

École polytechnique de Louvain

Development of a Computational Modelling Tool for Composite Megacolumns with Embedded Steel Profiles

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Use of Artificial Intelligence

Generative artificial intelligence tools were used in the preparation of this thesis, in accordance with the guidelines established by UCLouvain. In particular, ChatGPT was used as a support tool to help identify or correct issues in both code and written text. Similar to a search engine, it was also used to gather information and locate relevant documents for the development of this work.

For programming-related tasks, the AI assistant GitHub Copilot, integrated within the Visual Studio Code (VSCode) environment, was used to assist with code completion, debugging, and minor code improvements.

Additionally, non-generative AI-based language tools, including *DeepL* and *QuillBot*, were used for grammatical correction to improve the quality of the English language.

All technical analyses, modelling, interpretations, and conclusions were carried out and verified by the author, who remains fully responsible for the accuracy and integrity of the work.

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Finally, if this thesis represents the end of a chapter, it also reflects all the years that preceded it. For that reason, my deepest thanks go to my family and relatives, whose constant support, encouragement, and patience have accompanied me not only during this year, but during all my studies.

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Notations

The symbols and acronyms used in this document are defined below.

The following subscripts are used throughout the document: c for concrete, a for structural steel, and s for reinforcing steel. These three material subscripts may be denoted generically by j . The subscripts y and z refer to the principal axes of the cross-section, and i denotes a reinforcing bar or discretised element.

Acronyms

AISC	American Institute of Steel Construction
CISC	Canadian Institute of Steel Construction
CTBUH / CVU	Council on Tall Buildings and Urban Habitat / Council on Vertical Urbanism
EC	Eurocode
HSM	Hybrid Stiffness Method
LSM	Linear Stiffness Method
OFAT	One-Factor-At-a-Time
SmartCoCo	Smart Composite Components

Upper-case Latin letters

A_g	Total cross-sectional area
A_j	Cross-sectional area of material component j
$A_{j,\text{layer}}$	Area of material component j within a discretised layer
$A_{s,i}$	Cross-sectional area of reinforcing bar i
A_{curve}	Area enclosed by the N – M interaction diagram
B	Overall width of the cross-section
E	Young's modulus
E_j	Young's modulus of material component j
E_{cd}	Design value of the concrete modulus of elasticity
E_{cm}	Secant modulus of elasticity of concrete
$E_{c,\text{eff}}$	Effective modulus of elasticity of concrete accounting for long-term effects
EI	Flexural stiffness, often used as a shortened notation for the effective flexural stiffness
EI_{eff}	Effective flexural stiffness
$EI_{\text{eff},j}$	Effective flexural stiffness of material component j
$EI_{\text{eff},II,\text{lin}}$	Effective flexural stiffness for second-order analysis using the LSM

$EI_{\text{eff},II,\text{hyb}}$	Effective flexural stiffness for second-order analysis using the HSM
F_j	Axial force contribution of material component j
H	Overall height of the cross-section
I	Second moment of area
I_j	Second moment of area of material component j
I_g	Second moment of area of the total cross-section
I_y, I_z	Second moment of area about the y - and z -axes, respectively
K_0	Global stiffness reduction factor
K_j	Stiffness coefficient associated with material component j
K_e	Coefficient accounting for the effective contribution of concrete
$K_{e,II}$	Coefficient accounting for cracked concrete behaviour
K_{sa}	Stiffness coefficient for reinforcing steel and embedded structural steel
L	Member length
L_{cr}	Effective buckling length
M, M_y, M_z	Bending moment, and bending moments about the y - and z -axes, respectively
M_{max}	Maximum bending resistance on the N – M interaction diagram
$M_{b,Rd}$	Buckling-reduced design bending resistance
$M_{Ed,y}, M_{Ed,z}$	Design bending moments about the y - and z -axes, respectively
M_{Rd}	Design bending resistance
N	Axial force
N_{corr}	Axial force corresponding to the maximum bending resistance
N_{cr}	Euler critical load
$N_{cr,1st}$	Euler critical load obtained from first-order analysis
$N_{cr,2nd,\text{hyb}}$	Euler critical load obtained from second-order analysis using the HSM
$N_{cr,2nd,\text{lin}}$	Euler critical load obtained from second-order analysis using the LSM
$N_{b,Rd}$	Buckling-reduced design axial resistance
N_{Ed}	Design axial force
$N_{G,Ed}$	Design axial force due to permanent actions
N_j	Axial resistance of point j on the interaction diagram
$N_{\text{max}}, N_{pl,Rd}$	Maximum (plastic) axial resistance of the composite cross-section (i.e. $N_{\text{max}} = N_{pl,Rd}$)
N_{Rd}	Design axial resistance
V	Shear force
$V_{Ed}, V_{Ed,y}, V_{Ed,z}$	Design shear force, and design shear forces in the y - and z -directions, respectively

Lower-case Latin letters

b	Width of the structural steel profile
b_f	Flange width of the structural steel profile
b_{concrete}	Width of concrete within a discretised layer
b_{section}	Total width of the cross-section considered in the discretised model
b_{steel}	Width of structural steel within a discretised layer
$cover_s, c_s$	Concrete cover to the reinforcing bars

$c_{a,y}, c_{a,z}$	Concrete cover to the structural steel profile in the y - and z -directions, respectively
e	Load eccentricity
$f_{a,y}$	Yield strength of structural steel
$f_{a,u}$	Ultimate tensile strength of structural steel
$f_{a,yd}$	Design yield strength of structural steel
f_{cd}	Design compressive strength of concrete
f_{ck}	Characteristic compressive cylinder strength of concrete
$f_{s,y}$	Yield strength of reinforcing steel
$f_{s,yd}$	Design yield strength of reinforcing steel
h	Height of the structural steel profile
k	Second-order moment amplification factor
k_1	Concrete-strength coefficient used in the HSM
k_2	Load-level coefficient used in the HSM
n	Axial load ratio used in the HSM
r	Root radius of the structural steel profile
t_f	Flange thickness of the structural steel profile
t_w	Web thickness of the structural steel profile
w	Weighting factor used in the comparison score
x	Longitudinal coordinate along the member
y, z	Principal axes of the cross-section
z_j	Lever arm of material component j

Upper-case Greek letters

Δ	Relative difference between a reference value and a compared value
ΔA_{curve}	Relative difference in the area of the N – M interaction diagram
Δh	Discretisation layer thickness
$\Delta M_{b,\text{max}}$	Relative difference between the maximum bending resistance of the buckling-reduced and unbuckled interaction curves
ΔM_{max}	Relative difference in the maximum bending resistance of the N – M interaction diagram
$\Delta N_{b,\text{max}}$	Relative difference between the maximum axial resistance of the buckling-reduced and unbuckled interaction curves
ΔN_{max}	Relative difference in the maximum axial resistance of the N – M interaction diagram
ΔN_{corr}	Relative difference in the axial force corresponding to the maximum bending resistance of the N – M interaction diagram

Lower-case Greek letters

α	Buckling imperfection factor
β	Moment distribution factor
χ	Buckling reduction factor
δ	Steel contribution ratio or global deflection in second-order analysis, depending on the context
ε	Strain
ε_{c2}	Concrete strain at peak stress for the parabolic–rectangular law only
ε_{cu2}	Ultimate concrete compressive strain for the parabolic–rectangular law only
ε_j	Strain in material component j
ε_{ju}	Ultimate strain of material component j
ε_{jy}	Yield strain of material component j
γ_{ce}	Partial factor applied to the concrete modulus of elasticity
γ_j	Partial safety factor of material component j
λ	Global slenderness
$\bar{\lambda}$	Non-dimensional slenderness
ϕ	Intermediate buckling parameter
φ_t	Creep coefficient
ρ_a	Structural steel ratio
ρ_{sa}	Combined reinforcing steel and structural steel ratio
σ	Stress
σ_j	Stress in material component j
$\varnothing$	Diameter of reinforcing bars

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Chapter 1 Introduction

1.1 Context and Motivation

The construction of tall buildings has increased since around 2015. This has resulted in taller buildings and a greater number of skyscrapers. This trend can be attributed to several key factors: the global population increase; urban density, which promotes vertical rather than horizontal development; and the desire to push technological limits and break records for the tallest structures.

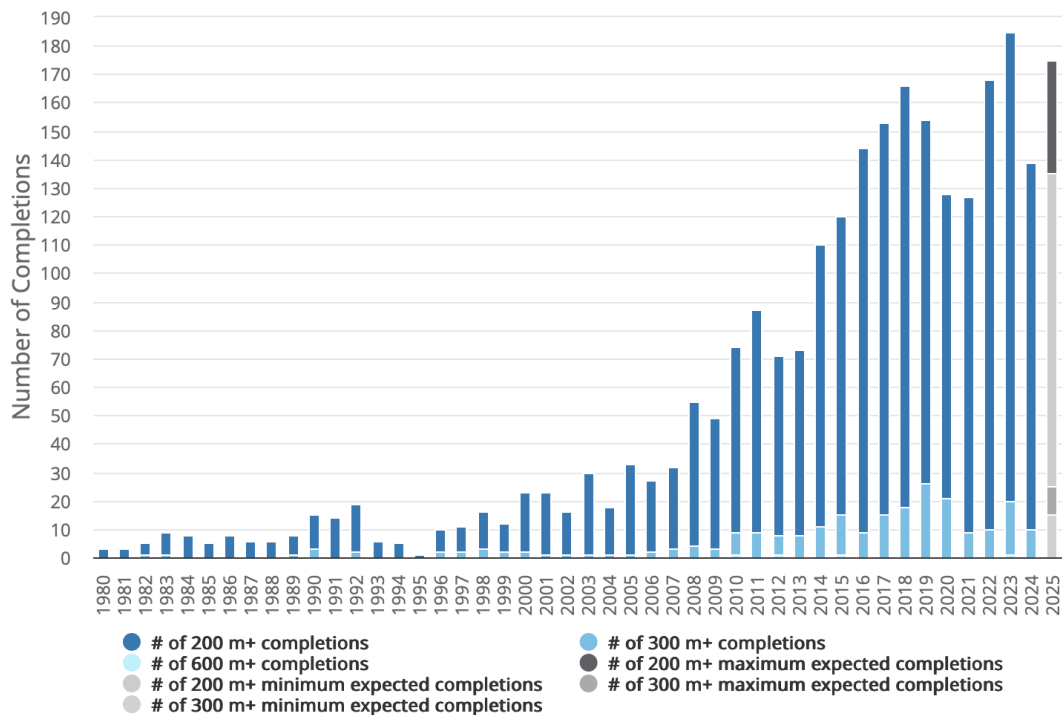


Figure 1.1: Global completion trends for buildings taller than 200 m worldwide [1].

Although this trend can be considered a natural consequence of humanity's continuous pursuit of innovation, another factor must now be taken into account, one that was not given sufficient attention in the past: global warming. The construction sector is responsible for over 35% of total greenhouse gas emissions, which means that engineers today must design and build in a more sustainable and efficient manner [2].

In this context, as buildings become taller, their self-weight inevitably increases. Consequently, the columns on the lower floors must either be larger in cross-sectional dimensions or greater in number. Both solutions lead to the same conclusion: the increase in structural material results

in higher embodied carbon. Therefore, beyond a certain height, constructing taller buildings becomes less optimal from the perspective of carbon emissions. From a structural perspective, reinforced concrete (RC) structures tend to be less suitable for high-rise buildings, as their significant self-weight increases gravity loads, leading to higher dead loads and a reduction in ductility, i.e. a limited capacity for deformation [3, 4].

To overcome the limitations of reinforced concrete columns in tall buildings, composite columns combining concrete with an embedded steel profile were developed. Conventional composite columns generally consist of a concrete section enclosing a single steel profile, as illustrated in Figure 1.2a).

An even more advanced solution consists of embedding several steel profiles within the same concrete section, forming what is commonly referred to as a *megacolumn*, as shown in Figure 1.2b). Compared with conventional composite columns, this configuration provides significantly higher structural capacity and improved structural efficiency.

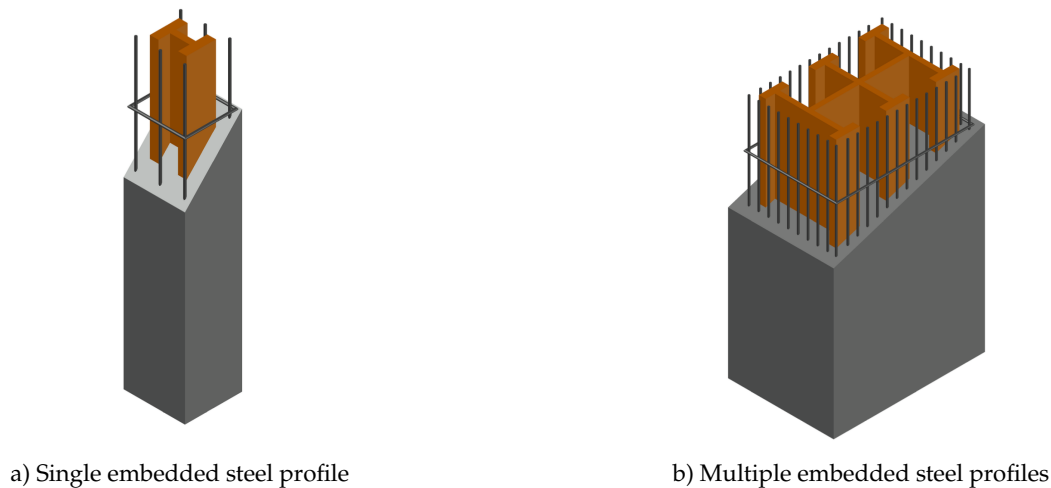


Figure 1.2: Conceptual illustration of a) a composite column with a single embedded steel profile and b) a composite megacolumn with multiple embedded steel profiles.

This concept offers several advantages from a global and conceptual perspective, beyond its purely structural performance. By concentrating the load-bearing capacity into fewer structural elements, megacolumns allow for a reduction in the number of columns and potentially smaller individual cross-sections. These characteristics translate into a range of architectural, functional, and economic benefits, which are discussed qualitatively in the following, while more detailed structural and mechanical aspects are addressed in the next chapter.

- **Increased architectural freedom**

Fewer columns provide more usable space and greater flexibility in interior design and functionality. Figure 1.3 highlights a conceptual and idealised reconfiguration of the ground floor of the *Willis Tower* in Chicago. This is intended for illustrative purposes only and is not a realistic design proposal. This solution enables architects to be less constrained by column layouts, thereby allowing greater creativity and versatility in the design of interior spaces.

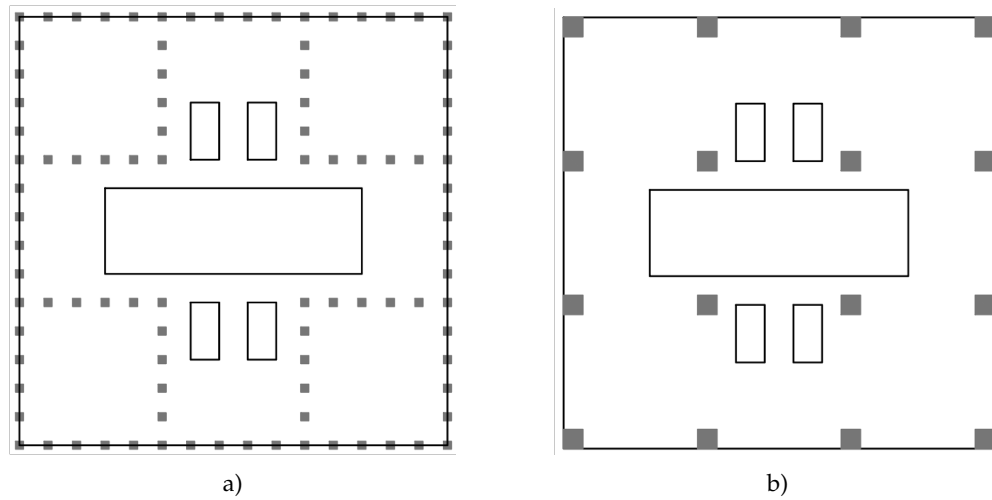


Figure 1.3: Comparison between a) the existing floor plan of the *Willis Tower* from levels 0 to 50 [5] and b) a conceptual layout using megacolumns.

- **Enhanced spatial quality and luminosity**

Reducing the number of columns and strategically positioning megacolumns allows for the creation of large open areas, such as the $30\text{ m} \times 30\text{ m}$ central space presented in Figure 1.4a), taken from the *One Bloor West Tower* in Toronto [6]. The 12 m height of the megacolumns located along the building perimeter, which corresponds to three floors in this example, highlights how these elements can efficiently support large spans, thereby improving both functionality and daylight penetration.

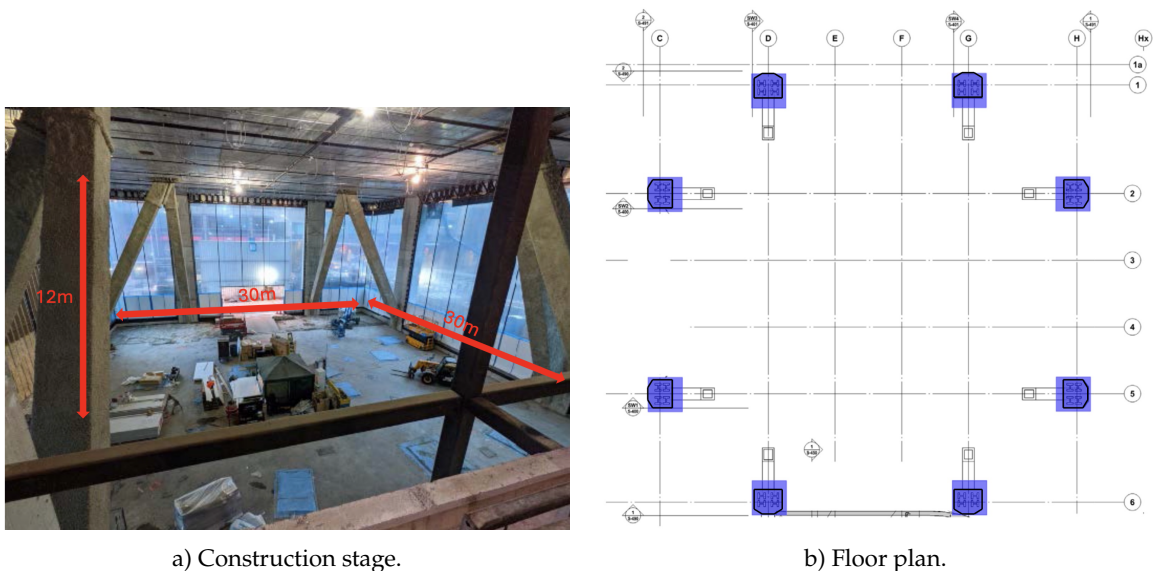


Figure 1.4: Illustration of the *One Bloor West Tower* megacolumn configuration: a) construction stage of the central megacolumn area and b) corresponding floor plan.

- **Material efficiency and economic benefits**

The use of megacolumns can significantly reduce overall material consumption. For instance, in the case of the *Canadian Supertall Building*, a study conducted by the design office *RJC Engineers* showed that replacing conventional reinforced concrete columns (with a typical cross-section of $2.5\text{ m} \times 2.5\text{ m}$) with megacolumns (of $1.8\text{ m} \times 2.0\text{ m}$) leads to substantial material savings and a reduction in embodied carbon.

As illustrated in Figure 1.4b), this optimization results in approximately 50 m^2 of additional usable floor area over just three storeys. Assuming an average rental price of about $40\text{ \$}/\text{m}^2$ in Toronto (2026) for residential use [7], this corresponds to an increase in annual revenue of approximately 24 000 \$.

Furthermore, from a construction perspective, concentrating the structural load transfer into eight megacolumns only led to an estimated saving of around 100 M\$ in steel material costs, according to *RJC Engineers* [8].

Hence, from both environmental and economic standpoints, megacolumns represent a highly advantageous structural solution.

1.2 Problem Definition

Following the introduction of the concept, a single megacolumn geometry will be used throughout this report. The geometry, plotted in Figure 1.5, is not explicitly covered by Eurocode 4¹, as the steel profiles within the composite section are not welded together. This configuration therefore falls outside the conventional composite column definitions, highlighting the need for dedicated numerical validation and design recommendations.

However, the fact that such structures fall outside the scope of Eurocode does not imply that they are unsafe or cannot be realised. Eurocodes provide standardised design rules for common structural configurations, but innovative or non-standard solutions can still be implemented, provided that their behaviour is adequately justified. When this happens, the structural behaviour must be justified using the right numerical modelling and supporting evidence.

This type of megacolumn is mainly used for very tall buildings, which are more common in North America and Asia than in Europe. In these regions, structural design is usually done using different standards, such as those established by the *American Institute of Steel Construction* (AISC)². For this reason, the practical implementation of such structural systems is more closely aligned with non-European regulatory frameworks.

Despite these considerations, the study remains relevant in a European context, as this master thesis was conducted in collaboration with ArcelorMittal. The company's activities extend not only to global steel production, but also to international engineering and consultancy, with projects worldwide. Through its *Steligen* department, the company promotes the development of optimized

¹Eurocodes are European standards that provide a common approach to the design of construction works and structural elements [9].

²The American Institute of Steel Construction (AISC) publishes design standards for steel and composite structures widely used in North America [10].

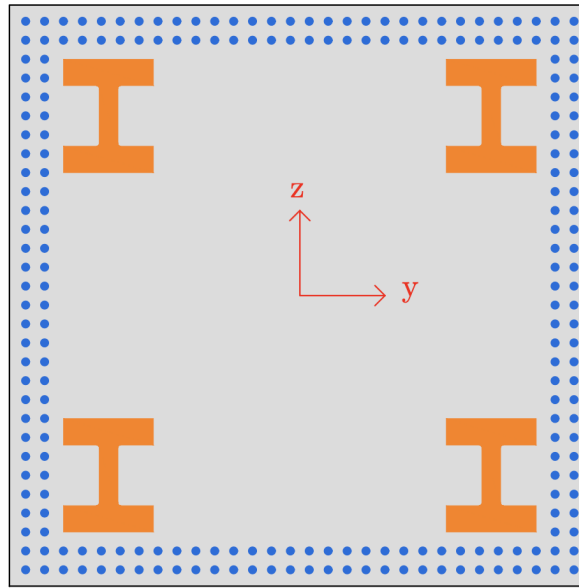


Figure 1.5: Cross-section of the composite megacolumn geometry considered in this study.

and innovative steel-based structural systems at a global scale. Consequently, the validation and improvement of design tools for megacolumn applications support engineering practice worldwide.

In this regard, dedicated numerical tools are required to analyse and design such unconventional composite configurations. The present work has **two main objectives**:

- to **validate the performance of an in-house software** developed by the ArcelorMittal Research and Development Department, hereafter referred to as *MegaColumn* [11];
- to **further investigate advanced theoretical aspects** that are not currently implemented in the *MegaColumn* software, and to assess whether their integration would be relevant and beneficial.

1.3 Study Methodology

The *MegaColumn* software, which constitutes the main subject of this thesis, provides several outputs related to the structural behaviour of composite columns, including the $N-M$ interaction diagram, the longitudinal shear forces, and the required number of mechanical connectors.

However, as the software is still under development, the present study focuses exclusively on the validation of the $N-M$ interaction diagram. The validation of other outputs, such as longitudinal shear and the number of mechanical connectors, is beyond the scope of this work and is left for future developments.

This output, as well as the fundamental concepts governing composite column behaviour, are first introduced in **Chapter 2**.

The overall structure of the present study is displayed in Figure 1.6.

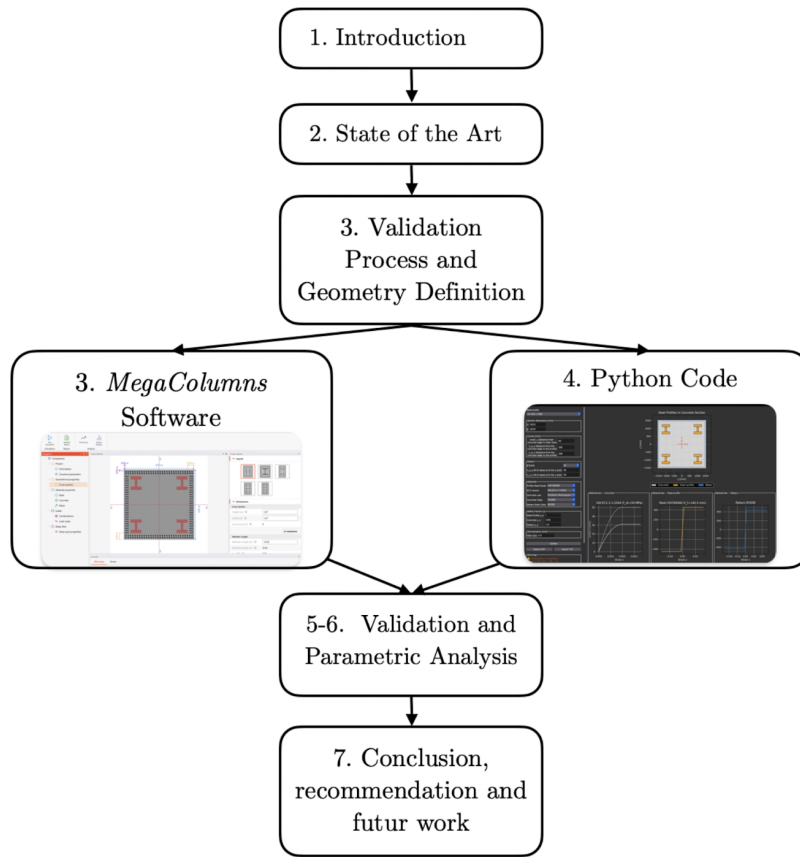


Figure 1.6: Schematic overview of the study methodology and relationships between the main chapters.

Chapter 3 introduces the *MegaColumn* software, providing an overview of its operation, the necessary inputs and the outputs produced. It also outlines the validation methodology adopted in this study and describes the reference geometry used for comparison.

Chapter 4 introduces an independent implementation developed in this thesis, which is based on the theories presented in the preceding chapters.

Although the objective was to validate all outputs, only the $N-M$ interaction diagram has been completed. The calculations performed by the *Python Code* for the remaining outputs are detailed in Appendix A, including longitudinal shear and the number of mechanical connectors.

Chapter 5 compares the results obtained with *MegaColumn* to those from other tools, while **Chapter 6** applies it to a real case study and presents a parametric analysis of advanced theoretical approaches that could be relevant for future implementation in the software.

Finally, the last chapter concludes the thesis by summarising the main findings obtained and by providing recommendations for future work, along with potential improvements to the software.

Chapter 2 State of the Art

Before delving into the new concepts introduced in this work, the fundamental background on which they are based must first be explained. The concept of a composite structure, and more specifically composite columns, must be introduced, together with the output to be validated: the N – M interaction diagram.

2.1 Composite Structures

The concept of a *composite structure* refers to the use of two or more distinct materials combined in such a way as to optimize their individual mechanical properties. In the context of civil engineering, the most common materials are concrete and steel, though other materials such as timber may also be used [4, 12].

The main advantage of combining concrete and steel is that these two materials complement each other. Each material has its own optimal mechanical characteristics [12].

- Concrete efficiently resists compressive stresses, while steel performs better in tension.
- Concrete limits the local instabilities of steel members, such as local buckling phenomena.¹
- Steel provides high ductility, enabling plastic deformation of the structure.

It is important to note that when referring to “steel–concrete composite structures,” the concrete component generally consists of a *reinforced concrete* section containing steel reinforcement bars made of *reinforcing steel*, while the steel component is composed of *structural steel* elements or profiles. Despite both being made of steel, they fulfil different structural roles and possess distinct mechanical and geometrical characteristics.

The overall behaviour of such composite members depends not only on the concrete and steel components themselves, but also on the interaction and load transfer mechanisms developed at their interface.

This combination offers several advantages [13, 14]:

- Reduced overall structural weight;
- Longer achievable spans;
- More compact and efficient cross-sections compared to reinforced concrete or purely steel members;

¹Buckling is a form of instability that may occur in slender steel elements subjected to compression. This concept is discussed in more detail in Section 6.7.

- Easier disassembly and potential reuse;
- Enhanced fire resistance compared to pure steel structures.

Composite behaviour can be applied to various structural elements, such as floors (composite slabs), beams (composite beams), and, in this case, composite columns, as shown in Figure 2.1.

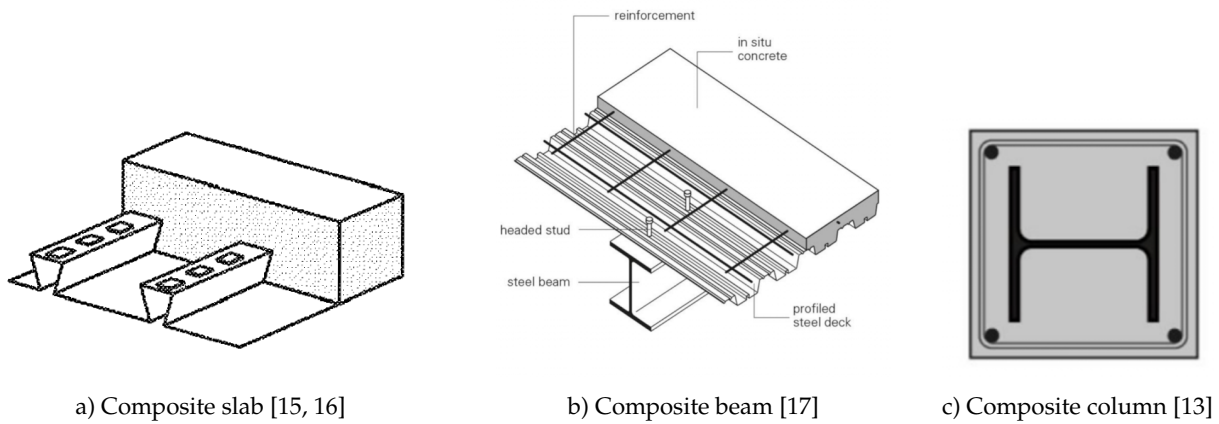


Figure 2.1: Examples of composite structural elements: a) composite slab, b) composite beam, and c) composite column.

2.1.1 Composite Column

Attention is now focused on composite columns. Eurocode 4, which governs composite steel–concrete structures, considers several types of composite column geometries, as presented in Figure 2.2.

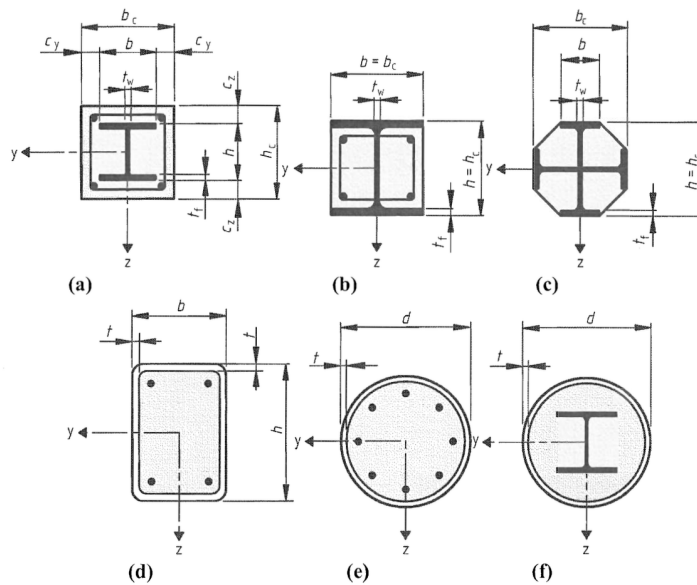


Figure 2.2: Typical composite column configurations according to Eurocode 4 [15].

While the fundamental principle remains the same, the construction methods and load-transfer mechanisms differ depending on whether:

- the steel profile is encased in concrete;
- the concrete is filled inside a steel tube (concrete-filled tube, CFT); or
- the steel and concrete are arranged in combined or cellular sections.

2.1.2 Megacolumns: Concept and Applications

The concept of *megacolumns* is not new; several high-rise buildings worldwide have already adopted this type of structural system [4]. As visualised in Figure 2.3, most existing applications rely on *Welded Reconstituted Profiles (WRP)*, in which multiple steel sections are welded together to form a large composite column. This welded configuration ensures full composite action between the steel elements and provides a uniform and predictable structural response [4, 18].

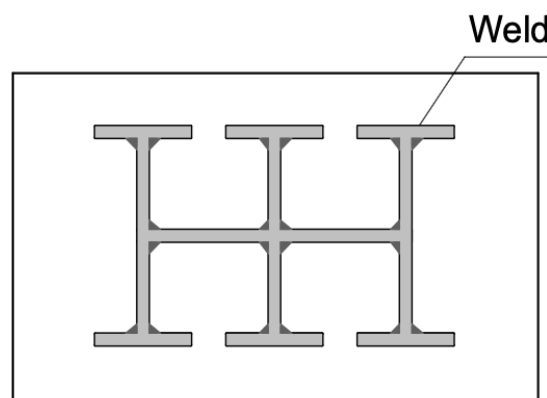


Figure 2.3: Typical megacolumn configuration using welded reconstituted steel profiles (WRP) [3].

However, welded megacolumns also present several drawbacks [13, 19].

- Fabrication is complex and requires precise alignment as well as highly skilled personnel. Strict quality control procedures must be implemented when welding operations are performed on site.
- These processes are sensitive to environmental conditions, such as temperature, humidity and wind, which may impact the quality of execution.
- The presence of residual stresses caused by welding can reduce the material's ductility and make it more susceptible to cracking.
- Once assembled, welded configurations are difficult to dismantle, modify, or reuse.
- When it comes to connection solutions, on-site welding operations can be more expensive than other options. This is because they take more time and labour to assemble.

To overcome these limitations, an alternative configuration may be adopted, in which the embedded steel profiles are connected to the surrounding concrete through mechanical shear connectors, rather than being directly welded together. Several possible configurations of such megacolumn systems are displayed in Figure 2.4 [20].

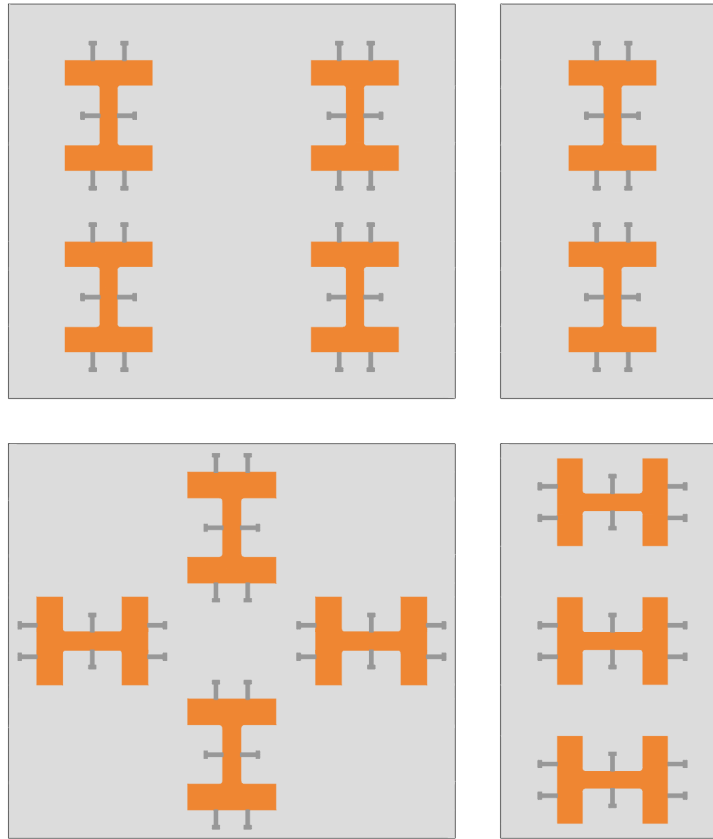


Figure 2.4: Examples of composite megacolumn configurations using non-welded embedded steel profiles connected to the surrounding concrete through mechanical shear connectors.

Compared to welded configurations and conventional reinforced concrete columns, composite megacolumns offer several advantages, particularly for high-rise applications [3, 21]:

- simplified fabrication and transportation;
- facilitated on-site assembly;
- reduced residual stresses associated with welding operations;
- compatibility with modular and prefabricated construction methods;
- more compact cross-sections;
- increased stiffness and resistance under combined loading;
- improved ductility and post-earthquake performance;

- enhanced efficiency during the construction phase;
- easier connection with steel or composite beams, allowing faster erection.

Even so, the non-welded configuration presents additional challenges. The independent behaviour of the steel profiles often results in only partial composite action. Additionally, modelling load transfer between the steel elements and the concrete core is more complex due to the irregular distribution of materials within the cross-section.

Furthermore, this type of composite megacolumn is not explicitly covered by the Eurocode 4 (EN 1994-1-1) [15, 16], which only addresses cases involving a single steel profile or fully welded assemblies.

2.2 *N–M Interaction Diagram*

When designing structural elements such as columns, it is necessary to verify their resistance under combined loading conditions. A simplified approach consists of checking each internal force independently by comparing the applied actions with the corresponding resistances for each load case, as demonstrated in Figure 2.5a). Still, this approach does not properly take into account the interaction between axial force and bending moment, which can significantly affect the structural behaviour [12].

A more general method consists of determining the complete range of axial force and bending moment combinations that the cross-section can resist. This relationship is represented by the *axial force–bending moment interaction diagram*, commonly referred to as the *N–M interaction diagram*, illustrated in Figure 2.5b).

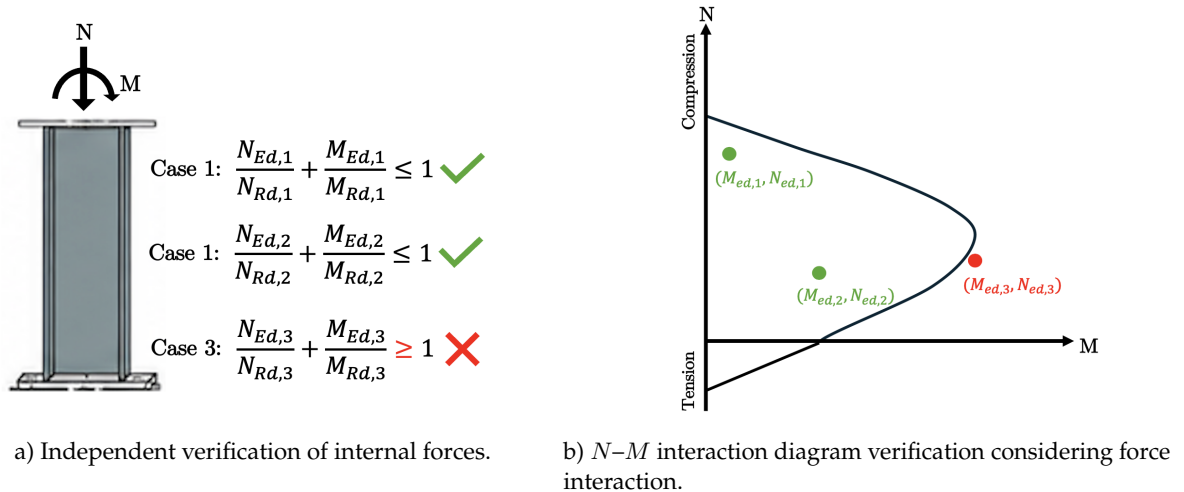


Figure 2.5: Comparison between a) the simplified verification of internal forces and b) the axial force–bending moment interaction diagram method.

This diagram represents the range of axial force and bending moment that a cross-section can safely resist. In this diagram, the bending moment M is plotted on the horizontal axis and the axial force N on the vertical axis. By convention, positive values of N correspond to compression and negative values to tension. Loading combinations (N, M) located inside the interaction curve are considered safe, whereas points outside the curve indicate failure of the section [22].

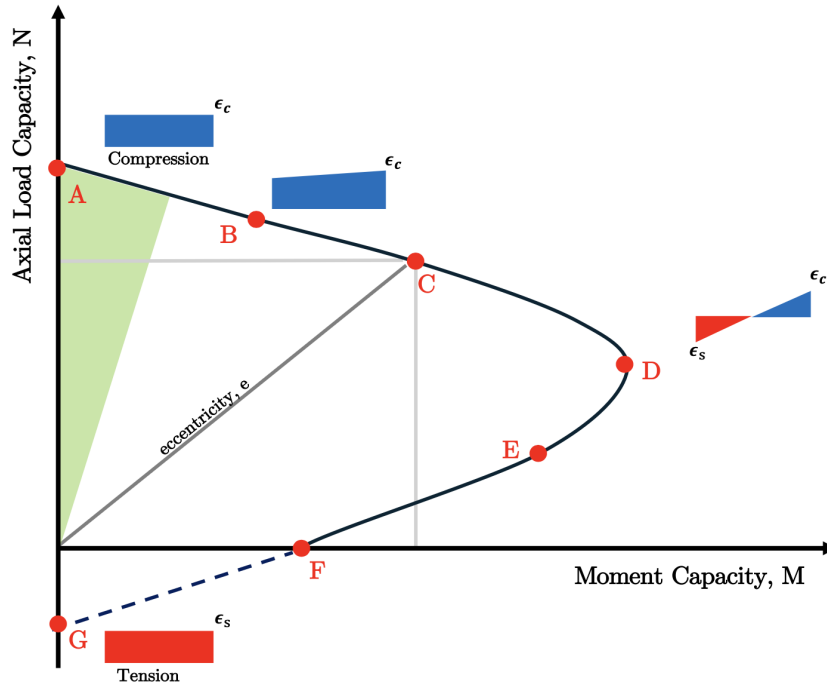


Figure 2.6: Axial force–bending moment interaction diagram illustrating the resistance domain of a composite column cross-section. Adapted from [23].

Several characteristic points can be identified on the N – M interaction diagram [3, 24, 25]:

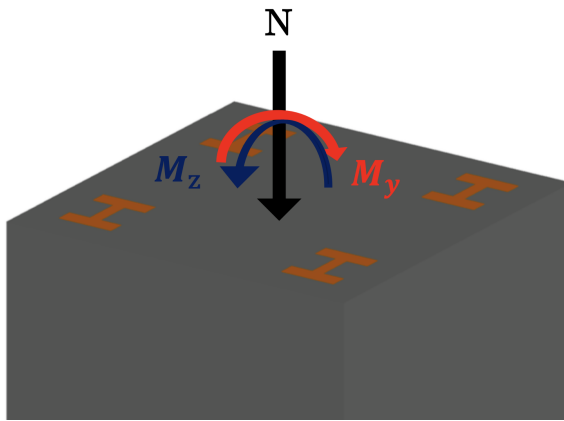
- **Point A – Pure axial compression:** The entire cross-section is under compression, meaning the bending moment is zero ($M = 0$).
- **Point F – Pure bending:** The axial force is zero ($N = 0$), so the resistance is entirely governed by bending.
- **Point G – Pure axial tension:** The entire cross-section is subject to tension, resulting in a zero bending moment ($M = 0$).
- **Point D – Balanced failure point:** The maximum moment occurs when the section is under compression in one part and traction in the other.
- **Intermediate points – Combined loading:** The interaction curve between these extreme cases defines all possible combinations of axial force and bending moment that can lead to rupture of the cross-section.

In practice, the columns in buildings are primarily subjected to compressive axial forces due to the weight of the structure. Nevertheless, bending can occur due to load eccentricities, geometric imperfections or lateral loads, such as those caused by wind or seismic activity. The eccentricity e is defined as follows:

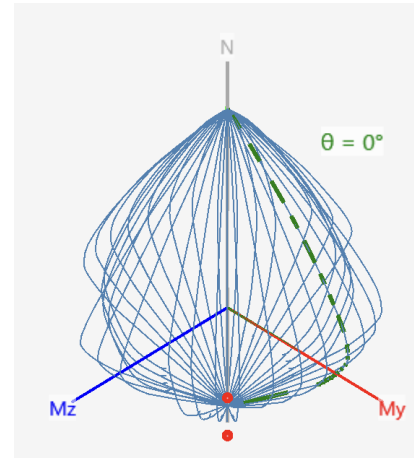
$$e = \frac{M}{N} \quad (2.1)$$

To account for imperfections, a minimum eccentricity is commonly considered in design. For instance, in an experimental study conducted in China [3], the eccentricity was taken as a fraction of the section height, with $\frac{e}{H} = 15\%$. This range typically corresponds to the green zone in Figure 2.6.

This N – M interaction diagram is valid for a given bending axis and cross-section orientation. In practice, a column may also be subjected to biaxial bending, as illustrated in Figure 2.7a), where bending moments act simultaneously about the y - and z -axes. In such cases, a three-dimensional interaction surface involving N , M_y , and M_z can be defined, as shown in Figure 2.7b). However, the generation of this surface requires significantly greater computational effort.



a) Composite column subjected to biaxial bending.



b) Three-dimensional interaction surface involving N , M_y , and M_z [11].

Figure 2.7: Extension of the axial force–bending moment interaction diagram to biaxial bending conditions.

2.3 Megacolumn research and modelling approaches

Before the 2010s, the use of composite columns with embedded steel profiles in supertall buildings remained largely undocumented, as these systems were mainly developed within engineering practice rather than academic research.

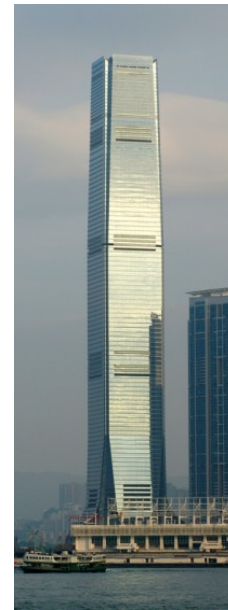
For example, the *China World Trade Center Tower III* in Beijing (Figure 2.8a)), the *Eton Center Dalian* (Figure 2.8b)), and the *Hong Kong ICC* (Figure 2.8c)) use composite megacolumn systems with several embedded steel profiles, mostly using WRP sections. Structural data for such buildings are mainly collected by the CVU (*Council on Vertical Urbanism*), formerly the CTBUH, though detailed information is generally only available when structural engineering companies, architectural companies, or building owners agree to share their design background. In these three cases, the structural design was carried out by Arup [4].



a) *China World Trade Center Tower III*.
Photo © Tim Griffith [26].



b) *Eton Place Dalian Tower*.
Photo © Terrence Zhang [27].



c) *Hong Kong ICC*.
Photo © Marshall Gerometta [28].

Figure 2.8: Examples of supertall buildings using composite megacolumn systems with welded reconstituted steel profiles (WRP).

