Louvain School of Management

Impact of behavioral biases on portfolio performance

Do biased investors unconsciously outperform theoretically better portfolios?

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“Wouldn't economics make a lot more sense if it were based on how people actually behave, instead of how they should behave?”

— Dan Ariely

« Traditional finance assumes that we are rational, while behavioral finance simply assumes we are normal. »

– Meir Statman

I would like to express my gratitude, first and foremost, to Mr. Vrins, Professor at the *Louvain School of Management* in the *Finance department* and supervisor of this thesis. The quality of our meetings, his guidance, and his teaching have allowed me to set objectives and progress step by step throughout the realization of this project.

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How could I forget my friends, with whom I experienced all the moments that a university cursus can offer.

And finally, to you, dear readers, for taking the time to read this work. I hope it will enlighten you.

Abstract

The modern portfolio theory developed by Harry Markowitz serves as a reference when discussing finance. However, over the past decades, behavioral finance has emerged because classical theory failed to explain all the phenomena occurring in financial markets.

By studying investors' behavior and their decision-making under uncertainty, behavioral finance aims to describe and reflect the world as it actually unfolds, disregarding simplifying assumptions. The purpose of this paper is to compare the investments made by market participants with their natural behavioral biases to those considered theoretically optimal according to modern portfolio theory.

To achieve this, optimal portfolios based on the theory were selected from a historical dataset of asset returns from the EuroStoxx 50. These portfolios were then compared to their alternatives, considered optimal in the eyes of investors. This difference arises because human behavior perceives things differently than they actually are, leading to unconscious modifications of data.

The results revealed that, in general, the future performance of behavioral portfolios was superior to their theoretically optimal counterparts. This can be partly explained by the fact that markets react on a large scale according to these behavioral patterns. Consequently, investors' choices are unconsciously better than the theory's predictions, as it is the aggregation of all these choices that defines the market outcomes.

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INTRODUCTION

In 1952, Harry Markowitz developed the modern portfolio theory, which aims to explain how rational investors optimize their portfolios through diversification and also demonstrates how to determine the price of an asset based on its risk relative to the market's average risk.

This theory, due to its practicality, has widely spread and become one of the main pillars of modern finance, still utilized by institutional investors today. More generally, standard finance also relies on the well-known concept of expected utility developed by Von Neumann and Morgenstern in 1944. Nevertheless, despite its easy understanding and generalization, this theory is built on quite strong assumptions.

In addition to assuming perfect markets (Hiro, Sofat, 2011), it also assumes a Gaussian distribution of asset returns and rational behavior of investors.

In this context, behavioral finance emerged in the 1970s, thanks in part to the works of Kahneman and Tversky. They drew on earlier work that identified financial anomalies that could not be explained by classical finance, such as the Allais paradox (1953 In order to find explanations for what at the time was still inexplicable, behavioral finance aims to go into more detail and to represent the world as it really is, without relying on simplifying assumptions.

Through the literature review, it becomes apparent that these assumptions of modern finance are not consistent with what is actually observed in financial markets. It is from this point that behavioral finance attempts to offer solutions and explanations for the observed phenomena. However, replacing simplifying assumptions with biased human behavior inevitably results in more complex situations to model and generalize.

The most concrete example is replacing the assumed rational behavior of investors by Markowitz with equations that describe how these same investors perceive outcomes or probabilities, as developed by Kahneman and Tversky in 1973 and 1992, respectively.

The spectrum of behavioral finance is quite vast, and this paper will primarily focus on the works of the pioneers in this branch of finance, namely Kahneman and Tversky. The former was even awarded the Nobel Prize in Economics in 2002 for his research in this field.

The objective of this paper is to compare optimal portfolios according to standard finance, specifically the mean-variance model supported by Markowitz, with portfolios that investors perceive as optimal, through modifications of reality modeled by Kahneman and Tversky in their cumulative prospect theory (1992).

These modifications of reality are, in fact, cognitive biases to which humans, even the most rational investors, are subject. Of course, other biases also exist and are not considered in this study, and we will mention them in the discussion of the results.

To achieve the research objective, the empirical part is divided into four steps. Firstly, 50,000 portfolios were randomly generated from the assets of EuroStoxx 50. Next, over a period of four years, the portfolios with the best performance according to the mean-variance model were selected, i.e., those with the highest Expected return-to-risk ratio, also known as the Sharpe ratio. After this, data was modified to reflect how a human perceives it, with transformations of outcomes and probabilities of these outcomes. From this, the previously selected portfolios were no longer perceived as optimal by the investor. Therefore, the objective was to find a behavioral alternative while maintaining the same perceived level of risk. The final step involved comparing the performances of these two families of portfolios out-of-sample. *Does the perception, the instinct, of investors surpass the theory?*

This paper is structured as follows:

Part I will review the essential concepts and limitations of the modern portfolio theory and its mean-variance framework.

Part II will provide an overview of the literature concerning the major trends in behavioral finance. This section is not exhaustive, but over the years it has become clear that some research has played a major role in the development of this field of finance, and often serves as the basis for new, more in-depth studies.

Part III will describe in more detail the empirical method applied in this work, as well as the database used.

Part IV will present the quantitative results obtained and offer an analysis of the observed trends while attempting to provide explanations.

Part V will conclude this paper and its results, explain the limitations of the work, and offer some suggestions for future research.

PART I: MODERN PORTFOLIO THEORY

The Modern Portfolio Theory (MPT, hereafter) was born in 1952 when Harry Markowitz developed his financial theory based on the mean-variance framework. In his work, investment in a portfolio is defined by its expected return and its risk, measured by the variance of its returns. This results in a key element of finance, namely diversification. This concept is based on the understanding that the portfolio should be viewed as a whole and not just as a sum of independent components. Indeed, there are covariances between assets, which means that risk can be reduced without impacting the expected return.

Let's briefly explain Markowitz's theory in practice, as it is the starting point for this research, which aims to make it more accurate to the realities of markets and investors.

1. Mean-Variance framework

As stated above, Markowitz's modern portfolio theory is based on the mean-variance framework, which aims to simplify and provide a guideline for random returns while structuring the construction of an optimal portfolio of securities. For this purpose, returns are considered to follow a Gaussian curve, which can be characterized by its first two moments, the mean and standard deviation.

Consequently, we obtain the following equation, wherein the expected return of the portfolio (E) is the weighted sum of the expected returns (μ_i) of the N securities that constitute it:

$$\boldsymbol{E = \sum_{i=1}^{N} X_i \mu_i}$$

With X_i being the weight of the i-th security in the portfolio, and $\sum X_i = 1$.

However, the variance of this portfolio is not simply the sum of the variances of each security. The mathematical equation of the variance of a sum implies considering a covariance factor of the returns of each security, as shown in this equation:

$$\boldsymbol{V = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i \cdot X_j \cdot \sigma_{ij}}$$

The covariance factor is represented by $\sigma_{ij} = \rho_{ij} \cdot \sigma_i \cdot \sigma_j$. Where ρ_{ij} gives the correlation between returns *i* and *j* and where σ_i and σ_j are their standard deviation.

As observed here, the variance of the portfolio depends, among other factors, on the correlation factor ρ_{ij}, which takes values between -1 and 1. Consequently, the volatility of a portfolio can decrease through accurate diversification, without affecting the expected returns (Markowitz, 1952). This risk reduction achieved through diversification is known as idiosyncratic risk, in contrast to market risk that cannot be eliminated through diversification.

That is why Markowitz (1959) advises rational agents to consider their portfolio as a whole rather than merely the sum of its individual components, recommending that they invest in different industries with diverse economic characteristics.

The portfolio efficiency frontier is defined on a graph combining a portfolio's risk, measured by its standard deviation of returns, and its expected return. Efficiency is reached when it is impossible to obtain a portfolio with a better expected return without increasing its standard deviation.

Once all these portfolios are mapped out, the investor chooses the one to which they are willing to allocate a certain level of risk, i.e., a certain level of volatility in the returns the portfolio will provide them.

2. Modern Portfolio Theory main assumptions

To better understand the rest of this study, it is important to recall and explain the three major assumptions on which MPT is based. It is from these that behavioral finance will emerge, in order to get rid of these assumptions and describe the world more accurately.

2.1. Efficient markets

The efficient market hypothesis posits that markets perfectly reflect all available information, accessible to all market participants. Therefore, for securities, it is assumed that their prices are justified by all known information, and there is no better estimation. This postulate asserts that it is impossible for an investor to consistently beat the market (Alphonse et al., 2013). However, this theory has been refuted on numerous occasions in the literature, revealing real anomalies that would not be expected to occur in perfectly efficient markets.

For instance, the finding that returns tend to move in a similar manner during a subsequent period after the announcement of financial results (Ball & Brown, 1968). Additionally, evidence of speculative bubbles and significantly higher volatilities than what would have been observed (Lobao, 2015) have challenged the efficient market hypothesis.

2.2. Investors rationality

As stated by Markowitz (1959), an efficient portfolio is the result of choices made by an investor with a rational risk profile. This means that agents make decisions by considering only two aspects: maximizing expected return while minimizing their acceptable level of risk. Rationality is defined by three properties. Firstly, agents clearly establish their preferences. Secondly, their goal is to maximize their utility without other considerations. Finally, they are capable of updating their expectations with new information (Alphonse et al., 2013).

Their preferences and utilities are defined according to the theory of expected utility, developed by Von Neummann and Morgenstein (1944). This theory is the foundation of rational behavior and decision-making under uncertainty for investors. Later in this work, we will see that a modification of the expected utility theory provides more accuracy regarding investors' real behavior and allows for more precise models.

Numerous studies have demonstrated, through experiments, that people do not always act rationally. For instance, we can mention the Allais paradox (1953), or the experiments conducted by Kahneman and Tversky (1979), which led to the development of prospect theory, which will be discussed further in this paper.

2.3. Normally distributed returns

This hypothesis holds that returns are symmetrical and entirely characterized by their first two moments, i.e. their mean and variance (Mills, 1956). This idea has been disproved in several works, notably by Mantegna and Stanley (1995). Mandelbrot (1963) also demonstrated that historical observations do not follow a normal distribution. He added that predictions based on the assumption of normality are inaccurate in relation to the significant price changes that take place.

