum-currency-area literature begins with Mundell (1961), who framed the trade-off between a single currency and the loss of country-specific monetary policy, and was extended by McKinnon (1963) and Kenen (1969) along the dimensions of openness and product diversification. A long tradition then studied when a common monetary policy is costly: when shocks are asymmetric, when relative prices adjust slowly, or when risk sharing is incomplete (Cole and Obstfeld, 1991). In most of this work heterogeneity across members is treated as a friction to be minimised, and the relevant question is whether shocks are symmetric enough for one policy rate to fit all. Kekre (2022) reopens this question from the labour-market side, showing (Proposition 4) that Mundell's business-cycle synchronicity criterion can break down when countries differ in labour-market structure: even common shocks need not leave a union efficient if they propagate differently across members. The present study takes this propagation view further, making heterogeneous wage institutions, rather than asymmetric shocks, the source of divergence.

The second literature concerns the slope of the Phillips curve. A large body of work documented a pronounced flattening of the inflation–activity relationship in advanced economies, including the euro area, where estimated slopes weakened markedly in the decades before the recent inflation episode (Moretti et al., 2019). The 2021–2022 surge prompted a reassessment: Ari et al. (2023) find that the curve re-steepened, particularly in tight labour markets, implying that its slope is not a fixed parameter but depends on the state of the economy. The slanted-L Phillips curve of Benigno and Eggertsson (2023, 2024), introduced above, gives this state dependence its structural form; the study supplies a wage-institutional microfoundation for it, with the nonlinearity arising at the new-hire wage threshold.

The third literature concerns wage-setting institutions and their role in inflation transmission. The search-and-matching tradition links labour-market tightness to hiring costs, job creation, and bargained wages (Pissarides, 2000; Shimer, 2005; Petrongolo and Pissarides, 2001), providing the mechanism through which tightness moves wage pressure. Comparative evidence on European wage-setting (Du Caju et al., 2008; Eurofound, 2024) shows that institutions differ sharply in how quickly labour-market conditions pass into wages: Belgium combines automatic indexation (Bijnens et al., 2023) with a fast pass-through of price pressure, while Germany's industry-region collective bargaining produces a more staggered adjustment of the wage stock (Jäger et al., 2022). These institutional differences are the empirical basis for the asymmetric propagation studied here.

The contribution of this study is to connect them, showing that wage-regime heterogeneity is not only a source of cross-country difference but a transmission mechanism through which a common shock becomes asymmetric inside a currency union. The study therefore shifts the interpretation of heterogeneity from a background friction to an active propagation channel linking wage institutions, nonlinear inflation dynamics, and country-specific policy stances.

3 The Model

The model is organised around the transmission mechanism. It combines a two-country currency-union environment with frictional labour markets, following the optimal-currency-area labour-market structure of Kekre (2022), and a nonlinear Phillips-curve mechanism in the spirit of Benigno and Eggertsson (2023, 2024). Since the central object is the regime-dependent new-hire wage, the wage block is introduced before the marginal-cost and pricing blocks. Standard equilibrium derivations are collected in the appendix.

The transmission chain is

$$u_t \longrightarrow \theta_t^i,\ w_t^{flex,i} \longrightarrow \mathbb{1}_t^i \longrightarrow w_t^{new,i} \longrightarrow mc_t^i \longrightarrow \pi_t^i \longrightarrow r_t^i \approx i_t^{ECB} - \mathbb{E}_t \pi_{t+1}^i.$$

Here mc_t^i denotes log-deviation real marginal cost and π_t^i denotes log inflation relative to steady state. A common cost-pressure shock u_t affects both countries, but its transmission depends on country-specific tightness, wage regimes, and inflation responses.

3.1 Model Environment and Wage Notation

The economy is a two-country New Keynesian currency union. Country H denotes Belgium and country F denotes Germany. Both countries face the common nominal interest rate i_t^{ECB}, with no nominal exchange rate and no independent national monetary-policy instrument.

Throughout the model, P_t^i denotes the country-i price index, $\Pi_t^i = P_t^i/P_{t-1}^i$ denotes gross inflation, and $\pi_t^i = \log \Pi_t^i - \log \Pi^i$ denotes log inflation relative to steady state. This compact notation is used in the household Euler equation, the Phillips curve, the terms-of-trade relation, and the monetary-policy rule.

Each country contains households, intermediate firms, retail firms, and a frictional labour market. Households choose consumption and bond holdings. Intermediate firms produce with labour and hire through a Diamond–Mortensen–Pissarides search-and-matching market, as formalised in Pissarides (2000) and used in the macro-labour literature (Shimer, 2005; Petrongolo and Pissarides, 2001). Retail firms set prices under nominal rigidity.

The shock u_t is a common cost-pressure disturbance, interpreted as a union-wide inflationary pressure such as an energy or import-cost shock. It hits both countries by construction. The stochastic process and calibration of u_t are specified in the simulation-design section.

The wage notation is introduced at the outset because it is central to the transmission mechanism. Lower-case wages are real wages and upper-case wages are nominal wages. In country i, $w_t^{ex,i}$ is the existing-worker wage norm, $w_t^{flex,i}$ is flexible wage pressure, and $w_t^{new,i}$ is the wage paid to new hires. The wage-setting block derives how these objects are related and how the new-hire wage is selected.

3.2 Households, Risk Sharing, and the Currency-Union Real Rate

The household block delivers the Euler equation and the country-specific real-rate channel. In each country $i \in \{H, F\}$, the representative household chooses final consumption C_t^i and a portfolio of one-period state-contingent nominal securities B_{t+1}^i. Employment and wages are taken as given because they are determined in the labour-market and wage-setting blocks. The household maximises

$$\max_{\{C_t^i, B_{t+1}^i\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where β is the discount factor, σ governs risk aversion and intertemporal substitution, χ is the welfare weight on employment disutility, and φ governs its curvature. Employment enters welfare but is not a household control. When $\sigma = 1$, the consumption utility term is understood as the logarithmic limit.

The final consumption basket combines Belgian and German goods. Under the Cole–Obstfeld benchmark (Cole and Obstfeld, 1991), $\varsigma = 1$, so the basket is Cobb–Douglas:

$$C_t^i = (c_{Ht}^i)^{\gamma} (c_{Ft}^i)^{1-\gamma}, \tag{2}$$

where γ is the expenditure weight on Belgian goods. Relative prices govern the composition of demand inside the final consumption basket.

The nominal budget constraint is

$$P_t^i C_t^i + \mathbb{E}_t \left[Q_{t,t+1} B_{t+1}^i \right] = W_t^i N_t^i + P_t^i b U_t^i + B_t^i + T_t^i. \tag{3}$$

The parameter b is a real unemployment benefit, so benefit income enters the nominal budget as $P_t^i b U_t^i$. The object $Q_{t,t+1}$ is the nominal state-contingent price of a payoff delivered at date $t+1$. The price of a one-period risk-free nominal payoff is

$$Q_t^{RF} \equiv \mathbb{E}_t[Q_{t,t+1}] = \frac{1}{1 + i_t^{ECB}}.$$

Solving the household problem gives the risk-free Euler equation

$$(C_t^i)^{-\sigma} = \beta \mathbb{E}_t \left[(C_{t+1}^i)^{-\sigma} \frac{1 + i_t^{ECB}}{\Pi_{t+1}^i} \right], \qquad \Pi_{t+1}^i \equiv \frac{P_{t+1}^i}{P_t^i}. \tag{4}$$

The Lagrangian, first-order conditions, and log-linearisation are reported in Appendix A.1. In log-linear form, the Euler equation implies

$$\widehat{C}_t^i = \mathbb{E}_t[\widehat{C}_{t+1}^i] - \frac{1}{\sigma} r_t^i, \qquad r_t^i \equiv i_t^{ECB} - \mathbb{E}_t[\pi_{t+1}^i]. \tag{5}$$

The nominal interest rate is common across the union, but inflation is country-specific. Hence one ECB nominal rate can imply different real-rate conditions across H and F.

Under complete markets, symmetric equal initial wealth, and the no-home-bias common-basket benchmark, the final consumption index is risk shared:

$$C_t^H = C_t^F = C_t. \tag{6}$$

This condition applies to the final utility-relevant consumption index. It does not imply identical quantities of Belgian and German goods inside the basket, since the composition of demand remains governed by relative prices.

3.3 Labour-Market Tightness and Matching

The labour-market block defines tightness, the state variable that later governs the wage-threshold condition. In each country $i \in \{H, F\}$, firms post vacancies and workers search for jobs. Employment evolves according to

$$N_t^i = (1 - \delta) N_{t-1}^i + M_t^i, \qquad M_t^i = q^i(\theta_t^i) V_t^i. \tag{7}$$

The parameter δ is the exogenous separation rate. A fraction $1 - \delta$ of last period's employment relationships survives into period t, while new matches M_t^i are created when vacancies V_t^i are filled with probability $q^i(\theta_t^i)$.

Labour-market tightness is defined as vacancies relative to the effective search pool:

$$\theta_t^i = \frac{V_t^i}{\widetilde{U}_t^i}. \tag{8}$$

Here $\widetilde{U}_t^i$ denotes workers searching for jobs within period t. A high value of θ_t^i means that vacancies are abundant relative to searching workers; a low value means that vacancies are scarce relative to searching workers.

The effective search pool follows the same-period search timing used in the model.

Workers separated at the beginning of period t can search within the same period, so

$$\widetilde{U}_t^i = U_{t-1}^i + \delta N_{t-1}^i, \tag{9}$$

where U_{t-1}^i is inherited unemployment. With the labour force normalised to one, $U_{t-1}^i = 1 - N_{t-1}^i$, and therefore

$$\widetilde{U}_t^i = 1 - (1-\delta) N_{t-1}^i. \tag{10}$$

Thus $\widetilde{U}_t^i$ is the pool available for matching within the period, not the end-of-period unemployment stock. The remaining labour-market accounting identities are reported in Appendix A.2.

Matches are produced by the Cobb–Douglas matching function

$$M_t^i = m_i (\widetilde{U}_t^i)^{\alpha} (V_t^i)^{1-\alpha}, \tag{11}$$

where $m_i > 0$ is matching efficiency and $\alpha \in (0,1)$ is the matching elasticity with respect to the search pool. This implies the vacancy-filling and job-finding probabilities

$$q^i(\theta_t^i) = \frac{M_t^i}{V_t^i} = m_i (\theta_t^i)^{-\alpha}, \qquad f^i(\theta_t^i) = \frac{M_t^i}{\widetilde{U}_t^i} = m_i (\theta_t^i)^{1-\alpha}. \tag{12}$$

The vacancy-filling rate falls with tightness, while the job-finding rate rises with tightness.

3.4 Wage Determination and the Tightness Threshold

This block determines the new-hire wage selected by labour-market tightness. The flexible wage follows the standard DMP bargaining logic:

$$w_t^{flex,i} = (1-\eta) b + \eta \left(p_t^i A_t^i + k\theta_t^i \right), \tag{13}$$

where η is worker bargaining power, b is the unemployment benefit, p_t^i is the real price of intermediate output in country i, $p_t^i A_t^i$ is the marginal revenue product of labour, and $k\theta_t^i$ captures the hiring-cost component of tightness. Since $\partial w_t^{flex,i} / \partial \theta_t^i = \eta k > 0$, tighter labour markets raise flexible wage pressure. In log-linear transmission form, flexible wage pressure is written as

$$\widehat{w}_t^{flex,i} = \varphi_a^i \widehat{A}_t^i + \varphi_\theta^i \widehat{\theta}_t^i + \psi_u^i u_t, \qquad \varphi_\theta^i > 0. \tag{14}$$

The first two terms are the log-linearised DMP wage-pressure components, while $\psi_u^i u_t$ is a cost-pressure shifter. The shock is common, but the regime outcome depends on whether the induced wage pressure crosses the country-specific threshold.

The existing wage norm is institutionally determined. For Belgium, the existing wage norm is represented as indexation-based (Eurofound, 2024; Du Caju et al., 2008):

$$W_t^{ex,H} = W_{t-1}^{ex,H} \Pi_{t-1}^H, \qquad w_t^{ex,H} = w_{t-1}^{ex,H} \frac{\Pi_{t-1}^H}{\Pi_t^H}. \tag{15}$$

For Germany, the existing wage norm is represented as a staggered wage stock generated by periodic collective bargaining (Du Caju et al., 2008; Jäger et al., 2022):

$$w_t^{ex,F} = (1-\lambda_w^F) w_{t-1}^{ex,F} + \lambda_w^F w_t^{flex,F}. \tag{16}$$

A lower λ_w^F implies stronger wage staggering because current labour-market pressure passes only gradually into the existing wage stock. For the threshold comparison, $w_t^{ex,i}$ is interpreted as the inherited wage norm relevant at the regime-selection stage; the law of motion describes how that norm is updated over time.

The new-hire wage is selected by comparing flexible wage pressure with the existing wage norm:

$$w_t^{new,i} = \max\{w_t^{flex,i}, w_t^{ex,i}\}. \tag{17}$$

The threshold $\theta_t^{\star,i}$ is the tightness level at which $w_t^{flex,i} = w_t^{ex,i}$. Using Eq. (13),

$$\theta_t^{\star,i} = \frac{w_t^{ex,i} - (1-\eta)b - \eta p_t^i A_t^i}{\eta k}. \tag{18}$$

Thus a higher existing wage norm raises the threshold, while higher productivity or a higher real output price lowers it.

Define the wage gap as

$$gap_t^i \equiv w_t^{flex,i} - w_t^{ex,i} = \eta k \left(\theta_t^i - \theta_t^{\star,i}\right). \tag{19}$$

Hence $gap_t^i > 0$ if and only if $\theta_t^i > \theta_t^{\star,i}$. The tight-regime indicator is

$$\mathbb{1}_t^i \equiv \mathbb{1}\{\theta_t^i > \theta_t^{\star,i}\}. \tag{20}$$

The new-hire wage can therefore be written as

$$w_t^{new,i} = \left(1 - \mathbb{1}_t^i\right) w_t^{ex,i} + \mathbb{1}_t^i w_t^{flex,i}. \tag{21}$$

When $\mathbb{1}_t^i = 0$, the existing wage norm binds for new hires. When $\mathbb{1}_t^i = 1$, the flexible wage becomes the new-hire wage.

The threshold is country-specific because the existing wage norm is country-specific:

$$\theta_t^{\star,H} \neq \theta_t^{\star,F}. \tag{22}$$

Belgium's threshold is shaped by indexation, while Germany's threshold is shaped by staggered collective bargaining. The local transmission link established in this block is therefore

$$u_t,\ \theta_t^i \longrightarrow w_t^{flex,i} \longrightarrow \mathbb{1}_t^i \longrightarrow w_t^{new,i}.$$

The next block shows why this new-hire wage is the relevant wage for firms' marginal cost.