Since the early 2010s, the rapid increase in tall building construction has led to a growing research interest in composite megacolumns with multiple embedded steel profiles. This has resulted in several experimental and analytical studies, progressively improving the understanding and modelling of these systems. Figure 2.9 summarises the main research developments and modelling approaches.



Figure 2.9: Overview of the main research contributions and modelling approaches related to composite megacolumns with multiple embedded steel profiles.

- 2013 – Experimental study on four encased steel profiles:** This study constitutes one of the first large-scale experimental investigations on composite columns with several embedded steel profiles subjected to axial and eccentric loading [3]. The work demonstrates that sufficient composite action can be achieved between concrete and multiple isolated steel sections, while also highlighting the limitations of existing design codes for such unconventional composite sections. For the present master’s thesis, this study provides an important experimental basis for understanding the global behaviour of megacolumns and the applicability of simplified approaches for the construction of the N – M interaction diagram.
- 2017 – SmartCoCo project (European Commission):** The *SmartCoCo* project represents a major European research initiative dedicated to *hybrid steel–concrete structures* reinforced by several embedded steel profiles [29]. The project investigates force-transfer mechanisms, longitudinal shear transmission, stability of slender hybrid columns, and the development of simplified design approaches compatible with Eurocode philosophy.
- 2019 – Chrzanowski PhD:** This PhD research focuses on shear-transfer mechanisms in composite columns with multiple encased steel profiles [30]. The work highlights the limitations of the classical assumption of fully rigid composite behaviour based on the Euler–Bernoulli beam theory. Experimental and numerical investigations show that shear deformation and partial interaction effects can significantly influence the behaviour of large composite columns. Although this research does not directly address the development of the N – M interaction diagram, it remains a highly relevant reference regarding the overall behaviour of megacolumns and composite interaction mechanisms.
- 2023 – Design of Hybrid Structures (Plumier & Degée):** This book synthesises more than a decade of research on hybrid steel–concrete structures, notably the outcomes of the *SmartCoCo* project, and translates them into practical design guidance [21]. It provides an insight into the theoretical principles behind force-transfer mechanisms, longitudinal shear resistance and the stability analysis of slender hybrid columns. It also presents practical approaches to designing walls and columns with several embedded steel profiles. Within this thesis, the book serves as a major reference for both the theoretical developments and the interpretation of the numerical results obtained with *MegaColumn*.

N–M interaction modelling

Across the reviewed studies, the determination of the N – M interaction diagram is generally based on simplified interaction formulations derived from Eurocode 4. These formulations are adapted to account for the specific behaviour of megacolumns, but are mostly intended for simplified hand calculations [31, 32].

As a different method is presented later in this thesis, these formulations will not be discussed further.

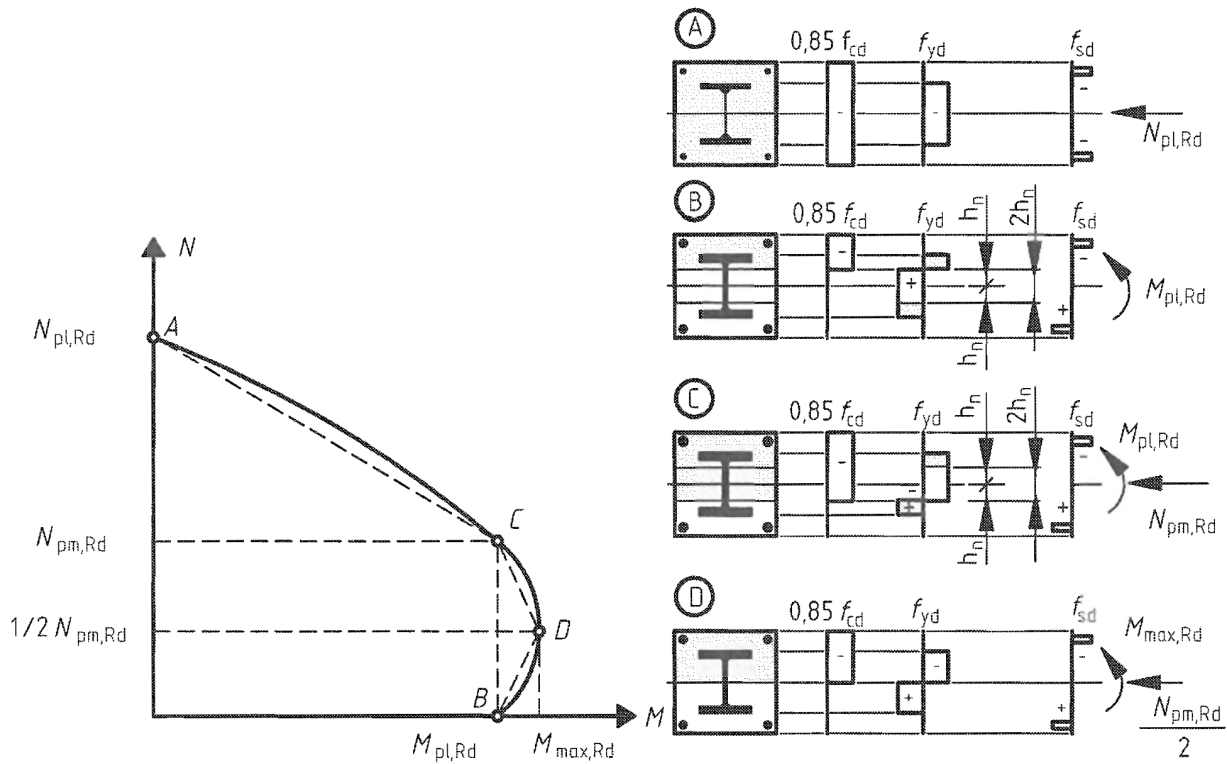


Figure 2.10: Simplified axial force–bending moment interaction curve according to Eurocode 4 [15].

Chapter 3 Validation Process, *MegaColumn* Software and Geometry Definition

3.1 Validation Strategy

To achieve the main objective of this thesis, the first step consists of validating the *MegaColumn* software before investigating additional theoretical developments that could later be implemented within it.

The validation process uses the *Python Code* implementation developed during this thesis. There are two main purposes behind this implementation:

- **Reproduction of the *MegaColumn* software behaviour:** The *Python Code* model aims to reproduce the behaviour of the *MegaColumn* software as closely as possible, based on theoretical formulations and modelling assumptions derived from the literature.
- **Support for the parametric analyses:** The *Python Code* implementation is also used for the parametric analyses presented later in this work, since the architectural source code of the *MegaColumn* software is not publicly accessible.

However, it remains possible that both the *MegaColumn* software and the independent *Python Code* implementation rely on similar assumptions and could thus reproduce the same modelling inaccuracies.

For this reason, a third-party software, *BiAxialPro*, based on the finite element method (FEM)¹, is introduced as an additional reference tool [34]. This numerical software provides an independent level of verification and helps ensure that any observed differences are more likely related to modelling assumptions rather than implementation errors.

In this study, *BiAxialPro* is therefore considered as the **reference software**, and the results obtained using the *MegaColumn* and the *Python Code* are compared.

The validation workflow adopted throughout this thesis is schematised in Figure 3.1.

¹The Finite Element Method is a numerical method used to obtain approximate solutions to engineering and physical problems by discretising a system into smaller interconnected elements [33].

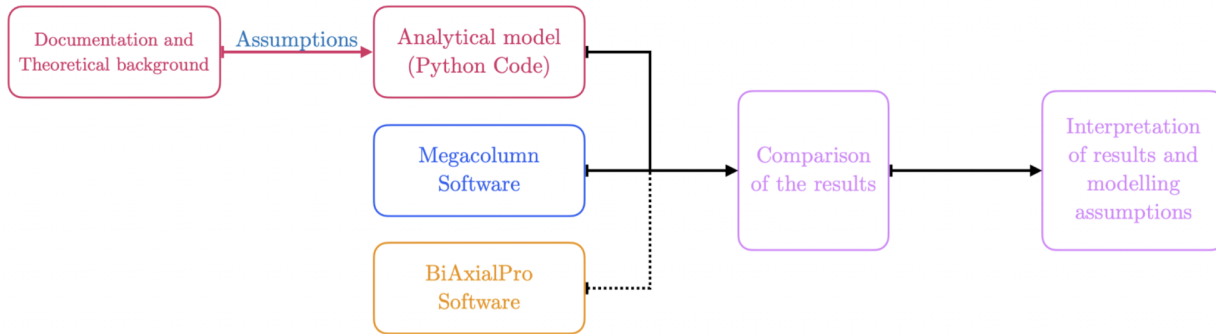


Figure 3.1: Schematic overview of the validation workflow used to assess the *MegaColumn* software.

3.2 *MegaColumn* Software

The *MegaColumn* software may be considered as a *black-box* tool, meaning that the internal modelling assumptions, numerical procedures, and calculation algorithms are not directly accessible to the user.

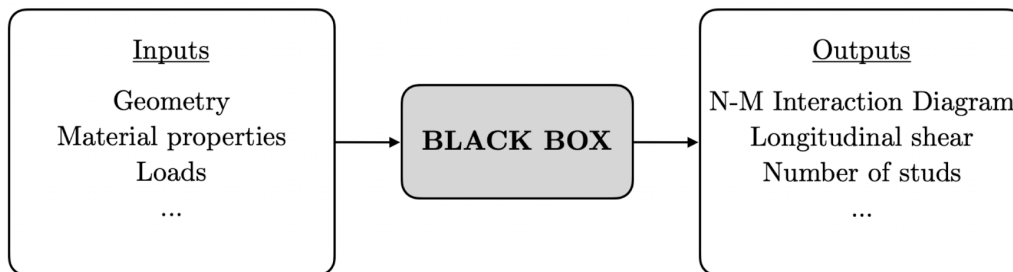


Figure 3.2: Schematic representation of the *MegaColumn* software as a black-box model, showing the input parameters and corresponding output quantities.

The user provides a set of input parameters (e.g. geometry, material laws, and loading conditions), after which the software performs the internal calculations and returns several output quantities related to the structural behaviour of the composite column, such as the N – M interaction diagram, the longitudinal shear forces, and the required number of shear studs.

The general organisation of the software interface is reported in Figure 3.3. The left-hand side of the interface contains the different input parameters defining the megacolumn configuration, while the upper part provides access to the main output results and analysis tools.

A more detailed description of the software interface and input parameters is provided in the software documentation [11].

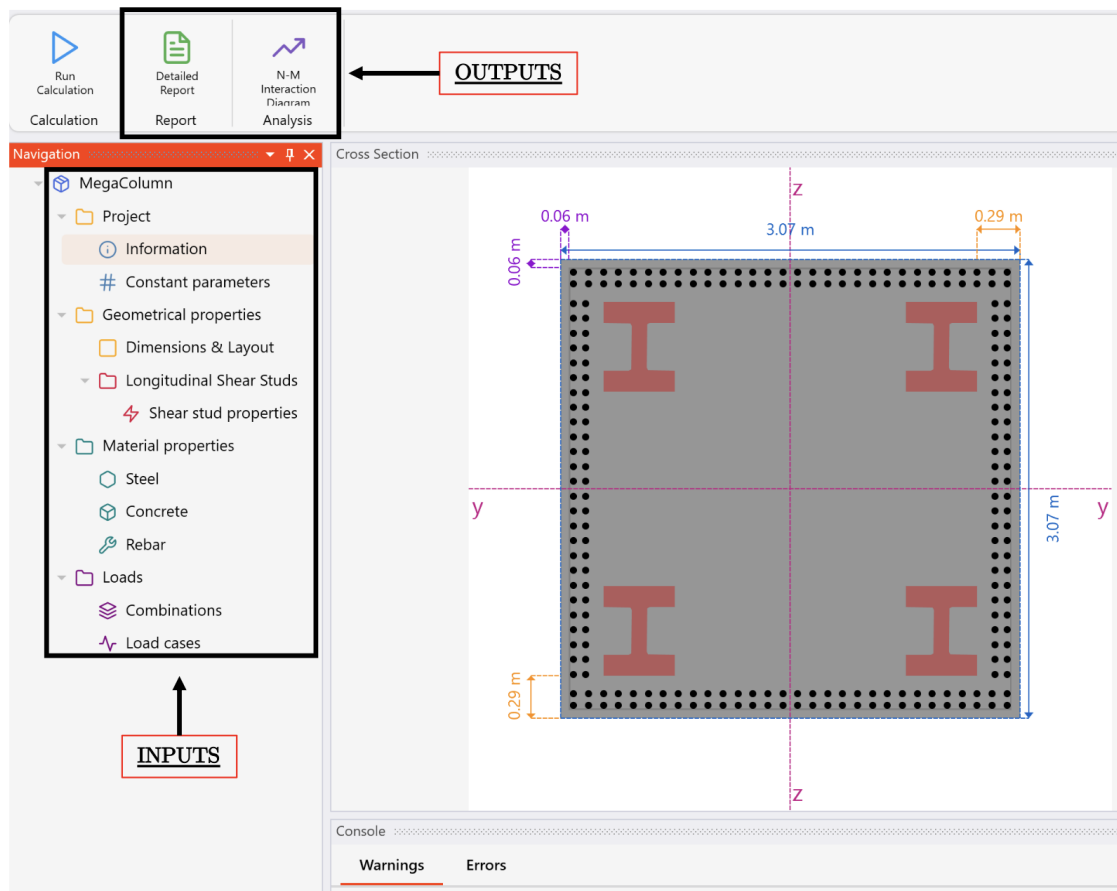


Figure 3.3: Graphical user interface of the *MegaColumn* software showing the main input parameters, geometrical definition of the composite section, and available output analyses [11].

Although advanced FEM-based software can be directly used for design verification, the development of the *MegaColumn* software is motivated by several practical reasons:

- **Suitability for early-stage design:** FEM-based software typically demands detailed structural models and significant input data, which are often unavailable during the initial design phase. In contrast, *MegaColumn* is specifically intended for preliminary design applications, using simplified analytical formulations and limited input parameters.
- **Fast calculations and practical use:** FEM analyses often require significant computational resources and longer calculation times. By comparison, *MegaColumn* is designed as a practical and user-friendly tool, providing quick results without the need for advanced numerical modelling.
- **Industrial development and dissemination:** From an industrial perspective, *MegaColumn* is intended to become an internally developed tool that could eventually be distributed worldwide as an open-access application. This objective is consistent with several design tools already provided by ArcelorMittal on its website, such as *ABC* or *A3C* [35].

3.3 BiAxialPro Software

The main reasons why FEM-based software is generally not used on a daily basis for the preliminary design of megacolumns were already discussed previously. Nevertheless, it is important to explain why the *BiAxialPro* software is adopted here as the reference tool for the validation process.

First, it is important to emphasise that no numerical model can perfectly reproduce the real structural behaviour of a megacolumn. This fundamental limitation of engineering modelling is well summarised by the statistician George Box:

“All models are wrong, but some are useful.” — George E. P. Box [36]

The objective is not to obtain a perfect model, but rather to develop models that are sufficiently accurate and useful for engineering applications.

The *BiAxialPro* software was selected as the reference tool for several reasons [34, 37]:

- **Independent development:** Unlike the *MegaColumn* software, *BiAxialPro* is completely independent of the ArcelorMittal development environment. This reduces the risk of reproducing the same modelling assumptions or implementation biases.
- **Finite element formulation:** As the software is based on FEM, it provides a detailed representation of the structural behaviour of the composite section. The geometry is divided into small triangular elements, enabling each part of the section to be analysed independently while ensuring an accurate overall structural response.
- **Flexible geometrical modelling:** The software is not limited to predefined megacolumn geometries. Two approaches can be used for the geometrical definition:
 - standard sections can be directly selected from the internal catalogue, with reinforcing bars added manually;
 - more complex geometries can be imported using *DXF* files², allowing the modelling of highly specific composite sections such as megacolumns.
- **User-defined modelling assumptions:** The software gives direct control to the user over all modelling parameters, such as material constitutive laws, stress–strain relationships, reinforcement properties, meshing strategy, and analysis assumptions, as observed in Figure 3.4. In contrast to simplified software, nearly every parameter must be explicitly defined by the user rather than being implicitly embedded in the program. This provides a high level of modelling flexibility and transparency, but also requires significantly more modelling effort and engineering expertise.
- **Existing software validation:** A set of validation benchmarks was already carried out by the developers before the release of the software. These validation examples are available in the software documentation [37].

²DXF files are vector-based drawing files commonly used in CAD software.

- **Dedicated cross-section analysis tool:** Compared to general-purpose FEM software such as *Abaqus*, *BiAxialPro* is specifically dedicated to the computation of cross-sectional structural behaviour, including the generation of $N-M$ interaction diagram interaction diagrams and related sectional outputs. This makes the software more suitable and efficient for structural cross-section analysis, whereas software such as *Abaqus* provides a much broader range of modelling capabilities and output information.

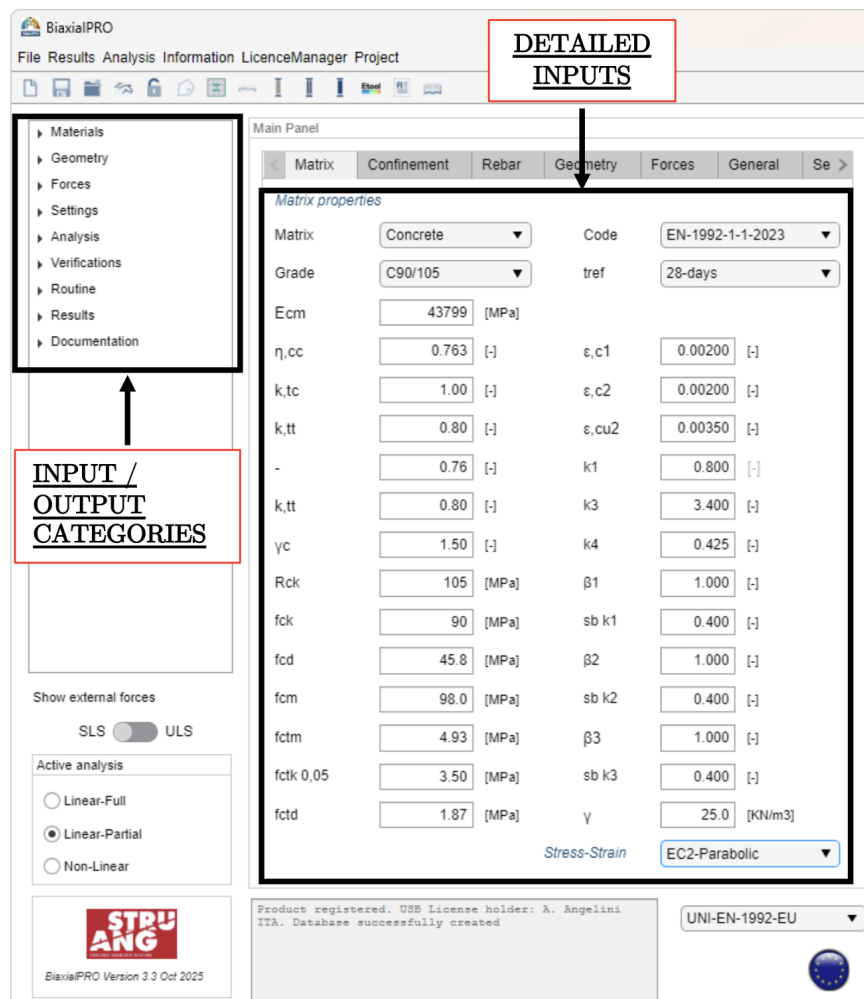


Figure 3.4: Graphical user interface of the *BiAxialPro* software showing the main input families [34].

Ultimately, the final argument supporting the use of *BiAxialPro* as the reference software remains the comparison of the results themselves. If the results obtained with the different approaches remain sufficiently close to one another, this strengthens the confidence in the relevance of the adopted reference methodology.

3.4 Geometry Definition and User Inputs

For the purpose of the comparison and validation, a unique representative megacolumn geometry is considered. The geometric parameters and their corresponding notations are illustrated in Figure 3.5.

The user input parameters available in the *MegaColumn* software, along with those used inside *Python Code*, are provided in a detailed summary in Table 3.1.

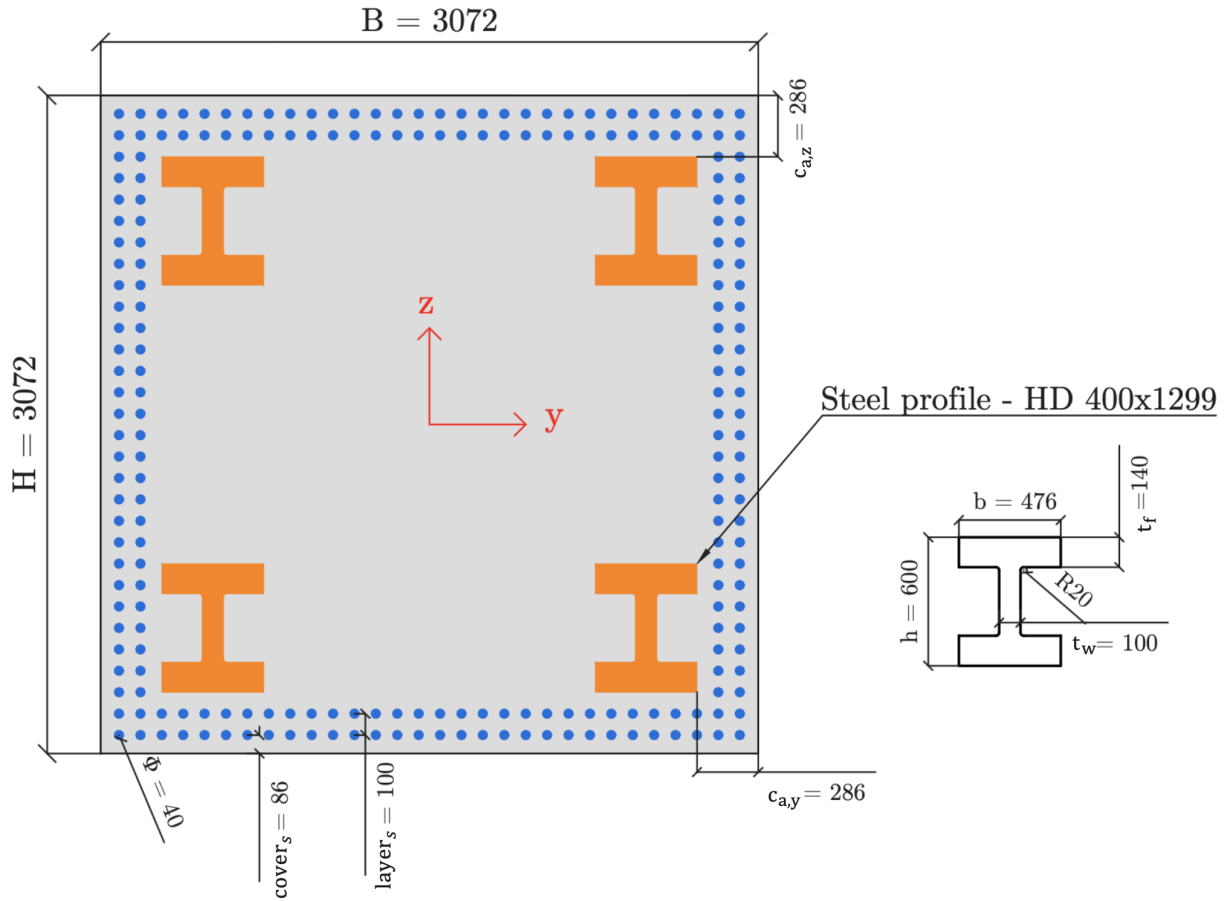


Figure 3.5: Geometric definition of the studied megacolumn cross-section and associated input parameters.

Category	Parameter	Description (Range)	Value
Steel profile	–	Profile type	HD 400 × 1299
Geometry	H	Section height	3 072 mm
	B	Section width	3 072 mm
	$\varnothing$	Rebar diameter (8–40 mm)	40 mm
	$n_{s,y}$	Number of rebars parallel to the y -axis	30
	$n_{s,z}$	Number of rebars parallel to the z -axis	30
Covers	n_{layer}	Number of reinforcement layers (1 or 2)	2
	$cover_s$	Distance from concrete edge to rebar face	66 mm
	$c_{a,y}$	Distance from concrete edge to steel profile (y -direction)	286 mm
	$c_{a,z}$	Distance from concrete edge to steel profile (z -direction)	286 mm
Studs type	–	Stud type according to the Stanley catalogue [38]	M19-100
Material properties	f_{ck}	Concrete strength class (C12/15 to C100/115)	50 MPa (C50/60)
	–	Eurocode 2 version (EC2-1-1:2004 or EC2-1-1:2023)	EC2-1-1:2023
	–	Concrete constitutive law (Parabolic–Rectangular / Non-linear)	Parabolic–Rectangular
	$f_{a,y}$ $f_{s,y}$	Yield strength of steel profile (S235–HISTAR 460)	460 MPa
Safety factors		Yield strength of reinforcing steel (B400–B700, classes A to C)	500 MPa (B500B)
	γ_a	Partial safety factor for structural steel	1.00
	γ_c	Partial safety factor for concrete	1.50
Loads	γ_s	Partial safety factor for reinforcing steel	1.15
	N_{Ed}	Axial force	–
	$M_{Ed,y}$	Bending moment about y -axis	–
	$M_{Ed,z}$	Bending moment about z -axis	–
	$V_{Ed,y}$	Shear force along y -axis	–
	$V_{Ed,z}$	Shear force along z -axis	–
Buckling effect	L_f	Effective length	15 m
	–	Stiffness model EI (Elastic or Hybrid)	Elastic Stiffness Method

Table 3.1: User input parameters and values used for the reference configuration

Chapter 4 *Python Code* Implementation of the N – M Interaction Diagram

This chapter presents the main calculation procedures and modelling concepts implemented in the *Python Code*. The details related to the code architecture and the user interface are provided in Appendix B.

The N – M interaction diagram is computed using the *General Method* described in Eurocode 2 [12, 24]. Figure 4.1 summarises the overall procedure.

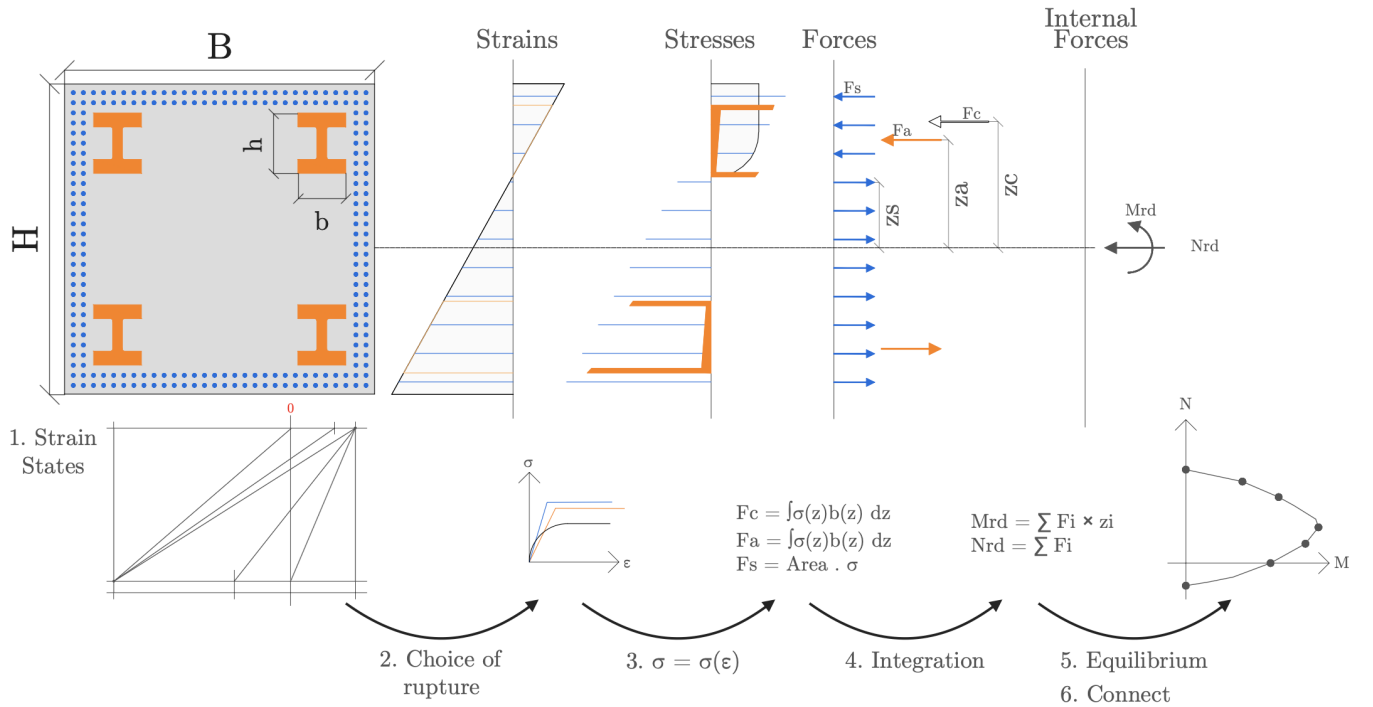


Figure 4.1: Schematic overview of the main steps used to compute the N – M interaction diagram.

- **Step 0: Discretisation:** For a given orientation, the section is divided into horizontal layers.
- **Step 1: Strain states:** Definition of ultimate strain states where at least one point of the section reaches failure.
- **Step 2: Strain profile:** For each strain state, computation of the strain in each layer.
- **Step 3: Stresses:** Computation of stresses from material stress–strain laws.

- **Step 4: Internal forces:** Integration of stresses over the area of each layer.
- **Step 5: Resultant forces:** Determination of axial force N and bending moment M from equilibrium.
- **Step 6: Diagram:** Connection of (N, M) points to form the interaction curve.

4.1 Step 0: Discretisation of the Section

The composite cross-section is discretized into horizontal layers of constant thickness. This discretisation transforms the heterogeneous cross-section, which contains concrete, structural steel profiles, and reinforcing bars, into a set of simpler elements that can be treated numerically.

Within each layer:

- Reinforcing bars are not discretised as layers but are instead modelled as concentrated points with a defined area. Therefore, it is assumed that this region is filled with concrete.
- The fillet radius of the steel profile is neglected.
- When both concrete and structural steel are present in the same layer, the layer is divided into two sub-areas corresponding to each material. Figure 4.2 schematically represents the width subdivision.

Thus, the total width of the section at a given level can be written as

$$b_{\text{section}} = b_{\text{concrete}} + b_{\text{steel}} \quad (4.1)$$

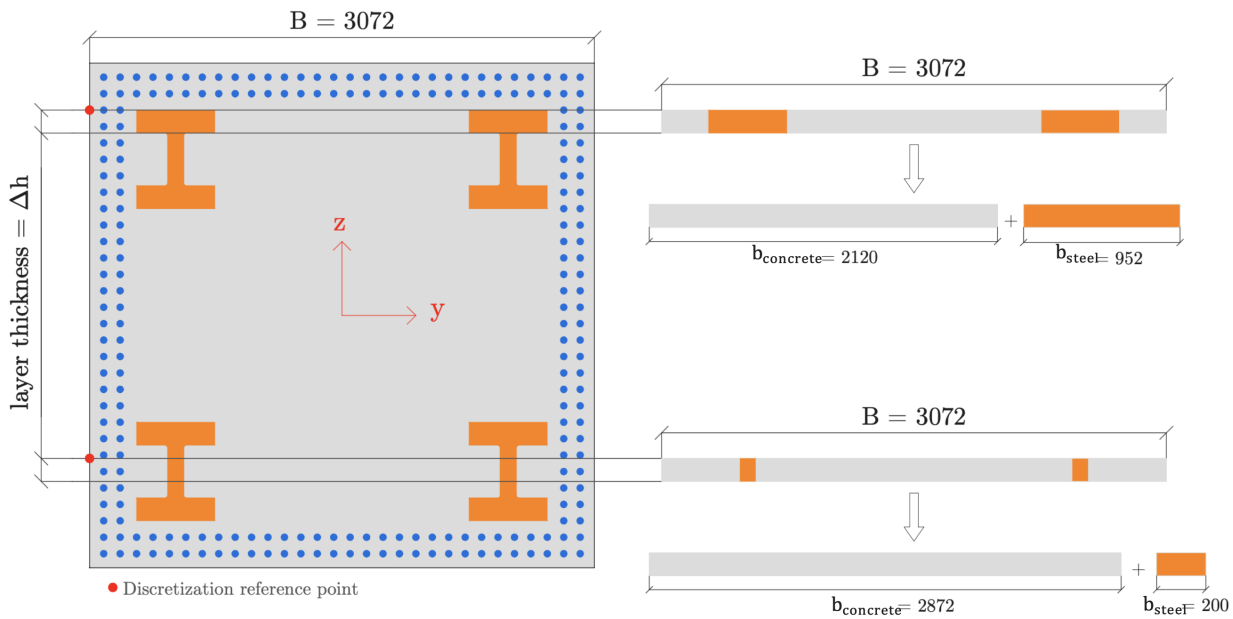


Figure 4.2: Illustration of the layer subdivision used when concrete and structural steel are present within the same discretised layer.

4.2 Step 1: Selection of the Strain State

A strain state is defined as a condition of the cross-section in which at least one point reaches failure. Failure is assumed to occur when a material reaches its ultimate strain. For instance, in the case of concrete modelled with a parabolic-rectangular law, failure in compression is reached at $\varepsilon_{cu2} = 3.5\text{‰}$ for a concrete class C50/60. It should be noted that concrete is assumed to have no tensile capacity.

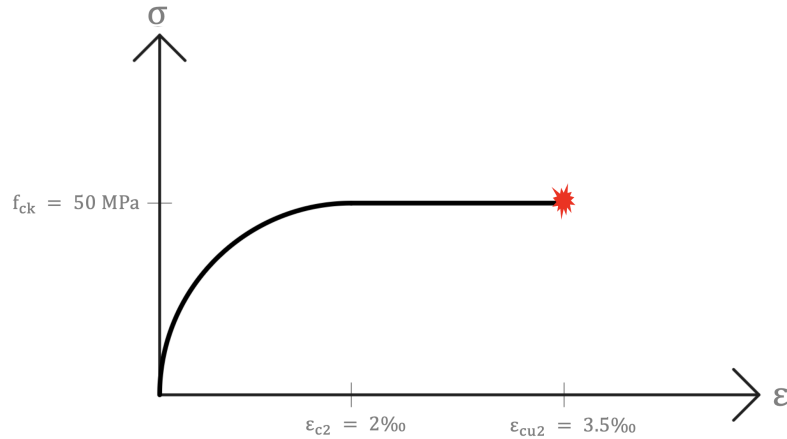


Figure 4.3: Concrete stress–strain relationship based on a parabolic-rectangular law, showing the characteristic strength $f_{ck} = 50\text{ MPa}$ and the ultimate strain $\varepsilon_{cu2} = 3.5\text{‰}$.

Beyond this limit, the concrete experiences a loss of stiffness and strength, and the section can no longer sustain additional load in a reliable manner. The structural response then becomes unstable [39].

Moreover, concrete is not the only material present in the cross-section. Reinforcing bars and structural steel profiles must also be considered. Unlike concrete, these materials exhibit significant ductility in both tension and compression, with ultimate strains much higher than those of concrete. Typically, reinforcing steel may reach ultimate strains ranging from 2.5% to 7.5%, as specified in the latest version of Eurocode 2 [25], whereas structural steel profiles may exhibit elongations of at least 15%, depending on the steel grade and manufacturer. In the present work, the material properties provided by *ArcelorMittal* are considered [40].

Additionally, as mentioned earlier, the column may also be subjected to bending. Hence, under pure compression, the behaviour is governed by the concrete, as it reaches its ultimate strain first. In contrast, under tensile conditions (neglecting the contribution of the concrete), the behaviour is governed by the reinforcing steel.

In order to account for all possible failure modes, a diagram of admissible strain distributions, referred here as the *strain profile diagram*, is constructed. Two reference points are considered at the extreme fibres of the section, as these locations are the most likely to reach failure first. The top point corresponds to failure of the concrete in compression, while the bottom point corresponds to failure of the reinforcing bars in tension.

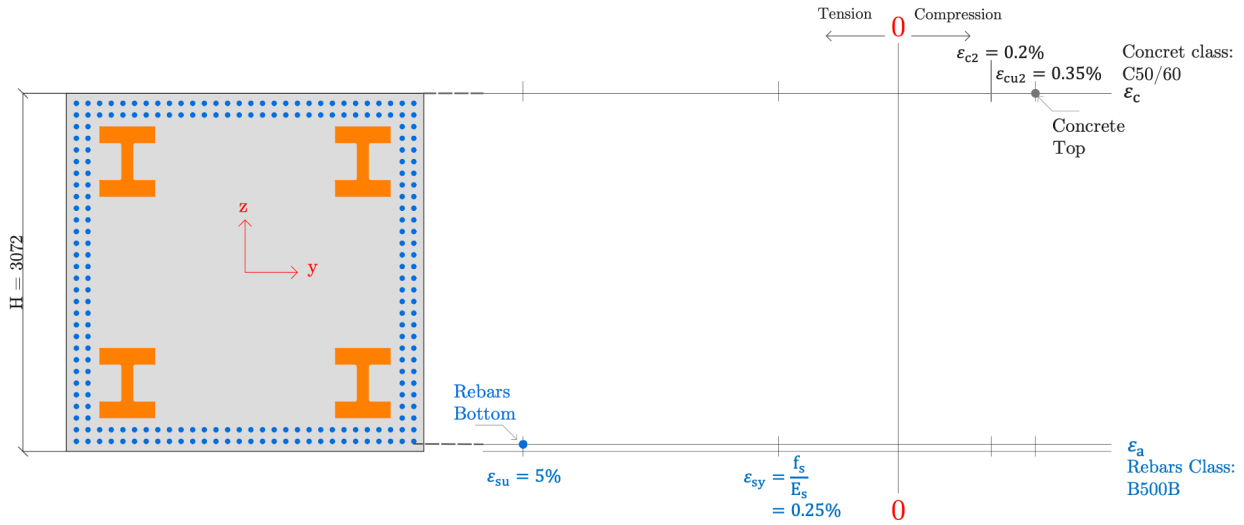


Figure 4.4: Definition of the reference strain limits at the extreme fibres of the cross-section.

A series of linear strain distributions can be defined from these two reference points, each of which corresponds to a potential failure mode.

Each line in the *strain profile diagram* represents a particular strain state of the section. For example, a fully vertical line, such as the one on the far left in Figure 4.5, indicates that the entire section is subjected to uniform tension, with a strain equal to $\varepsilon_{su} = 5\%$ throughout the section.

An inclined line represents a linear strain distribution over the section height, meaning that the strain varies depending on the vertical coordinate z . Consequently, different fibres of the section are subjected to different strain values.

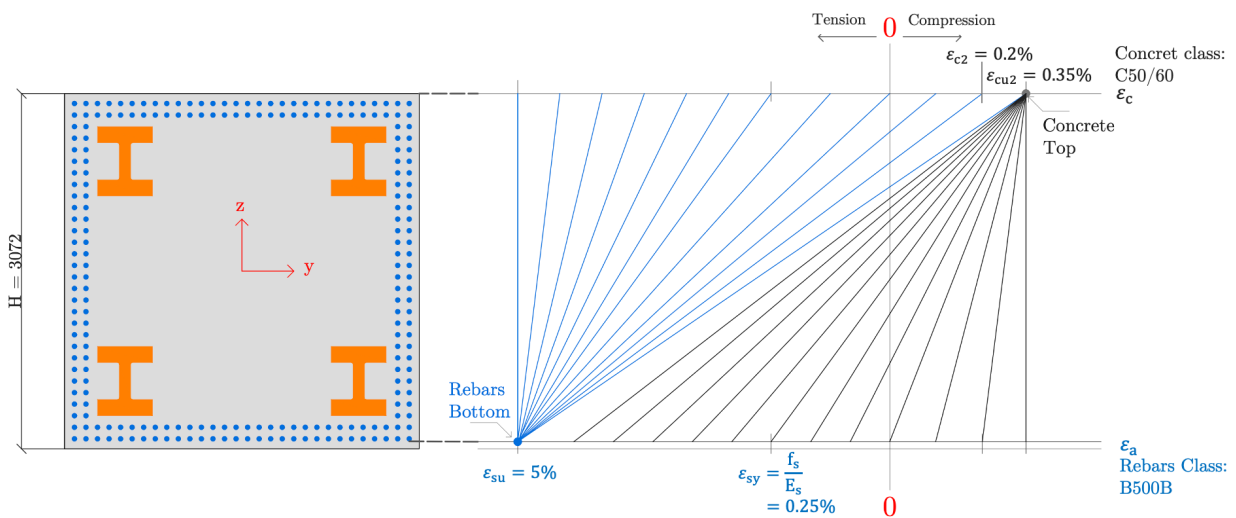


Figure 4.5: Linear strain distributions obtained by connecting the reference strain limits.

Although an infinite number of such strain states can be defined, only a limited number of representative cases are considered in practice, as highlighted in Figure 4.6. These correspond to critical and physically meaningful configurations, such as:

- **State 7:** The entire section reaches the ultimate concrete strain ε_{cu2} , meaning that the whole concrete section is at failure.
- **State 5:** The top concrete fibre reaches ε_{cu2} , while the reinforcing bars at the bottom reach their yield strain ε_{sy} .
- **State 4:** Both the top concrete fibre reaches ε_{cu2} and the reinforcing bars at the bottom reach their ultimate strain ε_{su} .

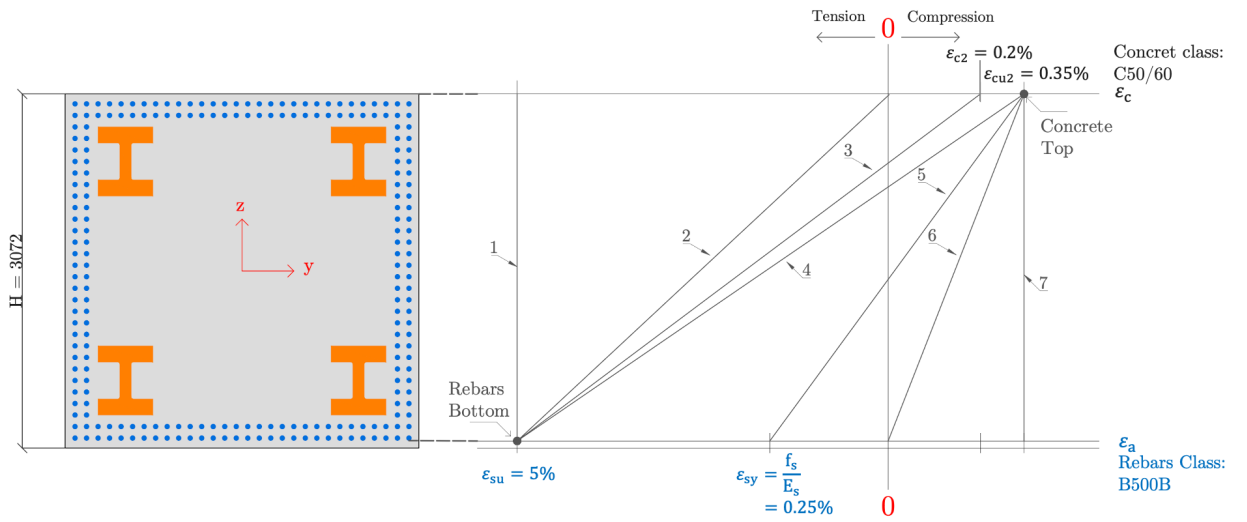


Figure 4.6: Typical strain states used to construct the $N-M$ interaction diagram.

4.3 Step 2: Computation of the Strain Profile

Once a strain state has been defined, the strain distribution over the entire section can be computed. Assuming a linear strain distribution according to the hypothesis of plane sections remaining plane, the strain at any position within the section can be determined from the strains defined at the top and bottom reference points.

Each discretised layer is assigned a vertical coordinate corresponding to its upper boundary. The strain in each layer is then determined from the linear strain profile at this position: positive values indicate compression, while negative values indicate tension.

As an illustration, Figure 4.7 presents strain state 4, considering reinforcing bars of ductility class B, for which $\varepsilon_{su} = 5\%$, and concrete with an ultimate compressive strain $\varepsilon_{cu2} = 3.5\%$.

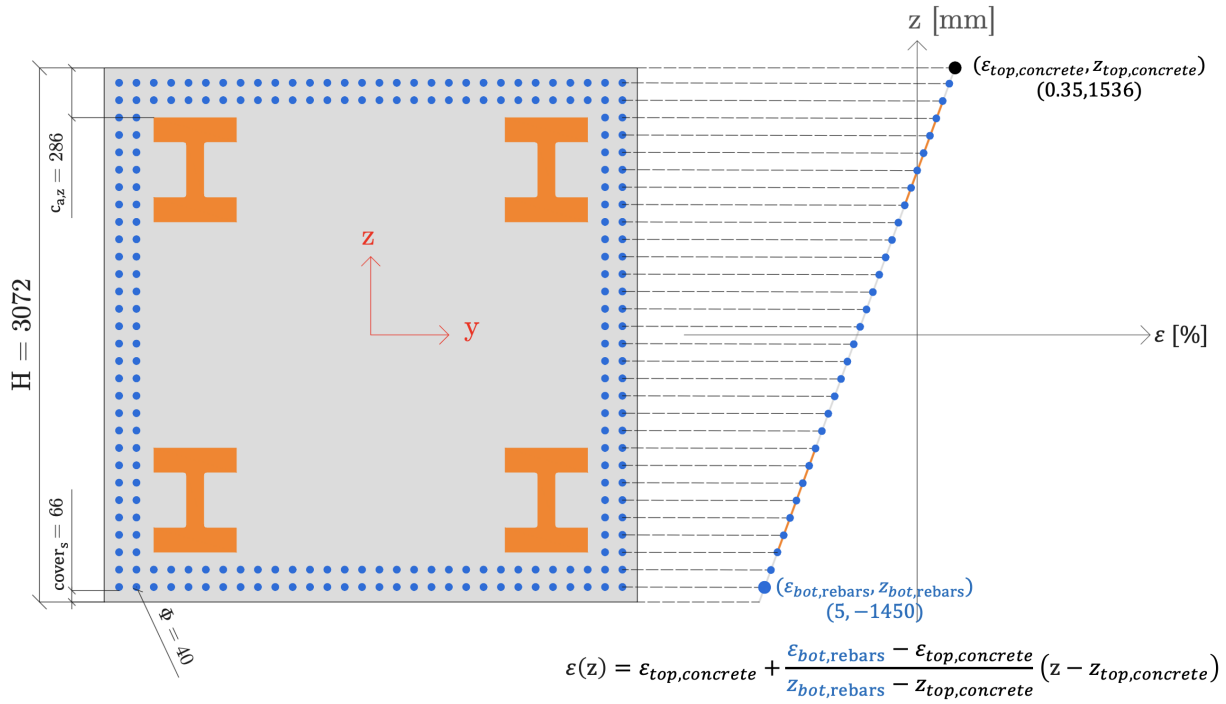


Figure 4.7: Linear strain distribution over the discretised cross-section.