PART II: BEHAVIORAL PORTFOLIO THEORY

This theory aims to have a portfolio theory that better corresponds to people's actual behavior. This has developed iteratively, each time bringing improvements to our understanding of investor behavior, whether in the way probabilities are actually perceived, or in the utility people attach to gains or losses.

1. Prospect Theory

In 1979, two economists specialized in behavioral analysis, Daniel Kahneman and Amos Tversky, developed prospect theory. This theory lays the foundation for behavioral finance and aims to demonstrate several behavioral biases associated with humans when making decisions under uncertainty, which contradicts the assumption of rational choice advocated by Markowitz.

Prospect theory seeks to describe the actual behavior of individuals, including investors, when faced with gains or losses. Its objective is to be more descriptive of reality. It shows that the expected utility theory, which assumes that investors' utility is symmetric for both gains and losses, is actually erroneous.

After observations on people's behavior and choices between certain lotteries, Kahneman and Tversky concluded that expected utility is not an adequate descriptive model and proposed an alternative model of decision-making under uncertainty. The two main revolutions of this model are the consideration of the value associated with gains or losses rather than the final outcome and the replacement of probabilities with decision weights. In the expected utility theory, utility of outcomes is calculated via their probabilities. However, through their experiments, they realized that people assigned greater utility to certain choices contrary to what the theory would have predicted. They, therefore, concluded that individuals did not perceive choices as they were theoretically assumed to.

They first demonstrate certain generalized behaviors that could have been considered irrational under the old model.

Firstly, individuals tend to overweight outcomes that are considered certain compared to those that are unlikely. This is called the certainty effect and represents a violation of the substitution axiom proposed in the expected utility theory. It is, in fact, the best-known violation of expected utility. It was initially discovered and presented by Allais in 1953 in his paradox.

A second behavior highlighted is the reflection effect, which is related to the previous one but also takes losses into account. The overweighting of a certain amount with certainty in a lottery with a higher expected value leads to risk aversion in the case of gains. However, the opposite is generally observed in the case of losses, meaning a preference for risk in a lottery with a worse expected value than the assured loss from it. This risk preference in case of losses had already been noted by Markowitz (1952).

The choice of individuals between different situations consists of two phases: a first phase of editing and a second phase of evaluation. The first phase is a preliminary analysis that the brain performs to simplify the choices presented. The second phase involves evaluating each solution and choosing the one with the highest assigned value.

This editing phase already truncates the actual choices offered and leads to initial preference anomalies. The context in which a choice appears already defines how it will be edited (framing effect). This editing phase was developed by Kahneman (2011) and referred to as "System 1 thinking." It involves mental shortcuts that aid humans in decision-making. These shortcuts have been part of our genes since ancient times and allow us to be more efficient and quick, helping us avoid cluttering the brain with unnecessary details (Howard, 2014). This process is very useful in everyday life but leads to choice inconsistencies when it comes to investment decisions.

The evaluation phase involves assigning a value to each situation. Here, the value of a situation is expressed using a decision weight (π) associated with a probability (p) and a subjective value (V) of the outcome relative to a reference point, thus the gain or the loss.

The value (V) assigned to a situation (x, p ; y, q) where x and y are outcomes, and p and q are probabilities is :

$$\boldsymbol{V(\mathrm{x}, \mathrm{p} \,;\, \mathrm{y}, \mathrm{q}) = \pi(\mathrm{p}) \,.\, v(\mathrm{x}) + \pi(\mathrm{q}) \,.\, v(\mathrm{y})}$$

A very important feature of this theory is that an outcome is considered in terms of its variation from a status quo situation, rather than a final state. This observation is totally consistent with what we experience in everyday life with tangible objects. An object with a given temperature, for example, may appear hot or cold, depending on the overall context. Stimuli are therefore perceived in relation to a reference point (Helson, 1964). So, rather than a final amount, the outcome is judged on the difference it makes compared to a situation where no investment would have been made.

Furthermore, as it is easier to detect a temperature change between 0°C and 30°C than between 100°C and 130°C, we can assume that the psychological response function to a change is concave with respect to its magnitude when it is positive and convex when the change takes negative values.

The value (*v*) assigned to an outcome is, therefore, a function of two arguments: the reference point that serves as a reference and the magnitude of the change, whether positive or negative.

Other special circumstances can alter investors' preferences, such as the need for a specific sum of money or a total aversion to a certain amount. Therefore, the value function, the utility of outcomes, is not always a generic attitude shared by all investors but can be influenced by individual personal situations.

A final characteristic of the value function is that, for the same magnitude of positive or negative change, the loss in assigned value is greater in the negative case than the gain of value in the positive case. This is referred to as loss aversion (Galanter & Pliner, 1974). This leads to a steeper function for losses than for gains, as evidenced by the following graph.

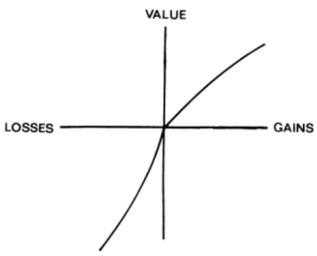

Figure 1*: Hypothetical value function*
(Prospect theory, p.279)

Its equation is as follows, where *k* is the reference point that separates losses and gains:

$$\mathbf{V}(\boldsymbol{y}) = \begin{Bmatrix} (\boldsymbol{y} - \boldsymbol{k})^{\mathbf{0.88}} & \boldsymbol{if\ y \geq k} \\ -\mathbf{2.25}\big(-(\boldsymbol{y} - \boldsymbol{k})\big)^{\mathbf{0.88}} & \boldsymbol{if\ y < k} \end{Bmatrix}$$

Like the outcomes value function V(*y*), the decision weight function (π) is the second major modification designed to reflect inversion behavior. The decision weight is not a probability as such, but rather the interpretation that people generally make of a probability.

First, it has 2 certain intersections with probabilities: $\pi(0) = 0$ et $\pi(1) = 1$.

Next, for very small probabilities p, $\pi(p) > p$. This demonstrates that low probabilities are overweighted. This overweighting is only a property of decision weights and does not account for the overestimation of probabilities of rare events. However, in many real-life situations, both overweighting and overestimation can play a role in increasing the impacts of rare events.

Another property of this decision weight function is that $\pi(p) + \pi(1-p)$ is less than unity, for any p. This property is called subcertainty and is demonstrated in some data by MacCrimmon and Larsson (1979). The slope of the decision weight function is a measure of the sensitivity of preferences to changes in probabilities. Preferences are generally less sensitive to variations in probabilities than suggested by the principle of expected utility.

An important limitation of this theory is that it can only calculate v and π on the basis of a comparison between situations. A mathematical generalization of these two variables is realized in the theory of cumulative perspectives, developed a decade later by the same 2 authors.

The reference point is an important element in this theory. Depending on this point, it determines whether the investor will be risk-seeking or risk-averse. For example, consider an individual who has already lost 2000 and is faced with a choice where either they gain 1000 with certainty or receive 2000 with a 50% chance. Their likely loss aversion from recent losses will make them perceive this choice as (-2000, 0.5) and (-1000) rather than (2000, 0.5) and (1000). Therefore, the value curve showing risk aversion in the case of gains would lead a rational individual to choose (1000). However, as this individual perceives the choice as negative values relative to their reference point, according to the same curve, they will have a risk preference and choose (2000, 0.50).

All the inefficiencies in choices described in the prospect theory are normally corrected by the decision-maker when they realize that their preferences are biased. Unfortunately, they often do not have the opportunity to notice that their choices are biased, and thus the anomalies identified by the prospect theory are expected to occur.

2. SP/A Theory

In 1952, Roy developed another measure of risk that does not rely on variance: the safety-first approach. Here, risk is measured by the probability of ruin, which is the probability that the total wealth of the investor falls below a certain level. This approach also assumes the normality of distributed returns but was developed to address the limitations of expected utility (Von Neumann & Morgenstern, 1947). Expected utility assumes a uniform behavior of investors regarding risk, with a concave utility function for all assets. However, Friedman and Savage (1948) demonstrated that investors can simultaneously buy insurance policies and lottery tickets. This shows that investors do not have a uniform behavior regarding risk (Shefrin & Statman, 2000). They, therefore, develop a concave utility function for insurance policies and a convex utility function for lottery tickets to illustrate this difference in attitude towards risk, which is consistent with Roy's safety-first theory. This utility function will be further refined in prospect theory, as seen earlier, and in cumulative prospect theory, as will be discussed in a following section.

After that, Telser (1955) developed this safety-first theory, stressing that risk is measured in terms of an acceptance that the event of ruin will occur, below a certain probability. This is represented by the equation :

$$\boldsymbol{E(R)\ u.c.\ p(W < s) < \alpha}$$

Where R is the portfolio return, s is the subsistence level, α is the acceptable probability of ruin, and W is the final wealth distribution.

This constraint is analogous to Value at Risk (VaR), a risk measure that calculates the possible loss in value of a security for a given confidence interval (Damodaran, 2011). This measure of risk via VaR appears to be more behaviorally appropriate, as it is logically assumed that investors are better at exposing their break-even point with the associated probability, rather than issuing their risk aversion coefficient according to MVT (Das et al, 2010). Those authors also demonstrate that a non-negligible loss of optimality occurs when risk tolerance is not well defined beforehand. Indeed, the expectation of gain deteriorates as a result of risk misspecification and can go as far as a possible loss of 40 basis points, for the least averse investors.

This vision of risk as represented by a VaR constraint develops further, ending up in the SP/A theory developed by Lopes (1987). This theory is a framework that explains how choices are made. It is also seen as an extension of the safety-first portfolio model developed by Roy, as the S represents safety. i.e. the investor does not wish to see the value of his portfolio fall below a certain threshold. Next, the P for Potential demonstrates the investor's desire for gain. Finally, the A for Aspiration reflects the investor's desired return in relation to the risks he has taken.

Agents' decisions are driven by 2 feelings: hope and fear (Lopes, 1987). These 2 feelings lead to a distortion of probabilities. Indeed, hope (or fear) overweighs the low probabilities of the best (or worst) outcomes. Lopes (1987) therefore also proposed a probability function transformation, a few years before the one developed in cumulative prospect theory, which would later be widely used in behavioral finance research.

Expected wealth is no longer calculated by aggregating probable outcomes, but by applying a weighting function that replaces probabilities with those actually perceived by investors (Broihanne et al., 2006).