3.5 Intermediate Firms, Job Creation, and Marginal Cost

The previous sections defined labour-market tightness and the wage-regime rule that selects $w_t^{new,i}$. This block shows why that wage matters for firms. In each country $i \in \{H, F\}$, intermediate firms produce with labour, post vacancies to create new matches, and sell their output to retail firms. The key object delivered by this block is real marginal cost. Intermediate output is produced with linear labour technology:

$$Y_t^i = A_t^i N_t^i, \tag{23}$$

where A_t^i is labour productivity and N_t^i is employment. In log deviations,

$$\widehat{Y}_t^i = \widehat{A}_t^i + \widehat{N}_t^i. \tag{24}$$

Vacancy creation is costly. The real cost of posting one vacancy is proportional to the new-hire wage:

$$k w_t^{new,i}, \tag{25}$$

where $k > 0$ governs recruiting costs. Since a vacancy is filled with probability $q^i(\theta_t^i)$, the expected cost of creating one filled job is $kw_t^{new,i}/q^i(\theta_t^i)$. Free entry into vacancy posting equates this cost to the expected discounted value of a filled job:

$$\frac{kw_t^{new,i}}{q^i(\theta_t^i)} = \mathbb{E}_t\left[Q_{t,t+1}\left(p_{t+1}^i A_{t+1}^i - w_{t+1}^{new,i} + (1-\delta)\frac{kw_{t+1}^{new,i}}{q^i(\theta_{t+1}^i)}\right)\right]. \tag{26}$$

The left-hand side is the expected hiring cost of one filled job. It rises when the new-hire wage increases or when tightness lowers the vacancy-filling probability. The right-hand side is the expected discounted value of the match: next period's revenue product, minus the new-hire wage, plus continuation value if the match survives. Here $Q_{t,t+1}$ is the state-contingent discount factor used to price next-period payoffs. The derivation from firm value and free entry is reported in Appendix A.4.

The marginal worker is a new hire. With linear production, one additional worker produces A_t^i units of output and costs $w_t^{new,i}$. Level real marginal cost is therefore

$$MC_t^i = \frac{w_t^{new,i}}{A_t^i}. \tag{27}$$

In log-deviation form, the marginal-cost object entering the Phillips curve is

$$mc_t^i = \widehat{w}_t^{new,i} - \widehat{A}_t^i. \tag{28}$$

The local transmission link established in this block is therefore

$$w_t^{new,i} \longrightarrow mc_t^i.$$

3.6 Retail Firms and the Nonlinear Phillips Curve

The previous block established that the wage selected by the wage regime enters firms' real marginal cost. The retail block maps that marginal cost into inflation. In country $i \in \{H, F\}$, retail firms buy the homogeneous intermediate good, differentiate it into varieties, and sell these varieties under monopolistic competition. Prices are sticky: each firm can reset its price only with probability $1-\iota$ in each period.

Demand for variety j is

$$y_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon} Y_t^i, \tag{29}$$

where $\varepsilon > 1$ is the elasticity of substitution across varieties. The Dixit–Stiglitz demand derivation and the Calvo reset-price problem are reported in Appendix A.5.

Let gross inflation be

$$\Pi_t^i \equiv \frac{P_t^i}{P_{t-1}^i},$$

and define log inflation as

$$\pi_t^i \equiv \log \Pi_t^i - \log \Pi^i.$$

The standard Calvo pricing problem implies

$$\pi_t^i = \beta\,\mathbb{E}_t[\pi_{t+1}^i] + \kappa_p\, mc_t^i, \tag{30}$$

where $mc_t^i \equiv \log MC_t^i - \log MC^i$ is the log deviation of real marginal cost and

$$\kappa_p = \frac{(1-\iota)(1-\beta\iota)}{\iota}. \tag{31}$$

The parameter κ_p is the Calvo slope and governs the response of inflation to real marginal cost for a given pricing friction.

The pricing structure is common across countries. The nonlinearity comes from the marginal-cost term inherited from the wage block. Since the marginal worker is a new hire,

$$mc_t^i = \widehat{w}_t^{new,i} - \widehat{A}_t^i. \tag{32}$$

Using the tight-regime indicator $\mathbb{1}_t^i$ defined in Section 3.4, marginal cost can be written as

$$mc_t^i = (1-\mathbb{1}_t^i)\left(\widehat{w}_t^{ex,i} - \widehat{A}_t^i\right) + \mathbb{1}_t^i\left(\widehat{w}_t^{flex,i} - \widehat{A}_t^i\right). \tag{33}$$

Substituting this expression into the Phillips curve gives

$$\pi_t^i = \beta\,\mathbb{E}_t[\pi_{t+1}^i] + \kappa_p\left[(1-\mathbb{1}_t^i)\left(\widehat{w}_t^{ex,i} - \widehat{A}_t^i\right) + \mathbb{1}_t^i\left(\widehat{w}_t^{flex,i} - \widehat{A}_t^i\right)\right]. \tag{34}$$

The Calvo pricing friction is the same across countries and regimes. What changes is the wage entering marginal cost. When $\mathbb{1}_t^i = 0$, marginal cost is constructed from the existing wage norm. When $\mathbb{1}_t^i = 1$, marginal cost is constructed from flexible wage pressure. The local transmission link established in this block is therefore

$$\theta_t^i \longrightarrow \mathbb{1}_t^i \longrightarrow w_t^{new,i} \longrightarrow mc_t^i \longrightarrow \pi_t^i.$$

Belgium and Germany solve the same pricing problem. Inflation differs because the wage regime determining marginal cost can differ across countries.

3.7 Common Monetary Policy, Relative Prices, and Shocks

The remaining closure problem is that Belgium and Germany share one nominal policy rate, while inflation, output, wages, and labour-market tightness are country-specific. The ECB sets the common nominal interest rate according to

$$i_t^{ECB} = \phi_\pi \bar{\pi}_t + \phi_y \bar{y}_t, \tag{35}$$

where union inflation and output are weighted averages:

$$\bar{\pi}_t = \omega\pi_t^H + (1-\omega)\pi_t^F, \qquad \bar{y}_t = \omega y_t^H + (1-\omega)y_t^F. \tag{36}$$

Let $y_t^i \equiv \widehat{Y}_t^i$ denote the output deviation. The parameter $\omega \in (0,1)$ is the weight on country H, while ϕ_π and ϕ_y govern the response to union inflation and output. The policy instrument is common:

$$i_t^{ECB,H} = i_t^{ECB,F} = i_t^{ECB}.$$

The country-specific real-rate condition follows from the household Euler equation:

$$r_t^i \approx i_t^{ECB} - \mathbb{E}_t\pi_{t+1}^i. \tag{37}$$

Hence

$$r_t^H - r_t^F = -\mathbb{E}_t\left(\pi_{t+1}^H - \pi_{t+1}^F\right). \tag{38}$$

A common nominal rate therefore implies country-specific real-rate conditions whenever expected inflation differs across countries. This is interpreted as policy-stance divergence within the currency union, not as a Ramsey-optimal policy wedge.

Relative prices close the goods-market side of the OCA model. Let

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}}. \tag{39}$$

denote the relative price of Belgian goods in terms of German goods, and let $tot_t \equiv \widehat{S}_t$. Since there is no nominal exchange rate inside the currency union, relative prices move through inflation differentials:

$$tot_t = tot_{t-1} + \pi_t^H - \pi_t^F. \tag{40}$$

The final consumption basket in country i combines Belgian and German goods. The associated CES goods demands are

$$c_{Ht}^i = \gamma \left(\frac{P_{H,t}}{P_t^i}\right)^{-\varsigma} C_t^i, \qquad c_{Ft}^i = (1-\gamma)\left(\frac{P_{F,t}}{P_t^i}\right)^{-\varsigma} C_t^i. \tag{41}$$

Here P_t^i is the country-i price index implied by the domestic and foreign goods prices. Under the Cole–Obstfeld benchmark, $\varsigma = 1$, so expenditure shares are constant:

$$P_{H,t}c_{Ht}^i = \gamma P_t^i C_t^i, \qquad P_{F,t}c_{Ft}^i = (1-\gamma)P_t^i C_t^i. \tag{42}$$

Goods markets clear for Belgian and German output:

$$Y_t^H = c_{Ht}^H + c_{Ht}^F + kw_t^{new,H}V_t^H, \qquad Y_t^F = c_{Ft}^H + c_{Ft}^F + kw_t^{new,F}V_t^F. \tag{43}$$

Thus inflation differentials move the terms of trade, relative prices allocate demand across Belgian and German goods, and output clears in each country after vacancy-posting resource costs.

The common cost-pressure shock follows

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \qquad 0 < \rho_u < 1, \qquad \varepsilon_t^u \sim \mathcal{N}(0, \sigma_u^2). \tag{44}$$

The innovation is common to both countries. Cross-country differences therefore come from the transmission of the same shock through wage institutions, tightness thresholds, marginal cost, inflation, and the common policy rule.

The model is now closed. A common shock affects wage pressure; wage-regime selection changes marginal cost; marginal cost feeds into inflation; the ECB responds to union aggregates; and inflation differentials move both relative prices and country-specific real rates. The resulting transmission chain is

$$u_t \longrightarrow \theta_t^i,\ w_t^{flex,i} \longrightarrow \mathbb{1}_t^i \longrightarrow w_t^{new,i} \longrightarrow mc_t^i \longrightarrow \pi_t^i \longrightarrow r_t^i \approx i_t^{ECB} - \mathbb{E}_t\pi_{t+1}^i. \tag{45}$$

4 Computational Implementation

The simulations use a reduced-form Dynare/OccBin implementation of the two-country OCA model of Section 3, built around the transmission chain in Eq. (45): a common cost-push shock shifts flexible wage pressure, flexible wages are compared with country-specific existing wage norms, the selected new-hire wage determines marginal cost and inflation, and the common ECB rate is evaluated against country-specific shadow-rate diagnostics. The occasionally binding object is the wage-floor condition of Section 3.4: the implementation uses the same max operator and tight-regime indicator defined in Eqs. (17), (20) and (21), and OccBin selects the active wage branch along the simulated path. The economic nonlinearity is the wage threshold; OccBin solves the regime switch it implies.

All endogenous variables are linear deviations from calibrated steady-state objects. The zero steady state reported by Dynare therefore refers to deviations, not to the levels of wages, employment, tightness, output, vacancies, or vacancy costs. These nonzero steady-state objects are computed separately and enter the linearised equations as parameters; Appendix B details these calculations. For example, the implementation variable n_t^i is the employment deviation corresponding to $\widehat{N}_t^i$. The associated Dynare solution checks are reported in Appendix C.4 and Table 4.

The household Euler channel enters through the real-rate term $i_t^{ECB} - \pi_{t+1}^i$ in the reduced job-creation equation. The firm-value problem is represented through a reduced DMP-style job-creation condition, while the CES goods allocation is summarised through net activity x_t^i, terms of trade tot_t, and relative demand $relD_t^{HF}$. The Phillips-curve pass-through parameter κ_p is constant; regime dependence enters through the wage selected by the tightness threshold. The implemented block map is reported in Table 2, the theory-to-implementation reduction in Table 3, and the implementation chronology in Appendix C.2 and Figure 7.

This implementation isolates one mechanism: whether asymmetric wage institutions generate inflation divergence and country-specific policy-stance diagnostics under a common ECB rule.

5 Calibration and Simulation Design

5.1 Calibration Principle

The model is calibrated rather than estimated: the aim is a transparent numerical environment for the wage-threshold mechanism, not a match to every moment of the Belgian and German economies. Parameter choices are disciplined in three ways: by standard macroeconomic and search-and-matching benchmarks, by country-specific empirical targets, and by robustness ranges for parameters not directly pinned down by data. The distinction matters: the country weight in the ECB rule and the steady-state unemployment targets are data-target objects, whereas the Phillips-curve pass-through, cost-push persistence, and wage-pressure loadings are benchmark objects that must be checked in robustness exercises. The calibration is therefore not used to tune one preferred simulation path.

5.2 Baseline Parameterisation

The benchmark values used in the baseline simulation are reported in Table 1 in Appendix C, which focuses on the parameters that matter for the main mechanism and records the source notes and alternative robustness values.

The implementation also computes steady-state and linearisation objects (n_{ss}^i, θ_{ss}^i, w_{ss}^i, kw_q^i, and s_V^i); these are not independent free parameters but are implied by the chosen labour-market targets and the reduced DMP steady-state equations.

5.3 Baseline Cost-Push Experiment

The baseline experiment uses a common cost-push shock

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \tag{46}$$

with a single first-period innovation,

$$\varepsilon_1^u = 0.001, \qquad \varepsilon_t^u = 0 \quad \text{for } t > 1. \tag{47}$$

The shock is deliberately common to both countries: the experiment imposes no cross-country asymmetry through different shocks, so the same disturbance interacts with different wage norms, threshold conditions, and labour-market responses. The choice is also aligned with the motivating episode ,i.e., a productivity shock is the standard OCA disturbance, but the recent euro-area inflation episode was more naturally associated with energy, import-price, and cost-pressure shocks, which activate the wage-pressure channel directly and can move the economy across the wage threshold.

5.4 Simulation Objects

The simulation tracks the four groups of objects that make up the transmission chain. The wage-regime objects show which branch each country is in and how far it sits from the threshold ($w_t^{flex,i}$, $w_t^{ex,i}$, $w_t^{new,i}$, and the threshold distance $\theta_t^i - \theta_t^{\star,i}$); these determine the regime that everything downstream inherits. The selected wage then feeds the price block through marginal cost mc_t^i and inflation π_t^i. The reduced OCA objects trace how the shock spreads through real activity and relative prices (θ_t^i, n_t^i, x_t^i, tot_t, and $relD_t^{HF}$), and the policy diagnostics close the chain by setting the common rate i_t^{ECB} against the country-specific shadow rates $i_t^{shadow,i}$ and their gaps $i_t^{shadow,i} - i_t^{ECB}$. The results section reports these objects through a small number of figures and summary moments. Appendix C provides the supporting verification layer: the calibration, theory-to-implementation mapping, Dynare solution checks, threshold and branch diagnostics, mechanism moments, policy-mismatch summary, and robustness screen.

6 Dynare Simulation Results

6.1 Baseline Experiment and Identification Logic

The baseline simulation follows the common cost-push experiment defined in Subsection 5.3, so the results below isolate how the same shock propagates through different wage institutions. This identification logic is central to the interpretation of the results: the model asks whether a common inflationary disturbance can generate different country-level outcomes when the same shock interacts with different wage-setting institutions. Belgium is represented by an indexation-style existing wage norm and Germany by a staggered wage-stock adjustment process. These norms determine how flexible wage pressure compares with the existing wage norm, and therefore which new-hire wage branch is selected by the occasionally binding wage threshold.

This interaction is the source of all asymmetry in what follows. The baseline experiment imposes no country-specific shocks, no country-specific monetary rules, and no exogenous policy wedges; one common shock interacts with different institutions, selects different new-hire wage regimes, generates inflation divergence, and maps into different country-level policy positions under a single ECB rate. The results are reported in this same order:

first wage-regime activation, in which the shock selects different new-hire wage branches; then price transmission, in which the selected wage affects marginal cost and inflation; and finally the monetary-policy diagnostic, in which the common ECB rate is compared with country-specific shadow-rate benchmarks. This ordering makes the policy-mismatch result the outcome of the wage and inflation mechanism rather than a separate object imposed on the simulation.

6.2 Wage-Regime Activation

The first step is to verify that the common shock activates the wage threshold in an economically meaningful way. The relevant object is the new-hire wage, since it is the variable that enters marginal cost and therefore transmits the wage-regime choice into price dynamics. In the implementation it is selected by the wage-determination rule of Eq. (17), $w_t^{new,i} = \max\{w_t^{flex,i}, w_t^{ex,i}\}$, so the figure reports not only wage movements but which branch is selected by the occasionally binding threshold in each country.

