ASSESSING THE CONSEQUENCES OF LOW RATES ENVIRONMENT ON ALM AND NON-MATURING LIABILITIES MODELLING

Thesis presented by
Antonio Liviello

Supervisor(s)
François Ducuroir (UCL)

Reader(s)
Yuliya Rychalovska (UNamur)

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Ecole d'économie de Louvain/UCL • Place Montesquieu 3 • 1348 Louvain-la-Neuve
Département des Sciences économiques/UNamur • Rempart de la Vierge 8 • 5000 Namur

ABSTRACT

For a decade, the ALM is facing to a low interest rate environment. This prolonged period affects negatively the profitability of a bank which induces it to increase its risk-taking. That led the regulators to change the interest rate risk in the banking book (IRRBB) framework. They increased the complexity of the interest rate risk measurement to get results in line with the current low rate environment. In this sense, the bank needs to compute more complex stress scenarios and a more complete way to describe its interest rate risk. There is another challenge for the ALM such as the interest rate risk measurement for the non-maturing liabilities. With this low interest rate environment, the volume of the non-maturing accounts is unstable. Indeed, the customer would withdraw his money because of a highly liquid environment and poor return on the accounts. Moreover, there is a strong competition in the supply of these non-maturing accounts. For the measurement, a replicating portfolio can be computed statically or dynamically. However, in a rising rate environment, the static replicating portfolio is not appropriate anymore. New comers could benefit from higher rates since this approach is backward-looking and uses historical observation with a moving average. This is why the bank must switch to stochastic modelling such the dynamic replicating portfolio or OAS models which are forward-looking. This resolves two problems: the interest rate risk assessment and the hedge against the new comers' threat in a rising rate environment.

ACKNOWLEDGMENT

First, I would like to thank to my Supervisor François Ducuroir. The risk management was a new world for me and it was not really easy to step in. Hopefully, I received all the support I needed to make this thesis. I would like also to thanks Yuliya Rychalovska to accept reading my work. Second, I would like to give thanks to the Economics School of Louvain for all the courses I sat. That gave me the opportunity to understand the risk management and to fulfill my passion for financial markets. Finally, I would like to thank my family and especially my girlfriend for the support during the writing time.

Lack of self-confidence and inspiration are really strong barriers between you and the end of your thesis. No matter what, my Supervisor and my family gave me back these powers to finish it.

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Glossary

ALCO	Asset and Liability Committee
ALM	Asset and Liability Management
BCBS	Basel Committee on Banking Supervision
CaR	Capital-at-Risk
CIR	Cox-Ingersoll-Ross
EaR	Earnings-at-Risk
EBA	European Banking Authority
ECB	European Central Bank
EVE	Economic Value of Equity
ICAAP	Internal Capital Adequacy Assessment Process
IRR	Interest Rate Risk
IRRBB	Interest Rate Risk Banking Book
IRF	Interest Rate Futures
IRS	Interest Rate Swaps
LMM	LIBOR Market Model
MRO	Main Refinancing Operations
NII	Net Interest Income
NIM	Net Interest Margin
NMD	Non-Maturing Deposits
OAS	Option Adjusted Spread
OLS	Ordinary Least Squares
RSA	Rate Sensitive Assets
RSL	Rate Sensitive Liabilities
SV DD LMM	Stochastic Volatility Displaced Diffusion LIBOR Market Model

Introduction

The Asset and Liability Management (ALM) represents "*any continuous management process that defines, implements, monitors and back tests financial strategies to jointly manage a firm's assets and liabilities*"[1]. In other terms, the ALM aims to reach a financial purpose for any level of risk in the current bank constraints. Due to the increasing importance of regulations and the highly complex models and technicalities that banks have to face to, ALM is essential for efficient financial strategies and for effective risk management.

One of the missions of the ALM's department is essentially to manage interest rate risks and, in some cases[2], liquidity risks in order to avoid any mismatch between the cash flows of the assets and those of the liabilities. By interest rate risk, we mean "*the current or prospective risk to both the earnings and capital of institutions arising from adverse movements in interest rates"*[3]. When interest rates change, the present value of future cash flows change too. This in turn will affect the underlying value of a bank's assets, liabilities and off-balance sheet items and thus its economic value. Movements in interest rates also affect a bank's earnings by altering interest rate-sensitive income and expenses, affecting in turn its net interest income. For instance, let us have a bank loan to a customer for three years at a fixed rate with a funding by means of a deposit re-priced monthly. If the interest rates rise, the bank's funding cost will increase at the next monthly reset but the rate on the loan will remain the same for the full three years, which will reduce the bank's profit.

In this work, we will focus on the low interest rate environment and the difficulties that this entails by three axes. The first one is the impact of the low interest rate environment on the bank value. We will see that this environment provokes negative effects on its profitability and increases its risk-taking. We will also show that a prolonged period of low interest rate environment increases the duration gap between assets and liabilities. To remediate to this problem, banks can either change the composition of its balance sheet or recur to interest rates derivatives. The second one relates to the change in the regulation. By having a low interest rate environment, the Basel Committee on Banking Supervision (BCBS) updated the rules of the Interest Rate Risk in the Banking Book (IRRBB) in April 2016. That represents more complex methods such for instance the inclusion of a dynamic approach for the net interest margin projections and sensitivity analysis. In this sense, banks must increase the granularity at the deal level in their simulations. In the new IRRBB framework, at least six scenarios must be defined for the possible changes in the interest rate curve. The third one is the interest rate risk measurement on the non-maturing liabilities. The interest rate risk on the non-maturing liabilities comes mainly from the problem in measuring the duration of non-maturing deposits (NMD). Indeed, the account has an embedded option which allows the customer to withdraw at any time his money. In consequence, the bank needs to estimate how much of its current account base is likely to be stable and how long. Nowadays, the future volume of the deposits is more difficult to predict than before. The low interest rate environment provokes smaller remuneration on deposits than before. If the interest rate environment is rising, new comers or competitors can propose higher rates and steal some customers. The low interest rate environment is a serious problem for bank since Millennials are ready to switch banks if the reward is higher.

In the second part, we will assess the different techniques to measure the duration for the interest rate risk on the non-maturing liabilities. For these techniques, we will mention replicating portfolios and the OAS models. Since the static replicating portfolio is backward-looking approach, it is not appropriate anymore. If the interest rates rise, the new comers will benefit from them and will propose higher rates. There also exists the forward-looking

[1] Marine Corlosquet-Habart (2015)
[2] Some banks will rather have a Treasury department in charge of Liquidity Management which consists of a different team from the ALM department which then primarily focuses on managing Interest Rates Risks.
[3] European Banking Authority's definition

approach with the stochastic modelling either by the dynamic replicating portfolio or the OAS models. We will see that in the low interest rate environment the forward-looking approach is more appropriate to measure the duration for the non-maturing liabilities.

Part I: Main challenges for the ALM in the low interest rate environment

Challenge n°1: Lower interest rates

1. Effect on bank profitability and risk-taking

The net interest income has historically been fundamental for banks. It is a key driver for commercial banks profitability and is thus used as a main performance metric by them. The *net interest income* (NII) corresponds to the difference between the interest income earned on the loans and the interest expenses coming from the paying liabilities. Since the interest rates are key drivers of NII, which are typically related to the absolute level of interest rates, low rates have an impact on banks' profitability. Then, to evaluate the cost and the efficiency of financial intermediation, the NII is divided by the total bank's earning asset which gives the *net interest margin* (NIM).

"Low-for-long" describes the period of low interest rates in which the economy plunged since the financial crisis. These low levels are in fact linked with unconventional monetary policy[4]. Even if banks can benefit from low interest rates, *e.g.* as non-performing loans will be lower because borrowers' debt will less burden, they can depress bank margins for many types of deposits. Indeed, banks are reluctant (or may be prohibited) to lower deposit rates especially below zero. When interest rates decline, banks margins compress, *e.g. floating rate loans will be offered against lower rates*. Since the current low interest rates environment is accompanied by a flattening of the yield curve, that also depresses the NIM of those banks which used to maintain a positive duration gap[5]. According to Claessens and al (2016), evidence for the United States finds that banks are adversely affected by interest rates that are low for an extended period through a narrower NIM. Then they showed that a decrease in the short-term interest rate lowers NIMs in both a low of high rate environment.
Borio and al. (2015) investigated the influence of monetary policy on bank profitability. They affirmed that both the level of interest rates and the slope of the yield curve are associated with higher net interest income. This relationship is likely to be especially strong at very low levels of nominal interest rates. Alessandri and Nelson (2015) confirmed the opinion by finding evidence of a systematic effect of market interest rates on bank profitability. In response to higher interest rates, bank raise their lending rates and reduce their lending volume, potentially by strengthening their lending standards. That means from the finding that the level of interest rates and yield curve slope affect the net interest margin and trading income in opposite directions. Finally, Bikker and Vervliet (2017) studied the effect of low interest rates on bank profitability. They found that bank performance is indeed impaired as a consequence of low interest rates. The ability of banks to generate profits from their traditional lending and funding practices is reduced as the net interest margin compressed by persistently low interest rates. In consequence, a "low-for-long" period provokes a negative impact for bank's profitability.

In case of risk-taking, Altunbas et al. (2010) find that an unusually low interest rate environment over an extended period contributes to an increase in bank risk-taking. Presumably, banks are inclined to assume greater risk mainly via two channels. First, they may generate more income from non-interest activities, by raising fee and trading income, as formulated by Rajan (2005). Weistroffer (2013) affirmed that there is an evidence of change in business by looking to the prime Japanese example of a long-lasting ultra-low interest rate environment. Banks indeed tried to change their business models and developed new sources of income to maintain profitability. The search for yield and increased risk appetite can also be explained by the finding of Manganelli & Wolswijk (2009). They found that lower interest rates may reflect less risk aversion.
The low interest rate environment can affect the risk exposure in loan portfolios in two contrasting ways. On the one hand, low interest rates might reduce the default probability on outstanding loans. On the other hand, banks might soften their lending standards lowering the loan portfolio quality, which in a rising rate environment could lead to higher credit losses.

[4] See the "interest rate theory" in the annex n°1
[5] See section dedicated to "Duration & Convexity" in the interest rate theory.

Dell'Ariccia and Marquez (2006) confirm this idea by finding that low interest rates reduce adverse selection problems and thereby may decrease bank screening, increasing the probability of granting loans to more risky debtors.

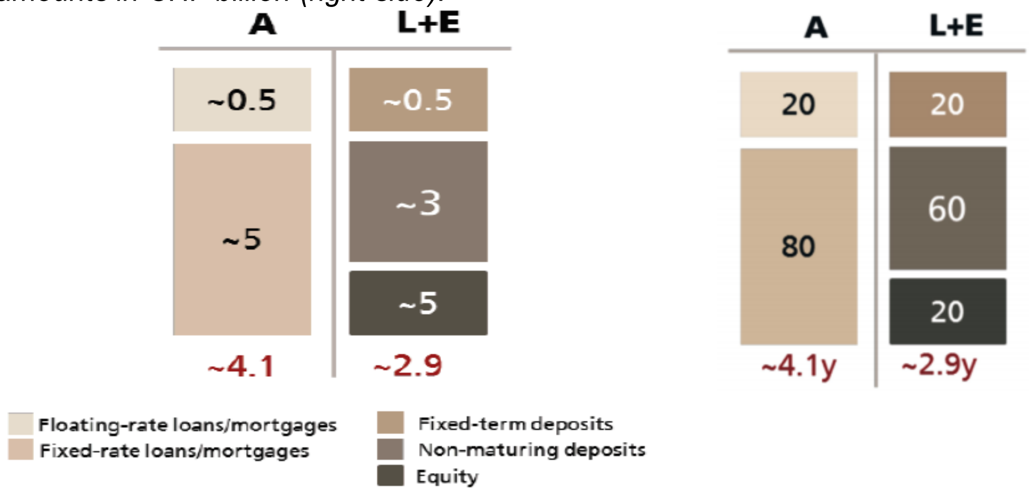
2. Effects on balance sheet and use of derivatives

2.1. Impact on balance sheet

To describe concretely what are the effects on balance sheet, an example inspired from Claude Moser and al (2015) is used. Even if the illustrative example does not represent an actual balance sheet, it must show how a bank's balance sheet changes when a low interest rate environment persists. By changes, we mean the changes in duration and hence in interest rate risk.

First, let us have a typical bank which holds two types of mortgages: floating-rate and fixed-rate mortgages. On the liability side, the bank holds equity and two types of deposits: CHF 20 billion of fixed-term deposits and CHF 60 billion of non-maturing deposits. On the asset side, let us have CHF 20 billion of floating-rate mortgages and CHF 80 billion of fixed-rate loans. For the interest rates sensitivity of the products, floating-rate mortgages and fixed-term deposits have the same duration, *i.e.* 6 months. Then, non-maturing deposits have a duration equal to 3 years. Finally, fixed-rate mortgages and equity have the biggest duration in the example as 5 years. By computing the weighted duration, the asset side is more sensible to interest rate changes than the liability side with a duration of 4.1 years and 2.9 years respectively. The example is illustrated in Figure 1.

Figure 1.– Typical banking balance sheet's example with duration in years (left side) and amounts in CHF billion (right side).



Source: Illustrative example prepared by UBS.

Then, because of a prolonged period of low interest rate environment, the customers have the incentive to switch from fixed-term to non-maturing deposits. Indeed, the term deposits are no more profitable at low rates and the clients would rather get their excess cash at any moments. In this way, the customers will wait until the future interest rates' rise before going back to term deposits and getting higher returns. Moreover, the customers require using more liquidity than before. This is because the world became highly liquid due to the unconventional monetary policy.

In reality, the current low interest rate environment gives the incentive to also contract other loans such the revolving loans. The revolving loan is an agreement which allows the borrower to draw down and repays a loan as often he need within an overall agreed loan limit and during an agreed term, without early repayment fees or reapplying for finance[6]. When arrives the repayment, the borrower will repay the amount with interest. If the interest rates surge, the payments will increase and in turn will increase the amount to repay. Interest income derived from revolving loans such credit card and overdraft[7] are sensitive to changes in market interest rates. If the cards have variable rate, then the decreasing market rates will affect the rate that the banks is charging on credit card and overdraft. When interest rates are low, the fees paid on these loans are low. That leads to a change in behavior from customers because the fees are lowered on overdrafts and credit card. However, there is a limit in the contraction of overdraft since unpaid amount of interest will be added to the existing unpaid amount and will rise the interest paid on the next overdraft. If the interest rates rise, the fees paid on overdraft will surge which increase the risk that a borrower cannot pay the amount. Failure to pay the interest charges on a regular basis can lead to a fall in credit score and increases interest rates for existing and future borrowing.

On the asset side, the persistent low interest rate environment might cause a higher demand for floating-rate mortgages than for fixed-rate mortgages because customers expect that the interest rates are going to remain at low levels or even to decrease but not to rise. Consequently, the monthly repayments are lower than if they contract a fixed-rate loan.

The switch from fixed to floating rates mortgages is not the only change. The low interest rates can raise another problem. In Belgium, the bank can face to a *prepayment risk*. A customer has the opportunity to reimburse or to prepay the credit during the contractual life by two ways: either by finding a new loan with a lower interest rate and to prepay the existing loan, or to renegotiate the existing loan. If the market rates go down, the bank will agree to reduce the loan interest rate in order to keep the customer away from the willingness to change of bank. Sometimes banks can ask these customers to pay penalties against a reduction of rates. As the customer repay earlier, the bank is faced to this prepayment risk[8]. This action modifies the credit schedule and is potentially costly for the ALM. First, the IRR will provoke a cost on monthly payments since they will be lower after the rates change.

There is another risk that arises from mortgages or other loans such the *pipeline risk*. This defines "*all the financial risk occurring from the moment a financial product is commercialized to the moment the product is booked and hedged*"[9]. One source of the pipeline risk can be the loan offers. If the interest rates go up, the customer will accept the credit proposed at the beginning of the decision period with an interest rate lower than the current one. On the contrary, if the interest rates go down, the customer may have the idea to switch to another bank to benefit a lower rate for the repayment.

Consequently, on the asset side, this increased demand for floating-rate mortgages makes the fixed-rate mortgages more sensible to interest rates changes and increases in turn their duration from 5 years up to 8 years[10].

On the liability side, the switch of deposits products by the customers makes a smaller increase in average duration but remains around the 3 years. Nevertheless, the low rates have

[6] Revolving credit facility definition from Barclays. Some revolving loans are without specified maturity.

[7] An overdraft facility is a short-term standby credit facility which allows you to write cheques or withdraw cash from your current account up to the overdraft limit (renewable on yearly basis). It is repayable on demand by the bank at any time. The overdraft limit is the maximum amount that you can overdraw. The interest is paid only on the amount overdrawn. If the amount of interest is unpaid, then it will be added on the overdrawn amount in the following month. The interest rate on an overdraft account is usually charge at a percentage over the bank's prime lending rate, e.g. prime + 3% per annum.

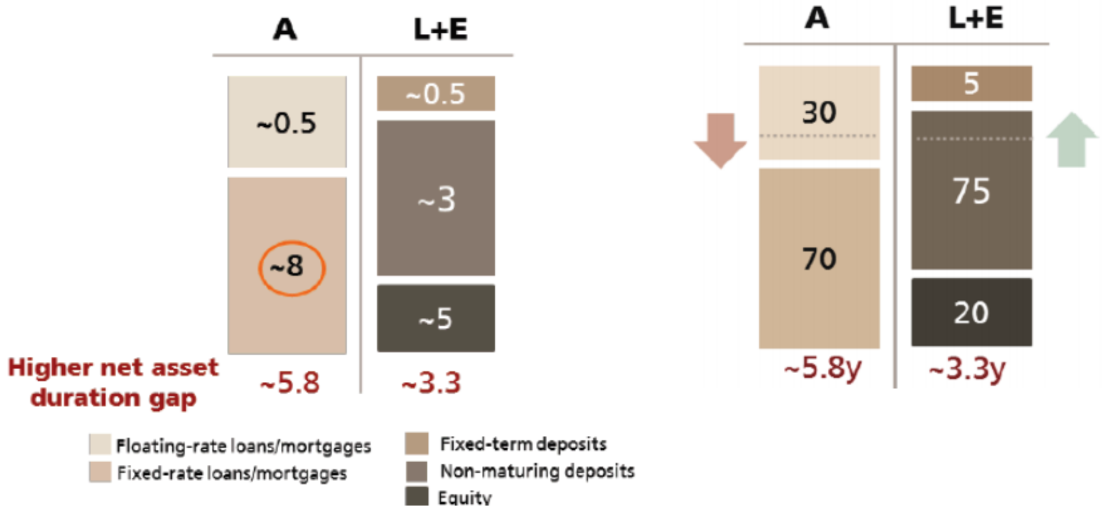
[8] In Belgium, the prepayment penalties are low and nearly fixed. In Netherlands, the prepayment penalty is equal to the difference in present value of the agreed interest rate flows and the market rate flows.

[9] A. ADAM (2007)

[10] These 3 years are only illustrative. That does not mean that a switch between the assets in persistent low interest rates' periods will lead to a duration's surge of 3 years.

induced a higher interest rate duration imbalance between assets and liabilities which is illustrated in Figure 2.