4.4 Step 3: Computation of the Stresses

The stresses are obtained from the stress–strain relationships of the materials, expressed as:

$$\sigma = \sigma(\varepsilon) \quad (4.2)$$

Concrete, structural steel, and reinforcing steel are evaluated separately. While the constitutive law adopted for concrete has already been introduced, the corresponding constitutive laws for reinforcing steel and structural steel are presented in Figure 4.8.

These simplified material models were selected to provide an appropriate balance between modelling simplicity and numerical accuracy. The implications and limitations of these assumptions are further discussed in the following chapter.

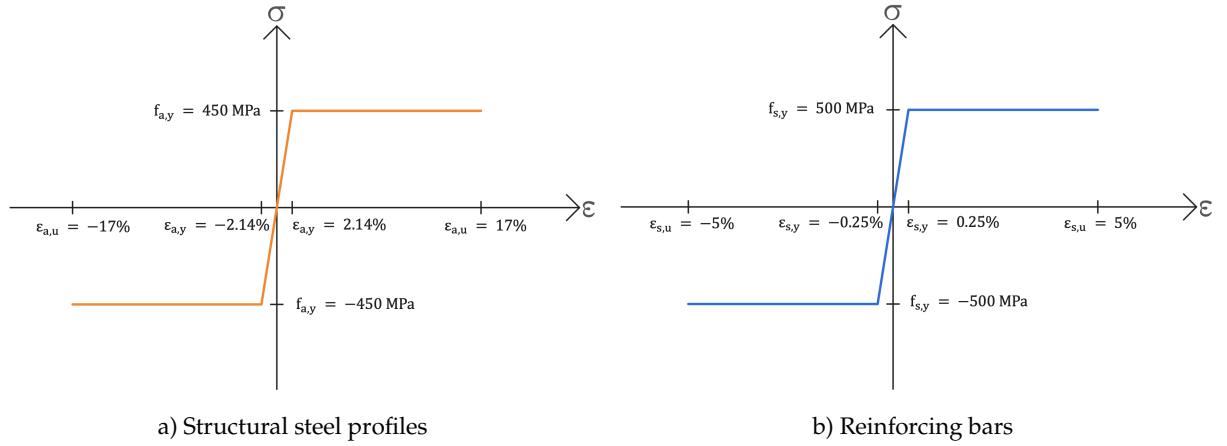


Figure 4.8: Steel components considered in the cross-section: a) structural steel profiles and b) reinforcing bars.

4.5 Step 4: Computation of the Internal Forces

The internal forces are obtained by integrating the stresses over the corresponding areas. For each discretised layer:

$$F_c = \sigma_c A_{c,layer} \quad (4.3)$$

where $A_{c,layer} = \Delta h \cdot b_{concrete}$.

$$F_a = \sigma_a A_{a,layer} \quad (4.4)$$

where $A_{a,layer} = \Delta h \cdot b_{steel}$.

Reinforcing bars are treated as concentrated points:

$$F_s = \sigma_s \frac{\pi \varnothing^2}{4} \quad (4.5)$$

This assumption leads to an overestimation of the concrete contribution, as the volume occupied by the reinforcing bars is effectively counted twice.

To correct this, the equivalent concrete force corresponding to the area occupied by the reinforcing bars is subtracted. Knowing the position of each bar and the strain distribution, this overestimated contribution can be expressed as:

$$F_{c,rebars} = \sigma_c \frac{\pi \varnothing^2}{4} \quad (4.6)$$

An illustrative example is shown in Figure 4.9, where an increased layer thickness and a reduced number of reinforcing bars are adopted to facilitate comprehension. The influence of the layer thickness is discussed in the following chapter.

The direction of the arrows indicates whether the stresses are in compression ($\leftarrow$) or tension ($\rightarrow$). As the area is always positive, the sign of the internal force is governed by the stress resulting from the strain.

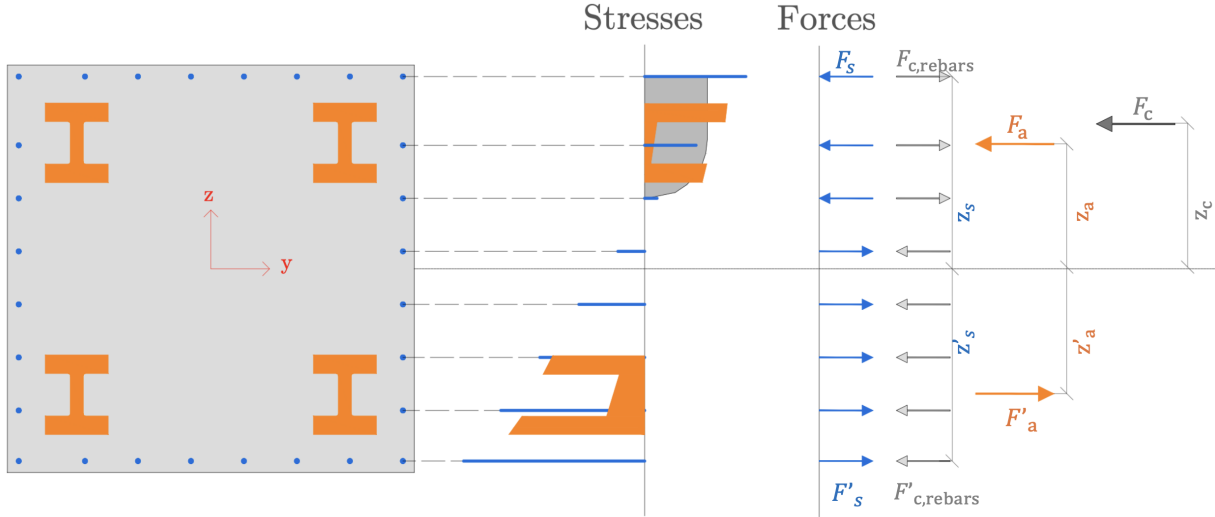


Figure 4.9: Illustration of the internal force distribution within the discretised cross-section.

4.6 Step 5: Determination of the resultant forces (N , M)

Finally, the axial force and bending moment corresponding to the considered strain state are obtained from equilibrium:

$$N_{Rd} = \sum_i F_i \quad (4.7)$$

$$M_{Rd} = \sum_i F_i z_i \quad (4.8)$$

where z_i represents the lever arm between the force F_i and the chosen reference axis (in this case, the y -axis).

4.7 Step 6: Assembly of the N – M Interaction Diagram

Repeating Steps 2 to 5 for all selected strain states allows the complete N – M interaction diagram to be constructed. As previously mentioned, increasing the number of strain states leads to a greater number of points and a more accurate representation of the interaction diagram.

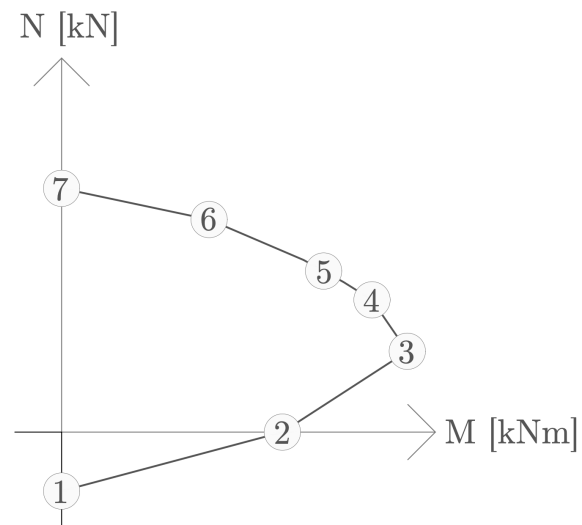


Figure 4.10: Construction of the N – M interaction diagram by connecting the points obtained from the selected strain states.

Chapter 5 Validation of N – M Interaction Diagram

This chapter presents a comparison of the N – M interaction diagrams obtained using the *Python Code* implementation, the *MegaColumn* software, and the *BiAxialPro* software.

This comparison has two objectives. Firstly, it evaluates the reliability of the *MegaColumn* software by comparing its results with those of the two other tools. Secondly, it assesses whether the Python model can reproduce *MegaColumn* behaviour, enabling its use for extended analyses in the next chapter.

5.1 Methodology of Comparison

The comparison is carried out using a *One Factor At a Time* (OFAT) approach. This method consists of varying a single parameter while keeping all other input variables constant [41].

A reference case is first defined (see Table 3.1). Each parameter used in the completion of the N – M interaction diagram is then modified individually in order to evaluate its influence on the results. This approach allows discrepancies between the different computational tools to be identified and highlights the parameters that have the most significant impact on the structural response.

The partial safety factors are kept identical for all simulations. All test cases are summarised in Table 5.1.

5.2 Comparison Criteria

For the N – M interaction diagram, the following quantities are evaluated:

- the maximum axial compressive force ($N_{\max}$);
- the maximum bending moment ($M_{\max}$) and the corresponding axial force (N_{corr});
- the area enclosed by the interaction curve (A_{curve});
- the overall shape of the interaction diagram.

The relative difference between two values is expressed as a percentage using:

$$\Delta = \frac{\text{reference value} - \text{compared value}}{\text{reference value}} \times 100 \quad (5.1)$$

Test No.	Name of the file	Steel profile	H	B	$\emptyset$	$n_{s,y}$	$n_{s,z}$	n_{layer}	C_s	$C_{a,y}$	$C_{a,z}$	Concrete class	Profile grade	Rebars class
0	GeneralCase	HD 400 × 1299	3072	3072	40	30	30	2	66	286	286	C50/60	HISTAR460	B500B
1_1	steel_profile_name_HE300B	HE 300 B	3072	3072	40	30	30	2	66	286	286	C50/60	HISTAR460	B500B
1_2	steel_profile_name_IPE750	IPe 750 × 134	3072	3072	40	30	30	2	66	286	286	C50/60	HISTAR460	B500B
2_1	H_2500	HD 400 × 1299	2500	3072	40	30	30	2	66	286	286	C50/60	HISTAR460	B500B
3_1	B_2500	HD 400 × 1299	3072	2500	40	30	30	2	66	286	286	C50/60	HISTAR460	B500B
4_1	diameter_10	HD 400 × 1299	3072	3072	10	30	30	2	66	286	286	C50/60	HISTAR460	B500B
4_2	diameter_20	HD 400 × 1299	3072	3072	20	30	30	2	66	286	286	C50/60	HISTAR460	B500B
5_1	n_rebars_z_10	HD 400 × 1299	3072	3072	40	30	10	2	66	286	286	C50/60	HISTAR460	B500B
5_2	n_rebars_y_10	HD 400 × 1299	3072	3072	40	10	30	2	66	286	286	C50/60	HISTAR460	B500B
5_3	n_rebars_zy_10	HD 400 × 1299	3072	3072	40	10	10	2	66	286	286	C50/60	HISTAR460	B500B
5_4	n_rebars_z_40	HD 400 × 1299	3072	3072	40	30	40	2	66	286	286	C50/60	HISTAR460	B500B
5_5	n_rebars_y_40	HD 400 × 1299	3072	3072	40	40	30	2	66	286	286	C50/60	HISTAR460	B500B
5_6	n_rebars_zy_40	HD 400 × 1299	3072	3072	40	40	40	2	66	286	286	C50/60	HISTAR460	B500B
6_1	n_layer_1	HD 400 × 1299	3072	3072	40	30	30	1	66	286	286	C50/60	HISTAR460	B500B
7_1	rebars_cover_30	HD 400 × 1299	3072	3072	40	30	30	2	30	286	286	C50/60	HISTAR460	B500B
7_2	rebars_cover_100	HD 400 × 1299	3072	3072	40	30	30	2	100	286	286	C50/60	HISTAR460	B500B
8_1	profile_cover_y_500	HD 400 × 1299	3072	3072	40	30	30	2	66	500	286	C50/60	HISTAR460	B500B
8_2	profile_cover_z_500	HD 400 × 1299	3072	3072	40	30	30	2	66	286	500	C50/60	HISTAR460	B500B
8_3	profile_cover_yz_500	HD 400 × 1299	3072	3072	40	30	30	2	66	500	500	C50/60	HISTAR460	B500B
9_1	concrete_class_C30	HD 400 × 1299	3072	3072	40	30	30	2	66	286	286	C30/37	HISTAR460	B500B
9_2	concrete_class_C90	HD 400 × 1299	3072	3072	40	30	30	2	66	286	286	C90/105	HISTAR460	B500B
10_1	profile_grade_S35J	HD 400 × 1299	3072	3072	40	30	30	2	66	286	286	C50/60	S355 J0/JR/J2/K2	B500B
10_2	profile_grade_S420M	HD 400 × 1299	3072	3072	40	30	30	2	66	286	286	C50/60	S420 M/ML	B500B
11_1	rebars_class_B450B	HD 400 × 1299	3072	3072	40	30	30	2	66	286	286	C50/60	HISTAR460	B450B
11_2	rebars_class_B700B	HD 400 × 1299	3072	3072	40	30	30	2	66	286	286	C50/60	HISTAR460	B700B

Table 5.1: OFAT test matrix used for the comparison between the computational tools. Each test case differs from the reference configuration by a single modified parameter, highlighted in red.

5.3 Results and Discussion

The objective of this section is not to present each individual test case in detail, as the complete dataset is not included here but can be requested from the author. Instead, the results are analysed globally by examining the distribution of differences for each quantity.

To achieve this, *box-and-whisker plots* are used. These plots provide a statistical overview of the results, including the minimum and maximum values, the median, and the interquartile range (i.e. the range containing 50% of the data) [42].

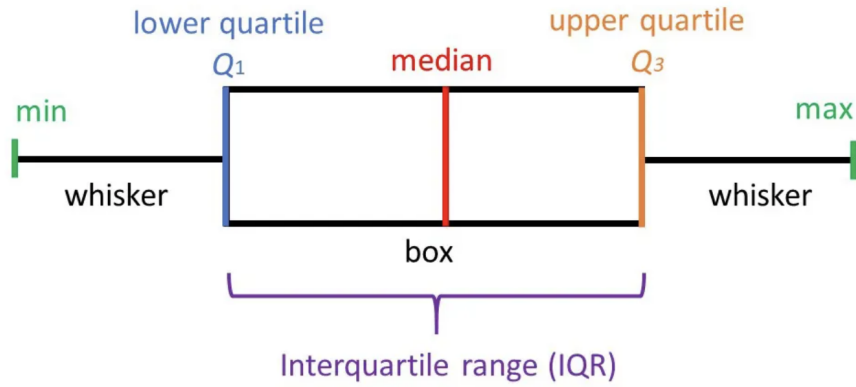


Figure 5.1: Example of a box-and-whisker plot representation [43].

The results presented correspond to bending about the y -axis. Results for bending about the z -axis are provided in the Appendix C.

For each figure, the left-hand side shows the relative differences for all test cases, considering three comparisons: *Python Code–MegaColumn* (Py vs MC), *Python Code–BiAxialPro* (Py vs BA), and *MegaColumn–BiAxialPro* (MC vs BA). According to Equation (5.1), a positive value indicates that the first software predicts a higher value than the second, and vice versa.

The right-hand side presents the statistical distribution using box-and-whisker plots.

5. Validation of N – M Interaction Diagram

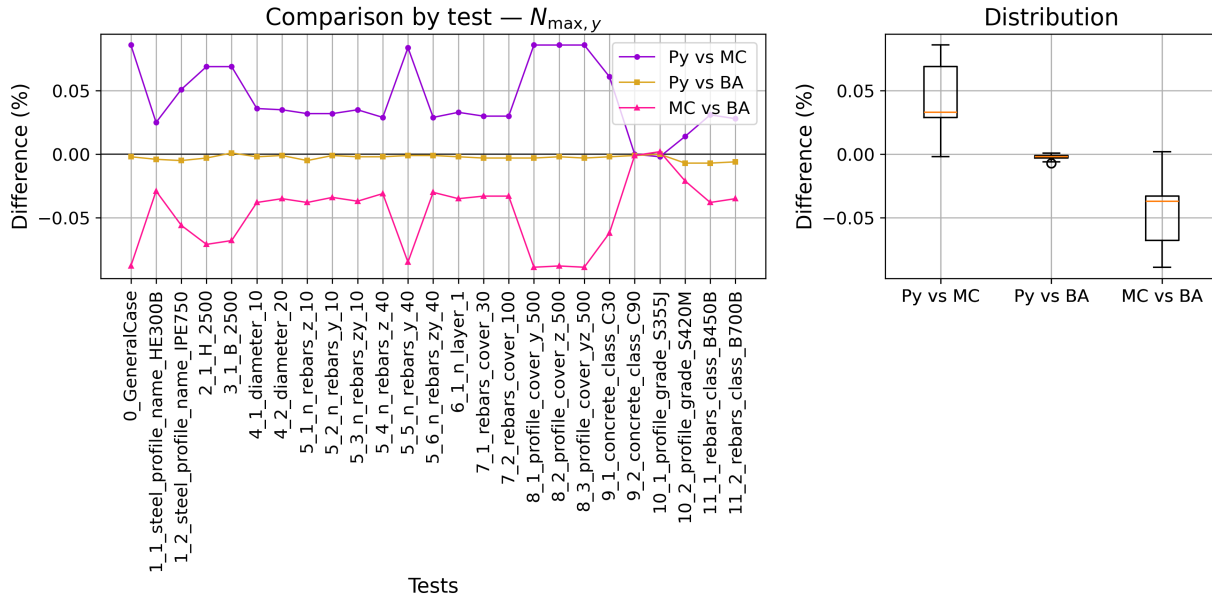


Figure 5.2: Relative differences in maximum axial force $N_{\max}$ for bending about the y -axis.

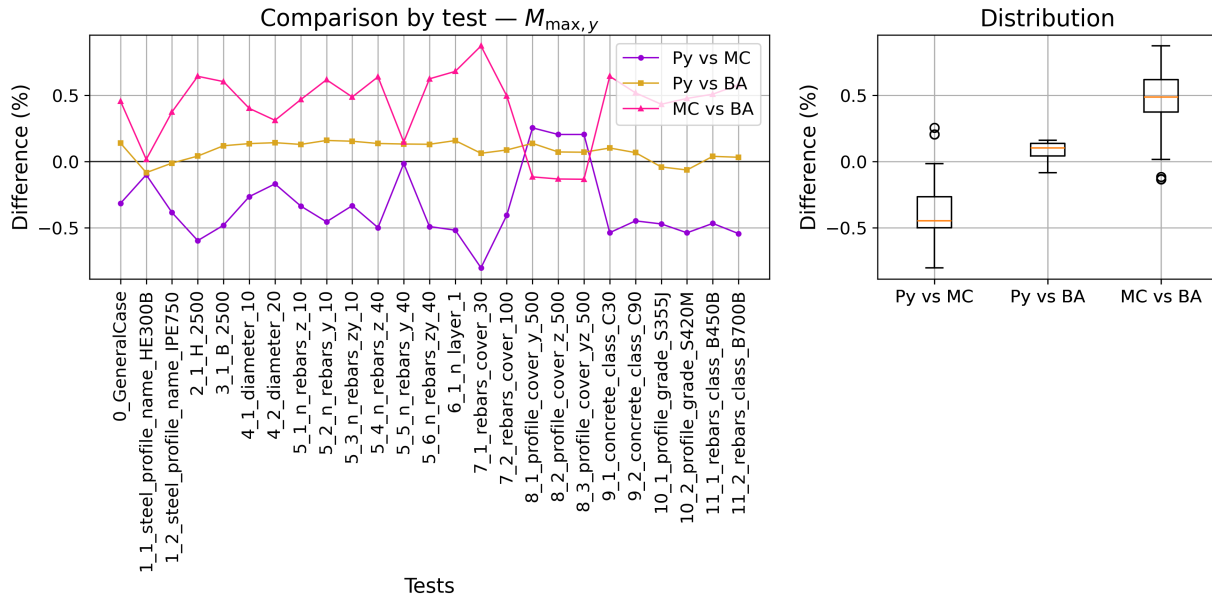


Figure 5.3: Relative differences in maximum bending moment $M_{\max}$ for bending about the y -axis.

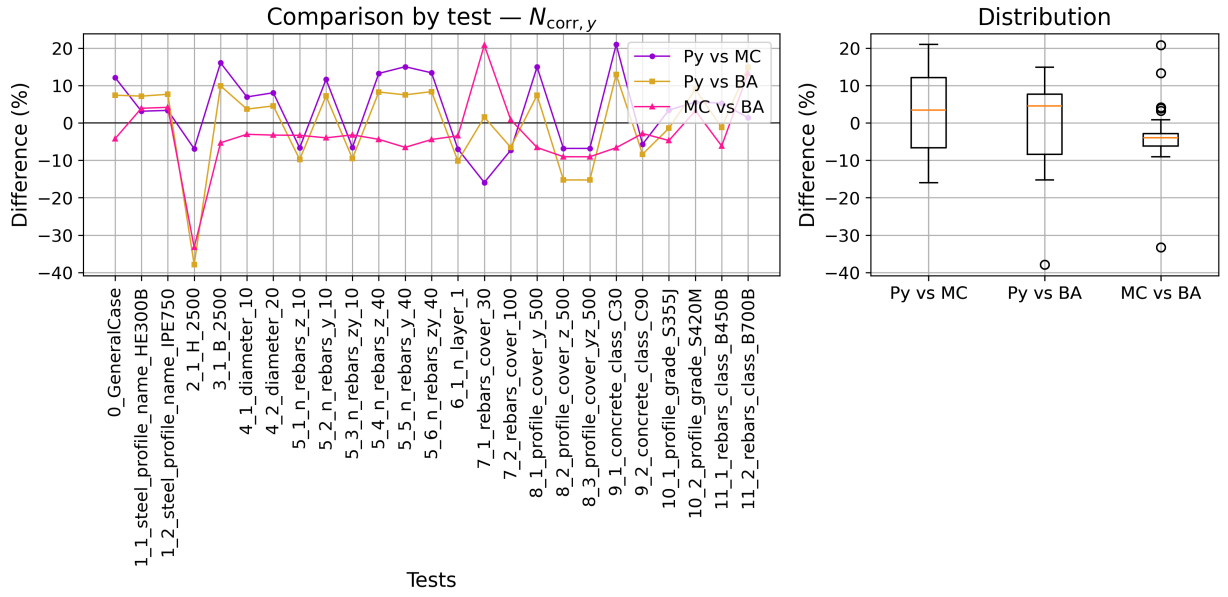


Figure 5.4: Relative differences in corresponding axial force N_{corr} for bending about the y -axis.

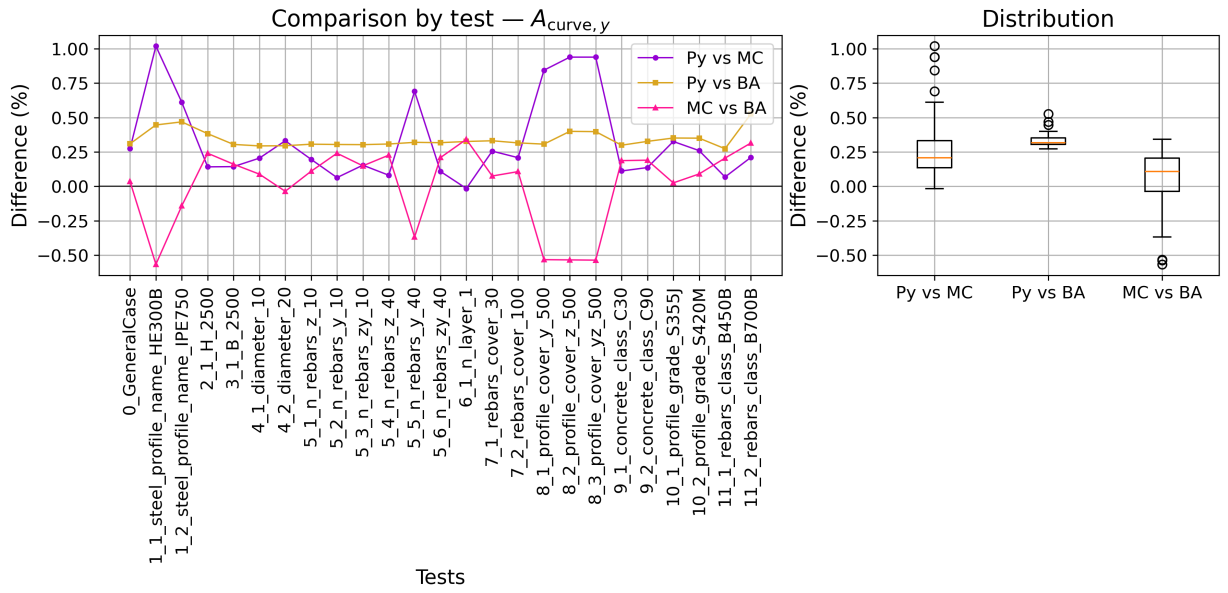


Figure 5.5: Relative differences in interaction curve area A_{curve} for bending about the y -axis.

Discussion

The main observations can be summarised as follows:

- **Consistency of global indicators and overall curve behaviour**

The comparison is based on four quantitative indicators ($N_{\max}$, $M_{\max}$, N_{corr} , and A_{curve}), complemented by a qualitative assessment of the overall shape of the interaction diagram. However, it is important to emphasise that the first three global quantities ($N_{\max}$, $M_{\max}$, and A_{curve}) already provide a robust indication of the consistency between the curves.

For these three parameters, the relative differences remain mostly within $\pm 1\%$, demonstrating a very strong agreement between the three tools. Since these quantities respectively characterise the axial capacity, the bending capacity, and the global extent of the interaction domain, it indicates that the overall shape and structural behaviour of the N – M interaction diagrams are accurately captured.

This conclusion is further supported by the direct visual comparison of selected curves plotted in Figure 5.6, where the three tools exhibit an almost identical response over the full interaction range.

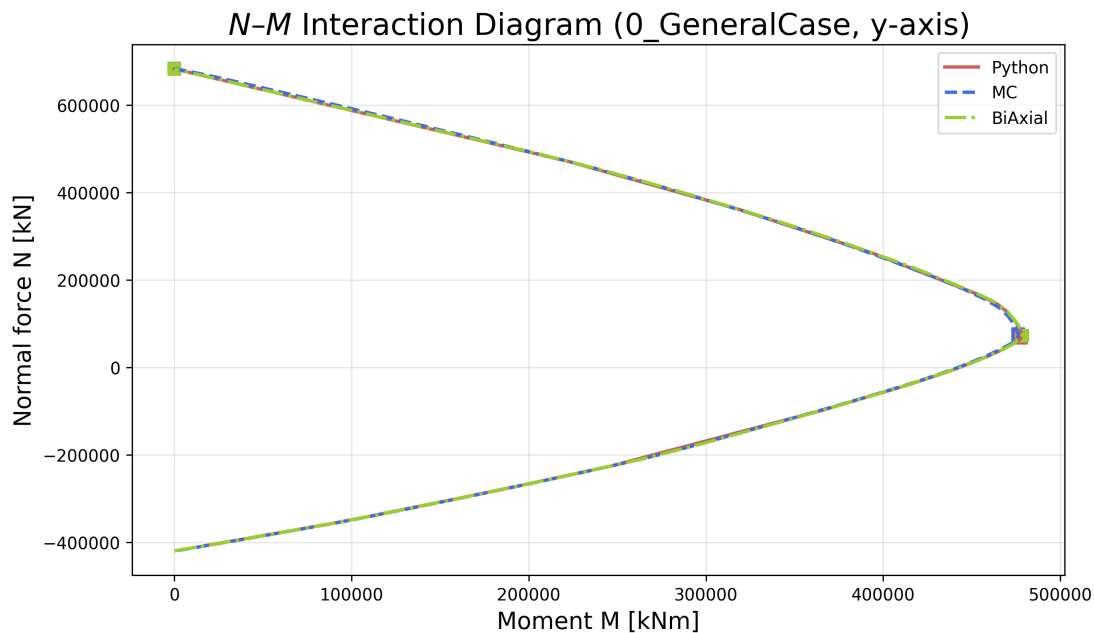


Figure 5.6: Comparison of the N – M interaction diagram obtained with *Python Code*, *MegaColumn*, and *BiAxialPro* for the reference configuration.

- **Validation of the modelling approaches across tools**

Based on the consistency of the global indicators, the *MegaColumn* software provides reliable predictions that are close with the finite element-based *BiAxialPro* software. Furthermore, the *Python Code* implementation successfully reproduces the behaviour of *MegaColumn*, thereby validating its use for extended parametric analyses in the following chapter.

- **Comparison between *Python Code* and *BiAxialPro***

A smaller and more uniform difference is observed between *Python Code* and *BiAxialPro*, which is illustrated by the reduced interquartile range in the box-and-whisker plots. This indicates that the deviation between these two tools is both limited in magnitude and consistent across all test cases.

As a result, both curves, *Py-MC* and *MC-BA*, appear to be symmetrical. This follows directly from the definition of the relative difference (Equation (5.1)). When the reference value is interchanged, the sign of the difference is reversed, leading to mirrored distributions.

This can be explained by the fact that the *Python Code* implementation developed in this work is fully controlled, meaning that all assumptions, material laws, and numerical procedures are explicitly known and consistently defined. Similarly, *BiAxialPro* allows the user to directly define the constitutive laws and modelling assumptions, making it possible to reproduce the same input parameters and material behaviour as those implemented in *Python Code*.

In contrast, *MegaColumn* may include implicit or embedded assumptions that are not fully accessible through the user interface. As a result, certain modelling choices or internal formulations cannot be controlled or reproduced exactly, which may explain the larger discrepancies observed in the comparisons involving *MegaColumn*.

- **Dispersion of N_{corr}**

In contrast, the results for N_{corr} exhibit a larger dispersion. This parameter is particularly sensitive, as it depends on the identification of the point corresponding to M_{max} along a discretised interaction curve. Differences in discretisation (30 points for *Python Code*, 52 for *BiAxialPro*, and 311 for *MegaColumn*) therefore directly affect the selected point.

Certain configurations (e.g. 2_1, 8_2, and 8_3) exhibit larger deviations due to more pronounced discretisation effects. In particular, the test case 2_1_H_2500 shows the largest differences. This sensitivity is illustrated in Table 5.2 and Figure 5.7, where small variations in M near its maximum are associated with significant differences in the corresponding axial force N .

Location	Python		MC		BiAxial	
	N [kN]	M [kNm]	N [kN]	M [kNm]	N [kN]	M [kNm]
Before M_{max}	70 946	328 392	64 448	326 897	76 219	328 203
	56 840	329 307	50 750	327 579	49 275	329 864
At (M_{max})	40 597	329 788	37 797	327 818	25 220	329 925
After M_{max}	31 341	329 771	25 556	327 674	6 375	327 351
	25 340	329 663	14 543	326 871	-7 035	322 337

Table 5.2: Comparison of axial force N and bending moment M for points located near M_{max} , illustrating the sensitivity of N_{corr} to curve discretisation.

This can be explained by the reduced section height H , while maintaining the same number of discretisation points, leading to a coarser representation of the interaction curve.

Nevertheless, these differences remain small relative to the magnitude of the forces involved and do not significantly affect the overall structural response. This is confirmed by the global agreement of the interaction diagrams, as observed in Figure 5.8.

5. Validation of N – M Interaction Diagram

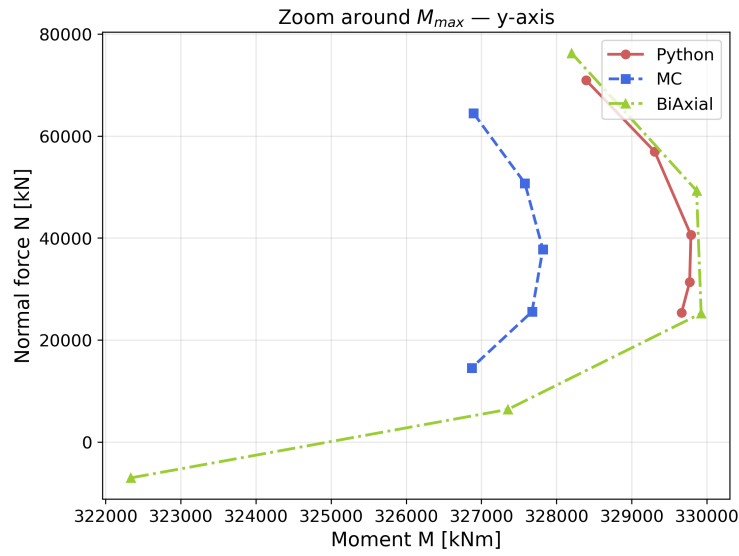


Figure 5.7: Zoom around the maximum bending moment $M_{\max}$ for the reference case, illustrating the sensitivity of N_{corr} to curve discretisation.

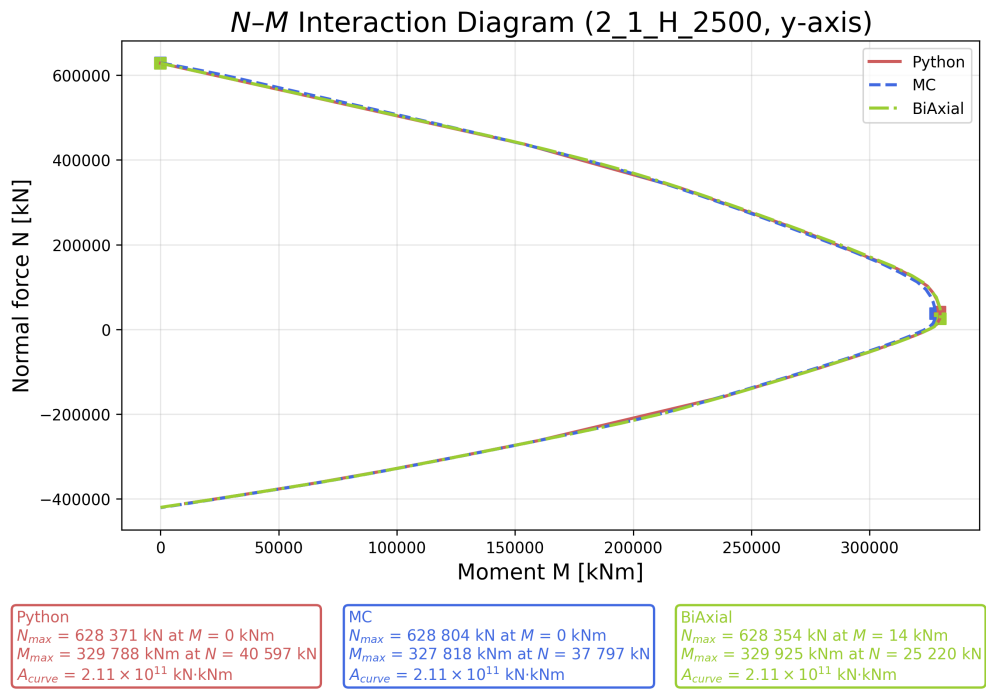


Figure 5.8: Comparison of N – M interaction diagrams obtained with *Python Code*, *MegaColumn*, and *BiAxialPro* for test case 2_1_H_2500.

- **Identification of the most and least reliable test cases**

Beyond the global comparison of indicators, it is useful to identify which test cases exhibit the largest discrepancies between the computational tools, despite the differences remaining small. To this end, a global discrepancy score is defined for each test case as:

$$\text{Score} = |\Delta N_{\max}| + |\Delta M_{\max}| + w |\Delta N_{\text{corr}}| + |\Delta A_{\text{curve}}| \quad (5.2)$$

where w is a weighting factor introduced to reduce the influence of N_{corr} , which is known to be more sensitive to discretisation effects.

This score provides a synthetic measure of the deviation between two tools for a given test case. It is evaluated separately for each comparison, as illustrated in Figure 5.9.

In addition, an overall discrepancy index is defined as the average of the three scores:

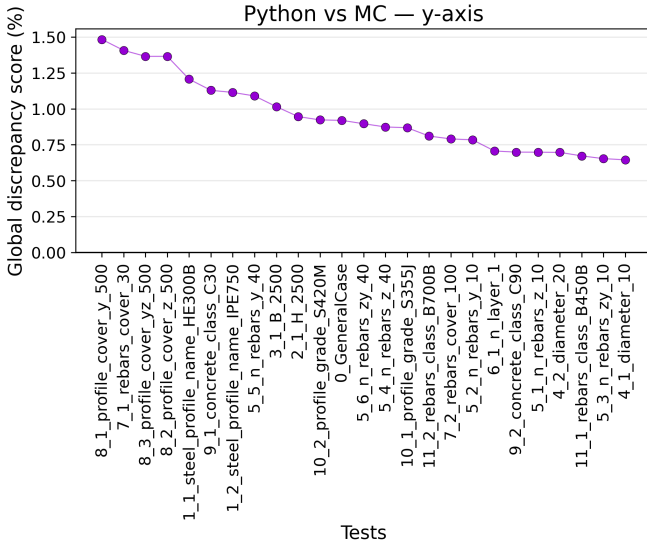
$$\text{Overall} = \frac{\text{Score}_{\text{Py-MC}} + \text{Score}_{\text{Py-BA}} + \text{Score}_{\text{MC-BA}}}{3} \quad (5.3)$$

The corresponding results are presented in Figure 5.9.

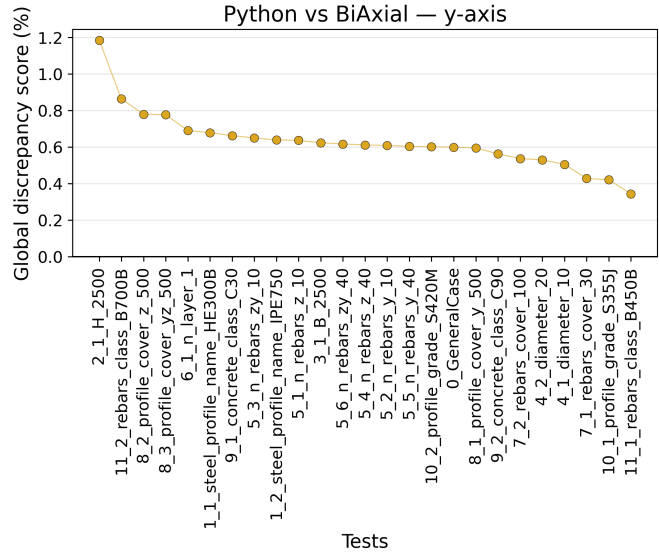
The analysis shows that the ranking of test cases varies depending on the comparison considered, which reflects the different sources of discrepancy between the tools. Nonetheless, the overall discrepancy remains low for all configurations, confirming the general consistency observed previously.

No specific family of test cases systematically exhibits larger errors, except for configurations with reduced section height H , for which the underlying reasons have already been discussed in the previous point.

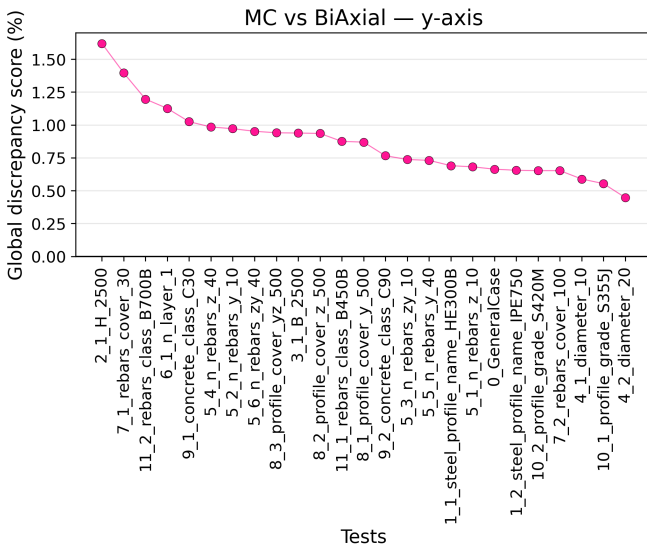
5. Validation of N - M Interaction Diagram



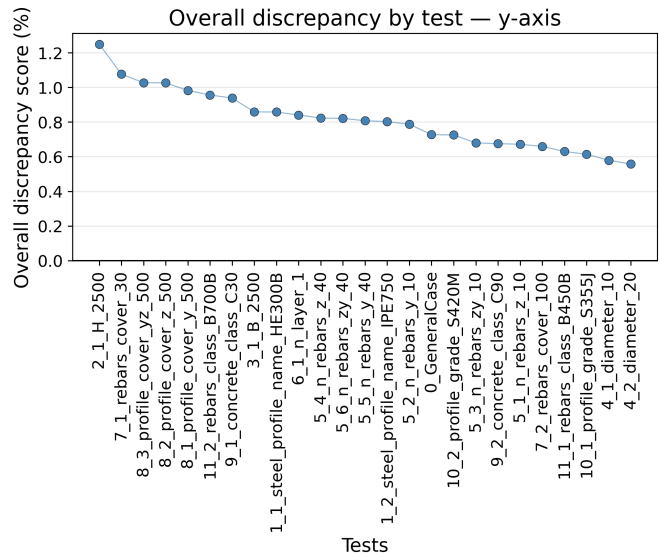
a) Pairwise discrepancy: *Python Code* vs *MegaColumn*



b) Pairwise discrepancy: *Python Code* vs *BiAxialPro*



c) Pairwise discrepancy: *MegaColumn* vs *BiAxialPro*



d) Overall discrepancy score

Figure 5.9: Global discrepancy score per test case for each pairwise comparison between *Python Code*, *MegaColumn*, and *BiAxialPro*, together with the overall discrepancy score defined as the average of the three pairwise comparison scores.

Chapter 6 Case Study and Parametric Analysis

The previous chapter demonstrated that the three softwares produce highly similar N – M interaction diagram results for a wide range of test cases. Nevertheless, as already mentioned, all numerical models remain simplified representations of reality. Consequently, the question is no longer only whether the softwares agree with each other, but also whether the implemented methodology is capable of representing realistic structural configurations.

While the use of a reference geometry is useful for validation purposes, real engineering projects can involve much more complex geometries and design constraints. It is therefore important to investigate how the proposed methodology performs when applied to a real megacolumn configuration.

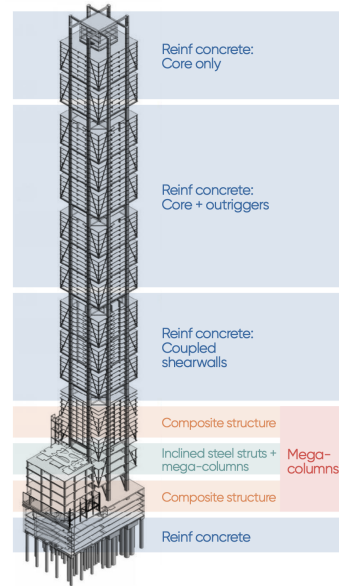
Fortunately, the megacolumn technology investigated in this study has already been implemented in the *One Bloor West* project. This provides an opportunity to compare the results obtained with the *Python Code* implementation against those developed by the project design office, *RJC Engineers*.

6.1 Case Study: *One Bloor West*

In order to briefly place the project into context, the *One Bloor West* tower is Canada's first supertall structure¹. This mixed-use tower, combining residential and commercial spaces, relies primarily on perimeter megacolumns, thereby eliminating the need for interior columns [6, 8, 45].

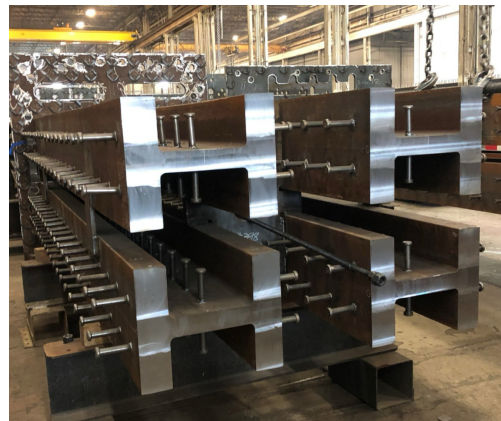
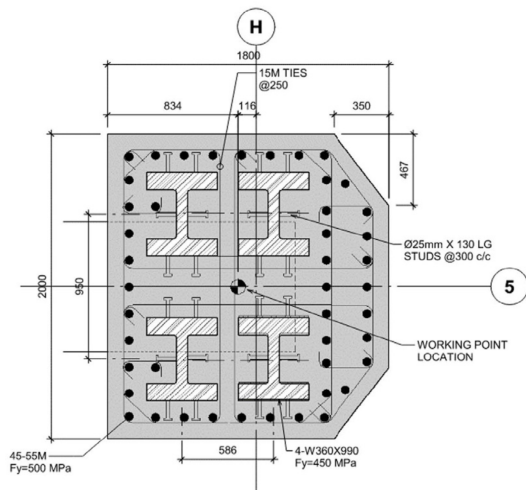
The megacolumn system is used from the second parking level up to the ninth residential floor. Despite the fact that the tower is still under construction, the structural design developed by *Foster + Partners* dates back to 2016. In collaboration with the design office, *ArcelorMittal* contributed to the development of the composite megacolumn system, leading to the cross-sectional configuration reported in Figure 6.2a).

¹According to the CVU, although the definition remains somewhat subjective depending on the surrounding urban context, a supertall structure is generally considered as a building exceeding 300 m in height [44].



a) Architectural visualisation of the *One Bloor West* tower, modelled by *Foster+Partners* [46] b) Structural compartmentalisation and megacolumn layout [6]

Figure 6.1: Overview of the *One Bloor West* project: a) architectural visualisation of the tower, and b) structural compartmentalisation illustrating the megacolumn system.



a) Original megacolumn cross-section used in the *One Bloor West* tower project [6]. b) Steel profile layout fabricated in the fabrication centre [6].

Figure 6.2: Overview of the *One Bloor West* megacolumn section: a) original megacolumn cross-section, and b) fabricated steel profile assembly.

The materials used for the megacolumn are summarised as follows:

- **Concrete:** 75 MPa self-consolidating concrete, with a tested Modulus of Elasticity (MOE) of approximately 45 GPa and a specified MOE of 38 GPa.
- **Steel profile:** ASTM A913 Grade 65 structural steel ($f_{a,y} = 450$ MPa).
- **Reinforcement bars:** S500B reinforcing bars.

