

École polytechnique de Louvain

Modeling complex objects within ray tracing simulations applied to beyond 5G communications

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Abstract

Over the years engineers have consistently tackled the challenge of satisfying the ever-growing demand for faster wireless data transfer rates and lower latency. One approach to achieve this, is to operate at higher frequencies; however, this introduces new challenges, due to higher frequency signals' increased susceptibility to interference from minor environmental elements. In turn, this increases the difficulty of channel modelling, which provides crucial information required by engineers to design their systems adequately.

Traditionally, engineers relied on costly and time-consuming measurement campaigns, or empirical models derived from extensive channel measurements, to gain insight into channel characteristics. However, at higher frequencies these empirical models often fail to capture the complexity of specific environments, due to the increasing influence of minor environmental details.

To address these concerns, ray tracers have been developed and are, in theory, able to provide channel characteristics as accurate as the model of the environment fed to them.

In this thesis, a ray tracer is developed¹ and validated against real world measurements at 12.5 GHz and 30 GHz. Our ray tracer uses an innovative pathfinding technique, *min path tracing*, which seamlessly handles diffraction and curved geometries. The fields propagated by the rays are computed using Geometrical Optics and the Uniform Theory of Diffraction.

The functionality of the ray tracer is showcased through an analysis of the channel characteristics of Place de La Vieille Halle aux Blés in Brussels, at 12.5 GHz, under two different transmitting antenna placements: at street level, or atop a building. Our findings suggest that placing the transmitting antenna at street level would lead to a much lower temporal dispersion of the channel. Additionally, we observed that spatial channel dispersion in the elevation plane is also substantially lower at street level. Overall, the lower channel dispersion achieved by street level antennas would facilitate higher potential wireless data transfer speeds, compared to antennas mounted atop buildings.

¹And is openly available at <https://github.com/MaximeLeu/RayTracing.git>

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Acronyms

GO	Geometrical Optics
GTD	Geometrical Theory of Diffraction
LOS	Line Of Sight
RMS	Root Mean Square
UTD	Uniform Theory of Diffraction
5G	Fifth generation technology standard for broadband cellular networks

Symbols

Symbol	Description	Unit
A	Vector magnetic potential	V s m^{-1}
B	Magnetic flux density	T
E	Electric field	V m^{-1}
H	Magnetic field strength	A m^{-1}
J	Current density	A m^{-2}
Δh_0	Standard deviation of surface height	m
ϵ	Permittivity	F m^{-1}
η	Wave impedance	Ω
$\Gamma_{\perp}, \Gamma_{\parallel}$	Perpendicular, parallel Fresnel reflection coefficients	-
λ	Wavelength	m
$\overline{\overline{\mathbf{D}}}$	Dyadic diffraction coefficient	-
$\overline{\overline{\mathbf{R}}}$	Dyadic reflection coefficient	-
μ	Magnetic permeability	H m^{-1}
ω	Angular frequency	rad s^{-1}
ϕ	Electric potential	V
ρ	Volume charge density	C m^{-3}
σ	Electrical conductivity	S m^{-1}
c	Speed of light	m s^{-2}

Notation

- Regular symbols denote scalar quantities. Ex. ϕ
- Boldface symbols denote vector quantities. Ex. $\boldsymbol{\phi}$
- Boldface symbols with a hat denote unit vectors. Ex. $\hat{\mathbf{e}}_i$
- Boldface symbols with two overline bars denote matrix quantities. Ex. $\overline{\overline{\mathbf{D}}}$
- $\triangleq$ means “is defined as”.

Chapter 1

Introduction

Characterizing the wireless propagation channel is an important task to ensure the proper design of a wireless system. Unfortunately, this task is made difficult by the costly and time consuming nature of channel measurements [1, 2, 3, 4, 5]. As a result, channel characterization is most often done using channel models. Three main types of channel models exist: empirical, stochastic and deterministic [1], though empirical and stochastic models are sometimes regrouped into one category.

Empirical channel models are based on extensive channel measurements in typical propagation scenarios [6]. They aim to provide reasonably accurate predictions based on a limited set of input data, such as frequency, antenna height, and type of environment (urban/suburban/rural/indoor/hilly/etc.) at a cheap computational cost. Unfortunately, these models are often limited to a narrow range of frequencies, and because they are strongly related to the measurement environment from which they are derived [1, 7], they are either invalid or require tuning when applied in other environments.

Stochastic channel models describe the channel through statistical parameters that are derived from measurement campaigns [1, 8]. These models are computationally inexpensive (compared to deterministic models) and capable of relatively accurately simulating a channel without requiring too many input parameters. However, just like empirical models, their validity is questionable when they are used in an environment which is not well represented in the measurement campaigns [7, 8, 9].

On the other hand, deterministic channel models rely on the simulation of the physical propagation of electromagnetic waves, and can provide channel characteristics as accurate as the model of the environment they are provided with. Nevertheless, providing an accurate representation of an environment to deterministic models can be challenging, and their computational cost can quickly become prohibitive. Despite these limitations, their high accuracy has already been demonstrated in many studies [2, 3, 4, 10], making them a valuable tool to avoid the tedious and expensive measurements campaigns.

The most accurate deterministic methods are those that directly solve Maxwell's equations, such as the Method of Moments or the Finite Difference Time Domain Method. Unfortunately the computational cost of those methods is excessive when applied to real world environments, and their use is therefore limited to very specific cases, or to circuits modelling [11]. For this reason, the most commonly used family of deterministic models used for channel modelling are the ray based approaches, which approximate Maxwell's equations following Geometrical Optics (GO) [1].

GO is a theory which relies on the assumption that the studied electromagnetic waves are very high frequency, such that all scatterers in the modelled environment are very large electrically (their dimensions vary negligibly over intervals on the order of a wavelength). Under this assumption GO postulates that the wave nature of electromagnetic waves can be ignored, and that their propagation may be described using rays. However, a significant consequence of representing electromagnetic waves as simple rays, is that it completely fails to model diffraction. This neglect was addressed by J.Keller in 1962 [12], who introduced the Geometrical Theory of Diffraction (GTD), an extension of GO, asserting the existence of diffracted rays, produced when rays illuminate the edges, corners, and vertices of scatterers. Keller also introduced a few laws enabling the computation of the electromagnetic fields propagated by these diffracted rays. However, the diffraction coefficients Keller introduced in his laws become singular on the incident and reflection shadow boundaries (illustrated on Figure 1.1), which could contain observation points of interest. This shortcoming was later addressed by Kouyoumjian and Pathak in 1982 [13] who introduced the Uniform Theory of Diffraction (UTD). In the UTD, the diffraction coefficients are multiplied by a transition function, which prevents points on the incident and reflection shadow boundaries from being singular, and thus enable a reliable computation of the diffracted field almost¹ everywhere in the scene [7]. Thanks to Keller's, Pathak, and Kouyoumjian's contributions, electromagnetic fields propagated by rays could accurately be computed, paving the way for the utilization of ray tracing techniques for channel modelling.

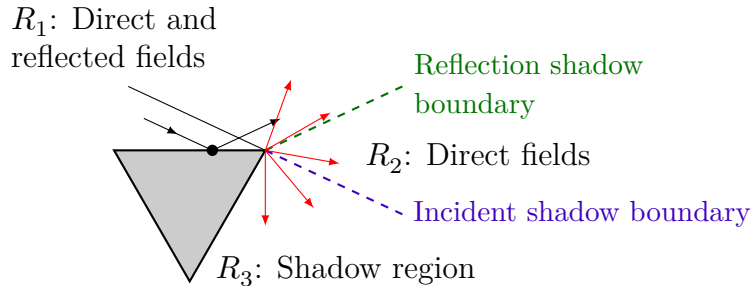


Figure 1.1: Geometrical optics representation of a diffraction on a wedge. Three regions should be considered: R_1 where direct, reflected and diffracted fields exist, R_2 where the direct and diffracted fields exist and R_3 where *only* diffracted fields exists.

¹More details about this in chapter 5.

Two types of ray tracing techniques are found in the literature: Shooting and Bouncing Rays and Ray-Tracing. In Shooting and Bouncing Rays, a finite amount of rays is shot from the emitter in all directions, according to a certain angular discretization. Then, the propagation of each ray is traced until it either reaches a small area around the receiver or is lost after a certain number of interactions (reflections, diffractions, transmissions). Conversely, in Ray-Tracing, *all* ray paths undergoing up to a given number of interactions and *exactly* linking the emitter and receiver are computed. Knowledge of these paths is then used to compute the strength of the electromagnetic field reaching the receiver and infer various channel characteristics. While Ray-Tracing often delivers better accuracy than Shooting and Bouncing Rays, it also comes at a higher computational cost [14].

In this thesis we implement a ray tracer in Python. To accomplish this, we need to develop three fundamental components. The first component's role is to build an accurate model environment, and store all the relevant physical and electromagnetic characteristics of the objects composing it. The second component is tasked with finding and storing all the paths connecting two points of the model environment, undergoing up to a given number of interactions. The last component is responsible for computing how the electromagnetic fields propagate along these paths and interact with the model environment.

This work elaborates on the theoretical concepts required to construct each of these components. First, Part I reviews some essential radiation and antenna theory, serving as a foundation for Part II. The latter presents GO and the UTD, pillars upon which the third component of our ray tracer is built. Following this, Part III provides a succinct explanation of the process of building the model environment and an overview of the methodology employed to identify all the possible paths between two of its points. We stay succinct because we are building upon [14], a previous work that developed these two components. Then, Part IV validates the implementation of our ray tracer² against real world measurements. Finally, before concluding, Part V demonstrates the functionality of the ray tracer at 12.5 GHz through an analysis of the channel characteristics of Place de La Vieille Halle aux Blés in Brussels, considering two different transmitting antenna positions: at street level, or atop a building.

²Openly available at <https://github.com/MaximeLeu/RayTracing.git>

Part I

Radiation & antenna theory

Chapter 2

Far field approximation

This chapter will walk through the computation of the electric field radiated by an antenna in the far field region. This is the first step of our journey to understand how electromagnetic waves can be represented by simple rays, and compute the electromagnetic fields propagating along those rays.

2.1 The wave equation

In *free space* Maxwell's equations may be written:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 0 && \text{Gauss' Law,} \\ \nabla \cdot \mathbf{B} &= 0 && \text{Gauss' magnetism Law,} \\ \nabla \times \mathbf{B} &= \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} && \text{Ampere's Law,} \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} && \text{Maxwell-Faraday's equation,}\end{aligned}\tag{2.1}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, t is time, $\nabla \cdot$ represents the divergence operator and $\nabla \times$ the curl operator, $c \triangleq \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ is the speed of light in free space, while μ_0 and ϵ_0 are the magnetic permeability and electric permittivity of free space.

Since the space and time coordinates are independent, we can take the curl of each side of the Maxwell-Faraday equation:

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \nabla \times \mathbf{B}.\tag{2.2}$$

Substituting Ampere's law on the right side of the equation we get:

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}.\tag{2.3}$$

Applying the following vector identity: $\nabla \times \nabla \times \mathbf{v} = \nabla(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$, we can write:

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}.\tag{2.4}$$

Finally, substituting Gauss' law on the left hand side leads us to the wave equation:

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \text{Wave equation.} \quad (2.5)$$

One of the solutions of this wave equation is a planar wave propagating in the direction $\hat{\mathbf{k}}$:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0 \cos(\hat{\mathbf{k}} \cdot \mathbf{r} - \omega t + \phi_0) \\ &= \text{Re} \left(\mathbf{E}_0 e^{j\phi_0} e^{j(\hat{\mathbf{k}} \cdot \mathbf{r} - \omega t)} \right), \end{aligned} \quad (2.6)$$

where $\mathbf{r}$ is the point of observation, $\mathbf{E}_0$ is a constant vector, $\omega \triangleq 2\pi f$ and ϕ_0 is a constant phase. Note that because electromagnetic waves are transverse, if $\hat{\mathbf{k}}$ is the direction of propagation of the wave, $\mathbf{E}$ and $\mathbf{H}$ must always be perpendicular to it.

So far we have proved that electromagnetic fields in free space obey the wave equation, however we may be interested to know how we can generate such a field. To this end, in the next section we will study the general formulation of Maxwell's equations.