Figure 2. – Impact of the low interest rate environment's persistency on duration (left side) and balance sheet value (right side)

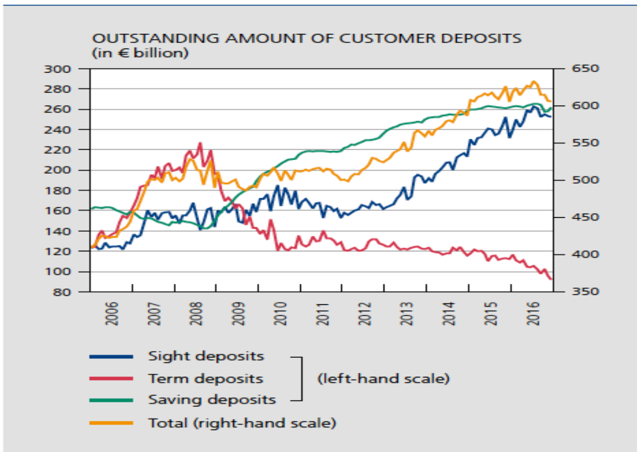


Source: Illustrative example prepared by UBS.

By having a bigger imbalance in duration between assets and liabilities, that leads the bank to rely more on external markets to hedge against interest rate risk via interest rate derivatives[11]. The low interest rate environment changes the composition of the liability side since there is a switch between term deposits and non-maturing deposits.

Figure 3 shows that the persistence of the low interest rate environment provokes effectively a decrease in the demand for term deposits and an increase in demand for non-maturing deposits such sight and saving deposits.

Figure 3. – Different types of deposits for Belgian customers



Source: NBB.
(1) Data from the monthly MIR survey in the case of new deposits. Deposits for a term of

[11] See the section 2.2. "Hedging with interest rates' derivatives

2.2. Hedging with interest rates' derivatives[12]

The risk can be controlled using a variety of techniques that can be classified into two kinds of methods. The first method is to restructure the balance sheet on changing contractual characteristics of assets and liabilities to achieve a particular duration or maturity gap. As it was said earlier, retail banks cannot charge negative rates on the deposits for regulatory reasons[13], which disconnect it economically from the external market. On one hand, the larger is this difference, less the external market is useful for managing its balance sheet mismatch between asset and liability duration[14]. On the other hand, that contradicts what it was said on the external market's reliance to hedge against interest rate risk. If it does not react, the difference existing between the actual negative market rate and the positive deposit rate paid to customers will induce losses and compress the NII. The impact depends on the model of the bank and its market power.

First, banks with strong market power can mitigate these effects to a certain extent by charging higher fees and applying very low/zero interest on deposits[15]. However, this mitigation technique is limited since a too big increase influences customers to withdraw their money to invest it in another bank with lower fees. This will be not the case in practice since customers do not want to change their habits just on the increasing costs' reason. Bank with lower market power can try to shift interest rate risk to their customers in the form of variable rate loans and variable rate deposits. But as for non-maturing deposits, the floating-rate mortgages' contract does not permit negative rates for contractual features. If the interest rates become negative, the bank will pay the monthly payments to the clients and will not be profitable. Hence, the floating-rate mortgages cannot be negative for economic reason.
There is a second method which relies on the use of interest rate derivatives. The advantage of derivatives over more traditional methods of ALM is that they can transform the duration of the balance sheet. On the contrary, Treasury securities have zero risk weight in the risk-based capital requirements, their presence on the balance sheet still increases the required "leverage ratio" or the ratio of capital to total assets. We will see below some interest rates' derivatives such the interest rate swaps, the interest rates futures and the caps, floors and collars.

Interest rate swap (IRS)

First, there is an interest rate swap for hedging against interest rate risk. The interest rate swap (IRS) represents a contractual agreement to exchange fixed payments against floating coupon payments between two counterparties for a specified period and based on a notional amount. IRS basically represents the case of a payer that pay fixed rate payments to a receiver in exchange for floating rate payments. The interest rate swap hedges against interest rate risk in two ways. The payment of the swap is then the difference between the fixed and the floating payments.

The present value an IRS (PV_{swap}) depends on the difference between present values of the fixed part (PV_{fixed}) and the floating part ($PV_{floating}$):

$$PV_{swap} = PV_{fixed} - PV_{floating} \qquad \textbf{(1)}$$

[12] "*Financial derivatives are financial instruments that are linked to a specific financial instrument or indicator or commodity, and through which specific financial risks can be traded in financial markets in their own right*" (International Monetary Fund: https://www.imf.org/external/bopage/pdf/98-1-20.pdf.)
In our case, the focus will be on interest rate derivatives.

[13] Here, we are speaking of the Belgian case.

[14] This is in the case of non-maturing liabilities where customers have the option to withdraw at any time its money.

[15] To remind, minimum legal rate imposed for Belgian banks to savings accounts is 0.11%.

The present value of the fixed part is defined as:

$$PV_{fixed} = \sum_{j=1}^{n_{fixed}} \alpha_j C P(0, T_j) \quad \textbf{(2)}$$

where C is the coupon rate
$P(0; T_j)$ are the discount factors and
α_j are the day count fractions applying to each semi-annual period[16]

On the other side, the present value of the floating part is defined as Equation (3).

$$PV_{floating} = \sum_{j=1}^{n_{floating}} \delta_j L_j P(0, T_j) \quad \textbf{(3)}$$

where $L_j = F(T_{j-1}, T_j) = \frac{1}{\delta_j}(\frac{1}{P(T_{j-1}, T_j)} - 1)$ is the forward rate for settlement at T_{j-1}
$P(0; T_j)$ are the discount factors and
δ_j are the day count fractions applying to periodic floating leg payment[17].

At break-even (or mid-market) swap has zero PV:

$$PV_{floating} = PV_{fixed} \quad \textbf{(4)}$$

And Equation (5) can be rewritten as:

$$PV_{fixed} = CL \quad \textbf{(5)}$$

where $L = \sum_{j=1}^{n_{fixed}} \alpha_j P(0, T_j)$ is the level of the swap. The level corresponds to the "Dollar" value of a one basis point parallel move of the rate curve[18].

The key characteristic of a swap is a spot settled floating rate bond, paying LIBOR and repaying the principal at maturity T_{mat} and is always valued at par. That gives Equations (6) and (7).

$$PV_{floating} + P(0, T_{mat}) = 1 \quad \textbf{(6)}$$
$$PV_{floating} = 1 - P(0, T_{mat}) \quad \textbf{(7)}$$

Finally, the break-even or mid-market swap rate is equal to:

$$S(T_{mat}) = \frac{1 - P(0, T_{mat})}{L} \quad \textbf{(8)}$$

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16 Number of days based on a 30/360-day count divided by 360
17 Number of days based on an act/360-day count divided by 360. Day-count convention is a market standard to express interest payments as the percentage of a certain notional amount per year. For any two dates d_1, d_2, a year fraction τ needs to be defined as the possible the number of years between them such that : $\tau = \tau_{1,2} = T(d_1, d_2)$
For instance : ACT/360: $T_{\frac{actual}{360}}(d_1, d_2) = \frac{n}{360}$ where *n* is the exact number of calendar days between the start date d_1 and the end date d_2.
18 Called also "Cash duration" of the swap. The dollar value of a one basis point parallel move can be expressed as *DV01*

Economically, an IRS is equivalent to holding a position in a fixed and floating rate bond simultaneously because of the same face value and maturity. Hence, receiving fixed on a swap corresponds to buying a fixed rate bond and selling short a floating rate note, while receiving floating corresponds to selling short a fixed rate bond and buying a floating rate note. In consequence, the IRR on a break-even IRS can be approximated using a duration approach[19]:

$$Duration_{swap} = Duration_{fixed\ rate\ bond} - Duration_{floating\ rate\ note} \quad \textbf{(9)}$$

Let us take a simple example where two banks get in an IRS. Bank A thinks that the interest rates will rise and would like to profit from the difference between floating and fixed payments. In contrast, Bank B thinks that interest rates will decrease and hence would like to profit from the difference between fixed and floating payments. Let us suppose that Bank A agrees to pay the fixed rate 5% on a notional amount of €100,000. In exchange, Bank B will pay floating payments based on LIBOR +1%. Assuming that the starting point is LIBOR at 4%, that gives the case where the fixed payments are equal to the floating payments since $Payments_{Fixed} = Payments_{Floating}$ because

$$Payments_{Fixed} = 5\% * 100{,}000 = 5{,}000 \quad \textbf{(10)}$$
$$Payments_{Floating} = (4\% + 1\%) * 100{,}000 = 5\% * 100{,}000 = 5{,}000 \quad \textbf{(11)}$$

Now, let us assume that the interest rates go up from 4% to 4.75%. Then, the Bank A will make a profit and Bank B a loss since $Payments_{Fixed} < Payments_{Floating}$ because

$$Payments_{Fixed} = 5\% * 100{,}000 = 5{,}000 \quad \textbf{(12)}$$
$$Payments_{Floating} = (4.75\% + 1\%) * 100{,}000 = 5.75\% * 100{,}000 = 5{,}750 \quad \textbf{(13)}$$

If the contractual agreement stipulates that only the net difference is paid to the appropriate party, then Bank B has to pay €750. On the opposite, let us assume that EURIBOR goes down from 4% to 3.25%. Then Bank B will make a profit and Bank A a loss since $Payments_{Fixed} < Payments_{Floating}$ because

$$Payments_{Fixed} = 5\% * 100{,}000 = 5{,}000 \quad \textbf{(14)}$$
$$Payments_{Floating} = (3.75\% + 1\%) * 100{,}000 = 4.75\% * 100{,}000 = 4{,}750 \quad \textbf{(15)}$$

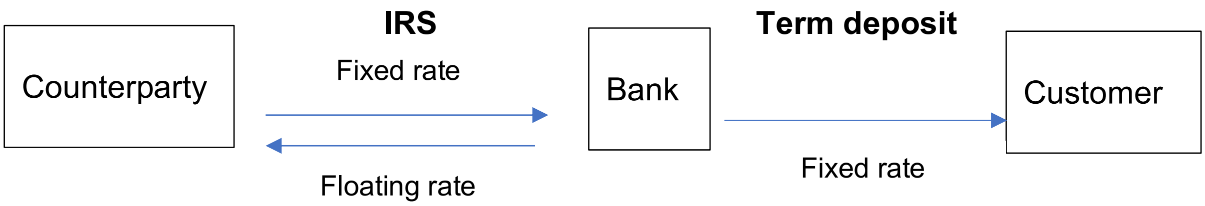
And hence, Bank A has to pay €250 to Bank B. The IRS can serve to match the duration between asset and liabilities.

The bank can decrease the duration gap by two possibilities. It might match the duration either by reducing the duration of the assets, or by increasing the duration of the debts. For the former, if the bank has a bond portfolio with fixed-coupon with a certain duration, then the IRS will swap against floating rate coupon and the combination of the bond with the swap will produce the cash-flows of a floating bond with a duration of zero. This change in duration mitigates the interest rate risk by modifying the interest rate risk profile. For the latter case, let us remind that in the example of Moser, the bank has term deposits with fixed interest rates and non-maturing deposits with floating rates. By switching from term to non-maturing deposits, the duration's liability increases. However, all the term deposits are not converted to non-maturing deposits and there remains potential losses in a decreasing interest rates scenario. Indeed, the bank will be hampered by decreasing rates since it has to pay higher rates to the customer than those of the market. Again, the interest rate of term deposits is

[19] In fact, since the duration of a floating rate note is close to zero, the duration of a swap tends to be the duration of the fixed-rate bond.

reached through an IRS with a counterpart where the bank receives a fixed interest rate and pays a floating interest rate. The purpose of this operation is to eliminate risk of fixed interest rates, getting exposure to a floating interest rate and hence increase the duration of the liabilities since the duration of non-maturing deposits is higher than duration of term deposits. If the interest rate decreases, there is a loss on client deposits and a gain on interest swap. On the opposite case, since the interest rates are pretty low, increasing rates represent a loss in the interest rate swap and a gain in the client deposit. In consequence, the two possible trajectories of the interest rate are hedged by IRS and is represented in the Figure 4.

Figure 4. – Interest rate hedging of term deposits with interest rate swap



Interest rate futures

Another method to hedge against interest rate risk with derivatives is through interest rate futures (IRF). A futures contract is an agreement between a buyer and a seller to exchange a fixed quantity of a financial asset at an agreed price on a specified date. The buyer of an IRF contract agrees to take delivery of an underlying bond when the contract expires, and the contract seller agrees to deliver the debt instrument. Such it will be said in the example, an IRF can be based on an underlying instrument such a Government bond. Hence, the hedging of the instrument through IRF aims to protect against adverse changes in interest rates.

IRF can be used to offset positions. Delivery of the asset in a future contract rarely occurs because traders can always close out the contracts. Since a future trader who has settled an initial position has both bought and sold futures, his profit depends on the prices of the futures he has bought and sold. The profit arises when the trader buys futures at a price lower than he sells them and the loss arises when the opposite case happens.

For instance, a trader buys a Government bond futures contract for €90 per €100 face value of a Government bond. A month later, he settles his position by selling this Government futures contract for €94. According to this case, he will make a profit of €4 per €100 face value of Government bond. On the contrary, if the Government bond futures contract falls to €87 per €100 face value of a Government bond, then the trader will suffer a loss of €3 per €100 face value of Government bond. In this sense, the value of the contract rises and falls inversely to change in interest rates. If the Government bond yields rise, then its price will fall and hence the futures contract on this bond will also fall in price.

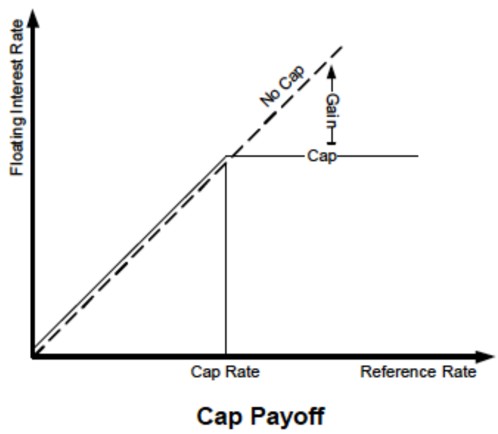
IRF are good hedging assets because its prices are closely related to the prices of fixed income assets when interest rates change. IRF can be used to control the risk associated with the duration GAP either at the macro-level or at micro-level. In the former case, IRF is used to protect the entire balance sheet. In the latter case, only individual assets or transactions are hedged by these derivatives. In general, an investor who suffers losses on his investment portfolio when interest rates rise hedges interest rate risk by selling IRF. When interest rates rise, interest rate futures prices fall which will lead an investor to make profit from falling futures prices. He needs then to gain on his futures contract to offset the loss on his original investment portfolio. Since sellers of futures make a profit when futures prices fall, the investor would hedge by selling futures. However, when interest rates fall, the losses on the futures offset the profits on the original investment portfolio. On the opposite case, if the investor suffers losses on his portfolio with falling rates, will hedge it by buying IRF. When interest rates will fall, IRF prices will rise. Since buyers of futures make a profit when futures prices rise, the investor will hedge by buying futures and the profit will offset the losses on the portfolio. However, if the interest rates rise, the gains on the portfolio will be offset by the losses on the futures prices. If a bank has to hedge a Government bond portfolio, it will hedge it by selling interest rate futures if it assumes that interest rates will rise. Since Government bond prices fall when interest rates rise, the losses' portfolio will be offset by the gains in the futures contract.

Cap, floors and collars

The last class of derivatives we would like to speak in this section is the class of caps, floors and collars. The *caps* and *floors* are basic products which aims to hedge against the floating rate risk. In more details, they are composition of individual options which are called *caplets* and *floorlets*. The combination of a cap and a collar into one product creates a *collar*. This section will bring the purpose of using these derivatives but will not discuss on the valuation techniques in low interest rate environment since it is not our scope.

First, a *cap* is a call option on the future realization of a given underlying market rate. More specifically, this is a collection of caplets, each of which is a call option on the market rate at a specific date in the future. We saw above that the persistence of a low interest rate environment will lead the customers and/or the bank to change the composition of the balance sheet. Therefore, banks will use the caps to limit their risk exposure to upward movements in short term floating rate debt. In the example, the maturities and duration of the assets exceed those of the liabilities, and short-term deposits are vulnerable to rises in short term interest rates as the cost of funding will rise without any comparable increase in earnings from mortgage assets. Since, rise in interest rates could be disastrous, the caps are used in situations where short term interest rates are expected to increase. For highly leveraged companies, rate caps are used to manage interest expense. The rate caps can be seen as insurance, ensuring that the maximum borrowing rate never exceeds the specified cap level. In exchange for the guarantee of a maximum level of the market rate, the purchaser pays a premium which is the client's maximum loss on a cap transaction. The purchaser has the right to receive a periodical cash flow, equal to the difference between the market rate and the strike, leading to a placement of a maximum limit on interest payments on floating rate debt. The payoff is represented in Figure 5. As the floating interest rate increases, the gain in *t* is the result of the difference between the rate without cap and the cap rate fixed as a limit. More the floating rate will increase, more the operation will provide a gain to the institution.

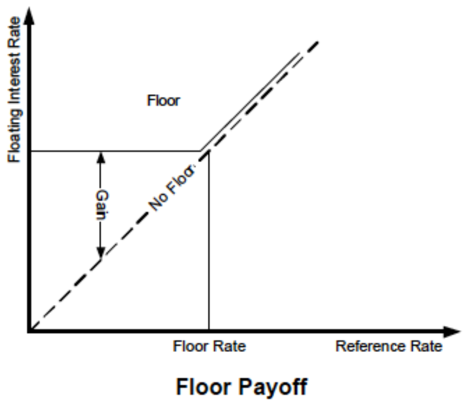
Figure 5. – Cap payoff in an interest rate cap



In this sense, cap is a guarantee of a future rate and its price is based on the sum of *n* caplets. The dominant pricing method in use is the Black-Scholes option pricing methodology and hence the forward rates. The implied forward rate will change over time as the market changes its view of futures rates. Therefore, the price of a cap depends on the likelihood that the market will change its view, which is measured by the volatility. An instrument expected to be volatile between entry and maturity will have a higher price than a low volatility instrument. In a stable market, the passing of time will lead to the cap falling in value. This phenomenon is called the "Time Decay" and represents the fall in value when we get closer to the maturity. The higher the strike compared to prevailing interest rates the lower the price of the cap. High strike or "out the money" (OTM) caps will be cheaper than "at the money" (ATM) or low strike "in the money" (ITM) because of the reduced probability of the caplets being in the money during the life of the option[20]. The price of the cap will increase with the length of the tenor as it will include more caplets to maturity. Further the strike is set "out the money", the cheaper is the cap, as the probability of payout is less and hence the cap is considered to be more leveraged. As rates rise, the cap will increase in value as it becomes closer to the money. In the market, traders will use volatility to quantify the probability of changes around interest rate trends. Higher volatility will increase the probability of a caplet being in the money and therefore the price of the cap. Since the market rates had reached historical low levels, there is an increasing risk of upwards movements in interest rates. The loss that a bank will face with rising rates can be offset with payments induced by a cap.