As it was not possible to reproduce the original megacolumn geometry exactly within the current *Python Code* implementation, an equivalent geometry was developed. The aim of this conversion was to preserve the overall geometry, concrete cover and reinforcement layout as closely as possible to the original configuration, as schematised in Figure 6.3.

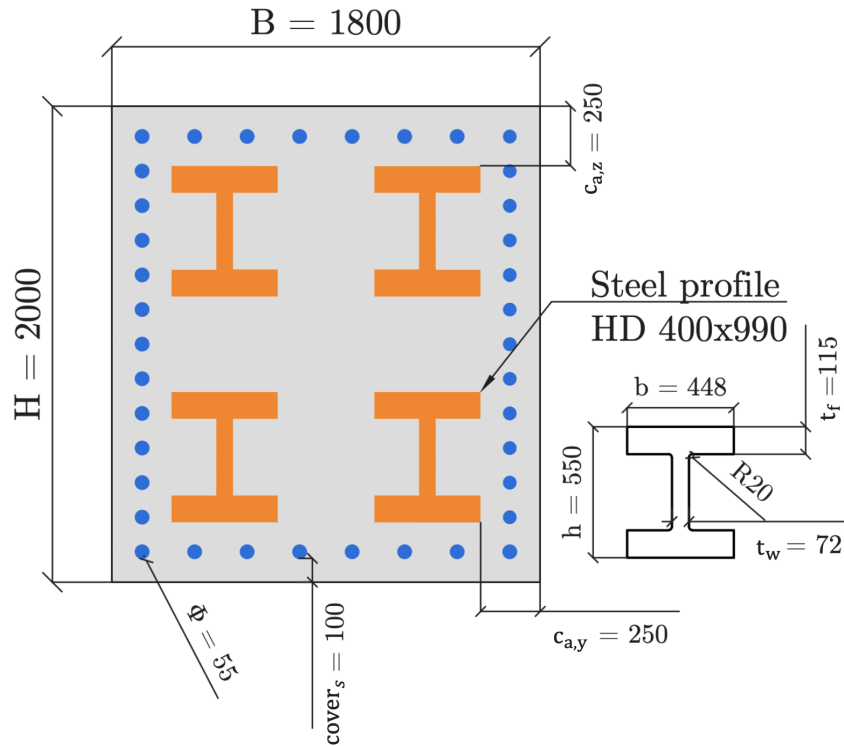


Figure 6.3: Equivalent megacolumn cross-section implemented in the *Python Code* model for the *One Bloor West* case study.

The selected equivalent geometry provides the closest axial compression resistance to the original megacolumn section. Although it contains a larger amount of concrete than the real section, the number of reinforcing bars is lower. The difference in axial compression resistance is only 0.17%.

Parameter	Original section	<i>Python Code</i> equivalent section
Maximum axial resistance $N_{\max}$ [kN]	403 056	403 757
Gross concrete area [m ²]	3.44	3.60
Number of reinforcing bars	45	38

Table 6.1: Comparison between the original *One Bloor West* megacolumn section and the equivalent geometry implemented in *Python Code*.

Now that the equivalent geometry has been established, the $N-M$ interaction diagram obtained with the *Python Code* implementation can be compared with the interaction diagrams developed during the design stage of the *One Bloor West* project.

Since the validation chapter demonstrated that the *MegaColumn* and *BiAxialPro* results are very close to those obtained with *Python Code*, only the *Python Code* results are presented here for clarity.

Comparison of N-M Interaction Diagram with Buckling Effect

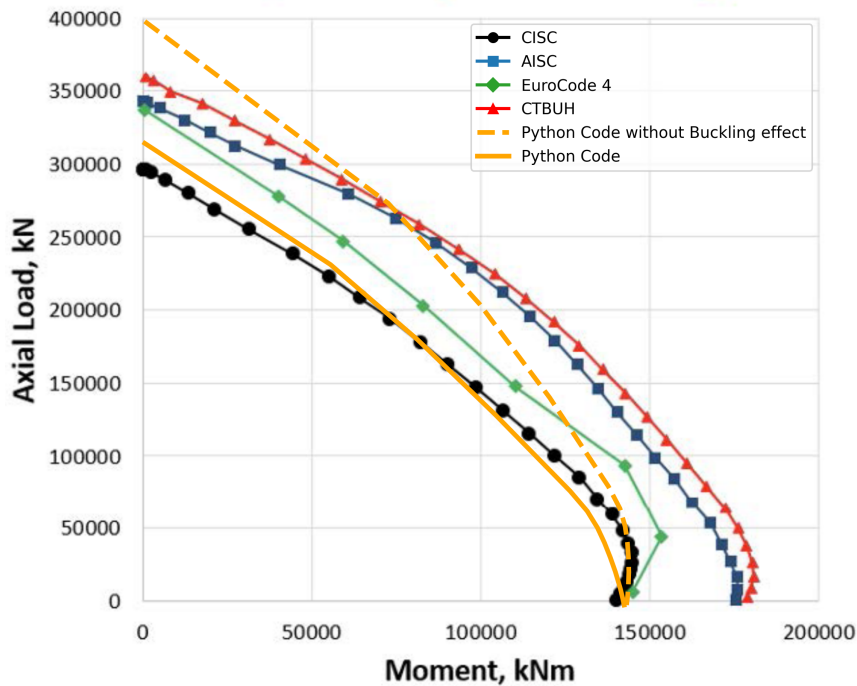


Figure 6.4: Comparison of $N-M$ interaction diagram results for the *One Bloor West* megacolumn configuration. Adapted from [6].

Discussion

The main observations can be summarised as follows:

- **Consistency with reference engineering results**

The *Python Code* implementation successfully reproduces the overall shape and behaviour of the $N-M$ interaction diagram obtained from the *One Bloor West* reference project. In particular, the *Python Code* curve including buckling effects remains very close to the CISC results (Canadian Institute of Steel Construction), where the CISC curve corresponds to the interaction diagram used by *RJC Engineers* for the structural design of the project.

- **Influence of modelling methodology**

Larger differences are observed between the *Python Code* results and the CTBUH results. This discrepancy can mainly be explained by the modelling philosophy adopted by each approach.

The CTBUH methodology is based on simplified analytical formulations using characteristic ABC points, meaning that the section is not discretised layer by layer (see Figure 2.10). In contrast, both the *Python Code* implementation and the CISC methodology rely on a more detailed sectional integration approach. Thus, the *Python Code* and CISC curves show closer similarity.

- **Importance of global buckling effects**

Most reference curves include the influence of global buckling, except the *Python Code* curve generated without buckling reduction. As illustrated in Figure 6.4, the curve without buckling effects predicts significantly higher resistance in the compression-dominated region compared with the other approaches.

This observation highlights the importance of including global instability effects in the generation of the N – M interaction diagram, since neglecting buckling may lead to an overestimation of the structural resistance. Although this feature is not currently implemented directly in the *MegaColumn* software, the present comparison demonstrates that it is an essential parameter for realistic megacolumn design.

This is further confirmed by the interaction curves developed by *RJC Engineers*, which already include global buckling effects, indicating that such information is considered relevant for practical structural design.

- **Influence of the equivalent geometry simplification**

More noticeable differences are observed near the pure bending region of the interaction diagram. This behaviour was expected due to the simplified equivalent geometry adopted in the *Python Code*.

The equivalent section was calibrated primarily to reproduce the maximum axial compression resistance $N_{\max}$. Consequently, certain geometric details affecting the bending behaviour, such as the exact lever arm and the detailed arrangement of the reinforcement on the right side, were simplified compared with the real section.

Nonetheless, for megacolumn applications, the design is generally governed mainly by compression-dominated behaviour rather than pure bending resistance. Therefore, the obtained results remain sufficiently close to the CISC reference curves and provide a realistic approximation of the structural response.

Overall, the similarity between the obtained results and the real case study provides additional confidence in both the developed *Python Code* implementation and the *MegaColumn* software. Nevertheless, the comparison also highlights several modelling aspects that are currently missing in *MegaColumn* and that may be relevant for engineering applications.

The following sections examine how key model assumptions affect the structural response of composite megacolumns. The aim is to identify the most influential parameters and assess whether alternative modelling approaches could enhance the accuracy and reliability of the *MegaColumn* software.

6.2 Methodology of Analysis

The study is organised according to different categories of modelling assumptions, including material behaviour, section discretisation, and buckling effects. For each category, several variations are considered in order to assess the sensitivity of the results. A summary of the investigated parameters is provided in Table 6.2.

Category	Parameter	Investigated variations
Material modelling	Concrete constitutive law	EN 1992-1-1:2004 vs EN 1992-1-1:2023; Parabolic-rectangular vs non-linear
Material modelling	Structural steel constitutive law	Elastic-perfectly plastic vs elastic with linear strain hardening
Material modelling	Reinforcing steel constitutive law	Elastic-perfectly plastic vs elastic with linear strain hardening; Ductility classes A, B, and C
Section discretisation	Layer thickness	0.1 mm to 500 mm
Global buckling	Flexural stiffness EI and Column height	Linear vs Hybrid stiffness models; Slenderness

Table 6.2: Summary of the modelling assumptions and parameter variations investigated in the parametric analysis of the N – M interaction diagram.

6.3 Influence of Material Modelling – Concrete

The compressive behaviour of concrete can be described using different constitutive models depending on the selected Eurocode formulation. In *Python Code*, the user can choose:

- **Eurocode version:** EN 1992-1-1:2004 or EN 1992-1-1:2023,
- **Constitutive law:** parabolic-rectangular diagram or non-linear stress–strain relationship.

In the former Eurocode formulation, additional expressions are provided for high-strength concrete when the characteristic compressive strength exceeds $f_{ck} = 50$ MPa. In contrast, the revised Eurocode restricts the range of concrete classes to C20/25 to C50/60.

Within the *MegaColumn* software, only the revised Eurocode formulation is implemented. Accordingly, when a concrete class higher than C50/60 is selected, the software issues a warning to inform the user that the selected strength class lies outside the recommended applicability range. However, the program still applies the parabolic-rectangular stress–strain relationship defined in EN 1992-1-1:2023, using the same ultimate strain value of $\varepsilon_{cu2} = 3.5\%$.

Figure 6.5 presents the stress–strain relationships for the different Eurocode formulations and constitutive laws across several concrete strength classes.

To examine this effect, a comparison between different concrete classes (e.g. C30/37, C60/75 and C90/105) is performed.

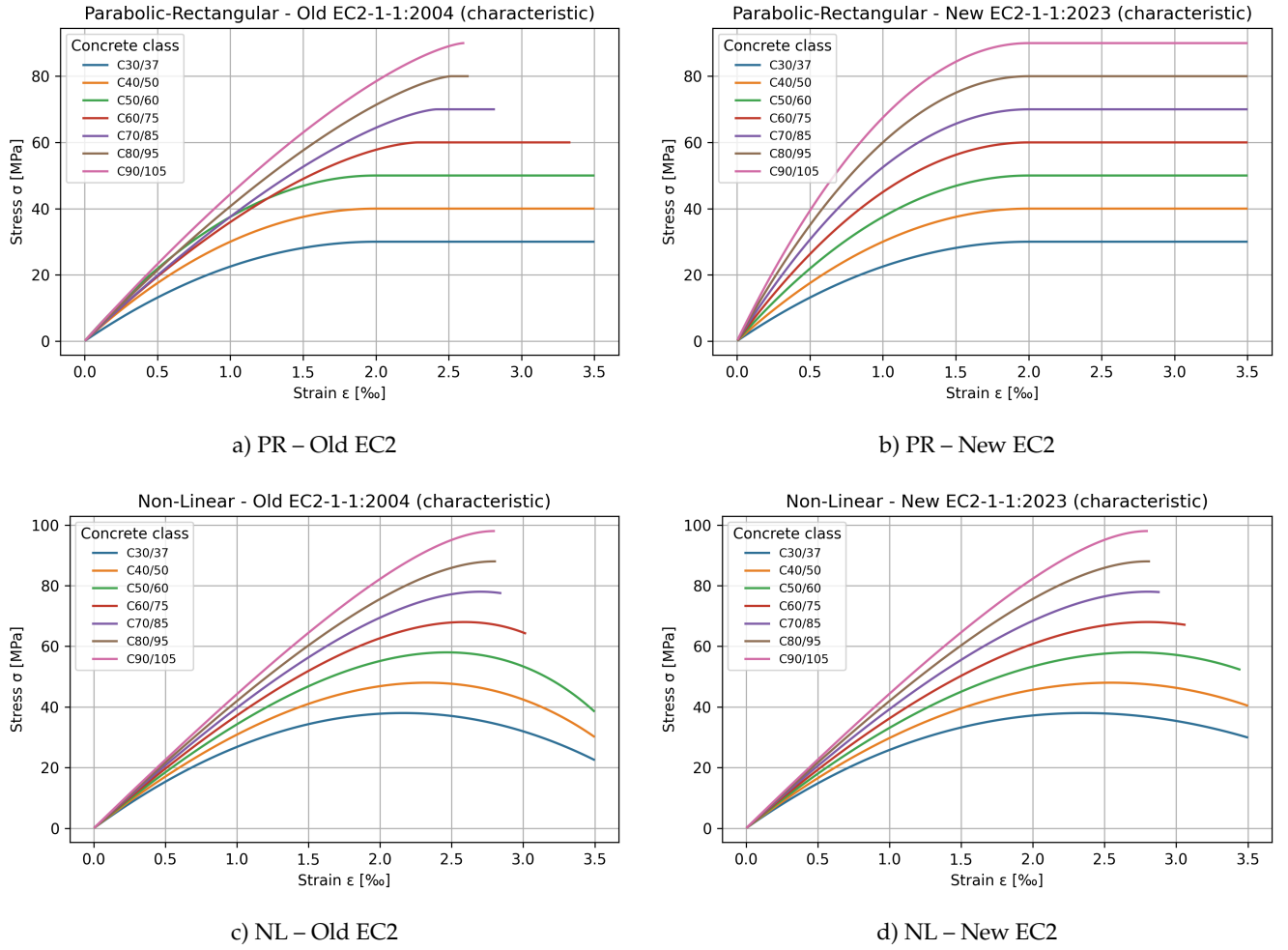


Figure 6.5: Comparison of concrete stress–strain relationships for increasing strength classes and different Eurocode formulations.

6.3.1 Concrete Class C30/37

For normal-strength concrete, only three distinct curves are visible in Figure 6.6, as the parabolic-rectangular stress–strain relationship is identical in both the former and revised Eurocode formulations. Thus, these two curves are fully overlying. Meanwhile, the non-linear laws exhibit increasing divergence with increasing strain ϵ , leading to noticeable differences in the material response.

This behaviour is reflected at the structural level, as observed in Figure 6.7, where the resulting N – M interaction diagrams show visible discrepancies.

The influence of the constitutive law can be further assessed through the key response quantities ($N_{\max}$, $M_{\max}$, N_{corr} , and A_{curve}), following the same approach as in the previous chapter. In this case, N_{corr} can be directly compared, as all results are obtained from a single computational model (*Python Code*).

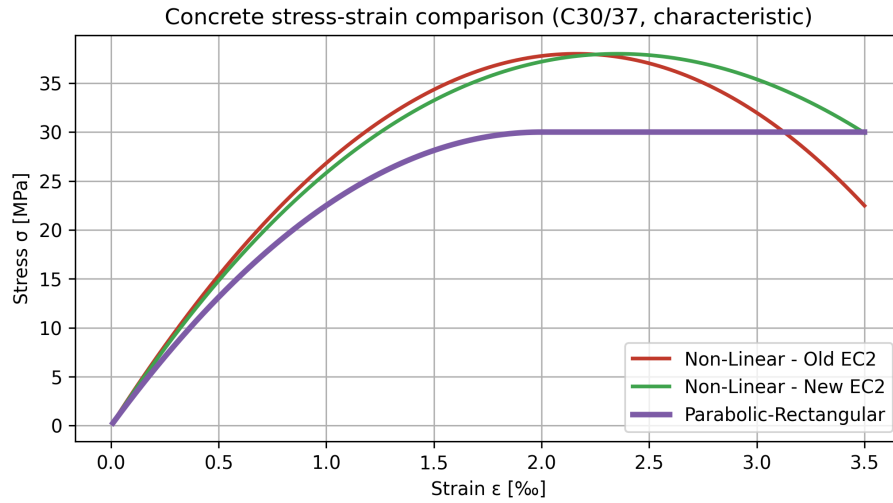


Figure 6.6: Comparison of concrete stress–strain relationships for $f_{ck} = 30$ MPa, showing the superposition of the parabolic-rectangular formulations.

Model	$N_{\max}$ [kN]	$M_{\max}$ [kNm]	N_{corr} [kN]	A_{curve}
NL – Old EC2	602 441	459 763	50 536	2.80×10^{11}
NL – New EC2	615 872	460 564	50 542	2.82×10^{11}
PR	564 297	437 819	25 087	2.50×10^{11}

Table 6.3: Comparison of $N_{\max}$, $M_{\max}$, N_{corr} , and interaction curve area for C30/37 concrete.

Specific observations for $f_{ck} = 30$ MPa

- Only three curves are observed, as the parabolic-rectangular laws from the former and revised Eurocode formulations are identical.
- The main differences are observed between the parabolic-rectangular and non-linear constitutive laws in compression-dominated regions (i.e. for positive values of N). The non-linear models lead to slightly higher values of $N_{\max}$, $M_{\max}$, and A_{curve} , indicating a moderately larger interaction domain.
- At $N_{\max}$, the differences remain limited. Although variations of the order of 50 000 kN are observed in axial resistance, this corresponds to a relative difference of approximately 8.4% according to Equation (5.1), which remains moderate compared with the magnitude of forces typically encountered in megacolumns.

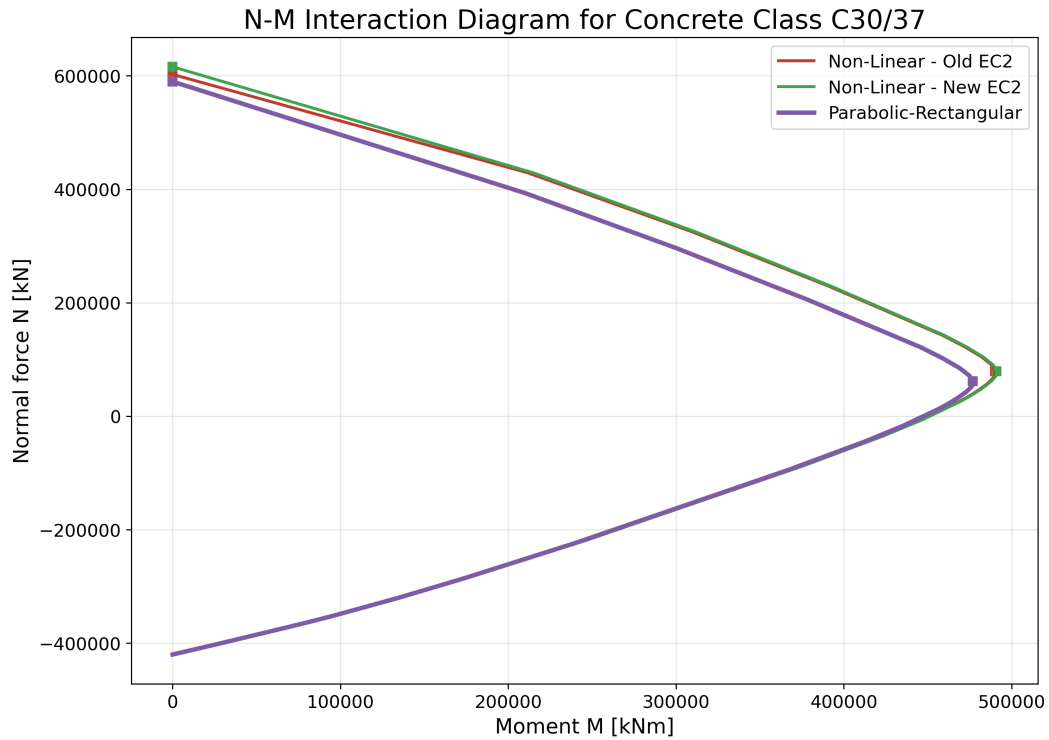


Figure 6.7: Comparison of the N – M interaction diagram for C30/37 concrete, illustrating the influence of the concrete constitutive law.

6.3.2 Concrete Class C60/75

For this intermediate concrete strength, the four constitutive laws can be clearly distinguished in Figure 6.8. At low strain levels ($\varepsilon < 0.8\text{‰}$), the parabolic-rectangular law from the revised Eurocode deviates from the other formulations, which remain relatively close. At higher strain levels ($\varepsilon > 2.2\text{‰}$), the parabolic-rectangular laws progressively converge towards a similar plateau, whereas the non-linear laws continue to exhibit slight differences.

Figure 6.9 and Table 6.4 reflect the structural behavior.

Model	$N_{\max}$ [kN]	$M_{\max}$ [kNm]	N_{corr} [kN]	A_{curve}
PR – Old EC2	708 705	476 877	71 005	3.17×10^{11}
PR – New EC2	716 717	489 359	94 562	3.29×10^{11}
NL – Old EC2	797 422	496 741	95 481	3.60×10^{11}
NL – New EC2	802 796	497 411	97 669	3.61×10^{11}

Table 6.4: Comparison of $N_{\max}$, $M_{\max}$, N_{corr} , and interaction curve area for C60/75 concrete.

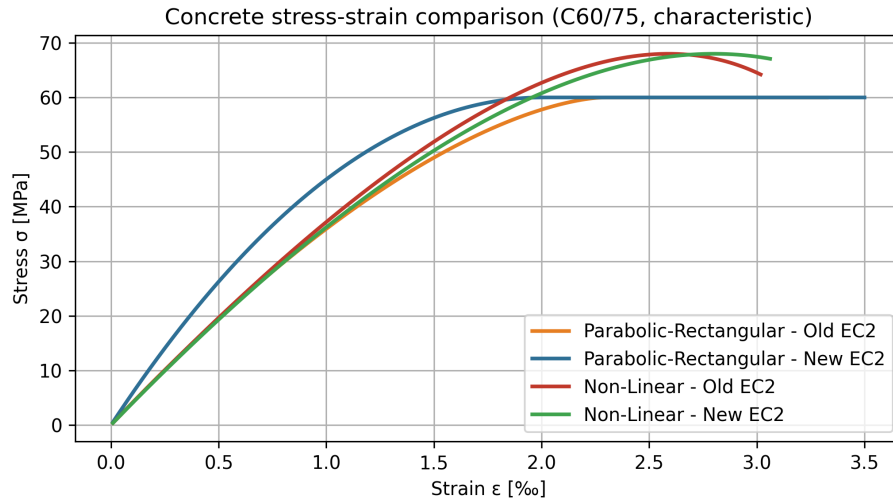


Figure 6.8: Comparison of concrete stress–strain relationships for $f_{ck} = 60$ MPa, illustrating the transition between Eurocode formulations.

Specific observations for $f_{ck} = 60$ MPa

- The four stress–strain relationships are visually distinguishable, the resulting N – M interaction diagrams tend to group into three main curves. In particular, the two non-linear formulations appear almost superimposed. This can be explained by the fact that the difference between their stress–strain curves remains very small, with a maximum deviation of only approximately 3 MPa.
- The results tend to cluster according to the constitutive law type rather than the Eurocode version. The non-linear laws generally provide a larger interaction domain than the parabolic-rectangular laws, as reflected by the higher values of A_{curve} , indicating that the constitutive law formulation has a greater influence than the code version.
- When the section is no longer governed by pure compression but increasingly influenced by bending, the values of M_{max} become more clearly differentiated. This suggests that the shape of the stress–strain relationship has a stronger influence when the strain distribution becomes non-uniform across the section.

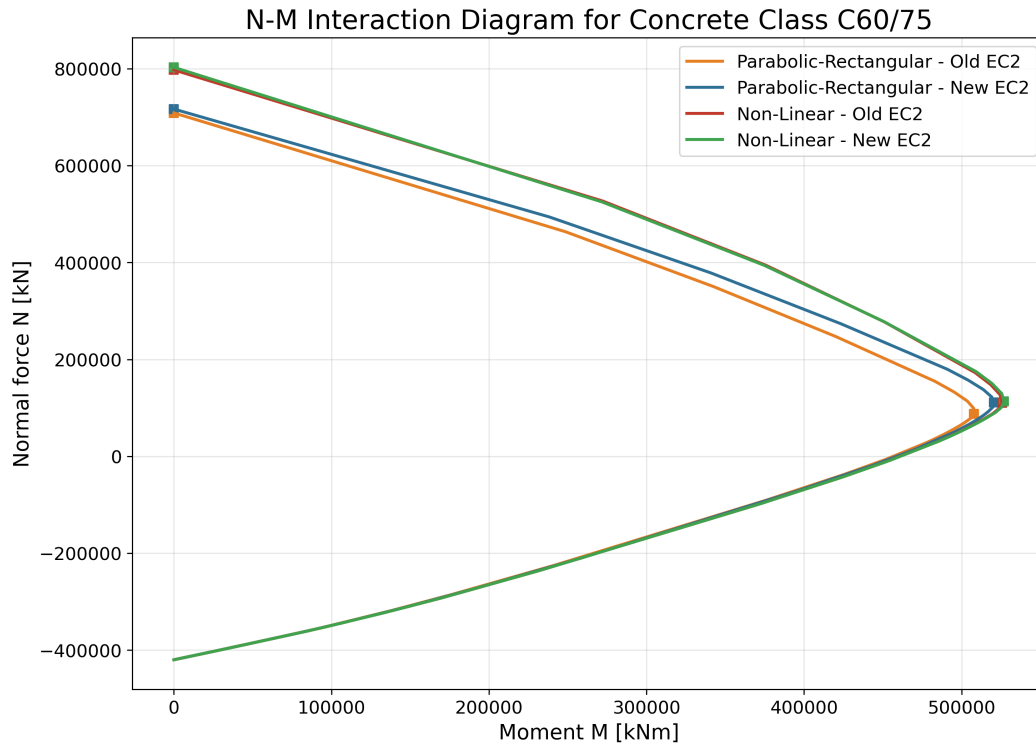


Figure 6.9: Comparison of the N – M interaction diagram for C60/75 concrete, illustrating the influence of the concrete constitutive law.

6.3.3 Concrete Class C90/105

For high-strength concrete, the situation differs from that observed for normal-strength concrete. In Figure 6.10, the non-linear stress–strain relationships from the former and revised Eurocode formulations are identical. In contrast, the parabolic-rectangular laws exhibit noticeable differences. These differences are most significant in the strain range between 0 and 2.5 ‰, leading to distinct material responses.

In particular, the revised Eurocode parabolic-rectangular law tends to exhibit a behaviour closer to a non-linear response, as the plateau region is absent.

This behaviour is reflected in the N – M interaction diagrams presented in Figure 6.11, where larger discrepancies are observed compared to the normal-strength case.

Model	$N_{\max}$ [kN]	$M_{\max}$ [kNm]	N_{corr} [kN]	A_{curve} [kN·kNm]
PR – Old EC2	853 113	467 713	76 859	3.48×10^{11}
PR – New EC2	808 844	520 973	128 954	3.82×10^{11}
NL	974 868	528 471	149 319	4.39×10^{11}

Table 6.5: Comparison of $N_{\max}$, $M_{\max}$, N_{corr} , and interaction curve area for C90/105 concrete.

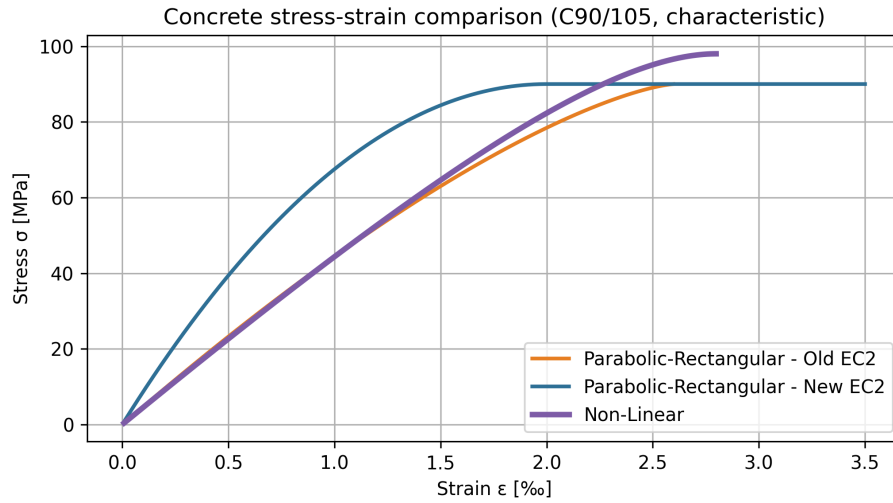


Figure 6.10: Comparison of concrete stress–strain relationships for $f_{ck} = 90$ MPa, showing the superposition of the non-linear formulations.

Specific observations for $f_{ck} = 90$ MPa

- For high-strength concrete, the influence of the constitutive law becomes significant. The differences between the parabolic-rectangular and non-linear models are clearly visible in the resulting N – M interaction diagrams.
- At $N_{\max}$, the differences become notable, exceeding 170 000 kN ($\approx 17\%$, according to Equation (5.1)). Such variations are no longer negligible. In addition, the parabolic-rectangular curves from the former and revised Eurocode formulations intersect. The revised Eurocode tends to provide lower values of $N_{\max}$ but higher values of $M_{\max}$, whereas the former Eurocode shows the opposite trend.
- The curves corresponding to the non-linear formulations exhibit the same behaviour since their stress–strain curves are identical.
- The difference between the revised parabolic-rectangular law and the non-linear formulation is particularly pronounced. The interaction curve area differs by approximately 6×10^{10} kN·kNm ($\approx 15\%$). This highlights a significant increase in the predicted resistance domain when using a non-linear model.
- An even larger difference is observed when comparing the former parabolic-rectangular law with the non-linear formulation, reaching approximately 26%. This discrepancy can be explained by differences in both the ultimate strain (of the order of 0.2 ‰) and the stress level (approximately 8 MPa). While these variations may appear limited at the material level, they have a significant effect at the structural scale, leading to a noticeably larger interaction domain.

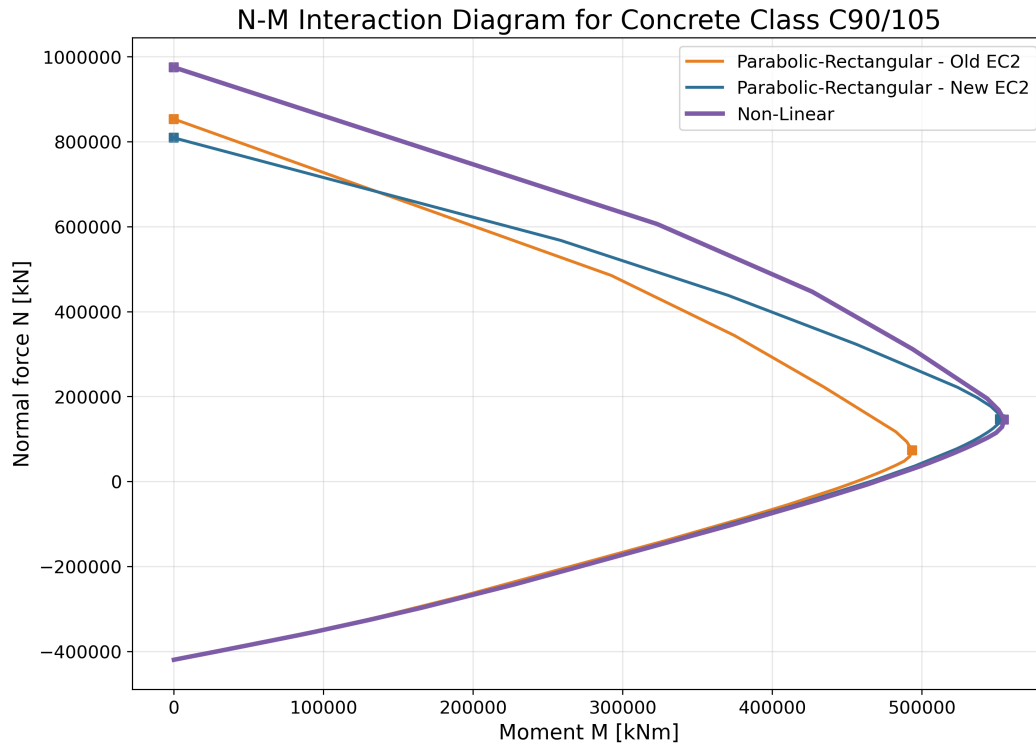


Figure 6.11: Comparison of the N – M interaction diagram for C90/105 concrete, illustrating the influence of the concrete constitutive law.

6.3.4 Discussion on Concrete Modelling

The main observations can be summarised as follows:

- **Influence of concrete strength level**

The impact of the constitutive law strongly depends on the concrete strength. For normal-strength concrete, the differences between the former and revised Eurocode formulations remain limited (generally below 15%). In contrast, for higher concrete strengths, the choice of constitutive model becomes increasingly significant, with differences exceeding 20% for the interaction curve area and up to 17% for $N_{\max}$.

- **Dominant effect of constitutive law type**

The distinction between parabolic-rectangular and non-linear laws has a greater influence on the structural response than the difference between Eurocode versions for concrete classes up to C60/75. The parabolic-rectangular law generally leads to a smaller interaction domain, resulting in a more conservative estimation of the section resistance.

- **Influence limited to compression regions** The influence of the concrete model is only observed in the compression-dominated regions of the N – M interaction diagram, as the tensile concrete is set to zero.

- **Limitations of constant strain parameters in the revised Eurocode**

The revised Eurocode adopts fixed strain limits (e.g. ε_{c2} and ε_{cu2}) for the parabolic-rectangular law, even for high-strength concrete. However, this assumption may lead to an inaccurate

representation of material behaviour beyond the recommended range (typically above C50/60). In contrast, the former Eurocode accounts for the evolution of the stress–strain relationship with increasing strength, leading to a reduced plateau and a response closer to a non-linear behaviour. This highlights the limitations of applying simplified models outside their intended range.

- **Influence of structural configuration**

The results presented are based on a reference configuration with a large cross-section ($3\,072\text{ mm} \times 3\,072\text{ mm}$). Still, the sensitivity to the constitutive law may vary depending on the structural characteristics, such as the section size and the proportion of steel profile and reinforcement. Hence, the magnitude of the observed differences may increase or decrease for other structural configurations.

Even so, the overall trends and conclusions discussed previously remain consistent. Indeed, two additional geometrically different configurations were also investigated, leading to similar behavioural observations. These supplementary case studies are presented in Appendix D.

- **Implications for modelling and design**

From a design perspective, the parabolic-rectangular law of the revised Eurocode remains suitable for normal-strength concrete. Nonetheless, its application to high-strength concrete should be treated with caution, as it may lead to unconservative or inconsistent results. Providing only a warning in design software may not be sufficient. Instead, allowing the user to adjust strain parameters or select alternative constitutive models would improve flexibility and reliability.

Moreover, the inclusion of non-linear constitutive laws appears beneficial, as they provide a more realistic representation of material behaviour, with differences of the order of 10% compared to simplified models. Ultimately, the choice of constitutive law should remain the responsibility of the engineer, taking into account the applicable standards, the design context, and potential international applications.

6.4 Influence of Material Modelling – Structural Steel

In design calculations, the stress–strain relationship of structural steel profiles is commonly simplified by neglecting the strain hardening branch of the material response. More advanced constitutive laws generally provide a representation closer to the real material behaviour. In particular, the inclusion of strain hardening leads to a more realistic stress–strain response, whereas its omission generally results in a conservative estimation of the structural resistance.

The *MegaColumn* software adopts an **elastic–perfectly plastic material model**. In contrast, the *Python Code* developed in this study allows the material behaviour to be modelled both **with and without strain hardening**, which enables a more detailed assessment of its influence on the structural response.

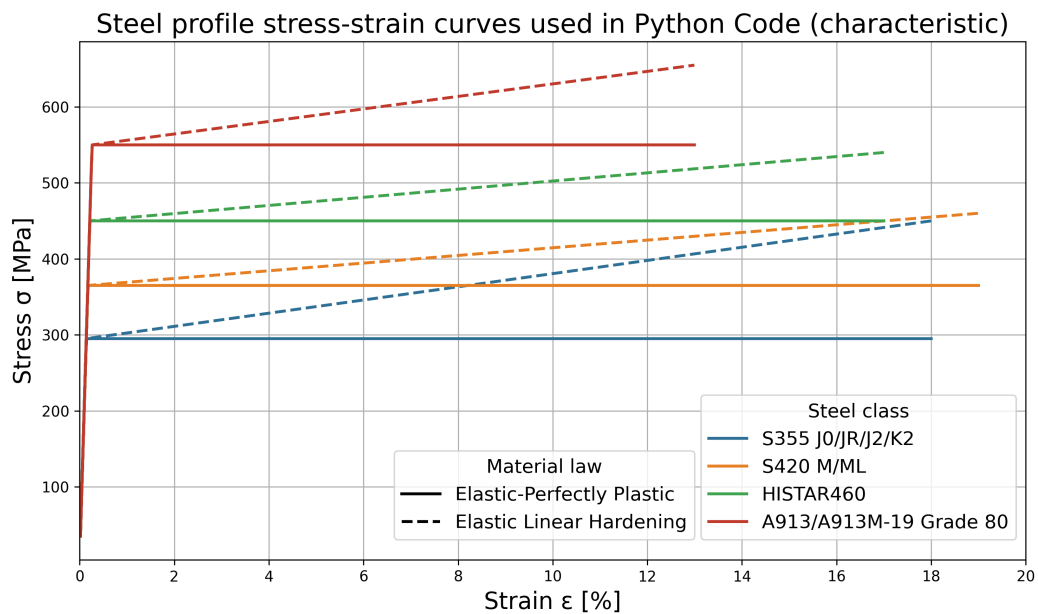


Figure 6.12: Comparison of stress–strain relationships for structural steel profiles with and without strain hardening.

The influence of strain hardening is analysed by comparing material models with and without hardening. Only two representative steel grades are presented, as the overall behaviour remains consistent across the other grades considered.

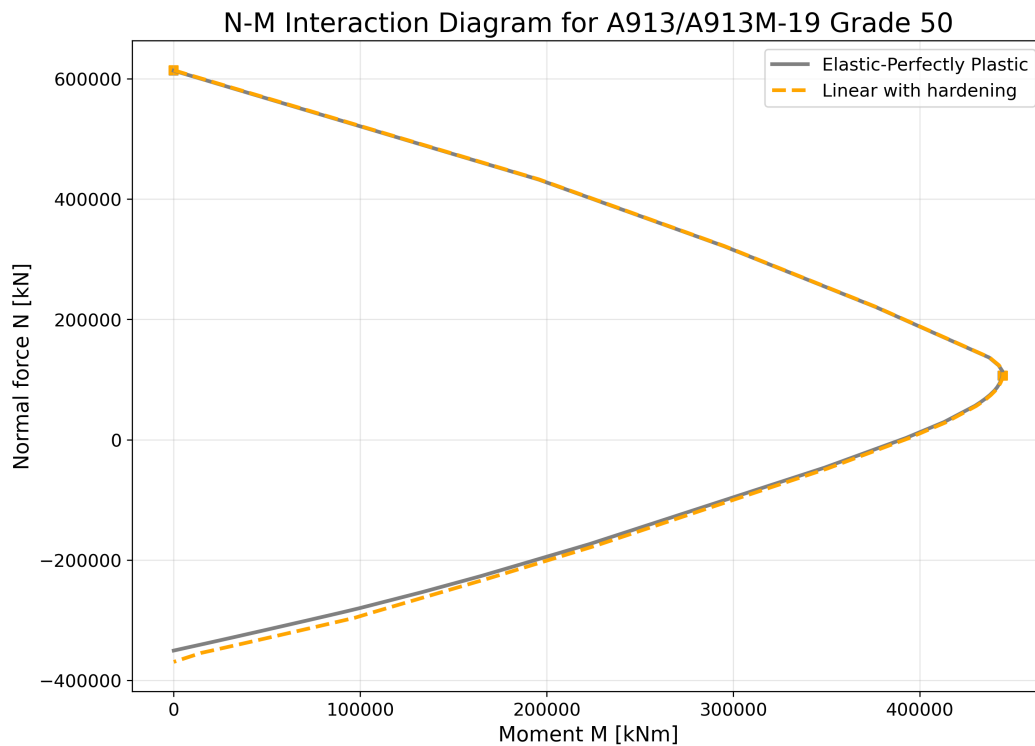
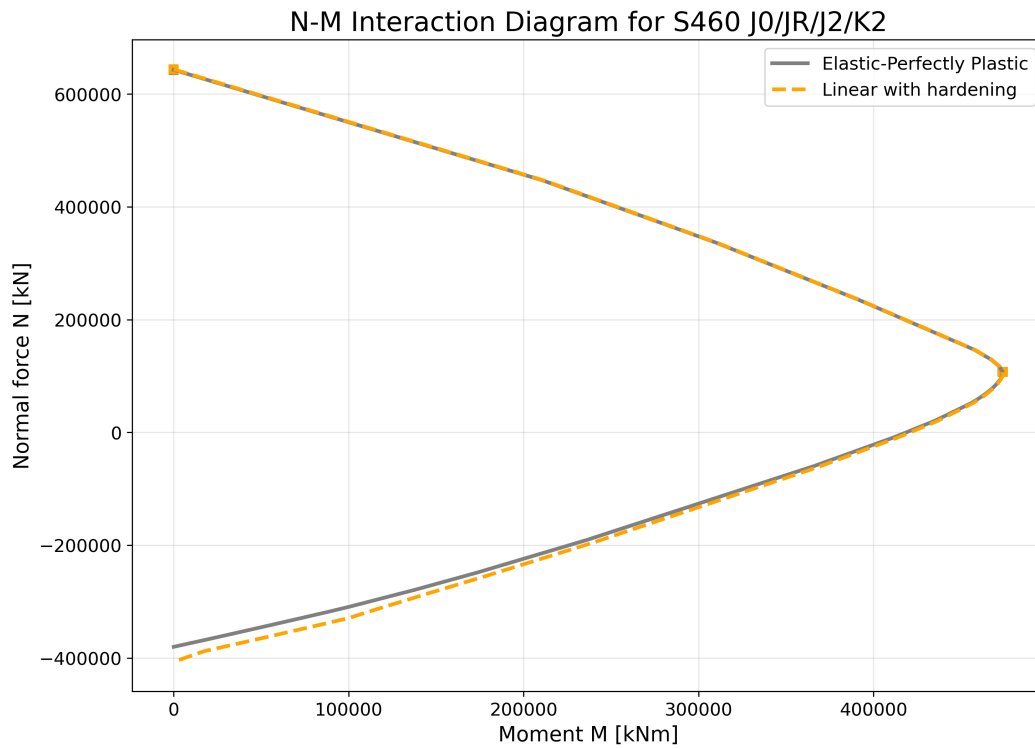


Figure 6.13: Comparison of N – M interaction diagram curves with and without strain hardening for two representative structural steel grades.

Discussion

The main observations can be summarised as follows:

- **Negligible effect in compression-dominated regions**

The influence of strain hardening is not noticeable in compression-dominated regions. This can be explained by two main factors.

(i) *Concrete governs the failure in compression*

Concrete reaches its ultimate strain (typically $\varepsilon = 0.0035$) before the steel profile can develop significant strain hardening.

For instance, for an S460 steel profile (with $t_f = 140$ mm), the yield strain is approximately $\varepsilon_y = 0.00183$, while the ultimate strain is around $\varepsilon_u = 0.17$. At $\varepsilon = 0.0035$, the steel is only slightly beyond yielding and remains far from the strain range where strain hardening becomes significant. At this strain level, the stress in the steel reaches approximately 391.4 MPa, compared to the yield strength $f_{a,y} = 385$ MPa, corresponding to a difference of only 6.4 MPa.

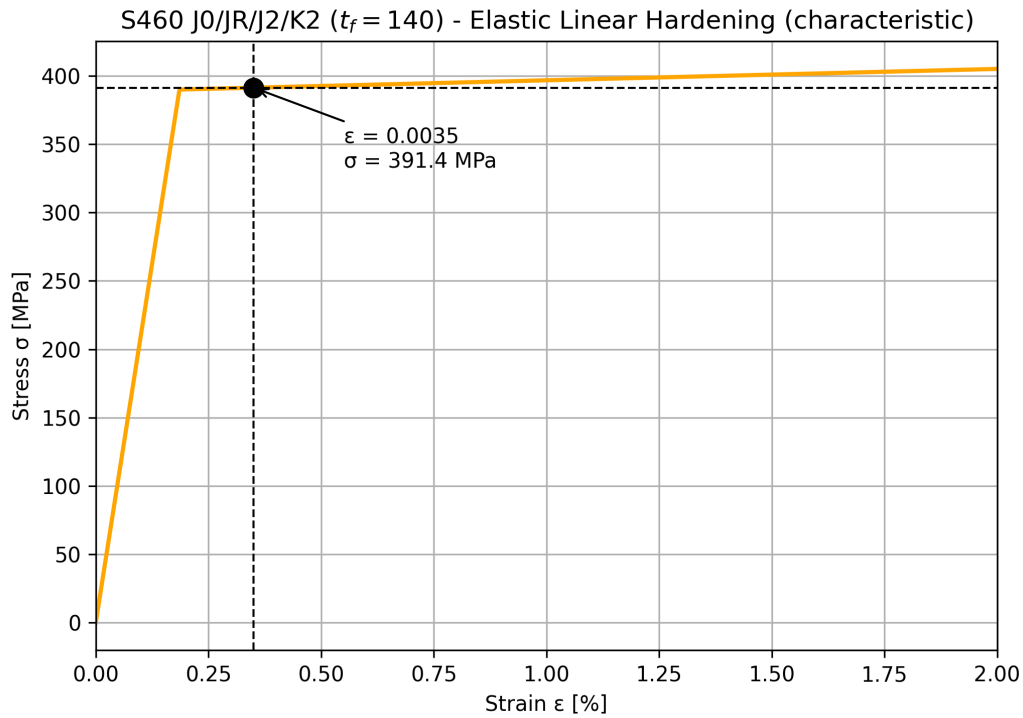


Figure 6.14: Stress–strain response of the structural steel profile at $\varepsilon = 0.0035$, illustrating the limited development of strain hardening.