3. Cumulative Prospect Theory (CPT)

In 1992, Kahneman and Tversky released an update to their prospect theory, calling it cumulative prospect theory.

Cumulative prospect theory is an improved version of the first theory, using cumulative rather than separable decision weights. It satisfies stochastic dominance, thus removing assumptions made in the first theory that were criticized. Nevertheless, it leads to similar predictions.

Risk appetite and risk aversion are determined by the value function and the cumulative weighting function.

This new theory reduces the sensitivity of decision weights to probabilities. Indeed, for an equal change in probability, the latter will have more impact close to the 0 and 1 boundaries than elsewhere. This results in a decision weight function that is concave close to 0 and convex close to 1 (see *Figure 2*).

Because, as described in prospect theory, investors place different values on losing or gaining the same amount, it seems logical that decision weights should also be different in the case of gains or losses. Their experiments led to these decision weight functions being very close in the case of gain or loss, although slightly more curved in the case of gain (*Figure 2*).

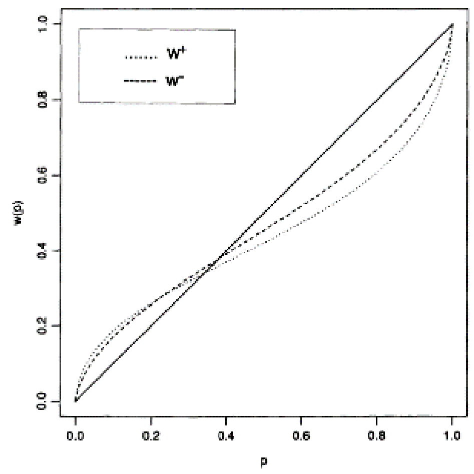

***Figure 2**: Weighting function for gains (w^+) and losses (w^-)*
(Advances in propect theory, p. 313)

The equations for these two weighting functions are as follows:

$$\boldsymbol{w^+(p) = \frac{p^{0.61}}{(p^{0.61} + (1-p)^{0.61})^{1/0.61}}}$$

$$\boldsymbol{w^-(p) = \frac{p^{0.69}}{(p^{0.69} + (1-p)^{0.69})^{1/0.69}}}$$

4. Behavioral Portfolio Theory (BPT)

Behavioral portfolio theory (BPT) was developed by Shefrin and Statman (2000), based on SP/A theory (Lopes, 1987) and part of Kahneman and Tversky's prospect theory. Investors maximize their expected wealth using SP/A theory and decision weights that transform the probabilities of outcomes. This ends up with an optimization like this:

$$\max E_\pi (W) \text{ u.c. } p(W < A) < \alpha.$$

In general, investors combine low and high levels of aspiration. They want to avoid poverty, i.e., seeing their portfolio fall below a certain threshold, but at the same time, they also want to have the opportunity to be wealthy. This type of portfolio can be seen as a multi-level pyramid, where investors place their money at the base of the pyramid, which prevents them from being poor, and then at the top of the pyramid, which allows them to have opportunities for significant gains. Tversky and Kahneman (1986) further demonstrate that individuals simplify their mental processes by separating joint probabilities into different mental accounts and layers in the pyramid.

These portfolios are therefore like a combination of bonds and lottery tickets. This is the result of the BPT investors' decision-making process, in which they build their portfolios like a pyramid of assets, with the least risky at the bottom on top of which are added fewer but increasingly risky assets.

Fisher and Statman (1997) analyzed mutual fund companies, which also recommended pyramid portfolios to their investors, with cash at the base, then bonds and finally equities. The aggressiveness of the portfolio was then regulated by the proportion of equities to bonds, as Canner, Mankiw and Weil (1997) also proved after reviewing investment advisors' recommendations. Nevertheless, this recommendation does not satisfy the CAPM model, as it contradicts the two-fund separation. Indeed, this concept stipulates that all efficient portfolios according to CAPM share the same equity/bond ratio, and that the attitude to risk is manifested in the proportion allocated to risk-free assets.

These so-called pyramid portfolios are therefore consistent with the behavioral model, rather than CAPM.

5. BPT$_{CPT}$

Bourachnikova et al. (2016) aimed to compare portfolios generated by BPT and MVT without any restrictions. Several studies, including some discussed above, had been conducted to examine whether portfolios with biases (i.e., incorporating behavioral aspects) could still be efficient in Markowitz's rational sense. However, all of these studies only considered certain aspects of behavioral portfolio theory each time. For example, Levy and Levy (2004) did not integrate the SP/A, while Das et al. (2010) and Alexander and Baptista (2011) did not take into account the transformation of probabilities into decision weights.

In addition to these limitations, all of these studies were based on a very strong assumption, which is the Gaussian distribution of asset returns. This assumption has been shown to be false by Mandelbrot (1963) and Mantegna and Stanley (1995).

Only one study appears to have attempted to compare behavioral theory to classical theory without assuming a Gaussian distribution, and that is Hens & Mayor (2014). However, this study is very limited, considering only a very small sample of data and not considering the SP/A theory.

It is in this context that Bourachnikova et al (2016) wanted to develop a comprehensive comparison between behavioral and "rational" portfolios, without making the assumption of normally distributed returns.

To carry out their study, Bourachnikova et al (2016) first considered the portfolio described by Shefrin and Statman (2000), the BPT. This incorporates the SP/A model of Lopes (1987), in which investors first satisfy their safety-first criterion reflecting the concept of fear. Then, they invest the remainder in a security with a high potential payoff, reflecting the concept of potential. This implies that the resulting portfolio has positive skewness, high risk and high returns. They also incorporate a transformation of probabilities into decision weights, but not the one developed in cumulative prospect theory.

She then developed these behavioral portfolios to propose one that is even closer to the human way of thinking. This portfolio, BPTCPT, still incorporates the SP/A aspect, but adds an update of the decision weight function, taking that developed in 1992 by Tversky and Kahneman. It also adds the subjective value of each monetary outcome. This value function reflects the fact that investors make decisions based on their fluctuating wealth rather than their final wealth, which can lead to risk-seeking behavior in the event of losses as we saw earlier.

Like the new version of decisión weights, the concept of outcome value was developed in CPT, hence the name of this portfolio BPT_{CPT}. The perceived value of outcome fluctuations is grafted onto the original BPT formula, giving rise to a new optimization function:

$$\mathbf{max\ E}_{\bar{\pi}}\mathbf{[}\boldsymbol{V}\mathbf{(W)]\ u.c.\ p(W < A) < \alpha}$$

Where $\bar{\pi}$ is the decision weighting function developed in CPT and V is the value function that transforms monetary outcomes into utility.

The reference point used to determine whether one is in loss or gain is an important element to determine, as it influences the optimization function. In this study, the authors decide to set it at the level of the long-term risk-free rate.

Here, losses and gains are treated differently by the value and decision weight function. As already explored in the CPT, the value associated with a positive or negative variation of the same magnitude will not be the same. This is because investors are risk averse (Erev et al., 2008). Losses are weighted almost twice as much as gains. In other words, a loss of €1 generates a loss of utility 2 times greater than the gain of the same euro. The same principle applies to probabilities linked to positive or negative events for the portfolio. They will not be perceived in the same way, depending on the sign of the fluctuation.

The results of the study show that optimal BPT portfolios are located just over 70% of the time on the MV optimal frontier. This result is robust to changes in *alpha* and aspiration levels. A very similar percentage is found for BPT_{CPT} portfolios.

In their simulations, BPT and BPT_{CPT} portfolios are identical between 50% and 90% of the time. For the non-identical portion, it turns out that the BPTCPT portfolios are less risky than the BPT ones. This is due to the risk aversion provided by the value function and the different decision weights in the event of gains or losses. BPT_{CPT} investors are considered more risk-averse and therefore seek out more secure portfolios.

Optimal BPT portfolios are often located close to the maximum return portfolio on MV, i.e. the one with the highest expected returns, but also the riskiest. These portfolios contain positively skewed returns, confirming the concept of hope in BPT theory. These investors will choose assets that resemble lottery tickets with high skewness, with the aim of obtaining high returns (Bali et al., 2011).

Thus, these behavioral portfolios, when they are optimal, lie on the upper right-hand side of the efficiency curve, demonstrating their fairly risky nature.

However, this study was carried out using a method that did not allow for a comparison of optimal portfolio behavior according to MVT, BPT and BPTCPT out-of-sample. This is because, with the conditions set, they were obliged to simulate different hypothetical states of nature in their model and could not assess the performance over real data.

PART III: EMPIRICAL METHODOLOGY

The aim of the empirical part is to apply in sample the various portfolio selection methods presented in the literature review above, and to compare the out-sample performance of these portfolios in terms of various factors such as average return or variance. The empirical methods were performed using R software.

1. Data and sample creation

The empirical study in this paper is based on the weekly total returns of "EuroStoxx 50" equities over the last 10 years, extracted from the Refinitiv platform. More specifically, a 4-year historical analysis will first be carried out, in order to constitute our sample. Based on these first 4 years, the 3 portfolio selection methods will be applied. Depending on the portfolios generated by each model, an analysis of the behavior of these portfolios over the following 6 years will be carried out. It will be interesting to see how each of these portfolios chosen using a different method behaves over time.

Two aspects of this methodology are interesting and innovative compared to previous research on behavioral portfolio theory.

Firstly, the artificial placement four years ahead and the analysis of the following six years as if they were in the future will allow a more meaningful comparison of each portfolio's performance. In other studies, portfolios were chosen based on the performance of assets over a defined period, but there was no subsequent analysis to observe the future performance of the portfolio. In this work, we put ourselves in a real-world scenario where, with a four-year historical data on the EuroStoxx 50, we want to construct the best portfolio for the next years.

Secondly, the temporal aspect of this work encompasses a period of unprecedented and unpredictable crisis, namely the Coronavirus pandemic. It will be interesting to observe which method of portfolio construction better withstands this crisis. However, this crisis may also bias our results and not fully represent a more "normal" situation. Yet, considering that we have already experienced three major crises recently (2000, 2008, and 2020), we can argue that our sample reasonably captures market behaviors in the 21st century.

In practical terms, the methodology is as follows.

Firstly, the Eurostoxx 50 refers to 50 of the largest market capitalizations in 9 eurozone countries, representing 60% of European market capitalization. The index's sector breakdown is quite good, despite the absence of a diversification policy. For example, 4 major sectors - consumer cyclicals, information technology, industry, and finance - each account for around 15% of total market capitalization, as shown in figure 3.