2.2 Radiation potentials

The general formulation of Maxwell's equations is:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{1}{\epsilon} \rho && \text{Gauss' Law,} \\ \nabla \cdot \mathbf{B} &= 0 && \text{Gauss' magnetism Law,} \\ \nabla \times \mathbf{B} &= \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} && \text{Ampere's Law,} \\ \nabla \times \mathbf{E} &= \mu \mathbf{J} - \frac{\partial \mathbf{B}}{\partial t} && \text{Maxwell-Faraday's equation,} \end{aligned} \quad (2.7)$$

where $\mathbf{J}$ is the current density, and ρ is the volume charge density. It is easier to solve these equations by working with electromagnetic potentials. Therefore, we define the electric scalar potential φ and the magnetic vector potential $\mathbf{A}$ such that:

$$\boxed{\begin{aligned} \mathbf{E} &= -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}, \\ \mathbf{B} &= \nabla \times \mathbf{A}. \end{aligned}} \quad (2.8)$$

It can be shown that as long as φ and $\mathbf{A}$ satisfy the above, $\mathbf{E}$ and $\mathbf{B}$ will satisfy Maxwell's equations. It is interesting to note the *gauge invariance* property of the potentials, meaning that defining $\varphi = f(x, y, z)$ or $\varphi = f(x, y, z) + k$ will leave $\mathbf{E}$ unchanged, and a similar conclusion is drawn for $\mathbf{A}$. Therefore, the potentials for a given electromagnetic field are not unique, and should be constrained by a gauge condition. In our case, the *Lorentz gauge* will be useful:

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0 \quad \text{Lorentz gauge.} \quad (2.9)$$

Notice that due to the two following vector identities, Gauss's magnetism law and Ampere's law are satisfied for any vector and scalar potential.

$$\nabla \cdot \nabla \times \mathbf{v} = 0, \quad \nabla \times \nabla \mathbf{v} = 0. \quad (2.10)$$

Substituting the potentials in the two remaining Maxwell's equations gives:

$$\begin{aligned} \nabla^2 \varphi + \frac{\partial}{\partial t} \nabla \cdot \mathbf{A} &= -\frac{1}{\epsilon_0} \rho, \\ \nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\mu \mathbf{J} + \nabla \left(\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} \right). \end{aligned} \quad (2.11)$$

And using the Lorentz gauge they can be written:

$$\begin{aligned} \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi &= \frac{1}{\epsilon} \rho, \\ \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} &= \mu \mathbf{J}. \end{aligned} \quad (2.12)$$

If ρ and $\mathbf{J}$ are known, the causal solutions of the above wave equations are called the retarded potentials and are given by:

$$\begin{aligned} \varphi(\mathbf{r}, t) &= \frac{1}{4\pi\epsilon} \int_V \frac{\rho(\mathbf{r}', t - \frac{R}{c})}{R} d^3\mathbf{r}', \\ \mathbf{A}(\mathbf{r}, t) &= \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}', t - \frac{R}{c})}{R} d^3\mathbf{r}', \end{aligned} \quad (2.13)$$

where $R = |\mathbf{r} - \mathbf{r}'|$ is the distance between the observation point $\mathbf{P}$ and the source point $\mathbf{P}'$. Assuming a sinusoidal time dependence $e^{j\omega t}$ for all quantities¹, the retarded solutions become:

$$\begin{aligned} \varphi(\mathbf{r}) &= \frac{1}{4\pi\epsilon} \int_V \frac{\rho(\mathbf{r}') e^{-jkR}}{R} d^3\mathbf{r}', \\ \mathbf{A}(\mathbf{r}) &= \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}') e^{-jkR}}{R} d^3\mathbf{r}', \end{aligned} \quad (2.14)$$

where $k \triangleq \frac{2\pi}{\lambda}$ is the free space wavenumber. Due to the R term present in them, these integrals are hard to solve. Thankfully, because we are interested in fields that have radiated far away from their current source we will be able to use the *far field approximation* to simplify them.

¹so $\varphi(\mathbf{r}, t) = \varphi(\mathbf{r}) e^{j\omega t}$, $\mathbf{A}(\mathbf{r}, t) = \mathbf{A}(\mathbf{r}) e^{j\omega t}$, $\rho(\mathbf{r}, t) = \rho(\mathbf{r}) e^{j\omega t}$, $\mathbf{J}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}) e^{j\omega t}$.

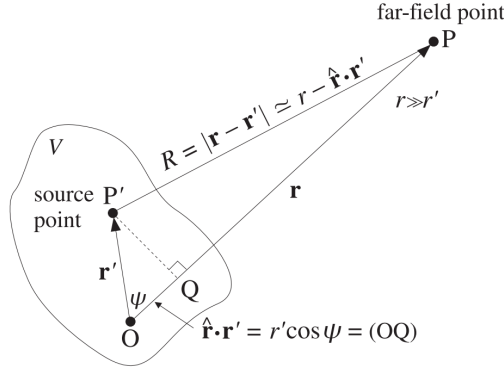


Figure 2.1: Far field approximation illustration: at far distances $|PP'| \approx |PQ|$ [15].

This approximation assumes that the observation point $\mathbf{P}$ is very far away from the source point $\mathbf{P}'$, where *very far* means “much further than the typical spatial extent $|\mathbf{r}'|$ of the current distribution ” [15].

Using Al-Kashi’s theorem on the triangle OPP' we can write:

$$\begin{aligned}
 R &= \sqrt{|\mathbf{r}'|^2 + |\mathbf{r}|^2 - 2|\mathbf{r}||\mathbf{r}'| \cos \psi} \\
 &= |\mathbf{r}| \sqrt{1 - 2 \frac{|\mathbf{r}'|}{|\mathbf{r}|} \cos(\psi) + \frac{|\mathbf{r}'|^2}{|\mathbf{r}|^2}} \\
 &\quad \downarrow \text{ 2nd order Taylor series expansion of } \sqrt{1+x} \\
 &\approx |\mathbf{r}| \left(1 - \frac{|\mathbf{r}'|}{|\mathbf{r}|} \cos \psi + \frac{|\mathbf{r}'|^2}{2|\mathbf{r}|^2} - \frac{|\mathbf{r}'|^2}{2|\mathbf{r}|^2} \cos^2 \psi \right) \\
 &\quad \downarrow \text{ Only keeping terms of the first order } \\
 &\approx |\mathbf{r}| - |\mathbf{r}'| \cos \psi \\
 &= |\mathbf{r}| - \mathbf{r}' \cdot \hat{\mathbf{r}},
 \end{aligned} \tag{2.15}$$

where the Taylor series approximation is justifiable when $|\mathbf{r}| \gg |\mathbf{r}'|$. Because R appears in the exponential term e^{-jkR} , in order to justify dropping the second order approximation terms we must also impose $k \frac{|\mathbf{r}'|^2}{|\mathbf{r}|} \ll 1$. Assuming $2|\mathbf{r}'| \approx d$, meaning that the size d of the aperture of the antenna is roughly twice the spatial extent $|\mathbf{r}'|$ of the current distribution, we reach² a more popular form of these two conditions:

$$|\mathbf{r}| \gg d, \quad \text{and} \quad |\mathbf{r}| \gg \frac{2d^2}{\lambda} \quad \text{Far field conditions,} \tag{2.16}$$

Using this approximation the expression of the retarded potentials simplifies to:

$$\begin{aligned}
 \varphi(\mathbf{r}) &\approx \frac{1}{4\pi\epsilon} \int_V \frac{\rho(\mathbf{r}') e^{-jk(|\mathbf{r}| - \mathbf{r}' \cdot \hat{\mathbf{r}})}}{|\mathbf{r}| - \mathbf{r}' \cdot \hat{\mathbf{r}}} d^3\mathbf{r}', \\
 \mathbf{A}(\mathbf{r}) &\approx \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}') e^{-jk(|\mathbf{r}| - \mathbf{r}' \cdot \hat{\mathbf{r}})}}{|\mathbf{r}| - \mathbf{r}' \cdot \hat{\mathbf{r}}} d^3\mathbf{r}'.
 \end{aligned} \tag{2.17}$$

²To be exact, we will reach $|\mathbf{r}| \gg \frac{d}{2}$, and $|\mathbf{r}| \gg \frac{\pi d^2}{2\lambda}$, which are then relaxed into their more popular forms by convention.

Further approximating $R \approx |\mathbf{r}| - \mathbf{r}' \cdot \hat{\mathbf{r}} \approx |\mathbf{r}|$ in the denominators but not the exponents³, we finally obtain:

$$\boxed{\begin{aligned}\varphi(\mathbf{r}) &\approx \frac{e^{-jk|\mathbf{r}|}}{4\pi\epsilon|\mathbf{r}|} \int_V \rho(\mathbf{r}') e^{jk\mathbf{r}' \cdot \hat{\mathbf{r}}} d^3\mathbf{r}', \\ \mathbf{A}(\mathbf{r}) &\approx \frac{\mu e^{-jk|\mathbf{r}|}}{4\pi|\mathbf{r}|} \underbrace{\int_V \mathbf{J}(\mathbf{r}') e^{jk\mathbf{r}' \cdot \hat{\mathbf{r}}} d^3\mathbf{r}'}_{\mathbf{F}(\mathbf{r})},\end{aligned}} \quad (2.18)$$

where $\mathbf{F}(\mathbf{r})$ is called the radiation vector, or radiation pattern.

2.3 Spherical coordinate system

It is easier to study radiation fields in a spherical coordinate system whose origin is the radiation source. The spherical coordinate system we use is represented on Figure 2.2 and conversion from cartesian to spherical coordinates is achieved from Equation 2.19.

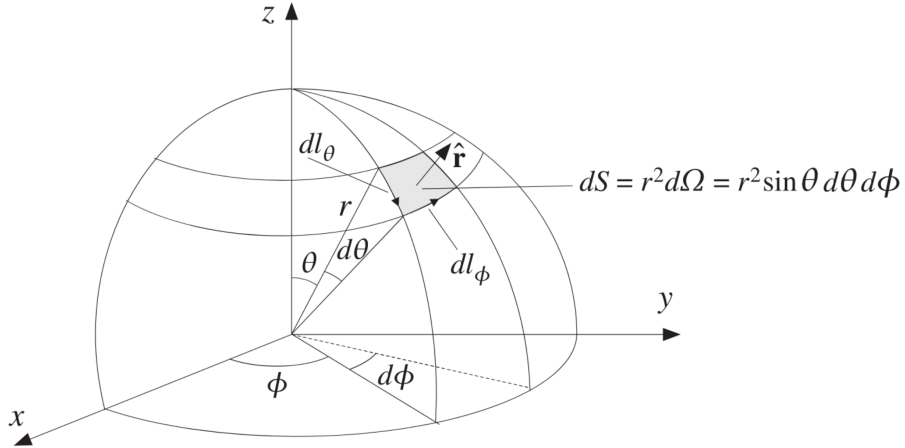


Figure 2.2: Spherical coordinate system and solid angle [15].

$$\begin{aligned}\hat{\mathbf{r}} &= \hat{\mathbf{x}} \cos \phi \cos \theta + \hat{\mathbf{y}} \sin \phi \cos \theta + \hat{\mathbf{z}} \sin \theta, \\ \hat{\boldsymbol{\theta}} &= \hat{\mathbf{x}} \cos \phi \sin \theta + \hat{\mathbf{y}} \sin \phi \sin \theta - \hat{\mathbf{z}} \cos \theta, \\ \hat{\boldsymbol{\phi}} &= -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi,\end{aligned} \quad (2.19)$$

where for a vector (x, y, z) of length r we have $\theta = \cos^{-1} \frac{z}{r}$ and $\phi = \tan^{-1} \frac{y}{x}$.

³In order to justify $e^{-jk(|\mathbf{r}| - \mathbf{r}' \cdot \hat{\mathbf{r}})} \approx e^{-jk|\mathbf{r}|}$, we should impose $k(\mathbf{r}' \cdot \hat{\mathbf{r}}) \approx 0$ which eventually reduces to $k|\mathbf{r}'| \ll 1$. This condition is hard to satisfy at high frequencies, hence why we avoid it.

2.4 Radiation fields

Since we have derived the expressions of the radiation potentials (Equation 2.18), we can plug them into their definitions (given in Equation 2.8) to compute the radiation fields. As a last approximation we will ignore all the terms of the radiation fields that fall off faster than $\frac{1}{r}$, because they become negligible in the far field. We find:

$$\mathbf{E}(r, \theta, \phi) = -jk\eta \frac{e^{-jk|\mathbf{r}|}}{4\pi|\mathbf{r}|} (\hat{\mathbf{r}} \times \mathbf{F}(\theta, \phi)) \times \hat{\mathbf{r}}, \quad (2.20)$$

where $\eta \triangleq \frac{|\mathbf{E}|}{|\mathbf{H}|} = \sqrt{\frac{\mu}{\epsilon}}$ is the intrinsic impedance of the medium. If we decompose the radiation vector into its radial and transverse part, as shown on Figure 2.3 we have:

$$\begin{aligned} \mathbf{F}(\theta, \phi) &\triangleq \hat{\mathbf{r}}F_r + \hat{\boldsymbol{\theta}}F_\theta + \hat{\boldsymbol{\phi}}F_\phi \\ &= \hat{\mathbf{r}}F_r + \mathbf{F}_\perp(\theta, \phi) \\ &= \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{F}(\theta, \phi)) + (\hat{\mathbf{r}} \times \mathbf{F}(\theta, \phi)) \times \hat{\mathbf{r}}. \end{aligned} \quad (2.21)$$

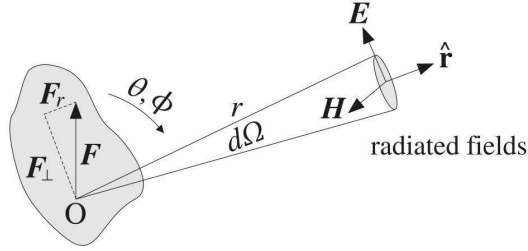


Figure 2.3: Decomposition of the radiation vector into its radial and transverse components [15].

Then, using this decomposition in Equation 2.20, the radiated field may be written:

$$\begin{aligned} \mathbf{E}(r, \theta, \phi) &= -jk\eta \frac{e^{-jk|\mathbf{r}|}}{4\pi|\mathbf{r}|} (\hat{\boldsymbol{\theta}}F_\theta + \hat{\boldsymbol{\phi}}F_\phi) \\ &= -jk\eta \frac{e^{-jk|\mathbf{r}|}}{4\pi|\mathbf{r}|} \mathbf{F}_\perp(\theta, \phi). \end{aligned} \quad (2.22)$$

Therefore highlighting the fact that the electric field is transverse. We conclude that the electric field radiated by an antenna in the far field region is given by:

$$\mathbf{E}(r, \theta, \phi) = \frac{-jk\eta}{4\pi} \cdot \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|} \cdot (\hat{\mathbf{r}} \times \mathbf{F}(\theta, \phi)) \times \hat{\mathbf{r}} \quad [\text{V m}^{-1}],$$

(2.23)

Chapter 3

Antennas and power

So far, we have computed the electric field radiated by an antenna in its far field region. In this chapter we provide a reminder of some essential antenna theory, which is required to understand how to combine all the electromagnetic waves arriving at an antenna, to compute the total power it gathers.

It is important to note that all the quantities given in this chapter, relate to the reference frame of the antenna. The reference frame of the antenna, illustrated in Figure 3.1, is an orthonormal reference frame whose origin is located in the middle of the aperture of the antenna. We will define the x-axis such that it is in the aperture's plane and parallel to the $\hat{\mathbf{x}}'\hat{\mathbf{y}}'$ plane, representing the ground. The y-axis is defined to be perpendicular to the x-axis, and in the aperture's plane, finally, the z-axis is normal to the aperture's plane formed by $\hat{\mathbf{x}}\hat{\mathbf{y}}$, pointing outwards. The conversion of vectors and points between the two reference frames is detailed in Appendix A.