(ii) *Limited contribution of the steel profile*

The contribution of the steel profile to the overall section is relatively small. For the reference configuration:

$$\rho_a = \frac{A_a}{A_g} = \frac{4 \times 165\,470}{9\,437\,184} \approx 7\% \quad (6.1)$$

In addition, the steel contribution ratio defined in Eurocode 4 (Eq. 6.38) [15] is:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}} = \frac{(4 \times 165\,470) \cdot 385}{643\,077\,000} \approx 0.39 \quad (6.2)$$

As a result, the effect of strain hardening at the structural level remains limited under compression, as steel represents a minority contribution within the composite section.

- **Limited influence in tension-dominated regions**

In tension-dominated regions, the effect of strain hardening becomes more visible, but remains limited. This can be explained by the governing role of the reinforcing bars.

In tension, concrete is neglected and the structural response is primarily governed by the reinforcement, defined here as B500B with a rupture strain of approximately 5%. At this strain level, the additional contribution from strain hardening in the steel profile remains relatively small, corresponding to an increase of approximately 45 MPa in stress.

At the structural level, this results in a variation of about 6.5% in the axial tensile capacity $N_{\max}$ (from $-380\,222$ kN to $-406\,723$ kN), which remains limited in comparison with the overall magnitude of forces involved.

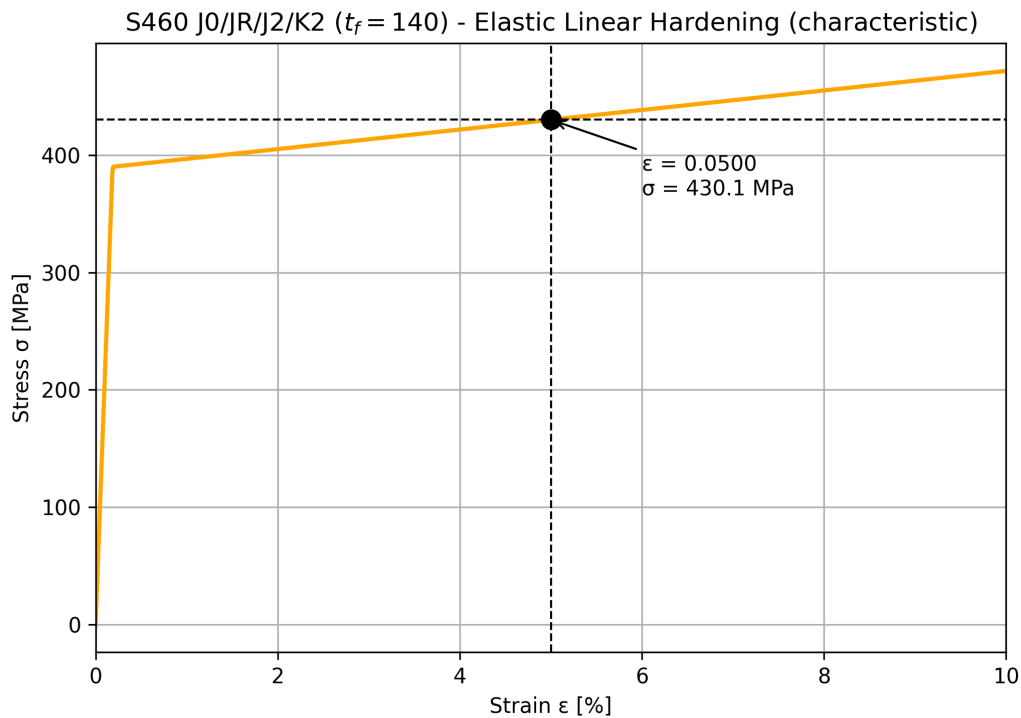


Figure 6.15: Stress–strain response of the structural steel profile at $\varepsilon = 5\%$, illustrating the limited additional contribution of strain hardening.

- **Minor impact on global structural behaviour**

Overall, the inclusion of strain hardening leads to only limited variations in the N – M interaction diagrams. The increase in stress due to hardening remains small compared

with the total stress levels involved, resulting in a negligible impact on the global structural response.

Furthermore, for megacolumn applications, the tension-dominated region is generally not utilised in design, which reduces the practical significance of strain hardening.

6.5 Influence of Material Modelling – Reinforcing Steel

The modelling approach adopted for reinforcing steel follows a similar philosophy as before. In the *MegaColumn* software, the reinforcing steel behaviour is represented using an elastic–perfectly plastic law defined by the yield strength of the selected reinforcement grade.

Typical grades available in the software range from B400 to B700 ($f_{s,y} = 400$ to 700 MPa), while a constant ultimate strain $\varepsilon_u = 0.05$ is adopted independently of the reinforcement ductility class, as shown in Figure 6.16a).

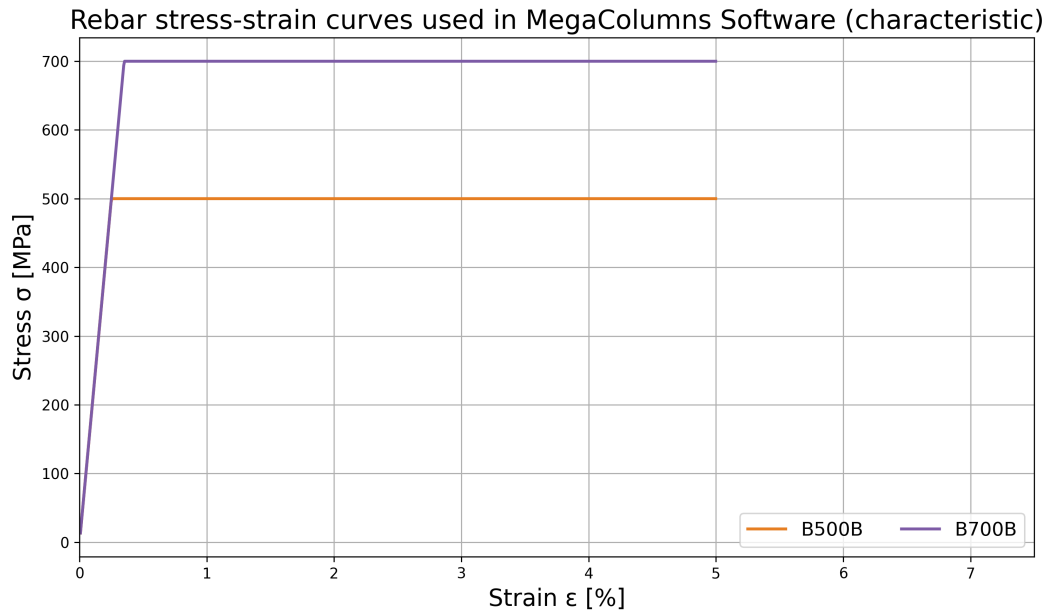
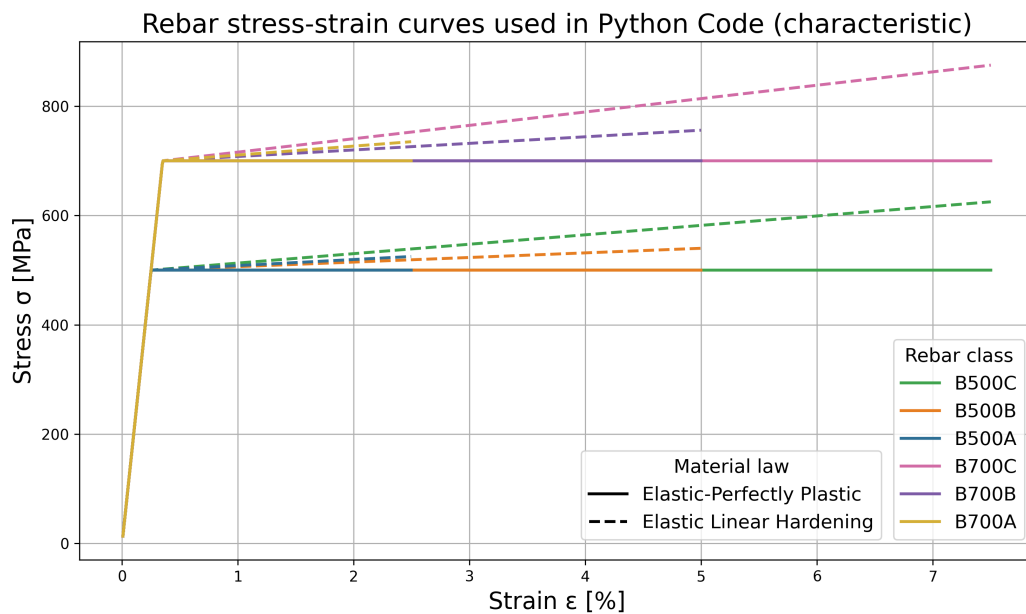
In the *Python Code* implementation (see Figure 6.16b)), the following modelling options are available:

- **Strain hardening:** inclusion or exclusion of strain hardening in the constitutive law.
- **Ultimate strain definition:** variation of the ultimate strain ε_u according to the revised Eurocode 2, which introduces ductility classes for reinforcing steel (A, B, and C) associated with different ultimate strain capacities.

Ductility class	Ultimate strain ε_u	Typical steel grades
A	$\varepsilon_u = 0.025$	Lower ductility reinforcement
B	$\varepsilon_u = 0.05$	Standard reinforcement
C	$\varepsilon_u = 0.075$	High ductility reinforcement

Table 6.6: Ultimate strain limits for reinforcing steel according to ductility class in EN 1992-1-1:2023 [25].

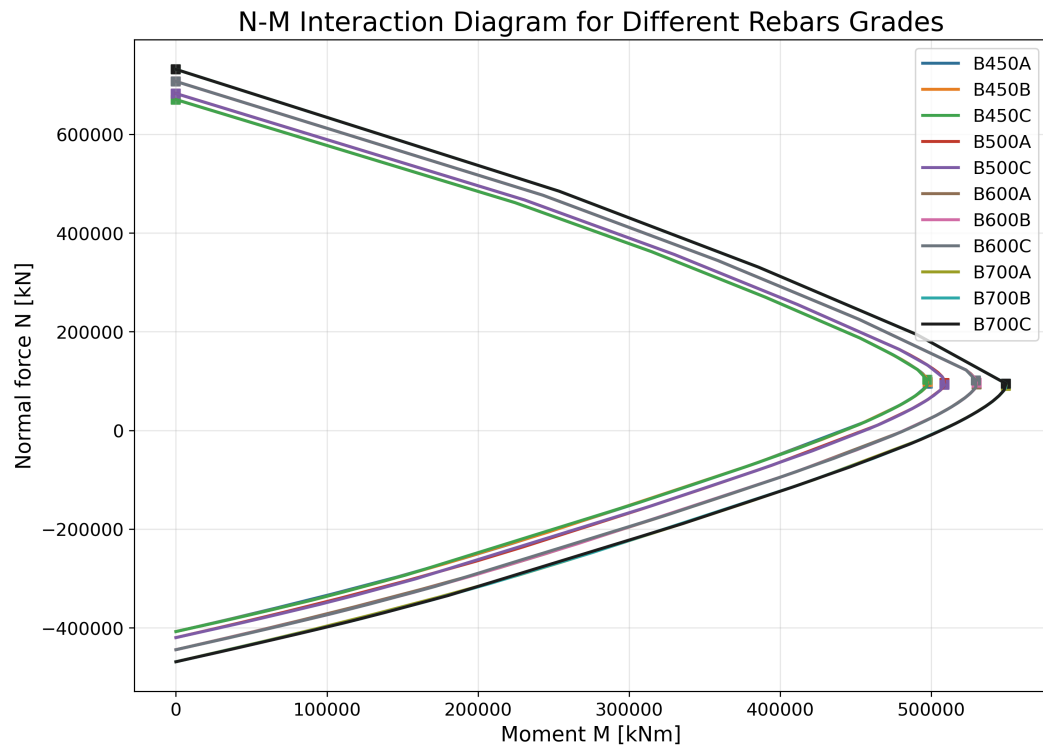
Consequently, depending on the selected reinforcement class, the ultimate strain capacity may differ from the constant value adopted in the *MegaColumn* software.


a) *MegaColumn* implementation


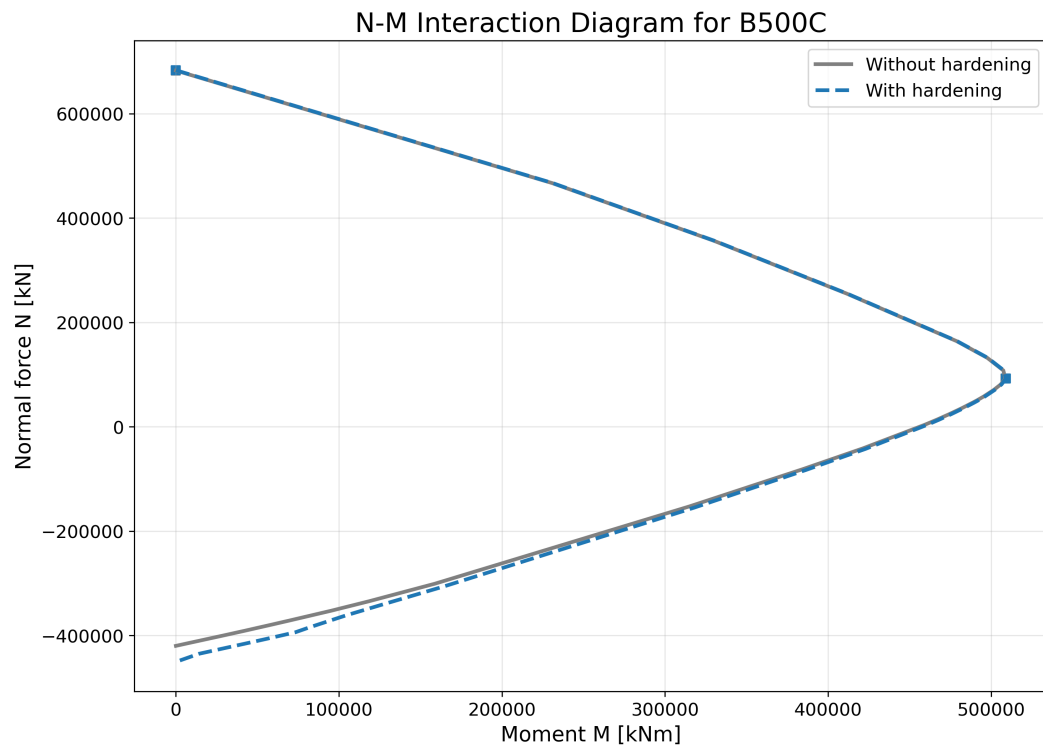
b) Python implementation

Figure 6.16: Comparison of reinforcing steel stress–strain models: a) implementation in *MegaColumn*, with constant ultimate strain ϵ_u , and b) implementation in *Python Code*, with ductility-dependent ultimate strain according to Eurocode 2.

The results for the general case and for the different reinforcing steel grades and ductility classes are presented in Figure 6.17a) for bending about the y -axis. Figure 6.17b) illustrates, for grade B500C, the difference between the response with and without hardening. The other hardening comparisons are not shown, as they exhibit the same overall behaviour.



a) all rebars class on the n-m interaction diagram



b) Hardening with and without for B500C

Figure 6.17: Comparison of N - M interaction diagram curves showing the influence of reinforcing steel grade, ductility class, and strain hardening.

Discussion

The main observations can be summarised as follows:

- **Limited influence of strain hardening**

Strain hardening has only a negligible effect on the N – M interaction diagrams. The elastic–perfectly plastic assumption adopted in the *MegaColumn* software remains thus sufficiently accurate for practical design.

- **Primary influence of reinforcement grade**

As expected, the choice of reinforcement grade directly influences both axial and bending capacities, whereas secondary parameters such as strain hardening and ductility class have a comparatively minor impact.

- **Minor influence of ductility class on column behaviour**

The ductility classes A, B, and C have only a negligible influence on the structural response of the composite column. The N – M interaction diagrams remain almost superposed, including near the maximum bending moment, as displayed in Figure 6.18.

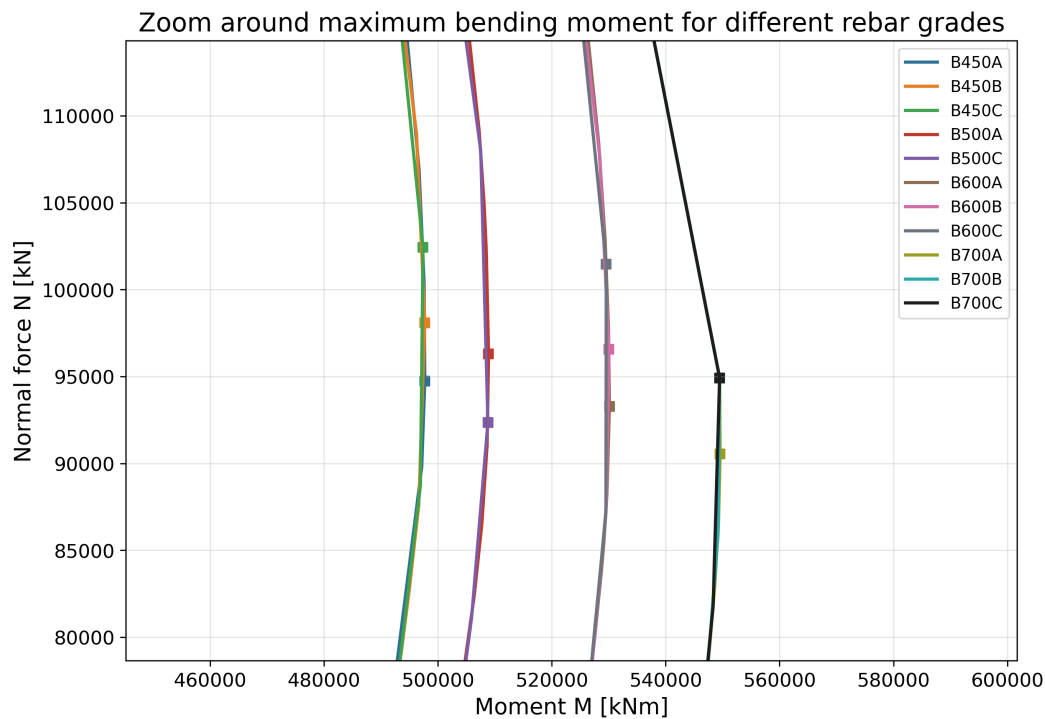


Figure 6.18: Comparison of the N – M interaction curves near $M_{\max}$ for different reinforcement ductility classes.

- **Implications for modelling and design**

From a modelling perspective, the simplified approach adopted in the *MegaColumn* software is appropriate for typical column design. However, it would be preferable to allow the user to choose the ductility class, especially since this option is now introduced in the new version of the Eurocode.

6.6 Influence of Discretisation Layer Thickness

The discretisation layer thickness is a key numerical parameter, as it directly affects the accuracy of the sectional analysis. Unlike geometric or material parameters, it is related to the numerical implementation of the model. The cross-section is discretised into horizontal layers, and the thickness of these layers influences the precision of the stress integration.

In general, a finer discretisation (i.e. thinner layers) leads to more accurate results, but at the expense of increased computational cost. This process is commonly referred to as *mesh convergence*. Due to this, it is important to assess whether the results converge towards a stable solution as the discretisation is refined. A lack of convergence would indicate that the numerical solution is not reliable [47].

In the *MegaColumn* software, the number of layers is fixed and not accessible to the user. Therefore, a parametric study is carried out using the *Python Code* implementation in order to evaluate the influence of layer thickness. The reference case corresponds to a square section of $3\,072\text{ mm} \times 3\,072\text{ mm}$, results are presented for both bending about the y -axis and the z -axis, as the orientation of the section affects the discretisation of the steel profile.

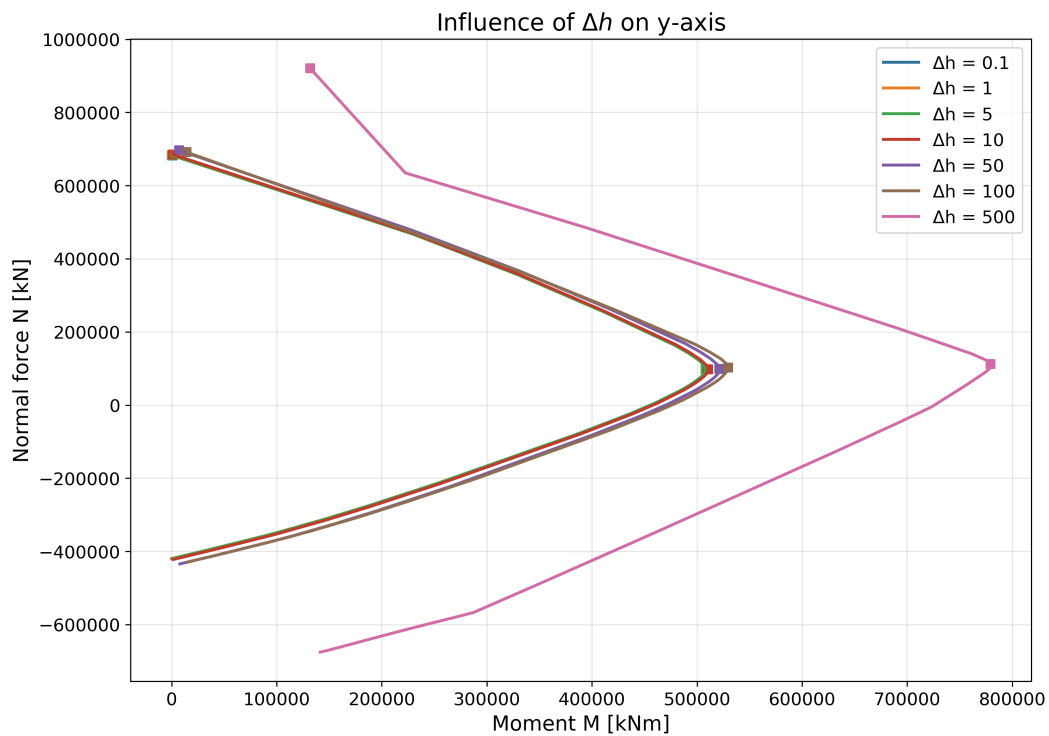


Figure 6.19: Comparison of N – M interaction diagram curves obtained with different discretisation layer thicknesses for bending about the y -axis.

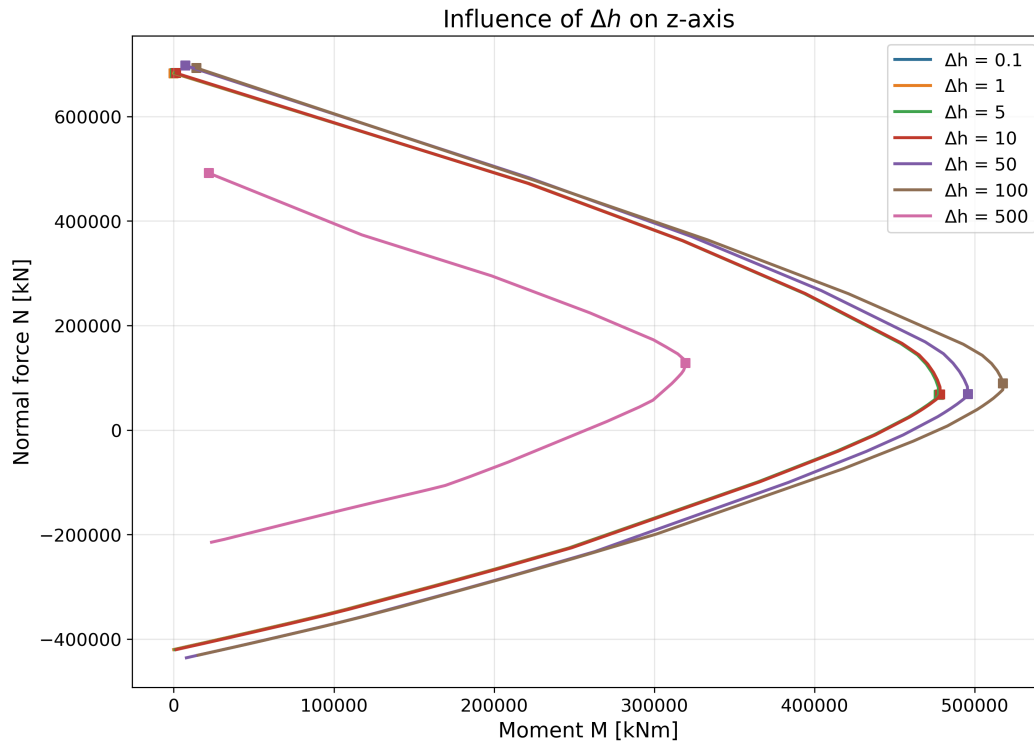


Figure 6.20: Comparison of N – M interaction diagram curves obtained with different discretisation layer thicknesses for bending about the z -axis.

Discussion

The main observations can be summarised as follows:

- **Strong influence of discretisation**

For large layer thicknesses (e.g. $\Delta h = 500$ mm), significant discrepancies are observed in the interaction diagrams. In particular, as the height of the layer decreases, the gap between the N – M interaction diagrams tends to close. This shows that discretisation matters and can lead to unreliable results, such as for the height of 500.

- **Influence of discretisation orientation**

The results show that the influence of discretisation strongly depends on the orientation of the layers.

In the case of bending about the y -axis, the layers are discretised from top to bottom. For large layer thicknesses, the geometry of the steel profile is poorly represented. For instance, with $\Delta h = 500$ mm, the discretisation becomes too rough relative to the profile height (approximately 600 mm for an HD 400×1299 section), leading to an overestimation of the steel contribution. In particular, the web is not captured, resulting in an exaggerated representation of the flanges. Consequently, the interaction curve exhibits a larger domain compared to more refined discretisation.

On the other hand, for bending about the z -axis, the layers are discretised from left to right. In this case, a low-resolution discretisation tends to underestimate the steel contribution. For

$\Delta h = 500$ mm, the layer thickness exceeds the characteristic width of the web, leading to its partial or complete omission in the discretised model. As a result, only the flanges are effectively represented, and the interaction curve is reduced compared to finer discretisation. These observations highlight that the discretisation strategy can either overestimate or underestimate the structural response, depending on the orientation.

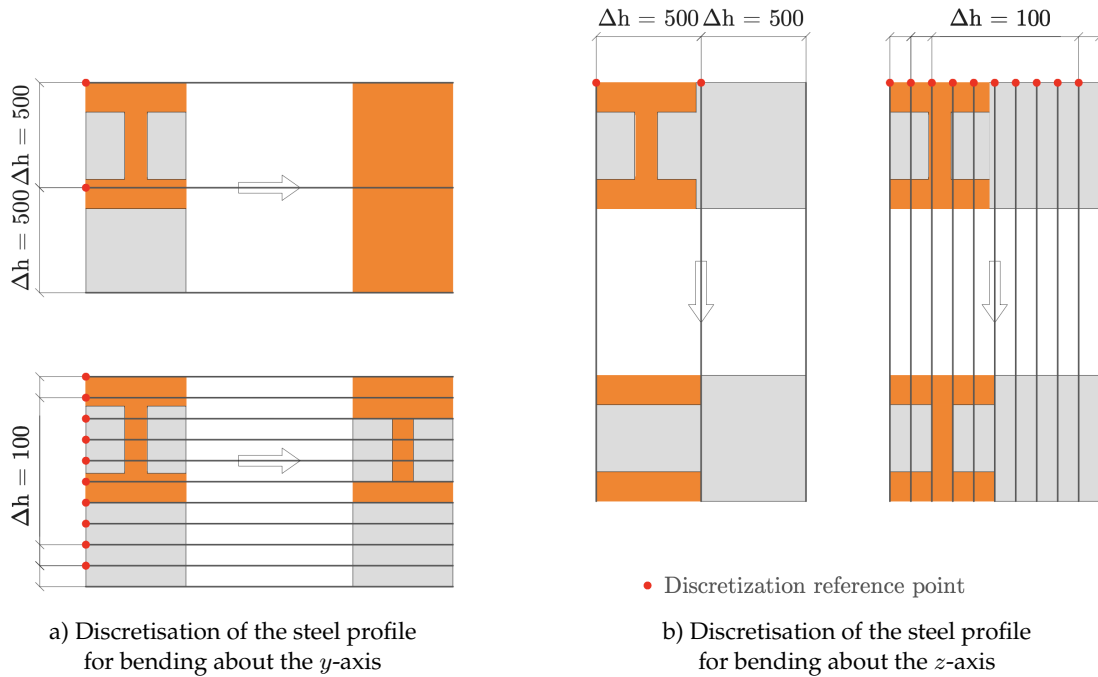


Figure 6.21: Illustration of the steel profile discretisation for both bending orientations and different layer thicknesses.

- **Convergence of results for fine discretisation**

As the layer thickness decreases, the interaction diagrams progressively converge towards a stable solution. Below a certain threshold (approximately 10 mm in this case, corresponding to about 0.3% of the section dimension), the differences between curves become negligible. This indicates that numerical convergence is achieved.

- **Practical implications for modelling**

The results demonstrate that the choice of discretisation layer thickness is critical for ensuring accurate predictions. Since the discretisation used in the *MegaColumn* software is not directly accessible to the user, providing control over this parameter would improve transparency and reliability.

As a practical guideline, a layer thickness not exceeding approximately 0.3% of the section height or width is recommended to ensure sufficient accuracy while maintaining reasonable computational cost.

6.7 Influence of Global Buckling

The final detailed investigation concerns the influence of global buckling on the structural response of composite megacolumns. As illustrated in the study case, this type of information can be useful and is often requested by design offices.

Unlike previous investigations, it is necessary to introduce the theoretical background required to fully understand the subject before analysing and discussing the phenomenon in detail.

6.7.1 Definition of Buckling

Buckling corresponds to a geometric instability of a compressed structural member (e.g. a column, wall, or plate), in which significant lateral deformations develop under compressive loading before the material resistance of the cross-section is fully reached [48, 49]. Several examples are presented in Figure 6.22, highlighting the progressive lateral deformation that may occur as instability develops.

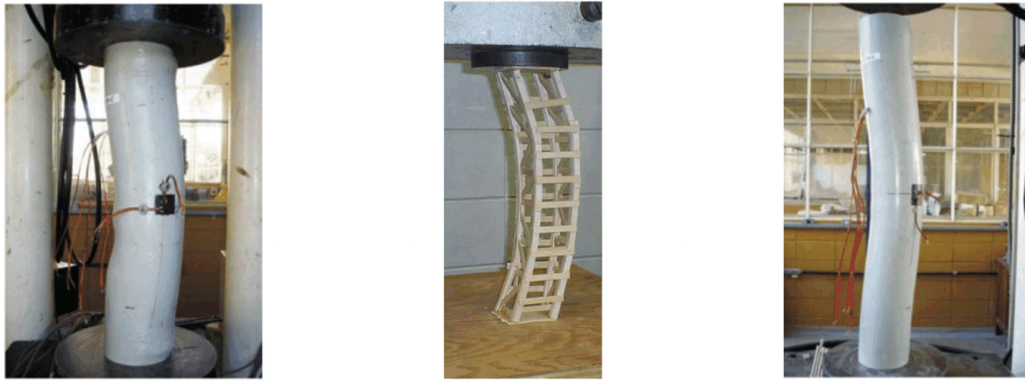


Figure 6.22: Illustrative examples of global column buckling, showing lateral deformation under compressive loading [50].

A key parameter governing buckling behaviour is the slenderness of the member. As the column becomes more slender, instability effects become increasingly significant. Although slenderness does not provide an exact prediction of buckling behaviour, it gives a useful first approximation of the sensitivity of the member to instability phenomena. The global slenderness can be computed easily from the global geometry of the column. In general, for values below approximately 20, buckling effects remain limited, whereas beyond values of around 50, the column becomes highly sensitive to buckling [22]:

$$\lambda = L_{cr} \sqrt{\frac{A_g}{I_g}} \quad (6.3)$$

This equation depends on the buckling length of the column which corresponds to the distance between two inflection points of the deformed shape. For example, if a column is pinned at both the bottom and the top, the buckling length is equal to the actual length of the column. In contrast, if the column is fixed at the base and free at the top, the buckling length becomes approximately twice the physical length of the member, as schematised in Figure 6.23.

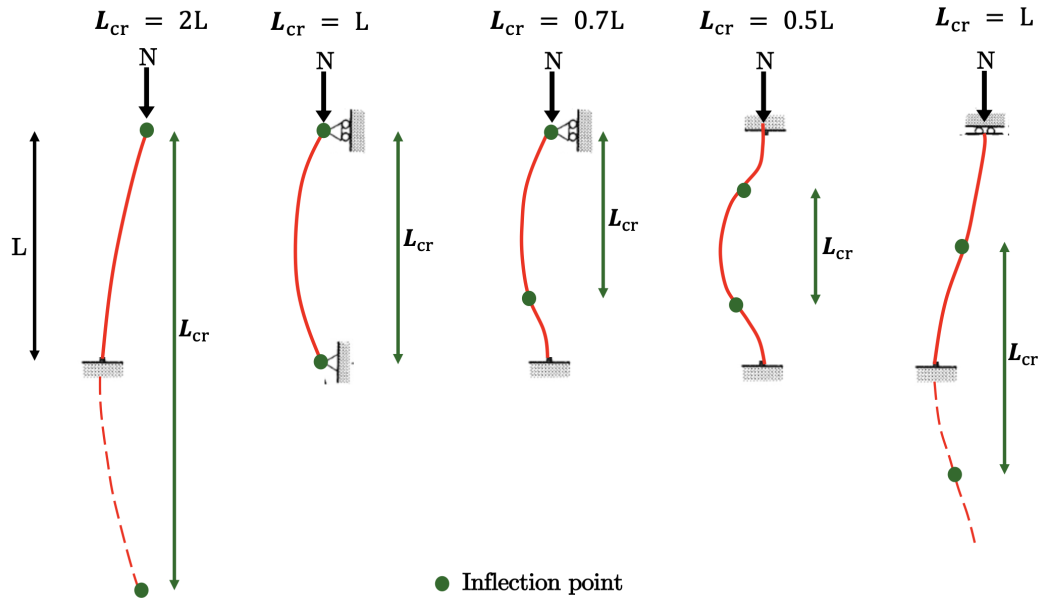
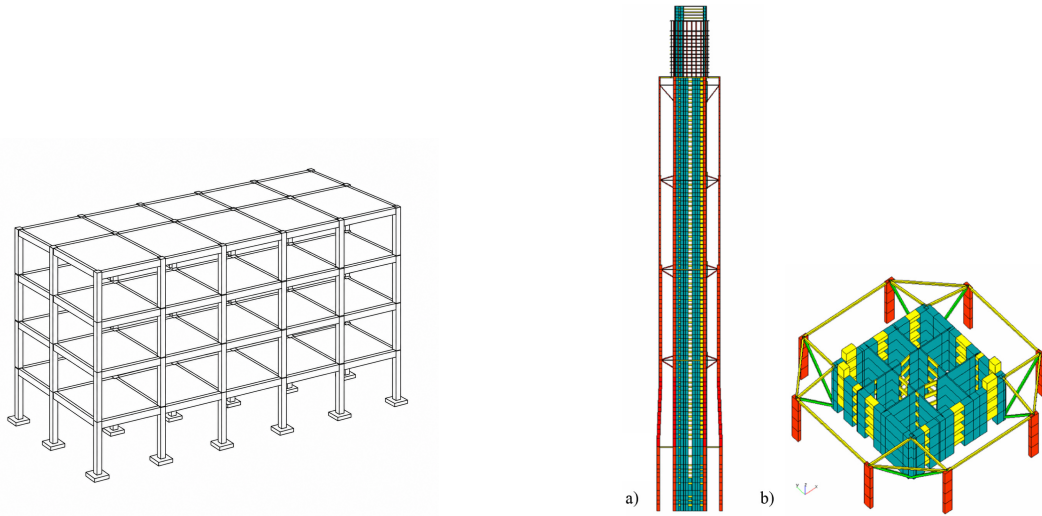


Figure 6.23: Examples of equivalent buckling lengths for different boundary conditions. Adapted from [22].

For conventional buildings with a regular structural arrangement, as shown in Figure 6.24a), the buckling length may often be approximated as the floor-to-floor height. In contrast, the situation becomes significantly more complex for megacolumns [51].

In tall buildings such as the Hong Kong ICC Tower, where the megacolumns are located along the perimeter of the structure rather than within the core, as illustrated in Figure 6.24b), the columns can reach very large heights. In such cases, the surrounding structural elements connected to the megacolumns may not provide sufficient stiffness to fully restrain their rotation.

Consequently, several questions arise regarding the appropriate buckling length to consider. *Should it correspond to the individual storey height between two floors? Should it correspond to the height between major outrigger levels? Or should the entire height of the assembled megacolumn be considered?* Unlike conventional buildings, where the effective buckling length is typically between 3.5 m and 5 m, megacolumns may exhibit effective lengths exceeding 20 m or even more. As a result, the global slenderness increases significantly, making buckling effects potentially critical for the structural response.



a) Typical building pattern highlighting a floor-to-floor buckling length. b) Megacolumn arrangement in the Hong Kong ICC Tower [4].

Figure 6.24: Comparison between a) a conventional building configuration with floor-to-floor buckling length and b) a tall building configuration with perimeter megacolumns.

Figure 6.25 demonstrates two general approaches that can be adopted to account for buckling effects. Similar to the approaches used to compute the N – M interaction diagram, buckling effects can either be evaluated through a pointwise safety check according to EC4 (§ 6.7.3.7) implemented within the *MegaColumn*, or through the generation of buckling-reduced N – M interaction diagrams implemented in the *Python Code* framework.

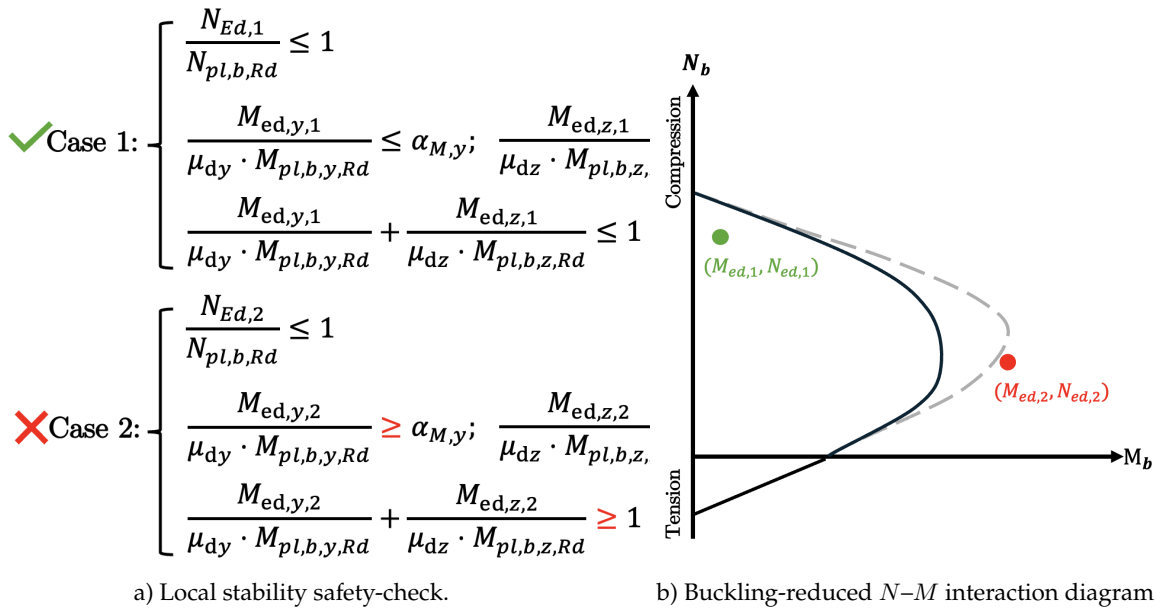


Figure 6.25: Comparison between a) a local stability safety-check approach and b) a buckling-reduced N – M interaction diagram approach.

6.7.2 Application to the Python Code

To account for buckling, the implemented methodology follows a stiffness-based Eurocode procedure in which:

- the axial resistance is reduced through a buckling reduction factor χ ;
- the bending resistance is reduced through a second-order amplification factor k .

Thus, for each point of the original N – M interaction diagram:

$$N_{\text{buckling,Rd}} = \chi N_{\text{Rd}} \quad (6.4)$$

$$M_{\text{buckling,Rd}} = \frac{M_{\text{Rd}}}{k} \quad (6.5)$$

This methodology transforms the full sectional resistance envelope into a reduced interaction domain that explicitly accounts for global instability effects [52].

6.7.2.1 General Eurocode-Based Computational Procedure

According to Eurocode, simplified buckling verification may be performed using two principal analytical approaches, depending on the loading configuration and the required level of refinement.

- **1st-Order Analysis:**

This simplified method is based on standard buckling curves and is primarily intended for members subjected to axial compression only. First-order analysis evaluates the equilibrium of the column in its undeformed configuration, meaning that the member is assumed to remain perfectly straight during loading. Consequently, additional bending moments generated by lateral deflections are neglected. This approach is suitable for simplified axial verification but does not explicitly account for moment amplification due to global instability effects [48, 53].

- **2nd-Order Analysis:**

This method is based on sectional verification including second-order moments and equivalent geometric imperfections. In contrast to first-order analysis, the equilibrium is evaluated in the deformed configuration of the member. Under compressive loading, lateral deflections develop and generate additional bending moments due to the increasing eccentricity of the applied axial force.

As illustrated in Figure 6.26, an axial compressive force N produces an initial lateral deflection. In first-order analysis, this deflection is neglected in the equilibrium calculation. In second-order analysis, on the other hand, the initial deflection, denoted by δ_0 , generates supplementary bending moments through the P – Δ effect. These additional moments further amplify the deformation until convergence toward a total lateral displacement denoted by δ .

This progressive interaction between axial force and lateral displacement provides a more realistic representation of slender column behaviour subjected to combined compression and bending [22, 54].

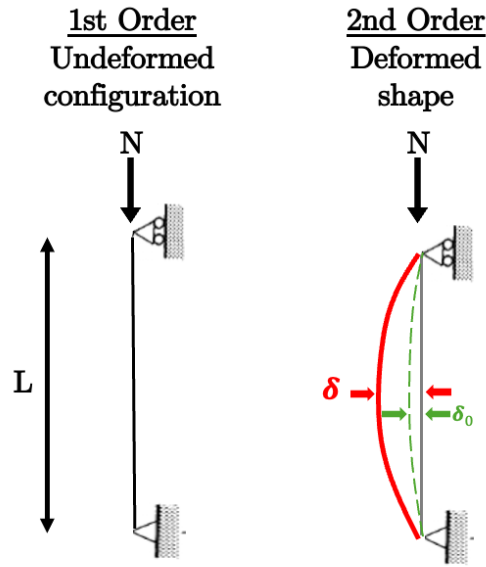


Figure 6.26: Illustration of first- and second-order analysis configurations, showing the influence of second-order effects on column behaviour. Adapted from [22, 55].

In addition to these two Eurocode approaches, a third advanced methodology is introduced in this study:

- **2nd Order Hybrid Analysis:**

Based on the *SmartCoCo* research project, this formulation extends the second-order Eurocode 4 stiffness philosophy by modifying the stiffness coefficients associated with concrete, structural steel, and reinforcement (e.g. K_c , K_a , and K_s).

The objective is to better represent the behaviour of megacolumns, for which standard Eurocode 4 assumptions may become insufficient. The *SmartCoCo* methodology suggests that, in certain large composite sections, embedded steel profiles may behave more similarly to very large reinforcing bars than to conventional structural steel sections, thereby requiring adapted stiffness assumptions [29].

6.7.2.2 Step-by-Step Methodology

For each methodology, the buckling verification of the composite column is performed point-by-point along the previously generated unbuckled N – M interaction curve.

Considering a given point with coordinates:

$$(N_{Rd}, M_{Rd}) \quad (6.6)$$

the implemented computational procedure is divided into two principal stages, summarised in Figure 6.27 and schematised in Figure 6.28:

1. **Evaluation of the reduced axial resistance;**
2. **Evaluation of the reduced bending resistance.** This stage is performed using two alternative approaches:
 - the *Linear Stiffness Method (LSM)*;
 - the *Hybrid Stiffness Method (HSM)*.

For a given (N_{Rd} , M_{Rd})	<u>Linear Stiffness Method</u> Eurocode 3 & 4	<u>Hybrid Stiffness Method</u> SmartCoCo
<u>Axial Reduction</u>	$EI_{1st} = E_a I_a + E_s I_s + K_e E_{c,eff} I_c$ $N_{cr,1st} = \frac{\pi^2 EI_{1st}}{L_{cr}^2}$ $\bar{\lambda} = \sqrt{\frac{N_{Rd}}{N_{cr,1st}}}$ $\phi = \frac{1}{2} [1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2]$ $\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$ $N_{b,Rd} = \chi N_{Rd}$	
<u>Bending Reduction</u>	$EI_{2nd,lin} = K_0 (E_a I_a + E_s I_s + K_{e,II} E_{c,eff} I_c)$ $N_{cr,2nd,lin} = \frac{\pi^2 EI_{2nd,lin}}{L_{cr}^2}$ $k_{lin} = \frac{\max(0.6 + 0.4r_m, 0.4)}{1 - \frac{N_{b,Rd}}{N_{cr,2nd,lin}}}$ $M_{b,Rd,lin} = \frac{M_{Rd}}{k_{lin}}$	$EI_{2nd,hyb} = K_{sa} E_a I_a + K_{sa} E_s I_s + K_c E_{cd} I_c$ $K_c \propto n = \frac{N_{Rd}}{N_{Rd,max}}$ $N_{cr,2nd,hyb} = \frac{\pi^2 EI_{2nd,hyb}}{L_{cr}^2}$ $k_{hyb} = \frac{\max(0.6 + 0.4r_m, 0.4)}{1 - \frac{N_{b,Rd}}{N_{cr,2nd,lin}}}$ $M_{b,Rd,hyb} = \frac{M_{Rd}}{k_{hyb}}$

Figure 6.27: Schematic overview of the buckling verification procedure using the Linear Stiffness Method (LSM) and the Hybrid Stiffness Method (HSM).