Second, interest rate floor is similar to a cap except that it is structured to hedge against decreasing interest rate, *i.e.* down-side risk. It can be purchased from a band at an amount to a contract where this amount is reimbursed to the buyer for the difference between the reference rate falling and the floor interest rate level. The difference with cap is that the reference rate must drop *below* the floor rate and not above. A typical interest rate floor is considered as a composition of *n* floorlets, which only yield payment on one period of time. If a bank is in a situation such that it has a loss on investments' portfolio from falling rates, the interest rate floor will be an ideal product for the firm to purchase in order to guarantee a minimum rate of return. If the floating rate runs below the minimum rate, the seller/bank will compensate the loss on falling interest rate. The floor payoff of an interest rate floor is represented in Figure 6.

Figure 6. – Floor payoff of an interest rate floor.



As we can see on Figure 6, the gain from this operation will occur when there is a positive difference between the floating rate without floor limit and the floor rate.

Finally, it might be possible to construct a combination of a cap and a floor into one product which is the *collar*. Since the premium for having cap or floor is often high, the collar was set up to permit hedging at lower cost. Buy a collar is buy a cap and sell a floor while sell a collar is sell a cap and buy a floor. Collars can benefit both borrowers and investors. In the former case, the collar protects against rising rates but limits the benefits of falling rates. In the case of an investor, the collar protects against falling rates but limits the benefits of rising rates.

[20] See the Annex n°2 "the volatility smile"

Unlike a cap or a floor, an up-front premium may or may not be required, depending upon where the strikes are set. The buyer and the seller agree upon the term (the tenor), the cap and floor strike rates, the notional amount, the start date and the settlement frequency. If at any time during the tenor of the collar, the index moves above the cap strike rate or below the floor strike rate, one party will owe the other a payment. The payment is calculated as the difference between the strike rate and the index times the notional amount.

For instance, a customer is borrowing €10 million at 1-month EURIBOR plus 200 bps, for a current rate of 7.75% as EURIBOR is currently at 5.75%, from Bank A. The customer wishes to cap EURIBOR so that it does not exceed 6%. In order to reduce the cost of the cap, the borrower sells a floor to Bank A with a strike of 4%. Bank A and the customer have created a "band" within which the customer will pay EURIBOR plus the borrowing spread of 200 bps. If EURIBOR drops below the floor, the customer compensates Bank A. If EURIBOR rises above the cap, Bank A compensates the borrower. The customer has foregone the benefit of reduced interest rates if EURIBOR falls below 4%.

However, there are risks using derivatives such the change in value. The value of a derivative instrument is dependent on both movements in the price of the underlying assets, and on the remaining time of the contract. Depending on the characteristics of the instrument, a movement in the price of the underlying asset will often result in a larger movement in the price of the derivative instrument. The relative change in the price of the derivative is therefore often larger than the change in the underlying asset. This is known as the leveraged effect, and it may lead to higher yields on the invested capital than the same amount invested in the underlying assets would have produced. Conversely, the leveraging effect may work to the investor's disadvantage, resulting in a bigger loss on the derivative instrument than would have been the case for an investment in the underlying asset, in our case the interest rate. If the price of the underlying assets moves in a different way than expected, the entire invested capital may be lost.
There is another risk which is linked with low interest rate environment: the valuation of the derivative. In the current negative rate environment, there is a number of challenges in the use of some traditional models. For example, according to a Black model the price of a simple cap option depends, among various other factors, on the logarithm of the forward rate. However, if the market-quoted forward rate is negative, then the logarithm is undefined. As a result, this model cannot give an answer. A related challenge is the fact that market quotes for volatilities of negative strikes do not always exist[21]. There exist additional risks such counterpart risks or risks induced by collateral posted to ensure the good outcome of the derivatives' contract.

[21] The solution to this challenge is a shifted SABR which is presented in section 5.2.2.

Challenge n°2: Change in the regulation

The second challenge that the ALM has to face is the regulation. In April 2016, the Basel Committee on Banking Supervision (BCBS) issued Final Standards on IRRBB that replace the 2004 Principles for the management and supervision of interest rate risk. Interest rate risk in the banking book (IRRBB) is part of the Basel capital framework's Pillar 2[22] and serves to the identification, the measurement, the monitoring and the control of this risk. The new standards set out the Committee's expectations on the management of IRRBB in terms of identification, measurement, monitoring, control and supervision.

This section aims to present the reason on how the change in the regulation is impacting the ALM. We will first assess the different characteristics of the regulation and then discuss on the impact of the change on the bank.

1. Definition and subdivision of the interest rate risk

EBA[23] subdivided into three sub-types of interest rate risk on the banking book:

- First, there is the *gap risk* which is resulting from the term structure of interest rate sensitive instruments. It arises from differences in the timing of their rate changes occurring consistently across the yield curve. The yield curve is the relationship between the interest rates and the remaining time to maturity of debt securities. In this thesis, the first part of the "interest rates theory" section in the annex will talk about yield curve definition, parallel risk and non-parallel risk. These risks are defined in the IRRBB, but we have first to define what is a yield curve to describe them.
- Second, there is the *Basis risk*. This is the risk related to the impact of relative changes in interest rates for financial instruments that have similar maturities but are priced using different yield curves[24]. In practice, there are different markets for interest rates which move differently from one another, giving rise to this basis risk. There are three different sub-types of basis risk.
    - There is first the *external reference rate basis risk*. This is where different financial items are linked to different external benchmark rates which lead to different yields for similar maturities in the same currency. For instance, there is basis risk between EURIBOR 1-month and the interest rate on the Main Refinancing Operations (MRO). The former rate is the interest rate at which European banks lend to one another with maturity of one month. The latter is the interest rate which provides the bulk of liquidity to the banking system and is by the Governing Council of the ECB.
    - The second sub-type of basis risk is called the *currency basis risk*. If the interest rates for one currency move differently to interest rates for another. Then a bank, which appeared fully matched after that all positions were converted into one currency, is exposed to this risk.
    - Last, the *maturity basis risk* is the risk that arises where actual amount by which items will be re-price could be different depending on the maturity of the next roll.
- Finally, the third sub-type of IRR is the *option risk* which arises from option derivative positions or from optional elements embedded in a bank's, assets, liabilities and/or off-balance sheet items. This risk can alter the level and timing of cash flow of the bank. In other terms, this is the risk that cash flows change due to embedded options as prepayment, extension in the residential mortgages. It can be also an implicit optionality as for instance a fixed rate mortgage where the customer has a right to re-pay early against a penalty. That is or can be less than the net present value of the benefits such customer would obtain by prepaying. For instance, let us assume that a

[22] European Banking Authority, (2015), "Guidelines on the management of interest rate risk arising from non-trading activities, EBA/GL/2015/08.
[23] The EBA is mandated by the EU Commission to translate the BCBS principles into the EU regulation.
[24] For instance, different interest rates indices

bank has a 15-year mortgage as asset and a 3-year time deposit. In period 1, the mortgage is paying 7% and the deposit 4% which gives a net interest income of 3%. Then let us assume a 2% decrease in mortgage interest rates which leads the customer to refinance his mortgage from 7% to 5%. The new net interest income is the difference between the 5% of the mortgage less the 4% of the deposit, *i.e.* 1%. Hence, the option risk presented in this example is the possibility of the customer to refinance in period 2.

For the measurement of IRRBB, it should be based on outcomes of both economic value and earnings-based measures. They are calculated using interest rate shock and stress scenarios. These scenarios are based on the internally selected interest rate shock scenarios addressing the bank's risk profile, according to its Internal Capital Adequacy Assessment Process (ICAAP)[25]. Moreover, historical and hypothetical interest rate stress scenarios have to be more severe than shock scenarios[26]. To capture the local rate environment, a historical time series for various maturities is used to derive each scenario for a given currency. A bank's stress testing framework for IRRBB will be used to assess the potential impact of the scenarios on the bank's financial condition. In developing the scenarios, banks consider many factors, such as the shape and level of the current term structure of interest rates and the historical and implied volatility of interest rates. In low interest-rate environments, banks also consider negative interest rate scenarios and the possibility of asymmetrical effects of negative interest rates on their assets and liabilities.

2. Earnings measures

2.1. Funding gap and duration gap analysis

Funding gap analysis

The gap analysis is used for identifying and estimating the interest rate exposure to repricing risk. That measures the arithmetic difference between nominal amounts of interest-sensitive assets and liabilities in absolute terms. GAP is computed in Equation (16).

$$GAP = RSA - RSL \tag{16}$$

where RSA are the rate sensitive assets and RSL the rate sensitive liabilities.

Multiplying GAP times the change in the interest rate immediately reveals the effect on bank income:

$$\Delta I = GAP * \Delta i \tag{17}$$

where ΔI is the change in bank income and Δi the change in interest rates.

Gaps with a larger volume of assets have a positive sign reflecting increasing value (income) of the banking book with rising value of liabilities. The principle bears on the allocation of all relevant RSA and RSL into a certain number of predefined time bands according to their next contractual repricing date. The EBA also ensures that a gap can be multiplied by an assumed change in interest rates to yield in net annualized interest income resulting from interest rate movement.
For instance, let us have a bank which wants to compute a gap analysis in time 1. On the RSA's side, it holds advances, investments and reserves amounting respectively 145,000, 100,000 and 85,000. On the RSL's side, we assume that the bank has only short-term deposits for 550,000 for simplicity. The resulting balance-sheet is represented in Table 1.

[25] Process which assesses the capital adequacy of a bank in relation of their risk profiles such a strategy to maintain their capital levels. For the different stress scenarios, see "Change in the IRRBB".

Table 1 – Bank balance sheet example

Liabilities			Assets	
Rate Sensitive Liabilities			**Rate Sensitive Assets**	
Short term deposits	550000		Advances	145000
			Investments	100000
			Reserves	85000

According to this example, the funding gap in time 1 amounts 220,000 which represents a negative gap as:

$$GAP = RSA - RSL = 330{,}000 - 550{,}000 = -220{,}000 \quad \textbf{(18)}$$

A 1% rise in interest rates will affect the bank income such:

$$\Delta I = \ -220{,}000 * 0.01 = -2{,}200 \quad \textbf{(19)}$$

With a positive gap, the interest margin would increase if short-term rates rose and decrease if short-term interest rates fell. On the contrary, with a negative, the interest margin would decline if short-term rates rose and increase if short-term rates fell.
The advantage in this tool is the simplicity of computation and understanding. The assumption of this tool is that all positions within a particular maturity segment mature or reprice simultaneously.
This technique suffers from the problem that many of the assets and liabilities that are not classified as rate-sensitive have different maturities. One refinement to deal with this problem, the *maturity bucket approach*, is to measure the gap for several maturity subintervals, called maturity buckets, so that effects of interest-rate changes over a multiyear period can be calculated. This last approach is presented in Table 2.
Another problem is that does not capture embedded options or complex instruments such the option risk. Moreover, the gap analysis focuses only on the effect of interest-rate change on income. Financial institutions care not only about change in interest rates on income but also on the economic value of the capital.
The illustrative example is taken from a paper of the Dutch National Bank (2005). The Loans, Investments and other assets are our RSA while the non-maturity deposits and other liabilities are our RSL. Based on the contractual maturities of the RSA and RSL, the portfolio of the product is segmented by maturity, *i.e.* <1 month, 1-3 months, 6-12 months…, >3years. This gap example reports the exposure that is released during a certain time period and the exposure that is outstanding during a particular time period.

Table 2. Example of maturity bucket approach

	<1m	1-3m	3-6m	6-12m	1-2y	2-3y	>3y	**Total**
Loans	100	10	20	45	5	20	30	**230**
Investments		5	5	10	20	20	50	**110**
Other assets	5						15	**20**
Total assets	**105**	**15**	**25**	**55**	**25**	**40**	**95**	**360**
NMD	-65				-30		-50	**-145**
Other liabilities	-35	-35	-45	-30	-10	-10	-20	**-185**
Total liabilities	**-100**	**-35**	**-45**	**-30**	**-40**	**-10**	**-70**	**-330**
Equity								**-30**
Net periodic gap	**5**	**-20**	**-20**	**25**	**-15**	**30**	**25**	
Cumulative gap	**5**	**-15**	**-35**	**-10**	**-25**	**5**	**30**	

Duration gap analysis

The gap analysis we mentioned is the *funding gap analysis* as well as the income view, while *the duration gap analysis* is a balance sheet view[27].

An alternative method for measuring interest-rate risk, called *duration gap analysis*, examines the sensitivity of the market value of the financial institutions net worth to changes in interest rates. Duration analysis is based on Macaulay's concept of duration, which measures the average lifetime of a security's stream of payments[28]. It provides a good approximation, particularly when interest-rate changes are small, of the sensitivity of a security's market value to a change in its interest rate using the formula in Equation (20).

$$DUR_{gap} = DUR_a - (\frac{L}{A} * DUR_l) \qquad \textbf{(20)}$$

where DUR_a is average duration of assets, DUR_l is the average duration of liabilities, L and A are the liabilities and the assets respectively.

The duration gap will give the change in the market value of net worth by Equation (21).

$$\frac{\Delta NW}{A} = -DUR_{gap} * \frac{\Delta i}{1+i} \qquad \textbf{(21)}$$

where ΔNW is the change in net worth, Δi the change in interest rate and i the interest rate.

For instance, if the bank has €100 million of assets, a $DUR_{gap} = 1.72$ and a rise in interest rates from 10% to 11%, that will give a fall in the market value of the net worth of €1.6 million. This result is obtained by Equation (22) and (23):

$$\frac{\Delta NW}{A} = -1.72 * \frac{0.01}{1+0.1} = -0.016 = -1.6\% \qquad \textbf{(22)}$$

$$\Delta NW = -1.6\% * €100\, million = €1.6\, million \qquad \textbf{(23)}$$

Although the duration gap analysis seems to be easy to compute, there are further complications.

One assumption that we have been using in our discussion of income gap and duration gap analyses is that when the level of interest rate changes, interest rates on all maturities change by exactly the same amount. In other terms, that means that the slope of the yield curve remains unchanged. In our calculation above, by assuming that interest rates for all maturities are the same, we assumed that the yield curve is assumed to be flat. It is obviously not the case if we refer to the "interest rate theory" section in the annex. The slope of the yield curve fluctuates and tends to change when the level of the interest rate changes. Thus, a financial institution has to assess what might happen to the slope of the yield curve when the level of the interest rate changes and then take this information into account when assessing IRR.

Another problem arises when comes the estimation of the average duration of assets and liabilities. Such the non-maturing liabilities, the future cash-flows are uncertain which complicates the estimation of the duration. Thus, the estimate of the duration gap might not be accurate either.

[27] This tool measured the change in equity against fluctuations in interest rate.
[28] Macaulay's duration is seen in the "Interest rate theory" section.

2.2. Earnings-at-Risk (EaR)

While the gap analysis is a static model, the Earnings-at-Risk is a dynamic model as it takes in account the embedded optionality relating to factors such interest sensitivity, prepayment and/or savings behavior. EaR measures the loss of NII over a certain time horizon resulting from interest rate changes. That estimated also the potential loss resulting from other adverse moves in market conditions such foreign exchange rates and market volatility. In this section, we will focus only on interest rate changes.
This tool is computed by allocating relevant assets and liabilities to time bands, by maturity or by repricing date as a starting date.

This is the difference in NII between a base scenario and alternative scenario. First, EBA defines the base scenario as the scenario where the interest rates used for repricing are derived from the forward rates by applying appropriate spreads and spot/forward rates[29]. In other words, the NII is calculated under the current or the forecast interest rate environment, based upon an assumed interest rate sensitivity of customer rates and forecast volumes of assets and liabilities. In the alternative scenario, the interest rate and spread shifts are added onto the forward rates used in the base scenario. In consequence, it will look to any change of the NII based on both scenarios by taking in account the different embedded options which were not considered in the previous static model. This method is used to calculate the yield curve risk, the basis risk, the repricing risk and obviously the option risk as it takes in account changes in savings and payments behavior. EaR is generally applied as a measure for a single shock or as a simulation method with many scenarios followed by computation of a maximum loss within predefined confidence interval. However, as it was explained in the introduction of the section, the earnings measure serves to calculate short-term effects.
Because of the lack of long-term scope, the EBA provides tools on economic value measures.

3. Economic value measures: Capital-at-Risk (CaR)/Economic Value of Equity (EVE)

CaR/EVE measures the theoretical change in the net present value of the current balance sheet and thus of its equity value resulting from interest rate changes. In other words, that evaluates the impact of interest rate changes on fair market values of asset, liabilities, and equity. For instance, we know from the inventory of exposure of interest rate changes that when interest rates go up, the price of a bond portfolio will fall. Then the EVE helps to compute the degree of change of the economic value[30] of the overall balance sheet provoked by the change in the bond portfolio value.

There exist two different models such a static and dynamic one. The static model resembles to the EaR as it compares a base scenario with alternative interest rate scenarios. The base interest rate scenario is the present value of the assets less the liabilities under the current or the forecast interest rate environment. After, the balance sheet is revalued under alternative scenarios and the difference between the value of equity under both scenarios is calculated. For internal purposes, EBA ensures that institutions should complement their static model with a model taking in account assumptions regarding equity capital. However, the net present value calculation will not take basis or option risk since that does not adjust for the impact on cash flows of the rate scenario and may underestimate the short-term effect of convexity and yield curve risk. The dynamic model will use a more sophisticated version where the cash flows are re-calculated dynamically to consider the fact that their size and the timing may differ under various scenarios because it takes the customer behavior reaction to the chosen scenario. In this way, that can measure the basis risk and can estimate the long-term effect of a change in a yield curve shape as long as the alternative stress scenarios are adequately designed.

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[29] See "interest rate theory" section in annex.
[30] The economic value of an asset or a liability corresponds to the ability to produce incomes in the future.

Both the measurement of NII and EVE involves the gap analysis for quantifying risks. This means that asset and liability's cash flows are slotted into time buckets based on the first date where the instrument is rate sensitive to until the repricing date. For an interest rate scenario i, the EVE calculations are according to (24) and (25).

$$EVE_j = \sum_{k=1}^{K} CF_j(t_k) e^{-R_j(0,t_k)t_k} = \sum_{k=1}^{K} CF_j(t_k) * DF_j(t_k) \quad \text{where } k = 1,2,\dots,K \tag{24}$$

$$\Delta EVE_i = \Delta EVE_0 - EVE_i \tag{25}$$

where $j = 0$ is the current interest rate term structure, $CF_j(t_k)$ is the net cash flow of instruments that reprice in the time bucket t_k, $R_j(0, t_k)$ the interest rate between 0 and t_k, with both being computed under scenario j.

An example of EVE based on Dutch National Bank figures (2005) is illustrated in Table 3.

Table 3 – Example of an EVE report

Interest rate scenario (Bp)	EVE	Change relative to base scenario	% change
+300	11,991	-6,103	-33.73
+200	14,669	-3,425	-18.93
+100	17,005	-1,089	-6.02
0	18,094	-	0.0
-100	17,864	-230	-1.27
-200	17,645	-449	-2.48
-300	17,157	-947	-5.18

As the interest rates go up, the economic value of equity decreases and the change relative to the base scenario, *i.e.* EVE under 0 change and the EVE under an interest rate change, increase.