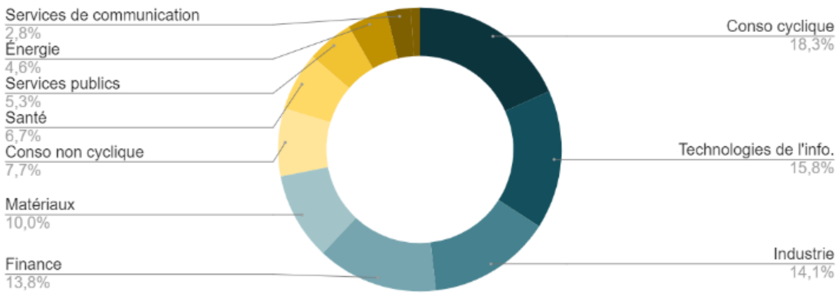

***Figure 3**: Sector breakdown of EuroStoxx 50 equities*
(https://finance-heros.fr/euro-stoxx-50/)

This index is updated every year using a method that ensures a degree of stability. Every year, at the end of August, market capitalizations are ranked in descending order. The 40 largest are included in the index, and then the components that were included but are not among the top 40 are added. Even if this method avoids too frequent changes in components, it is unlikely that our 4-year sample of the EuroStoxx 50 refers only to the same 50 stocks. Therefore, a first part of the methodology will be to keep only the components common to the 4 historical years of the Eurostoxx 50. For the years 2013 to 2017, 46 stocks common[1] were retained to form our sample.

After this initial data cleaning, portfolios will be determined according to each of the 3 selection methods, and performance over the following 6 years will be analyzed independently of whether or not stocks are still present in the index.

2. Portfolios creation

The main idea of this empirical study was initially to compare, out of sample, the optimal portfolios retained by the three portfolio selection methods, namely Mean-Variance theory

[1] Air Liquide, Allianz, Anheuser-Busch InBev, ArcelorMittal, ASML Holding, AXA, Banco Santander, BASF, Bayer, BBVA, BMW, BNP Paribas, Carrefour, Daimler, Danone, Deutsche Bank, Deutsche Börse, Deutsche Telekom, E.ON, ENI, Essilor International, GDF Suez, Generali, Iberdrola, Inditex, ING, Intesa Sanpaolo, L'Oréal, LVMH, Munich Re, Nokia, Philips, PSA Peugeot Citroën, Renault, Repsol, Rio Tinto, Royal Dutch Shell, Saint-Gobain, Sanofi, SAP, Schneider Electric, Siemens, Société Générale, Telefonica, Total, Unilever, Vinci.

(MVT), Value at Risk (VaR), and BPT_{CPT} which combined VaR and CPT behavioral theories. These three methods are the most important trends and developments in portfolio theory.

However, after developing a model that optimized the asset weights in a portfolio under Value at Risk (VaR) constraints, it became apparent that the sample composed of EuroStoxx 50 assets was not appropriate. This was because the asset with the highest average return (Nokia) had a total return that barely dipped below 0% over the duration of the sample. As a result, the VaR optimization model invested entirely in this asset, regardless of the realistic values given to the constraint. This result contradicted the principle of diversification, and it would have been necessary to use this model on a sample of more volatile assets for the VaR constraint to be binding. Since the BPT_{CPT} method also relies in part on a non-binding VaR constraint in our case, this aspect of the theory is not considered in this study.

This unexpected feature actually enabled the empirical method to follow a much more precise and achievable goal on the scale of a thesis. The empirical objective was thus ultimately to compare optimal portfolios according to the classical MVT method with their alternatives found according to the behavioral CPT method.

To achieve this goal, 50,001 portfolios were created in RStudio. Specifically, 50,000 random weight vectors were generated, to which was added the equally weighted portfolio, i.e. assigning the same weight to each asset in the portfolio. The random portfolios are long-only, i.e. there is an assumption that short-selling is not allowed, in order to simplify the methodology. Each vector therefore contains 46 weights (because there are 46 assets), each of these weights is between 0 and 1 and their sum is 1.

3. CPT portfolios

It is appropriate to begin the empirical explanation with the more complex portfolio choice method of the two. This will enable a direct understanding of the few modifications made to the mean-variance theory to ensure consistency between these two methods.

Optimal BPT_{CPT} portfolios satisfy this equation :

$$\max \mathrm{E}_{\bar{\pi}}[V(\mathrm{W})] \text{ u.c. } \mathrm{p}(\mathrm{W} < \mathrm{A}) < \alpha$$

As the VaR-type constraint "p(W < A) < α" is not binding in our case, we will focus on the changes that "$\bar{\pi}$" and "V" imply compared to a classical mean-variance maximization (MVT).

Firstly, it should be clarified that W represents the distribution of the investor's final wealth. The data used for the study are the Total Returns of the stocks, recorded on a weekly basis over a period of 10 years (*Figure 4*). This type of data has two essential characteristics for the study. On the one hand, they are the most comprehensive as they consider not only the price variation of the stock but also any dividends that the stock may have yielded. On the other hand, these Total Returns are also cumulative with respect to the reference date. For example, if an action has gained 15% after 3 weeks, this figure will be noted in the 3rd row of the Total Returns table. This allows for the calculation of the total value of each portfolio week after week, and thus to construct the distribution of their final wealth.

	AXAF.PA	ABI.BR	ALVG.DE	MT.AS	ASML.AS
2013-06-24	0.023	-0.074	-0.041	-0.137	-0.033
2013-06-17	0.011	-0.101	-0.099	-0.121	-0.074
2013-06-10	0.047	-0.045	-0.026	-0.071	-0.042
2013-06-03	0.064	-0.031	-0.002	-0.044	-0.026
2013-05-27	0.059	-0.037	0.023	-0.004	0.025

Figure 4*: Example of the database (46 assets, Total returns until May 2023)*

3.1. First modification: Total Return perception

The modification function « V » is applied to Total Returns (W), which transforms the monetary outcome into utility. This CPT value function is defined as:

$$\boldsymbol{V(y)} = \begin{Bmatrix} (\boldsymbol{y} - \boldsymbol{k})^{\mathbf{0.88}} & \boldsymbol{if\ y \geq k} \\ -\mathbf{2.25}(-(\boldsymbol{y} - \boldsymbol{k}))^{\mathbf{0.88}} & \boldsymbol{if\ y < k} \end{Bmatrix}$$

This first modification of the data is pretty straightforward and does not require any further explanation. However, a discussion can be held in relation to the threshold *k* that decides whether the outcome is experienced as a loss or a gain by the investor. In some works, this threshold is not equal to 0 but rather to the long-term risk-free rate (i.e., 10-year U.S. Treasury Bond) (Bourachnikova, 2016). For the sake of time and simplicity, it is assumed in this study that this threshold will remain at 0, which is still the clearest boundary between loss and gain for everyone.

3.2.Second modification: Weighting function

As described in the literature review, behavioral finance work by Tversky and Kahnemann led to the conclusion that human beings perceive probabilities in a way that is distorted to reality. With this in mind, the function "$\overline{\pi}$" was modeled by cumulative prospect theory. This function is articulated as follows:

$$\boldsymbol{w^+(p) = \frac{p^{0.61}}{(p^{0.61} + (1-p)^{0.61})^{1/0.61}}}$$

$$\boldsymbol{w^-(p) = \frac{p^{0.69}}{(p^{0.69} + (1-p)^{0.69})^{1/0.69}}}$$

w(p) is then the weighting function that modifies the real probability p of an outcome of a portfolio into the decision weight, i.e. the probability actually perceived by an investor.

Specifically, in the empirical method, this weighting function required a few modifications to the returns data, which had already been modified with the "*V*" function. For each of the 50,001 portfolios, a returns density function was calculated and simplified by the histogram function to obtain the 208 weekly total returns in around 10 bins (the histogram function optimizes the number of bins needed for each portfolio to lose as little information as possible).

This brings us to an intermediate stage where the final wealth distribution of each portfolio is contained in a table (*Table 1*). A quick comparison of the expected final wealth of each portfolio calculated by the sum of returns*density and the one calculated by the weighted average of the return of each asset showed that the modification into a density function gave a result almost similar to that before transformation.

	Returns pf 1	p	Returns pf 2	p	Returns pf 3	p
1	-0.250	0.005	-0.250	0.005	-0.250	0.005
2	-0.150	0.014	-0.150	0.019	-0.150	0.014
3	-0.050	0.029	-0.050	0.029	-0.050	0.029
4	0.050	0.062	0.050	0.082	0.050	0.048
5	0.150	0.216	0.150	0.303	0.150	0.207
6	0.250	0.250	0.250	0.279	0.250	0.351
7	0.350	0.173	0.350	0.120	0.350	0.091
8	0.450	0.183	0.450	0.149	0.450	0.212
9	0.550	0.053	0.550	0.014	0.550	0.043
10	0.650	0.014				

Table 1: *Returns example of some portfolios.*

To go a step further and apply the probability transformation, we need to do it on the cumulative distribution function, as set out by Kahneman and Tversky in their update of Prospect Theory. Applying the transformation on the cumulative function also keeps the sum of probabilities equal to 1. *Table 2* shows the cumulative function of a portfolio before and after the "$\bar{\pi}$" transformation of its probabilities. We can then take the difference between each line, which allows us to return to a more classical density function (*Table 3*) and construct the parameters useful for analysis.

Cumulative Returns pf 1	Cumulative probabilities	Cumulative transformed probabilities
-0.250	0.005	0.024
-0.400	0.019	0.061
-0.450	0.048	0.109
-0.400	0.111	0.181
-0.250	0.327	0.345
0	0.577	0.461
0.350	0.750	0.568
0.800	0.933	0.761
1.350	0.986	0.892
2	1	1

Table 2: *Transformation of cumulative probabilities.*

Returns pf 1	Transformed p
-0.250	0.024
-0.150	0.036
-0.050	0.048
0.050	0.072
0.150	0.165
0.250	0.116
0.350	0.107
0.450	0.193
0.550	0.131
0.650	0.108

Table 3: *Transformed probabilities.*

3.3. Expected return and risk

From Table 3, we can calculate the perceived expected return and the perceived risk of each portfolio.