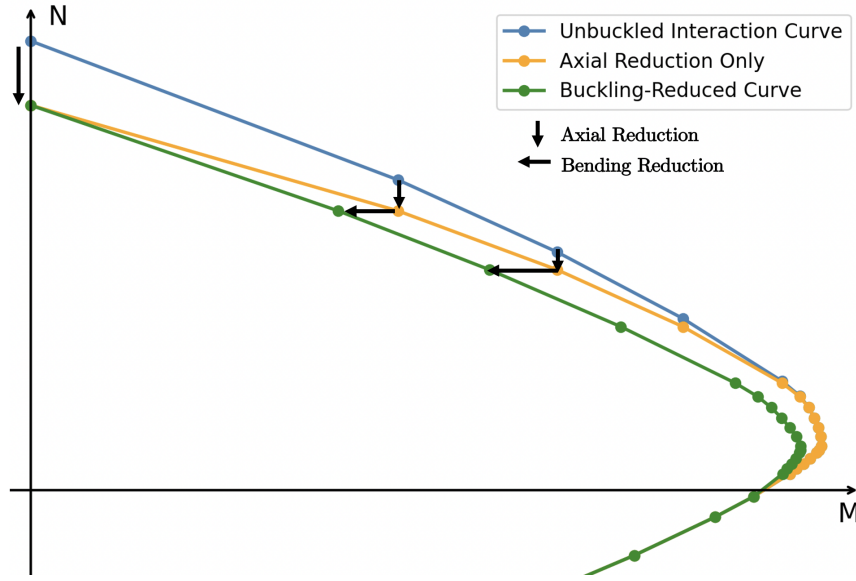


Figure 6.28: Illustration of the progressive reduction of the N – M interaction diagram during the buckling verification procedure.

6.7.2.3 Evaluation of the Axial Buckling Reduction Factor χ

The axial buckling reduction factor is determined on the basis of a first-order elastic analysis. The equations below are taken from Eurocodes 3 and 4 [56, 57].

First-Order Elastic Stiffness The effective flexural stiffness is evaluated using the simplified elastic formulation provided in Eurocode 4, Equation (6.40) [15]:

$$EI_{1st} = E_a I_a + E_s I_s + K_e E_{c,eff} I_c \quad (6.7)$$

where $K_e = 0.6$ accounts for the effective contribution of concrete to the global flexural stiffness.

The influence of long-term effects on the buckling resistance is considered by reducing the mean concrete modulus E_{cm} to an effective modulus $E_{c,eff}$, expressed as:

$$E_{c,eff} = E_{cm} \left(\frac{1}{1 + \frac{N_{G,Ed}}{N_{Ed}} \varphi_t} \right) \quad (6.8)$$

where:

- φ_t is the creep coefficient;
- $\frac{N_{G,Ed}}{N_{Ed}}$ represents the ratio between permanent and total axial load.

According to the large-scale experimental campaign conducted in China on similar composite megacolumns, a representative value of $\varphi_t = 1.5$ may be adopted [3].

In high-rise buildings, a significant proportion of the axial load is generally sustained over long periods. Consequently, the ratio $\frac{N_{G,Ed}}{N_{Ed}}$ may reasonably be assumed equal to 0.75, indicating that approximately 75% of the total axial load is permanent.

Euler Critical Load N_{cr} This parameter is calculated using the first-order elastic stiffness:

$$N_{cr,1st} = \frac{\pi^2 EI_{1st}}{L_{cr}^2} \quad (6.9)$$

Non-Dimensional Slenderness The non-dimensional slenderness is evaluated as:

$$\bar{\lambda} = \sqrt{\frac{N_{Rd}}{N_{cr,1st}}} \quad (6.10)$$

For tensile points, where $N_{Rd} < 0$, no buckling reduction is applied.

For compression points, the Eurocode reduction factor is determined through:

$$\phi = \frac{1}{2} [1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2] \quad (6.11)$$

$$\chi = \min \left[1.0, \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \right] \quad (6.12)$$

with $\alpha = 0.49$ This value corresponds to the selected Eurocode 4 buckling curve from Table 6.5 of EN 1994-1-1 [15].

The reduced axial resistance is obtained as:

$$N_{b,Rd} = \chi N_{Rd} \quad (6.13)$$

For tensile points:

$$N_{b,Rd} = N_{Rd} \quad (6.14)$$

6.7.2.4 Evaluation of the Second-Order Moment Amplification

Calculating the second-order amplification requires the determination of both the effective stiffness and the corresponding Euler critical load. Accordingly, the adopted formulation depends on the analysis methodology considered.

Effective Flexural Stiffness

Second-Order Elastic Linear Stiffness The stiffness expression becomes:

$$EI_{2nd,lin} = K_0 (E_a I_a + E_s I_s + K_{e,II} E_{c,eff} I_c) \quad (6.15)$$

where:

- $K_0 = 0.9$ is the global stiffness reduction factor;
- $K_{e,II} = 0.5$ accounts for the cracked concrete behaviour.

Hybrid Nominal Stiffness Formulation for Megacolumns For the hybrid stiffness formulation, the stiffness is evaluated individually for each point along the interaction curve:

$$EI_{2nd,hyb} = K_c E_{cd} I_c + K_{sa} E_s I_s + K_{sa} E_a I_a \quad (6.16)$$

where:

- $K_c = \frac{k_1 k_2}{1 + \varphi_{eff}}$ accounts for the effective concrete contribution and depends on the axial load level associated with the considered interaction point.

where:

- $k_1 = \left(\frac{f_{ck}}{20} \right)^{0.5}$
- $k_2 = \min \left(0.2, \frac{n\lambda}{170} \right)$, where $n = \frac{N_{Rd}}{N_{pl,Rd}}$ is the normalised axial load ratio. Here, $N_{pl,Rd}$ corresponds to the maximum axial resistance of the unbuckled interaction curve. This quantity is also denoted as N_{max} ; both notations refer to the same value.
- $E_{cd} = \frac{E_{cm}}{\gamma_{ce}}$, the design modulus of elasticity of concrete, where γ_{ce} is recommended to be set to 1.2 [29].
- The coefficient K_{sa} is obtained as

$$K_{sa} = \begin{cases} 1.0, & \rho_{sa} \leq 0.04 \\ \frac{0.45\lambda^{0.147}}{1 + 1.433\varphi_{eff}e^{-0.027\lambda}}, & \rho_{sa} > 0.04 \end{cases} \quad (6.17)$$

where $\rho_{sa} = \frac{A_s + A_a}{A_c}$ is the combined reinforcement and structural steel ratio.

Equation (6.17) reflects the progressive transition between reinforced concrete behaviour and composite steel–concrete behaviour.

For low values of ρ_{sa} , the contribution of structural steel remains limited relative to the concrete core. The structural response is closer to reinforced concrete behaviour, consistent with the design philosophy of Eurocode 2.

On the other hand, for larger values of ρ_{sa} , the contribution of steel becomes increasingly dominant, and the behaviour progressively approaches that of conventional composite steel–concrete columns, more closely aligned with the assumptions adopted in Eurocode 4.

Since n depends directly on N_{Rd} , the hybrid stiffness must be evaluated individually for each point of the unbuckled interaction diagram. Consequently, an interaction curve composed of 30 points leads to 30 different values of n , K_c , $EI_{2nd,hyb}$, and subsequently N_{cr} .

Second-Order Euler Critical Load Two different formulations of the Euler critical load are considered:

$$N_{cr,2nd,lin} = \frac{\pi^2 EI_{2nd,lin}}{L_{cr}^2} \quad (6.18)$$

$$N_{cr,2nd,hyb} = \frac{\pi^2 EI_{2nd,hyb}}{L_{cr}^2} \quad (6.19)$$

Second-order moment amplification This variable is evaluated using the reduced axial resistance $N_{b,Rd}$ and the moment distribution factor is calculated as:

$$\beta = \max(0.6 + 0.4r_m, 0.4) \quad (6.20)$$

The value $r_m = 1.0$ is considered to be the most critical case. Other values of β can be directly obtained from Table 6.4 of EN 1994-1-1 [15].

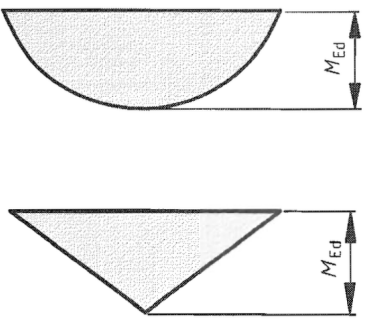
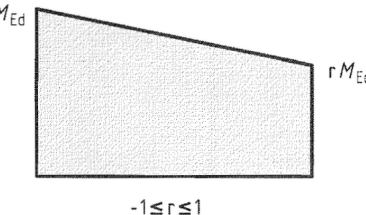
Moment distribution	Moment factors β	Comment
	First-order bending moments from member imperfection or lateral load: $\beta = 1,0$	M_{Ed} is the maximum bending moment within the column length ignoring second-order effects
	End moments: $\beta = 0,66 + 0,44r$ but $\beta \geq 0,44$	M_{Ed} and $r M_{Ed}$ are the end moments from first-order or second-order global analysis

Figure 6.29: Factor β used for the determination of moments according to second-order theory [15].

The k factor is defined for the two different methods as:

$$k_{lin} = \frac{\beta}{1 - \frac{N_{b,Rd}}{N_{cr,2nd,lin}}} \quad (6.21)$$

$$k_{hyb} = \frac{\beta}{1 - \frac{N_{b,Rd}}{N_{cr,2nd,hyb}}} \quad (6.22)$$

Finally, the moment resistance is reduced as follows:

$$M_{b,Rd,lin} = \frac{M_{Rd}}{k_{lin}} \quad (6.23)$$

$$M_{b,Rd,hyb} = \frac{M_{Rd}}{k_{hyb}} \quad (6.24)$$

For tensile points, no second-order moment amplification is applied, and the original bending resistance is retained. In other words, for tensile points $k = 1$.

6.7.3 Analysis

Both approaches employ the same axial buckling reduction procedure based on the first-order stiffness, EI_{1st} . However, they differ in the evaluation of the second-order moment amplification.

In the Linear Stiffness Method (LSM), a constant second-order Euler critical load is adopted over the entire interaction curve. In contrast, in the Hybrid Stiffness Method (HSM), both the effective stiffness and the corresponding Euler critical load are evaluated individually at each interaction point along the $N-M$ interaction diagram.

To investigate the influence of buckling, a two-factor parametric study is performed. The following parameters are considered:

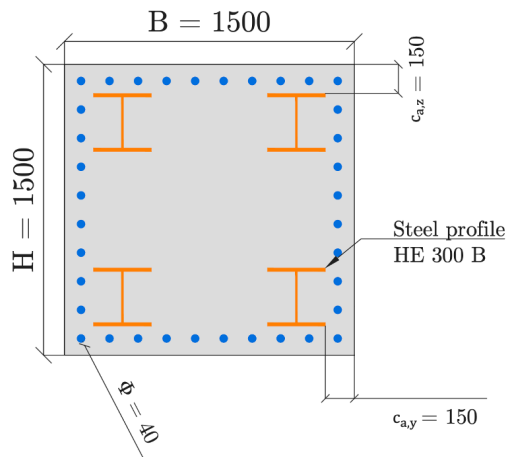
- **Adopted effective stiffness formulation:** Linear Stiffness Method (LSM) or Hybrid Stiffness Method (HSM);
- **Effective buckling length:** L_{cr} (1, 5, 10, 15, 20, 30, 40, and 50 m), directly governing the global slenderness of the composite column, as defined in Equation (6.3).

Case 1 corresponds to the reference configuration presented in Figure 3.5, whereas Cases 2 and 3 are schematised in Figure 6.30.

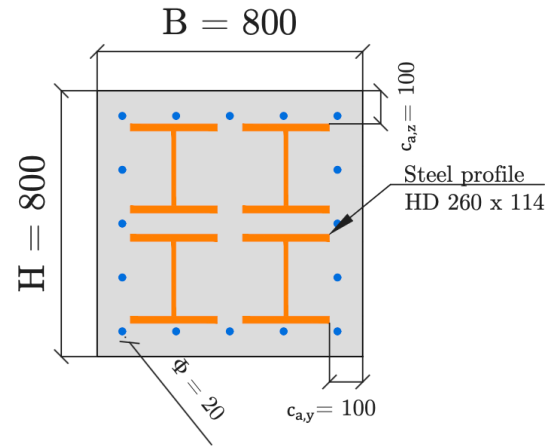
The results are presented for each case using graphs for both the LSM and the HSM, together with Table 6.7, which summarises all the data.

L_{cr}	Case 1: 3072 x 3072				Case 2: 1500 x 1500				Case 3: 800 x 800			
	λ	$\Delta N_{b,max}$	$\Delta M_{b,max}$		λ	$\Delta N_{b,max}$	$\Delta M_{b,max}$		λ	$\Delta N_{b,max}$	$\Delta M_{b,max}$	
1 m	1.13	0.0	0.0	0.0	2.31	0.0	0.0	0.3	4.33	0.0	0.1	0.3
5 m	5.64	0.0	0.1	0.4	11.55	0.0	0.8	3.7	21.65	7.6	0.8	1.3
10 m	11.28	0.0	0.3	1.2	23.09	7.3	2.9	9.8	43.30	27.3	1.4	2.4
15 m	16.91	1.9	0.6	2.4	34.64	16.5	5.9	15.9	64.95	48.6	2.3	3.7
20 m	22.55	5.9	1.1	3.7	46.19	26.7	9.6	18.2	86.60	64.8	3.4	5.3
30 m	33.83	14.3	2.6	6.3	69.28	47.7	17.1	20.3	129.90	86.8	6.8	9.2
40 m	45.11	23.4	4.4	7.7	92.38	64.1	19.2	22.9	173.21	89.1	11.0	14.0
50 m	56.38	33.2	6.4	9.0	115.47	74.6	21.2	25.8	216.51	92.8	16.1	19.7

Table 6.7: Comparison of the maximum differences in axial force and bending moment between the Linear Stiffness Method (LSM) and the Hybrid Stiffness Method (HSM) for the three analysed cases.

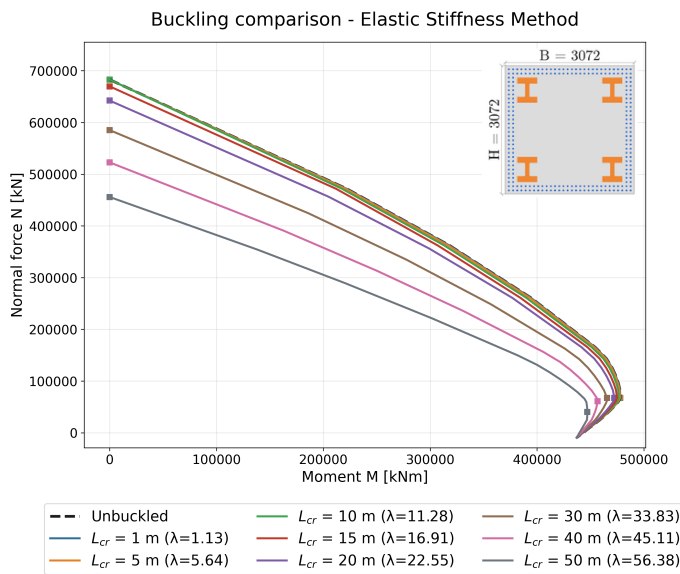


a) Alternative megacolumn geometry — Case 2

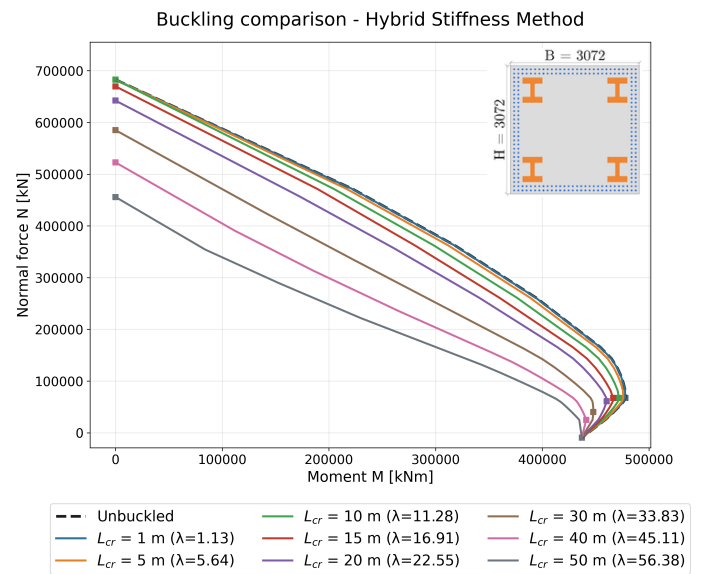


b) Alternative megacolumn geometry — Case 3

Figure 6.30: Alternative megacolumn geometries considered for the buckling analysis: a) Case 2 with a $1.5 \text{ m} \times 1.5 \text{ m}$ cross-section and b) Case 3 with a $0.8 \text{ m} \times 0.8 \text{ m}$ cross-section.



a) Linear Stiffness Method



b) Hybrid Stiffness Method

Figure 6.31: Comparison of buckling-reduced N – M interaction diagrams for Case 1: a) LSM results and b) HSM results for different effective buckling lengths.

6. Case Study and Parametric Analysis

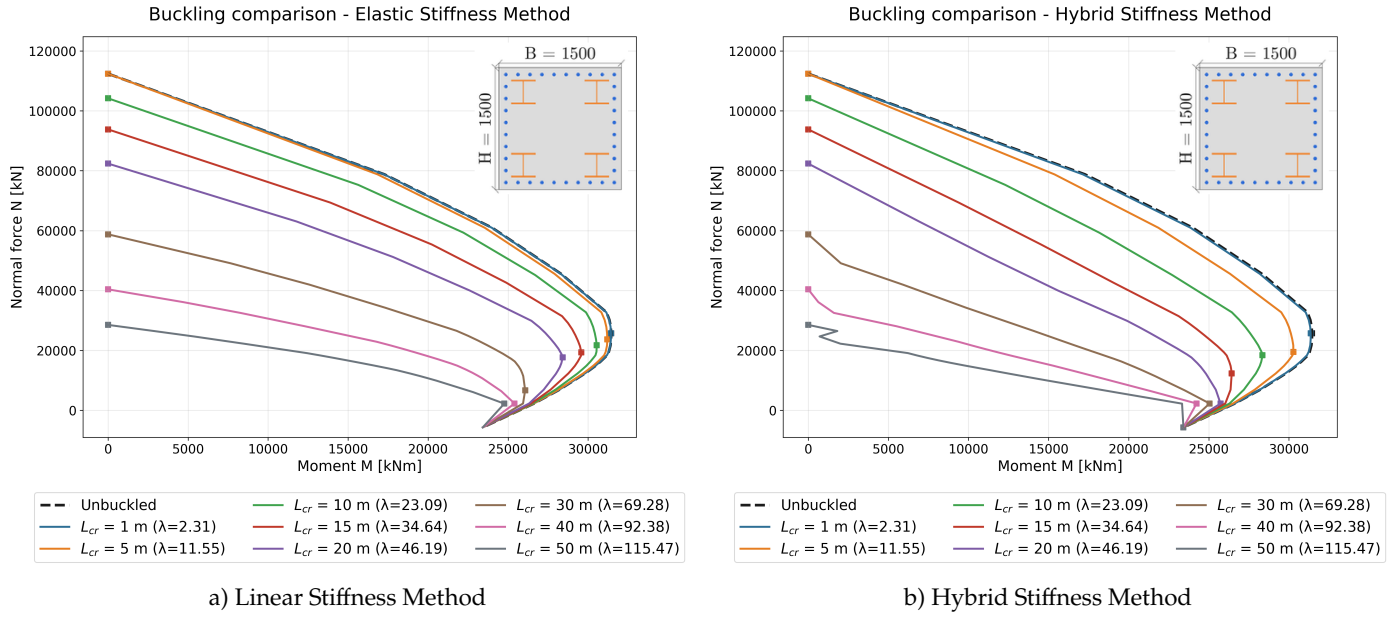


Figure 6.32: Comparison of buckling-reduced N - M interaction diagrams for Case 2:
a) LSM results and b) HSM results for different effective buckling lengths.

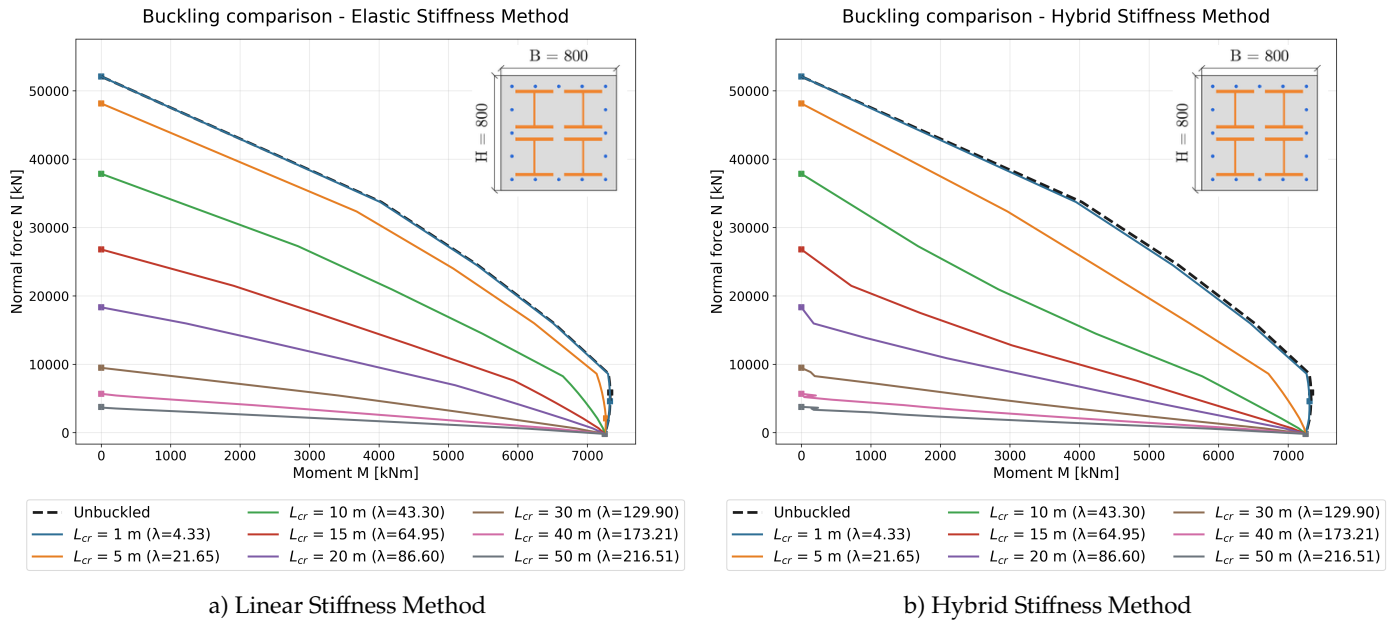


Figure 6.33: Comparison of buckling-reduced N - M interaction diagrams for Case 3:
a) LSM results and b) HSM results for different effective buckling lengths.

Discussion

The main observations may be summarised as follows. Note that the analysis was carried out for bending about the y -axis. Similar observations can also be made for bending about the z -axis, and the corresponding data may be obtained from the author upon request.

- **Influence of the effective buckling length on axial resistance**

As expected, the column height directly influences the buckling response through the effective buckling length L_{cr} .

Increasing the effective buckling length reduces the Euler critical load and increases the non-dimensional slenderness:

$$L_{cr} \uparrow \Rightarrow N_{cr,1st} \downarrow \Rightarrow \bar{\lambda} \uparrow \Rightarrow \phi \uparrow \Rightarrow \chi \downarrow \quad (6.25)$$

In consequence, the buckling reduction factor decreases, leading to a reduction in axial resistance.

The influence of second-order effects becomes noticeable when the global slenderness exceeds approximately $\lambda = 21$ ($\Delta N_{b,max} \geq 5\%$), which is consistent with the conclusions presented in the previous section.

- **Influence of the effective buckling length on bending resistance**

Increasing the effective buckling length also amplifies second-order effects:

$$L_{cr} \uparrow \Rightarrow N_{cr,2nd} \downarrow \Rightarrow k \uparrow \quad (6.26)$$

In consequence, the bending resistance progressively decreases as the global slenderness increases.

For the investigated configurations, the maximum reduction in bending resistance remains below approximately 16% for the LSM and 26% for the HSM.

In addition, the location of the maximum bending resistance M_{max} progressively changes with increasing slenderness. Simultaneously, the corrected axial resistance N_{corr} decreases and the interaction curve gradually evolves towards a more linear shape between the pure bending point $(M; 0)$ and the pure compression point $(0; N)$.

This behaviour reflects the increasing influence of second-order effects, which become more significant in regions associated with larger bending moments.

- **Influence of the stiffness formulation**

Overall, the Hybrid Stiffness Method (HSM) predicts larger reductions in bending resistance compared with the Linear Stiffness Method (LSM).

This behaviour is directly related to the adopted stiffness formulation:

$$EI_{2nd,hyb} \uparrow \Rightarrow N_{b,Rd} \uparrow \Rightarrow k_{hyb} \uparrow$$

For this reason, the HSM generally predicts larger effective stiffness values. However, this also increases the sensitivity of the section to second-order effects, resulting in a larger reduction of the interaction domain.

Since the reduction in axial resistance $\Delta N_{b,\max}$ remains identical for both methods, whereas the reduction in bending resistance $\Delta M_{b,\max}$ differs, the interaction domain predicted by the HSM is systematically smaller than that obtained with the LSM.

This behaviour is illustrated below for Case 1 with effective buckling lengths of $L_{cr} = 20$ m.

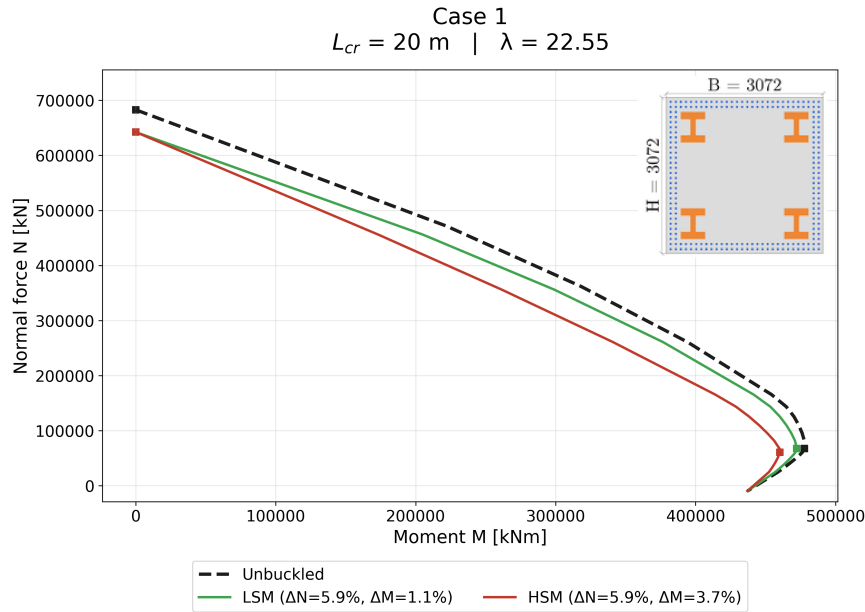


Figure 6.34: Comparison of buckling-reduced N – M interaction diagrams for Case 1 with $L_{cr} = 20$ m: a) Linear Stiffness Method (LSM), and b) Hybrid Stiffness Method (HSM).

• Limitations of the HSM formulation

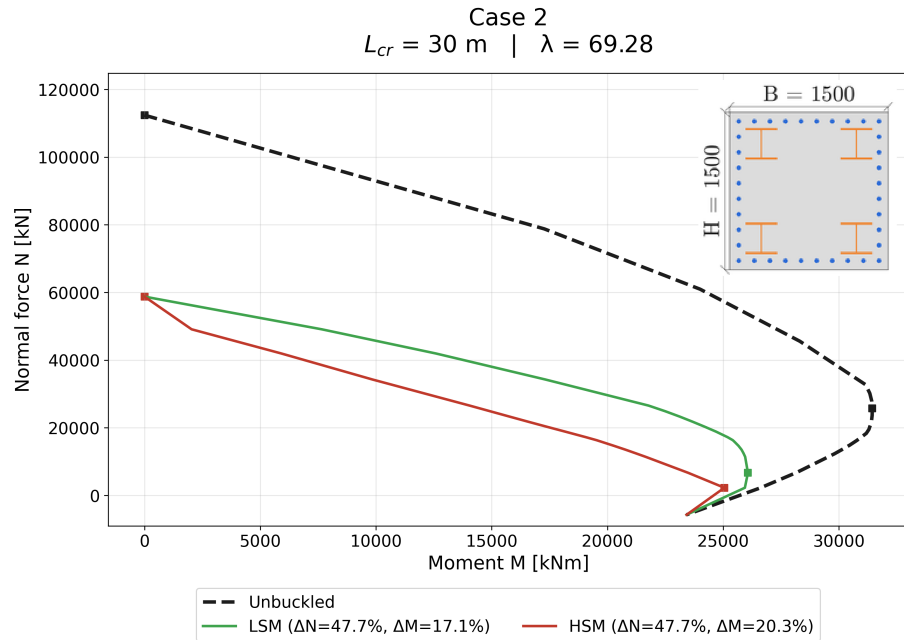
For the HSM, particularly in Cases 2 and 3, the interaction diagrams do not evolve as smoothly as expected when the global slenderness increases.

A local angular behaviour progressively appears within the compression-dominated region of the interaction curve, leading to a sudden change in slope. In contrast, the LSM interaction curves remain smooth over the entire loading range.

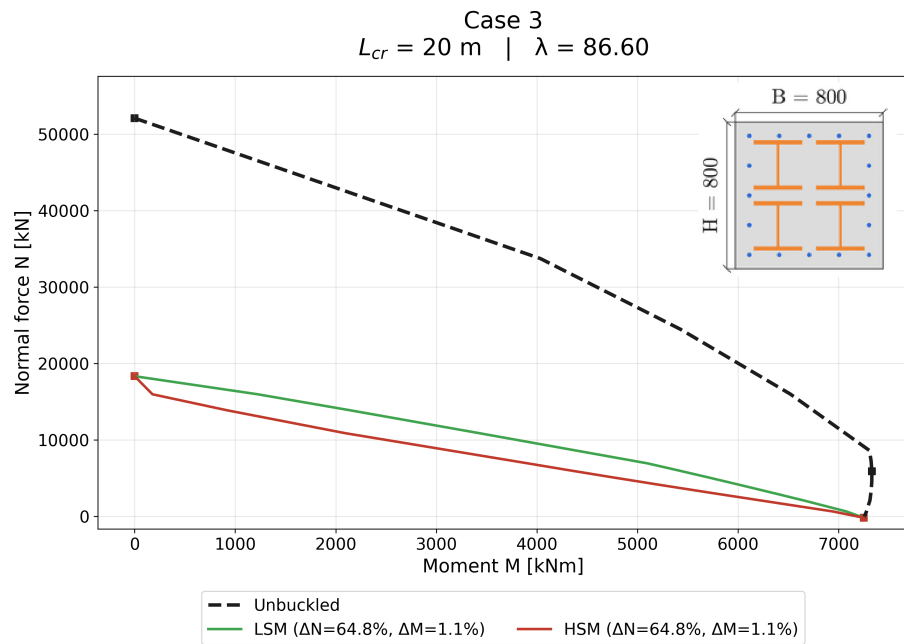
For global slenderness values of around 45, this phenomenon is relatively limited, but it becomes clearly noticeable when the global slenderness reaches approximately 55. At this level of slenderness, the reduction in the compression-dominated region already exceeds approximately 30%, which is structurally significant.

This behaviour is likely related to the formulation adopted for the effective stiffness $EI_{2nd,hyb}$, which was calibrated using statistical data. Since the available database does not include very slender members, the formulation may become less reliable outside its original calibration range [58].

The phenomenon is presented below for the two investigated cases, using the lowest effective buckling length at which this behaviour becomes visible.



a) Case 2



b) Case 3

Figure 6.35: Local angular behaviour observed in the HSM interaction diagrams at high slenderness for Case 2 and Case 3.

- **Practical considerations**

As previously discussed, the effective buckling length of a column is not directly equal to the storey height, since it depends on the global structural system and the associated boundary conditions.

For this reason, it may be more relevant to formulate the problem inversely. Instead of prescribing the effective buckling length as an input parameter, it could be more practical to define a target reduction in axial resistance and subsequently determine the corresponding admissible buckling length.

This procedure was implemented in the developed *Python Code* tool. For the reference configuration (Case 1), a target reduction of $\Delta N_{b,\max} = 20\%$ corresponds to an effective buckling length of approximately **36 m**.

Such a value is not unrealistic for tall buildings. For example, assuming a configuration with a restrained base and a free top, an effective buckling length of 36 m corresponds to a physical column height of approximately 18 m. Although, this height remains realistic for megacolumn applications. As discussed in the Introduction, the open-storey transfer zone of the *One Bloor West* tower already reaches approximately 12 m in height.

This observation further highlights the importance of accounting for global buckling effects in the design of very large composite columns. It also emphasises the necessity of carefully evaluating the actual boundary conditions and the global stiffness provided by the surrounding structural system, particularly for megacolumn applications.

6.8 Synthesis of Parametric Analyses

The parametric analyses performed in this chapter demonstrated that not all modelling assumptions influence the structural response to the same level. While some parameters have a limited impact on practical megacolumn applications, others play a more significant role and may justify the use of advanced modelling approaches.

The main observations and recommendations for future developments of the *MegaColumn* software are summarised below.

- **Concrete constitutive law: major influence — should be implemented in *MegaColumn*.**

The concrete constitutive law was identified as one of the most influential parameters affecting the N – M interaction diagram, particularly for high-strength concrete. In some cases, the difference in axial resistance exceeded 15%, together with noticeable modifications of the interaction curve geometry.

These results demonstrate that the constitutive modelling of concrete becomes increasingly important as the concrete strength increases.

MegaColumn recommendations:

- Allow the user to select different concrete constitutive laws;
- Distinguish between the former and revised Eurocode formulations;
- Extend the applicability of the software to high-strength concrete classes by allowing the modification of the ultimate concrete strain values.

- **Structural steel strain hardening: limited influence — not essential for *MegaColumn*.**

The influence of strain hardening in the embedded structural steel profiles remained relatively limited for the investigated megacolumn configurations. The main differences were observed in the tension-controlled regions of the interaction diagrams. These regions are not relevant because megacolumns are predominantly subjected to compression. This behaviour is mainly explained by the relatively small steel ratio within the composite section.

Consequently, the elastic–perfectly plastic assumption currently adopted in *MegaColumn* appears sufficiently accurate for standard megacolumn applications.

MegaColumn recommendations:

- No immediate implementation appears necessary;
- The current elastic–perfectly plastic model provides a sufficiently accurate and conservative representation.

- **Reinforcing steel strain hardening and ductility class: secondary influence — optional improvement for *MegaColumn*.**

Similar observations were obtained for reinforcing steel. The inclusion of strain hardening and the variation of the reinforcement ductility class produced only minor modifications of the interaction diagrams.

MegaColumn recommendations:

- The implementation of reinforcement ductility classes could be considered as an optional advanced parameter;
- Such an addition would improve compatibility with the revised Eurocode formulations.

- **Layer discretisation thickness: secondary influence — warning or information to be added in *MegaColumn*.**

The discretisation layer thickness was found to influence the numerical accuracy when the mesh size exceeded approximately 5% of the section dimension, leading to noticeable deviations in the predicted interaction diagrams.

***MegaColumn* recommendations:**

- Display the mesh size or the number of discretisation layers to the user;
 - Allow the user to modify the mesh size;
 - Provide warning messages when the selected layer thickness exceeds recommended limits.
- **Global buckling effects: major influence for slender columns — should be implemented in *MegaColumn*.**

The results showed that buckling effects become increasingly significant for slender columns, particularly when the global slenderness approaches values of approximately 45, for which the reduction in axial resistance exceeds 20%.

Furthermore, the Hybrid Stiffness Method (HSM) produced larger reductions in bending resistance compared with the Linear Stiffness Method (LSM). Nonetheless, the results also indicated that the applicability of the HSM may be limited to geometries with a global slenderness lower than approximately 45.

***MegaColumn* recommendations:**

- Include second-order analysis by reducing the N – M interaction diagram;
- Allow the user to choose between the Linear and Hybrid stiffness methods;
- Provide guidance regarding the selection of appropriate buckling lengths;
- Add an optional feature allowing the user to define a target reduction in axial resistance and automatically determine the corresponding buckling length.

Overall, the study showed that the current *MegaColumn* software already provides sufficiently accurate results for most practical megacolumn applications, as demonstrated in the previous validation chapter. Nevertheless, the additional modelling approaches investigated in this study could further improve the accuracy of the predicted structural response.

More importantly, these additions do not make the software significantly heavier or excessively “machine-oriented”, which are often among the main drawbacks of advanced FEM tools and are not the objective of *MegaColumn*. The proposed improvements remain relatively simple to implement and can be integrated without fundamentally modifying the overall architecture of the current code.

Chapter 7 Conclusions and Perspectives

7.1 Summary of Findings and Contributions

This thesis investigated the structural behaviour and numerical modelling of composite mega-columns composed of multiple embedded steel profiles that are not welded together. The work was conducted in the context of developing and validating the *MegaColumn* software, an in-house pre-design tool developed by ArcelorMittal for the analysis of unconventional composite mega-column configurations. The study focused primarily on the computation and validation of the axial force–bending moment interaction diagram (N – M interaction diagram), addressing two main research questions by the end of the work:

- *Is the MegaColumn software reliable, and can its results be trusted by the user?*
- *Which advanced modelling approaches could be integrated into the MegaColumn software in future developments?*

To answer these questions, an independent *Python Code* implementation was developed to reproduce the behaviour predicted by the *MegaColumn* software using classical strain compatibility and equilibrium principles for the computation of N – M interaction diagrams. Additional modules related to longitudinal shear and headed stud connectors were also implemented, mainly inspired by the *SmartCoCo* European Commission project.

The validation phase compared the results obtained with the *Python Code* to those produced by the *MegaColumn* software and by the third-party finite element software *BiAxialPro*. The comparison revealed a strong similarity between the various approaches. The global shape of the interaction diagrams, the maximum axial resistance, and the maximum bending resistance were found to be highly consistent for most investigated cases, with differences generally limited to approximately $\pm 1\%$ over more than 25 parametric tests carried out using a *One-Factor-at-a-Time* methodology.

Local discrepancies were nevertheless observed near the peak bending resistance, particularly for the corresponding axial force values, where small numerical variations in the interaction curves occasionally produced differences exceeding 20% in three cases out of twenty-five investigated configurations. These differences were attributed primarily to variations in strain-state discretisation, numerical sampling sensitivity and interpolation procedures, rather than to fundamental errors in the modelling assumptions.

The final verification step consisted of a comparison with the real-case study of the *One Bloor West* tower in Canada. The obtained N – M interaction diagrams showed very good correlation with those provided by an external design office.

Overall, two major conclusions can be drawn:

- the *Python Code* successfully reproduces the behaviour of the *MegaColumn* software.;
- the *MegaColumn* software itself provides highly reliable results when compared with a finite element approach, such as *BiAxialPro*.

The second major part of the thesis focused on the parametric analysis of the N – M interaction diagrams, highlighting the influence of several modelling assumptions on the predicted structural response. The main investigated parameters were the material constitutive laws, the discretisation layer thickness, and the global buckling effects.

Among these parameters, the concrete constitutive law and the buckling effects were identified as the two most influential aspects and should therefore be prioritised for future integration into the *MegaColumn* software. The influence of the other investigated parameters remained of secondary importance under the considered assumptions.

Based on these observations, several recommendations were formulated for the future development of *MegaColumn*. Concerning the concrete modelling, the software should allow the user to select both the Eurocode version and the type of simplified constitutive law adopted for the concrete behaviour. Regarding global buckling, the software should provide a buckling-reduced N – M interaction diagram while also enabling the user to select the stiffness formulation to be adopted, as well as the percentage reduction in axial and bending resistance used to derive the effective buckling length.

Finally, the *MegaColumn* software could also display the number of discretisation layers or fibres used in the analysis, allowing users to better monitor potential mesh convergence issues and numerical accuracy.

7.2 Limitations and Future Work

Despite the significant results obtained throughout this thesis, several limitations and perspectives for future developments must be acknowledged.

First, the validation performed in this work focused exclusively on the two-dimensional N – M interaction diagrams. Several functionalities currently available or planned within the *MegaColumn* software were not validated, including:

- three-dimensional N – M interaction surfaces;
- alternative megacolumn configurations with multiple embedded steel profiles;
- longitudinal shear forces;
- and the determination of the required number of mechanical shear connectors.

Secondly, the numerical implementation developed in this thesis relies on simplified analytical assumptions based on classical beam theory and strain compatibility. While these assumptions are

suitable for pre-design applications, they neglect several phenomena that may become important in real structures, such as stress concentrations, cracking propagation, creep, shrinkage, thermal effects, seismic loading, fatigue, and fire performance. Investigating these effects could improve the robustness and predictive capability of future software developments, despite the fact that the main purpose of the *MegaColumn* software is to facilitate the initial design of such columns.

From a computational perspective, future developments could also focus on optimisation strategies for identifying efficient megacolumn configurations. Since the problem involves many interacting parameters, it could be interesting to define only a limited number of global inputs, such as the maximum applied loads or the target gross section dimensions, and allow the software to automatically determine suitable steel profiles, concrete classes, reinforcement layouts, or connector densities. Optimisation algorithms could then be used to identify configurations that minimise material consumption and embodied carbon while satisfying structural, economic, and environmental requirements.

Finally, the continued development of the *MegaColumn* software itself represents an important perspective. Improving the graphical user interface, expanding the available geometries, integrating additional material models, and automating verification procedures could progressively transform the software into a more comprehensive and accessible design tool for engineers worldwide.

Overall, this master's thesis represents an important contribution to the understanding, validation, and practical implementation of innovative composite megacolumn systems. While several challenges and areas for improvement remain, the results obtained throughout this work highlight the strong potential of these structural solutions for the design of future tall buildings. It is expected that the developments and observations presented in this thesis will contribute, even modestly, to the continued evolution of more efficient, sustainable, and high-performance composite structures.

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Appendix A

Longitudinal Shear and Shear Connectors

This Appendix presents the work carried out on longitudinal shear, from the theoretical background to the *Python Code* implementation developed during this master's thesis.

A.1 Theoretical Background

A.1.1 Shear Behaviour

Shear forces play a fundamental role in the mechanical behaviour of structural members. Shear stresses develop whenever there is a variation of bending moment along the member length or when forces must be transferred between different materials. In composite structures, shear is particularly important because it governs the interaction between steel and concrete components [47].

The advantages of composite action are illustrated in Figure A.1. The upper element is made of concrete, while the lower element is a steel beam. When a unit load is applied at mid-span, as shown on the left side of the figure, both elements behave independently. Their stress distributions are therefore separate and smaller, and slip occurs at the interface between the two materials [13, 22].

In contrast, on the right side, the stress distribution is continuous, indicating that the two materials act compositely. This results in an increased bending stiffness due to a larger effective moment of inertia (I) and a higher plastic section modulus (W). However, this composite behaviour also generates longitudinal shear stresses at the interface between the two materials.

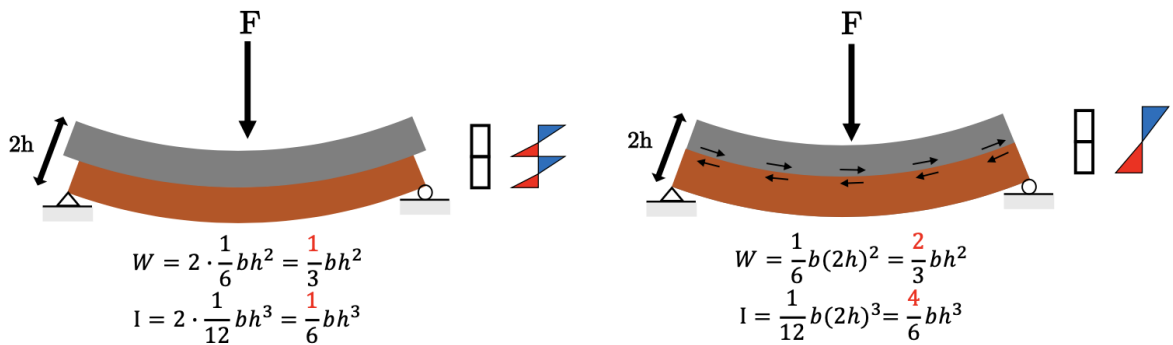


Figure A.1: Comparison between non-composite beam behaviour without shear transfer (left) and composite beam behaviour with longitudinal shear transfer at the interface (right). Adapted from [13, 22].

Shear behaviour in composite members can be divided into two main categories: transversal shear and longitudinal shear.

A.1.1.1 Shear Definition

Shear corresponds to the action of forces parallel to a surface, which tend to produce sliding between adjacent parts of a material or between different materials in contact [22].

In beam and column theory, the shear force V is directly related to the variation of bending moment M along the longitudinal axis:

$$V = \frac{dM}{dx} \quad (\text{A.1})$$

where x represents the longitudinal coordinate of the structural element.

This relationship shows that shear forces are inherently linked to bending behaviour. When bending moment varies, internal shear stresses develop within the cross-section to maintain equilibrium [59].

A.1.1.2 Transversal Shear

Transversal shear refers to shear stresses acting perpendicular to the longitudinal axis of the member and distributed over the cross-section [60].

These stresses are caused by external shear forces acting on the element, such as transverse loads or support reactions. The distribution of it depends mainly on the cross-sectional geometry and the properties of the material.

In columns subjected primarily to axial compression, transversal shear is generally less critical than axial force and bending moment. Still, it may become significant in regions where lateral forces are introduced, such as at beam-column connections or under horizontal loading conditions.

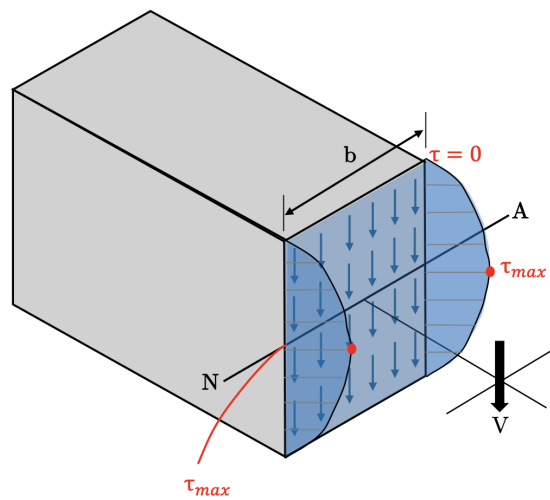


Figure A.2: Schematic representation of transversal shear stresses in a composite member. Adapted from [61].