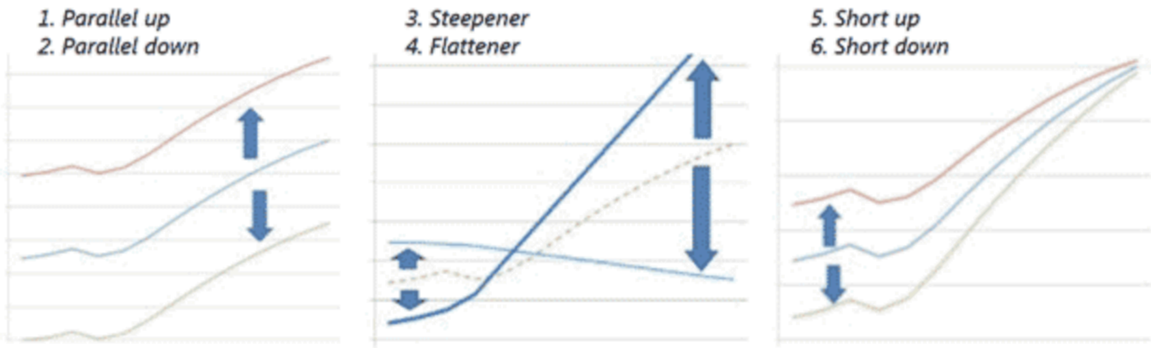
The benefits of such approach are that the interest rate risk is measured in terms of economic value and can evaluate the long-term effects. However, the model evaluates change in the overall balance sheet. This is a problem for off-balance sheet items or assets and liabilities which bankers cannot trade.

4. Changes in the IRRBB

As it was said in the introduction of this section, the IRRBB updated its principles in April 2016. The update reflects change in market and supervisory practices due to the current low interest rate environment. It also provides methods and models to be used by banks in a wider and enhanced risk management framework.

The first change is the inclusion of a dynamic approach for the net interest margin projections and sensitivity analysis. In this sense, banks must increase the granularity at the deal level in their simulations. The new IRRBB framework has defined at least six scenarios representing the possible changes in the interest rate curve. These six scenarios allowing for the measure changes of the EVE are represented in Figure 7.

Figure 7. – Interest rate scenarios in the new IRRBB



These shocks provide significant improvement in the coverage of shock patterns which the market could face. They make the shocks more realistic and still understandable for the governing body of each bank. Not only are these standardized interest rate shocks more instructive, but they are now better linked to the actual local market situation. The new standardized framework and interest rate shocks get even more value, such as the impacts of changes on very low interest rates, which are more difficult to evaluate.

The second change is that the renewed IRRBB principles require banks to implement policy limits that keep IRRBB exposure within their risk appetite. This limit framework is a control mechanism and lays the groundwork for IRRBB exposures. A bank must determine policy limits based on its nature, size, complexity and capital adequacy, as well as its ability to measure and manage its risk.

The last main change is the change in the governance structure. That includes a stronger involvement of the management and stricter definitions of responsibilities. The new IRRBB ensures an extended responsibility of the management by being more active in the business steering and decision-making. The Asset-Liability Committee (ALCO) should be better linked to the monitoring of limits and validation of the behavioral models which will extend the responsibility of the committee. The improved requirements of the IRRBB will have a major impact on systems and processes, and models for determining the interest rate risks in the banking book.

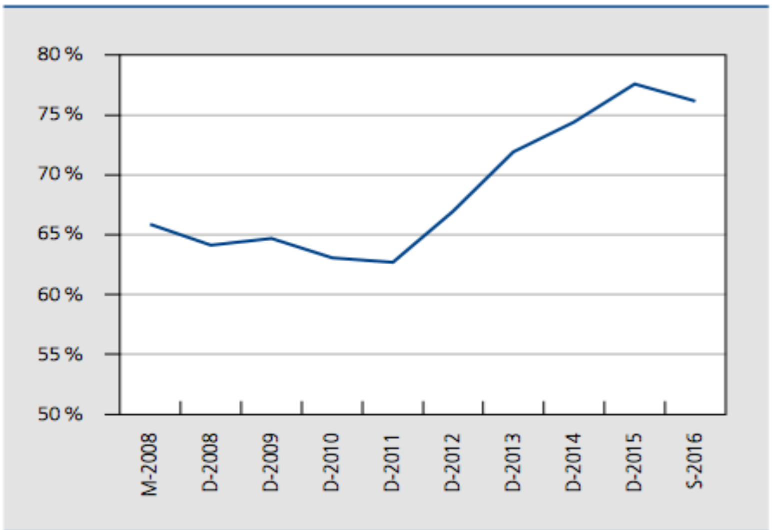
Challenge n°3: Non-Maturing Liabilities Modelling

1. Importance and evolution

Deposits are at the core of bank's financial intermediation. By issuing deposits, banks are the intermediate between small savers looking for high liquidity and low risk, and needs of investors, who require stable funding typically for riskier, larger, and longer-term projects. By transforming the liquid deposits into longer term assets, banks are exposed among others to interest rate risks.

There exist different kinds of deposits accounts. First, there are the *sight deposits* which can be withdrawn at any time and may be used as means of payments, even if it offers a poor positive return. Then, there are *term deposits* which offer higher market consistent returns. However, these deposits cannot be withdrawn before their defined contractual term expires. Finally, there are *savings deposits*. These deposits aim to preserve a high degree of liquidity, while offering a relatively attractive rate of return. The deposits are an important source of funding of the Belgian banks. In 2012, deposits amounted €490.9 billion, representing 1.3 times the gross domestic product (GDP)[31]. The importance of savings in bank's balance sheets has risen in recent years, from 45.38% to 62.45%. According to Huyghebaert (2013), the deposits remained stable even if the crisis was a big turmoil in the world economy. According to the National Bank of Belgium, the share of deposits in total assets has increased from 65.9% in Q1 2008 to 76.2% in Q3 2016 thanks to the rise in customer deposits[32].
Due to the imposition of liquidity requirements, banks were designed to reduce their dependence on short-term wholesale funding. Moreover, this increase is also due to the refocusing by Belgian banks on more traditional activities, such customer deposits which the key source of funding for banks are involved in the real economy financing. Figure 8 shows the increasing importance of total deposits compared to the liabilities for Belgian banks between 2008 and 2016.

Figure 8.- Evolution of total deposits compared to total Liabilities in Belgium



Source : NBB, Supervisory Reporting Scheme, consolidated data.

[31] HUYGHEBAERT N. (2013), "Study of the importance of deposits for the Belgian economy and the relationship with the financial crisis"
[32] National Bank of Belgium, "Financial Stability Report 2017".

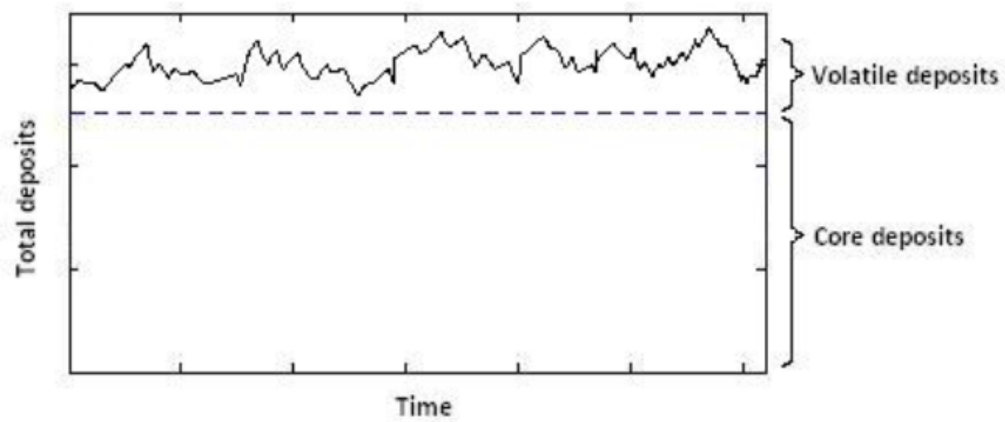
2. Problem in measuring interest rate risk for non-maturing deposits

The need to model these non-maturing liabilities is critical because there are no industry benchmark or implied parameters which can be used for this purpose. The behavior of depositors may vary from bank to bank and additionally based on the customer profile, product characteristics, … Hence, it is imperative for each institution to understand and analyze these products and their depositor's behavior before making any assumptions regarding how to agree to treat them.

For the modelling of the duration, the deposits can be subdivided into two categories[33]. First, there is the core part which is quite stable through time. Indeed, a typical retail customer will pay into his account a monthly salary and will make living expenses. The volume of his deposit will fluctuate during the month but on average will keep a stable balance. Then, there is a volatile part which is the part fluctuating during the time. Such von Feilitzen (2011) explained, the volatile part is assumed to have short maturity while the former part is assumed to have long maturity as it is illustrated in Figure 9.

From an ALM's point of view, the core part of the deposit volume can be invested in interest-bearing assets. To make the investments, the appropriate re-pricing maturity for the invested funds need to be modelled. But there are some difficulties.

Figure 9. – Evolution of the total volatile and core deposits through time.



First, the bank needs to estimate how much of its current account base is likely to be stable and how long. For the investment's side, they can make one assumption[34]: they will keep their current volume of current account balances for the foreseeable future because they assume that their market share is assumed to remain stable, or the economic growth will still ensure the total aggregate balance will not fall materially. This assumption is not the only one since banks can work on the basis of scenarios or strategic planning. However, since the financial crisis, a lot of new "challengers" enter the market and eroded market share of the more established players.

Moreover, some factors may change the fact that current account balances are stable. Banks have been forced by regulators to make the switching of accounts easier and quicker. The decision to withdraw or deposit money from the accounts are more or less necessarily driven by market conditions. The inclination for customers to switch banks is going to increase even if this behavior was historically low. According to the Financial Brand, 83% of the Millennials would switch banks if one offered more or better rewards than another[35]. When the interest

[33] In their replicating portfolio, we will see in the interest rate theory, Maes and Timmermans (2005) assumed that there are three categories: the core part, the volatile part and the remaining balances. The last part of the deposits volume is the part which needs to be modelled.
[34] Newson (2017), "Interest rate risk in the banking book"
[35] Millenials (ages 18-34) are a crucial market segment for financial institutions to attract, engage and retain. Survey here: https://thefinancialbrand.com/64222/millennial-switching-banking-marketing-checking-accounts/. Study made in USA, as of 2017.

rates will begin to rise, new comers or other banks can offer better rates paid on deposit accounts. In the second part, we will see that a limit of the static replicating portfolio is that it is based on a backward-looking approach. By looking backward, new comers will offer higher rates than the bank which relies on past rates. Hence, in a rising interest rate environment, we might see an increase in switching of banks.
Then, there is an increasing part of banking product which can be digitally bought, for instance by Internet and mobile phones. The digitalization is even a big problem for banks to retain their core part. Since it is easier to get products by Internet, Internet providers might provide banking services in the future. This is the case for instance of Orange which is basically a fixed telephony and mobile telecommunications provider and now, provide also online banking via their brand Orange Bank. In consequence, the assumption of stable current account balance cannot be based on past observations. Hence, customers could rely on internet services to lower their cost such distance to branches, fees, etc.

Second, most banking books products are relatively straightforward from modelling point of view. With most required financial parameters determined on a contractual basis, it is fairly simple to compute results like future cash flows, interest rate sensitivities and key rate durations. However, with non-maturing positions the cash flow and interest rate sensitivity generation process are a bit more complicated since they do not have fixed maturity. Further, they do not have fixed interest rate of any sort of floating benchmark-based re-pricing: the rate changes are either regulatory controlled or at the discretion of the banks.

To explain the dynamics of the Belgian savings deposit rate, Maes and Timmemans (2005) followed the US retail deposit rates dynamics modelled by O'Brien (2000). In the model, the deposit rate changes depend on whether the deposit rates are above or below a potential time-varying long-run equilibrium or target deposit rate. The model also explained that there is an asymmetry in the reaction speed at which the deposit rate is expected to mean-revert to its long-run equilibrium level. The upward change is more sluggish since deposit rates are below the long-term equilibrium while the downward change is faster since the deposit rates are above this level. In that dynamic, it is a real challenge for the ALM to model the non-maturing liabilities. Since we are in a low interest rate environment, the short market rates are negative while the deposit rates are always positive because of the Belgian minimum legal rate at 0.11%. That means that if they invest the core balance in shorter maturities, the return will be negative. Then for the modelling, the future evolution of deposit rates cannot use the historical path of the rates since they have been floored at zero for a long time. Since the incomplete adaption of the client rate to markets rates, the margin will be often unstable.

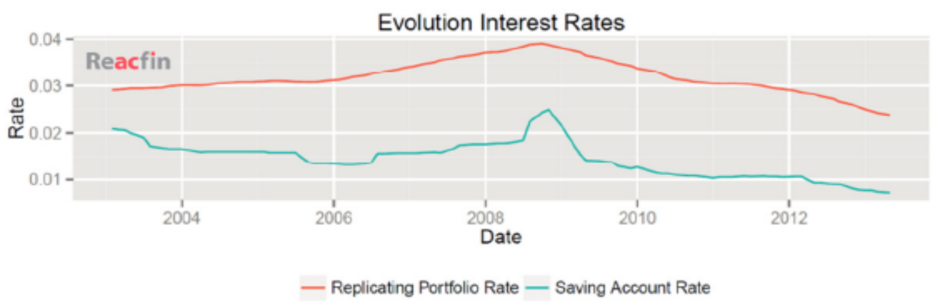
Part II: How to measure the interest rate risk for non-maturing liabilities

Method n°1: Static replicating portfolio

The duration is an interest rate risk measure which reflects the potential impact of a small parallel shift of the yield curve on the profit of future cash-flows[36]. For instance, if the bank's balance sheet contains a bond portfolio which has a duration of 5 years, that means that a 0.01% increase in interest rates will decrease the bond portfolio by approximately 0.05%. The contractual duration for non-maturing account is close to zero. However, the duration is much larger because it will depend on the behavior of the depositors. Depending on their profile, they can have a low sensitivity or a high sensitivity to any change in market rates. An extreme sensitivity to changes in market rate will give a zero duration, whereas the opposite will give a much longer duration like a consol.[37] To measure interest rate risk on non-maturing liabilities, Belgian, German and Dutch banks often use *replicating portfolio*. This approach aims to model demand deposits and define their replacement schedule by replicating the interest margin dynamics. With that concept, you will build an investment strategy of fixed-income securities which replicates, as accurately as possible, the future cash-flow of the liabilities. The replicating portfolio, we can compute it in statically or dynamically. In this section, we will bring the characteristics of each replicating portfolio and underline which is the most appropriate in a rising rate environment.

Replicating portfolios is one approach to model the duration of demand deposits and to define their replacement schedule. This methodology assumes that there is a portfolio that closely "replicates" the interest margin dynamics. The aim of this method is to build an investment strategy of fixed incomes securities, based on the duration of the non-maturing liabilities, that replicate the cash-flows of these liabilities. The methodology of a static replicating portfolio[38] is based on the approach of François Ducuroir and Wim Konings (2014). For a static replicating portfolio, the portfolio of the fixed income securities is defined as a buy-and-hold portfolio of "risk-free" zero-coupon bonds. Then, these securities are distributed across different maturity buckets and are renewed after being matured. The reinvestment is done in a limited number of maturity buckets, i.e. "reinvestment buckets". The reinvestments are computed following a fixed allocation rule called "reinvestment rule". The optimization of this portfolio can follow two criteria: the minimization of the standard deviation of the margin or the maximization of the risk-adjusted margin measured by the Sharpe ratio. However, most of Belgian banks use the first criterion. To compute this static replicating portfolio, large set of potential reinvestment rules is first generated. Then, for each reinvestment rule, the composition of the replicating portfolio and its portfolio rate are simulated. Finally, the optimal reinvestment weights are determined by focusing on the reinvestment rule that results in the most stable margin between portfolio rate and the non-maturing instrument rate. In consequence, we get the following replicating portfolio rate in Figure 10.

Figure 10. –Replicating rate obtained for the portfolio with the most stable margin



[36] For more details, see "interest rate theory in the annex n°1.
[37] A consol is a government bond where the issuer never repays the principal like a perpetual bond.
[38] A static approach means that the replicating portfolio is constructed using a "Buy & Hold until maturity" strategy for the replicating bond portfolio.

The replicating portfolio can also be dynamically. The difference is that the maturing funds in the replicating portfolio are renewed at the same maturity. The dynamic replicating portfolio models allow the bank to react more quickly by adapting the portfolio to changes in client behavior and the market environment.

1. Static replicating portfolio example : Maes and Timmermans (2005)

According to them, most large Belgian banks rely on static replicating portfolio model to estimate the duration or interest rate sensitivity of the non-maturing liabilities. They base their idea on the investment of the volume from the accounts in a portfolio of fixed-income assets to replicate the dynamics of the deposit balance over an historic sample period. The optimization is based on the selection of the portfolio which yields the most stable margin, represented by the minimization of the margin standard deviation of the spread between the portfolio return r_p and the deposit rate R:

$$Min\ std\ (r_p - R) \tag{26}$$

According to the constraint that $\sum_{i=1}^{n} w_i r_i = r_p$ where the sum of the weight w_i is 1 and each weight is positive[39]. An alternative objective criterion is to maximize the risk-adjusted margin, measured by the margin's Sharpe ratio. This ratio is computed as the ratio of the expected margin to the standard deviation of the margin:

$$S_m = \frac{\mathrm{E[m]}}{\sqrt{var[m]}} = \frac{\bar{m}}{\sigma_m} \tag{27}$$

where $\bar{m} = \overline{r_p - R}$ is the average margin and $\sigma_m = std(m) = std(r_p - R)$ is the standard deviation of the margin.

Maes and Timmermans also divide the total volume into three parts: the core deposits, the volatile deposits and the remaining balances. The core deposits are invested at a chosen long horizon (in their study 7 years), the volatile deposits are invested at the interest rate risk free short horizon (one month) and the remaining balances are replicated by the portfolio.

2. Limit of the static replicating portfolio

In this section, we will present a limit of the static replicating portfolio. Since the static replicating portfolio is a backward-looking approach, the bank is threatened by new comers. These new comers can profit from rising rates environment and propose higher paying rates on deposits. The illustrative example aims to illustrate this limit. I used the Online App *Athena* from *Reacfin* to simulate a replicating portfolio for non-maturing liabilities and *Fortuna* for simulating future paths of interest rates[40].
First, we have to choose a replicating portfolio under the most stable margin's assumption. I simulate 10,000 different portfolios containing the time buckets 1M, 2M, 3M, 6M, 1Y, 5Y and 10Y. I chose 10,000 in order to have sufficient samples to get an optimal replicating portfolio. However, the time buckets were randomly chosen and are not illustrating the reality. I could use 2Y, 3Y or 7Y time buckets but it will not change the outcome. All the portfolios are represented in the Figure 11, *i.e.* shows the plane of the different portfolios simulated. On the x-axis is the standard deviation of the margin and the y-axis represents the average of the margin. We could choose the higher average margin of the portfolio, but it will not be optimally stable. Indeed, the optimal portfolio will be the one which is as left as possible since the selection criteria is the most stable margin. The different colors represent the average duration of each portfolio with red being 1 year of duration and dark blue for 4 years.