The expected return is obtained directly by taking the sum of products between returns and associated probability. For risk, the standard deviation of each density function is calculated using the weighted.sd function.

For each of the 50,001 portfolios, we thus obtain the expected final return and the risk over the sample period, as perceived by the investor.

4. Mean-variance portfolios

Concerning the Mean-variance framework, it requires to be consistent with the previous method. So, as the expected returns and risks of the portfolios were calculated using the density function modified with CPT, here too we have to generate the density function. We don't modify it afterwards, but this allows us to compare the two methods. Indeed, the only two things that change between them are the « v » et « $\bar{\pi}$ » modifications.

We therefore go through the same steps as before, and obtain the final wealth distribution of each portfolio, with their associated probabilities. Their expected return and standard deviation are calculated in the same way as for the CPT method.

After this, we construct a comparison table (*Table 4*) for each of the 50,001 portfolios. This shows the actual expected returns and risks of each portfolio, as well as the return and risk perceived by the investor.

	MVT ExpRet	MVT risk	CPT 'felt' Return	CPT 'felt' risk
1	0.284	0.149	0.310	0.235
2	0.251	0.128	0.259	0.209
3	0.284	0.141	0.293	0.214
4	0.280	0.139	0.302	0.223

***Table 4**: Data of the first 4 portfolios.*

5. Comparison MVT efficient and CPT-alternative

From this comparison table, we can start building the tools that will allow us to achieve the objective of this empirical part. To do this, we will look at the 10 most efficient portfolios according to MVT. This can be done by analyzing the Sharpe Ratios of each portfolio. The Sharpe Ratio divides the portfolio's return by its risk, which allows us to identify those that offer the best return relative to the risk incurred. To this panel of 10 portfolios, we add the equally weighted remarkable portfolio.

These 11 portfolios possess the characteristics of MVT, but also the ones transformed into CPT. The next step is then to find, for each of these portfolios, the alternative that is efficient according to CPT. So, at a fixed perceived risk level, we search among the 50,001 portfolios to find the one that will provide the maximum perceived return.

The comparison can now take place. We will compare the out-of-sample performances of the efficient portfolios according to MVT as well as their efficient alternatives according to CPT. It should be noted that these alternatives do not have the best Sharpe Ratios according to MVT and are therefore theoretically less performing. It is this logic that we will seek to verify in the next part of this study.

6. Out-of-sample analysis

To perform the analysis on the 6 out-of-sample years, it was necessary to pre-process the return data. In order to show what interests the investor, which is the expected final wealth of their portfolio, the Total Returns of the assets need to be treated separately from the sample. Therefore, we take the last date of the 4-year sample as the reference, as if it were at that moment that the investor had to choose in which portfolio to invest their money.

The weekly cumulative Total Returns of the assets are then calculated with the new reference, for the 6 years. So, if an action has gained 30% between week 1 and week 55, its Total Return in week 55 will be 30%. This allows, once again, to estimate the total value of each portfolio every week.

PART IV: EMPIRICAL ANALYSIS

Before analyzing the out-of-sample results, it is interesting to graph the scatter plot representing the 50,001 portfolios (*Figure* 5). The colored circles represent the portfolios with the 10 highest Sharpe Ratios (1: highest Sharpe Ratio, 10: 10th highest Sharpe Ratio), as well as the equally weighted portfolio (1/N, N=46). The colored diamonds represent the alternative portfolios to the circles, which are efficient under CPT. These alternatives are also located quite close to the efficiency frontier, but they are not considered the best portfolios according to MVT. In fact, the diamond portfolio number 3 has the highest Sharpe Ratio among the alternative portfolios, but it ranks only 32nd in terms of all portfolios. The other alternatives are tens or hundreds of places lower in the ranking (see *Table 5*).

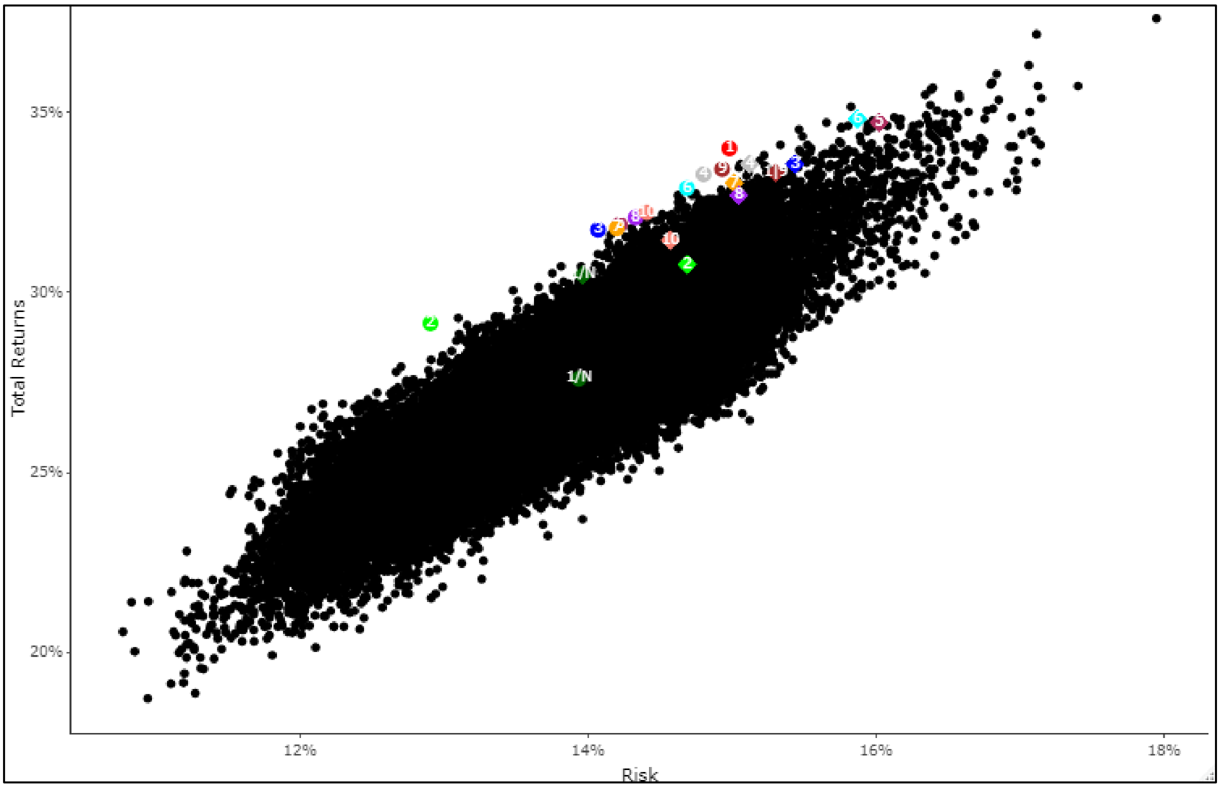

Figure 5: *Plot of the features in sample of the 50 001 portfolios.*

The initial intention was to add 2 other special portfolios to the data: the Global Minimum Variance (GMV) and the tangent portfolio. Unfortunately, with the database, the GMV portfolio had a negative expected return, so it was impossible to calculate the tangent portfolio as well. No viable explanation was found, but we can nonetheless say that the 50,001 portfolios provide a large enough panel for a consistent analysis.

The graph also shows that the alternative to portfolios 1 and 9 is the same. This is because the perceived risk of these 2 portfolios was the same to within 10^{-4}.

MVT ranking Sharpe Ratio •	CPT alternative ranking Sharpe Ratio ◆
1	235
2	3970
3	272
4	32
5	380
6	99
7	73
8	284
9	235
10	555

Table 5: *Sharpe ratio of the chosen portfolios.*

1. Out of sample analysis

Now that we have explained the method in detail, we can proceed with the out-of-sample analysis of these portfolios. *Figure 6* shows in a minimalist way the expected Total Return and volatility of each portfolio chosen in-sample. The efficiency frontier was drawn using the performances of the portfolios that performed best out-of-sample.

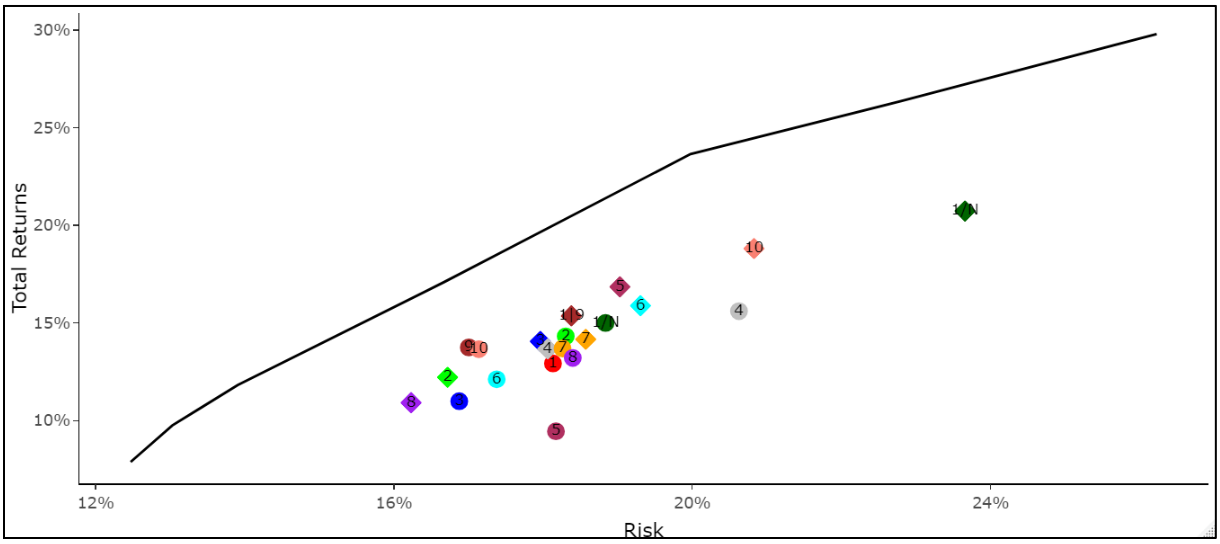

Figure 6: Plot of the *features out-sample of the chosen portfolios.*

Graphically, we can already see that some diamond portfolios (CPT alternatives) perform better than their MVT counterparts, such as portfolio 5 or portfolio 1, for example.