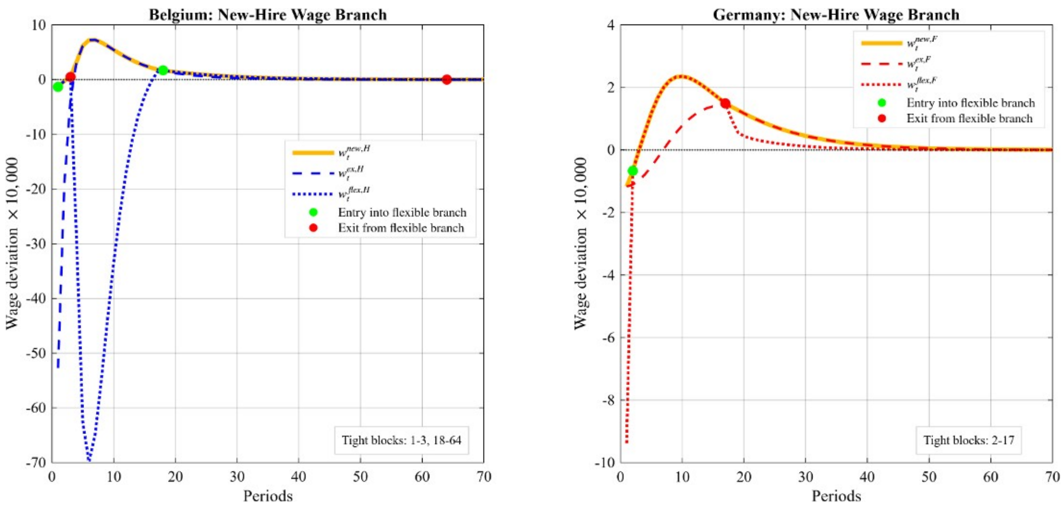


Figure 1: New-hire wage branch selection in the baseline simulation

Figure 1 shows that the same cost-push shock generates different wage-regime paths in Belgium and Germany. Belgium is in the tight branch for 50 periods over the 80-period diagnostic horizon, with tight blocks in periods 1–3 and 18–64; Germany is in the tight branch for 16 periods, with the main block in periods 2–17. The common shock thus activates the threshold in both economies, but the timing and persistence of activation differ. The divergence does not come from different shocks or different cost-push exposure: Belgium's indexation-style norm and Germany's staggered wage-stock adjustment imply different comparisons between $w_t^{flex,i}$ and $w_t^{ex,i}$, and therefore different paths for $w_t^{new,i}$. This is the first link in the mechanism; the corresponding threshold-distance diagnostic in Appendix C.5 (Figure 8, Table 5) confirms that the simulated regime path is mechanically consistent with the threshold condition.

6.3 Inflation and Marginal-Cost Transmission

The wage-regime result matters only if the selected new-hire wage reaches the price-setting block. As in the marginal-cost relation of Eq. (28), marginal cost is $mc_t^i = w_t^{new,i} - a_t^i$. Since productivity innovations are zero in the baseline experiment, the cross-country marginal-cost difference is driven by the selected new-hire wage and the associated labour-market response rather than by different productivity disturbances.

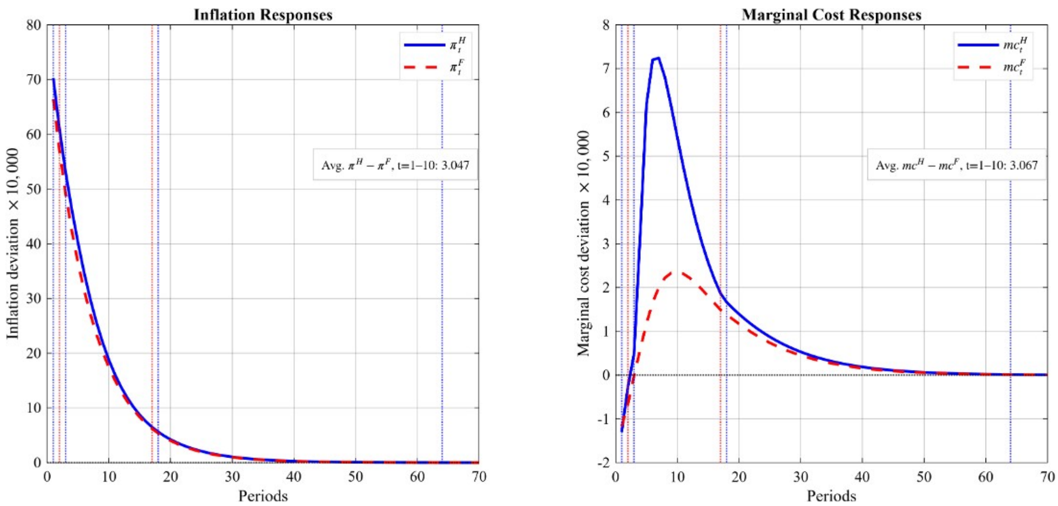


Figure 2: Inflation and marginal-cost transmission in the baseline simulation

Figure 2 shows that the wage-regime difference translates into a marginal-cost difference. Over periods 1–10 the average marginal-cost gap is $mc_t^H - mc_t^F = 3.07$ (model units $\times$10,000, as in the figures), and the inflation response follows the same ranking, with an average gap of $\pi_t^H - \pi_t^F = 3.05$; peak inflation is also higher in Belgium, at 70.3 against 66.4 in Germany. This is the second link in the mechanism: the common shock selects different new-hire wage paths, the selected wage moves marginal cost, and marginal cost feeds into inflation, so the price divergence is produced by the wage-regime channel rather than by country-specific inflation shocks. The same regime difference also moves real activity: output, employment, and net activity all fall more in Belgium than in Germany, and by considerably more than the price gaps, with the terms of trade and relative demand adjusting accordingly (Appendix C.7).

6.4 Country-Specific Policy Mismatch

The final step translates the inflation and activity responses into a monetary-policy diagnostic. Both countries face a single ECB rate, set from union-wide inflation and output as in Eqs. (35) and (36). To compare this common stance with country-level conditions, the simulation computes a country-specific shadow rate $i_t^{shadow,i} = \phi_\pi \pi_t^i + \phi_y Y_t^i$ and the policy-mismatch object $gap_t^{policy,i} = i_t^{shadow,i} - i_t^{ECB}$. The shadow rates are not alternative policy rules imposed on the model: they apply the same Taylor-rule coefficients as the ECB rule to country-specific inflation and output.

Figure 3 shows that the common ECB rate does not coincide with the country-specific shadow-rate benchmarks. Over periods 1–10 the average ECB rate is 25.44, against an average Belgian shadow rate of -13.40 and an average German shadow rate of 30.74, implying average policy gaps of $i_t^{shadow,H} - i_t^{ECB} = -38.84$ and $i_t^{shadow,F} - i_t^{ECB} = 5.30$ (all model units $\times$10,000); the corresponding summary table is reported in Appendix C.8 (Table 7). The signs of these gaps are the main policy result: the common rate is above the Belgian benchmark and slightly below the German one, so Belgium faces a stance that is too tight relative to its own diagnostic benchmark while Germany faces one that is slightly too loose. The result does not imply that the ECB should mechanically follow either country; it shows how one union-wide interest rate can map into different country-level policy positions when wage institutions transmit the same shock asymmetrically.

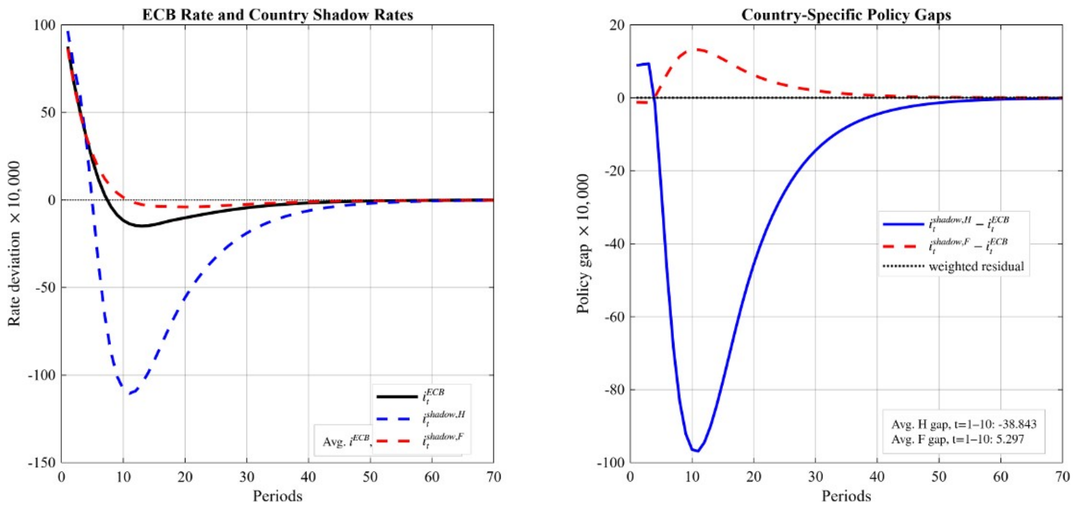


Figure 3: ECB policy mismatch under country-specific shadow-rate diagnostics

This policy mismatch is the end point of the mechanism rather than an additional assumption: one common shock interacting with different wage institutions is enough to make a common nominal instrument correspond to different country-level policy stances—a reduced-form computational illustration of the OCA argument.

7 Robustness Analysis

The baseline result should not depend on one exact calibration, so the mechanism is rechecked one parameter at a time around the baseline values. The question is not whether every numerical moment is fixed, since these parameters should change the size and timing of the responses, but whether the same qualitative chain survives: the common shock creates asymmetric wage-regime paths, Belgian inflation stays above German inflation in the initial window, and the policy gaps keep their opposite signs.

The checks support the mechanism across the screened ranges. Scaling the shock from 10^{-5} up to 0.002 changes only the magnitude of the responses, leaving the policy-gap sign pattern intact. Varying the Phillips-curve slope κ_p from 0.05 to 0.17 changes the size of inflation pass-through but not the transmission into policy mismatch, and varying shock persistence ρ_u from 0.60 to 0.95 changes the timing and magnitude of the adjustment without overturning the qualitative result. The institutional and trade parameters are equally reassuring: varying the German wage-updating parameter λ_w^F from 1/6 to 1/12 does not remove the policy-mismatch result, so the German wage block does not rely on a knife-edge updating value, and varying the trade elasticity ξ_{trade} from 0.75 to 1.50 changes the reduced terms-of-trade closure while leaving the wage, inflation, and policy-gap results essentially unchanged. This is what the model structure predicts, since the main result runs through wage-selected marginal cost and the common ECB rule rather than through the terms-of-trade closure.

The separation rate δ is the only mixed case. Moderate values from 0.030 to 0.100 solve and preserve the mechanism, but the extreme low-separation case ($\delta = 0.016$) fails numerically in Dynare/OccBin and is treated as a stress-test boundary rather than an economic counterexample. Taken together, the screen shows that the magnitudes are not invariant, since they move with the calibration in the expected directions, but the qualitative chain from asymmetric wage regimes to inflation divergence and opposite-signed policy gaps survives every economically reasonable perturbation. The full one-at-a-time screen, with the solve status and key moments for each individual case, is reported in Appendix C.9 (Table 8).

8 Data and Empirical Strategy

8.1 Data, measurement, and model mapping

The empirical section complements the simulation results by estimating reduced-form dynamic responses using local projections (Jordà, 2005). The objective is not to estimate the full structural Dynare/OccBin model, but to ask whether the main transmission chain it generates is visible in the data: a common cost-pressure shock is followed by country-specific inflation responses; under a common ECB nominal rate, inflation divergence implies different ex-post real-rate conditions; and wage/labour-market variables are used to assess whether the wage-pressure channel is empirically plausible.

The empirical analysis uses quarterly data for country H and country F, where H denotes Belgium and F denotes Germany. The common cost-pressure shock is measured using euro-area HICP energy inflation, while national inflation is measured using total HICP inflation for H and F. The ECB nominal policy rate provides the common monetary-policy instrument. Ex-post real rates are constructed as

$$r_t^i = i_t^{ECB} - \pi_t^i,$$

so that, under the common nominal rate, inflation differences translate directly into country-specific real-rate conditions:

$$r_t^H - r_t^F = (i_t^{ECB} - \pi_t^H) - (i_t^{ECB} - \pi_t^F) = -(\pi_t^H - \pi_t^F).$$

This identity links the inflation-response exercise to the policy-stance implication of the model.

The wage and labour-market variables provide reduced-form evidence on the transmission mechanism. LCI wage growth is used as an aggregate wage/labour-cost pressure proxy. This is not a direct measure of the model's new-hire wage, but it captures observed labour-cost adjustment. Labour-market tightness is proxied by the vacancy-unemployment ratio,

$$\theta_t^i = \frac{V_t^i}{U_t^i},$$

where V_t^i denotes vacancies and U_t^i denotes unemployed persons. Full details on variable definitions, sources, transformations, and estimation samples are reported in Appendix Table 9.

8.2 Local-projection design

For each variable, the local projection is estimated horizon by horizon:

$$y_{t+h}^i = \alpha_h^i + \beta_h^i shock_t^{common} + \gamma_h^i y_{t-1}^i + \delta_h^i shock_{t-1}^{common} + \varepsilon_{t+h}^i, \qquad h = 0, 1, \ldots, 12.$$

The coefficient β_h^i measures the response of variable y^i at horizon h to a one-standard-deviation common cost-pressure shock. Standard errors are HAC/Newey–West. The exercises are conducted at quarterly frequency, and effective samples differ across exercises because inflation, policy-rate, wage, vacancy, and unemployment series are not available over identical periods.

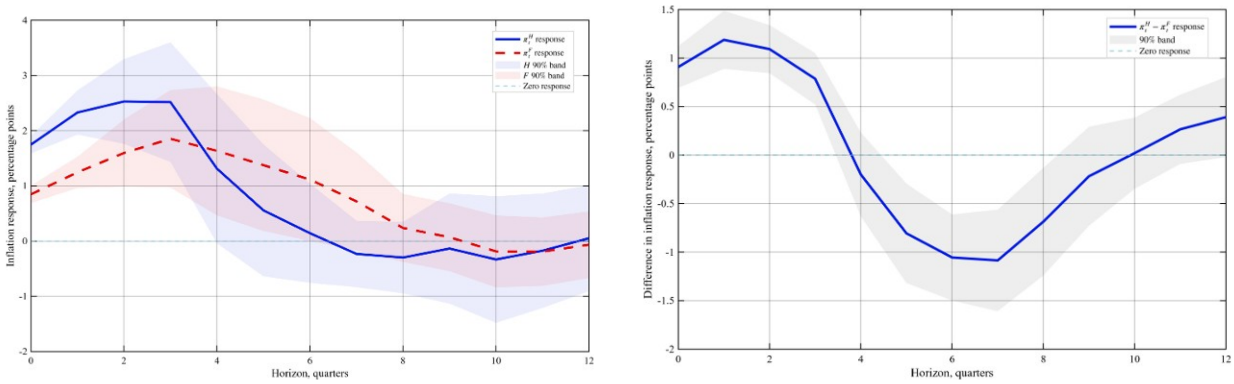
To evaluate cross-country asymmetry, the same local-projection structure is also estimated for difference variables such as $\pi_t^H - \pi_t^F$, $r_t^H - r_t^F$, and $w_t^{LCI,H} - w_t^{LCI,F}$, which test directly whether the response of H differs from that of F after the same common shock. The results are reported in the next section; they are read as external reduced-form support for the central mechanism rather than as a structural estimation of the model.

9 Reduced-Form Empirical Results

The reduced-form results follow the same transmission chain as the simulation results. The first empirical object is inflation: the data are used to check whether a common cost-pressure shock is followed by different national inflation paths. The second object is the ex-post real rate: under a common ECB nominal rate, inflation divergence should translate into different country-level real-rate conditions. The final object is the labour-market channel: tightness and LCI wage growth are used to assess whether the nonlinear wage-pressure mechanism has a visible empirical counterpart. Detailed data construction and coefficient tables are reported in Appendix D; the main text reports the figures and key estimates.

9.1 Inflation divergence after a common shock

The first empirical implication is that a common cost-pressure shock does not necessarily generate common national inflation responses. Figure 4 reports the country-level inflation responses and the corresponding response difference, $\pi_{t+h}^H - \pi_{t+h}^F$.