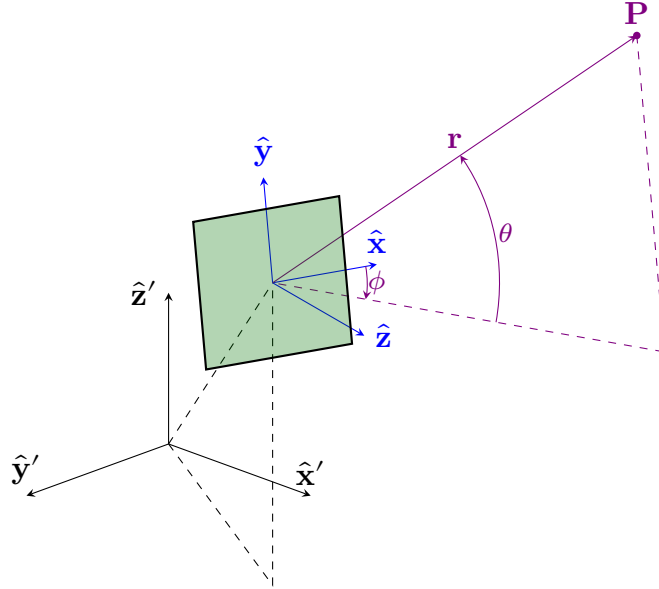


Figure 3.1: Antenna's aperture, and associated reference frame, relative to the world reference frame.

3.1 Antenna theory

Principle of reciprocity the properties of an antenna as an emitter are the same¹ as its properties as a receiver, and vice versa.

In the following definitions, let $\mathbf{r} = (r, \theta, \phi)$ be a vector in the direction of propagation of the electromagnetic wave, as represented on Figure 3.1.

Poynting vector noted $\mathbf{S}(\mathbf{r})$, is defined by:

$$\mathbf{S}(\mathbf{r}, t) \triangleq \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t). \quad (3.1)$$

The above is the instantaneous Poynting vector, taking the time average of $\mathbf{S}(\mathbf{r}, t)$ gives us a more useful quantity called the *time averaged Poynting vector* defined as:

$$\mathbf{S}(\mathbf{r}) \triangleq \frac{1}{2} \text{Re} \left[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r}) \right], \quad (3.2)$$

where the factor $\frac{1}{2}$ appears because $\mathbf{E}$ and $\mathbf{H}$ are peak to peak quantities, but should be removed when using RMS values. Computing the cross product leads us to:

$$\begin{aligned} \mathbf{S}(\mathbf{r}) &= \frac{1}{2} \cdot \text{Re} \left[|\mathbf{E}(\mathbf{r})| \cdot |\mathbf{H}^*(\mathbf{r})| \right] \hat{\mathbf{r}}, \quad \text{since } \mathbf{E} \perp \mathbf{H} \\ &= \frac{1}{2Z_0} |\mathbf{E}(\mathbf{r})| \cdot |\mathbf{E}^*(\mathbf{r})| \hat{\mathbf{r}}, \quad \text{since } Z_0 \triangleq \frac{|\mathbf{E}|}{|\mathbf{H}|} \\ &= \frac{|\mathbf{E}(\mathbf{r})|^2}{2Z_0} \hat{\mathbf{r}} \quad [\text{W m}^{-2}]. \end{aligned} \quad (3.3)$$

Where $Z_0 \simeq 120\pi$ is the impedance of free space and $\hat{\mathbf{r}}$ is a unit vector in the direction of propagation of the electromagnetic wave. The length of the Poynting vector is called the *radiation power density*.

Radiation intensity noted $U(\mathbf{r})$, is defined as the power radiated from an antenna per unit solid angle:

$$U(\mathbf{r}) = r^2 |\mathbf{S}(\mathbf{r})| \quad [\text{W sr}^{-1}]. \quad (3.4)$$

When the radiation intensity is plotted in a two-dimensional plane, with one axis representing the azimuthal plane, and the other the elevation plane, the resulting plot is called the *antenna radiation pattern*. The total power P_{rad} radiated by an antenna is related to the radiation intensity by:

$$P_{rad} = \int_0^{2\pi} \int_0^\pi U(\mathbf{r}) \sin \theta d\theta d\phi \quad [\text{W}]. \quad (3.5)$$

¹when operating in a non-magnetic medium.

Antenna directivity noted $D(\mathbf{r})$, is defined as the ratio of the radiation intensity $U(\mathbf{r})$ in a particular direction to the radiation intensity $U_0(r)$ of an isotropic antenna.

$$D(\mathbf{r}) \triangleq \frac{U(\mathbf{r})}{U_0(r)} = \frac{U(\mathbf{r})}{P_{rad}/(4\pi r^2)}. \quad (3.6)$$

It can be shown that the maximum available directivity of an antenna is

$$D_{max} = \frac{4\pi}{\lambda^2} A, \quad (3.7)$$

where A is the aperture area of the antenna.

Antenna gain noted $G(\mathbf{r})$, is analogous to directivity, but normalized to the total power absorbed by the antenna instead of the total radiated power:

$$G(\mathbf{r}) = \eta_{rad} D(\mathbf{r}), \quad (3.8)$$

where η_{rad} is the radiation efficiency of the antenna.

3.1.1 Antenna efficiency

Radiation efficiency is defined by the ratio of the power P_t provided to the antenna to the power P_{rad} actually radiated by the antenna:

$$\eta_{rad} \triangleq \frac{P_{rad}}{P_t}. \quad (3.9)$$

It may be represented by the product of other sub efficiencies:

$$\eta_{rad} = \eta_r \eta_{abs} \eta_{pol}, \quad (3.10)$$

where η_r represents the impedance mismatch between the antenna and its feed line, η_{abs} the ohmic losses in the antenna and η_{pol} the *polarization efficiency*. In the rest of this work we will assume perfect matching: $\eta_r = 1$, and lossless antennas: $\eta_{abs} = 1$.

Polarization efficiency “measures the loss in system sensitivity that results when the polarization of an incident field is not matched to the receiving antenna polarization” [16]. It is given by

$$\eta_{pol} = |\hat{\mathbf{e}}_r \cdot \hat{\mathbf{e}}_t|^2, \quad (3.11)$$

where $\hat{\mathbf{e}}_t$ and $\hat{\mathbf{e}}_r$ are unit vectors representing the polarisation of the receiving antenna, and of the incoming electromagnetic wave respectively.

Aperture efficiency also known as effective aperture area, is a useful metric when the aperture of the antenna is not clearly defined. It is defined as “the ratio between the total power P_r received at the antenna port, and the power density $|\mathbf{S}|$ of the plane wave coming in from the pointing direction (θ, ϕ) of the antenna” [17]:

$$A_e(\theta, \phi) \triangleq \frac{P_r}{|\mathbf{S}|} = A \cdot \frac{\eta_{rad} \cdot D(\theta, \phi)}{D_{max}} = A \cdot \frac{G(\theta, \phi)}{D_{max}} = \frac{\lambda^2}{4\pi} \cdot G(\theta, \phi) \quad [\text{m}^2]. \quad (3.12)$$

3.2 Power at the receiver

Using the definitions given in section 3.1 it follows that the power harvested by the receiving antenna from an electromagnetic wave coming from the direction (θ, ϕ) is given by:

$$\begin{aligned} P_{rx}(\theta, \phi) &= A_{e,rx}(\theta, \phi) \cdot |\mathbf{S}| \cdot \eta_{pol} \\ &= A_{e,rx}(\theta, \phi) \cdot \frac{|\mathbf{E}|^2}{2Z_0} \cdot \eta_{pol} \quad [\text{W}], \end{aligned} \quad (3.13)$$

where $A_{e,rx}$ is the effective aperture area of the receiver antenna, and $|\mathbf{S}|$ is the power density of the incoming electromagnetic wave.

The total power $P_{rx,tot}$ at the receiving antenna may be computed by considering the contribution of the electromagnetic fields originating from each of the n incoming electromagnetic waves, called multipaths, individually:

$$P_{rx,tot} = \sum_{i=0}^n P_{rx}(\theta_i, \phi_i) = \frac{1}{2Z_0} \cdot \sum_{i=0}^n |\mathbf{E}_i|^2 A_{e,rx}(\theta_i, \phi_i) \cdot \eta_{pol,i} \quad [\text{W}]. \quad (3.14)$$

To be exact we should combine all the multipath components before calculating their norm. However, because geometrical variations of the order of a wavelength already lead to significant phase change of each multipath component, we average the received power by considering each multipath independently. This is because at the frequencies we work with, achieving a level of accuracy of the order of a wavelength when creating our geometry is difficult as we will explain in section 7.1.

3.2.1 Normalization

For comparison purposes, in all further plots of this thesis, we will normalize the received power to the transmit power P_t :

$$P_{rx,\text{dB}} = 10 \log_{10} \frac{P_{rx,\text{watts}}}{P_{t,\text{watts}}} \quad [\text{dB}] \quad (3.15)$$

3.3 Friis transmission formula

In line of sight cases, it can be useful to compare the results of our ray tracer to Friis transmission formula.

Friis formula relates the power received at the receiver to the power transmitted by the transmitter, when the two antennas are separated by a distance $d > \frac{2D^2}{\lambda}$, where D is the largest dimension of either antenna:

$$\frac{P_{rx}}{P_{tx}} = (1 - |\Gamma_{tx}|^2) \cdot (1 - |\Gamma_{rx}|^2) \cdot |\hat{\mathbf{e}}_t \cdot \hat{\mathbf{e}}_r|^2 \cdot G_{tx}(\theta, \phi) \cdot G_{rx}(\theta, \phi) \cdot \left(\frac{\lambda}{4\pi d} \right)^2.$$

In practice when computing path loss, we will assume that the antennas are perfectly matched to their respective line or load: $|\Gamma_{tx}|^2 = |\Gamma_{rx}|^2 = 0$. Moreover, we will

also ignore polarisation mismatch between the incoming wave and the RX antenna: $|\hat{\mathbf{e}}_t \cdot \hat{\mathbf{e}}_r|^2 = 1$. Finally, we will consider that the antennas are aligned for maximum radiation between them: $G_{tx}(\theta, \phi) = G_{tx}$ and $G_{RX}(\theta, \phi) = G_{rx}$, where G_{rx} and G_{tx} are the peak gains. Therefore, the formula reduces to:

$$\frac{P_{rx}}{P_{tx}} = G_{tx} \cdot G_{rx} \left(\frac{\lambda}{4\pi d} \right)^2. \quad (3.16)$$

Part II

Ray optics

Chapter 4

Geometrical optics

In this chapter we explain how the propagation of high frequency electromagnetic waves may be described using simple rays. We then link this concept to Maxwell's equations, and introduce the notion of amplitude, polarization and phase to it. We continue by detailing how the phase, amplitude and polarization of those rays are affected when they reflect specularly on a surface, before finishing the chapter with a discussion about diffuse reflections.

4.1 The concept of rays

Although Maxwell's equations provide a rigorous way of solving the problem of light propagation in a scene, exact solutions can rarely be found. This is where approximations such as the one made by ray optics, also known as geometrical optics (GO), come into play. This approximation is based on the hypothesis that “when light propagates around objects that are much larger than its wavelength, the wave nature of light may not be revealed, and its propagation may simply be described using rays” [18].

Geometrical optics relies on four postulates [18]:

1. Light rays travel out of a source independently.
2. In a homogeneous medium, light rays travel along straight lines
3. Light rays obey the law of reflection: $\theta_i = \theta_r$.
4. Light rays obey the law of refraction: Snell's law.

The last three postulates may all be derived from Fermat's principle, stating that the path that a ray of light will choose, to travel between two points, is the path that can be travelled in the least time. The principle of reversibility, stating that a light ray whose direction is reversed will follow the same path backwards, extends Fermat's principle to account for the direction of propagation.

The laws of reflection and refraction are a direct consequence of Fermat's principle. This is illustrated for the law of reflection on Figure 4.1: any reflection path linking

S and O whose reflection angle θ_r is different from the incident angle θ_i , would be longer than the path where $\theta_r = \theta_i$, and therefore would not be the path traversed in the least time, which would break Fermat's principle. The law of refraction can be found similarly.

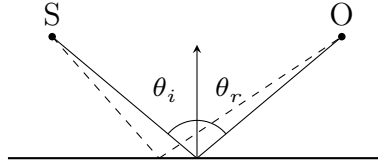


Figure 4.1: The shortest reflection path between S and O is the one following the law of reflection ($\theta_r = \theta_i$).

Let us consider a ray $\mathbf{r}(x(s), y(s), z(s))$ and its length s . The ray equation:

$$\frac{d}{ds} \left(n \frac{d\mathbf{r}}{ds} \right) = \nabla n \quad \text{Ray equation.} \quad (4.1)$$

describes the path the ray will follow in a medium with a refractive index $n(x, y, z)$, and is a consequence of Fermat's principle. The rectilinear propagation of rays in a homogeneous medium ($\nabla n = 0$) is evident from this equation, because $\nabla n = 0$ implies that $n \frac{d\mathbf{r}}{ds}$ is a constant, and thus that $\mathbf{r}$ is a first order polynomial.

The fact that high frequency electromagnetic waves (called rays) reflect specularly after an interaction with an object which is electrically large¹, is a consequence of Huygens principle. This principle states that every point on a wavefront is itself the source of spherical wavelets. Therefore, when a wavefront hits an object, scattered fields will be reradiated in all directions. If the object is an infinite plane surface, the contribution of all scattered fields will interfere destructively, except for those coming from the point of reflection.

4.2 Linking electromagnetism theory to GO

Geometrical optics can be linked to electromagnetism theory. Since GO is based upon the assumption that objects are large compared to the electromagnetic wave's wavelength, in the following we will assume an asymptotically high frequency field, that is: $f \rightarrow \infty$.