After a superficial graphical analysis, it is necessary to statistically compare the performances of these portfolios. To do this, an analysis of the Sharpe ratios will allow us to see for each

MVT portfolio versus its CPT alternative, which one performed better over the 6-year out-of-sample period.

Beyond simply observing whether one Sharpe ratio is higher than the other, it is also important to ensure that this difference is statistically significant using a hypothesis test. For this purpose, the SharpeTesting function in RStudio was used. *Table 6* shows, for each portfolio comparison, the MVT Sharpe ratio, the CPT alternative Sharpe ratio, the difference between the two, and the p-value testing whether the difference is statistically significant or not. Remember that a p-value less than 0.05 indicates that a Sharpe ratio is significantly higher than the other one.

	MVT Sharpe	CPT Sharpe	Sharpe Difference	p-value
1	0.713	0.836	-0.123	0***
2	0.782	0.731	0.052	0***
3	0.651	0.782	-0.131	0***
4	0.756	0.757	-0.0003	0.960
5	0.520	0.885	-0.365	0***
6	0.697	0.823	-0.125	0***
7	0.750	0.762	-0.013	0.003
8	0.718	0.673	0.045	0***
9	0.808	0.836	-0.027	0.00003
10	0.797	0.903	-0.106	0***
1/N	0.796	0.876	-0.080	0***

Table 6*: Characteristics of the chosen portfolios.*

The analysis of this relatively simple table forms the core of the research. Indeed, we can observe that out of the 11 portfolio comparisons, the Sharpe difference is negative 9 times out of 11. This means that the Sharpe ratio of the CPT alternative is higher than its MVT counterpart in 9 out of 11 cases. However, considering the p-values, we realize that no conclusion can be drawn regarding portfolio number 4.

Ignoring the 1/N portfolio, which is not optimal under MVT, we still find that the CPT alternative significantly outperforms the optimal portfolio in 7 out of 10 cases. This is despite the fact that these same portfolios were sometimes far from being considered the best in-sample.

By observing the naive equally weighted portfolio, which occasionally performs well out of sample, we can see that indeed its Sharpe Ratio is among the highest of the optimal MVT portfolios. However, its CPT alternative, found through the applied transformations, once again outperforms it.

Table 7 provides another perspective in the comparison. The first two columns show the total growth of each portfolio during the entire out-of-sample period, from May 22, 2017, to May 22, 2023. Although this method is less mathematically consistent as it evaluates the performance of a portfolio at a specific moment, it allows us to quickly see which portfolio would have grown our money the most over the 6 years. Once again, we observe that CPT portfolios outperform their efficient MVT counterparts 7 times out of 10. Regarding the 1/N portfolio, it is also less performing than its CPT alternative. It is worth noting that the 3 portfolios that perform better under MVT compared to CPT (2, 4, and 8) are the same ones that had either a higher or non-significantly smaller Sharpe ratio in the previous analysis.

The rest of the columns simply provide the details of the Sharpe ratios obtained in the previous table, which is the mean return divided by the volatility (standard deviation).

	MVT Final Return	CPT Final Return	MVT Mean Return	CPT Mean Return	MVT Volatility	CPT Volatility
1	0.539	0.576	0.129	0.154	0.181	0.184
2	0.547	0.505	0.143	0.122	0.183	0.167
3	0.480	0.521	0.110	0.141	0.169	0.180
4	0.583	0.551	0.156	0.137	0.206	0.181
5	0.471	0.565	0.095	0.169	0.182	0.190
6	0.501	0.586	0.121	0.159	0.174	0.193
7	0.532	0.548	0.137	0.142	0.183	0.186
8	0.532	0.470	0.132	0.109	0.184	0.162
9	0.538	0.576	0.137	0.154	0.170	0.184
10	0.500	0.640	0.137	0.188	0.171	0.208
1/N	0.558	0.717	0.150	0.207	0.188	0.237

Table 7: *Additional information about the chosen portfolios.*

2. Robustness test

Before analyzing the reasons for this almost recurring better performance of CPT portfolios compared to MVT portfolios, it is first necessary to show that this result is consistent over time.

The previous results were obtained from a 4-year sample (May 2013 to May 2017) and an out-of-sample analysis covering the following 6 years. Now, we will reproduce the previous process but with modified sample durations. Specifically, we will conduct the study using, on one hand, a 5-year sample and a 5-year out-of-sample period, and on the other hand, a 6-year sample and a 4-year out-of-sample period. So, with the same dataset of weekly total returns from the last 10 years of EuroStoxx 50 stocks, we want to verify if the previously discovered trend is insensitive to changes in the sample periods.

2.1.Sample: 5 years, Out-Sample: 5 years

The same methodology as before is followed, calculating the expected returns, risk, "perceived" expected return, and "perceived" risk of each of the 50,001 portfolios. Then, for the 10 portfolios with the highest Sharpe ratios, optimal alternatives are found based on "perceived" expected return and risk.

During this search for alternatives, a case that did not occur previously arose. Some portfolios that were optimal according to MVT were also optimal under CPT and therefore had no alternatives. This was the case for the 1st, 2nd, 8th, 9th, and 10th portfolios with the highest Sharpe ratios. The other 5, as well as the equally weighted portfolio, indeed had a different CPT alternative. In the graph (*Figure 7*), we highlight only the portfolios that are different under MVT and CPT to reanalyze the comparison.

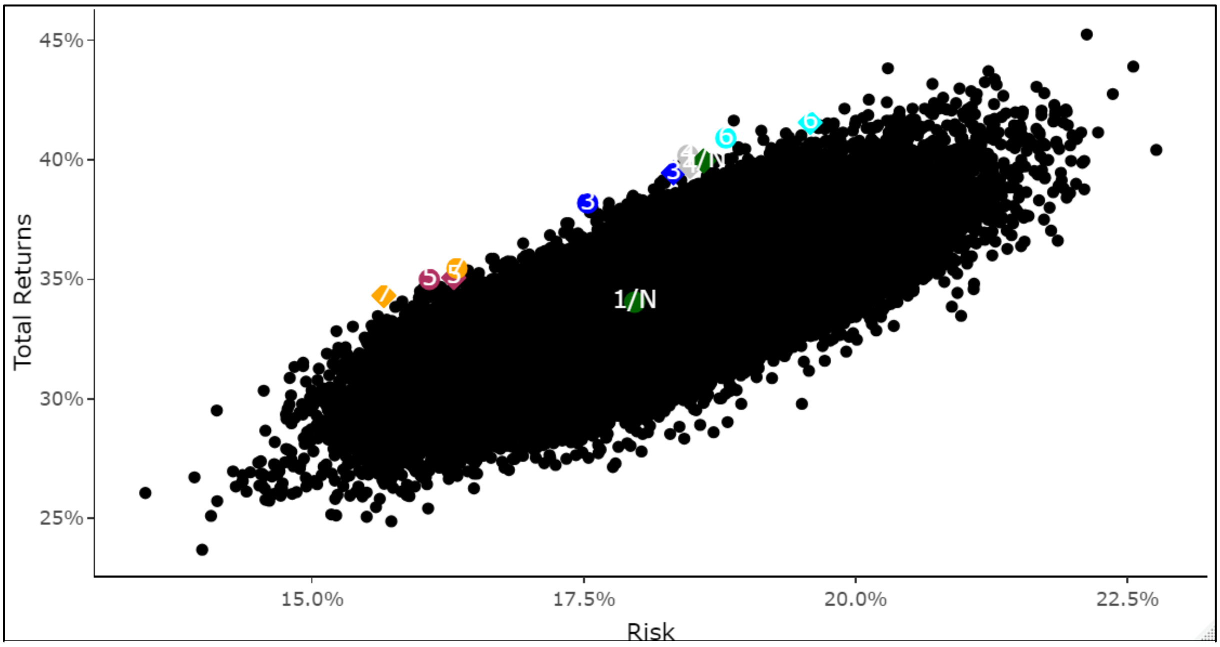

Figure 7: *Plot of the features in sample of the 50 001 portfolios*

Figure 7 highlights the in-sample characteristics of the 6 MVT wallets and their diamond-shaped CPT alternative.

Figure 8 shows the performance of out-of-sample portfolios. This graph is supported by *Table 8*, which shows precisely which portfolio outperforms the other.

The observation shows that 3 times out of 6, the CPT portfolio is significantly more performing than its MVT counterpart. In 2 other cases, the difference is too small to affirm that one portfolio performs better than the other. Additionally, there is the MVT portfolio number 5 that performs better than its CPT alternative.

By contrast, on May 22, 2023, half the comparisons saw the MVT portfolio outperform its CPT alternative by 5 years. But as explained earlier, this measure is less meaningful because it only takes into account growth between the first and last day out of sample, and nothing that has happened in between.