(a) Inflation responses **(b)** $H - F$ inflation response

Figure 4: Inflation responses to a common cost-pressure shock

Inflation in H rises more strongly than inflation in F in the first quarters after the common cost-pressure shock. The difference, $\pi^H - \pi^F$, is positive and statistically significant from horizon 0 to horizon 3. The initial difference is about 0.80 percentage points and reaches about 1.14 percentage points at horizon 1. After horizon 4, the difference changes sign: from horizons 5 to 8, inflation in F responds more strongly than inflation in H. The detailed estimates are reported in Appendix Table 10. Thus, the same common shock is followed by different national inflation paths, with a stronger short-run response in H and a later relative response in F.

9.2 Real-rate policy-stance implication

The real-rate response is the direct policy-stance implication of the inflation result. Since both countries face the same ECB nominal policy rate, the identity $r_t^H - r_t^F = -(\pi_t^H - \pi_t^F)$ implies that the real-rate gap is the exact mirror of the inflation gap. The response of

$r^H - r^F$ is therefore negative and statistically significant from horizon 0 to horizon 3, reaching approximately -1.14 percentage points at horizon 1: the ex-post real rate in H falls relative to F in the short run, because inflation in H rises more strongly under the common ECB nominal rate. The sign becomes positive and significant from horizon 5 to horizon 8, reflecting the later reversal in the inflation gap. The responses are plotted in Appendix Figure 11, with detailed estimates in Appendix Table 11. The result supports the policy-stance mechanism: a common nominal interest rate does not imply common real-rate conditions when national inflation responses differ.

9.3 ECB nominal-rate timing diagnostic

The ECB nominal-rate diagnostic checks whether the common nominal policy rate adjusts at the same time as the national inflation responses; it is used only as a timing diagnostic, not as a Taylor-rule estimation.

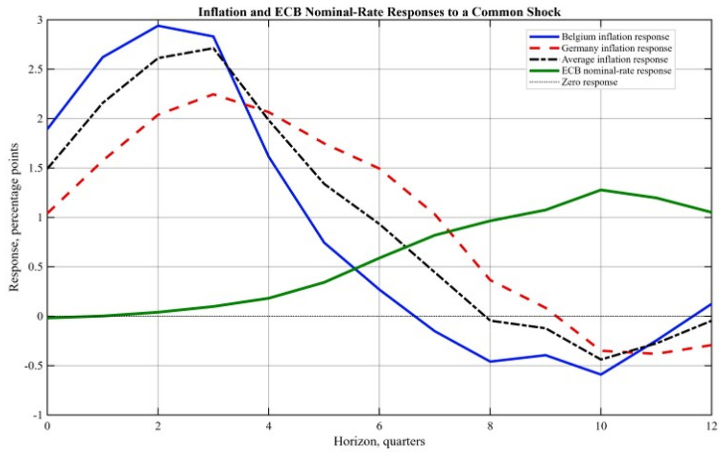


Figure 5: Inflation and ECB nominal-rate responses to a common shock. The nominal-rate response lags the national inflation responses, turning significant only around horizon 6.

The nominal-rate response is small and statistically insignificant over the first five horizons, becoming positive and significant only from around horizon 6 onward. This timing matters because the national inflation responses occur earlier than the common nominal-rate adjustment: the early inflation asymmetry is not immediately offset by monetary policy, which is what allows the short-run real-rate divergence documented above to emerge. The detailed estimates are reported in Appendix Table 12.

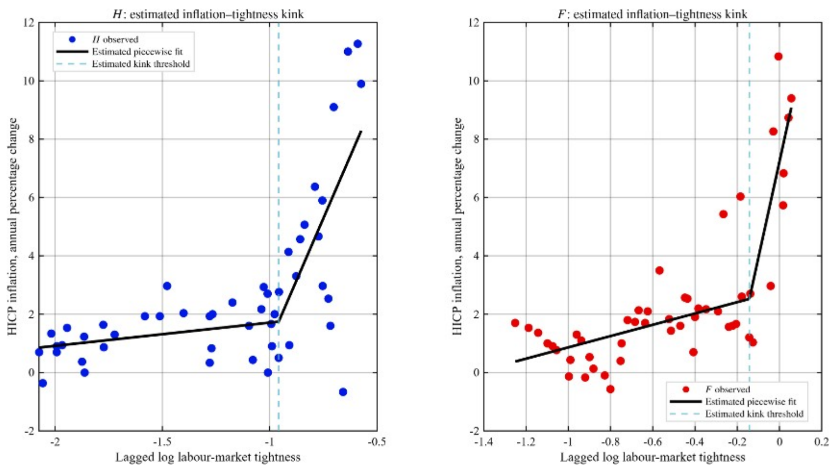
9.4 Wage, tightness, and nonlinear transmission

The final part of the empirical section examines whether wage and labour-market variables support the nonlinear transmission mechanism. The LCI wage-growth response is interpreted as an aggregate wage/labour-cost pressure proxy rather than a direct measure of the model's new-hire wage and it is weaker and more delayed than the inflation and real-rate responses: the $H - F$ wage-growth difference is insignificant in the first horizons but becomes positive and significant around horizons 5 and 6, with weaker significance at horizon 7. This is consistent with LCI wage growth being an aggregate labour-cost measure that adjusts more slowly than inflation or marginal wage pressure. The responses are plotted in Appendix Figure 12, with detailed estimates in Appendix Table 13.

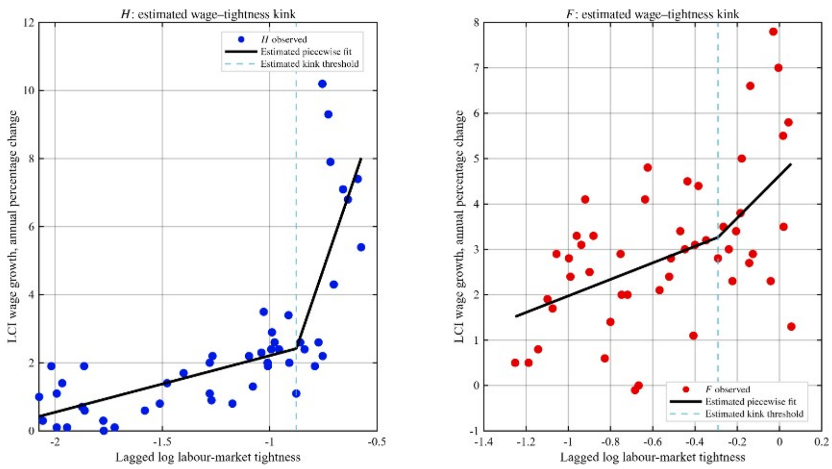
The nonlinear tightness mechanism is examined more directly by estimating the kink point in the relationship between inflation and lagged labour-market tightness. Since the structural threshold $\theta_t^{*,i}$ is not observed in the data, the empirical threshold is estimated as the value of lagged log tightness that minimizes the residual sum of squares in a piecewise specification; it is therefore not interpreted as the structural OccBin threshold, but as the observed tightness level at which the reduced-form inflation–tightness slope changes most sharply.

The estimated-threshold diagnostic in Figure 6a provides reduced-form evidence of a nonlinear inflation–tightness relationship. For H, the kink occurs around the 64th percentile of the lagged tightness distribution; for F, it occurs later, around the 80th percentile. In both countries the additional high-tightness slope is positive and statistically significant, supporting the Benigno-style mechanism that inflation becomes more sensitive to labour-market tightness once tightness crosses a country-specific threshold. The detailed estimates are reported in Appendix Table 14.

The wage-channel version of the same diagnostic estimates the kink point between LCI wage growth and lagged labour-market tightness.



(a) Inflation–tightness kink



(b) Wage–tightness kink

Figure 6: Estimated Benigno-style kinks: inflation–tightness (top) and wage–tightness (bottom), H (left) and F (right) within each panel.

The wage–tightness diagnostic in Figure 6b shows a sharper cross-country difference. For H, the estimated kink occurs around the 72nd percentile; the low-tightness slope is positive and significant, and the additional high-tightness slope is large and highly significant. For F, the low-tightness slope is also positive, but the additional high-tightness slope is not significant. The detailed estimates are reported in Appendix Table 15. Both countries therefore display a positive wage–tightness relationship, while the nonlinear wage/labour-cost response is much stronger in H—reduced-form support for the wage-regime channel in the model.

10 Conclusion

This study examines how heterogeneity in wage and labour-market transmission can turn a common shock into different country-level outcomes within a currency union. The starting point is a standard OCA problem: a common monetary authority sets one nominal rate while member economies differ in how shocks pass through wages, marginal costs, inflation, and real rates. The central argument is that this heterogeneity is not only a background friction to control for; it can itself be the mechanism through which a common disturbance becomes asymmetric.

The model develops this in a two-country currency union with occasionally binding wage-regime dynamics. A common cost-push shock hits both economies, but interacts with different wage-setting structures: an indexation-style response in Belgium and a more staggered wage-stock adjustment in Germany. The key mechanism is the selection of the new-hire wage branch: when flexible wage pressure exceeds the existing wage norm the economy moves into the tight branch, and this selection feeds marginal cost, then inflation, and finally, through the common ECB rate, country-specific real-rate and policy-stance outcomes. The Dynare/OccBin simulations confirm that the mechanism operates: Belgium spends more time in the tight branch than Germany, marginal cost and inflation are higher in Belgium over the initial window, and the common ECB rate ends up above the Belgian shadow-rate benchmark and slightly below the German one. The policy mismatch is thus the end point of the wage-and-inflation chain, not an imposed wedge.

The reduced-form evidence supports the same ordering. Local projections show that a common euro-area cost-pressure shock is followed by stronger short-run inflation in Belgium than Germany, which under the shared nominal rate maps directly into a short-run real-rate gap that later reverses; the ECB nominal rate responds with delay, leaving the early divergence unoffset. The labour-market evidence reinforces the mechanism: inflation and wage growth become more sensitive to tightness once it crosses a country-specific empirical threshold, and that nonlinearity is sharper in Belgium, consistent with the wage-regime channel in the model.

The contribution is to connect three objects usually treated separately: nonlinear Phillips-curve dynamics, wage-regime heterogeneity, and OCA policy mismatch. A common shock need not remain common in its consequences: once economies differ in how tightness affects wages and inflation, one nominal rate can produce different national inflation paths and different real policy stances. Natural extensions would connect the empirical tightness thresholds to a structural counterpart of $\theta_t^{*,i}$, widen the framework to a euro-area panel, and link the aggregate evidence to micro-level data on new-hire and renegotiated wages. The central point stands: in a currency union, heterogeneity is often not what complicates transmission but the reason transmission differs in the first place.

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A Theoretical Model Details

A.1 Household Block: Derivations and OCA Risk Sharing

This appendix derives the household Euler equation and the complete-markets risk-sharing condition used in Section 3.2. Following the complete-markets notation in Kekre, households trade one-period state-contingent nominal securities. The date-t price of a payoff delivered at date $t+1$ is denoted by $Q_{t,t+1}$. The reduced Dynare implementation does not carry the full asset-market allocation explicitly; it keeps the Euler-implied country-specific real-rate channel.

A.1.1 Lagrangian and first-order conditions

Let λ_t^i denote the multiplier on the nominal budget constraint. The household in country $i \in \{H, F\}$ chooses final consumption C_t^i and a portfolio of one-period state-contingent nominal securities B_{t+1}^i. The date-t budget constraint is

$$P_t^i C_t^i + \mathbb{E}_t \left[Q_{t,t+1} B_{t+1}^i \right] = W_t^i N_t^i + P_t^i b U_t^i + B_t^i + T_t^i.$$

The asset B_t^i is the payoff of securities purchased in the previous period. The benefit b is measured in real terms, so benefit income enters the nominal budget as $P_t^i b U_t^i$.

The household Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t^i)^{1+\varphi}}{1+\varphi} + \lambda_t^i \left(W_t^i N_t^i + P_t^i b U_t^i + B_t^i + T_t^i - P_t^i C_t^i - \mathbb{E}_t \left[Q_{t,t+1} B_{t+1}^i \right] \right) \Bigg].$$

The first-order condition with respect to consumption is

$$(C_t^i)^{-\sigma} = \lambda_t^i P_t^i,$$

so

$$\lambda_t^i = \frac{(C_t^i)^{-\sigma}}{P_t^i}.$$

For each date-$t+1$ state, the first-order condition with respect to state-contingent securities is

$$\lambda_t^i Q_{t,t+1} = \beta \lambda_{t+1}^i.$$

Substituting the consumption first-order condition gives the nominal stochastic discount factor:

$$Q_{t,t+1} = \beta \frac{(C_{t+1}^i)^{-\sigma}}{(C_t^i)^{-\sigma}} \frac{P_t^i}{P_{t+1}^i}.$$

Using

$$\Pi_{t+1}^i = \frac{P_{t+1}^i}{P_t^i},$$

this can be written as

$$Q_{t,t+1} = \beta \frac{(C_{t+1}^i)^{-\sigma}}{(C_t^i)^{-\sigma}} \frac{1}{\Pi_{t+1}^i}.$$

The price of a one-period risk-free nominal payoff is the state-contingent price integrated over next-period states:

$$\frac{1}{1+i_t^{ECB}} = \mathbb{E}_t[Q_{t,t+1}].$$

Therefore the risk-free Euler equation can be written as

$$1 = \beta(1+i_t^{ECB})\mathbb{E}_t\left[\frac{(C_{t+1}^i)^{-\sigma}}{(C_t^i)^{-\sigma}}\frac{1}{\Pi_{t+1}^i}\right].$$

Equivalently,

$$(C_t^i)^{-\sigma} = \beta(1+i_t^{ECB})\mathbb{E}_t\left[(C_{t+1}^i)^{-\sigma}\frac{1}{\Pi_{t+1}^i}\right].$$

A.1.2 Log-linearisation and the country-specific real rate

Let $\widehat{x}_t \equiv \ln x_t - \ln x^*$. The steady-state Euler condition is

$$1 = \beta\frac{1+i^*}{\Pi^*}.$$

Log-linearising the risk-free Euler equation around the steady state gives

$$-\sigma\widehat{C}_t^i = -\sigma\mathbb{E}_t[\widehat{C}_{t+1}^i] + i_t^{ECB} - \mathbb{E}_t[\pi_{t+1}^i].$$

Here i_t^{ECB} and π_t^i denote deviations from steady state in the linearised system. Rearranging,

$$\widehat{C}_t^i = \mathbb{E}_t[\widehat{C}_{t+1}^i] - \frac{1}{\sigma}\left(i_t^{ECB} - \mathbb{E}_t[\pi_{t+1}^i]\right).$$

Thus the country-specific real-rate component is

$$r_t^i \equiv i_t^{ECB} - \mathbb{E}_t[\pi_{t+1}^i].$$

A.1.3 Complete-markets risk sharing

Complete markets imply that the same state-contingent security price $Q_{t,t+1}$ prices payoffs for households in both countries. Using

$$\lambda_t^i Q_{t,t+1} = \beta\lambda_{t+1}^i,$$

for $i = H, F$, the ratio of marginal utility values of nominal wealth must be constant across dates and states:

$$\lambda_t^H = \Xi\lambda_t^F,$$

where Ξ is determined by initial wealth. Using

$$\lambda_t^i = \frac{(C_t^i)^{-\sigma}}{P_t^i},$$

the general risk-sharing condition is

$$\frac{(C_t^H)^{-\sigma}}{P_t^H} = \Xi\frac{(C_t^F)^{-\sigma}}{P_t^F}.$$

Under the symmetric equal-wealth normalisation, $\Xi = 1$. Under the common-basket no-home-bias benchmark, this reduces to

$$C_t^H = C_t^F = C_t.$$

This equality applies to the final utility-relevant consumption index; the composition of Belgian and German goods inside the basket remains governed by relative prices and the CES demand system.