A.1.1.3 Longitudinal Shear

Longitudinal shear corresponds to the shear stresses developing along the interface between steel and concrete when the internal bending moment varies along the member length. These stresses ensure the transfer of longitudinal forces between the different components of the composite section [29, 62]. This phenomenon was schematised previously in Figure A.1.

From equilibrium considerations, the longitudinal shear per unit length, also referred to as the shear flow v_L , can be expressed as:

$$v_L = \frac{dN}{dx} \quad (\text{A.2})$$

Alternatively, in bending members, the longitudinal shear flow can also be related to the global shear force V :

$$v_L = \frac{VS}{I} \quad (\text{A.3})$$

where:

- V is the transversal shear force,
- S is the first moment of area of the considered part of the section,
- I is the second moment of area of the entire composite section.

This longitudinal shear flow represents the force per unit length that must be transferred by the shear connection to ensure compatible deformation between steel and concrete.

In composite columns with multiple embedded steel profiles, this mechanism plays a key role, as it controls how loads are transferred between the steel profiles and the surrounding concrete. The shear connection must therefore be designed so that its resistance exceeds the longitudinal shear forces that develop along the interface.

A.1.2 Shear Connection

The shear connection between steel and concrete is a key aspect governing the mechanical behaviour of composite members. Its primary role is to ensure compatibility of longitudinal deformations between the different components. If slip occurs at the interface, the composite action is reduced, leading to a decrease in bending stiffness and resistance.

Slip at the steel–concrete interface can be classified into two distinct categories, as described in [13].

- **Micro-slip:** This corresponds to very small relative displacements at the interface, which are not visible to the naked eye. Despite their limited magnitude, these displacements mobilise the bond mechanisms between steel and concrete and contribute to the transfer of shear forces.
- **Macro-slip:** This refers to larger and measurable relative displacements between the steel and concrete components. Macro-slip results in a partial loss of composite action and significantly affects the global structural behaviour.

The transfer of longitudinal shear forces at the steel–concrete interface is governed by three fundamental mechanisms, the first two of which are plotted in Figure A.3 [15]:

- **Chemical bond:** This effect comes from the adhesion between the cement paste and the steel surface. It is mainly active at the early stages of loading, but it is difficult to quantify and tends to decrease over time.
- **Friction:** Friction develops when relative displacement occurs at the interface between steel and concrete. Its contribution depends on the compressive force acting between the two materials and becomes more important once the chemical bond has weakened.
- **Mechanical connection:** Mechanical connectors, such as embossments, ribs, welded plates, or headed studs, provide a direct transfer of shear forces. In practice, these connectors are the main elements that ensure composite action.

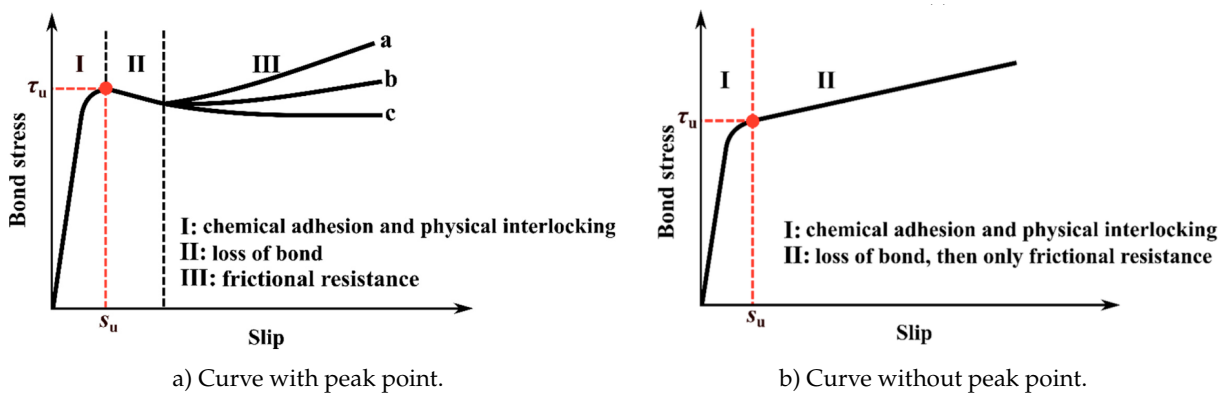


Figure A.3: Typical bond stress–slip relationships at the steel–concrete interface: a) response with peak bond stress and softening, and b) response governed by progressive frictional resistance [63].

The type, number, and capacity of shear connectors have a significant influence on the structural response of composite members, as they govern the degree of interaction between steel and concrete. Rigid connectors, such as welded plates, and sufficiently strong and numerous connectors can prevent significant slip at the interface, resulting in **full shear connection**. In this case, full composite action develops, allowing the member to reach its maximum stiffness and resistance.

On the other hand, when the connectors are more flexible or insufficient in number or capacity, relative displacement occurs at the interface, leading to **partial shear connection**. This partial interaction reduces the efficiency of composite action and results in lower stiffness and resistance.

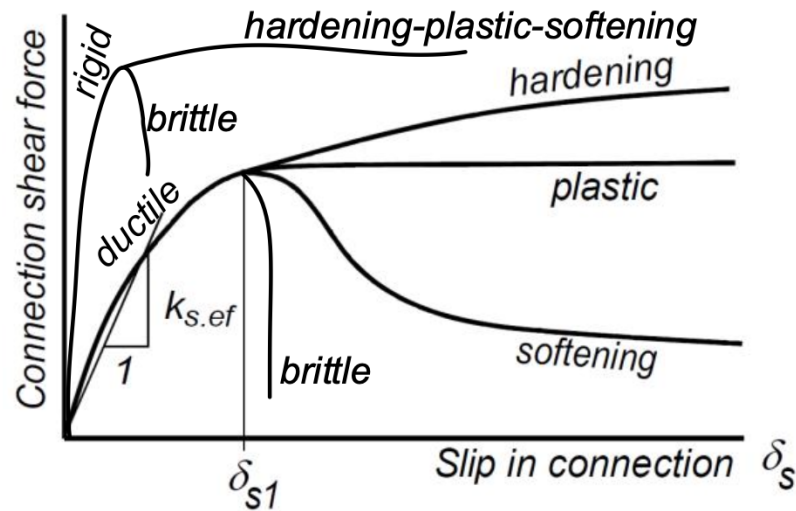


Figure A.4: Typical load-slip behaviour of shear connectors used in composite structures [30].

A.1.2.1 Mechanical Connectors – Headed Studs

Among the different types of mechanical connectors, headed studs are the most widely used solution in composite construction due to their efficiency, reliability, and ease of installation.

Headed studs are cylindrical steel poles welded to a steel profile and embedded in surrounding concrete. They primarily serve to transfer longitudinal shear forces across the steel–concrete interface, preventing excessive relative displacement between the materials.

The resistance of a stud connector depends on several parameters, including [15]:

- the stud diameter and height;
- the mechanical properties of steel and concrete;
- the concrete confinement conditions;
- and the geometric configuration of the composite section.

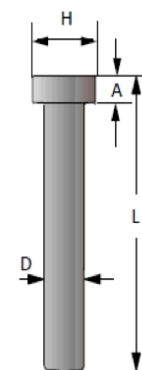
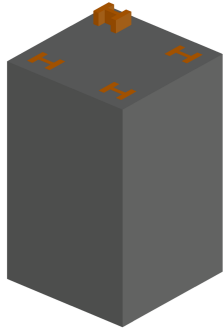


Figure A.5: Headed stud shear connectors: a) examples of stud geometries and b) definition of the main geometric parameters [38, 64].

The total number of studs required in a composite column must be sufficient to transfer the longitudinal shear forces developing along the steel–concrete interface. In composite megacolumns with multiple embedded steel profiles, an adequate shear connection is essential to prevent relative displacement between the profiles and to ensure effective composite behaviour. Otherwise, the steel profiles may behave independently, significantly reducing the structural efficiency of the megacolumn concept.



a) Relative displacement between adjacent steel profiles.



b) Headed stud connectors used in the One Bloor West Tower [6].

Figure A.6: Illustration of shear interaction in composite megacolumns: a) relative displacement between adjacent steel profiles and b) headed stud connectors ensuring mechanical shear transfer.

A.1.3 Longitudinal Shear Modelling for the Specific Megacolumn

Four main modelling strategies can be identified in the literature.

Elastic beam method The elastic beam method consists of modelling the megacolumn as a one-dimensional beam element based on Euler–Bernoulli beam theory. The composite cross-section is discretized into its individual components, and compatibility and equilibrium conditions are enforced to evaluate the distribution of internal forces and longitudinal shear. This approach is computationally efficient and particularly well suited for pre-design applications, although it neglects shear deformation and local effects [3, 21, 29].

Strut-and-tie method The strut-and-tie method models the internal force transfer mechanism using an equivalent truss system. In this representation, the concrete is idealised as compression struts, while the steel profiles and reinforcing bars act as tension ties. This method provides a simplified mechanical interpretation of force transfer and is especially useful for understanding load paths and designing shear connections [21, 24, 65, 66].

Corotational finite element method More advanced numerical models are based on the finite element method using a corotational formulation. This approach allows the analysis of geometrically non-linear behaviour by introducing local reference frames that follow the deformation of the structure. It enables the consideration of large displacements, rotations, and non-linear kinematics, providing a highly accurate representation of the structural response. However, this

method requires significantly greater computational effort and is primarily used for research purposes rather than pre-design [58, 67, 68].

Mixed beam–truss finite element method An intermediate approach consists of combining beam theory and truss modelling within a finite element framework. Typically, Timoshenko beam elements are used to represent the global behaviour of the composite member, while truss or Vierendeel-type elements are introduced to model the internal force transfer mechanisms between steel and concrete. This method provides a more refined representation than classical beam theory while maintaining reasonable computational efficiency [30, 69].

A.2 Python Code Implementation for the Longitudinal Shear

A.2.1 Computation of the Longitudinal Shear

The evaluation of the longitudinal shear force presented in this study is based on classical *Elastic Beam Theory*. The adopted methodology follows the approach described in *Design of Hybrid Structures* [21], with several adaptations introduced to ensure consistency with the present numerical implementation.

In the following, the procedure is detailed for a transverse shear force acting along the z -axis, denoted $V_{Ed,z}$. The corresponding formulation for the y -axis is provided in the next section.

In contrast to the previous section, the present computation is based on a given design action rather than on the determination of a resistant capacity. The objective is therefore to evaluate the distribution of the applied shear force within the cross-section and to determine the resulting longitudinal shear at the critical steel–concrete interfaces, where shear connectors (e.g. welded headed studs) may be required.

Since the composite cross-section is heterogeneous, it cannot be treated as a fully homogenised section at this stage. Instead, the applied shear force is distributed among the different zones of the section according to their relative flexural stiffness. As *stiffness attracts forces*, regions with higher stiffness attract a larger portion of the applied shear force.

A.2.1.1 Step 0: Division of the Cross-Section into Subsections

The *Elastic Beam Theory* (plane sections remain plane) allows the cross-section to be divided into distinct zones with approximately homogeneous behaviour:

- **BC3**: edge regions with a higher density of reinforcement, treated as reinforced concrete sections;
- **BC4**: central region, also treated as reinforced concrete;
- **BS**: regions containing structural steel profiles, reinforcement, and concrete.

This subdivision allows the mechanical behaviour of each zone to be characterised independently.

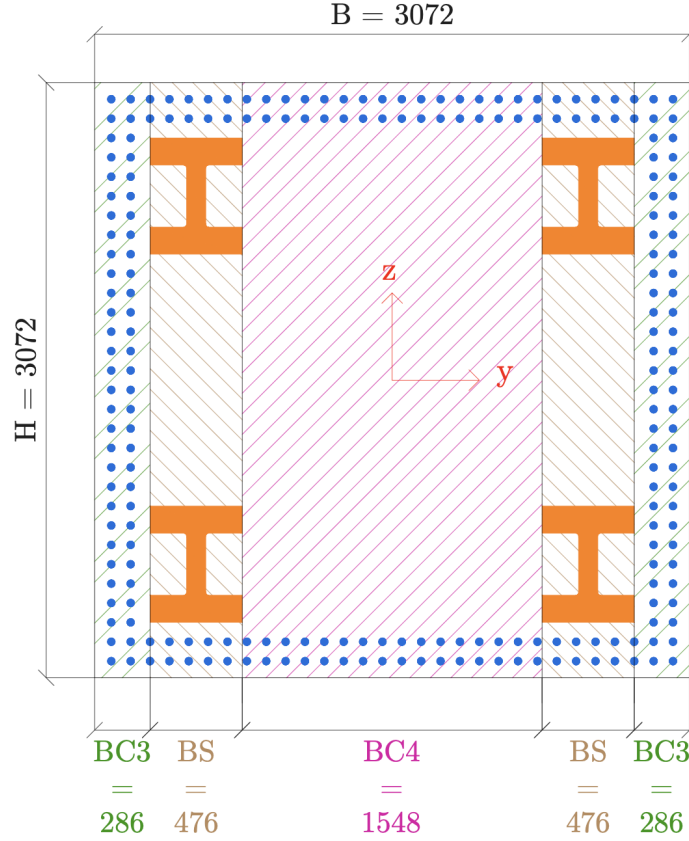


Figure A.7: Division of the cross-section into BC3, BS, and BC4 zones for the evaluation of longitudinal shear along the z -axis.

A.2.1.2 Step 1: Computation of the Effective Flexural Stiffness EI

The effective flexural stiffness of each zone is determined in accordance with Eurocode 4 [15] (Eq. 6.41), accounting for the long-term effects of concrete, while second-order effects are neglected, as follows:

$$EI_{\text{eff},i} = E_a I_{a,i} + E_s I_{s,i} + K_e E_{c,\text{eff}} I_{c,i} \quad (\text{A.4})$$

This expression combines the contributions of structural steel, reinforcing steel, and concrete, where:

- $K_e = 0.6$ accounts for cracking effects in concrete;
- $E_{c,\text{eff}}$ accounts for creep effects and was previously introduced in Equation (6.8). In the present case:

$$E_{c,\text{eff}} = \frac{36\,773.3}{1 + 1.5 \times 0.75} = 17\,305.1 \text{ MPa} \quad (\text{A.5})$$

Geometrical properties The second moments of area are evaluated for each material.

For reinforcing bars, using the parallel axis theorem:

$$I_{s,i} = \frac{\pi R^4}{4} + A_{s,i} z_{s,i}^2 \quad (\text{A.6})$$

For structural steel profiles:

$$I_{a,i} = 2 (I_y + A_a z_{a,cg}^2) \quad (\text{A.7})$$

In zones BC3 and BC4, the contribution of the structural steel profile is none, i.e. $I_{a,i} = 0$.

The concrete contribution is obtained by subtracting the steel contributions from the total inertia:

$$I_{c,i} = I_{\text{total},i} - I_{s,i} - I_{a,i} \quad (\text{A.8})$$

Global stiffness The total effective flexural stiffness of the cross-section is obtained by summing the contributions of each zone:

$$EI_{\text{eff}} = 2 EI_{\text{eff},\text{BC3}} + 2 EI_{\text{eff},\text{BS}} + EI_{\text{eff},\text{BC4}} \quad (\text{A.9})$$

Based on this global stiffness, the applied shear force is distributed among the different zones in proportion to their respective stiffness:

$$V_{Ed,z,i} = V_{Ed,z} \cdot \frac{EI_{\text{eff},i}}{EI_{\text{eff}}} \quad (\text{A.10})$$

The numerical values obtained for each zone are summarised in Table A.1.

Parameter	Unit	BC3	BS	BC4
E_a	MPa	210 000	210 000	210 000
I_a	mm ⁴	–	3.14×10^{11}	–
E_s	MPa	200 000	200 000	200 000
I_s	mm ⁴	6.73×10^{10}	4.00×10^{11}	1.60×10^{11}
K_e	–	0.6	0.6	0.6
$E_{c,\text{eff}}$	MPa	17 305	17 305	17 305
I_c	mm ⁴	6.23×10^{11}	8.199×10^{11}	3.58×10^{12}
$EI_{\text{eff},i}$	Nmm ²	1.99×10^{16}	8.22×10^{16}	6.92×10^{16}
EI_{eff}	Nmm ²	2.734×10^{17}		
$V_{Ed,z}$	kN	20 000	20 000	20 000
$V_{Ed,z,i}$	kN	1 458.2	6 010.9	5 061.7

Table A.1: Mechanical properties and effective flexural stiffness of the zones considered in the longitudinal shear calculation along the z -axis.

A.2.1.3 Step 2: Homogenisation of the Steel Profile Zone

The BS zone is transformed into an equivalent homogeneous concrete section in order to apply classical shear flow formulas. This transformation is performed by ensuring equivalence of flexural stiffness:

$$E_{cm} I_c^* = E I_{\text{eff,BS}} \quad (\text{A.11})$$

which leads to:

$$I_c^* = \frac{E I_{\text{eff,BS}}}{E_{cm}} = \frac{8.22 \times 10^{16}}{36\,773.3} = 2.23 \times 10^{12} \text{ mm}^4 \quad (\text{A.12})$$

The steel profile is converted into an equivalent concrete section using modular ratios, with the concrete between the flanges being neglected. Figure A.8 schematically represents the conversion process:

$$b_s^* = b_s \cdot \frac{E_s}{E_{cm}} = 476 \cdot \frac{210\,000}{36\,773.3} = 2\,718.2 \text{ mm} \quad (\text{A.13})$$

$$t_w^* = t_w \cdot \frac{E_s}{E_{cm}} = 100 \cdot \frac{210\,000}{36\,773.3} = 571.1 \text{ mm} \quad (\text{A.14})$$

This allows the use of standard elastic shear flow expressions on an equivalent homogeneous section.

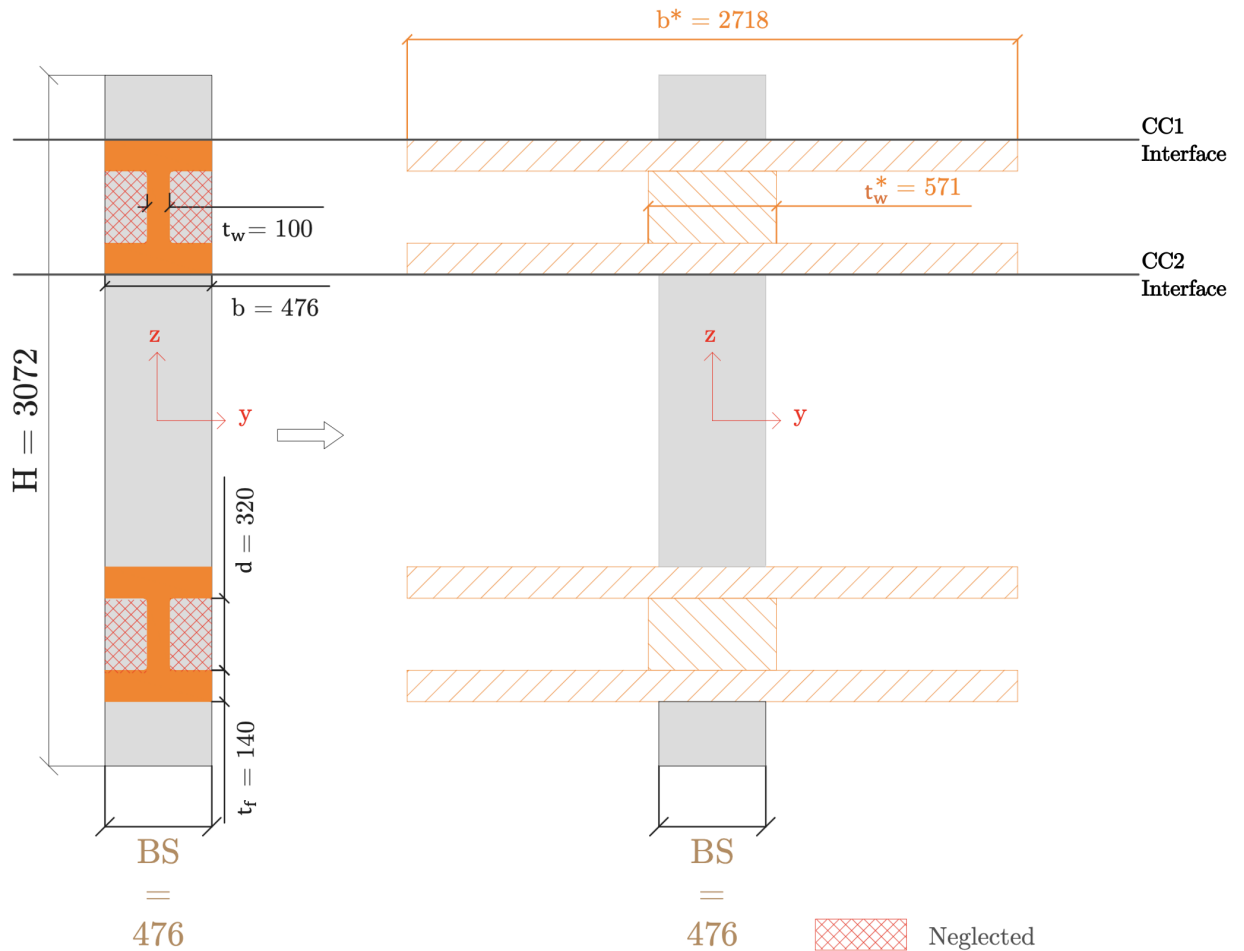


Figure A.8: Transformation of the BS zone into an equivalent homogeneous concrete section for the evaluation of shear flow.

A.2.1.4 Step 3: Computation of the Longitudinal Shear

The longitudinal shear flow at a given interface is computed as:

$$v_{L,Ed} = \frac{V_{Ed,BS} \cdot S}{I_c^*} \quad (\text{A.15})$$

The remaining parameter to be determined is S , which represents the first moment of area of the portion of the cross-section located above the considered interface. This quantity is defined by the general expression:

$$S_y = \int_A z \, dA \quad (\text{A.16})$$

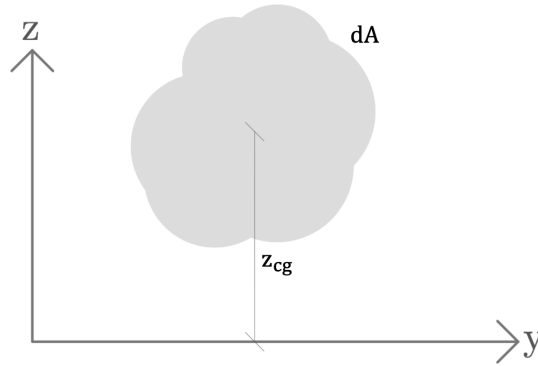


Figure A.9: Definition of the first moment of area S_y for a given interface.

Two interfaces are critical: CC1 and CC2, where shear connectors can be provided.

Longitudinal shear at interface CC1

$$A_{CC1} = b_{BS} \cdot c_{a,z} = 476 \cdot 286 = 1.36 \times 10^5 \text{ mm}^2 \quad (\text{A.17})$$

$$z_{CC1} = \frac{H}{2} - \frac{c_{a,z}}{2} = \frac{3072}{2} - \frac{286}{2} = 1393 \text{ mm} \quad (\text{A.18})$$

$$S_{CC1} = A_{CC1} \cdot z_{CC1} = 1.36 \times 10^5 \cdot 1393 = 1.89 \times 10^8 \text{ mm}^3 \quad (\text{A.19})$$

Thus:

$$v_{L,Ed,CC1} = \frac{V_{Ed,BS} \cdot S_{CC1}}{I_c^*} = \frac{6010.9 \times 10^3 \cdot 1.89 \times 10^8}{2.23 \times 10^{12}} = 510.2 \text{ kN/m} \quad (\text{A.20})$$

Longitudinal shear at interface CC2

The first moment of area is composed of three contributions, as shown in Figure A.10b):

$$S_{CC2} = S_{CC1} + S_{\text{steel}} + S_{c,CC2}^* \quad (\text{A.21})$$

The contribution of the steel profile is transformed into an equivalent concrete area:

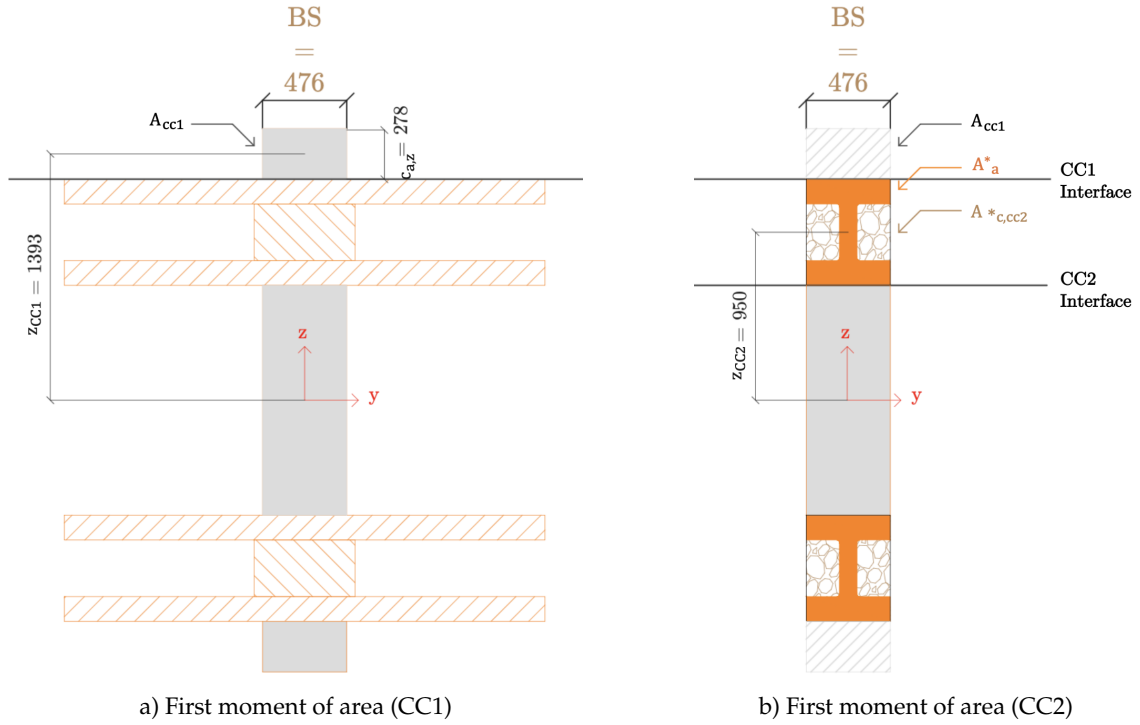


Figure A.10: First moment of area evaluated at the critical steel–concrete interfaces CC1 and CC2.

$$A_a^* = A_a \cdot \frac{E_s}{E_{cm}} = 165\,470 \cdot \frac{210\,000}{36\,773.3} = 9.45 \times 10^5 \text{ mm}^2 \quad (\text{A.22})$$

$$S_{\text{steel}} = A_a^* \cdot z_{CC2} = 9.45 \times 10^5 \cdot 950 = 8.97 \times 10^8 \text{ mm}^3 \quad (\text{A.23})$$

The concrete contribution between the flanges is:

$$A_{c,CC2}^* = b_{BS} \cdot h_{\text{steel}} - A_a = 476 \cdot 600 - 165\,470 = 1.20 \times 10^5 \text{ mm}^2 \quad (\text{A.24})$$

$$S_{c,CC2}^* = A_{c,CC2}^* \cdot z_{CC2} = 1.20 \times 10^5 \cdot 950 = 1.14 \times 10^8 \text{ mm}^3 \quad (\text{A.25})$$

Thus:

$$S_{CC2} = 1.89 \times 10^8 + 8.97 \times 10^8 + 1.14 \times 10^8 = 1.20 \times 10^9 \text{ mm}^3 \quad (\text{A.26})$$

Finally:

$$v_{L,Ed,CC2} = \frac{V_{Ed,BS} \cdot S_{CC2}}{I_c^*} = \frac{6\,010.9 \times 10^3 \cdot 1.20 \times 10^9}{2.23 \times 10^{12}} = 3\,232.2 \text{ kN/m} \quad (\text{A.27})$$

This procedure provides the longitudinal shear flow at the critical steel–concrete interfaces, which is subsequently used for the design of shear connectors.

A.2.2 Determination of the Number of Studs

The determination of the required number of shear connectors (headed studs) follows the provisions of Eurocode 4 [15].

A.2.3 Failure Modes of Mechanical Connectors

The load transfer mechanism of a stud connector evolves as the applied load increases. In the initial stage, shear forces are primarily transmitted through bearing stresses at the base of the stud, close to the weld collar. As illustrated in Figure A.11, with increasing load (P), the displacement (w) increases and local crushing of the surrounding concrete occurs. This leads to a redistribution of stresses along the height of the stud, inducing bending and tensile stresses within the stud shank.

Simultaneously, compressive forces (dark grey area) develop beneath the stud head, which enhances the frictional resistance at the interface between the steel and the concrete [21, 70].

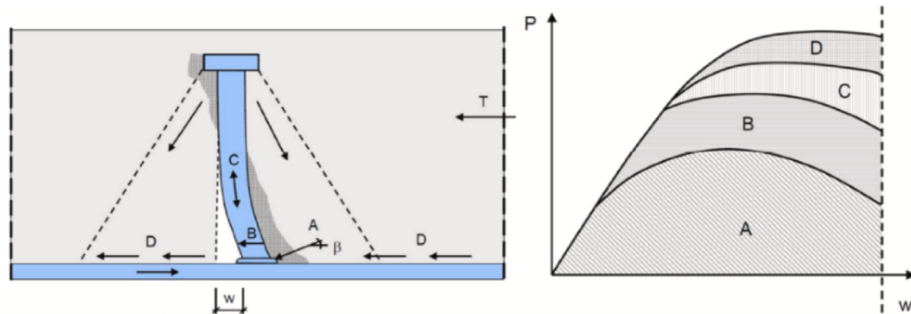


Figure A.11: Schematic representation of the load transfer mechanism of a headed stud connector embedded in concrete [71].

Failure typically occurs near the weld collar. However, alternative failure modes may also develop, as illustrated in Figure A.12.

1. Shearing of the connector just above the weld bead;
2. Local crushing of the concrete at the base of the connector;
3. Pull-out failure associated with the formation of a concrete cone;
4. Shear failure of the concrete rib;
5. Combined shear and tensile failure due to large deformations.

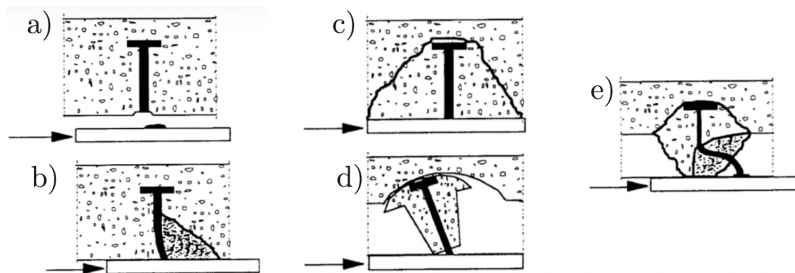


Figure A.12: Typical failure modes of headed stud connectors embedded in concrete [13].

A.2.4 Computation of the Design Shear Resistance

To account for the different failure modes, Eurocode 4 (§6.6.3.1) provides corresponding design expressions [15]. Failure may occur either in the stud or in the surrounding concrete. Therefore, the design shear resistance of a headed stud is defined as the minimum of these two contributions:

$$P_{Rd} = \min(\text{stud failure ; concrete failure})$$

$$= \min \left(\frac{0.8 f_u \pi d^2}{4 \gamma_v} ; \frac{0.29 \alpha d^2 \sqrt{f_{ck} E_{cm}}}{\gamma_v} \right) \quad (\text{A.28})$$

$$\alpha = \begin{cases} 1 & \text{if } \frac{h}{d} > 4 \\ 0.2 \left(\frac{h}{d} + 1 \right) & \text{if } 3 < \frac{h}{d} \leq 4 \end{cases} \quad (\text{A.29})$$

$$\gamma_v = 1.25 \quad (\text{A.30})$$

Stud properties used in the example

Parameter	Symbol	Value
Stud type	–	M25-106
Diameter	d	25 mm
Height	h	100 mm
Ultimate strength	f_u	450 MPa
Concrete strength	f_{ck}	50 MPa
Mean modulus of elasticity of concrete	E_{cm}	36 773.3 MPa

Table A.2: Geometrical and mechanical properties of the headed stud considered in the numerical example.

Thus, the governing failure mode corresponds to failure of the shear stud:

$$P_{Rd} = \min(P_{Rd,\text{stud}} ; P_{Rd,\text{concrete}}) = \min(141.4 ; 196.6) = 141.4 \text{ kN} \quad (\text{A.31})$$

A.2.4.1 Determination of the Number of Studs per Unit Length

The required number of studs per metre is obtained by relating the longitudinal shear force acting over a length of 1 m to the design resistance of a single stud:

$$n_{\text{studs}} = \frac{v_{L,Ed} \cdot 1 \text{ m}}{P_{Rd}} \quad (\text{A.32})$$

For the two critical interfaces:

$$n_{\text{studs,CC1}} = \frac{510.2 \cdot 1 \text{ m}}{141.4} \approx 4 \text{ studs/m} \quad (\text{A.33})$$

$$n_{\text{studs,CC2}} = \frac{3\,232.2 \cdot 1 \text{ m}}{141.4} \approx 23 \text{ studs/m} \quad (\text{A.34})$$

These values define the required density of shear connectors to ensure adequate transfer of longitudinal shear at the steel–concrete interfaces.

The above expressions are valid when only one stud is provided per row. In certain design situations, such as composite beams, two studs may be arranged within the same row. In such cases, the expression becomes:

$$n_{\text{studs}} = \frac{v_{L,Ed} \cdot 1 \text{ m}}{n_{\text{studs,row}} \cdot P_{Rd}} \quad (\text{A.35})$$

where $n_{\text{studs,row}}$ is the number of studs per row.

For the case of two studs per row, the required densities become approximately 2 and 12 studs per metre for interfaces CC1 and CC2, respectively.

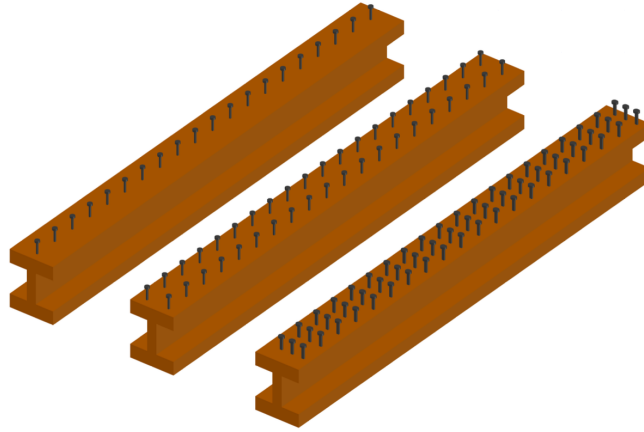


Figure A.13: Illustration of stud arrangements with one, two, and three studs per row along the steel profile.

A.2.5 Longitudinal Shear along the y -Axis

This separate section presents the differences associated with the computation of longitudinal shear along the y -axis. The methodology remains consistent with that described for the z -axis, although the subdivision of the cross-section and the associated geometrical parameters differ.

The subdivision of the section into distinct zones is visualised in Figure A.14.

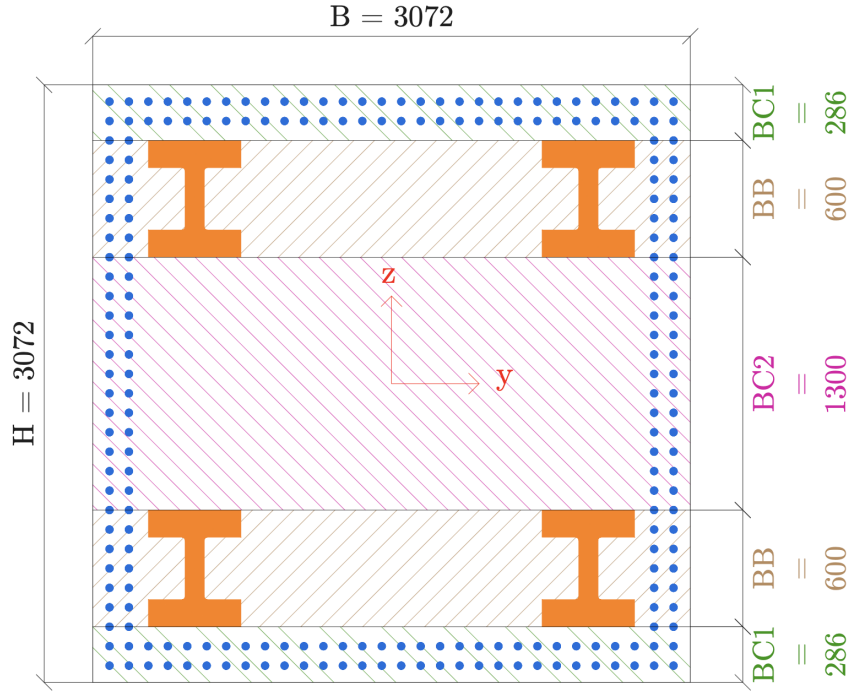


Figure A.14: Division of the cross-section into BC1, BB, and BC2 zones for the evaluation of longitudinal shear along the y -axis.

The second moments of area are computed about the y -axis. For reinforcing bars:

$$I_{s,i} = \frac{\pi R^4}{4} + A_{s,i} y_{s,i}^2 \quad (\text{A.36})$$

For the structural steel profile:

$$I_a = 2 (I_z + A_a y_{a,cg}^2) \quad (\text{A.37})$$

The effective flexural stiffness of each zone is summarised in Table A.3.

For the zone containing the steel profile (zone BB), homogenisation is performed in the same manner as previously described:

$$I_c^* = \frac{E I_{\text{eff, BB}}}{E_{cm}} = \frac{9.51 \times 10^{16}}{36\,773.3} = 2.59 \times 10^{12} \text{ mm}^4 \quad (\text{A.38})$$

The steel profile is transformed into an equivalent concrete section using modular ratios:

$$h^* = h \cdot \frac{E_s}{E_{cm}} = 600 \cdot \frac{210\,000}{36\,773.3} = 3\,426.4 \text{ mm} \quad (\text{A.39})$$

Parameter	Unit	BC3	BS	BC4
E_a	MPa	210 000	210 000	210 000
I_a	mm ⁴	–	3.44×10^{11}	–
E_s	MPa	200 000	200 000	200 000
I_s	mm ⁴	6.73×10^{10}	6.00×10^{10}	1.20×10^{11}
K_e	–	0.6	0.6	0.6
$E_{c,\text{eff}}$	MPa	17 305	17 305	17 305
I_c	mm ⁴	6.24×10^{11}	1.05×10^{12}	3.02×10^{12}
$EI_{\text{eff},i}$	Nmm ²	1.99×10^{16}	9.51×10^{16}	5.54×10^{16}
EI_{eff}	Nmm ²	2.855×10^{17}		
$V_{Ed,y}$	kN	20 000		
$V_{Ed,y,i}$	kN	1 396.5	6 663.5	3 880

Table A.3: Mechanical properties and effective flexural stiffness of the zones considered in the longitudinal shear calculation along the y -axis.

$$t_f^* = 2 \cdot t_f \cdot \frac{E_s}{E_{cm}} = 2 \cdot 140 \cdot \frac{210\,000}{36\,773.3} = 1\,598.9 \text{ mm} \quad (\text{A.40})$$

The resulting homogenised section is illustrated in Figure A.15.

The longitudinal shear is then evaluated at the critical interfaces.

Interface CC1

$$A_{CC1} = b_{BB} \cdot c_{a,y} = 1.72 \times 10^5 \text{ mm}^2 \quad (\text{A.41})$$

$$z_{CC1} = \frac{B}{2} - \frac{c_{a,y}}{2} = \frac{3072}{2} - \frac{286}{2} = 1\,393 \text{ mm} \quad (\text{A.42})$$

$$S_{CC1} = A_{CC1} \cdot z_{CC1} = 1.72 \times 10^5 \cdot 1\,393 = 2.39 \times 10^8 \text{ mm}^3 \quad (\text{A.43})$$

$$v_{L,Ed,CC1} = \frac{6\,663.5 \times 10^3 \cdot 2.39 \times 10^8}{2.59 \times 10^{12}} = 615.9 \text{ kN/m} \quad (\text{A.44})$$

Interface CC2

$$S_{\text{steel}} = A_a^* \cdot y_{cg} = 9.45 \times 10^5 \cdot 1012 = 9.56 \times 10^8 \text{ mm}^3 \quad (\text{A.45})$$

$$S_{c,CC2}^* = A_{c,CC2}^* \cdot y_{cg} = 1.20 \times 10^5 \cdot 1012 = 1.21 \times 10^8 \text{ mm}^3 \quad (\text{A.46})$$

$$S_{CC2} = S_{c,CC2}^* + S_{\text{steel}} + S_{CC1} = 1.21 \times 10^8 + 9.56 \times 10^8 + 2.39 \times 10^8 = 1.32 \times 10^9 \text{ mm}^3 \quad (\text{A.47})$$

$$v_{L,Ed,CC2} = \frac{6\,663.5 \times 10^3 \cdot 1.32 \times 10^9}{2.59 \times 10^{12}} = 3\,392.9 \text{ kN/m} \quad (\text{A.48})$$

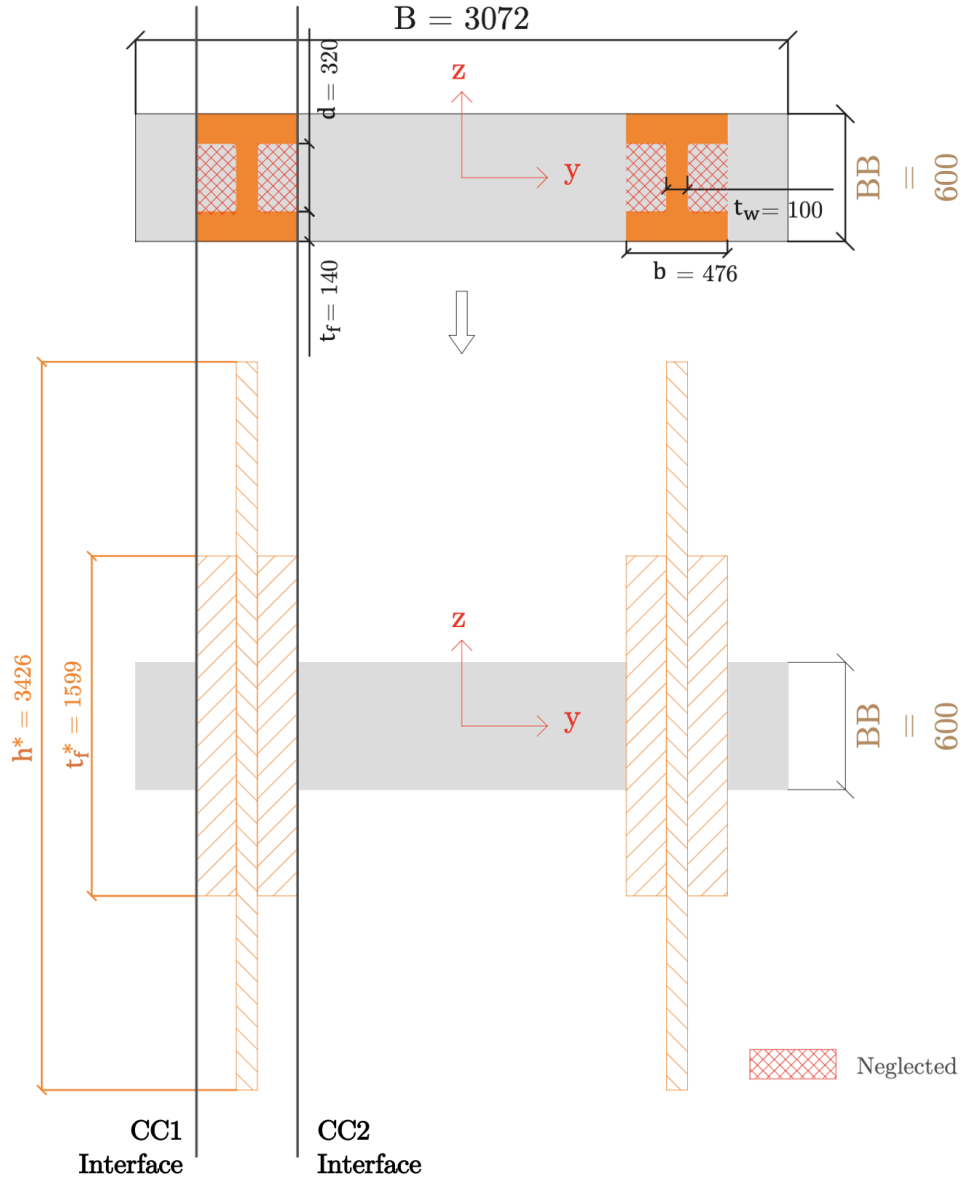


Figure A.15: Transformation of the BB zone into an equivalent homogeneous concrete section for the evaluation of shear flow.

The obtained values correspond to the longitudinal shear flow along the steel–concrete interfaces in the y -direction and are subsequently used for the design of shear connectors. The corresponding number of studs per unit length in the y -direction, assuming the same type of studs as those used for the z -axis, is given by:

$$n_{\text{studs,CC1}} = \frac{615.9 \cdot 1 \text{ m}}{141.4} \approx 5 \text{ studs/m} \Rightarrow 3 \text{ studs/m (for 2 studs per row)} \quad (\text{A.49})$$

$$n_{\text{studs,CC2}} = \frac{3392.9 \cdot 1 \text{ m}}{141.4} \approx 25 \text{ studs/m} \Rightarrow 12 \text{ studs/m (for 2 studs per row)} \quad (\text{A.50})$$

Appendix B

Structure of the *Python Code*

B.1 Code Architecture

For clarity, this appendix presents the structure of the *Python Code* developed to plot the $N-M$ interaction diagram, compute the longitudinal shear, and determine the required number of shear studs.