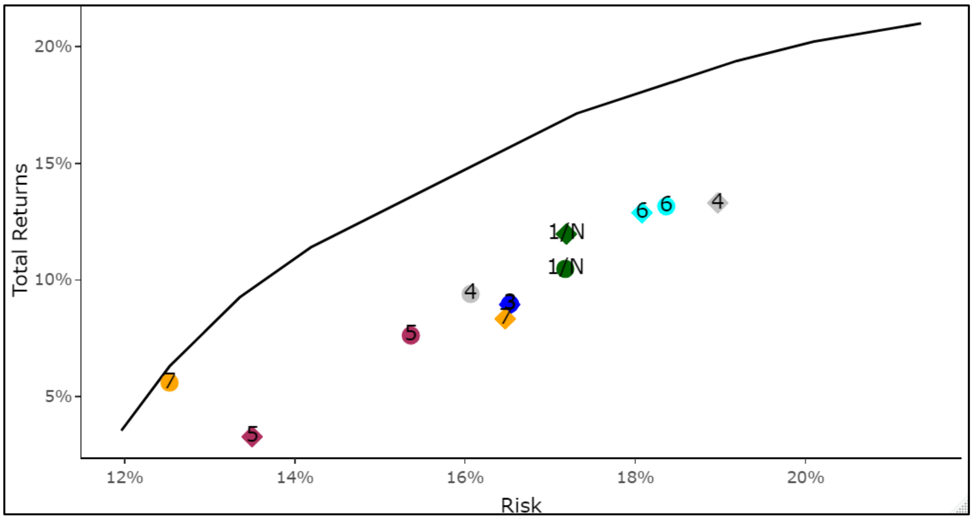

Figure 8: *Plot of the features out-sample of the chosen portfolios.*

	MVT Sharpe	CPT Sharpe	Sharpe Difference	p-value	MVT Final Return	CPT Final Return
3	0.542	0.541	0.001	0.580	0.454	0.443
4	0.585	0.701	-0.116	0***	0.443	0.520
5	0.495	0.242	0.253	0***	0.412	0.343
6	0.716	0.712	0.004	0.371	0.537	0.516
7	0.446	0.505	-0.060	0***	0.344	0.445
1/N	0.609	0.696	-0.086	0***	0.468	0.506

Table 8: *Features out-sample of the chosen portfolios.*

2.2. Sample: 6 years, Out-Sample: 4 years

In this case, three optimal MVT portfolios did not have alternatives because they were also optimal according to CPT. These portfolios are the ones with the first, third, and tenth largest Sharpe ratios. The comparison in this timeframe will, therefore, be conducted on the 8 other portfolio alternatives, whose in-sample behaviors can be seen in *Figure 9.*

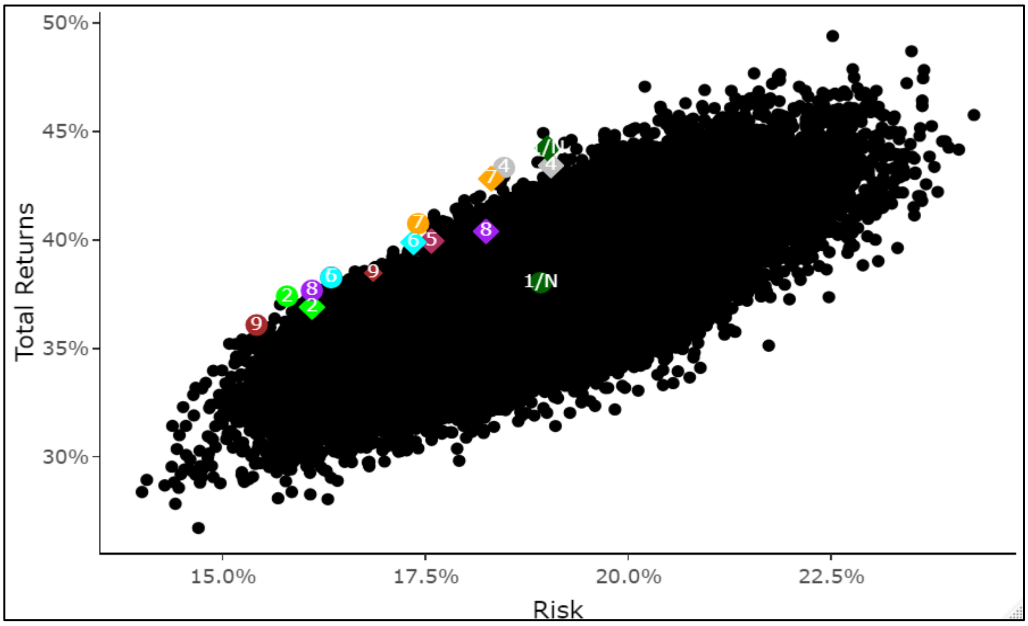

Figure 9: *Plot of the features in sample of the 50 001 portfolios*

For the out-of-sample analysis, as before, we will use the graph (*Figure 10*) and the summary table (*Table 9)*.

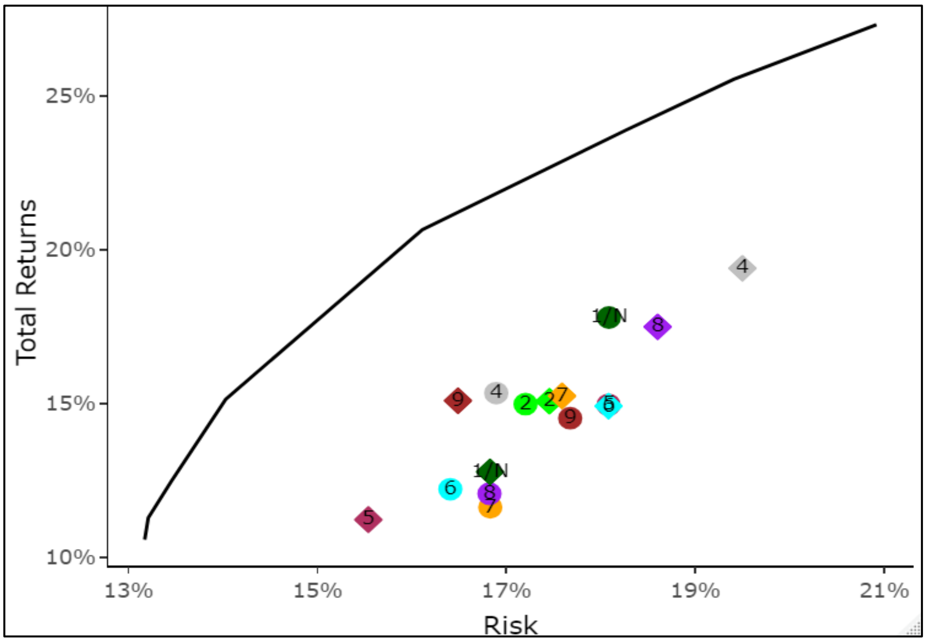

Figure 10: *Plot of the features out-sample of the chosen portfolios.*

	MVT Sharpe	CPT Sharpe	Sharpe Difference	p-value	MVT Final Return	CPT Final Return
2	0.871	0.864	0.007	0.137	0.436	0.447
4	0.908	0.995	-0.087	0***	0.453	0.514
5	0.827	0.723	0.105	0***	0.465	0.397
6	0.745	0.825	-0.080	0***	0.396	0.469
7	0.691	0.867	-0.176	0***	0.398	0.416
8	0.718	0.940	-0.223	0***	0.418	0.485
9	0.822	0.916	-0.094	0***	0.452	0.436
1/N	0.984	0.760	0.224	0***	0.486	0.430

Table 9*: Features out-sample of the chosen portfolios.*

We once again observe that, in the majority of cases, CPT portfolios outperformed their MVT counterparts. Specifically, in 5 out of 8 cases, the CPT portfolios significantly outperformed the MVT portfolios during the out-of-sample period. In 2 cases, it was the MVT portfolios that performed better, and in the last case, no significant difference in performance could be concluded.

In terms of total returns, the alternative CPT portfolios would have resulted in higher returns in 5 out of 8 cases.

3. Cause of the difference

We will now seek to understand the reasons why CPT alternatives outperform their theoretically best MVT portfolios in the majority of cases. To do this, we will examine the characteristics that distinguish these two classes during the in-sample period, which may indicate better future performance during the out-of-sample period.

To begin this analysis, we will first assume that the total returns follow a Gaussian curve. Although we have seen previously that this assumption is not necessarily true, it will nevertheless allow us to study other parameters, such as skewness and kurtosis. These third and fourth moments provide information that is not captured by the mean (first moment) and variance (second moment).

Skewness is a measure of the distribution's symmetry. A negative skewness coefficient indicates a distribution with a tail stretched towards the left, and thus, in our case, towards returns close to or below 0. It is worth noting that a coefficient of zero does not guarantee a symmetric distribution, although a symmetric distribution will have a skewness coefficient of zero.

Regarding the fourth moment, kurtosis, it is important to note that the smaller its value, the flatter the distribution will be, but also the thinner the tails of the distribution will be, reducing the probability of outliers.

When we analyze these in-sample moments, the first thing we notice is that skewness is often close to or slightly below 0. This applies to both MVT and CPT portfolios, and to all sample timeframes. There is therefore no striking difference at this level, which could explain the trend discovered. Moving on to the 4th moment, the kurtosis, we notice a difference that could help explain the results. CPT portfolios have a significantly smaller kurtosis (p-value = 0.003) than their MVT counterparts. This could explain the better out-of-sample performance, as a lower kurtosis reduces the chances of having outliers in the portfolio.

However, this explanation should be taken with caution for two reasons. The first reason is that this result regarding kurtosis could not be replicated in the robustness samples. In other words, when considering samples consisting of the first five years and the first six years, there was no significant difference in kurtosis between the two classes of portfolios. The second reason is that in the sample where there is a significant difference in kurtosis, the kurtosis of CPT portfolios 2 and 8 are smaller than those of their MVT counterparts, while these CPT portfolios are the two that perform less well than their counterparts out of sample.

Since no consistent mathematical explanation can be found, other hypotheses must be considered. Since it has been proven that returns do not follow a Gaussian distribution, it even makes sense to find explanations elsewhere than in the analysis of assumed distributions.

Several lines of thought can be considered in this direction, particularly by revisiting the psychological aspect of this study. It has been observed that human beings have various perception biases, two of which were studied in the context of CPT, namely value perception and probability perception. When investors analyze all the information available to them, they may perceive the CPT portfolio as better than its theoretically superior MVT counterpart. It is conceivable that when operating out of sample, other investors also make similar analyses and perceive the same biased information. Consequently, investments are made in assets that benefit from these biases, leading to an increase in their prices and improving their out-of-sample returns.

This explanation may seem somewhat limited, assuming that individual investor biases would influence the market more than algorithmic and automated investments that are immune to these distortions of reality. The proportion of volumes traded by high-frequency trading compared to traditional brokerage is difficult to estimate. This proportion was reported to have increased from 9% to 40% between 2007 and 2011 (Tous, 2021), remaining estimated at 40% in Europe in 2018 (Bouton, 2018), but would be close to 80% of traded volumes today (Tous, 2021). Based on this data, it can be inferred that during all our out-of-sample periods, traditional brokerage still had a significant share, thus potentially exerting an influence on asset prices. Moreover, quantitative algorithmic analysis would aim to improve the understanding of market players by examining certain operational and financial performance factors. It is not, as many believe, based solely on a crude analysis of data linked to the underlying assets. It is believed that "market cycles are partially induced by market participants' emotions" (BNP Paribas Wealth Management, 2021).