A.2 Labour-Market Accounting and Search Timing

This appendix derives the steady-state labour-market accounting identities associated with the same-period search timing used in Section 3.3.

The effective search pool is

$$\widetilde{U}_t^i = U_{t-1}^i + \delta N_{t-1}^i.$$

With the labour force normalised to one, $U_{t-1}^i = 1 - N_{t-1}^i$, so

$$\widetilde{U}_t^i = 1 - (1-\delta)N_{t-1}^i.$$

Thus $\widetilde{U}_t^i$ is the within-period search pool, while $U_t^i = 1 - N_t^i$ is the end-of-period unemployment stock.

Let M_t^i denote new matches. Employment evolves as

$$N_t^i = (1-\delta)N_{t-1}^i + M_t^i.$$

Matches can be written either from the vacancy side or from the worker-search side:

$$M_t^i = q^i(\theta_t^i)V_t^i = f^i(\theta_t^i)\widetilde{U}_t^i.$$

At steady state,

$$\delta N^i = f^i \widetilde{U}^i, \qquad \widetilde{U}^i = 1 - (1-\delta)N^i.$$

Substituting the second equation into the first gives

$$\delta N^i = f^i \left[1 - (1-\delta)N^i\right].$$

Solving for employment,

$$N^i = \frac{f^i}{\delta + f^i(1-\delta)}.$$

Therefore,

$$U^i = 1 - N^i = \frac{\delta(1-f^i)}{\delta + f^i(1-\delta)}, \qquad \widetilde{U}^i = \frac{\delta}{\delta + f^i(1-\delta)}.$$

These expressions differ from the alternative timing convention in which newly separated workers search only in the next period, which would imply the familiar $U^i = \delta/(\delta + f^i)$. The model uses same-period search, so the relevant matching object is the effective search pool $\widetilde{U}^i$. These accounting identities are independent of the wage regime.

A.3 Wage Determination and Threshold Derivations

This appendix provides the derivational details behind Section 3.4. The main text reports the flexible wage, existing wage norms, new-hire wage rule, and threshold condition. This appendix collects the Nash-bargaining derivation, log-linear wage equations, complementary-slackness representation, and threshold derivatives.

A.3.1 Flexible wage from Nash bargaining

Let $\mathcal{W}_t^i$ and $\mathcal{U}_t^i$ denote the worker values of employment and unemployment, and let J_t^i denote the firm value of a filled job. Nash bargaining chooses the flexible wage to solve

$$\max_{w_t^{flex,i}} \left(\mathcal{W}_t^i - \mathcal{U}_t^i\right)^{\eta} \left(J_t^i\right)^{1-\eta},$$

where $\eta \in (0,1)$ is worker bargaining power. The sharing condition is

$$\eta J_t^i = (1-\eta)\left(\mathcal{W}_t^i - \mathcal{U}_t^i\right).$$

Under the standard DMP surplus-sharing logic, this gives

$$w_t^{flex,i} = (1-\eta)b + \eta\left(p_t^i A_t^i + k\theta_t^i\right).$$

The derivative with respect to tightness is

$$\frac{\partial w_t^{flex,i}}{\partial \theta_t^i} = \eta k > 0.$$

Around steady state, the log-linear flexible wage can be written as

$$\widehat{w}_t^{flex,i} = \frac{\eta p^i A^i}{w^{flex,i}}\left(\widehat{p}_t^i + \widehat{A}_t^i\right) + \frac{\eta k\theta^i}{w^{flex,i}}\widehat{\theta}_t^i.$$

The transmission specification used in the main text writes this as

$$\widehat{w}_t^{flex,i} = \varphi_a^i \widehat{A}_t^i + \varphi_\theta^i \widehat{\theta}_t^i + \psi_u^i u_t, \qquad \varphi_\theta^i > 0,$$

where

$$\varphi_a^i \equiv \frac{\eta p^i A^i}{w^{flex,i}}, \qquad \varphi_\theta^i \equiv \frac{\eta k\theta^i}{w^{flex,i}}.$$

The term $\psi_u^i u_t$ is a cost-pressure shifter.

A.3.2 Log-linear existing wage norms

For Belgium, nominal indexation implies

$$W_t^{ex,H} = W_{t-1}^{ex,H}\Pi_{t-1}^H.$$

Dividing by P_t^H gives

$$w_t^{ex,H} = w_{t-1}^{ex,H}\frac{\Pi_{t-1}^H}{\Pi_t^H}.$$

Therefore, in log-linear form,

$$\widehat{w}_t^{ex,H} = \widehat{w}_{t-1}^{ex,H} + \pi_{t-1}^H - \pi_t^H.$$

For Germany, the staggered wage stock is

$$w_t^{ex,F} = (1-\lambda_w^F)w_{t-1}^{ex,F} + \lambda_w^F w_t^{flex,F}.$$

Its log-linear counterpart is

$$\widehat{w}_t^{ex,F} = (1-\lambda_w^F)\widehat{w}_{t-1}^{ex,F} + \lambda_w^F \widehat{w}_t^{flex,F}.$$

A.3.3 Complementary-slackness representation

The max operator can be represented as an occasionally binding wage-floor constraint:

$$w_t^{new,i} - w_t^{ex,i} \geq 0, \qquad \mu_t^{w,i} \geq 0,$$

with complementary slackness

$$\mu_t^{w,i}\left(w_t^{new,i} - w_t^{ex,i}\right) = 0.$$

In the existing-wage branch,

$$w_t^{new,i} = w_t^{ex,i}, \qquad \mu_t^{w,i} \geq 0,$$

with the multiplier strictly positive away from the threshold. In the flexible-wage branch,

$$w_t^{flex,i} > w_t^{ex,i}, \qquad \mu_t^{w,i} = 0, \qquad w_t^{new,i} = w_t^{flex,i}.$$

Thus the max operator can be read as the equilibrium outcome of a wage-floor constraint.

A.3.4 Threshold derivatives

The threshold satisfies

$$w_t^{flex,i} = w_t^{ex,i}.$$

Using the flexible wage equation,

$$(1-\eta)b + \eta\left(p_t^i A_t^i + k\theta_t^{\star,i}\right) = w_t^{ex,i}.$$

Solving for $\theta_t^{\star,i}$ gives

$$\theta_t^{\star,i} = \frac{w_t^{ex,i} - (1-\eta)b - \eta p_t^i A_t^i}{\eta k}.$$

The comparative statics are

$$\frac{\partial \theta_t^{\star,i}}{\partial w_t^{ex,i}} = \frac{1}{\eta k} > 0, \qquad \frac{\partial \theta_t^{\star,i}}{\partial A_t^i} = -\frac{p_t^i}{k} < 0,$$

and

$$\frac{\partial \theta_t^{\star,i}}{\partial p_t^i} = -\frac{A_t^i}{k} < 0.$$

Thus a higher existing wage norm raises the threshold, while higher productivity or a higher real output price lowers it.

A.3.5 Wage-gap representation

Define

$$gap_t^i \equiv w_t^{flex,i} - w_t^{ex,i}.$$

Using the threshold definition,

$$w_t^{ex,i} = (1-\eta)b + \eta p_t^i A_t^i + \eta k \theta_t^{\star,i}.$$

Substituting into the gap gives

$$gap_t^i = \eta k\left(\theta_t^i - \theta_t^{\star,i}\right).$$

Therefore,

$$gap_t^i > 0 \quad \Longleftrightarrow \quad \theta_t^i > \theta_t^{\star,i}.$$

This is the analytical equivalence between the wage-gap condition and the tightness-threshold condition used in the main text.

A.4 Intermediate Firms: Job-Creation Derivations

This appendix derives the job-creation condition and marginal-cost expression used in Section 3.5. The main text reports the production technology, the vacancy-posting condition, and the marginal-cost equation.

A.4.1 Firm value and free entry

Let J_t^i denote the value to the firm of a filled job in country i. A filled job produces $p_t^i A_t^i$, pays the marginal new-hire wage $w_t^{new,i}$, and survives with probability $1-\delta$. Hence

$$J_t^i = p_t^i A_t^i - w_t^{new,i} + (1-\delta)\mathbb{E}_t\left[Q_{t,t+1}J_{t+1}^i\right].$$

The state-contingent discount factor $Q_{t,t+1}$ is defined in the household block:

$$Q_{t,t+1} = \beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{1}{\Pi_{t+1}^i}.$$

In log-linear form,

$$\widehat{Q}_{t,t+1} = -\sigma\left(\widehat{C}_{t+1}^i - \widehat{C}_t^i\right) - \pi_{t+1}^i.$$

The one-period risk-free nominal rate satisfies

$$\mathbb{E}_t[Q_{t,t+1}] = \frac{1}{1+i_t^{ECB}},$$

which yields the country-specific real-rate component

$$r_t^i = i_t^{ECB} - \mathbb{E}_t[\pi_{t+1}^i].$$

Free entry into vacancy posting implies that the expected value of a vacancy is zero. Posting a vacancy costs $kw_t^{new,i}$, and the vacancy is filled with probability $q^i(\theta_t^i)$. Therefore,

$$-kw_t^{new,i} + q^i(\theta_t^i)\mathbb{E}_t\left[Q_{t,t+1}J_{t+1}^i\right] = 0,$$

or

$$\frac{kw_t^{new,i}}{q^i(\theta_t^i)} = \mathbb{E}_t\left[Q_{t,t+1}J_{t+1}^i\right].$$

Using the filled-job value one period ahead gives the nonlinear job-creation condition:

$$\frac{kw_t^{new,i}}{q^i(\theta_t^i)} = \mathbb{E}_t\left[Q_{t,t+1}\left(p_{t+1}^i A_{t+1}^i - w_{t+1}^{new,i} + (1-\delta)\frac{kw_{t+1}^{new,i}}{q^i(\theta_{t+1}^i)}\right)\right].$$

A.4.2 Steady-state job creation

In steady state, the job-creation condition becomes

$$\frac{kw^*}{q^*} = \frac{1}{1+r^*}\left[p^*A^* - w^* + (1-\delta)\frac{kw^*}{q^*}\right].$$

Rearranging,

$$\frac{kw^*}{q^*}(r^*+\delta) = p^*A^* - w^*.$$

The steady-state flow surplus from a filled job equals the annuitised cost of creating the match.

A.4.3 Log-linear job creation

Since

$$q^i(\theta_t^i) = m_i(\theta_t^i)^{-\alpha},$$

the log-linear vacancy-filling rate is

$$\widehat{q}_t^i = -\alpha\widehat{\theta}_t^i.$$

Therefore, the log-linear movement in the expected cost of creating a filled job is

$$\widehat{w}_t^{new,i} - \widehat{q}_t^i = \widehat{w}_t^{new,i} + \alpha\widehat{\theta}_t^i.$$

Log-linearising the job-creation condition around the country-specific steady state gives

$$\begin{aligned}\frac{kw^i}{q^i}\left(\widehat{w}_t^{new,i} + \alpha\widehat{\theta}_t^i\right) = \frac{1}{1+r^i}\mathbb{E}_t\Bigg[&p^iA^i\left(\widehat{p}_{t+1}^i + \widehat{A}_{t+1}^i\right) - w^i\widehat{w}_{t+1}^{new,i} \\ &+ (1-\delta)\frac{kw^i}{q^i}\left(\widehat{w}_{t+1}^{new,i} + \alpha\widehat{\theta}_{t+1}^i\right)\Bigg] \\ &- \frac{kw^i}{q^i}\left(i_t^{ECB} - \mathbb{E}_t\pi_{t+1}^i\right).\end{aligned}$$

A.4.4 Marginal cost

With linear production,

$$Y_t^i = A_t^iN_t^i,$$

the marginal worker produces A_t^i units of output. Since the marginal worker is a new hire, the relevant wage cost is $w_t^{new,i}$. Level real marginal cost is therefore

$$MC_t^i = \frac{w_t^{new,i}}{A_t^i}.$$

The log-deviation marginal-cost object entering the Phillips curve is

$$mc_t^i = \widehat{w}_t^{new,i} - \widehat{A}_t^i.$$

A.5 Retail Pricing and the Calvo Phillips Curve

This appendix derives the variety-demand equation and the Calvo Phillips curve used in Section 3.6. The nonlinear wage-regime substitution into marginal cost is reported in the main text.

A.5.1 Demand for differentiated varieties

In country i, households consume a CES basket of varieties $j \in [0,1]$:

$$Y_t^i = \left[\int_0^1 y_t^i(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ is the elasticity of substitution across varieties. Cost minimisation for a given Y_t^i gives

$$y_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon} Y_t^i,$$

with aggregate price index

$$P_t^i = \left[\int_0^1 P_t^i(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}.$$

Under Calvo pricing, a fraction ι of firms keep last period's price, while a fraction $1-\iota$ reset their price to P_t^{*i}. The aggregate price index becomes

$$P_t^i = \left[\iota(P_{t-1}^i)^{1-\varepsilon} + (1-\iota)(P_t^{*i})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

A.5.2 Reset-price problem

A firm selected to reprice chooses P_t^{*i} to maximise expected discounted profits while the reset price remains in effect:

$$\max_{P_t^{*i}} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\iota)^\tau \frac{\lambda_{t+\tau}^i}{\lambda_t^i}\left[\frac{P_t^{*i}}{P_{t+\tau}^i} - MC_{t+\tau}^i\right] y_{t+\tau}^i(j).$$

Here $MC_{t+\tau}^i$ is the level of real marginal cost. Using demand, the reset-price first-order condition can be written as

$$P_t^{*i} = \frac{\varepsilon}{\varepsilon-1}\,\frac{\mathbb{E}_t \sum_{\tau=0}^{\infty}(\beta\iota)^\tau \frac{\lambda_{t+\tau}^i}{\lambda_t^i}(P_{t+\tau}^i)^\varepsilon Y_{t+\tau}^i MC_{t+\tau}^i}{\mathbb{E}_t \sum_{\tau=0}^{\infty}(\beta\iota)^\tau \frac{\lambda_{t+\tau}^i}{\lambda_t^i}(P_{t+\tau}^i)^\varepsilon Y_{t+\tau}^i}.$$

Thus the reset price is a markup, $\varepsilon/(\varepsilon-1)$, over a weighted average of current and expected future marginal costs.

A.5.3 Log-linear Phillips curve

At the zero-inflation steady state,

$$P_t^{*i} = P_t^i = P^*, \qquad \Pi^* = 1, \qquad MC^* = \frac{\varepsilon-1}{\varepsilon}.$$

Log-linearising the reset-price condition and the Calvo price index gives

$$\pi_t^i = \beta\,\mathbb{E}_t[\pi_{t+1}^i] + \kappa_p\, mc_t^i, \qquad \kappa_p = \frac{(1-\iota)(1-\beta\iota)}{\iota}.$$

Here π_t^i and mc_t^i denote log-deviation inflation and real marginal cost. The Calvo derivation is common across countries. State dependence enters through marginal cost, since the wage block determines which wage enters mc_t^i.

A.5.4 Note on Rotemberg equivalence

Benigno and Eggertsson (2023) use Rotemberg quadratic price adjustment costs. Around a zero-inflation steady state, Rotemberg and Calvo pricing both imply a forward-looking Phillips curve in which inflation depends on expected future inflation and real marginal cost. The study uses the Calvo formulation, while the Benigno-style nonlinearity enters through the wage-threshold mechanism that changes marginal cost across regimes.

A.6 Goods-Market Closure and Relative-Price Derivations

This appendix derives the bilateral goods demands and the terms-of-trade law of motion used in Section 3.7. The ECB rule, real-rate condition, shock process, and goods-market clearing equations are stated in the main text.