4.2.1 The eikonal equation

Although we have shown that the ray equation may be derived from Fermat's principle, a more rigorous derivation can be obtained from Maxwell's equations. Doing so will allow us to introduce the eikonal equation, which is a critical component of GO.

¹Meaning large compared to the electromagnetic wave's wavelength.

Luneberg and Kline [19] proposed the following ansatz to obtain an asymptotic solution to Maxwell's equations in a source free region, occupied by an isotropic medium:

$$\mathbf{E}(\mathbf{r}, \omega) \sim e^{-jk\Psi(\mathbf{r})} \sum_{n=0}^{\infty} \frac{\mathbf{E}_n(\mathbf{r})}{(j\omega)^n} \quad \text{Luneberg-Kline asymptotic expansion,} \quad (4.2)$$

where k is the wavenumber, $\omega \triangleq 2\pi f$ and $\sim$ means equal in the asymptotic sense (when $\omega \rightarrow \infty$). As ω tends to infinity, only the first term ($n = 0$) of the expansion remains significant, and the field is then called a GO field.

The phase function $\Psi(\mathbf{r})$, also called the eikonal function, is a real scalar. Surfaces along which $\Psi(\mathbf{r})$ is constant are the wavefronts of the electromagnetic field. Thus, having knowledge of the eikonal function is primordial as it allows us to describe the motion of electromagnetic waves. Fortunately, by substituting the Luneberg Kline expansion into Maxwell's equations, an equation involving the eikonal function can be reached:

$$|\nabla\Psi(\mathbf{r})|^2 = n(\mathbf{r})^2 \quad \text{eikonal equation.} \quad (4.3)$$

where $n(\mathbf{r})$ is the refraction index of the medium in function of the position. This equation is the foundation of GO, because its solutions describe the wavefronts surfaces, also called eikonal surfaces. Because we know that electromagnetic waves propagate normally to their wavefront, knowledge of the wavefront leads us directly to the knowledge of the rays along which the wave propagates, and inversely.

To be rigorous, we can prove this by showing that the ray equation can be obtained from the eikonal equation. We know that the electromagnetic energy of a harmonic wave will propagate along $\hat{\mathbf{t}}$: a unit vector normal to its wavefront, observe that this vector can be written $\hat{\mathbf{t}} = \nabla\Psi(\mathbf{r})/n$. Now consider a ray path $\mathbf{r}(s)$, where s is the distance along the path, such that $\hat{\mathbf{t}}$ is tangent to it at any given point. The differential equation describing this path is [18] :

$$n\hat{\mathbf{t}} = \nabla\Psi(\mathbf{r}) = n\frac{d\mathbf{r}}{ds}. \quad (4.4)$$

Notice that taking the directional derivative (d/ds) of this equation leads directly to Equation 4.1: the ray equation.

4.2.2 Amplitude of GO fields

In order to introduce the notion of amplitude into GO, we can impose the conservation of energy flux inside a tube of rays. Consider the astigmatic ray tube represented in Figure 4.2. The tube is said to be astigmatic because the rays do not meet at a single point. Two equiphase surfaces of areas σ_0 and σ are positioned along that ray tube and spread by a distance s . The principal radii of curvature of the wavefronts are ρ_1 and ρ_2 .

Due to conservation of energy flux inside the ray tube, the radiation densities, noted S and S_0 , of the two equiphase surfaces are related by:

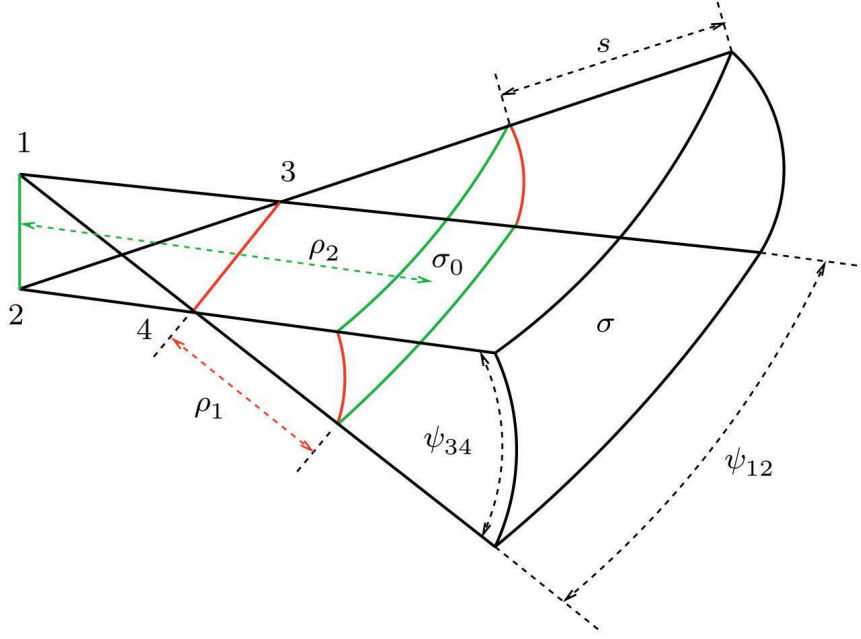


Figure 4.2: Astigmatic ray tube. Segments [1,2] and [3,4] are called caustics [20]

$$S_0 \sigma_0 = S \sigma. \quad (4.5)$$

Using the definition of the radiation power density given in Equation 3.3 to develop this expression we obtain:

$$|\mathbf{E}(\sigma_0)|^2 \cdot \sigma_0 = |\mathbf{E}(\sigma)|^2 \cdot \sigma. \quad (4.6)$$

We can now define the spread factor $A(s)$ as the ratio of the amplitudes of the electric fields at the equiphase surfaces of area σ and σ_0 . Using the notations of Figure 4.2 we have:

$$A(s) \triangleq \frac{|\mathbf{E}(\sigma)|}{|\mathbf{E}(\sigma_0)|} = \sqrt{\frac{S}{S_0}} = \sqrt{\frac{\sigma}{\sigma_0}} = \sqrt{\frac{\rho_1 \psi_{34} \rho_2 \psi_{12}}{(\rho_1 + s) \psi_{34} (\rho_2 + s) \psi_{12}}} = \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}}. \quad (4.7)$$

Note that in the case of a spherical wavefront, $\rho_1 = \rho_2$, and $\frac{|\mathbf{E}(\sigma)|}{|\mathbf{E}(\sigma_0)|} = \frac{\rho_1}{\rho_1 + s}$.

Also, in the case of a planar wavefront $\rho_1, \rho_2 \rightarrow \infty$, and $\frac{|\mathbf{E}(\sigma)|}{|\mathbf{E}(\sigma_0)|} = 1$.

4.2.3 Polarization and phase of GO fields

It can now be seen that the electric field at the end of an astigmatic ray tube like the one shown in Figure 4.2 is given by:

$$\mathbf{E}(\sigma) = \underbrace{\mathbf{E}(\sigma_0)}_{\text{field at reference point}} \underbrace{\sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}}}_{\text{spread factor}} \underbrace{e^{-jks}}_{\text{phase shift}}, \quad (4.8)$$

where the factor e^{-jks} represents the phase shift along the ray path. Note that the polarisation of the electric field at σ_0 and σ is the same, because the medium is assumed homogeneous and isotropic. When the medium is inhomogeneous the rays will bend and the polarisation will be affected. Also note that when $s = -\rho_1$ or $s = -\rho_2$ the equation is singular, hence this equation cannot describe the field at or near caustics.

4.3 Specular reflection

Now that we have established the foundations of GO, one may wonder what would be the expression of the electric field after a reflection. Let the superscripts i and r refer to the variables related to the incident and reflected fields respectively.

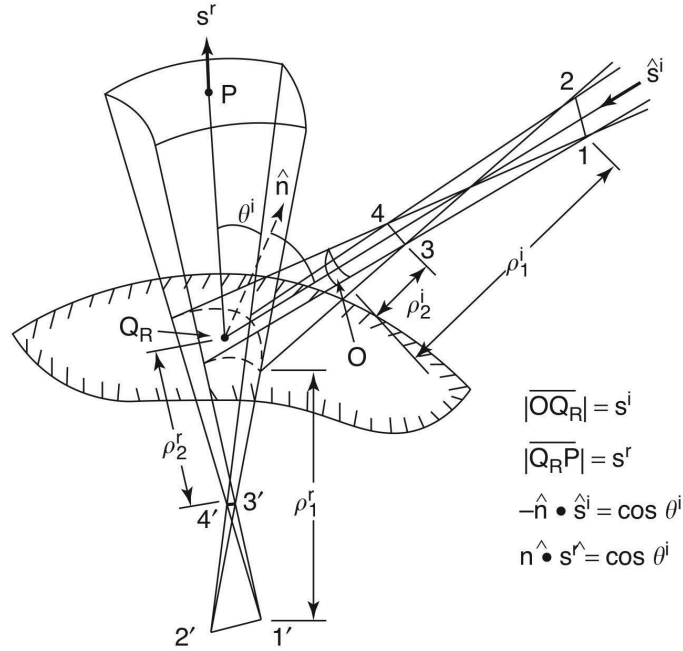


Figure 4.3: Reflected ray tube from a curved surface [21].

Let us consider a ray tube reflecting on a surface at point Q_R , and reaching an observation point P , as shown on Figure 4.3. We introduce a reflection coefficient $\overline{\mathbf{R}}$, called the Fresnel dyadic reflection coefficient (which we will define in subsection 4.3.1), which describes how much of the incident field at the reflection point $\mathbf{E}^i(Q_R)$ is reflected:

$$\mathbf{E}^r(Q_R) = \mathbf{E}^i(Q_R) \cdot \overline{\mathbf{R}}. \quad (4.9)$$

Taking into account the spreading of the rays and phase shift, the reflected field at point P is then given by:

$$\boxed{\mathbf{E}^r(P) = \mathbf{E}^i(Q_R) \cdot \overline{\overline{\mathbf{R}}} \sqrt{\frac{\rho_1^r \rho_2^r}{(\rho_1^r + s^r)(\rho_2^r + s^r)}} e^{-jks^r} \quad [\text{V m}^{-1}].} \quad (4.10)$$

The spreading factor can often be simplified, because “at high frequencies reflections and diffractions can be considered local phenomenons, depending largely on the geometry of the surface in the immediate neighborhood of the reflection point. Therefore the reflecting surface can be approximated by a plane, tangent at Q_R , and the incident wavefront can be assumed planar” [11]. With these hypotheses the spreading factor simplifies to $A^r(s^r) = 1$.

To study the impact of surface roughness on the rays, or when studying lower frequencies, it may be interesting to consider the case of reflection on a spherical surface whose radius of curvature is sufficiently small compared to the wavelength. In that case, because the reflection of a plane wave on a spherical surface is a spherical wave we can write $\rho_1^r = \rho_2^r$. Using the method of images as it is done in Figure 4.3, it is easy to see that $\rho_1^r = s^i$, and the spreading factor of the reflected wave therefore simplifies to $A^r(s^i, s^r) = \frac{s^i}{s^i + s^r}$.

Let us now compute the Fresnel dyadic reflection coefficient.

4.3.1 Fresnel dyadic reflection coefficient

Ray fixed coordinate system

The incident and reflected fields can be decomposed in a ray fixed coordinate system, shown in Figure 4.4.

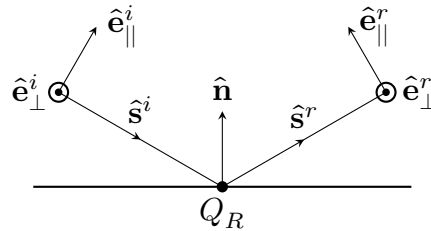


Figure 4.4: Example of the ray fixed coordinate system during a reflection.

Using the notations of Figure 4.4 the incident and reflected fields are written²:

$$\begin{aligned} \mathbf{E}^i &\triangleq E_{\parallel}^i \hat{\mathbf{e}}_{\parallel}^i + E_{\perp}^i \hat{\mathbf{e}}_{\perp}^i, \\ \mathbf{E}^r &\triangleq E_{\parallel}^r \hat{\mathbf{e}}_{\parallel}^r + E_{\perp}^r \hat{\mathbf{e}}_{\perp}^r, \end{aligned} \quad (4.11)$$

²To be complete we should write $\mathbf{E} = (\mathbf{E} \cdot \hat{\mathbf{e}}_{\parallel}) \hat{\mathbf{e}}_{\parallel} + (\mathbf{E} \cdot \hat{\mathbf{e}}_{\perp}) \hat{\mathbf{e}}_{\perp} + (\mathbf{E} \cdot \hat{\mathbf{s}}) \hat{\mathbf{s}}$ where $\hat{\mathbf{s}}$ is a unit vector in the direction of propagation of $\mathbf{E}$. However the electric field vector is always perpendicular to the direction of propagation, therefore $\mathbf{E} \cdot \hat{\mathbf{s}}$ is always 0 and we can omit it from the equations.

where the parallel and perpendicular components of the incident and reflected fields are found with:

$$\begin{aligned}\hat{\mathbf{e}}_{\perp}^i &= \hat{\mathbf{e}}_{\perp}^r = \hat{\mathbf{s}}^i \times \hat{\mathbf{s}}^r, \\ \hat{\mathbf{e}}_{\parallel}^i &= \hat{\mathbf{e}}_{\perp}^i \times \hat{\mathbf{s}}^i, \\ \hat{\mathbf{e}}_{\parallel}^r &= \hat{\mathbf{e}}_{\perp}^r \times \hat{\mathbf{s}}^r.\end{aligned}\tag{4.12}$$

The Fresnel dyadic reflection coefficient is given by [21]:

$$\overline{\overline{\mathbf{R}}} = \Gamma_{\perp} \hat{\mathbf{e}}_{\perp} \hat{\mathbf{e}}_{\perp} + \Gamma_{\parallel} \hat{\mathbf{e}}_{\parallel}^i \hat{\mathbf{e}}_{\parallel}^r.\tag{4.13}$$