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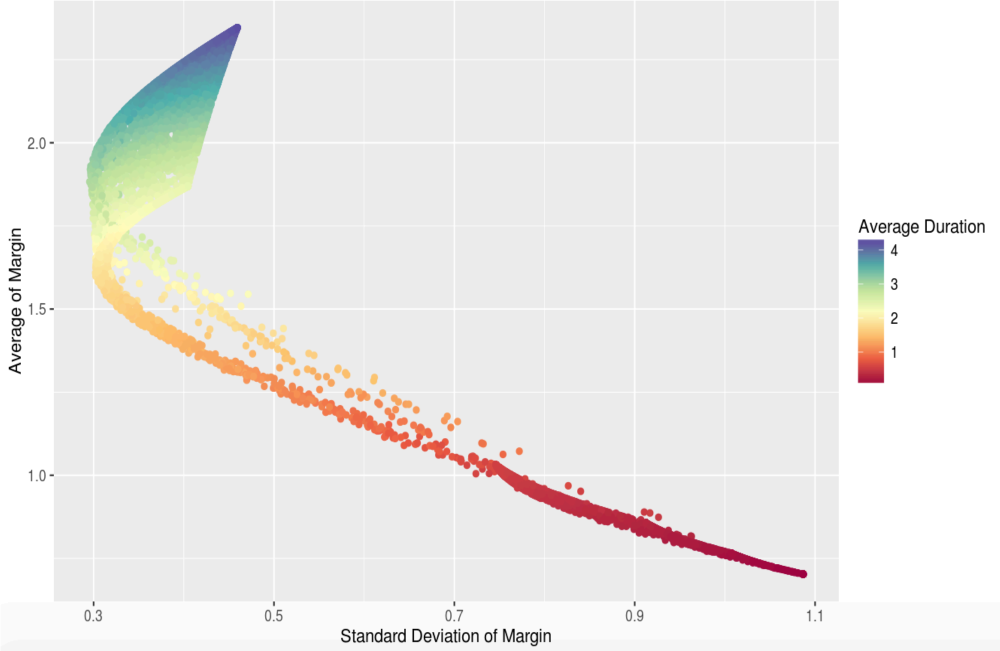
[39] no short sales
[40] Available here https://www.reacfin.com/en/know-how-to-risk/online-apps

Figure 11. –Risk-return plane for the 10,000 portfolios



The Table 4 illustrates the five most optimal portfolio according to their standard deviation and their investment rule with the different weights.

Table 4. – Selection of the optimal replicating portfolio based on the most stable margin's criterion.

Investment Rule	Standard Deviation Margin	Average Margin	Duration
1135	0.0029634	0.0188041	2.98
4706	0.0029665	0.0190988	3.05
2073	0.0029689	0.0188326	2.94
5163	0.0029689	0.0190153	3.04
3896	0.0029715	0.0190307	3.04

Investment Rule	Weight 1M	Weight 2M	Weight 3M	Weight 6M	Weight 1Y	Weight 5Y	Weight 10Y
1135	6%	20.89%	48.76%	0	17.29%	0	7.06%
4706	0	0	34.26%	28.38%	26.37%	0	10.99%
2073	0	24.58%	35.07%	0	30.11%	1.54%	8.70%
5163	39.52%	0	28.02%	10.96%	14.66%	0	6.84%
3896	47.99%	0	24.41%	6%	15.23%	0	6.36%

Depending on the standard deviation margin, the most stable margin follows the investment rule 1135.

Now, let us have our bank that is investing depending on the investment rule 1135. For the example, the investment begins in January 2018 for simplicity and is *before* the rising rate environment. The replicating rate in time *t* is Γ_t and is computed with a weighted average such in Equation (28).

$$\Gamma_t = \frac{\mathrm{w}_1 * \mathrm{r}_1 + \mathrm{w}_2 * \mathrm{r}_2 + \cdots + \mathrm{w}_n * \mathrm{r}_n}{n} \quad \textbf{(28)}$$

Where w_i are the weights and r_i are the interest rates with $i = 1,2, \ldots, n$.

In the low interest rate environment, the bank will have the rates presented in Table 6.
For instance, in January or May, the bank will get the rate presented in Equation (29) and (30) respectively.

$$Bank_{January} = \frac{6\% * -0.234\% + 20{,}89\% * -0.212\% + \cdots + 7.06\% * 1.12\%}{5} = -0.011\% \quad \textbf{(29)}$$

$$Bank_{May} = \frac{17.29\% * 0.010\% + 7.06\% * 1.12\%}{2} = 0.040\% \quad \textbf{(30)}$$

Table 5. – Rate obtained by the bank in a low interest rate environment.

Month	Year	EUR_1M	EUR_2M	EUR_3M	EUR_1Y	EUR_10Y	Bank
Jan	2018	-0.234%	-0.212%	-0.201%	0.124%	1.120%	-0.011%
Feb	2018		-0.212%	-0.201%	0.010%	1.120%	-0.015%
Mar	2018			-0.201%	0.010%	1.120%	-0.006%
Apr	2018				0.010%	1.120%	0.040%
May	2018				0.010%	1.120%	0.040%
Jun	2018				0.010%	1.120%	0.040%
Jul	2018				0.010%	1.120%	0.040%
Aug	2018				0.010%	1.120%	0.040%
Sep	2018				0.010%	1.120%	0.040%
Oct	2018				0.010%	1.120%	0.040%
Nov	2018				0.010%	1.120%	0.040%
Dec	2018				0.010%	1.120%	0.040%

Now, let us have two new comers in a rising rate environment. I assumed they followed the same investment rule, *i.e.* the investment rule 1135. I also assumed that New comer 1 enters the market in October 2018 and New comer 2 in December 2018. For the future paths of interest rates, I used a CIR2+ with a displacement such that it is named on the online app "DCIR" with two risk drivers. I simulated 1000 different paths for a 1y-simulation for the different EURIBOR maturities' rates except for the EUR-10y since it was not available. For this last, I invented some figures which could be following the simulated path. For the other maturities, I used an average of the 1000 paths to find a reference path. Hence, I get the different interest rates in Table 6. In green are the rates at which New comer 1 will invest his replicating portfolio and in dark orange the rates of New comer 2. The simulation of the future interest rates begins in June since rates for May are actual data.

If we compute the rate for New comer 1 and New comer 2, we will obtain the results in Equation (31) and (32).

$$New\ comer\ 1_{October} = \frac{6\%*0.043\%+20{,}89\%*0.057\%+\cdots+7.06\%*2.68\%}{5} = 0.06\% \qquad \textbf{(31)}$$

$$New\ comer\ 2_{December} = \frac{6\%*0.055\%+20{,}89\%*0.071\%+\cdots+7.06\%*2.92\%}{5} = 0.069\% \qquad \textbf{(32)}$$

The example is showing that backward-looking approach can threaten the bank against new comers. Since the interest rate environment is rising, each new comer will benefit from higher rates and hence can propose higher deposit rates as we have in the example above:

$$Bank_{January} < \ New\ comer\ 1_{October} < New\ comer\ 2_{December} \qquad \textbf{(33)}$$

Table 6. – Evolution of the different EURIBOR rates and simulation starting in June.

Month	EUR_1M	EUR_2M	EUR_3M	EUR-6M	EUR_1Y	EUR_10Y
Jan	-0.234%	-0.212%	-0.201%	0.020%	0.124%	1.120%
Feb	-0.204%	-0.184%	-0.181%	0.070%	0.124%	1.150%
Mar	-0.156%	-0.131%	-0.122%	0.102%	0.124%	1.190%
Apr	-0.038%	-0.023%	-0.019%	0.134%	0.124%	1.340%
May	0.006%	0.020%	0.084%	0.180%	0.334%	1.57%
Jun	0.013%	0.024%	0.089%	0.184%	0.342%	1.61%
Jul	0.021%	0.031%	0.094%	0.189%	0.352%	1.94%
Aug	0.029%	0.042%	0.103%	0.194%	0.361%	2.01%
Sep	0.036%	0.045%	0.103%	0.200%	0.372%	2.67%
Oct	**0.043%**	**0.057%**	**0.107%**	**0.205%**	**0.382%**	**2.68%**
Nov	0.050%	0.062%	0.108%	0.210%	0.392%	2.84%
Dec	**0.055%**	**0.071%**	**0.111%**	**0.210%**	**0.395%**	**2.92%**

There are however some drawbacks in the illustrative example. First, the bank will not invest in securities which are getting negative returns. The example allows them however to emphasize the impact of negative rates in the modelling of the static replicating portfolio. Second, the new comers do maybe not follow the same investment rule of the bank. Maybe that they use not the same tool for the modelling of the duration for non-maturing liabilities. Indeed, new comers could not be financial institutions and propose only online accounts due to their daily business, *e.g.* Amazon or Google. Last but not least, the EUR-10y path was randomly set. Maybe its value compared to the EUR_1Y value is less pronounced and hence the difference between the rate of the new comers and the one proposed by the bank could be smaller.
Nevertheless, this example shows an evidence: in rising interest rate environment, backward-looking approach for the modelling of non-maturing liabilities' duration is not the optimal method. On the contrary, the dynamic replicating portfolio is forward-looking and hence can simulate future paths in order to avoid having smaller rates than new comers in a rising rate environment.

Method n°2: Dynamic replicating portfolio

To overcome any potential inefficiencies arising from this "lack of foresight", future responses to changes in the risk factors should also be considered in the determination of the current decision. We can achieve this by multistage stochastic programming techniques such the dynamic replicating portfolio. That is called *multistage* programming because the weights at which new tranches are allocated will be readjusted from stage to stage along each scenario. Instead of fitting the weights of the replicating portfolio to a single historic scenario only, the optimal allocation of new tranches for a given planning horizon is calculated at all point in time along each scenario for future market rates, client rate and volume. These three variables are the risk factors that we need to define to compute a dynamic replicating portfolio. "Optimal" can be specified in a number of ways such minimization of the tracking error, maximization of the margin subject to risk limits,… Additional restrictions such minimum margin, loss, portfolio composition can be integrated if desired.
In this section, we will first define the three risk factors mentioned above and then show an example of dynamic replicating portfolio.

1. Risk factors definition

1.1. Market rates

The market rate model describes the evolution of those interest rates that are relevant for the reinvestment of the deposit position. The "low-for-long" interest rate environment is hurting the net interest margin and provokes tremendous economic effects. However, they can cause technical problems as for instance the inability of functions to deal with negative inputs from mathematical models for simulating different paths. These technical problems can lead to computation problems for the modelling where the financial position of assets and/or liabilities cannot be valued. If that happen, the positions cannot be hedged effectively. Banks need to take into account the negative levels of the interest rates by choosing the most relevant interest rate model.
The market rate model has to consider the negative values of the term structure. In other terms, it will be more relevant to take into account scenarios were the market rates are negative, although non-maturing deposits ensure that customers will always be paid positively.

However, some models do not permit negative levels. Thus, a model such the LIBOR Market Model is no longer relevant because it models forward and zero-coupon (ZC) rates in a strictly positive way. For some models, the diffusions are assumed to be normally distributed and permit simulation of negative rates while they also reproduce the initial ZC curve. For instance, the 1-factor Hull-White model and the G2++ model[41] are relevant model for this kind of assumption. To remind, the G2++ can model parallel curve risk and slope change risk while 1-factor model do not afford the modelling of the second risk.
Empirically, the 1-factor proved containing too few parameters to properly replicate observed implied volatilities[42]. On the contrary, the G2++ seems to reproduce at-the-money (ATM) volatilities well, without reproducing perfectly the out-of-the-money (OTM) volatilities. This inaccuracy produces a problem for computing volatility smiles. Then, some models adapted from the forward diffusion LIBOR Market Model appear to be particularly robust. They provide negative interest rates, fit the initial ZC curve and the volatility surfaces such smiles. For instance, the Displaced Diffusion – LIBOR Market Model (DD-LMM) and the LMM+ (which is an adaptation of the DD-LMM with a stochastic volatility) offer better fit to observe volatility smiles. Then, the CIR2++ are based on a shifted χ^2 diffusion for the instantaneous rate and they can also reproduce volatility smiles. Moreover, it can take in account two states of the world as the high and the low interest rate environments and thus can identify shock whatever they are. However, the DD-LMM and the LMM+ can produce explosive IR scenarios when volatilities are high together with a low shift value. These explosive interest rate situations

[41] To remind, it is a 2-factor hull-White model
[42] Implied volatility and volatility smile are presented in annex n°2

cannot be accepted by ALM models and must integrate an IR cap to temper them. But the introduction of the cap has to integrate a floor to keep concern about the martingality[43] of the process which is applied to the generated scenarios.
If you do not take in account the negative levels of the interest rates for the modelling, the ALM will face to problems when it comes situations like today. If the interest rate model used assumes only positive interest rates such the LMM model, it becomes impossible to reproduce the actual spot interest rate curve such it was showed in the actual yield curve in the "interest rate theory" section in the annex. Indeed, the very short maturities which reflect negative interest rates would be not effectively reproduced by the model. In this sense, it will be interesting to use the LMM+ or the G2++ to allow negative interest rate levels for the modelling and hence the use of scenarios simulation.

1.1.1. G2++

G2++ is one application of the Hull-White 2 factors model. The main characteristic of the HW models is its ability to match the initial yield curve by using a shift function.
First, the One-factor Hull-White, which is a generalization of the Vasicek model, follows the short rate dynamics as in Equation (34).

$$dr_t = (\theta_t - ar_t)dt + \sigma dW_t \tag{34}$$

where W_t a Wiener process, θ_t a deterministic function of time and *a* is a factor which multiplies the short-term rate r_t.

The differential stochastic equation reflects a mean reversion where the speed at which r_t returns to its long-term mean[44] is defined by *a*. Bigger the value of *a* is, faster r_t returns to its mean. Since r_t is normally distributed, the Hull-White also affords negative rates. However, as the rates afford negative values, this model is not relevant to model market rates in non-maturing liabilities model even if negative rates were observed for the Swiss overnight rates. However, since this is a one-factor model, the correlation with entire term structure will be by 100% which did not reflect the reality. Moreover, the moves across the curve are determined by one single factor which creates a deterministic relation between all points of the curve. This relation cannot be deterministic otherwise the correlation between the rates will be 100%.
Second, a stochastic component u_t is added with a mean reversion *b* which is lower than *a*. A correlation factor ρ between the Wiener processes is added to drive the dynamics of r_t and u_t. That leads us to the Two-factor Hull-White model in (35) and (36).

$$dr_t = (\theta_t + u_t - \bar{a}r_t)dt + \sigma_1 dW_{1,t} \tag{35}$$

$$du_t = -\bar{b}u_t dt + \sigma_2 dW_{2,t} \tag{36}$$

with $dW_{1,t}\, dW_{2,t} = \rho dt$

The difference between the two models is the introduction of a non-trivial correlation between forward rates, which ensures a more realistic view of the market rates.

1.1.2. CIR2++

The Cox-Ingersoll-Ross (1985) model is an equilibrium asset pricing model for the term structure of interest rates. The model is important for linking intertemporal asset pricing theory and term structures of interest rates, preserving nonnegative values for interest rates, and producing simplicity in solutions for bond prices. CIR proposed an alternative to the Vasicek model in Equation (37).

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dW_t \tag{37}$$

[43] It is the ability of the scenarios to exhibit the martingale property and hence deliver arbitrage-free convergent Monte Carlo estimators for any set of cash-flows. A martingale M_t is a stochastic process such that $M_O = E(M_t)$
[44] This is a long-term mean if the process assumes a positive constant θ. This assumption was discussed to explain a general case of mean reversion.

where a is the speed at which the process goes back to its mean, b the long-term mean, r_t the short-term rate, W_t a Wiener process and σ is the volatility.

Having the same mean-reverting drift as Vasicek, the diffusion coefficient contains a volatility proportional to the short-term rate level. This means that if r_t increases, then the volatility will increase too.
As we explained in the previous model, a one-factor model is a deterministic function which is not the relation that exists between rates. In consequence, there is an extension to illustrate a more realistic view of the market rates moves. In this sense, we get:

$$r_t = x_t + y_t \quad \textbf{(38)}$$
$$dx_t = k_x(\theta_x - x_t)dt + \sigma_x\sqrt{x_t}dW_{1,t} \quad \textbf{(39)}$$
$$dy_t = k_y(\theta_y - y_t)dt + \sigma_y\sqrt{y_t}dW_{2,t} \quad \textbf{(40)}$$

with $dW_{1,t}\, dW_{2,t} = \rho dt$
x_t and y_t are two short-term rates.

As it is shown in the CIR equation, the diffusion part of the stochastic differential equation depends on the market rate. That means that if the interest rate is low, the volatility computed will be also low. This is a major problem when "low-for-long" period for the computation of replicating portfolio since the first risk factor to compute is the market rate. Because the sample contains low interest rates data during few years, that will influence the diffusion part prediction to the bottom.

1.1.3. LIBOR Market Model and LMM+

The LIBOR Market Model (LMM) is an interest rate model based on evolving LIBOR market forward rates. In contrast to models such the Hull-White or the Black-Karasinski models, the objects modelled using LMM are market-observable quantities such LIBOR forward rates. The LMM is also famous for the consistency with the market standard approach for pricing caps for the Black's formula which are used to hedge against interest rates risk[45]. The LMM can be used to price any instrument whose pay-off can be decomposed into a set of forward rates by assuming that the evolution of each forward rate is lognormal. Each forward rate has a time dependent volatility and time dependent correlations with other forward rates. The interest rate curve is seen as a subsequent sequence of forward rates $F_k(t)$ with $k = 1,2, \dots, n$ and $\sigma_k(t)$ is the volatility of each forward rate, with $t = 1,2, \dots, T$. The LMM is then represented by the Geometric Brownian Motion (GBM):

$$dF_k(t) = \mu_k(t)F_k(t)dt + \sigma_k(t)F_k(t)dW_k \quad \textbf{(41)}$$
$$cov(dW_i dW_j) = \rho_{ij}dt \quad \textbf{(42)}$$

Where dW_k is a standard Wiener process and $cov(dW_i dW_j)$ represents the covariance between all forward rates.

The drift $\mu_k(t)$ is arbitrage-free and can be calculated using the other forward rates with their instantaneous volatilities. These instantaneous volatilities can in turn be computed by their correlations and will help to describe their evolution in the future. For the instantaneous volatilities and correlations, there are three different approaches. First, they can be obtained from the analyzing historical data. Second, they can be obtained by calibrating the model to current market prices of interest rates derivatives. And last, the user can explicitly specify volatilities and correlations on their evolution in the future according to his opinion. The main

[45] The most famous interest rates derivatives including the caps will be assessed in the next section

reason for the popularity of the LIBOR market model is the recovery of the Black-Scholes prices taken from the formula.