A.6.1 CES goods demand

The final consumption basket in country i is

$$C_t^i = \left[\gamma^{1/\varsigma}(c_{Ht}^i)^{\frac{\varsigma-1}{\varsigma}} + (1-\gamma)^{1/\varsigma}(c_{Ft}^i)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}.$$

The associated country-i price index is

$$P_t^i = \left[\gamma(P_{H,t})^{1-\varsigma} + (1-\gamma)(P_{F,t})^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}}.$$

Cost minimisation for a given C_t^i gives

$$c_{Ht}^i = \gamma\left(\frac{P_{H,t}}{P_t^i}\right)^{-\varsigma} C_t^i, \qquad c_{Ft}^i = (1-\gamma)\left(\frac{P_{F,t}}{P_t^i}\right)^{-\varsigma} C_t^i.$$

Under the Cole–Obstfeld benchmark, $\varsigma = 1$, the basket becomes

$$C_t^i = (c_{Ht}^i)^{\gamma}(c_{Ft}^i)^{1-\gamma},$$

and expenditure shares are constant:

$$P_{H,t}c_{Ht}^i = \gamma P_t^i C_t^i, \qquad P_{F,t}c_{Ft}^i = (1-\gamma)P_t^i C_t^i.$$

A.6.2 Terms of trade

The terms of trade is

$$S_t = \frac{P_{H,t}}{P_{F,t}}.$$

Using

$$\Pi_t^H = \frac{P_{H,t}}{P_{H,t-1}}, \qquad \Pi_t^F = \frac{P_{F,t}}{P_{F,t-1}},$$

the level law of motion is

$$S_t = S_{t-1}\frac{\Pi_t^H}{\Pi_t^F}.$$

In log deviations, with $tot_t \equiv \widehat{S}_t$, this gives

$$tot_t = tot_{t-1} + \pi_t^H - \pi_t^F.$$

Thus inflation differentials inside the currency union accumulate into relative-price movements.

B Steady-State Calculations and Linearisation Objects

The Dynare/OccBin implementation described in Section 4 is written in linear deviations from calibrated steady-state objects. The zero steady state reported by Dynare therefore does not mean that economic levels such as wages, employment, vacancies, tightness, output, or vacancy costs are zero. These nonzero objects are computed before solving the linear model and enter the implemented equations as parameters. This appendix records where the steady-state objects used in the Dynare equations come from, so that the coefficients used in Section 4 can be traced back to their steady-state calculations.

The steady-state nominal interest-rate object is computed from the discount factor:

$$r_{ss} = \frac{1}{\beta} - 1.$$

The steady-state relative price of intermediate output is set from the desired markup:

$$p_{ss} = \frac{\varepsilon - 1}{\varepsilon}.$$

In the implementation this object is also used as the steady-state productivity normalisation in the reduced job-creation block,

$$A_{ss} = p_{ss}, \qquad pa = p_{ss}.$$

For each country $i \in \{H, F\}$, the labour force is normalised to one. Given a country-specific unemployment target U^i_{ss}, steady-state employment is

$$n^i_{ss} = 1 - U^i_{ss}.$$

The model uses same-period search timing. Workers separated at the beginning of the period can search within the same period, so the effective search pool is

$$\widetilde{U}^i_{ss} = 1 - (1 - \delta) n^i_{ss}.$$

The steady-state tightness object is not chosen independently. It is solved from the reduced steady-state condition used in the code. For country i, with unemployment target U^i_{ss} and benefit parameter b^i, θ^i_{ss} solves

$$0 = \frac{A_{ss}}{1 + k(r_{ss} + \delta)\frac{U^i_{ss}(\theta^i_{ss})^{\alpha}}{\delta(1-U^i_{ss})}} - (1-\eta)b^i - \eta(A_{ss} + k\theta^i_{ss}).$$

This is the analytical counterpart of the 'fzero' equation used to compute θ^H_{ss} and θ^F_{ss}.

Given θ^i_{ss}, the matching-efficiency object implied by the steady-state unemployment target is

$$m^i_{ss} = \frac{\delta(1 - U^i_{ss})}{U^i_{ss}(\theta^i_{ss})^{1-\alpha}}.$$

The steady-state vacancy-filling and job-finding probabilities are

$$q^i_{ss} = m^i_{ss}(\theta^i_{ss})^{-\alpha}, \qquad f^i_{ss} = m^i_{ss}(\theta^i_{ss})^{1-\alpha}.$$

The steady-state wage used in the implemented linear system is

$$w^i_{ss} = \frac{A_{ss}}{1 + k(r_{ss} + \delta)\frac{(\theta^i_{ss})^{\alpha}}{m^i_{ss}}}.$$

This wage is the reference wage around which the flexible wage, existing wage, and new-hire wage deviations are written.

The expected steady-state cost of creating one filled job is

$$kw^i_q = \frac{kw^i_{ss}}{q^i_{ss}}.$$

This object appears in the reduced job-creation equation. In the Dynare implementation, the corresponding linear equation contains

$$kw_q^i(w_t^{new,i} + \alpha\theta_t^i)$$

on the left-hand side and the country-specific real-rate term

$$-kw_q^i(i_t^{ECB} - \pi_{t+1}^i)$$

on the right-hand side.

The employment law in the implemented model is

$$n_t^i = \lambda_n^i n_{t-1}^i + \lambda_\theta^i \theta_t^i.$$

The coefficients are computed from the steady-state matching objects:

$$\lambda_n^i = 1 - \delta,$$

and

$$\lambda_\theta^i = \frac{f_{ss}^i \widetilde{U}_{ss}^i}{n_{ss}^i}(1 - \alpha).$$

Thus the coefficient on tightness in the employment equation is not a free parameter; it is implied by the matching block and the calibrated labour-market steady state.

Steady-state vacancies are

$$V_{ss}^i = \theta_{ss}^i \widetilde{U}_{ss}^i.$$

The linearised vacancy equation follows from the tightness identity and the same-period search-pool definition. In the implementation it is written as

$$v_t^i = \theta_t^i - \frac{(1-\delta)n_{ss}^i}{1-(1-\delta)n_{ss}^i} n_{t-1}^i.$$

This is the source of the coefficient multiplying lagged employment in the Dynare vacancy equation.

Steady-state output is

$$Y_{ss}^i = A_{ss} n_{ss}^i.$$

The vacancy-cost share entering the reduced net-activity equation is

$$s_V^i = \frac{kw_{ss}^i V_{ss}^i}{Y_{ss}^i}.$$

The implemented net-activity object is therefore written as

$$x_t^i = Y_t^i - s_V^i(w_t^{new,i} + v_t^i),$$

where s_V^i is computed from the steady-state wage, vacancies, and output.

The flexible-wage pressure equation also uses steady-state coefficients. In the implemented model,

$$w_t^{flex,i} = A_i a_t^i + B_i \theta_t^i + \psi_u^i u_t,$$

where

$$A_i = \frac{\eta pa}{w_{ss}^i}, \qquad B_i = \frac{\eta k \theta_{ss}^i}{w_{ss}^i}.$$

The tightness threshold is then written in deviation form as

$$\theta_t^{\star,i} = \frac{w_t^{ex,i} - A_i a_t^i - \psi_u^i u_t}{B_i},$$

and the threshold-distance object is

$$\theta_t^{bind,i} = \theta_t^i - \theta_t^{\star,i}.$$

The OccBin constraints are written in tightness units using the same conversion factor B_i, so that the wage-gap tolerance is translated into a tightness-threshold tolerance.

Finally, all endogenous variables inside the `steady_state_model` block are set to zero. In grouped form, the implemented steady state is

$$\begin{aligned}
&a_t^i = u_t = w_t^{flex,i} = w_t^{ex,i} = gap_t^i = 0, \\
&\theta_t^{\star,i} = \theta_t^{bind,i} = w_t^{new,i} = mc_t^i = \pi_t^i = 0, \\
&\theta_t^i = n_t^i = v_t^i = Y_t^i = x_t^i = 0, \\
&tot_t = relD_t^{HF} = i_t^{ECB} = 0.
\end{aligned}$$

This is because the Dynare variables are deviations from the nonzero steady-state objects computed above. The zero steady state in Dynare is therefore a statement about the deviation system, not about the economic levels of wages, employment, tightness, vacancies, output, or vacancy costs.

C Computational Diagnostics and Additional Simulation Figures

This appendix reports the verification layer behind the reduced-form Dynare/OccBin mechanism exercise. The objects collected here support the simulation results in Sections 4–6: they document the calibration, the theory-to-implementation mapping, the Dynare solution diagnostics, the OccBin branch-selection checks, the reduced OCA closure, the policy-mismatch summary, and the robustness screen. The purpose is not to add separate results, but to show that the baseline mechanism is solved consistently and that the main transmission chain is numerically transparent.

C.1 Baseline Calibration

Table 1 reports the benchmark calibration used in the mechanism experiment, together with the source references that discipline each parameter.

Table 1: Baseline calibration and source references

Parameter	Value	Role	Source reference
A. Discounting, shocks, and price setting			
β	0.99	Quarterly discount factor	(Gomme and Rupert, 2007)
ρ_u	0.85	Cost-push shock persistence	(Moretti et al., 2019; Benigno and Eggertsson, 2023)
κ_p	0.10	Phillips-curve marginal-cost pass-through	(Calvo, 1983; Moretti et al., 2019; Benigno and Eggertsson, 2023)
λ_u^H, λ_u^F	1.00, 1.00	Symmetric direct cost-push exposure	(Benigno and Eggertsson, 2023; Moretti et al., 2019)
ψ_u^H, ψ_u^F	2.00, 2.00	Cost-push pass-through into flexible wage pressure	(Benigno and Eggertsson, 2023; Eurofound, 2024)
B. Monetary policy and union aggregation			
ϕ_π	1.50	ECB response to union-wide inflation	(Taylor, 1993)
ϕ_y	0.30	ECB response to union-wide activity	(Taylor, 1993)
ω	0.12	Belgium weight in the two-country ECB aggregate	(Kekre, 2022; European Central Bank, 2024)
C. Labour-market primitives			
δ	0.10	Quarterly separation rate	(Shimer, 2005; Kekre, 2022)
α	0.50	Matching elasticity	(Petrongolo and Pissarides, 2001)
η	0.50	Worker bargaining weight	(Hosios, 1990; Petrongolo and Pissarides, 2001)
k	4.00	Vacancy-cost scale	(Shimer, 2005; Kekre, 2022)
D. Wage institutions and reduced OCA closure			
ρ_w	0.50	Belgian existing-wage persistence	(Eurofound, 2024; Du Caju et al., 2008)
χ_H	0.75	Belgian indexation-style adjustment	(Eurofound, 2024; Du Caju et al., 2008)
λ_w^F	1/8	German staggered wage-stock adjustment	(Du Caju et al., 2008)
ξ_{trade}	1.00	Reduced terms-of-trade and relative-demand elasticity	(Cole and Obstfeld, 1991; Kekre, 2022)

Notes: The table reports the baseline calibration used in the mechanism experiment. Source references indicate the literature or institutional evidence used to discipline each parameter. Robustness checks around the baseline calibration are reported in Table 8.

C.2 Implementation Mechanism Map

Table 2: Model block map

Block	**Main object**	**Role in mechanism**	**Source / implementation**
Shock process	$u_t = \rho_u u_{t-1} + \varepsilon_t^u$. Code: `u = rho_u*u(-1) + eps_u;`	Common cost-push shock that starts the wage-pressure mechanism.	Reduced-form shock process.
Flexible wage pressure	$w_t^{flex,i} = A_i a_t^i + B_i \theta_t^i + \psi_u^i u_t$. Code: `w_flex_raw_i = A_i*a_i + B_i*theta_i + psi_u_i*u;`	Maps productivity, labour-market tightness, and cost-push pressure into the flexible wage.	Benigno-style wage-pressure object, adapted to two countries.
Tightness threshold	$\theta_t^{\star,i} = (w_t^{ex,i} - A_i a_t^i - \psi_u^i u_t)/B_i$, with $\theta_t^{bind,i} = \theta_t^i - \theta_t^{\star,i}$.	Defines the state-dependent threshold. The flexible branch is active when $\theta_t^i > \theta_t^{\star,i}$.	Derived from $w_t^{flex,i} > w_t^{ex,i}$. Not an additional calibrated threshold.
New-hire wage selection	$w_t^{new,i} = w_t^{ex,i}$ in slack and $w_t^{new,i} = w_t^{flex,i}$ in tight.	Creates the nonlinear wage mechanism. New-hire wages switch between the institutional wage norm and the flexible wage.	Tightness-threshold wage-selection mechanism.
Marginal cost	$mc_t^i = w_t^{new,i} - a_t^i$. Code: `mc_i = w_new_i - a_i;`	Transfers the wage-regime switch into firms' marginal costs.	Reduced-form marginal-cost channel.
Phillips curve	Slack: $\pi_t^i = \beta E_t \pi_{t+1}^i + \kappa_p (w_t^{ex,i} - a_t^i) + \lambda_u^i u_t$. Tight: $\pi_t^i = \beta E_t \pi_{t+1}^i + \kappa_p (w_t^{flex,i} - a_t^i) + \lambda_u^i u_t$.	Converts the wage-regime difference into country-specific inflation dynamics.	Benigno-style wage-kink NKPC, implemented in reduced form.
Job creation	$kw_q^i (w_t^{new,i} + \alpha \theta_t^i)$ equals discounted expected surplus minus the real-rate term $kw_q^i (i_t^{ECB} - \pi_{t+1}^i)$.	Links wages, expected real rates, and labour-market tightness.	Reduced DMP-style job-creation condition.
Labour-market accounting	$n_t^i = \lambda_i^n n_{t-1}^i + \lambda_i^\theta \theta_t^i$. Also, $v_t^i = \theta_t^i - \frac{\lambda_i^n n_i^{ss}}{1-\lambda_i^n n_i^{ss}} n_{t-1}^i$.	Maps tightness into employment and vacancies.	Reduced DMP accounting. Unemployment is implicit through employment.
Production and net activity	$Y_t^i = a_t^i + n_t^i$. $x_t^i = Y_t^i - s_i^V (w_t^{new,i} + v_t^i)$.	Creates the activity object used in the terms-of-trade equation.	Kekre-style reduced Dynare closure.
Terms of trade	$tot_t = tot_{t-1} + (\pi_t^H - \pi_t^F) + \xi_{trade}^{-1} [(x_t^H - x_{t-1}^H) - (x_t^F - x_{t-1}^F)]$.	Closes the two-country relative-price side without adding a full household asset-market block.	Kekre-style reduced terms-of-trade closure.
ECB rule	$i_t^{ECB} = \phi_\pi [\omega \pi_t^H + (1-\omega) \pi_t^F] + \phi_y [\omega Y_t^H + (1-\omega) Y_t^F]$.	A single common monetary policy responds to union-wide inflation and output.	Currency-union policy rule.
Policy mismatch	$i_t^{shadow,i} = \phi_\pi \pi_t^i + \phi_y Y_t^i$. Policy gap: $i_t^{shadow,i} - i_t^{ECB}$.	Diagnoses the common ECB rate relative to country-specific shadow-rate benchmarks.	Thesis diagnostic object.

Notes: The table maps each block of the reduced-form Dynare/OccBin implementation to its main equation, its role in the transmission mechanism, and its theoretical or computational source.