In the above $\hat{\mathbf{e}}_{\perp} \hat{\mathbf{e}}_{\perp}$ and $\hat{\mathbf{e}}_{\parallel}^i \hat{\mathbf{e}}_{\parallel}^r$ are dyads. The dyad can be interpreted as a matrix operator and quickly computed, that is:

$$\hat{\mathbf{a}} \hat{\mathbf{b}} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}.\tag{4.14}$$

Therefore Equation 4.10 may be written either using a convenient matrix notation (Equation 4.15), or completely explicitly (Equation 4.16):

$$\begin{bmatrix} E_{\parallel}^r(P) \\ E_{\perp}^r(P) \end{bmatrix} = \begin{bmatrix} \Gamma_{\parallel} & 0 \\ 0 & \Gamma_{\perp} \end{bmatrix} \begin{bmatrix} E_{\parallel}^i(Q_R) \\ E_{\perp}^i(Q_R) \end{bmatrix} \cdot A(s^i, s^r) \cdot e^{-jks^r}.\tag{4.15}$$

$$\begin{aligned}\mathbf{E}^r &= E_{\parallel}^r \hat{\mathbf{e}}_{\parallel}^r + E_{\perp}^r \hat{\mathbf{e}}_{\perp}^r \\ &= (\Gamma_{\parallel} E_{\parallel}^i \hat{\mathbf{e}}_{\parallel}^r + \Gamma_{\perp} E_{\perp}^i \hat{\mathbf{e}}_{\perp}^r) \cdot A(s^i, s^r) \cdot e^{-jks^r} \\ &= (\Gamma_{\parallel} (\mathbf{E}^i \cdot \hat{\mathbf{e}}_{\parallel}^i) \hat{\mathbf{e}}_{\parallel}^r + \Gamma_{\perp} (\mathbf{E}^i \cdot \hat{\mathbf{e}}_{\perp}^i) \hat{\mathbf{e}}_{\perp}^r) \cdot A(s^i, s^r) \cdot e^{-jks^r}.\end{aligned}\tag{4.16}$$

Reflection coefficients

For a perfectly conducting reflection surface, the Fresnel reflection coefficients are given by $\Gamma_{\perp} = \Gamma_{\parallel} = \pm 1$. For a non perfectly conducting reflection surface with a large radius of curvature compared to the wavelength (such that it may be considered locally plane), $\Gamma_{\perp}$ and $\Gamma_{\parallel}$ are given by³:

$$\Gamma_{\perp} \triangleq \frac{E_{\perp}^r}{E_{\perp}^i} = \left| \frac{Y_1 \cos \theta_i - Y_2 \cos \theta_t}{Y_1 \cos \theta_i + Y_2 \cos \theta_t} \right|, \quad \Gamma_{\parallel} \triangleq \frac{E_{\parallel}^r}{E_{\parallel}^i} = \left| \frac{Y_2 \cos \theta_i - Y_1 \cos \theta_t}{Y_2 \cos \theta_i + Y_1 \cos \theta_t} \right|,\tag{4.17}$$

where θ_i , and θ_t are the incident and transmission angles respectively, as shown in Figure 4.5, and $Y \triangleq \frac{|\mathbf{H}|}{|\mathbf{E}|} = \sqrt{\frac{\epsilon_{eff}}{\mu}}$ is the complex intrinsic admittance of the medium.

Assuming a non magnetic medium ($\mu_1 = \mu_2 = \mu_0$) and using Snell's law, these coefficients simplify to:

$$\Gamma_{\perp} = \left| \frac{\cos \theta_i - \sqrt{\frac{\epsilon_{2,eff}}{\epsilon_{1,eff}} - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\frac{\epsilon_{2,eff}}{\epsilon_{1,eff}} - \sin^2 \theta_i}} \right|, \quad \Gamma_{\parallel} = \left| \frac{\frac{\epsilon_{2,eff}}{\epsilon_{1,eff}} \cos \theta_i - \sqrt{\frac{\epsilon_{2,eff}}{\epsilon_{1,eff}} - \sin^2 \theta_i}}{\frac{\epsilon_{2,eff}}{\epsilon_{1,eff}} \cos \theta_i + \sqrt{\frac{\epsilon_{2,eff}}{\epsilon_{1,eff}} - \sin^2 \theta_i}} \right|,\tag{4.18}$$

³The notations R_s and R_h for the soft and hard coefficients are often found in the literature and are equivalent to $\Gamma_{\perp}$ and $\Gamma_{\parallel}$ respectively

where $\epsilon_{1,eff}$ and $\epsilon_{2,eff}$ are the complex effective relative permittivities of the incident and transmission mediums, defined as:

$$\epsilon_{k,eff} = \epsilon_{r,k} - j \frac{\sigma_k}{2\pi f \epsilon_0}, \quad (4.19)$$

with $\epsilon_{r,k}$ the relative permittivity of medium k , and σ_k its conductivity in $[\text{S m}^{-1}]$. In the context of a ray tracing algorithm, the normal $\hat{\mathbf{n}}$ to the reflecting surface and the incident ray $\hat{\mathbf{s}}^i$ are known, and the incident angle θ_i is conveniently found using a dot product

$$\theta_i = -\cos^{-1}(\hat{\mathbf{n}} \cdot \hat{\mathbf{s}}^i). \quad (4.20)$$

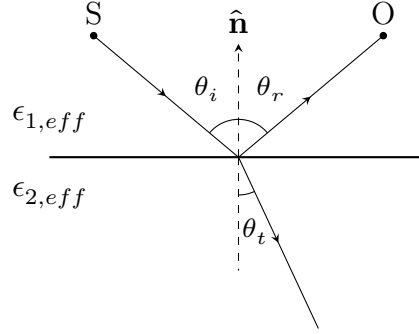


Figure 4.5: Snell's law

4.4 Diffuse reflection

When the surface is rough, we will have to consider the diffuse characteristic of the reflection. The reflected energy in the specular direction will be attenuated, and there will be scattered fields in all directions. In this work we will limit ourselves to Rayleigh scattering, that is: scattering by objects much smaller than the wavelength.

4.4.1 Rayleigh roughness criterion

The Rayleigh roughness criterion lets us determine when to consider a surface as rough, and thus when to consider a reflection as diffuse. Let us demonstrate this criterion.

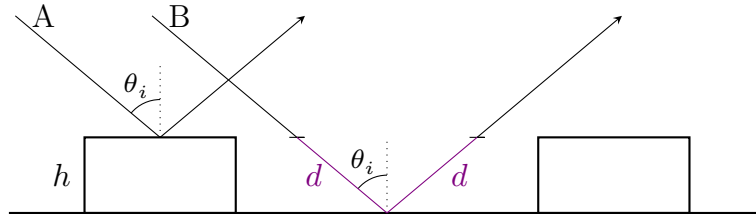


Figure 4.6: Illustration of Rayleigh's roughness criterion.

As shown in Figure 4.6, ray A which hits an irregularity of height h travels a shorter distance than ray B. This travel distance difference of $2d = 2h \cos \theta$ will lead to an

additional phase difference $\Delta\phi = \frac{2\pi}{\lambda} \cdot 2d$ between the two rays. If the additional phase difference is small enough, the surface can be considered smooth. However, if it becomes too large ($\Delta\phi \geq \frac{\pi}{2}$) the surface will be considered rough, the roughness criterion follows directly:

$$h \geq \frac{\lambda}{8 \cos \theta_i}. \quad (4.21)$$

In practice h is the standard deviation of surface height in the Fresnel zone, noted Δh_0 .

4.4.2 Reduction of energy in the specular direction

When the surface is rough, a common way to consider the reduction of the reflected energy in the specular direction due to scattering in all directions, is to multiply the Fresnel reflection coefficients by an empirical attenuation factor, called the Rayleigh roughness factor and noted χ [22, 23, 24]:

$$\chi = \exp(-2k^2 \Delta h_0^2 \cos^2 \theta_i). \quad (4.22)$$

Computing the contribution of the fields scattered in the non specular direction, to the power at the receiver, is computationally expensive because the ray tracer would have to recompute all the paths linking the scattering point to the receiver. Moreover, conflicting conclusions have been made in previous studies about the importance of taking this contribution into account for millimeter wave channel characterisation. Indeed, in 2015, at 30 GHz in an urban outdoor scenario [25] showed the diffuse component contribution may be neglected with respect to the specular one. Three years later in the same environment and using the same frequency [1] compared measurements to results provided by his ray tracer and found that diffuse scattering played an important role in achieving a good accuracy for path loss predictions.

In this study we will neglect the contribution from scattered fields.

Chapter 5

Uniform theory of diffraction

In the previous chapter, which introduced Geometrical Optics, we failed to talk about diffraction. This is because GO is limited to reflections. For this reason, Keller introduced the Geometrical Theory of Diffraction (GTD), which addressed this. Then Kouyoumjian and Pathak improved the GTD in what is called the Uniform Theory of Diffraction (UTD), to prevent singularities on the incident and reflection shadow boundaries. In this chapter, we will detail the UTD.

We will be focusing on the case of diffraction by a straight wedge such as one formed by two flat planes, however the following expressions are also valid for a wedge formed between a curved surface and a flat plane, such as the wedge found at the base of a cylinder.

The reason we will not dive into diffraction by wedges formed by two curved surfaces is because doing so would only be useful if accurate knowledge about the radii of curvature of objects in the scene are available. For our application, such accurate knowledge is hardly available.

Corner diffractions are also a special case, for which solutions [26] exist, but that are out of the scope of this work.

Bibliographical note The expressions we will present in this chapter were all found by Kouyoumjian and Pathak in 1970, who are the authors of the UTD. They first developed the UTD for a straight perfectly conducting wedge in 1970 [13], and later extended it to a perfectly conducting curved edge in 1974 [27]. They then summed up their results in a 1974 journal article [28]. We aim to present the main results of the aforementioned works, which were useful to develop our ray tracer.

5.1 Edge diffraction

An edge diffracted ray tube is represented in Figure 5.1.

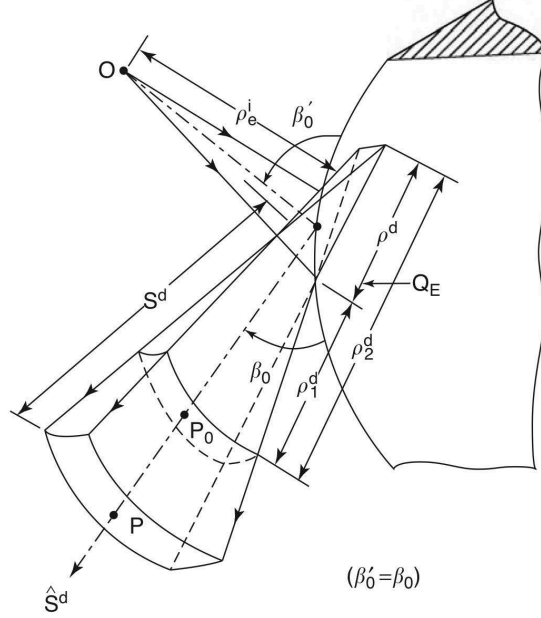


Figure 5.1: Edge diffracted ray tube [21]

Let the superscripts i and d refer to the variables related to the incident and diffracted fields respectively

The relation between the diffracted fields at points P and P_0 , separated by a distance s_0^d may be written as the product of the field at P_0 , a spreading factor, and a phase compensation:

$$\mathbf{E}^d(P) = \mathbf{E}^d(P_0) \cdot \sqrt{\frac{\rho_1^d \rho_2^d}{(\rho_1^d + s_0^d)(\rho_2^d + s_0^d)}} \cdot e^{-jks_0^d}. \quad (5.1)$$

Moving the reference point P_0 to the point of diffraction (by letting $\rho_1^d \rightarrow 0$) allows us to relate the field at the observation point to the field at the point of diffraction:

$$\begin{aligned} \mathbf{E}^d(P) &= \lim_{\rho_1^d \rightarrow 0} \left[\mathbf{E}^d(P_0) \cdot \sqrt{\frac{\rho_1^d \rho_2^d}{(\rho_1^d + s_0^d)(\rho_2^d + s_0^d)}} \cdot e^{-jks_0^d} \right] \\ &= \lim_{\rho_1^d \rightarrow 0} \left[\mathbf{E}^d(P_0) \sqrt{\rho_1^d} \right] \cdot \sqrt{\frac{\rho_2^d}{s_0^d(\rho_2^d + s_0^d)}} \cdot e^{-jks_0^d}. \end{aligned} \quad (5.2)$$

This limit exists, must be finite, and is defined as:

$$\lim_{\rho_1^d \rightarrow 0} \left[\mathbf{E}^d(P_0) \sqrt{\rho_1^d} \right] \triangleq \mathbf{E}^d(Q_E), \quad (5.3)$$

where $\mathbf{E}^d(Q_E)$ itself is defined as the diffracted field at the point of reflection:

$$\mathbf{E}^d(Q_E) \triangleq \mathbf{E}^i(Q_E) \cdot \overline{\overline{\mathbf{D}}}. \quad (5.4)$$

Similarly to what was done in the case of reflection, we introduced a diffraction coefficient $\overline{\overline{\mathbf{D}}}$, called the dyadic diffraction coefficient, describing how much of the incident field is diffracted.

Notice that as ρ_1^d tends to 0, $s_0^d \rightarrow s^d$ and $\rho_2^d \rightarrow \rho^d$. Taking this into account, we obtain an expression for the diffracted field at an observation point P , which is similar to the expression found for the reflected field:

$$\boxed{\mathbf{E}^d(P) = \mathbf{E}^i(Q_E) \cdot \overline{\overline{\mathbf{D}}} \cdot \underbrace{\sqrt{\frac{\rho^d}{s^d(\rho^d + s^d)}}}_{A(s^d, \rho^d)} \cdot e^{-jks^d} \quad [\text{V m}^{-1}].} \quad (5.5)$$

In Equation 5.5 $\mathbf{E}^i(Q_E)$ is the incident electric field at the diffraction point, $\overline{\overline{\mathbf{D}}}$ is the dyadic diffraction coefficient, $A(s^d, \rho^d)$ is a spreading factor where ρ^d is the distance between the two caustics of the diffracted ray, and e^{-jks^d} represents the phase shift along the ray path.