There remain some problems for this type of model. First, the standard lognormal LMM does not reproduce the market-observed volatility smile because the volatility of forward rates is assumed to be deterministic as $\sigma_{inst} = \sigma_{inst}(T, t)$ where t denotes the calendar time and T is the maturity of the forward rate. To get this smile, there is the other model such the LMM+ which will add a stochastic volatility. The second and most important problem bears on the title of this section: the negative interest rates level. Such it was said above, the standard LMM only produces positive inputs due to the Geometric Brownian Motion. To understand this, suppose that $\boldsymbol{Z} = \{Z_t : t \in [0, \infty)\}$ is a standard Brownian motion and that $\mu \in \mathbb{R}$ and $\sigma \in (0, \infty)$. The standard Brownian motion is then defined as:

$$Z_t = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma Z_t \quad \textbf{(43)}$$

Then, the Geometric Brownian Motion is the exponential of this stochastic process such:

$$X_t = \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma Z_t\right] \quad \textbf{(44)}$$

By being an exponential, the GBM cannot output negative values. At the moment of Monte Carlo simulation, the short maturities of the yield curve will not be reproduced.

Then, the LMM+ corresponds to a displaced diffusion LIBOR Market Model with a stochastic volatility. The stochastic volatility is important to reproduce the smile in order to compute the market prices for interest rate risk derivatives. But in low interest rate environment, the use of a displacement coefficient, allowing to forecast interest rates in the negative region, is becoming a market standard. That leads to the Displaced Diffusion SV-LMM denoted DD-SV-LMM[46].
This model is such that:

$$\frac{d(f_i(t))}{f_i(t)+\alpha} = \mu_i^{(\alpha)}(\boldsymbol{f}, t)dt + \sigma_i^{(\alpha)}(t) g_i(t) dZ_i(t) \quad \textbf{(45)}$$

and

$$E[\mathrm{dZ}_i \mathrm{dZ}_j] = \rho_{i,j} \quad \textbf{(46)}$$

First the displaced diffusion is represented by changing $f_i(t)$ in $f_i(t) + \alpha$ as a diffusion. Indeed, the forward rate is diffused by adding to it some constant α which is called the displacement coefficient. If the parameter α is put posteriori, then reasonable values of α gives a rather small probability of experiencing negatives rates. If it is put a priori, then it is necessary to impose a cap and a floor to avoid explosive scenarios such it was explained above. Moreover, the parameter α permits to control the forward rates distribution for targeting the skew of the volatility.

For the stochastic volatility part of the model, it contains the stochastic volatility itself, $g_i(t)$ and it adds a Wiener process. The purpose of $g_i(t)$ is to keep the volatility stable. The volatility form takes a mean reverting process such the CIR. The mean reversion causes the average volatility to be more variable for shorter maturities than longer. As a lower variability in the volatility will result in a shallower smile, the mean reversion will result in the smile being shallower for longer maturities than shorter.

[46] Which in turns will be named LMM+.

When we compare both models, it seems that the LMM+ is more intuitive. First, by adding the displaced diffusion via the diffusion coefficient α, the volatility of the percentage increments is not constant as in the standard model but decreasing for an increasing forward rate. Then, the stochastic volatility permits to maintain the parametric form of the volatility but lets the coefficients of the function be stochastic. In consequence, the displaced diffusion will result in a skew in implied volatilities and the stochastic volatility in a smile. This allows to replicate the volatility smile and is convenient for derivatives pricing. In conclusion, we will say that the model is *market consistent*, *i.e.* the ability to reproduce prices and hence implied volatilities.
The LMM+ seems to be the best model we proposed in this work since it follows the stylized facts of the market:

- A long-term mean reverting: the interest rates are stationary;
- A proportional volatility: the high interest rates increase the probability that another extreme movement happens. The opposite case for low interest rates holds;
- Multi factors: the difference between rates' maturities shows imperfect correlation. On the contrary, a Hull-White 1 factor shows a correlation with entire term structure of 100%;
- Stochastic volatility: the interest rates are subject to extreme uncertainty periods;
- Negative interest rates.

1.2. Deposit rate and deposit volume

The client rate represents the "benchmark" in the optimization model and has direct impact on the composition of the replicating portfolio. Therefore, particular attention must be paid to the specification of its evolution and dependency on interest rates. After larger changes in the level of markets rates, it is then adjusted by the bank with some delay. Often, an asymmetric response to market rate changes can be observed, *i.e.* deposit rates move more rapidly when market rates drop than they rise. Moreover, the deposit rate does not exceed a certain level even if market rates are still increasing.
Since examples of market rates model were seen previously, this section will explain the two latter components in the literature. We will not talk about the modelling of Frauendorfer and Schürle since we described the client rate and volume evolutions as illustrative for a dynamic replicating portfolio in the interest rate theory part. The structure of NMDs modelling in the literature follows Andreas Kordel (2017). Then, we need to show how to define the deposit rate and the deposit volume. This section will show first different models such linear or behavioral ones and then go to the different replicating portfolios.

1.2.1. Linear models

First, let us begin with linear modelling for the NMDs. Nyström (2008) suggests using three components we mentioned above as market rates, deposit rates and deposit volumes. Market rates are modelled using an extended one-factor Vasicek model while deposit volume is modelled under behavioral assumptions under which individual customers income are supposed to be based on their behaviors. The volume is assumed to target a certain volume which is a fraction of their average monthly income and is supposed to cover the customer's liquidity needs. Nyström ensured that deposits rates are function of market rate and/or volume but without adding an example. Although he did not illustrate his article with a model, he afforded that the deposit rate d_t followed the market rate r_t as in Equation (47).

$$d_t = \beta_1 r_t \quad \textbf{(47)}$$

The deposit rate function is a policy function that is determined by the bank which determined β_1 as constant. The aim of the paper of Nyström is to find on an optimal policy function with a deposit rate framework that was created.
Then, Elkenbracht and Nauta (2006) modelled deposit rates for stabilizing the margin between the investment return and the deposit rates.

Similarly, to the previous approach, they did not create a model, but they described the deposit rate linearly such:

$$d_t = \beta_0 + \beta_1 r_t \tag{48}$$

where β_0 and β_1 are supposed constant and need to be fitted.

The advantage of using linear models is the simplicity for the specification as it does not allow for the consideration of asymmetric adjustments. They are very simple to implement, and they give a rather good fit. However, Hutchison (1995) find a clear evidence that the linear models have an incomplete adjustment of the product rate for US negotiable order of withdrawal[47] and money market deposit accounts[48] from 200 banks. Moreover, the analysis is static and does not account rigidity or asymmetry in response to market rate changes. According to Ausubel (1992), this is because limited information on market investment alternatives and search costs influence the deposit rate setting that are observed. These models also ensure that the bank will lose deposits when deposit rates are low or if the spreads between the client rate and market rates are narrow which this is actually not the case. As described in the inventory of exposure in interest rate risk, the customers have not the incentive to switch of banks since they do not want to move their deposits. Then, the asymmetry is illustrated by O'Brien (2000) and adapted to Belgian savings by Maes and Timmermans (2005). The description of the model was seen above in the interest rate theory but to remind quickly, the deposit rate changes depend on the level of the current deposit rate in comparison with its long-term level. The deposit rates are particularly sluggishly when deposit rates are below and are adjusted more swiftly when they are above this level. In their paper, they modelled changes in deposit rates such Equation (49).

$$\Delta R_t = \left(\lambda^+ I_t + \lambda^-(1 - I_t)\right)(br_t - g - R_{t-1}) + e_t \tag{49}$$

$$I_t = 1 \qquad \text{if } br_t - g - R_{t-1} > 0 \tag{50}$$
$$I_t = 0 \qquad \text{otherwise} \tag{51}$$

where λ^+, λ^-, b and *g* are the parameters to be estimated.

The variable R_t stands for the deposit rate and $br_t - g$ is defined as the unobserved time-varying equilibrium deposit rate, itself a function of the market rate r_t such 3 months Euribor. Then, *b* can be considered to be a proxy for 1 minus the marginal reserve requirement is such a requirement applies, and *g* reflects the positive servicing cost. I_t is an indicator variable that signals whether actual deposit rates are above or below the long-run equilibrium deposit rate level, which is assumed to be at its break-even level. The parameters λ^+, λ^- reflect the speed at which the deposit rates adjust to the equilibrium level. Finally, e_t is the error term. What they found for largest Belgian banks by using a sample between 1996 and 2004 is that λ^+ is estimated to be substantially smaller than λ^- which confirms that the deposit rates react rather sluggish when being higher than long-run equilibrium deposit rate and more swiftly when being lower[49]. What it could be interesting is to test this model with a sample between the end of the economic crisis and today in order to verify if this dynamic is explained by the more or less same difference between the two parameters.
Kordel (2017) confirmed what we said in the introduction of this section about the moving average. Indeed, he proved by implementing the different linear models with the observed

[47] Negotiable order of withdrawal account (NOW) is "an interest-earning account on which chechs may be drawn" Campbell R. Harvey (2012). This is a checking account that earns interest where an account holder can write checks against the money he holds in his account.
[48] "An account, such as checking or savings account, that pays a certain (low) interest rate to the holder and from which funds may be withdrawn on demand". Farlex Financial Dictionary (2012).
[49] λ^+ was estimated between 0.000 and 0.002 while λ^- was estimated between 0.072 and 0.267

market and deposit rates that the moving average reacts too slowly to new events even if he shorts the moving average from 6 months to 1 month. Although the models work well in a non-negative interest rate environment, they have a poor fit in a negative interest rate environment since the models blindly follows the reference rate although the deposit rate is floored at zero[50].

1.2.2. Jarrow and van Deventer (1998)

The study of Jarrow and van Deventer (1998) focus on two-sides of the market such the banks and the individuals with an arbitrage free approach to model the NMDs. They assumed that there are entry barriers for banks to supply NMDs, although both banks and individuals have unrestricted access to the treasury market. With these assumptions, they described the value of the NMDs Ψ_0 such:

$$\Psi_0 = E_0^Q\left[\sum_{t=0}^{\tau-1} \frac{V_t(r_t - d_t)}{B_{t+1}}\right] \quad \textbf{(52)}$$

where V_t is the volume at time *t* and τ is the number of periods used. The value of the money market account at time *t* is denoted B_t and is obtained by (53) and (54).

$$B_t = B_{t-1}(1 + r_{t-1}) \quad \textbf{(53)}$$

$$B_0 = 1 \quad \textbf{(54)}$$

The expectation $E_0^Q[\cdot]$ is taken under the unique risk neutral martingale measure *Q* generated by the term structure[51]. Since the Ψ_0 has to be martingale in a specific measure, and hence by a risk-neutral measure because of the arbitrage-free assumption. The economic interpretation of Ψ_0 is such that the value at time 0 is like an exotic interest rate swap that receives the floating short market rate and pays the deposit rate with a principal V_t in period *t*. Jarrow and van Deventer described the present value of the NMD liability to the bank at time 0 equals the initial demand deposit less the net present value of the NMD as in Equation (55).

$$V_0 - \Psi_0 \quad \textbf{(55)}$$

This value can be hedged by investing V_0 in the bond with shortest maturity available and by shorting the value of the NMD Ψ_0. Since the deposit dynamics are more realistic to simulate in continuous time, the value of NMDs is equal to:

$$\Psi_0 = E_0^Q\left[\int_0^\tau \frac{V_t(r_t - d_t)}{B_t}\partial t\right] \quad \textbf{(56)}$$

Then, they modelled the deposit rate d_t that has a short market rate as the driving factor in a continuous time:

$$\partial d_t = (\beta_0 + \beta_1 r_t)\partial t + \beta_2 \partial r_t \quad \textbf{(57)}$$

or equivalently

$$d_t = d_0 + \beta_0 t + \beta_1 \int_0^t r_s \partial s + \beta_2(r_t - r_0) \quad \textbf{(58)}$$

where β_0, β_1 and β_2 are parameters that should be estimated.

[50] If we modelled our deposit rate level, we would have to stick the level at 0.11% instead of 0.

The model of Jarrow and van Deventer is quite simple to implement and has one of the best goodness of the fit between tested models of Kordel (2017). However, the model has the same problem as the linear models when it comes to handling negative interest rate environments. When the deposit rate is floored the model may still predict a negative deposit rate which need to be avoid from a regulatory point of view. In the out-of-sample analysis, the model falls far from the real deposit rate after having performed well few times. The reason bears on the low level of interest rates and on the deposit rate zero-level over a long-time period. Indeed, an error in the beginning of the prediction period has consequences for the whole prediction.

1.2.3. Castagna and Manenti (2013): a behavioral model

The last model I would like to present is the NMD model Castagna and Manenti presented in their study in 2013. The advantage of this study is that they took into consideration low interest rates levels for modelling in an optimal way. They described stochastic factor models as most significant and effective approaches for interest rate and liquidity risk management of these balance-sheet items. According to them, the volumes depend on the liquidity preference and risk-aversion of the depositors for which the opportunity cost between different allocations is conducting their behavior. Stochastic factor approaches are then *behavioral* because they try to explain the dynamics of these behaviors with respect to market rates and deposit rates movements. For market rates, they used CIR++ model such the model is capable to perfectly match the current observed term structure of risk-free zero rates.

Then, the deposit rate evolution is linked to the pricing policy of the banks since increasing the deposit rate can be increased in order to avoid an incentive from the depositors to withdraw their money. This increase could even increase the amount which is deposited. They adopted a deposit rate d_t in a linear relationship with the market short rate r_t:

$$d_t = \alpha + \beta r_t + u_t \tag{59}$$

where $E(u_t) = 0 \quad \forall t$ and u_t the error term.

From there, they modelled the deposit volumes in a linear behavioral function and in a non-linear one. In the linear model, they ensured that the deposit volume's evolution depends on the deposit rate such that this is discretionary for the bank for driving the deposit volume and defining liquidity strategies. Since the deposit volumes depend on behaviors, they modelled them as linear relationship between its log-variations and the risk factors, *i.e.* market interest and deposit rates. They also added an autoregressive component and imposed that the log-variation of the previous period with a given factor. They finally included a relationship with time *t* for detecting time-trends:

$$logD_t = \gamma_0 + \gamma_1 \log D_{t-1} + \gamma_2 t + \gamma_3 \Delta r_t + \gamma_4 \Delta d_t + \epsilon_t \tag{60}$$

with Δ being the first-order difference operator and ϵ_t the idiosyncratic error term with zero mean. The parameters presented in this formula which seems to be in practice the same as in Jarrow and van Deventer can be estimated easily on historical data via standard OLS algorithm.

They took a non-linear model from Nyström's work as the linear relationship does not capture the empirically observed depositors' reactions to market and deposit rates' movements. This is due to the non-linearity of the actual behavior observed when they react to market and deposit rates' movements. They based their model on the main idea that depositors prefer to keep their investments in liquidity for low levels of market rates. As market rates increase, the preference for liquidity is counterbalanced such that the depositors transfer money to fewer liquid investments.

They begin in this way to consider that the total depositors' income *I* is growing at an annual rate ρ. Then, they assumed that each depositor modifies his balance on the deposit account targeting a given fraction $\bar{\lambda}$ of their income which can be interpreted as the amount they need to cover their short-term liquidity needs. Given the current fraction λ_t which happens in time *t*, this fraction $\bar{\lambda}$ is targeted at a speed ζ. After, there is an interest rate's strike level *E*, with which the customer will compare the market rate. When the level is below the market rate, they reconsider the target level and redirects a higher amount to other investments by a fraction γ of *I*. Finally, they assumed that there is a deposit rate's strike level *F*, specific to the customer. When the rate paid on deposit is above *F*, then they are more reluctant to withdraw money by a fraction δ of *I*. With these assumptions, they modelled the deposit volume as

$$D_t = \lambda_t + I_t \tag{61}$$

with the evolution of the fraction λ_t of the income allocated in sight deposit is

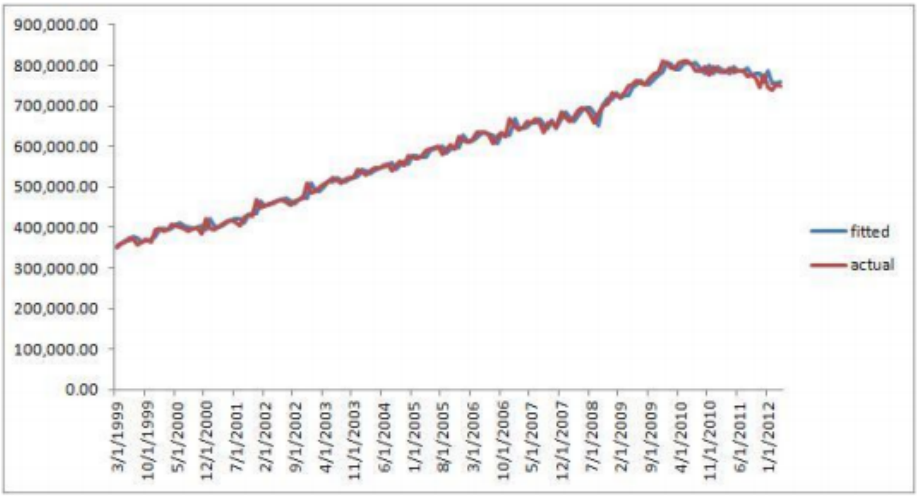
$$\lambda_{t+\Delta t} - \lambda_t = \zeta\left(\bar{\lambda} - \lambda_t\right)\Delta t + \gamma\, \mathbf{1}_{[E,\infty]}(r_t) + \delta \mathbf{1}_{[F,\infty]}(r_t) \tag{62}$$

where $\mathbf{1}_{[E,\infty]}$ is the indicator function equal to 1 when the condition in the subscript is verified. Finally, the evolution of the income *I* is:

$$I_{t+\Delta t} - I_t = I_t \rho \Delta t \tag{63}$$

Then, they underlined that each depositor has different level of strike rates *E* and *F* due to their preferences for liquidity such that there is a distribution of strike rates reflecting their heterogeneity in behavior.
They plotted the actual time series of deposit volumes and the fitted values for the linear behavioral model and show that the model explained well the data. The plot is presented in Figure 12.

Figure 12. Actual time series of deposit volumes vs fitted values for the linear behavioral model



The behavioral models such Castagna and Manenti modelled are important since they represent one of the core elements of the new IRRBB framework. Indeed, the interest rate risk calculation can be approached by the banks with a certain level of freedom.

2. Example from Frauendorfer and Schürle (2003)

In a rising interest rate environment, the dynamic replication has a big advantage on the static one since the latter is using a moving average. At the period where the rates go up, the moving average will be computed on previous observations, *e.g.* on last three months, which are at low levels. In this reasoning, the banks will face to a difference between current market rates and the rate that the static replicator has modelled. Hence new comers will profit from this difference by proposing higher rates as it will be illustrated in the example of Frauendorfer and Schürle (2007). They ensured that stochastic programming models provide higher and simultaneously less volatile margins than the static replicating portfolio approach even if they were not constructed to "track" the NMD exactly.

Frauendorfer and Schürle rejected the static replicating portfolio for two reasons. First, they explained that the weights and the relevant maturities at which maturing instruments and a volume change are reinvested must be recalculated frequently. Indeed, they observe that the weights depend heavily on the sample period and the corresponding margins are unstable. That means that the significant fluctuations in the portfolios would make hedging strategies inefficient. Then, they added that the replicating portfolio is not derived from a single historical scenario but from many scenarios for the possible future outcomes of the relevant risk factors. In consequence, they achieved a dynamic replication by a multistage stochastic programming model and test it on Swiss market rates.