However, it is essential not to limit ourselves to the two biases studied in CPT, as there are other cognitive biases at play, such as the familiarity bias, for instance. This well-known bias states that individuals tend to invest in stocks and brands they are familiar with (capital.com, n.d.). Consequently, "bankable" companies experience faster price increases for their stocks over time due to this bias. In our case, all the companies in the EuroStoxx 50 index enjoy significant brand recognition, leading to less variability in this factor. Nevertheless, this bias can still manifest, even to a lower extent.

A quick analysis can indeed demonstrate some results. The MVT portfolio with the highest Sharpe ratio in our 4-year sample and the one with the 10th highest Sharpe ratio were both clearly outperformed by their respective CPT alternatives. By examining the weights assigned to different assets in these CPT portfolios compared to their MVT counterparts, we can see that they allocate higher weights to well-known companies such as Telefonica and Generali Assurance for the first portfolio and Axa or Shell for the second one. Those are names that are familiar to the general public and undoubtedly more recognizable than other companies in the EuroStoxx 50 index.

CONCLUSION

Behavioral finance brings the irrational aspects of human behavior to standard finance. Through the literature review, it has been observed that many assumptions of modern portfolio theory have been proven false over time. However, these assumptions are still useful for understanding and generalizing the theory. Therefore, it is not only a matter of refuting them but also of replacing them with other equations that aim to be more accurate and reflect the real behavior of investors and the market.

A number of equations have therefore been developed as a result of psychological experiments, in order to make sense of the irrationality of human choices. These include the function for modifying the perceived values of outcomes, the function for modifying the perception of probabilities, and risk seen as a Value at Risk rather than a risk aversion coefficient, which remains highly conceptual. These major contributions from behavioral portfolio theory have provided alternatives to what was proposed by modern portfolio theory. With these alternatives also comes the complexity of modeling and unifying all this.

In any case, we have observed the importance of seeking to understand the thought processes of human beings. Understanding this leads to comprehending certain market movements and, ultimately, improving predictions. Currently, the results of this study show that biased human behavior actually leads to better investment choices than a non-biased method. This is because the markets on which investments are made are subject to the same cognitive biases that are partially modeled by CPT.

Several limitations should be acknowledged in this paper. As explained in the methodological section, the fully comprehensive BPTCPT model could not be utilized due to the non-binding VaR constraint in our data sample. A potential avenue for future research could be to develop a consistent and complete model and apply it to a larger dataset, such as EuroStoxx 500. This would likely allow the VaR constraint to play a role in optimization. However, a challenge will be to compare portfolios with similar risk levels as they are expressed differently in MVT and BPT_{CPT}.

Another limitation of this study is related to the robustness test. Conducting a robustness test on different samples was a crucial step in confirming the results. However, it would have been insightful to perform a robustness test on the parameter *k*, which defined the boundary between gains and losses. This parameter was arbitrarily set to 0, as it seemed the most logical value to separate gains from losses. However, it was noted that Bourachnikova (2016) chose to parameterize this value as the long-term risk-free rate. Therefore, it would have been interesting to observe the potential changes in optimal CPT alternatives based on modifications of the parameter *k*.

And of course, any other contribution to the discovery and modelling of new cognitive biases would be a significant advance for behavioral finance in its frenetic quest to understand the world around us ever better.

BIBLIOGRAPHY

Alexander, G. J., & Baptista, A. M. (2011). Portfolio selection with mental accounts and delegation. *Journal of Banking & Finance*, *35*(10), 2637–2656. https://doi.org/10.1016/j.jbankfin.2011.02.020

Allais, M. (1953). Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'Ecole Americaine. *Econometrica*, 21(4), 503–546.

Alphonse, P., Desmuliers, G., Grandin, P., & Levasseur, M. (2013). *Gestion de portefeuille et marchés financie*r. Montreuil: Pearson France.

Bali, T. G., Cakici, N., & Whitelaw, R. F. (2011). Maxing out: Stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics*, *99*(2), 427–446. https://doi.org/10.1016/j.jfineco.2010.08.014

Ball, R., & Brown, P. (1968). An Empirical Evaluation of Accounting Income Numbers. *Journal of Accounting Research*, 6(2), 159-178.

Blume, L., and D. Easley. Evolution and Market Behavior. *Journal of Economic Theory*, 58 (1992), 9-40.

BNP Paribas Welath Management, *La finance comportementale et la vertu des perspectives à long terme*. (n.d.). Retrieved June 12, 2023, from https://wealthmanagement.bnpparibas/fr/expert-voices/finance-comportementale-vertu-perspectives-long-terme.html

Bourachnikova, O., Pfiffelmann, M., & Roger, T. (2016). When behavioral portfolio theory meets Markowitz theory. *Economic Modelling*, 53(1), 419–435.

Bouton, M. (2018, April 20). *Bourse: Les robots ont-ils pris le pouvoir sur les marchés ?* . Le Revenu. https://www.lerevenu.com/bourse/bourse-les-robots-ont-ils-pris-le-pouvoir-sur-les-marches

Broihanne, M.-H., Merli, M., & Roger, P. (2006). Théorie comportementale du portefeuille. Intérêt et limites. *Revue économique*, *57*(2), 297–314. https://doi.org/10.3917/reco.572.0297

Canner, N., N. G. Mankiw, and D. Weil. "An Asset Allocation Puzzle." *American Economic Review*, 87(1997), 181-191.

Damodaran, A. (n.d.). *Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2011 Edition*.

Das, S., Markowitz, H., Scheid, J., & Statman, M. (2010). Portfolio Optimization with Mental Accounts. *Journal of Financial and Quantitative Analysis*, *45*(2), 311–334. https://doi.org/10.1017/S0022109010000141

Erev, I., Ert, E., & Yechiam, E. (2008). Loss aversion, diminishing sensitivity, and the effect of experience on repeated decisions. *Journal of Behavioral Decision Making*, *21*(5), 575–597. https://doi.org/10.1002/bdm.602

Familiarity Bias Definition: What Is It and How It Works. (n.d.). Retrieved August 1, 2023, from https://capital.com/familiarity-bias

Fisher, K. L., & Statman, M. (1997). Investment Advice from Mutual Fund Companies. *The Journal of Portfolio Management*, *24*(1), 9–25. https://doi.org/10.3905/jpm.1997.409626

Friedman, M., & Savage, L. J. (1948). The Utility Analysis of Choices Involving Risk. *Journal of Political Economy*, 56(4), 279–304.

Galanter, E., and P. Pliner. Cross-Modality Matching of Money Against Other Continua. in *Sensation and Measurement*, ed. by H. R. Moskowitz et al. Dordrecht, Holland: Reidel, 1974, pp. 65-76

Helson, H.*: Adaptation-Level Theory*. New York: Harper, 1964.

Hens, T., & Mayer, J. (2014). *Cumulative Prospect Theory and Mean Variance Analysis: A Rigorous Comparison* (SSRN Scholarly Paper 2417191). https://doi.org/10.2139/ssrn.2417191

Howard, C. T. (2014). *Behavioral Portfolio Management: How successful investors master their emotions and build superior portfolios*. Petersfield: Harriman House LTD.

Jorion, P. (1994). Value at Risk: The New Benchmark for Managing Financial Risk. McGraw-Hill.

Kahneman, D. (2011). *Thinking, fast and slow*. Farrar, Straus and Giroux.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. Econometrica: *Journal of the Econometric Society*, 47(2), 263-291.

Kahneman, D., & Tversky, A. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5(4), 297-323.

La bourse est-elle contrôlée par des algorithmes ? | ComparatifsBanques.CA. (2021, August 23). https://www.comparatifsbanques.ca/la-bourse-est-elle-controlee-par-des-algorithmes/

Levy, H., & Levy, M.. (2004). Prospect Theory and Mean-Variance Analysis. The Review of Financial Studies, 17(4), 1015–1041.

Lobão, J. (2015). 2FI14 - *Behavioral finance*. Faculdade de Economia da Universidade do Porto.

Lopes, L. (1987). Between hope and fear: The psychology of risk. *Advances in experimental social psychology*, 20(3), 255-295.

MacCrimmon, K. R., and S. Larsson. Utility Theory: Axioms versus Paradoxes. *in Expected Utility Hypothesis and the Allais Paradox*, ed. by M. Allais and 0. Hagen, forthcoming in *Theory and Decision*.

MacCrimmon, K.R., Larsson, S. (1979). Utility Theory: Axioms Versus 'Paradoxes'. In: Allais, M., Hagen, O. (eds) Expected Utility Hypotheses and the Allais Paradox. Theory and Decision Library, vol 21. Springer, Dordrecht. https://doi.org/10.1007/978-94-015-7629-1_15

Mandelbrot, B. (1963). *The variation of certain speculative prices*. Journal of Business, 36, 394–419.

Mantegna, R. N., & Stanley, H. E. (1995). Scaling behaviour in the dynamics of an economic index. *Nature*, *376*(6535), Article 6535. https://doi.org/10.1038/376046a0

Markowitz, H. M. (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77–91.

Markowitz, H. M. (1959). *Portfolio Selection: Efficient diversification of investements*. United States of America: Cowles Foundation for Research in Economics at Yale University.

Mills, F. C. (1956). *Introduction to Statistics*. New-York: Henry Holt & co.

Morgenstern, O., & Von Neumann, J. (1944): *Theory of Games and Economic Behavior*, Second edition, 1947; third edition, 1953. Princeton, New Jersey: Princeton University Press.

Polkovnichenko, V. (2005). Household Portfolio Diversification: A Case for Rank-Dependent Preferences. *The Review of Financial Studies*, *18*(4), 1467–1502. https://doi.org/10.1093/rfs/hhi033

Roy, A. D. (1952). Safety First and the Holding of Assets. Econometrica, 20(3), 431–449.

Shefrin, H., & Statman, M. (2000). Behavioral Portfolio Theory. *The Journal of Financial and Quantitative Analysis*, 35(2), 127–151.

Sofat, Rajni & Hiro, Preeti. (2010). Strategic financial management. published by prentice hall of India, New Delhi.

Telser, L. G. (1955). Safety First and Hedging. *The Review of Economic Studies*, 23(1), 1–16.

Thaler, R. (1985). Mental Accounting and Consumer Choice. *Marketing Science*, *4*(3), 199–214. https://doi.org/10.1287/mksc.4.3.199

Tous, L. F. P. (2021, October 20). *Trading haute fréquence*. La finance pour tous. https://www.lafinancepourtous.com/decryptages/marches-financiers/produits-financiers/trading-haute-frequence/

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