Equation 5.5 is only valid for source and field points far from the edge (s^i and s^d must be large) and cannot be used to compute the field at a caustic of the diffracted ray. Moreover, it is assumed that the incident field is slowly varying at the point of diffraction [28]. This expression is not valid for corner diffractions.

5.1.1 Spreading factor

As it can be expected, the spreading factor presented in Equation 5.5 can perhaps be found more easily by applying conservation of power flow in an ray tube whose caustic is located on the diffracting edge, as shown in Figure 5.2.

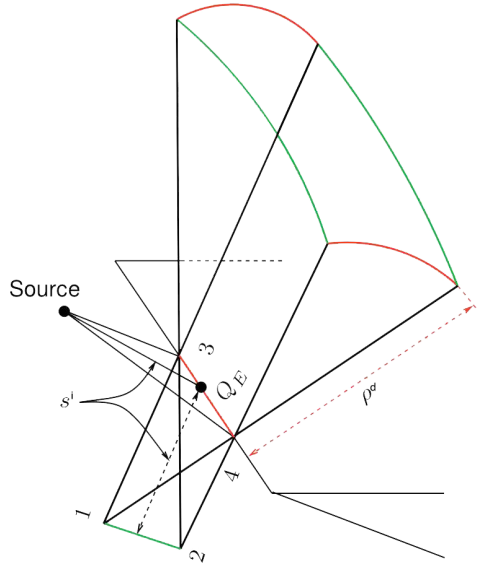


Figure 5.2: Edge diffracted ray tube [20].

Although the distance ρ^d is a function of the radii of curvature of the incident field, the radius of curvature of the diffracting edge at the point of diffraction, and the incident and diffraction angles, it is easily found for the following specific cases:

- For spherical wave incidence on a straight edge, $\rho^d \rightarrow s^i$ and the spreading factor reduces to:

$$A(s^i, s^d) = \sqrt{\frac{s^i}{s^d(s^i + s^d)}}. \quad (5.6)$$

- For plane wave incidence, $\rho^d \rightarrow \infty$ and the spreading factor reduces to:

$$A(s^i, s^d) = \frac{1}{\sqrt{s^d}}. \quad (5.7)$$

5.1.2 Dyadic diffraction coefficient

Edge fixed coordinate system

Although we computed the dyadic reflection coefficient using a ray fixed coordinate system, doing so to compute the dyadic diffraction coefficient is not suitable, because the diffraction coefficients would be composed of the sum of seven dyads [29].

Therefore we define an edge fixed coordinate system, as shown in Figure 5.3.

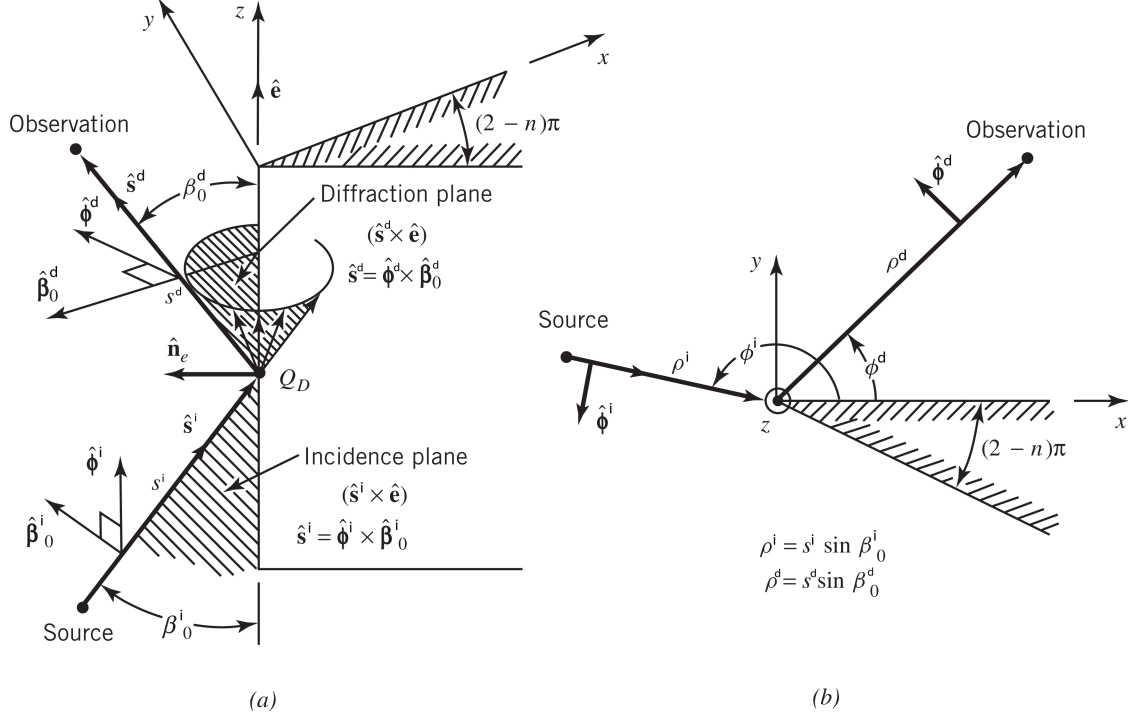


Figure 5.3: Diffraction from a straight wedge and edge fixed coordinate system (a): oblique incidence, (b): top view. [11]

The incident field $\mathbf{E}^i$ is thus decomposed into $\hat{\beta}_0^i$: a component parallel to the plane of incidence; and $\hat{\phi}^i$: a component perpendicular to the plane of incidence. The same is done for the diffracted field $\mathbf{E}^d$, which is decomposed into its $\hat{\beta}_0^d$ and $\hat{\phi}^d$ components, who are parallel and perpendicular to the plane of diffraction respectively. Let $\hat{\mathbf{e}}$ be an unit vector tangential to the edge, then the components of $\mathbf{E}^i$ in this edge fixed coordinate system are given by [29]:

$$\hat{\beta}_0^i = \hat{\mathbf{s}}^i \times \hat{\phi}^i, \quad \text{and} \quad \hat{\phi}^i = \hat{\mathbf{s}}^i \times \hat{\mathbf{e}}. \quad (5.8)$$

While those of $\mathbf{E}^d$ are:

$$\hat{\beta}_0^d = \hat{\mathbf{s}}^d \times \hat{\phi}^d, \quad \text{and} \quad \hat{\phi}^d = \hat{\mathbf{e}} \times \hat{\mathbf{s}}^d. \quad (5.9)$$

Using those components we define:

$$\begin{aligned} \mathbf{E}^i &\triangleq E_{\parallel}^i \hat{\beta}_0^i + E_{\perp}^i \hat{\phi}^i, \\ \mathbf{E}^d &\triangleq E_{\parallel}^d \hat{\beta}_0^d + E_{\perp}^d \hat{\phi}^d. \end{aligned} \quad (5.10)$$

The dyadic diffraction coefficient in the edge fixed coordinate system is given by¹

$$\overline{\overline{\mathbf{D}}} = -D_{\perp} \hat{\beta}_0^i \hat{\beta}_0^d - D_{\parallel} \hat{\phi}^i \hat{\phi}^d. \quad (5.11)$$

And a convenient matrix notation can be used to rewrite Equation 5.5:

$$\begin{bmatrix} E_{\parallel}^d(P) \\ E_{\perp}^d(P) \end{bmatrix} = \begin{bmatrix} -D_{\perp} & 0 \\ 0 & -D_{\parallel} \end{bmatrix} \begin{bmatrix} E_{\parallel}^i(Q_D) \\ E_{\perp}^i(Q_D) \end{bmatrix} \cdot A(s^i, s^d) \cdot e^{-jks^d}. \quad (5.12)$$

Diffraction coefficients

The diffraction coefficients found in Equation 5.11 are given by:

$$D_{\perp, \parallel} = D_1 + D_2 + \Gamma_{\perp, \parallel} (D_3 + D_4), \quad (5.13)$$

where $\Gamma_{\parallel} = \Gamma_{\perp} = \pm 1$ as we will consider the edge to be a perfect conductor². For the special case of grazing incidence ($\phi^i = 0$ or $n\pi$), the diffraction coefficients should be multiplied by a factor $\frac{1}{2}$ [11]. For angles near grazing incidence, other theories, such as the Extended Spectral Theory of Diffraction (ESTD) should be applied. We will restrict the scope of this thesis to non grazing incidence rays to avoid dealing with the ESTD.

¹Similarly to what was said for the dyadic reflection coefficient, note that $\hat{\beta}_0^i \hat{\beta}_0^d$ and $\hat{\phi}^i \hat{\phi}^d$ are dyads that can be computed with a matrix product.

²Not doing so involves much more complicated computations which will bring negligible gains of accuracy for our purpose.

Diffraction coefficient components

Each of the four components (defined in Equation 5.14) of the diffraction coefficients, has been carefully engineered to ensure that the diffracted field remains continuous at and around the incident and reflection shadow boundaries.

Let us refer to the faces of the wedge the ray diffracts on as the 0-face and the n -face. The designation of which face is the 0-face and which is the n -face is arbitrary.

The presence of four diffraction coefficient components arises from the existence of both the 0-face and the n -face, each producing an incident and a reflection shadow boundary. Specifically, D_1 (resp. D_2) compensates for the discontinuity in the GO field when the 0-face (resp. n -face) is shadowed, and D_3 (resp. D_4) compensates for the discontinuity in the GO field when there is reflection from the n -face (resp. 0-face) [29]. Obviously this is not true for corner diffractions, where other expressions should be used.

Within each diffraction coefficient component, the transition function $F(kLa^\pm)$ plays the role of ensuring the continuity of the total fields across the shadow boundaries. Consequently, when the angle ϕ is sufficiently far from the shadow boundaries, the transition function $F(kLa^\pm)$ tends to unity [29].

The components of the diffraction coefficients are given by:

$$\begin{aligned}
D_1 &= \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k} \sin \beta_0} \cdot \cot \left(\frac{\pi + (\phi^d - \phi^i)}{2n} \right) \cdot F[kL^i \cdot a^+(\phi^d - \phi^i)], \\
D_2 &= \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k} \sin \beta_0} \cdot \cot \left(\frac{\pi - (\phi^d - \phi^i)}{2n} \right) \cdot F[kL^i \cdot a^-(\phi^d - \phi^i)], \\
D_3 &= \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k} \sin \beta_0} \cdot \cot \left(\frac{\pi + (\phi^d + \phi^i)}{2n} \right) \cdot F[kL^{rn} \cdot a^+(\phi^d + \phi^i)], \\
D_4 &= \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k} \sin \beta_0} \cdot \cot \left(\frac{\pi - (\phi^d + \phi^i)}{2n} \right) \cdot F[kL^{ro} \cdot a^-(\phi^d + \phi^i)].
\end{aligned} \tag{5.14}$$

Let us enumerate the terms found in Equation 5.14:

- β_0^i (resp. β_0^d) is the angle between the incident (resp. diffracted) ray and the tangent to the edge. From Keller's diffraction law $\beta_0^i = \beta_0^d = \beta_0$. It is found with:

$$\sin \beta_0 = |\hat{\mathbf{s}}^i \times \hat{\mathbf{e}}| = |\hat{\mathbf{s}}^d \times \hat{\mathbf{e}}|.$$

But because this is a double valued function we should rather use

$$\beta_0 = \cos^{-1}(\hat{\mathbf{s}}^i \cdot \hat{\mathbf{e}}). \tag{5.15}$$

- $a^\pm(\xi)$ is a function defined as:

$$a^\pm(\xi) = 2 \cos^2 \left(\frac{2n\pi N^\pm - \xi}{2} \right). \quad (5.16)$$

Where:

- n is a parameter defined such that α : the inner angle between the 0-face and the n -face, is $\alpha = (2 - n)\pi$. Knowing the normal vectors to the 0-face and the n -face, noted $\hat{\mathbf{n}}_0$ and $\hat{\mathbf{n}}_n$ respectively, when $\hat{\mathbf{n}}_0$ (resp. $\hat{\mathbf{n}}_n$) points outwards of the 0-face (resp. n -face), this angle is easily found with $\alpha = \pi - \cos^{-1}(\hat{\mathbf{n}}_0 \cdot \hat{\mathbf{n}}_n)$.
- $N^\pm$ are integers most nearly satisfying $2n\pi N^\pm - \xi = \pm\pi$.

When $a(\xi)$ is evaluated at $\xi = \phi^d \pm \phi^i$ it gives a measure of the angular separation between the field point and an incident shadow boundary or a reflection shadow boundary [27].