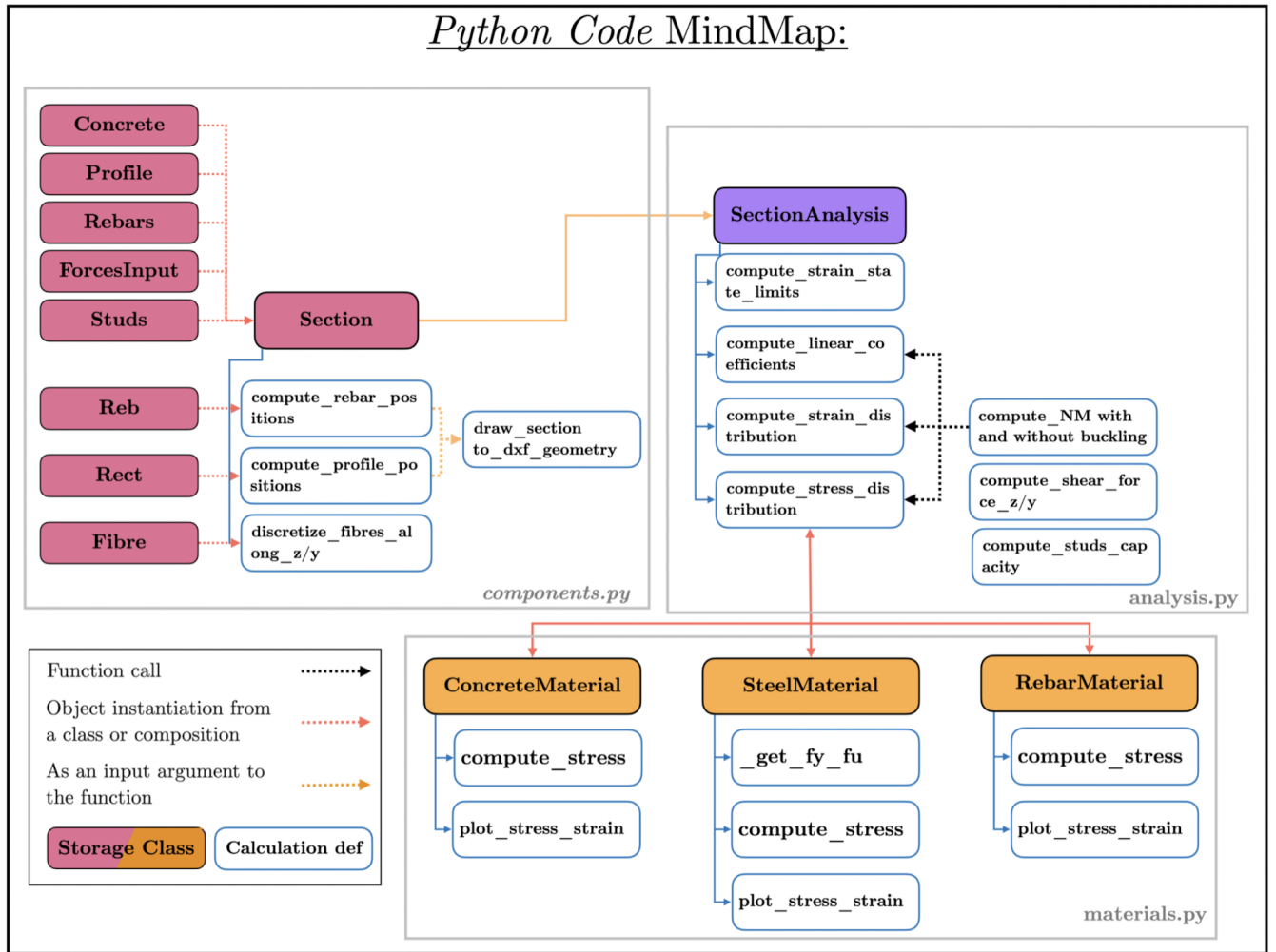


Figure B.1: Schematic overview of the *Python Code* architecture developed for the computation of $N-M$ interaction diagram curves, longitudinal shear forces, and shear stud verification.

The organisation of the code can be summarised as follows:

- Classes are represented as boxes in which the user-defined input parameters are stored.
- Functions (defined using `def`) perform the calculations indicated by their names.
- Arrows represent the interactions and data flow between the different components.
- The `.py` files correspond to the main modules of the implementation.

B.2 Graphical User Interface

To facilitate its use, a Graphical User Interface (GUI) has been implemented. The following figures present the interface and illustrate the different user inputs and corresponding outputs.

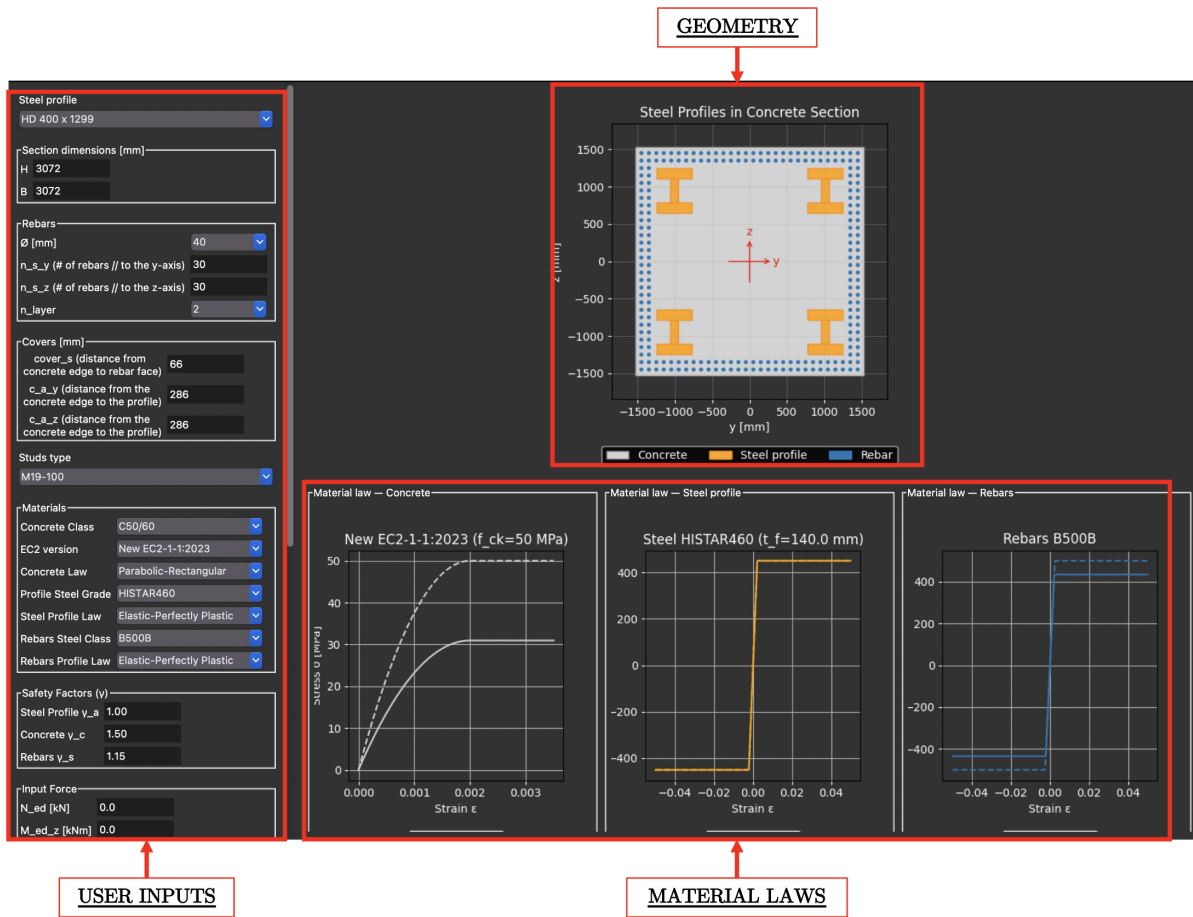


Figure B.2: Graphical user interface showing the general input parameters, cross-section geometry, and constitutive material laws.

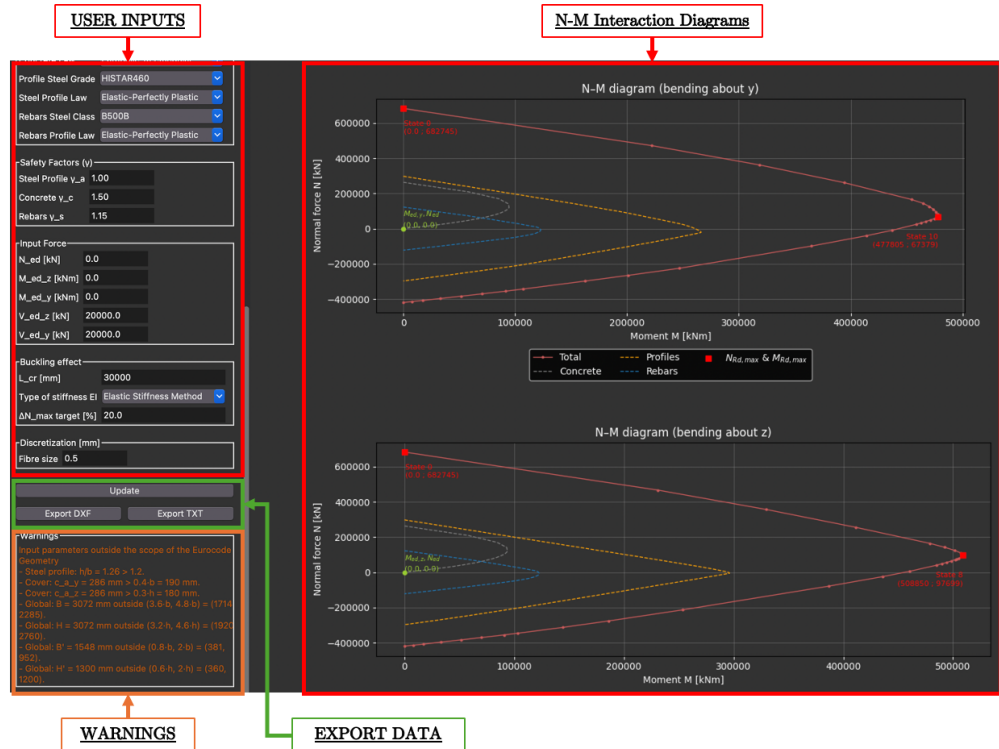


Figure B.3: Graphical user interface showing the advanced analysis settings, export options, automatic warning system, and generated $N-M$ interaction diagram for bending about the y - and z -axes.

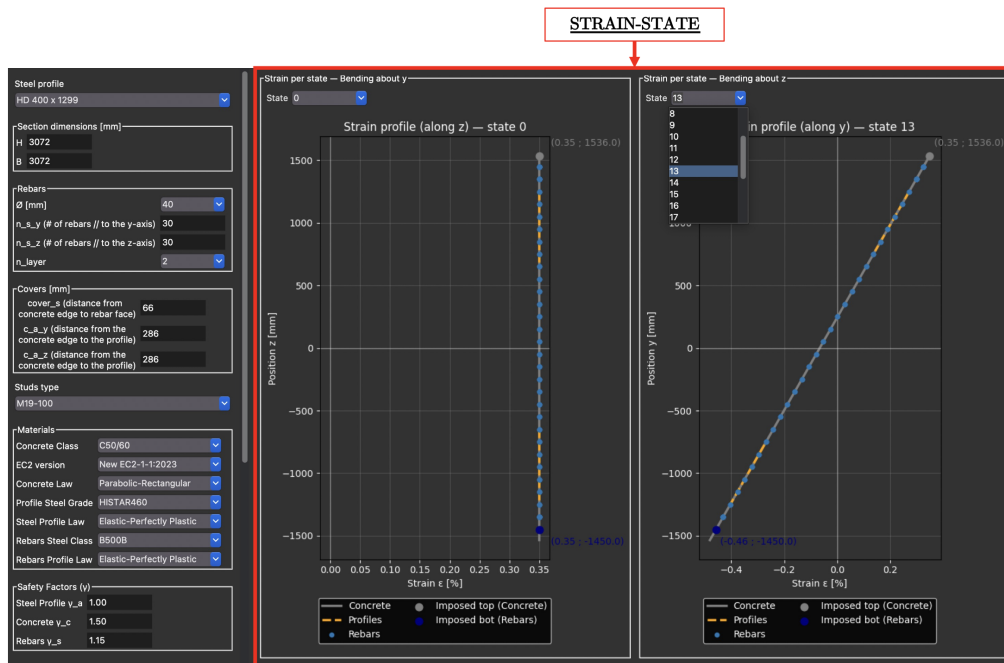


Figure B.4: Graphical user interface showing the strain-state visualisation and selection tool used for the construction of the $N-M$ interaction diagram.

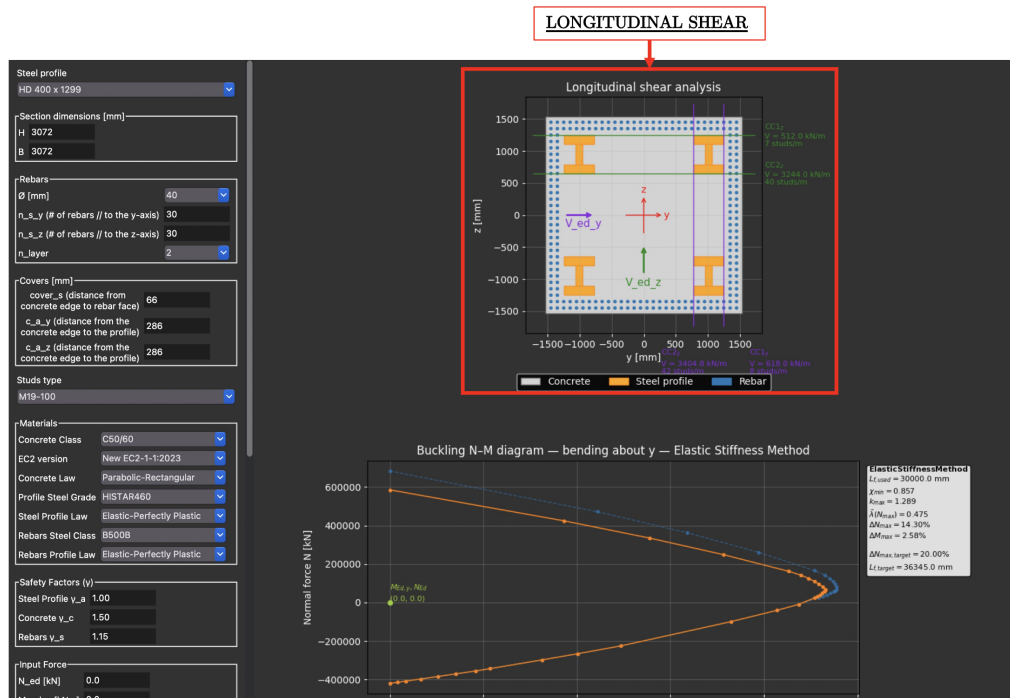


Figure B.5: Graphical user interface showing the longitudinal shear verification procedure and the required number of shear studs at the steel–concrete interfaces.

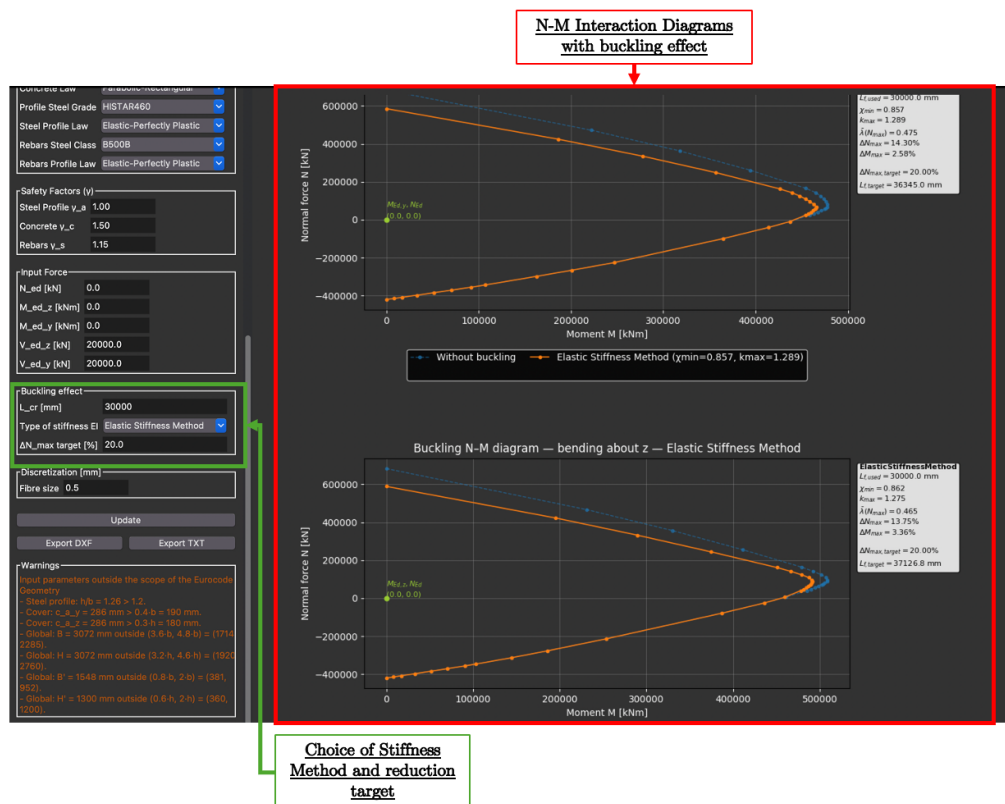


Figure B.6: Graphical user interface showing the buckling input parameters and the resulting buckling-reduced N – M interaction diagram curves.

Warnings and Assumptions in the GUI

To assist the user during the modelling process, the graphical user interface (GUI) includes an automatic warning system. These warnings are displayed dynamically whenever the input parameters fall outside the recommended ranges or assumptions defined in the Eurocode provisions and practical design guidelines (see the bottom-left corner of Figure B.3).

The purpose of these warnings is not to prevent the calculation, but rather to inform the user that the selected configuration may lead to unrealistic geometries, non-standard material combinations, or results outside the validated scope of the implemented formulation.

The warnings are grouped into different categories.

Geometry Warnings

Geometry warnings are triggered when the section dimensions, reinforcement layout, or steel profile proportions do not comply with the recommended limits adopted in the implementation.

Typical checks include:

- steel profile proportions (h/b);
- minimum and maximum profile dimensions;
- excessive concrete cover values;
- insufficient number of reinforcing bars;
- reinforcement congestion;
- global section proportions;
- spacing between embedded steel profiles.

Material Law Warnings

Additional warnings are associated with the constitutive material laws. Certain combinations of concrete classes and stress–strain relationships may not be recommended within specific versions of Eurocode 2.

For example, the GUI warns the user when the *Parabolic-Rectangular* concrete law is combined with concrete strength classes outside the preferred range of applicability in EN 1992-1-1:2023.

Computation Warnings

The software also verifies the compatibility between the selected steel profile and the chosen steel grade using the *ArcelorMittal* catalogue database. If a non-standard combination is detected, a computation warning is displayed.

Appendix C

Validation of the N – M Interaction Diagram for Bending about the z -Axis

This appendix presents the results of the OFAT analysis for bending about the z -axis. The corresponding results for bending about the y -axis are discussed in Chapter 5.

Overall, the same trends and conclusions as those observed for the y -axis remain valid. The agreement between the three tools is similarly strong, and the general behaviour of the N – M interaction diagrams is consistent.

A notable difference concerns the dispersion of N_{corr} . While, for bending about the y -axis, this variation was mainly associated with changes in the section height H , it is here primarily related to variations in the section width B . A similar observation can be made for the influence of the profile cover, which becomes more significant when $c_{a,y} = 500$ (test 8_1). The N – M interaction diagrams are also presented below. Despite the local differences discussed previously, the overall structural response remains very similar across the three tools.

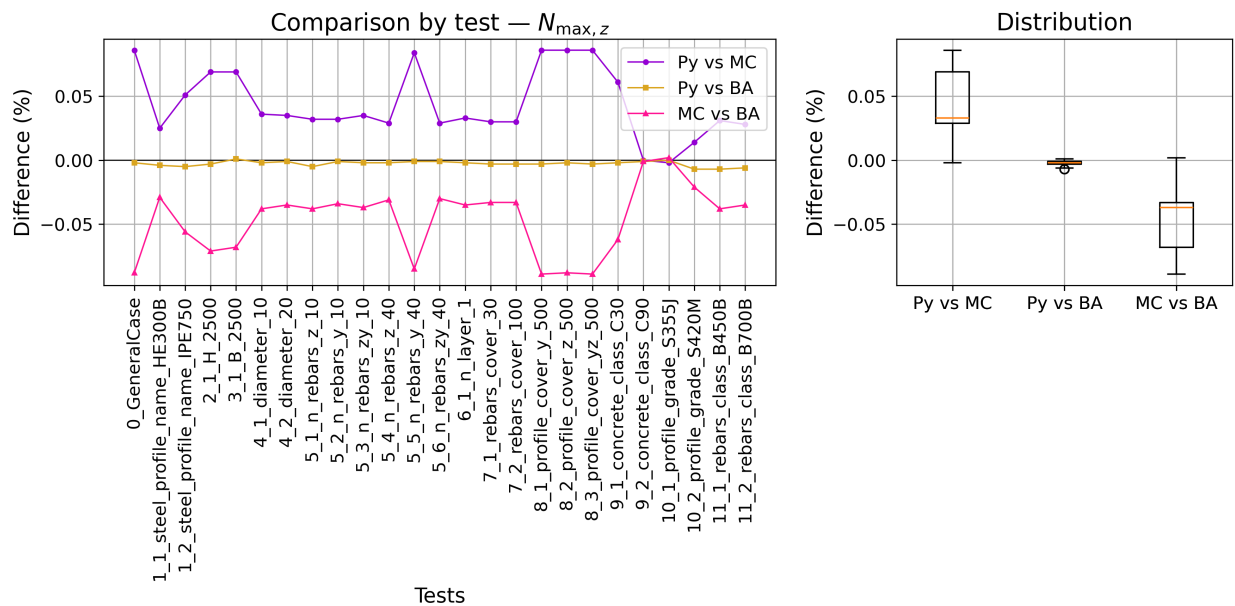


Figure C.1: Relative differences in maximum axial force N_{max} for bending about the z -axis.

C. Validation of the N – M Interaction Diagram for Bending about the z -Axis

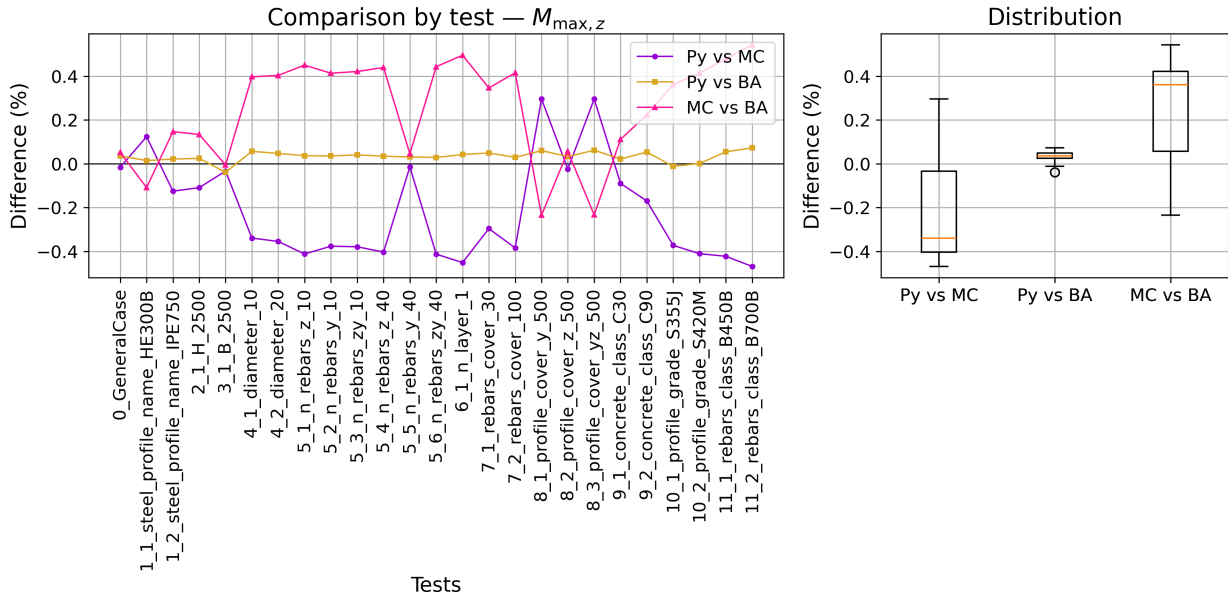


Figure C.2: Relative differences in maximum bending moment $M_{\max}$ for bending about the z -axis.

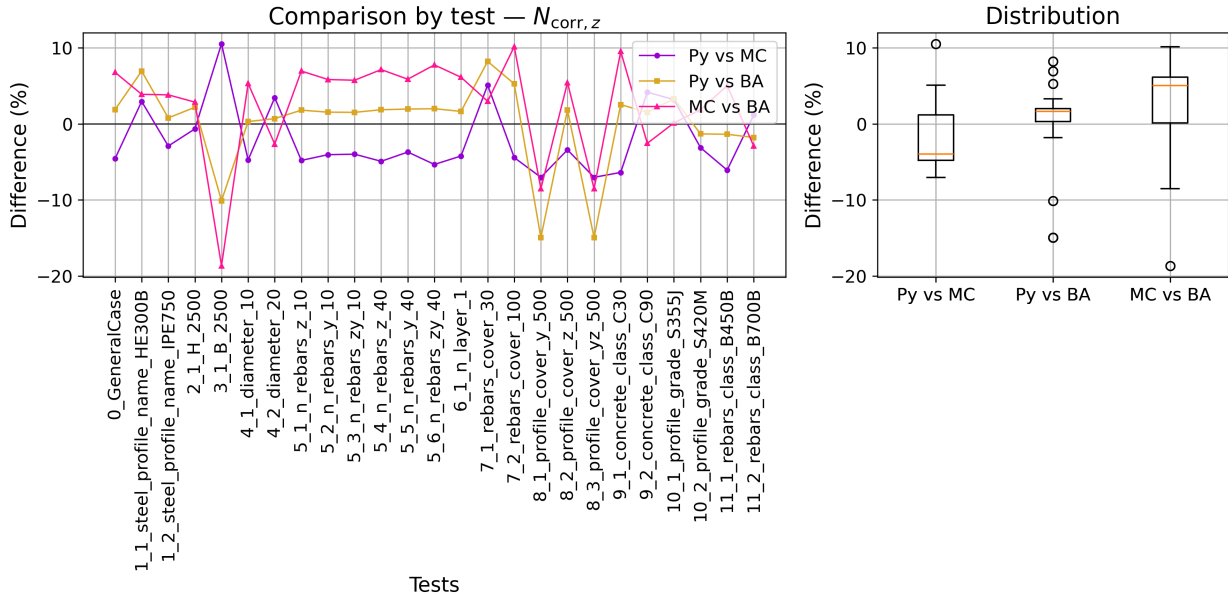


Figure C.3: Relative differences in corresponding axial force N_{corr} for bending about the z -axis.

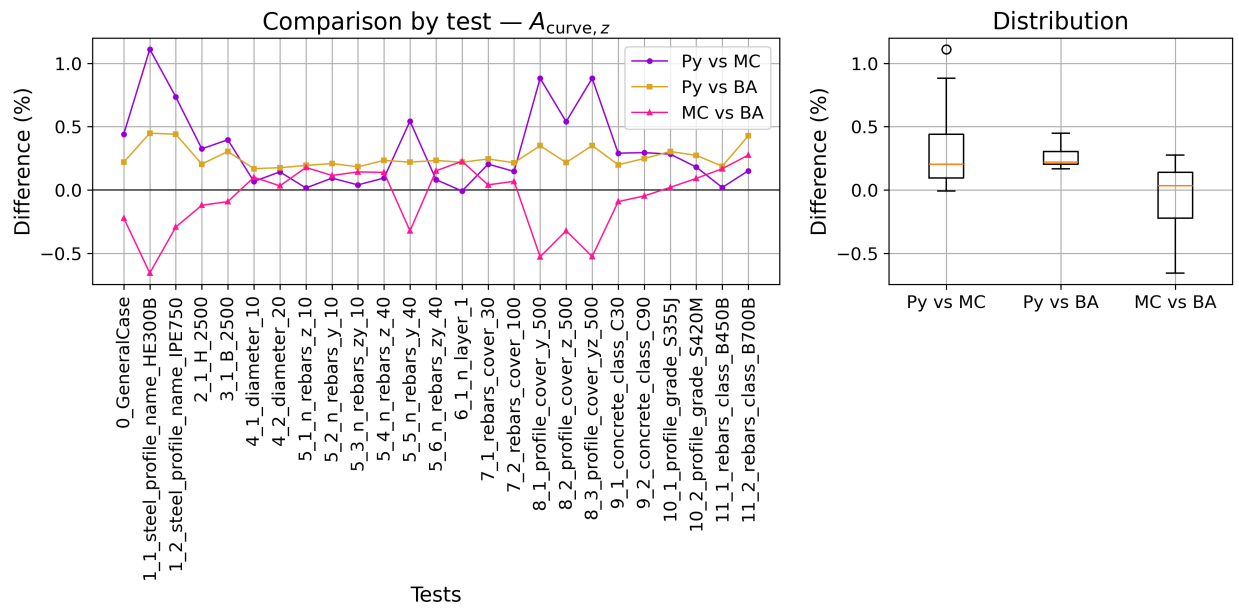


Figure C.4: Relative differences in interaction curve area A_{curve} for bending about the z -axis.

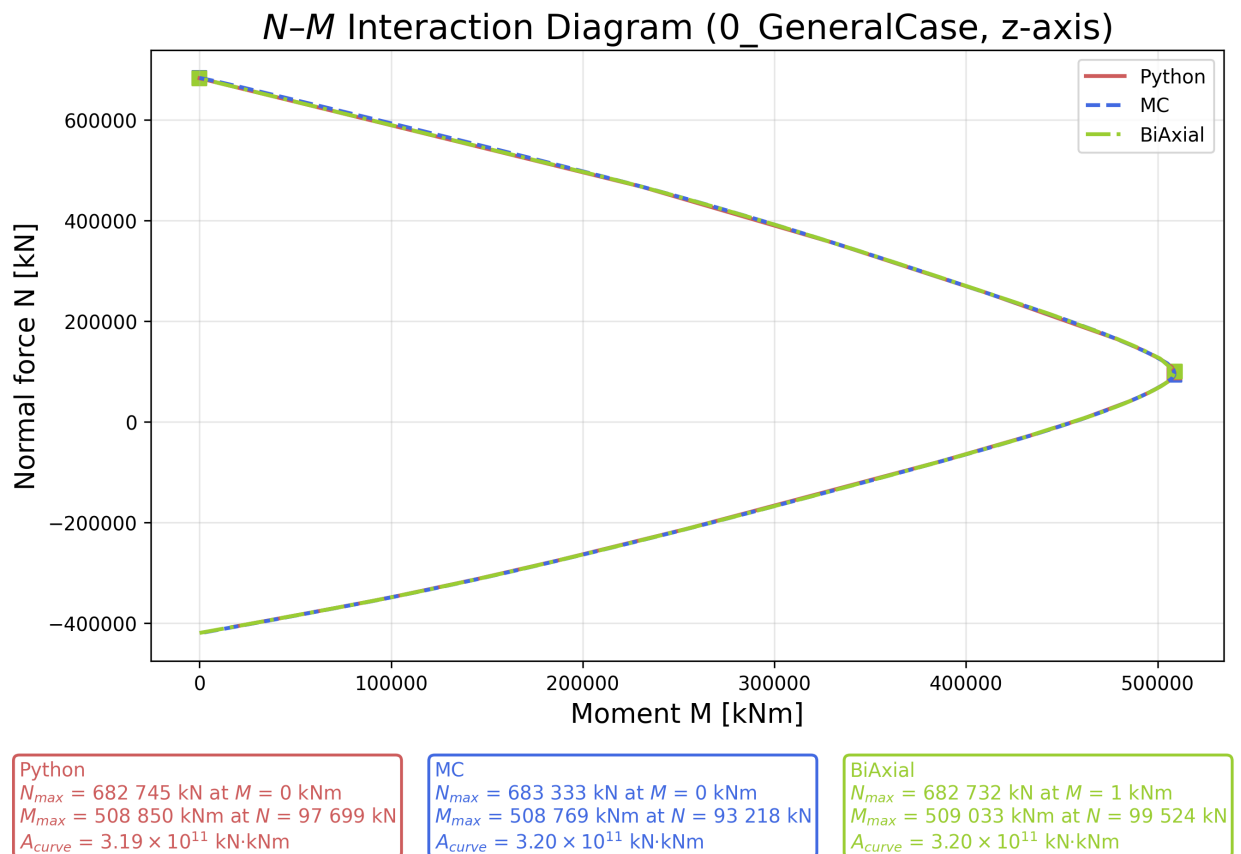


Figure C.5: Comparison of the N – M interaction diagram obtained with *Python Code*, *MegaColumn*, and *BiAxialPro* for the reference configuration under bending about the z -axis.

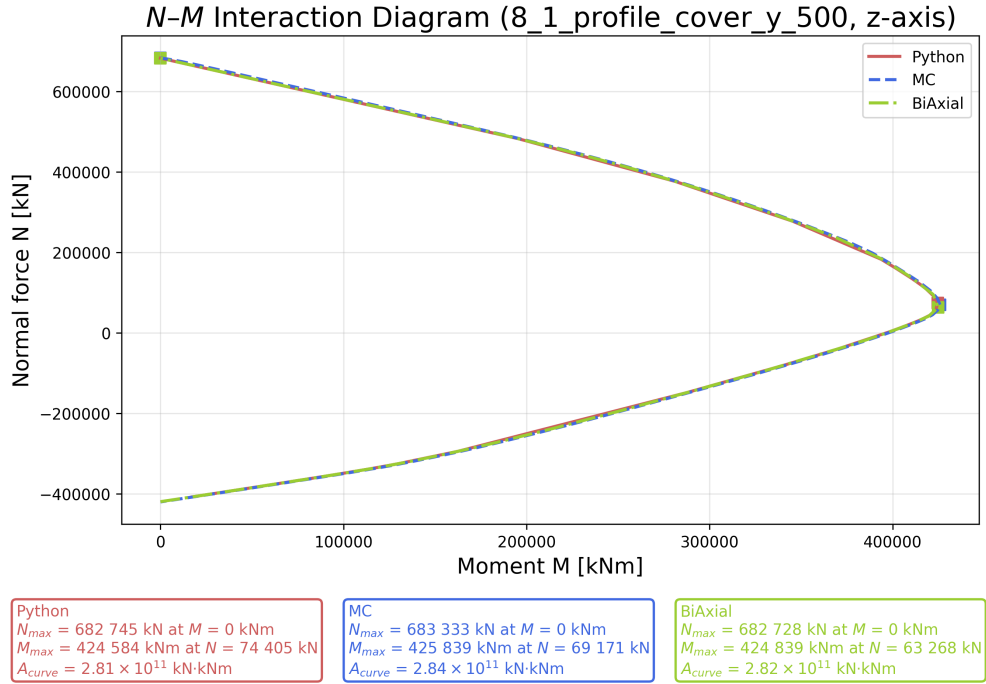


Figure C.6: Comparison of the N – M interaction diagram obtained with *Python Code*, *MegaColumn*, and *BiAxialPro* for test case 8_1 under bending about the z -axis.

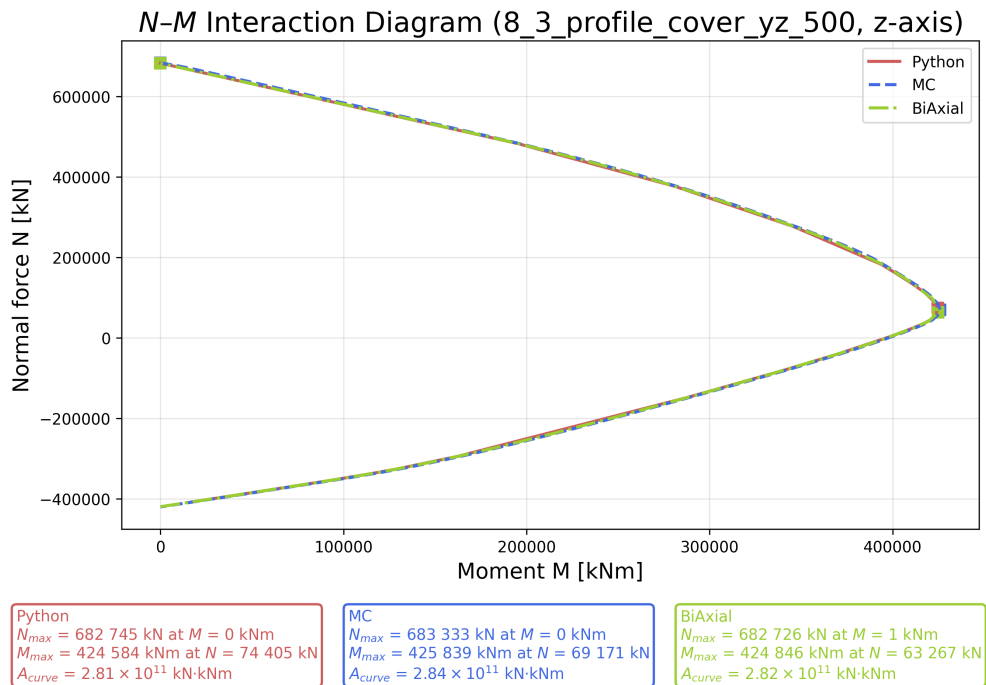


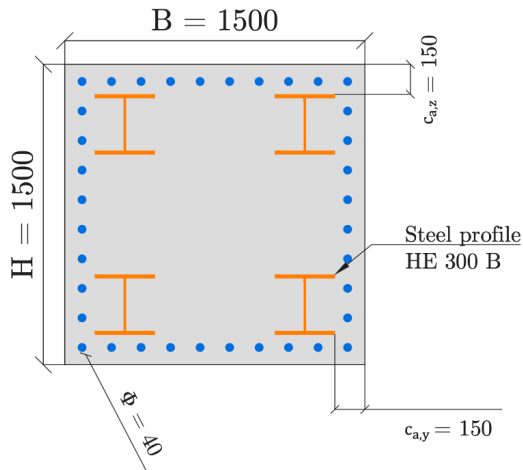
Figure C.7: Comparison of the N – M interaction diagram obtained with *Python Code*, *MegaColumn*, and *BiAxialPro* for test case 8_3 under bending about the z -axis.

Appendix D

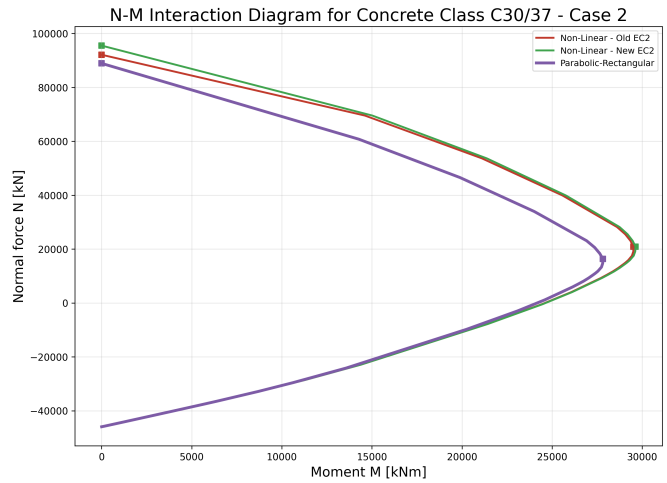
Influence of Material Modelling – Concrete for Alternative Geometries

This appendix presents additional results obtained for alternative cross-section geometries in order to verify whether the conclusions drawn in the main study remain valid. Although small numerical differences can be observed depending on the geometry and the concrete strength class, the overall structural behaviour and the general trends remain consistent with the conclusions presented in Section 6.3.

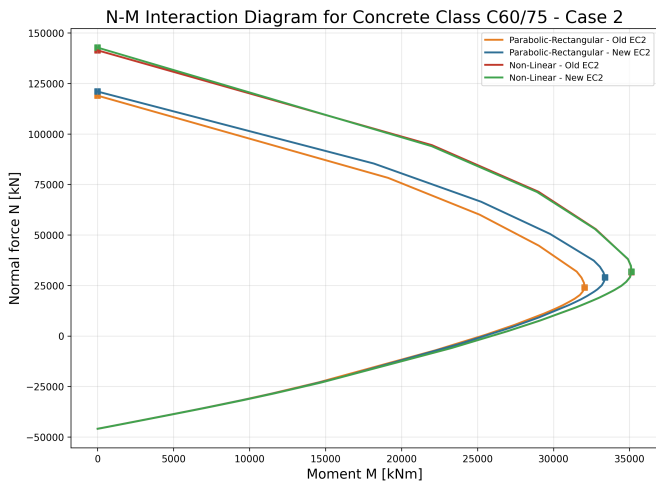
Figures D.1 and D.2 present the corresponding N – M interaction diagrams for Cases 2 and 3, respectively, considering different concrete strength classes.



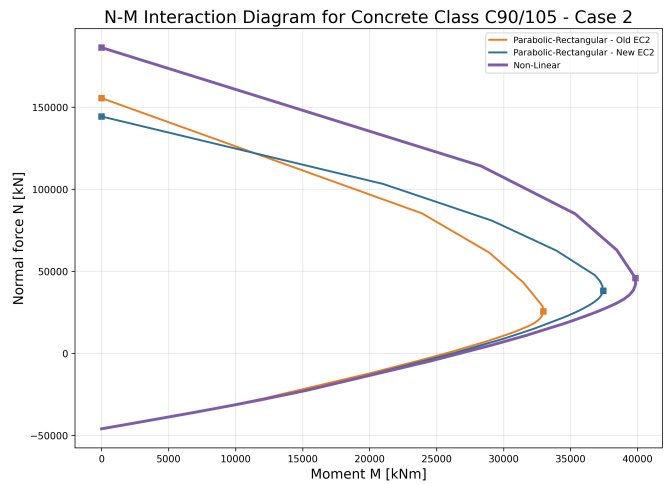
a) Case 2 geometry



b) N - M interaction diagram for concrete class C30/37

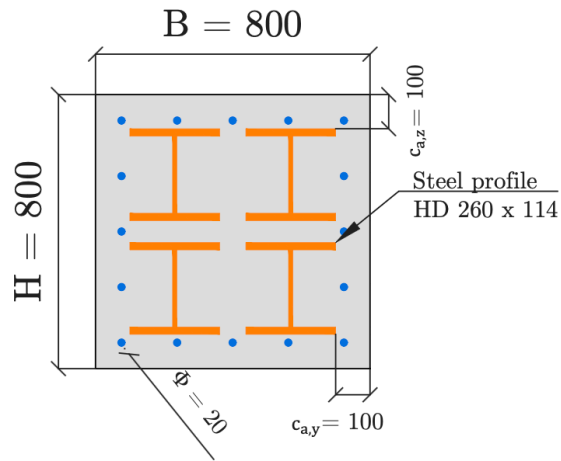


c) N - M interaction diagram for concrete class C60/75

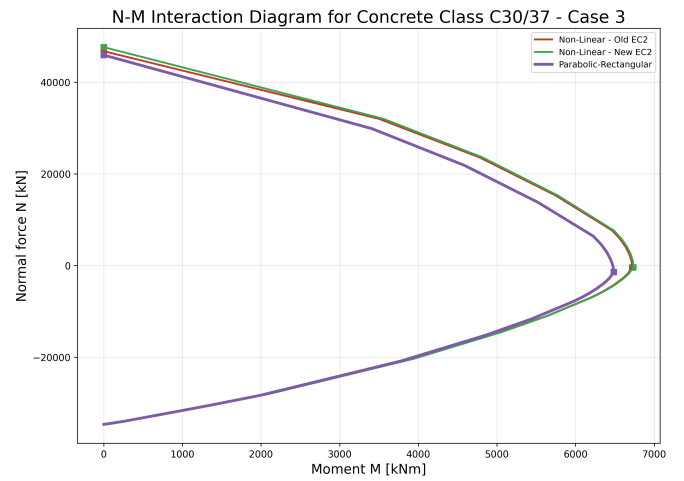


d) N - M interaction diagram for concrete class C90/105

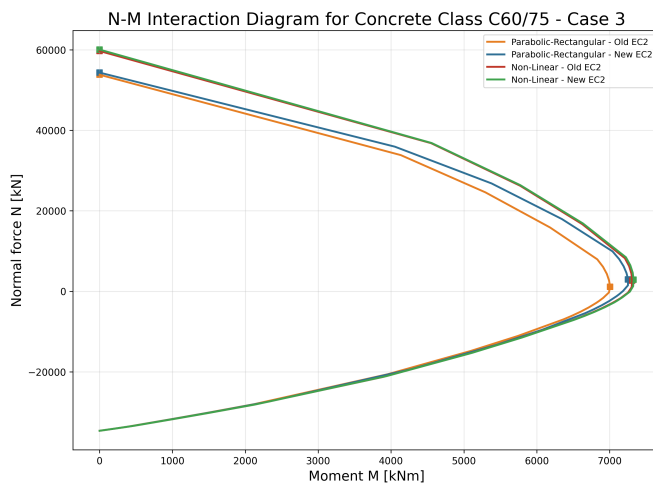
Figure D.1: Comparison of N - M interaction diagram results obtained for the Case 2 geometry considering different concrete strength classes and constitutive laws.



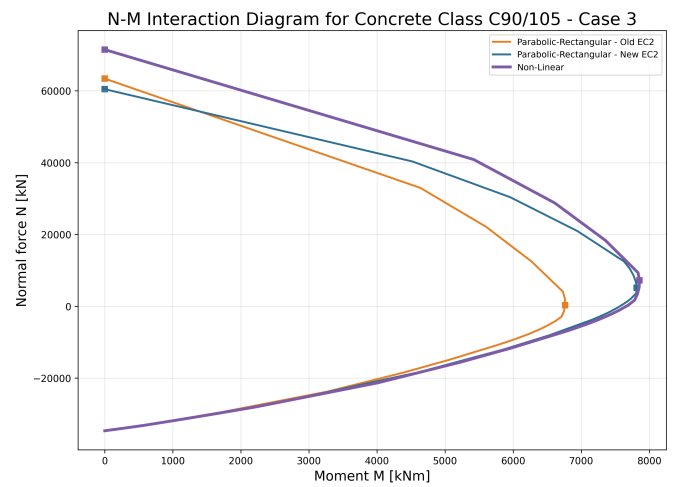
a) Case 3 geometry



b) N - M interaction diagram for concrete class C30/37



c) N - M interaction diagram for concrete class C60/75



d) N - M interaction diagram for concrete class C90/105

Figure D.2: Comparison of N - M interaction diagram results obtained for the Case 3 geometry considering different concrete strength classes and constitutive laws.

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