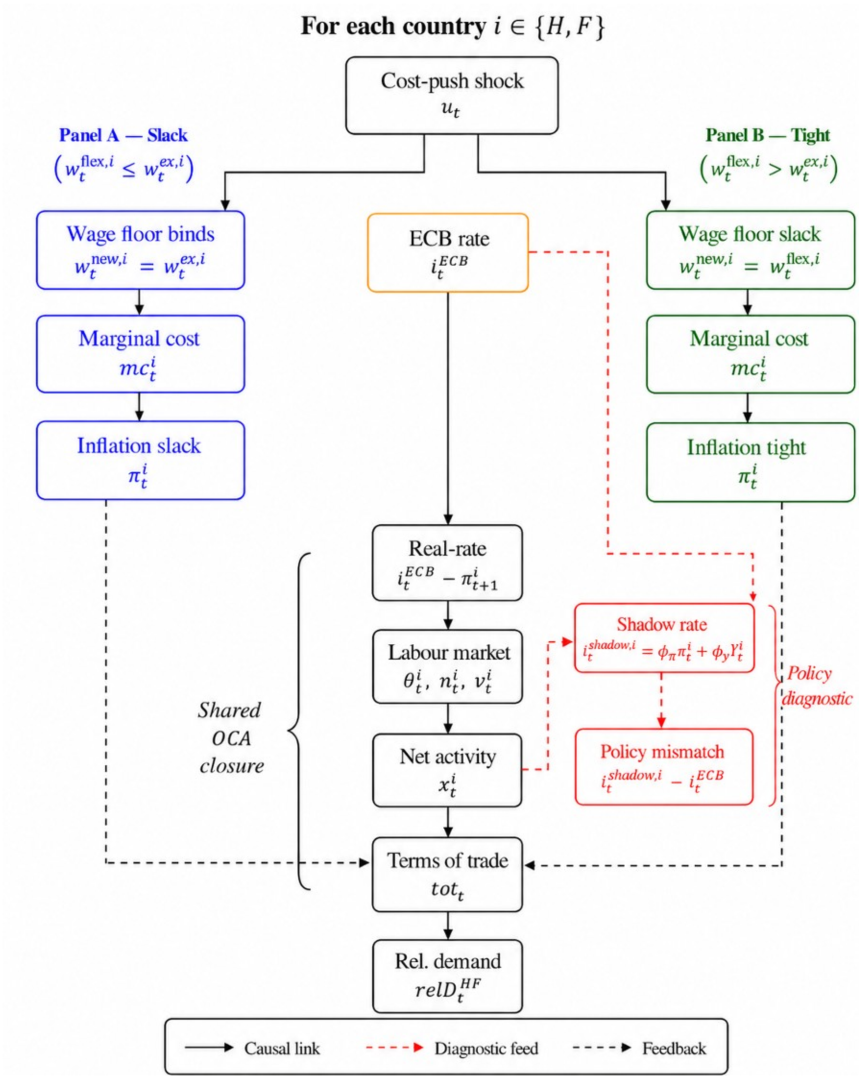


Figure 7: Schematic chronology of the reduced-form Dynare/OccBin implementation. The cost-push shock affects country-level wage pressure, marginal cost, inflation, and activity. The ECB rate is determined by union-wide inflation and output, while country-specific shadow rates are reported only as diagnostics. Solid arrows denote implemented transmission channels, red dashed arrows denote diagnostic calculations, and black dashed arrows denote reduced relative-price closure links.

C.3 Theory-to-Implementation Reduction

Table 3: Mapping from complete theory to reduced-form implementation

Complete theoretical object	**Reduced-form implementation**
Household Euler equation and asset-market allocation	Represented through the country-specific real-rate term $i_t^{ECB} - \pi_{t+1}^i$ in the reduced job-creation equation.
Firm value function J_t^i	Replaced by a reduced DMP-style job-creation equation using $w_t^{new,i}$, θ_t^i, productivity, continuation terms, and the country-specific real-rate term.
Full CES goods allocation	Summarised by reduced net activity x_t^i, terms of trade tot_t, and relative demand pressure $relD_t^{HF}$.
Full Benigno-style state-dependent $\kappa(\theta_t^i)$	Represented through the wage selected by the max operator. The implemented Phillips-curve pass-through κ_p is constant, while marginal cost changes by regime through $w_t^{new,i}$.

Notes: The table shows how the complete theoretical objects developed in Section 3 are represented in the reduced-form computational implementation.

C.4 Dynare Solution Diagnostics

Table 4: Dynare solution diagnostics

Diagnostic object	Value	Interpretation
Dynare version	7.0	The baseline reduced-form OCA model is solved using Dynare/OccBin.
Number of model equations	34	Dynare detects a square dynamic system with 34 model equations.
Steady-state values	All zero	This is expected because the implemented endogenous variables are written as deviations from the calibrated steady state.
Unstable eigenvalues	2	The number of eigenvalues larger than one in modulus equals the number of forward-looking variables.
Forward-looking variables	2	The dynamic system contains two forward-looking variables.
Blanchard–Kahn conditions	Verified	Dynare verifies both the order and rank conditions for the baseline linearized system.
Computing time	22 seconds	The baseline model solves without a Dynare/BK failure.

Notes: The zero steady state reflects the fact that the implemented variables are expressed as deviations from steady state. The Blanchard–Kahn diagnostics refer to the baseline linearized system before the OccBin regime path is simulated.

C.5 Threshold and Branch Diagnostics

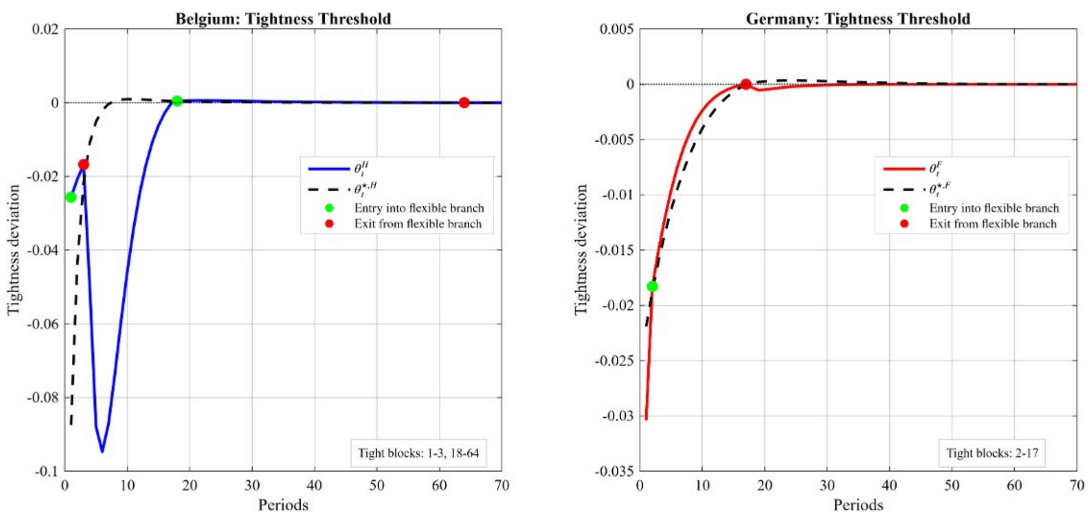


Figure 8: Tightness-threshold diagnostic for wage-regime activation

Table 5: Regime summary and threshold diagnostics

Country	Tight	Slack	Border	Tight blocks	Avg. $\theta - \theta^\star$	Avg. gap	Threshold resid.	Branch resid.
Belgium	50	14	16	1–3, 18–64	-0.039287	-0.003266	6.51e−18	3.90e−18
Germany	16	47	17	2–17	0.000563	0.000054	3.52e−19	4.47e−19

Notes: Averages are computed over periods 1–10. The threshold residual checks $gap_t^i = B_i(\theta_t^i - \theta_t^{\star,i})$. The branch residual checks that the selected new-hire wage is consistent with the OccBin regime. Borderline periods are close to the threshold tolerance and are not interpreted as separate economic regimes.

C.6 Benigno-Style Kink Illustration

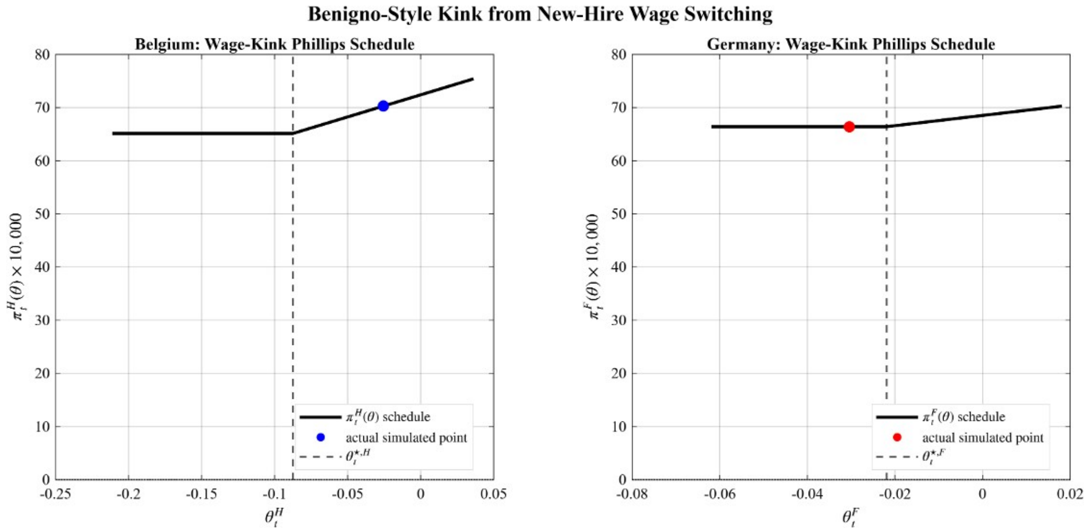


Figure 9: Benigno-style kink interpretation generated by new-hire wage switching, where the horizontal axis: Labour-market tightness deviation, θ_t^i, and the vertical axis: Inflation deviation, $\pi^i(\theta) \times 10{,}000$

C.7 Mechanism Moments and Reduced OCA Closure

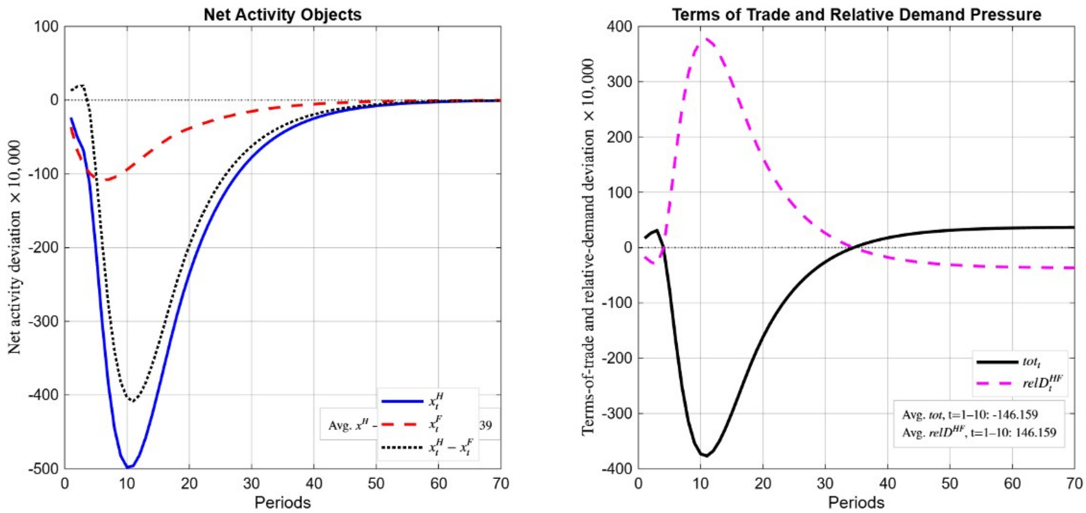


Figure 10: Reduced terms-of-trade and relative-demand closure

Table 6: Mechanism moments, periods 1–10

Object	**Mean** (×10,000)
$w_t^{new,H} - w_t^{new,F}$	3.067
$mc_t^H - mc_t^F$	3.067
$\pi_t^H - \pi_t^F$	3.047
$\theta_t^H - \theta_t^F$	-453.69
$n_t^H - n_t^F$	-162.37
$Y_t^H - Y_t^F$	-162.37
$x_t^H - x_t^F$	-165.44
tot_t	-146.16
relD_t^{HF}	146.16
i_t^{ECB}	25.440

Notes: Values are averages over periods 1–10 of the baseline simulation, scaled by 10,000.

C.8 Policy-Mismatch Summary

Table 7: Policy mismatch summary, periods 1–10

Object	Mean ×10,000
ECB rate, i_t^{ECB}	25.440
Belgium shadow rate, $i_t^{shadow,H}$	-13.400
Germany shadow rate, $i_t^{shadow,F}$	30.740
Belgium policy gap, $i_t^{shadow,H} - i_t^{ECB}$	-38.840
Germany policy gap, $i_t^{shadow,F} - i_t^{ECB}$	5.300

Notes: Values are averages over periods 1–10, scaled by 10,000. Shadow rates are computed using the same Taylor-rule coefficients as the ECB rule, but using country-specific inflation and output. A negative policy gap means the common ECB rate is above the country-specific shadow rate. A positive policy gap means the common ECB rate is below the country-specific shadow rate.

C.9 Full Robustness Screen

Table 8: One-at-a-time robustness screen

Case	**Screen**	**Value**	**Status**	**Tight H**	**Tight F**	**Avg.** $\pi_H - \pi_F$	**Avg.** $Y_H - Y_F$	**Policy gap H**	**Policy gap F**
1	Shock size, ε_u	1×10^{-5}	Pass	3	9	3.0009×10^{-6}	-0.000163	-0.000039	0.000005
2	Shock size, ε_u	2×10^{-5}	Pass	3	12	6.0514×10^{-6}	-0.000325	-0.000078	0.000011
3	Shock size, ε_u	5×10^{-5}	Pass	17	15	1.5193×10^{-5}	-0.000812	-0.000194	0.000027
4	Shock size, ε_u	0.0001	Pass	26	15	3.0428×10^{-5}	-0.001624	-0.000389	0.000053
5	Shock size, ε_u	0.0002	Pass	34	16	6.0897×10^{-5}	-0.003248	-0.000777	0.000106
6	Shock size, ε_u	0.0005	Pass	43	16	0.000152	-0.008119	-0.001942	0.000265
7	Shock size, ε_u	0.0010	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530
8	Shock size, ε_u	0.0020	Pass	56	16	0.000609	-0.032473	-0.007769	0.001059
9	Phillips slope, κ_p	0.05	Pass	51	16	0.000135	-0.016238	-0.004109	0.000560
10	Phillips slope, κ_p	0.086	Pass	50	16	0.000253	-0.016260	-0.003959	0.000540
11	Phillips slope, κ_p	0.10	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530
12	Phillips slope, κ_p	0.17	Pass	46	16	0.000620	-0.015791	-0.003351	0.000457
13	Shock persistence, ρ_u	0.60	Pass	42	10	0.000073	-0.009217	-0.002337	0.000319
14	Shock persistence, ρ_u	0.75	Pass	47	13	0.000148	-0.012352	-0.003066	0.000418
15	Shock persistence, ρ_u	0.85	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530
16	Shock persistence, ρ_u	0.95	Pass	4	26	0.001287	-0.016421	-0.002637	0.000360
17	German wage updating, λ_w^F	1/6	Pass	47	14	0.000305	-0.016510	-0.003955	0.000539
18	German wage updating, λ_w^F	1/8	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530
19	German wage updating, λ_w^F	1/12	Pass	54	18	0.000303	-0.015826	-0.003778	0.000515
20	Trade elasticity, ξ_{trade}	0.75	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530
21	Trade elasticity, ξ_{trade}	1.00	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530
22	Trade elasticity, ξ_{trade}	1.50	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530
23	Separation rate, δ	0.016	Dynare fail	–	–	–	–	–	–
24	Separation rate, δ	0.030	Pass	69	29	0.000553	-0.004479	-0.000453	0.000062
25	Separation rate, δ	0.050	Pass	69	23	0.000391	-0.007873	-0.001563	0.000213
26	Separation rate, δ	0.075	Pass	65	19	0.000329	-0.011972	-0.002727	0.000372
27	Separation rate, δ	0.100	Pass	50	16	0.000305	-0.016237	-0.003884	0.000530

Notes: The table reports one-at-a-time robustness checks around the baseline calibration. The averages are computed over periods 1–10. The policy gap is $i_t^{shadow,i} - i_t^{ECB}$. Case 7 is the baseline cost-push experiment. Case 23 is retained as a stress-test failure and is not used as an economic robustness result. The tight-country columns report the number of tight-regime periods over the 80-period diagnostic horizon. Inflation, output and policy-gap averages are reported in model units and are computed over periods 1–10.