- It is conventional to consider that ϕ^i is the angle between the incident ray and the 0-face, while ϕ^d represents the angle between the diffracted ray and the 0-face, as shown in Figure 5.3(b). To compute these angles, let us define a few quantities:
 - Let $\hat{\mathbf{n}}_0$ be an unit vector normal to the 0-face. And consider $\hat{\mathbf{e}}$: an unit vector tangential to the edge. Then we can let $\hat{\mathbf{t}}_0 \triangleq \hat{\mathbf{n}}_0 \times \hat{\mathbf{e}}$ be an unit vector tangential to the 0-face. Note that the direction of $\hat{\mathbf{e}}$ should be such that $\hat{\mathbf{t}}_0$ points away from the wedge between the 0-face and the n -face.
 - Let $\hat{\mathbf{s}}_t^i$ (resp. $\hat{\mathbf{s}}_t^d$) be the component of $\hat{\mathbf{s}}^i$ (resp. $\hat{\mathbf{s}}^d$) lying in the plane of incidence (resp. diffraction) and perpendicular to the edge:

$$\begin{aligned} \hat{\mathbf{s}}_t^i &= \frac{\hat{\mathbf{s}}^i - (\hat{\mathbf{s}}^i \cdot \hat{\mathbf{e}})\hat{\mathbf{e}}}{|\hat{\mathbf{s}}^i - (\hat{\mathbf{s}}^i \cdot \hat{\mathbf{e}})\hat{\mathbf{e}}|}, \\ \hat{\mathbf{s}}_t^d &= \frac{\hat{\mathbf{s}}^d - (\hat{\mathbf{s}}^d \cdot \hat{\mathbf{e}})\hat{\mathbf{e}}}{|\hat{\mathbf{s}}^d - (\hat{\mathbf{s}}^d \cdot \hat{\mathbf{e}})\hat{\mathbf{e}}|}. \end{aligned} \quad (5.17)$$

Using those quantities, the angles ϕ^i and ϕ^d are found with:

$$\begin{aligned} \phi^i &= \pi - [\pi - \cos^{-1}(-\hat{\mathbf{s}}_t^i \cdot \hat{\mathbf{t}}_0)] \cdot \text{sgn}(-\hat{\mathbf{s}}_t^i \cdot \hat{\mathbf{n}}_0), \\ \phi^d &= \pi - [\pi - \cos^{-1}(\hat{\mathbf{s}}_t^d \cdot \hat{\mathbf{t}}_0)] \cdot \text{sgn}(\hat{\mathbf{s}}_t^d \cdot \hat{\mathbf{n}}_0), \end{aligned} \quad (5.18)$$

where sgn denotes the sign function. Obviously, ϕ^i and ϕ^d should fall between 0, and $n\pi$, otherwise they would be inside the wedge.

- $F(x)$ is a transition function given by:

$$F(x) = 2j\sqrt{x}e^{jx} \int_{\sqrt{x}}^{\infty} e^{-ju^2} du. \quad (5.19)$$

See appendix B for details on how this function is computed in Python.

- L^i is a distance parameter associated with the incident shadow boundaries and depending on the type of illumination. L^{ro} and L^{rn} are distance parameters associated with the reflection shadow boundaries of the 0-face and the n -face respectively. “They are found by imposing that the sum of the geometrical optics field and the diffracted fields is continuous along the incident and reflection shadow boundaries” [11]. For diffraction on a wedge formed between a curved plane and a flat plane, such as the base of a cylinder, or between two flat planes, it can be shown that $L^i = L^{ro} = L^{rn} = L$ and is given by:

$$L = \frac{s^d(\rho_e^i + s^d)\rho_1^i\rho_2^i}{\rho_e^i(\rho_1^i + s^d)(\rho_2^i + s^d)} \cdot \sin^2 \beta_0. \quad (5.20)$$

where ρ_1^i and ρ_2^i are the principal radii of curvature of the incident wavefront at the point of diffraction, and ρ_e^i is the radius of curvature of the incident wavefront in the edge fixed plane of incidence.

For spherical wave incidence $\rho_1^i = \rho_2^i = \rho_e^i = s^i$ and L reduces to:

$$L = \frac{s^i \cdot s^d}{s^i + s^d} \cdot \sin^2 \beta_0. \quad (5.21)$$

For plane wave incidence L further reduces to $L = s^i \sin^2 \beta_0$.

In the case of diffraction on a wedge formed by two curved surfaces, the distance parameter is not the same anymore for each of the four diffraction coefficient components, and their expression are given in [27]. We won't discuss them further here because their implementation would require knowledge about the radii of curvature of objects in the scene, which is hardly available.

For clarity, let us reiterate all the assumptions behind Equation 5.5 and all the components of Equation 5.13 described above:

- The incident field should be generated from a source sufficiently far away from the reflecting surface, such that the polarization of the incident electromagnetic wave can be considered plane.
- The incident field's wavelength should be sufficiently small compared to the objects around which it propagates, such that the geometrical optics approximation holds.
- The diffraction wedge should be straight, or formed by the intersection of a flat surface and a curved one.
- The diffraction wedge should be a perfect conductor, otherwise more complicated solutions involving the computation of the surface currents induced on the wedge surface should be used, but are out of the scope of this thesis.
- Corner diffractions are a special case for which the above is not valid, and are out of the scope of this thesis.

- The above is not valid at or near caustics. The uniform asymptotic theory of diffraction [30] fixes this problem, but is out of the scope of this thesis.
- The medium through which the field propagates should be homogeneous and isotropic.
- The expression is not valid at or near grazing incidence, which is out of the scope of this thesis.
- This expression is only valid on structures whose edges are sufficiently far apart from each other, that coupling between them can be neglected. If it is not the case, other expressions are provided in [11] but are out of the scope of this thesis.

Chapter 6

Vegetation attenuation

Many studies [2, 3, 4, 31] have shown the importance of modelling tree foliage attenuation in ray tracers, even at low frequencies (1 GHz), due to the significant underestimation of path loss that result when it is not taken into account. In this chapter we present the attenuation model we implemented in our ray tracer, and justify the choice of the model by a small literature review.

6.1 Literature review

The literature proposes different models depending on the foliage penetration distance. In our urban scenario we expect spaced, single trees on the sidewalk, in close proximity to buildings, so the penetration distance will only span the length of one, or eventually a few trees.

In [32] and [33] factors significantly affecting signal attenuation in vegetation have been identified as canopy thickness, operating frequency, density of tree components and antenna height relative to tree canopy height, while factors such as tree species, and leaf shape were found less significant.

Although the ITU R 833-10 [34] suggests a rather theoretical model for attenuation of an electromagnetic wave above 1 GHz by a single vegetative obstruction, it is difficult to implement, and a simpler empirical model will be used instead.

Surveying of the literature reveals three widely recognized empirical models: the Weissberger model [35], the Fitted ITU-R model (FITU) [36], and the COST235 model [37].

The Weissberger and COST235 models are both exponential decay models. The Weissberger model is known to consistently underestimate path loss at millimeter wave frequencies resulting in very optimistic predictions [2, 33, 38, 39]. Conversely, the COST235 model, designed to address the shortcomings of the Weissberger model, tends to rather overestimate path loss, leading to pessimistic predictions [33, 39].

In [39] and [33], the FITU-R model is found to be a good middle ground between the Weissberger and COST235 models, providing the most accurate predictions. It is therefore the preferred model for this study.

6.2 Attenuation model

The FITU-R model is recommended for frequencies in the 10 GHz to 40 GHz range and gives the attenuation loss due to vegetation, in excess to free space losses. It is given as:

$$L(dB) = \begin{cases} 0.39f^{0.39}d^{0.25} & \text{in leaf} \\ 0.37f^{0.18}d^{0.59} & \text{out of leaf} \end{cases} \quad (6.1)$$

where f is the frequency in MHz, and d is the vegetation depth in meters. The model differentiates two states, in leaf: when the tree is in full foliage, and out of leaf: when the tree does not have foliage.

Part III

Ray tracing

Chapter 7

Path finding

In the previous chapters, we treated the purely electromagnetic aspect of ray propagation, but not the geometrical one. In this chapter, we start by describing how we create the geometry of an environment. We then succinctly present the algorithm used to compute all the possible paths between two points of the geometry, undergoing a given number of interactions.

7.1 Geometry construction

The accuracy of ray tracers is directly tied to the quality of the model they are provided with. Consequently, building a precise and accurate model of the environment is critical, though often the most difficult part. This is because accurate environmental data, especially elevation data, is not readily available, and because each additional detail included in the model will increase the computational cost of the path finding algorithm, which can quickly become prohibitive. Due to these limitations our model environment is coarsely built.

To build our model, we extract environmental data from OpenStreetMap via <https://overpass-turbo.eu>. The query provided in Figure 7.1 allows buildings shapes and positions, along with the position of some trees in a selected area to be easily exported in a .geojson file. Buildings can also be manually selected (see Appendix E for an example).

```
1 [out:json][timeout:30];
2 (
3   way["building"]({{bbox}});
4   relation["building"]["type"]="multipolygon"({{bbox}});
5   node["natural"]="tree"({{bbox}});
6 );
7 out body;
8 >;
9 out skel qt;
```

Figure 7.1: Overpass-turbo query used to export building shapes and positions along with the positions of some trees from OpenStreetMap.

Our Python program then uses this `.geojson` file, containing for each building a list of coordinates representing its corners, to build the geometry using custom functions who rely on the Shapely library. The buildings are modelled as right prisms, composed of n flat faces. Due to the nature of the data extracted from OpenStreetMap, curved surfaces are represented by n -faced polygons.

Electromagnetic properties of each building are fixed according to a given building type. According to the building type, facade material is chosen following Table 7.1, and electromagnetic properties (permittivity, permeability, conductivity, standard deviation of surface height) are set. Detailed information about electromagnetic properties of various materials are provided in Appendix F.

Building type	Facade material
Apartments	Brick
Offices	Glass
Garage	Concrete
Road	Concrete

Table 7.1: Building types and corresponding facade materials. Building roofs are always considered to be flat concrete surfaces.

In the future, additional building types could easily be implemented. Moreover, facades could be tiled, while staying flat in order not to increase the complexity of the path finding algorithm, to distinguish windows from walls. A faster implementation of the path finding algorithm would enable more environmental details to be accounted for, such as the topography of building facades.

7.2 Path finding algorithm

The literature often relies on the image based ray tracing method to solve the path finding problem. While this method benefits from simplicity and extensive application in prior research, it is limited to finding specular reflections on planar surfaces. Moreover, it is hard, if not impossible due to computational complexity, to use it to take into account interactions other than specular reflections and a single diffraction, i.e., multiple diffractions, refractions, diffuse reflections [14].

Improving on the image method, this work relies on *min path tracing*, which generalizes the computation of all the possible paths between two nodes regardless of the geometries in the three-dimensional scene, or the number of diffractions encountered along the path [14].

The min path tracing algorithm can be split into three parts. The first is to preprocess the geometry to gather data about the surfaces in the scene, and build a visibility matrix. The second one is to provide candidate paths to the algorithm. The last one is to run a minimizer that will find the path coordinates for each can-

didate path, and verify that the candidate paths respects the laws of reflection and diffraction.

7.2.1 Geometry preprocessing

Although min path tracing works with curved geometries, in the context of this work we represent curved surfaces as n -faced polygons. Contrarily to curved surfaces, the normal vector of a planar surface is the same for every point of the surface, therefore we can compute the unit normal vector of each of the surfaces of the geometry during preprocessing and store them for later use. It must be noted that the unit normal vectors are oriented outwards of the polyhedrons that the surfaces compose. We will also find all the edges on which diffraction can occur.

Finding all the paths between an emitter and a receiver can be computationally expensive, as it requires evaluating if a path *that follows the laws of reflection and diffraction* exists between all the different combinations of faces and edges in the geometry. To speed up the process, a visibility matrix that simply encodes for each face and each edge the list of faces and edges it can see, is constructed. As objects in the geometry often block each other's view the matrix is typically sparse, which greatly reduces the amount of paths the algorithm has to test. Details on how to compute this visibility matrix are provided in [14].

7.2.2 Verifying path validity

Reflection

One can easily verify that a given path respects the law of reflection because the incident vector $\hat{\mathbf{s}}^i$, the reflected vector $\hat{\mathbf{s}}^r$ and the normal to the surface at the interaction point $\hat{\mathbf{n}}$ are related by [14]:

$$\hat{\mathbf{s}}^r = \hat{\mathbf{s}}^i - 2(\hat{\mathbf{s}}^i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}, \quad (7.1)$$

note that this is valid for any surface: planar, or not. Notice how Equation 7.1 can be rewritten as a function of three points: the point of departure from the previous interaction $\mathbf{X}_{i-1}$, the interaction point $\mathbf{X}_i$, and the point of arrival at the next interaction $\mathbf{X}_{i+1}$. The location of the transmitter is noted $\mathbf{X}_0$ and the location of the receiver is noted $\mathbf{X}_{n_t+1}$, where n_t is the number of interactions. Using these notations Equation 7.1 becomes:

$$\frac{|\mathbf{s}^i|}{|\mathbf{s}^r|}(\mathbf{X}_{i+1} - \mathbf{X}_i) = (\mathbf{X}_i - \mathbf{X}_{i-1}) - 2\left((\mathbf{X}_i - \mathbf{X}_{i-1}) \cdot \hat{\mathbf{n}}_i\right)\hat{\mathbf{n}}_i, \quad (7.2)$$

which we will write:

$$\underbrace{\frac{|\mathbf{s}^i|}{|\mathbf{s}^r|}(\mathbf{X}_{i+1} - \mathbf{X}_i) - \left((\mathbf{X}_i - \mathbf{X}_{i-1}) - 2\left((\mathbf{X}_i - \mathbf{X}_{i-1}) \cdot \hat{\mathbf{n}}_i\right)\hat{\mathbf{n}}_i\right)}_{\triangleq \mathcal{I}_i^r(\mathbf{X}_{i-1}, \mathbf{X}_i, \mathbf{X}_{i+1})} = \mathbf{0}. \quad (7.3)$$

Diffraction

One can confirm that a path respects the principles of diffraction by ensuring it follows Keller's diffraction law, stating that “an incident vector $\mathbf{s}^i$ diffracts on an edge with local direction vector $\hat{\mathbf{e}}$ into a multitude of rays $\mathbf{s}^d$ that build-up a cone such that $\mathbf{s}^i$ and $\mathbf{s}^d$ make the same angle with $\hat{\mathbf{e}}$ ” [14]. This is illustrated on Figure 7.2. They therefore must respect the following relation:

$$\frac{\mathbf{s}^i \cdot \hat{\mathbf{e}}}{|\mathbf{s}^i|} = \frac{\mathbf{s}^d \cdot \hat{\mathbf{e}}}{|\mathbf{s}^d|}. \quad (7.4)$$

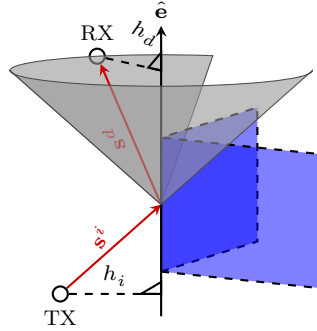


Figure 7.2: Illustration of keller's diffraction cone [14].