First, they provide an extension of the Vasicek model with $K = 2$ factors represented in Equation (64) and (65).

$$d\eta_{1t} = \kappa_1(\theta - \eta_{1t})dt + \sigma_1 dz_{1t} \quad \textbf{(64)}$$
$$d\eta_{2t} = -\kappa_2\eta_{2t}dt + \sigma_2 dz_{2t} \quad \textbf{(65)}$$

Both state variables exhibit mean reversion around long term means of θ and zero, respectively. The parameters $\kappa_i, i = 1{,}2$, control the speed at which the factors revert to these levels. σ_i is the instantaneous volatility of the *i*-th process and z_1 and z_2 are two uncorrelated Wiener processes. The first factor η_{1t} moves parallel to the 5Y rate, η_{2t} is highly correlated with the spread between short and long rates (3M-5Y). $\eta_{1t} + \eta_{2t}$ is equivalent to the instantaneous short rate of a single-factor model (3M). Second, they modelled deposit rates and volume as follows: as they observed that the deposit rate does not exceed a certain level even if market rates are increasing, they didn't regress the estimation with OLS[52]. Even if the markets rates freely fluctuate, the deposit rates remain dependant each other. Thus, the assumption of the normal distribution of the error term will lead to biased and inconsistent estimators. Moreover, they used previous values of the deposit rate as well as the current and past observations of the level factor. As they assumed that $\delta_0 < \cdots < \delta_n$ are the deposit rate increments, the actual change at time *t* depends on the outcome of a control variable:

$$c_t^* = \beta_0 c_{t-1} + \beta_1\eta_{1t} + \cdots + \beta_{m+1}\eta_{1,t-m} \quad \textbf{(66)}$$

and follows the rule

[52] Ordinary least square

$$\Delta c_t = \begin{cases} \delta_0 & \text{if } c_t^* \leq \gamma_1 \\ \delta_1 & \text{if } \gamma_1 < c_t^* \leq \gamma_2 \\ \vdots & \\ \delta_{1-1} & \text{if } \gamma_{n-1} < c_t^* \leq \gamma_n \\ \delta_n & \text{if } \gamma_n < c_t^*, \end{cases} \tag{67}$$

where $\gamma_1 < \cdots < \gamma_n$ represent some threshold values and c_t is the client rate paid per period for holding the deposit account.

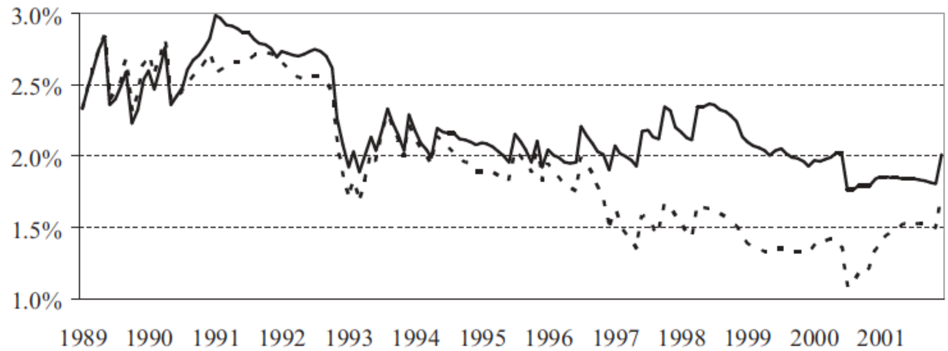
Finally, they modelled the volume according to the dependency on interest rates. They observed that when the market rates are above the average, the volume of savings deposits is low, and the contrary occurs if the markets rates are below. Based on this observation, they modelled relative changes in the deposit volume v_t by Equation (68).

$$\ln v_t = \ln v_{t-1} + e_0 + e_1 t + e_2 \eta_{1t} + e_3 \eta_{2t} + \xi_t \tag{68}$$

where the constant e_0 and the time component $e_1 t$ reflect that the total deposit volume exhibits a positive trend.

The factors $\eta_1$and η_2 of the market rates model are included to reflect the dependency of the interest rates. ξ is an additional uncorrelated factor which do not fully explain the observed evolution of the balance. By looking to the time series of the market rates time series from the calibration, the volume model was estimated by OLS regression with the stochastic factors η_{1t}, η_{2t}, and ξ_t multivariate normally distributed. They finally simulate a large set of scenarios and obtained their replicating portfolio by minimizing the tracking error at any time when the reinvestments time occur (at maturity or change in volume). They illustrated that the dynamic portfolio outperformed static portfolio represented in Figure 13.

Figure 13. – Comparison of the margin between static replicating portfolio (dotted line) and dynamic replicating portfolio (straight line)



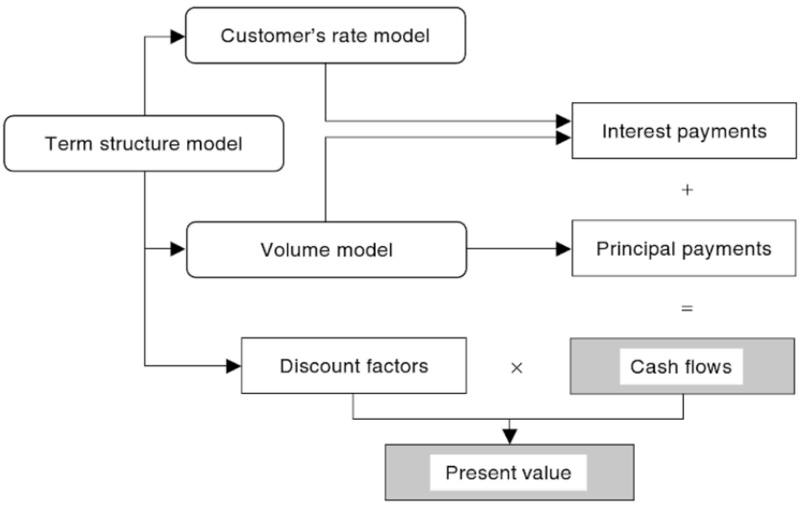
They compared the two methods and showed that the margin is higher with a reduced volatility. and outlined that the dynamic management don't reflect fluctuations of the time buckets and exhibits a higher duration (0.5y longer than the static replication). Then, they also observed that the volume risk is reflected more appropriately thanks to having considered the correlations with interest rates.

Method n°3: OAS models

The option-adjusted spread can be explained as a spread that represents the added value of having an option. Consider for example the difference between a callable and a non-callable bond. The callable bond is more expensive due to the fact that there is an embedded option. The OAS describes the spread the holder of the bond receives for providing this option. In other words, the OAS is the spread that must be added to the market rate so that the market value of the option equals the option value. For non-maturing liabilities, an OAS model aims to capture and model the options that are embedded in them, *i.e.* the customers' option to add or withdraw money anytime. That is also the option of the bank to change the deposit rate. Hence, the OAS models attempts to capture the value of these options. So, it is a way of viewing non-maturing products as highly complicated options and applying option pricing theory to deal with them.

OAS models are stochastic models and their foundation is the term structure model which will generate possible interest rate scenarios. The different term structure models were described in the "market rates" section with the CIR2+/, the G2++ or the LMM+. Since we are talking about "options", the LMM+ is more appropriate to capture volatility smiles. To capture the options in an OAS model, the further requirements are a deposit rate model and a volume model. The deposit rate model typically depends on the market rate with usually some lag. That might be as the deposit rate functions described in the "deposit rate and deposit volume" section. The volume model aims to depict the volume changes on the accounts and also shows the customer behavior, *e.g.* the model of Castagna and Manenti.
From this interest rate scenarios can be generated stochastically by use of Monte Carlo simulation and future cash flows can be estimated. Then, when the cash flows are estimated, the present value of the non-maturity liabilities could be calculated. The OAS models calculate the value of the embedded options by discounting the expected future cash flows at a discount rate which includes a spread to account for the riskiness of the cash flows. Figure 14 explains the functionality of the OAS model starting from the term structure model until the present value calculation for the non-maturing liabilities. The term structure permits to compute the three models for risk factors such the customer's rate, the volume and the discount factors. The cash flow is found by the interest payments from the customer's rate model and the volume model, and the principal payments come from the volume model. By discounting the expected future cash flow at a discount rate as said above, the present value for non-maturing liabilities is calculated.

Figure 14. – OAS model (Bardenhewer 2007)



Conclusion

In the first part, we talked about the three main challenges that the ALM is facing due to the low interest rate environment: lower interest rates, the change in the regulation for measuring the interest rate risk and the non-maturing liabilities modelling.

The first challenge is the lower interest rates. We studied the impact of the low interest rate environment on the profitability of the bank and its risk-taking. To evaluate that, we evaluated the impact on the NII. Even if banks can benefit from low interest rates, *e.g.* as non-performing loans will be lower because borrowers' debt will less burden, they can depress bank margins for many types of deposits. Indeed, banks are reluctant (or may be prohibited) to lower deposit rates especially below zero. When interest rates decline, banks margins compress, *e.g. floating rate loans will be offered against lower rates*. Since the current low interest rates environment is accompanied by a flattening of the yield curve, that also depresses the NIM of those banks which used to maintain a positive duration gap[53]. Empirical evidences show that banks are adversely affected by interest rates that are low for an extended period of time through narrower NIM. Both the level of interest rates and the slope of the yield curve are associated with higher net interest income. This relationship is likely to be especially strong at very low levels of nominal interest rates. In case of risk-taking, unusually low interest rate environment over an extended period of time contributes to an increase in bank risk-taking. The banks can increase its risk-taking by either non-interest activities and by raising fees.

Then, we studied the effect of the low interest rate environment on the balance sheet. With an example of Claude Moser, we showed that the "Low-for-long" period provoke a bigger duration imbalance between assets and liabilities. By having a bigger imbalance, banks need to rely more on external markets to hedge against interest rate risk with interest rates derivatives. The bigger duration gap between assets and liabilities is due to changes in the composition of the balance sheet. In the liabilities' side, the customers have the incentive to switch from fixed-term to non-maturing deposits. Indeed, the term deposits are no more profitable at low rates and the clients would rather get their excess cash at any moments. In this way, the customers will wait until the future interest rates' rise before going back to term deposits and getting higher returns. Moreover, the customers require using more liquidity than before. This is because the world became highly liquid due to the unconventional monetary policy.

On the asset side, the persistent low interest rate environment provokes a higher demand for floating-rate mortgages than for fixed-rate mortgages because customers expect that the interest rates are going to remain at low levels or even to decrease but not to rise. Consequently, the monthly repayments are lower than if they contract a fixed-rate loan.

The switch from fixed to floating rates mortgages is not the only change. The low interest rates can raise another problem. The bank can face to a prepayment risk. A customer had the opportunity to reimburse or to prepay the credit during the contractual life by two ways: either by finding a new loan with a lower interest rate and to prepay the existing loan, or to renegotiate the existing loan. If the market rates go down, the bank will agree to reduce the loan interest rate in order to keep the customer away from the willingness to change of bank. Sometimes banks can ask to these customers to pay penalties against a reduction of rates. As the customer repay earlier, the bank is face to prepayment risk. This action modifies the credit schedule and is potentially costly for the ALM. First, the IRR will provoke a cost on monthly payments since they will be lower after the rates change.

The second challenge is the change in the regulation. The Basel framework aims to measure the interest rate risk in a low interest rate environment for the bank. The regulation ensures that more there are assumptions, more the measurement will be effective. This environment complicates the measurement, especially for the duration estimation of the non-maturing liabilities. In this sense, the BCBS updated the IRRBB rules to be conformed to the low interest rate environment. The regulation aims to be stricter for the stress scenarios and complexifies the assumption for the measurement of economic value and earnings. They are calculated

[53] See section dedicated to "Duration & Convexity"

using interest rate shock and stress scenarios. More interest rate stress scenarios have to be studied compared to before. In this sense, the bank must follow six prescribed interest rate shock scenarios: parallel shock up/down, steepener/flattener shock and short rate shock up/down.
In the earnings measures, they promote funding gap analysis to identify and estimate the interest rate exposure for the repricing risk. This technique suffers from the problem that many of the assets and liabilities, which are not rate-sensitive, have different maturities. One solution to deal with this problem is the maturity bucket approach. But this approach has a big disadvantage since it plugs assets/liabilities with fixed and unfixed maturities in the same pool. An alternative method to measure interest rates risk is the duration gap analysis. It examines the sensitivity of the market value of the financial institutions net worth to change in interest rates.
Further complications were found. First, the gap analysis, the funding and the duration gap analysis, assume that interest rates on all maturities change by exactly the same amount. In other terms, that means that the slope of the yield curve remains unchanged. A flat yield curve is obviously not a realistic assumption. A second problem arises when comes the estimation of the average duration of assets and liabilities. Such the non-maturing liabilities, the future cash-flows are uncertain which complicates the estimation of the duration for the duration gap.

The third challenge for the ALM is the non-maturing liabilities modelling. The need to model these non-maturing liabilities is critical because there are no industry benchmark or implied parameters which can be used for this purpose. First, the bank needs to estimate how much of its current account base is likely to be stable and how long. Banks have been forced by regulators to make the switching of accounts easier and quicker. The decision to withdraw or deposit money from the accounts are more or less necessarily driven by market conditions. The interest rate risk on the non-maturing liabilities comes mainly from the problem in measuring the duration of non-maturing deposits (NMD). The future volume of the deposits is difficult to predict. The low interest rate environment provokes smaller remuneration on deposits than before. If the interest rate environment is rising, new comers or competitors can propose higher rates and steal some customers. There is also an increasing trend of buying deposits online which decrease the administration fees. The online account has another advantage: the customer can stay at home to manage his money. For these reasons, barriers to enter the market are lower which give the opportunity to companies such Orange or Amazon to propose online deposits with lower fees and higher technology skills. The low interest rate environment is a serious problem for bank since Millennials are ready to switch banks if the reward is higher.
Another problem of low interest rate environment that was enlightened was the rate at which the core part of the non-maturing deposits is invested. Indeed, the core part is a part in the deposit volume that is pretty stable. This part is invested in long maturities and hence bear more interest rate risk than previously assumed. The increasing IRR is due to longer maturities in the investment since the short maturities pay negative rates.

In the second part, we assessed how banks can measure the interest rate risk on non-maturing liabilities in this low interest rate environment. There are two ways to model them such the replicating portfolio's and the OAS models. The replicating portfolio's aim to replicate at best the non-maturing liabilities to model a replicating portfolio to get a most stable margin as possible. This is also possible to get a replicating portfolio on a criterion of "Sharpe ratio", but it was not our assumption in this thesis. We see two types of replicating portfolio's as the static replicating portfolio and the dynamic replicating portfolio.
The static replicating portfolio implements a backward-looking approach by using historical observations. The reinvestment rule that has the smallest standard deviation is the optimal replicating portfolio. However, in a low interest rate environment, backward-looking exhibits some limits. First, if the interest rates environment is rising, new comers might profit from higher rates than the bank with its reinvestment rule. Second, the weights and the relevant maturities at which maturing instruments and a volume change are reinvested must be

recalculated frequently. Indeed, the weights depend heavily on the sample period and the corresponding margins are unstable. That means that the significant fluctuations in the portfolios would make hedging strategies inefficient. Last, the static replicating portfolio is not derived from a single historical scenario but from many scenarios for the possible future outcomes of the relevant risk factors.
On the contrary, the dynamic replicating portfolio aims to replicate at best the non-maturing liabilities by simulating many scenarios and using three risk factors: the market rate, the deposit rate and the deposit volume. We began by showing three different market rates models for the first risk factor. CIR and Hull-White 1 factor or Libor Market Model do not permit negative levels. For some models, the diffusions are assumed to be normally distributed and permit simulation of negative rates while they also reproduce the initial ZC curve. For instance, the 1-factor Hull-White model and the G2++ model[54] are relevant model for this kind of assumption. To remind, the G2++ can model parallel curve risk and slope change risk while 1-factor model do not afford the modelling of the second risk. However, these two models do not reproduce the volatility smile for the options such in the OAS models. An extension of the Libor Market Model with a displaced diffusion and a stochastic volatility can model negative values and volatility smile at the same time. This model is called Stochastic Volatility Displaced Diffusion Libor Market Model (SV DD LMM or LMM+). The displaced diffusion will output negative rates and the stochastic volatility will experience the volatility smile. The model is then *market consistent* as it has the ability to reproduce prices and implied volatilities. To avoid outputting explosive scenarios, a cap and a floor need to be added.
For the deposit rate, it is often represented linearly with the market rate. Last, the deposit volumes were defined differently across studies. The most realistic assumptions were in the behavioral model of Castagna and Manenti. Their model assumed that the deposit volume's dynamics were totally behavioral and depend on the liquidity needs of the customers. We finished by illustrating this dynamic replicating portfolio made by Frauendorfer and Schürle. The dynamic replicating portfolio is a forward-looking approach which avoids the threats of new comers with the rising rates. However, it exhibits a higher duration than the static replicating portfolio which says that it is more sensitive to interest rates change. Moreover, the dynamic replicating portfolio is more complex to implement than the static replicating portfolio. In a low interest rate environment, reinvestment rates are low, but the future path of the market rates can influence the replicating portfolio we would choose. In a decreasing rate environment, the static replicating portfolio with its backward-looking approach is more relevant for its simplicity. In addition, a forward-looking approach will offer lower rates since reinvestment rates are lower than the previous ones. In a rising interest rate environment, the forward-looking approach is more efficient since the reinvestment rates will be higher than the previous ones.

The third way to model the non-maturing liabilities is the OAS models. The option-adjusted spread can be explained as a spread that represents the added value of having an option. For non-maturing liabilities, an OAS model aims to capture and model the options that are embedded in them, *i.e.* the customers' option to add or withdraw money anytime. That is also the option of the bank to change the deposit rate. Hence, the OAS models attempts to capture the value of these options. So, it is a way of viewing non-maturing products as highly complicated options and applying option pricing theory to deal with them. OAS models are stochastic models and their foundation is the term structure model which will generate possible interest rate scenarios. The OAS models calculate the value of the embedded options by discounting the expected future cash flows at a discount rate which includes a spread to account for the riskiness of the cash flows.

It will be interesting for further works to implement the dynamic replicating portfolio with the following three risk drivers. First, a deposit rate that depends linearly on the market rate. Then, a market rate model such the LMM+ that can model negative values with a cap and a floor for

[54] To remind, it is a 2-factor hull-White model

the avoidance of infinite scenarios. Last, a deposit volume such the behavioral model of Castagna and Manenti for the realistic assumptions they set. Last, it should be interesting to compare the prediction of the static replicating portfolio, the dynamic replicating of Frauendorfer and Schürle, the new dynamic replicating portfolio, and an OAS model with these risk factors.

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Annex n°1 : Interest rates theory

In order to understand the duration's modelling of non-maturing liabilities or different interest models, the interest rates' theory need to be briefly introduced. This section will begin by explaining the yield curve with spot and forward rates. Then, we will see the determinant of the interest rates level to understand how we reached a low interest rate environment. Finally, we will end this section by explaining the fundamentals of duration, convexity and immunization.