D Empirical Appendix

This appendix reports the data construction details and detailed coefficient tables corresponding to the reduced-form empirical results in Section 9. The main text reports the figures and key estimates, while the appendix collects the variable definitions, sources, transformations, estimation samples, and full horizon-by-horizon estimates. The purpose is to make the empirical exercise transparent without interrupting the main transmission narrative.

D.1 Empirical Data Sources and Sample Construction

Table 9: Data sources, variable construction, and estimation samples

Empirical exercise	**Model object**	**Empirical measure and source**	**Transformation / use**	**Effective sample**	**Obs.**
Inflation response to common shock	Common shock; π_t^H, π_t^F	Euro-area HICP energy inflation and national HICP total inflation for H and F. Source: ECB Data Portal, HICP.	Monthly annual rates converted to quarterly averages. Common shock standardized before local projections.	1997Q1–2026Q2	118
Country-specific shock robustness	Country-specific shocks; π_t^H, π_t^F	HICP energy inflation for H and F, plus national HICP total inflation. Source: ECB Data Portal, HICP.	Monthly annual rates converted to quarterly averages. Country-specific shocks standardized separately. Appendix robustness only.	1997Q1–2026Q2	118
Real-rate response	i_t^{ECB}, π_t^i, r_t^i	ECB main refinancing operations fixed rate and national HICP inflation. Source: ECB Data Portal.	ECB daily date-of-change series converted to quarterly averages. Real rate constructed as $r_t^i = i_t^{ECB} - \pi_t^i$.	1999Q1–2025Q2	106
ECB nominal-rate timing diagnostic	Common shock; i_t^{ECB}; π_t^i	Euro-area HICP energy inflation, ECB main refinancing rate, and national HICP inflation. Source: ECB Data Portal.	Reduced-form timing diagnostic. Not interpreted as a Taylor-rule estimation.	2001Q1–2025Q2	98
LCI wage response	$w_t^{LCI,H}, w_t^{LCI,F}$	Labour Cost Index, wages and salaries. Source: Eurostat.	Quarterly annual rate of change. Used as aggregate wage/labour-cost pressure proxy.	2001Q1–2025Q2	98
Labour-market tightness diagnostics	$V_t^i, U_t^i, \theta_t^i, w_t^{LCI,i}$	Job vacancy statistics, unemployment, and LCI wages. Source: Eurostat.	Tightness constructed as $\theta_t^i = V_t^i/U_t^i$. Used for wage–tightness and piecewise kink diagnostics.	2013Q1–2025Q2	50

Notes: Country H denotes Belgium and country F denotes Germany. All empirical exercises are conducted at quarterly frequency. HICP variables are originally monthly and are converted to quarterly averages. The ECB nominal policy-rate series is converted from a daily date-of-change series to quarterly averages. The real-rate exercise begins in 1999Q1, corresponding to the start of the common ECB monetary-policy regime. The LCI wage series is interpreted as an aggregate wage/labour-cost pressure proxy rather than as a direct measure of new-hire wages. Labour-market tightness is constructed as a vacancy-unemployment ratio.

D.2 Inflation and real-rate responses

Table 10: Inflation-response asymmetry after a common cost-pressure shock

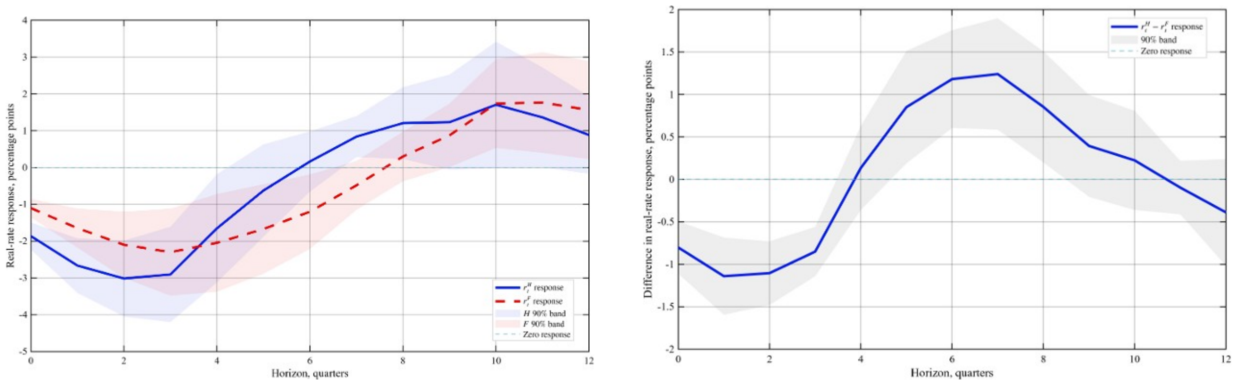
Horizon (quarters)	$\pi^H_{t+h} - \pi^F_{t+h}$ (pp)	HAC s.e.	t-statistic	p-value
0	0.802	0.188	4.270	0.000***
1	1.140	0.276	4.129	0.000***
2	1.104	0.228	4.838	0.000***
3	0.851	0.176	4.826	0.000***
4	-0.135	0.301	-0.448	0.654
5	-0.851	0.402	-2.114	0.034**
6	-1.181	0.350	-3.375	0.001***
7	-1.241	0.398	-3.115	0.002***
8	-0.854	0.398	-2.148	0.032**
9	-0.394	0.365	-1.081	0.280
10	-0.224	0.355	-0.631	0.528
11	0.096	0.191	0.504	0.614
12	0.388	0.382	1.018	0.309

Notes: The table reports local-projection estimates of the difference between the inflation response in country H and country F after a one-standard-deviation common cost-pressure shock. Country H denotes Belgium and country F denotes Germany. Standard errors are HAC/Newey–West. The sample contains 106 quarterly observations after removing missing values. Significance levels are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.10$.

Table 11: Real-rate response asymmetry after a common cost-pressure shock

Horizon (quarters)	$r^H_{t+h} - r^F_{t+h}$ (pp)	HAC s.e.	t-statistic	p-value
0	-0.802	0.188	-4.270	0.000***
1	-1.140	0.276	-4.129	0.000***
2	-1.104	0.228	-4.838	0.000***
3	-0.851	0.176	-4.826	0.000***
4	0.135	0.301	0.448	0.654
5	0.851	0.402	2.114	0.034**
6	1.181	0.350	3.375	0.001***
7	1.241	0.398	3.115	0.002***
8	0.854	0.398	2.148	0.032**
9	0.394	0.365	1.081	0.280
10	0.224	0.355	0.631	0.528
11	-0.096	0.191	-0.504	0.614
12	-0.388	0.382	-1.018	0.309

Notes: The table reports local-projection estimates of the difference between the ex-post real-rate response in country H and country F after a one-standard-deviation common cost-pressure shock. The ex-post real rate is defined as $r^i_t = i^{ECB}_t - \pi^i_t$. Country H denotes Belgium and country F denotes Germany. Standard errors are HAC/Newey–West. The sample contains 106 quarterly observations after removing missing values. The common shock has mean 4.289 and standard deviation 9.571. Average real rates over the estimation sample are -0.286 for H and 0.081 for F. Significance levels are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.10$.



(a) Real-rate responses **(b)** $H - F$ real-rate response

Figure 11: Ex-post real-rate responses to a common cost-pressure shock. By the identity $r^H_t - r^F_t = -(\pi^H_t - \pi^F_t)$, panel (b) is the mirror image of the inflation gap in Figure 4.

D.3 ECB nominal-rate timing diagnostic

Table 12: ECB nominal-rate response after a common cost-pressure shock

Horizon (quarters)	i_{t+h}^{ECB} response (pp)	HAC s.e.	t-statistic	p-value
0	-0.018	0.103	-0.172	0.864
1	0.002	0.236	0.009	0.993
2	0.040	0.293	0.137	0.891
3	0.098	0.290	0.340	0.734
4	0.182	0.289	0.629	0.529
5	0.342	0.313	1.095	0.274
6	0.589	0.355	1.661	0.097*
7	0.820	0.398	2.060	0.039**
8	0.965	0.432	2.236	0.025**
9	1.075	0.457	2.353	0.019**
10	1.278	0.585	2.183	0.029**
11	1.198	0.584	2.049	0.040**
12	1.051	0.555	1.894	0.058*

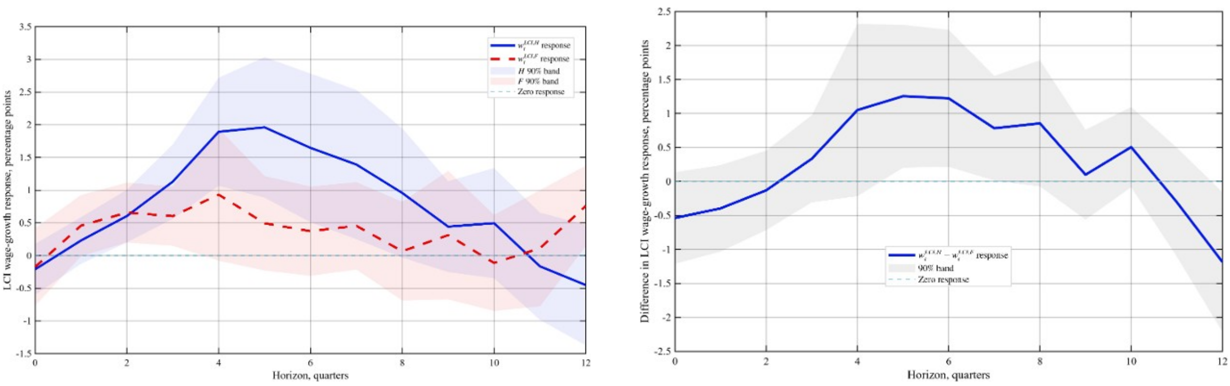
Notes: The table reports local-projection estimates of the ECB nominal-rate response after a one-standard-deviation common cost-pressure shock. The exercise is a reduced-form policy-response diagnostic and is not interpreted as an estimated Taylor rule. Standard errors are HAC/Newey–West. The sample contains 98 quarterly observations after removing missing values. The average ECB nominal rate over the estimation sample is 1.993. Average inflation is 2.416 for H and 2.091 for F. Significance levels are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.10$.

D.4 Wage and labour-market diagnostics

Table 13: LCI wage-growth response asymmetry after a common cost-pressure shock

Horizon (quarters)	$w_{t+h}^{LCI,H} - w_{t+h}^{LCI,F}$ (pp)	HAC s.e.	t-statistic	p-value
0	-0.537	0.409	-1.313	0.189
1	-0.395	0.386	-1.023	0.307
2	-0.130	0.355	-0.366	0.715
3	0.335	0.390	0.861	0.389
4	1.051	0.770	1.365	0.172
5	1.255	0.638	1.966	0.049**
6	1.224	0.613	1.997	0.046**
7	0.783	0.468	1.674	0.094*
8	0.855	0.564	1.517	0.129
9	0.100	0.401	0.248	0.804
10	0.506	0.359	1.411	0.158
11	-0.303	0.487	-0.623	0.533
12	-1.184	0.624	-1.896	0.058*

Notes: The table reports local-projection estimates of the difference between the LCI wage-growth response in country H and country F after a one-standard-deviation common cost-pressure shock. Country H denotes Belgium and country F denotes Germany. The LCI wage series are year-on-year percentage changes and are interpreted as aggregate wage/labour-cost pressure proxies rather than direct measures of new-hire wages. Standard errors are HAC/Newey–West. The sample contains 98 quarterly observations after removing missing values. The common shock has mean 4.007 and standard deviation 9.745. Average LCI wage growth over the estimation sample is 2.706 for H and 2.613 for F. Significance levels are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.10$.



(a) LCI wage-growth responses

(b) $H - F$ LCI wage response

Figure 12: LCI wage-growth responses to a common cost-pressure shock

Table 14: Estimated Benigno-style inflation–tightness kink

Statistic	H	F
Estimated threshold, $\widehat{c}^i$	-0.958	-0.143
Threshold percentile	64th	80th
Low-tightness slope, $\beta_{1,i}$	0.797	1.937**
HAC s.e.	(0.606)	(0.934)
Extra high-tightness slope, $\beta_{2,i}$	16.219**	31.095***
HAC s.e.	(6.943)	(5.525)
Implied high-tightness slope, $\beta_{1,i} + \beta_{2,i}$	17.016	33.032
Observations	49	49

Notes: The table reports an estimated-threshold inflation–tightness diagnostic. The estimated equation is

$$\pi_t^i = \alpha_i + \beta_{1,i} \log(\theta_{t-1}^i) + \beta_{2,i} \max\{0, \log(\theta_{t-1}^i) - \widehat{c}^i\} + \varepsilon_t^i,$$

where $\widehat{c}^i$ is the empirical kink threshold selected by grid search to minimize the residual sum of squares. Country H denotes Belgium and country F denotes Germany. The estimated threshold is expressed in lagged log labour-market tightness. The threshold percentile reports the location of the estimated threshold in each country's lagged tightness distribution. Standard errors are HAC/Newey–West and are reported in parentheses. The estimated threshold is not interpreted as the structural model threshold $\theta_t^{*,i}$, but as the observed tightness level at which the reduced-form inflation–tightness slope changes most sharply. The dependent variable is annual HICP inflation in percentage points. The reported slopes therefore measure the percentage-point change in annual inflation associated with a one-unit increase in lagged log tightness. Significance levels are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.10$.

Table 15: Estimated wage–tightness kink

Statistic	H	F
Estimated threshold, $\widehat{c}^i$	-0.876	-0.291
Threshold percentile	72nd	66th
Low-tightness slope, $\beta_{1,i}$	1.663***	1.822**
HAC s.e.	(0.540)	(0.889)
Extra high-tightness slope, $\beta_{2,i}$	16.842***	2.857
HAC s.e.	(3.706)	(2.598)
Implied high-tightness slope, $\beta_{1,i} + \beta_{2,i}$	18.505	4.678
Observations	49	49

Notes: The table reports an estimated-threshold wage–tightness diagnostic. The estimated equation is

$$w_t^{LCI,i} = \alpha_i + \beta_{1,i} \log(\theta_{t-1}^i) + \beta_{2,i} \max\{0, \log(\theta_{t-1}^i) - \widehat{c}^i\} + \varepsilon_t^i,$$

where $\widehat{c}^i$ is the empirical kink threshold selected by grid search to minimize the residual sum of squares. Country H denotes Belgium and country F denotes Germany. The estimated threshold is expressed in lagged log labour-market tightness. The threshold percentile reports the location of the estimated threshold in each country's lagged tightness distribution. Standard errors are HAC/Newey–West and are reported in parentheses. The estimated threshold is not interpreted as the structural model threshold $\theta_t^{*,i}$, but as the observed tightness level at which the reduced-form wage–tightness slope changes most sharply. The dependent variable is annual LCI wage growth in percentage points. The reported slopes therefore measure the percentage-point change in annual wage growth associated with a one-unit increase in lagged log tightness. Significance levels are denoted by $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.10$.