1. Interest rates and yield curves

1.1. The yield curve

According to the ECB, a yield curve represents "the relationship between market remuneration (interest rates) and the remaining time to maturity of debt securities"[55]. The yield curve is also known as the *term structure of interest rates* which is a representation of interest rates as a function of time to maturity.
Yield to maturity (YTM) is the return an investor get by holding his bond until it matures. For instance, the YTM of a fixed coupon bond is computed in Equation (69).

$$PV = \sum_{t=1}^{M} \frac{C_t}{(1+y)^t} \tag{69}$$

with $PV = \text{present, and } C_t = \text{bond cash flow, } \text{y} = \text{annualized yield to maturity and M} = \text{maturity}$.

The YTM is found by solving y in the equation. For instance, a 10y-bond having a 9% annual coupon which is trading $938.5 has a YTM of 10%.[56]

The Yield curve risk is the risk that the general level of interest rates may change, and that this consequently results in some adverse financial impact on the bank. This is the most common type of IRR affecting banks. A bank can only be subject to yield curve risk if there is a mismatch existing between assets and liabilities. Moreover, the yield curve risk is most represented by considering a parallel change in interest rates. The *parallel yield curve risk* is the parallel change that the interest rates for all fixed maturities move by the same amount. On the contrary, a *non-parallel yield curve risk* is the risk of some change to the shape of the yield curve, *i.e*. the yield curve become steeper if longer maturities go up more than the shorter one and flatter if the opposite happen.

In order to understand the different interest rate models, we have to differ the spot and the forward rates. Let us have a tenor structure with possible maturities T_1, T_2, ..., T_N and a tenor is defined as $\tau_i = T_{i+1} - T_i$. Then, the *discount rates*, meaning the price at time *t* of a zero-coupon bond paying 1 unit of currency at maturity time T_i, is denoted by $P(t, T_i)$. The *spot interest rate* at time *t* for maturity T_i is denoted by $R(t, T_i)$. The relationship between the discount and the spot rates is defined in Equation (70).

$$P(t, T_i) = \frac{1}{\left(1+\frac{R(t,T_i)}{m}\right)^{T,m}} \tag{70}$$

where *m* is the compounding frequency[57] of the spot rate

[55] http://faculty.baruch.cuny.edu/ryao/fin3710/pimco_yield_curve_primer.pdf
[56] https://www.fidelity.com/learning-center/investment-products/fixed-income-bonds/yield-maturity-bond
[57] The compounding frequency is for example *annually* or *semi-annually*. A semi-annually compounding frequency is 2 while a *monthly* compounding frequency is 12.

There subsists one important interest rate to complement these concepts which is the *forward rate*. This is the interest rate which will be contract for a time *t* for a loan between future time T_i and T_{i+1}. Given a set of zero-coupon bonds, we get Equation (71).

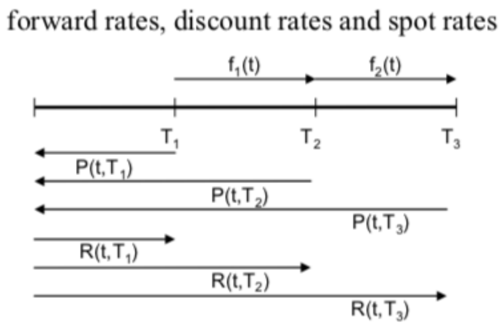
$$f(t, T_i, \tau_i) = \frac{\frac{P(t,T_i)}{P(t,T_i+\tau_i)} - 1}{\tau_i} \quad \textbf{(71)}$$

T_i and T_{i+1} are called the reset and the maturity of the *I*'th forward rate. (71) can be illustrated as it is the case in (72)

$$f(T_i, T_i, \tau_i) = R(T_i, T_i + \tau_i) \quad \textbf{(72)}$$

The relationship above relates that the forward rate for a given period can change as time passes. But it is fixed at the reset time, *i.e.* resets, and equal to the spot rate for the same time period. The interest is due for payment at the end of the period which is the maturity time of the forward rate. The timing convention and the relations between the forward rates, discount rates and spot rates are showed in the Figure 15.

Figure 15. – Discount, spot and forward rates.



1.2. Determinants of interest rates

First, the interest rate is fundamentally a price at which the lender charges a borrower for the temporary use of funds that he lent to him. The interest rate will not only reflect the credit risk of the borrower and the time value of money, that also represents any operational cost that the lender will support by making this loan. If the credit risks are subtracted, then it will be called "risk-free rate", which mean the theoretical rate that the lender would charge the borrower if he had no credit risk such that the repayment was assured. Theoretically, the level of risk-free rates is determined by demand and supply with the perfect market's assumption.

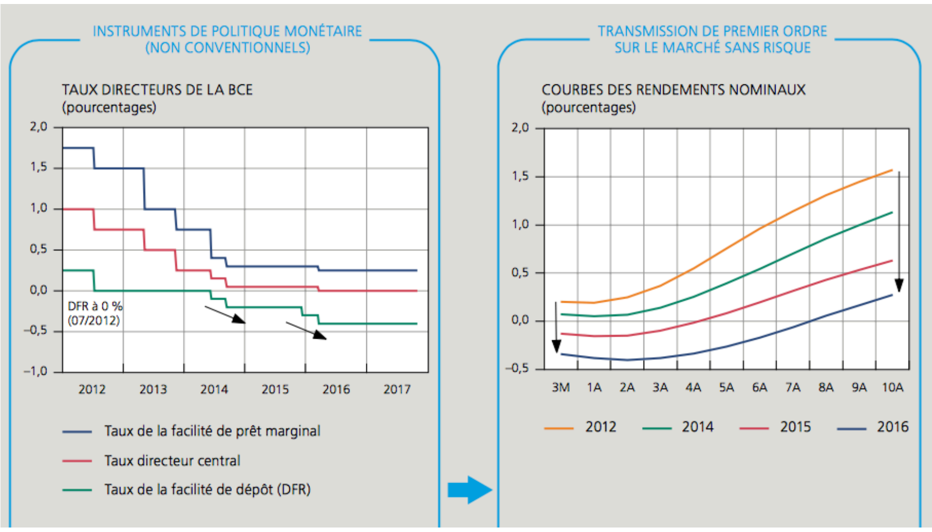
In this thesis, we speak about "low interest rates environment" but we have to know why we reached this level. When bunch of banks such Lehmann Brothers bankrupted[58], there was a hit on the countries such GDP growth decline and a surge in risk-aversion. A distrust against international investors resulted in overnight lending. Moreover, because huge default happened, the banks restrict their credit access to protect their balance sheets against the collapse. The banking balance sheet was affected mainly by the stock prices shrink, sharp declines in house prices, which cause an even worse balance sheet for firms and households which reinforce the dynamics. As consequence, in the Eurozone the interest rates were decreased by the ECB in order to stimulate activity.
Then came a second main negative shock in the Eurozone : the sovereign debt crisis. The sovereign borrowing costs spiked because of a surge in credit spread for bond yields, the banks exposures to the selected government was under scrutiny. In this case, with already

[58] https://www.ecb.europa.eu/pub/pdf/scpops/ecbocp130.pdf

inefficient low interest rates effect on the economy, the ECB put some measures in place to directly influence the effect of interest rates. The first measure is the *negative interest rates* on deposit facility. In a context of excess reserves and a long low inflation period, this policy has brought overnight rates even more down to provide positive stimuli for the economic activity. This effect is that the short term and medium term was flattened to give easier credit conditions.

The second measure is the *asset purchase programme* which has helped on depress the term structure of the yield curve by compressing the risk premium along it. That lead to lower interest rates for borrower and higher assets prices for banks which stimulate the economy on the both sides. In the Figure 16, we can observe the effect of the negative interest rate policy on the yield curve we spoke in the previous section. On the left side, there are three different rates: the lending facility rate[59] is in blue and represents the ceiling rate; the deposit facility rate[60] in green which represents the floor rate and finally the key interest rate[61] in red with which the European Central Bank influences the economy. In this sense, we call "negative interest rate policy" if the deposit facility rate is below the zero-bound. Then on the right side, we have the risk-free yield curve between 2012 (orange) and 2016 (blue). It appeared that the measures not only lowered the curve, but also flattened it. The flattening is due to the sensibility of the long-term rates to the assets purchases program.

Figure 16. – Effect of unconventional monetary policy measures on yield curve[62].



Then, in a depositor's point of view, we can observe that the deposits rate goes not under the zero-bound. The rigidity of the retail deposit rates is mainly due to the existance of fiat money – such coins and banknotes – which has a rate of return equal to 0%. If the deposit rate goes below zero, the households will withdraw their money from deposits to get it in fiat money such that they look after the highest return. The consequence for the banks is dramatic: when there is an erosion of the deposit volumes, this is problematic since the deposits are the main stable funding source of the banks. Nevertheless, the assumption of money withdrawing is not realistic. Indeed, the costs related to the possession of only physical cash at home is high for obvious reasons. According to a Royal Decree, the minimum legal rate that a bank can apply is 0.11% with 0.01% of base rate and 0.10% of fidelity.

[59] Permits to the banks to borrow liquidities to the Central Bank
[60] Permits to the banks to put their liquidity surplus to the Central Bank
[61] This is the rate for the main refinancing operations
[62] National Bank of Belgium, December 2017

2. Duration and convexity

We saw in the previous section dedicated to IRRBB the different risks that the ALM has to face. The duration the convexity are the main risk indicators to help taking decisions. This section will be presented as the way Bergendhal and Janssen did (1999).

First, we consider $CF = \{CF_j, t_j\}$ the financial future cash flow of an asset or liability where CF_j, t_j are the amount and the time of the *j*th movement of this flow, on a time scale $[0, \infty)$ with non-negative amounts. With a fixed annual rate *i,* the present value of the flow *PV* is given by Equation (73).

$$PV(i) = \sum_{j=1}^{n} CF_j\,(1+i)^{-t_j} \tag{73}$$

Now let us take the same example as they took. Let's take the case of a bank which has customer loans for a total of € 100,000,000 generating an annual interest income of 5% for 3 years. The administrative costs amount 1% per year and, due to credit losses, the loans will be repaid at 98%. These loans are refunded on the interbank at rate of 3.5%. In this sense, the results are presented in Table 7.

Table 7. – Example for customer loans (in €million)

Year	Interest income	Administrative costs	Repayment
1	0.05 x 100	-0.01 x 100	4
2	0.05 x 100	-0.01 x 100	4
3	0.05 x 100	-0.01 x 100	98+4=102

Then the market value of the flow at time 0 is computed as in Equation (74).

$$PV = \frac{4}{1.035} + \frac{4}{(1.035)^2} + \frac{102}{(1.035)^3} = €99{,}596{,}533 \tag{74}$$

which gives us a loss of € 403,467, that is 4.04%.

Now let's consider that the interbank rate decreases by 1 basis point (BPS), then the new market value at time 0 is represented in Equation (75)

$$PV = \frac{4}{1.025} + \frac{4}{(1.025)^2} + \frac{102}{(1.025)^3} = €102{,}426{,}836 \tag{75}$$

which gives us a profit of €2,426,836, that is 2.42%.

What I want to illustrate is the consequence of the profit caused by the movements in interest rates. This influence is called the *duration*. To define it, let us assume that the interest rate has a variation (positive or negative) Δi; using the Taylor formula to the value $PV(i)$ defined by the relation (73) we obtain Equation (76)

$$\Delta PV = PV(i+\Delta i) - PV(i) \cong PV'(i)\Delta i + \frac{1}{2}PV''(i)(\Delta i)^2 \tag{76}$$

and for the relative increment:

$$\frac{PV(i+\Delta i)-PV(i)}{PV(i)} \cong -D_m(i)\Delta i + \frac{1}{2}CV(i)(\Delta i)^2 \quad \textbf{(77)}$$

with $-D_m = \frac{PV\prime(i)}{PV(i)}$, $(1+i)D_m(i) = D(i)$, $CV(i) = \frac{PV\prime\prime(i)}{PV(i)}$ where $D(i), D_m(i), \text{and } CV(i)$ are respectively *duration (Macaulay), modified duration and convexity*.

From the definition of Equation (77), we can define each indicator as follow:

$$D(i) = \frac{1}{PV(i)}\sum_{j=1}^{n} t_j\, CF_j(1+i)^{-t_j} \quad \textbf{(78)}$$

$$D_m(i) = \frac{1}{(1+i)PV(i)}\sum_{j=1}^{n} t_j\, CF_j(1+i)^{-t_j} \quad \textbf{(79)}$$

$$CV(i) = \frac{1}{PV(i)}\sum_{j=1}^{n} t_j\, CF_j(1+i)^{-t_j-2} \quad \textbf{(80)}$$

The main difference between the duration and the modified duration is that the former is quoted in years although the latter is quoted in percentage change in price. Moreover, the duration is an absolute measure while the modified duration is a relative measure. To take the case of a bond, the coupon rate, the interest rates and the maturity contribute to the duration measures. If we referred to the definitions of duration, modified duration and convexity, when the maturity increases, the duration increases and the bond's price becomes more sensitive to interest rate changes. Then, if the coupon increases, the duration will decrease and in consequence the bond becomes less sensitive to interest rate changes. Finally, if the interest rates increase, the duration will decrease and the bond becomes less sensitive.
What is important to understand is, these concepts represent the influence of interest rates movements and can be used as a measure of risk.

3. Duration matching : immunization

A solution to the reduce the interest rate sensitivity is to match the duration as well as present values of the portfolio and the future cash obligations. This process is called *immunization*, *i.e.* protection against change in yield. By matching duration, portfolio value and present value of cash obligations will respond identically to a change in yield.
Let us consider two flows : an asset flow *A* and a liability flow *L* represented by the following future cash flows:

$$A = \{(A_j, t_j), j = 1, \dots, n\} \qquad L = \{(L_j, t_j, j = 1, \dots, n\} \quad \textbf{(81)}$$

Here we don't consider that there is an equity cash-flow.
With a constant interest rate, the present market value called $S(i)$ is given by Equation (82).

$$S(i) = \sum_{j=1}^{n}(A_j - L_j)(1+i)^{-t_j} \quad \textbf{(82)}$$

Let define $D_{m,A}$(i), $D_{m,L}$(i), CV_A(i), and CV_L(i) be respectively the modified duration of asset and liabilities, and the convexity of the assets and the liabilities. Then, from (76) and (82), $\Delta S(i) = S(i+\Delta i)) - S(i)$, *i.e.* the variation of present market value is defined in Equation (83).

$$\Delta S(i) \cong -[A(i)D_{m,A}(i) - L(i)D_{m,L}(i)]\Delta i + \frac{1}{2}[CV_A(i)A(i) - CV_L L(i)](\Delta i)^2 \quad \textbf{(83)}$$

Finally, at the first-order, the position {A,B} will be immunized against small variations of the interest rate if and only if:

$$A(i)D_{m,A}(i) = L(i)D_{m,L}(i) \quad \textbf{(84)}$$

which says that the cash flows of the assets hedge the cash flow of the liabilities. If we are at time 0 with a null position, *i.e.* $A(i) = L(i)$, then the last condition is that the duration of assets corresponds to the duration of the liabilities as in Equation (85).

$$D_{m,A}(i) = D_{m,L}(i). \quad \textbf{(85)}$$

From (28), this immunization is reinforced if the condition in (86) is respected.

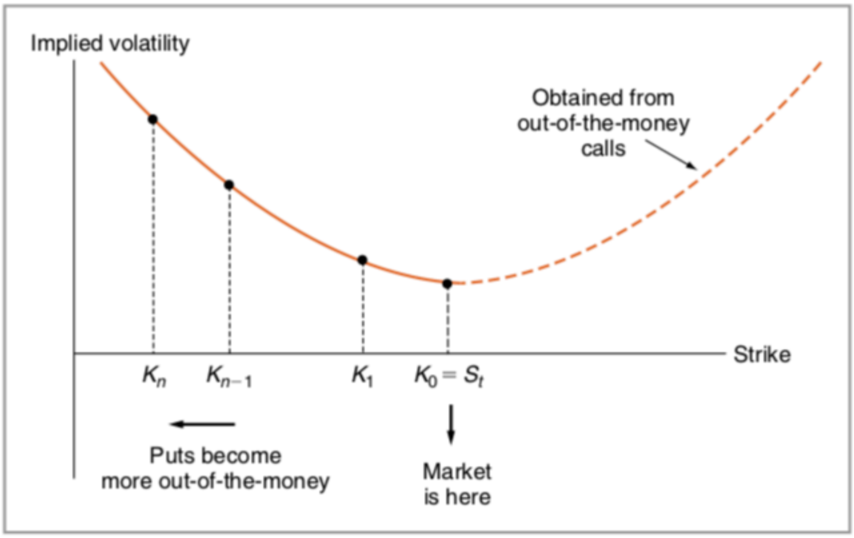
$$[CV_A(i)A(i) - CV_L L(i)] \geq 0 \quad \textbf{(86)}$$

There are difficulties with immunization procedure in practice. First, it is necessary to rebalance or re-immunize the portfolio from time to time since the duration depends on yield. Second, the immunization method assumes that all yields are equal. This is not quite realistic to have bonds with different maturities to have the same yield. Last, when the prevailing interest rate changes, it is unlikely that the yields on all bonds all change by the same amount.

Annex n°2: the volatility smile

In the section of interest rate models in negative interest rate environment, we saw that the volatility smile need to be correctly modelled in order to get a relevant model. First, the *implied volatility* need be explained. In the options, there is the observed market price which relies on the Black-Scholes formula to price them. Then the implied volatility is the volatility which must be plugged into the formula in order to recover the fair market price. It differs from the realized volatility if the volatility is stochastic and if a risk premium needs to be added to volatility quotes. Before explaining the volatility smile, I need to differentiate for options in-the-money, at-the-money, and out-of-the-money. For a call/put option, being *in-the-money* (ITM) means that the strike price is lower/higher than the underlying price. On the contrary, being *out-of-the-money* (OTM) means that the strike price is higher/lower than the underlying price. Finally, being *at-the-money* (ATM) means that the strike price is equal than the underlying price. It turns out that everything else being the same, an OTM put or call has a higher implied volatility than an ATM call/put. The more out-of-the-money you a call/put option is, the higher is the corresponding implied volatility. This ensures that there is an empirical fact which is called *volatility smile* or *volatility skew* which is represented in the Figure 17.

Figure 17. – Representation of the volatility smile/skew



On the left side, more you have an OTM put, more the implied volatility will increase. Then on the right side, this is the same reasoning but for a call option. On the x-axis, the K_0 is ATM and as K_i decreases[63], the puts go deeper OTM.

The volatility smile is an empirical phenomenon that violates the assumptions of the Black-Scholes world. Taking in account a new volatility process such the volatility smile is more relevant for the modelling. First, the volatility smile dynamics offer trade opportunities for market professionals since the volatility smile is associated with all the risk factors and offers to traders the opportunity to take spread positions and arbitrage it. Then, the volatility smile contain information about the dynamics of the underlying realized volatility processes.

[63] K_i represents the *i*th strike of the option series and S_t is the equity price index