Keller's law relies on the fact that electromagnetic wave propagation is a local phenomenon, and therefore his law is only valid for sufficiently small wavelengths.

As we have done in the case of reflection, we can rewrite Equation 7.4 as a function of three points:

$$\frac{(\mathbf{X}_i - \mathbf{X}_{i-1}) \cdot \hat{\mathbf{e}}_i}{|\mathbf{X}_i - \mathbf{X}_{i-1}|} = \frac{(\mathbf{X}_{i+1} - \mathbf{X}_i) \cdot \hat{\mathbf{e}}_i}{|\mathbf{X}_{i+1} - \mathbf{X}_i|}, \quad (7.5)$$

which we will write:

$$\underbrace{\frac{(\mathbf{X}_i - \mathbf{X}_{i-1}) \cdot \hat{\mathbf{e}}_i}{|\mathbf{X}_i - \mathbf{X}_{i-1}|} - \frac{(\mathbf{X}_{i+1} - \mathbf{X}_i) \cdot \hat{\mathbf{e}}_i}{|\mathbf{X}_{i+1} - \mathbf{X}_i|}}_{\mathcal{I}_i^d(\mathbf{X}_{i-1}, \mathbf{X}_i, \mathbf{X}_{i+1})} = \mathbf{0}. \quad (7.6)$$

Refraction

Accurately modelling the interiors of buildings or objects can be a challenging task, and rays refracting within buildings prior to reaching the receiver are typically heavily attenuated, often resulting in a negligible contribution to the total electromagnetic field reaching the receiver. Consequently, this work will disregard refraction, although it could easily be considered by the min path tracing algorithm by encapsulating the vectorial form of Snell's law into a function $\mathcal{I}_i^{refr}(\mathbf{X}_{i-1}, \mathbf{X}_i, \mathbf{X}_{i+1})$, similarly to what was done for reflection and diffraction.

7.2.3 Min-Path-Tracing

The min path tracing method views the path finding problem as a minimization problem. Indeed, at each interaction the function $\mathcal{I}_i^r(\mathbf{X}_{i-1}, \mathbf{X}_i, \mathbf{X}_{i+1})$ or $\mathcal{I}_i^d(\mathbf{X}_{i-1}, \mathbf{X}_i, \mathbf{X}_{i+1})$ depending on the nature of the interaction, must be minimized under the constraint that each of the interaction points must lie on its corresponding surface or edge. This constraint is considered by verifying that the implicit equation $f_i(\mathbf{X}_i)$ of the surface or edge that each interaction point lies on is satisfied. In [14] it is shown that the problem boils down to finding a minimizer to the following optimization problem:

$$\text{minimize } \mathcal{C}(\mathcal{X}) := \|\mathcal{J}(\mathcal{X})\|^2 + \|\mathcal{F}(\mathcal{X})\|^2, \quad (7.7)$$

where the vector functions $\mathcal{J}(\mathcal{X})$ and $\mathcal{F}(\mathcal{X})$ are defined as:

$$\begin{aligned} \mathcal{J}(\mathcal{X}) &\triangleq [\mathcal{I}_1(\mathbf{X}_0, \mathbf{X}_1, \mathbf{X}_2)^T, \dots, \mathcal{I}_{n_t}(\mathbf{X}_{n_t-1}, \mathbf{X}_{n_t}, \mathbf{X}_{n_t+1})^T]^T, \\ \mathcal{F}(\mathcal{X}) &\triangleq [f_1(\mathbf{X}_1), \dots, f_{n_t}(\mathbf{X}_{n_t})]^T, \end{aligned} \quad (7.8)$$

and functions $\mathcal{I}_i(\mathbf{X}_{i-1}, \mathbf{X}_i, \mathbf{X}_{i+1})$ are either $\mathcal{I}_i^r$ or $\mathcal{I}_i^d$ depending on the nature of the interaction (reflection or diffraction).

This optimization problem can be solved by guessing an interaction point (taking advantage of the visibility matrix), and using gradient descent to find its exact coordinates. In [14] it was found that gradient descent will always converge for configurations involving planar surfaces and straight edges. For other configurations multiple minima to Equation 7.7 can exist and those where $\mathcal{C}(\mathcal{X}) \neq 0$ should be discarded.

Part IV

Validation

Chapter 8

Theoretical validation

Using the theory developed in the previous chapters, we built a ray tracer. To validate the reflection part of our ray tracer, we will compare its results in a simple outdoor open scenario to those obtained using an analytical two ray ground reflection model. Further validation is required for the diffraction component of our ray tracer, and more general reflections. This will be addressed in the next chapter by comparing our ray tracer's results to actual measurements.

8.1 Analytical two ray ground reflection model

When there is LOS between a TX and a RX antenna, the two most significant radio propagation paths between them are typically the direct LOS path and the ground reflection path. The power provided by these paths can be analytically computed using what is commonly referred to as a “two ray ground reflection model”.

Two ray models are almost exact for outdoor open areas, but their accuracy diminishes in other situations, where additional rays must be considered. For instance, [40] showed that at least five rays should be accounted for in order to achieve reasonable accuracy in an indoor corridor.

8.1.1 Geometry

The geometry behind the two ray ground reflection model is shown on Figure 8.1. It illustrates the TX and RX antennas mounted on poles of differing heights and separated by a flat, smooth ground. Note that the TX and RX antennas are aligned to the LOS path.

Given the distance L between the bases of the antennas, the heights z_{tx} and z_{rx} at which the antennas are mounted, all the other quantities shown in Figure 8.1 can be found using the method of images and basic trigonometry.

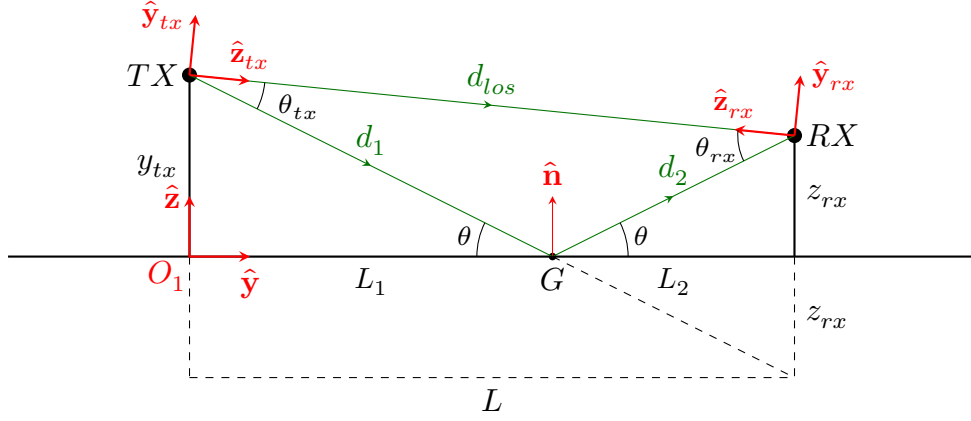


Figure 8.1: Two ray ground reflection model.

We find:

- The length d_{los} of the LOS path, and the total length $d_{ref} \triangleq d_1 + d_2$ of the reflection path:

$$d_{los} = \sqrt{(z_{tx} - z_{rx})^2 + L^2}, \quad d_{ref} = \sqrt{(z_{tx} + z_{rx})^2 + L^2}. \quad (8.1)$$

- The lengths d_1 and d_2 of the parts of the reflection path:

$$d_1 = d_{ref} \cdot \frac{z_{tx}}{z_{tx} + z_{rx}}, \quad d_2 = d_{ref} - d_1. \quad (8.2)$$

- The distance L_1 and L_2 between the bases of the antennas and the reflection point G :

$$L_1 = L \cdot \frac{z_{tx}}{z_{tx} + z_{rx}}, \quad L_2 = L - L_1. \quad (8.3)$$

- The angle of reflection θ , along with the emission angle θ_{tx} and reception angle θ_{rx} of the reflected path:

$$\theta = \tan^{-1} \left(\frac{z_{tx} + z_{rx}}{L} \right), \quad \theta_{tx} = \sin^{-1} \left(\frac{2z_{rx} \cdot L}{d_{los} \cdot d_{ref}} \right), \quad \theta_{rx} = 2\theta - \theta_{tx}. \quad (8.4)$$

8.1.2 Received power

In section 3.2 we have shown that the total power $P_{rx,tot}$ at the receiving antenna may be computed by individually by considering the contributions of each ray individually:

$$\begin{aligned} P_{rx,tot} &= P_{rx,los}(\theta_{los}) + P_{rx,ref}(\theta_{rx}) \\ &= \frac{1}{2Z_0} |\text{Re}(\mathbf{E}_{los})|^2 A_{e,rx}(\theta_{los}) + \frac{1}{2Z_0} |\text{Re}(\mathbf{E}_{ref})|^2 A_{e,rx}(\theta_{rx}), \end{aligned} \quad (8.5)$$

where $P_{rx,los}$ and $P_{rx,ref}$ represent the powers coming from the LOS, and ground reflected ray respectively, while $\mathbf{E}_{los}$ and $\mathbf{E}_{ref}$ represents their respective electric fields. $A_{e,rx}$ is the aperture efficiency of the receiving antenna (computed using Equation 3.12), $\theta_{los}=0$ because the receiving antenna is aligned to the LOS path, and Z_0 is the impedance of free space.

Field radiated by the antenna

We have shown in chapter 2 that the field radiated by an antenna (in the far field region) is given by

$$\mathbf{E}(r, \theta, \phi) = \underbrace{\frac{-jk\eta}{4\pi} \left((\hat{\mathbf{r}} \times \hat{\mathbf{F}}(\theta, \phi)) \times \hat{\mathbf{r}} \right)}_{\triangleq \mathbf{E}_0(\theta, \phi)} \cdot \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}, \quad (8.6)$$

where $\mathbf{r}$ is a vector pointing from the TX antenna to the observation point (r, θ, ϕ) , $\hat{\mathbf{F}}(\theta, \phi)$ is the normalized¹ radiation pattern of the antenna, and $k = \frac{2\pi}{\lambda}$ is the wavenumber. We will ignore ϕ in the rest of the computations, as it is the same for every ray considered in this situation.

Field from the LOS ray

The field propagated by the LOS ray is given by:

$$\mathbf{E}_{los}(\theta_{los}) = \mathbf{E}_0(\theta_{los}) \cdot \frac{e^{-jk d_{los}}}{d_{los}}, \quad (8.7)$$

where $\theta_{los} = 0$ because the TX antenna is aligned to the LOS path.

Field from the ground reflected ray

The field propagated by the reflected ray is given by:

$$\begin{aligned} \mathbf{E}_{ref}(\theta_{tx}) &= \mathbf{E}_{ref,\parallel}(\theta_{tx}) + \mathbf{E}_{ref,\perp}(\theta_{tx}) \\ &= \mathbf{E}_{0,\parallel}(\theta_{tx}) \cdot \Gamma_{\parallel} \cdot \frac{e^{-jk(d_1+d_2)}}{d_1+d_2} + \mathbf{E}_{0,\perp}(\theta_{tx}) \cdot \Gamma_{\perp} \cdot \frac{e^{-jk(d_1+d_2)}}{d_1+d_2} \\ &= \left(\mathbf{E}_{0,\parallel}(\theta_{tx})\Gamma_{\parallel} + \mathbf{E}_{0,\perp}(\theta_{tx})\Gamma_{\perp} \right) \cdot \frac{e^{-jk(d_1+d_2)}}{d_1+d_2}. \end{aligned} \quad (8.8)$$

Where $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ are the Fresnel reflection coefficients, and the decomposition of the reflected field into its parallel and perpendicular components is given by:

$$\begin{aligned} \mathbf{E}_{0,\perp}(\theta_{tx}) &\triangleq \mathbf{E}_0(\theta_{tx}) \cdot (\hat{\mathbf{d}}_1 \times \hat{\mathbf{n}}), \\ \mathbf{E}_{0,\parallel}(\theta_{tx}) &\triangleq \mathbf{E}_0(\theta_{tx}) \cdot \left((\hat{\mathbf{d}}_1 \times \hat{\mathbf{n}}) \times \hat{\mathbf{d}}_1 \right). \end{aligned} \quad (8.9)$$

¹Here by normalized we mean $\hat{\mathbf{F}}(\theta, \phi) = \frac{\mathbf{F}(\theta, \phi)}{|\mathbf{F}_{max}|}$

8.1.3 Validation of the analytical two ray model

Figure 8.2, which compares the analytical two ray ground reflection model derived above with the Friis model (introduced in section 3.3) and a simplified two ray model (described in Appendix C), validates its derivation when isotropic antennas are used. The purpose of this comparison is to perform a simple sanity check, ensuring that the analytical two ray model has been correctly computed. As the comparison shows, the analytical two ray model discussed in this section (red curve) perfectly matches the simplified two ray model (yellow) and the Friis model (blue).

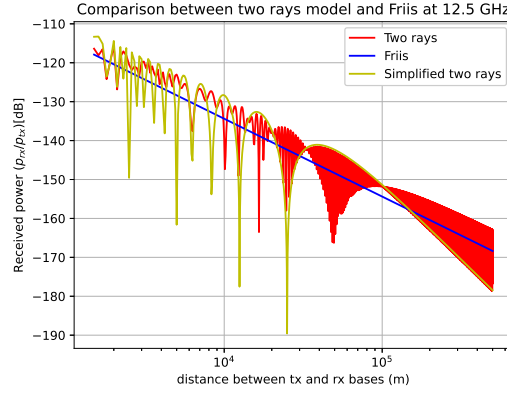


Figure 8.2: Comparison between Friis model, the analytical two ray model described in this chapter, and a simplified two ray model, at 12.5 GHz (antennas heights: TX=30 m, RX=10 m).

8.2 Analytical validation of the ray tracer

Figure 8.3 compares the results obtained from the analytical two ray model to those of our ray tracer when applied to the canonical scenario shown on the right of that Figure. The perfect match between the results provides confidence that the ray tracer accurately computes fields issued from reflections.

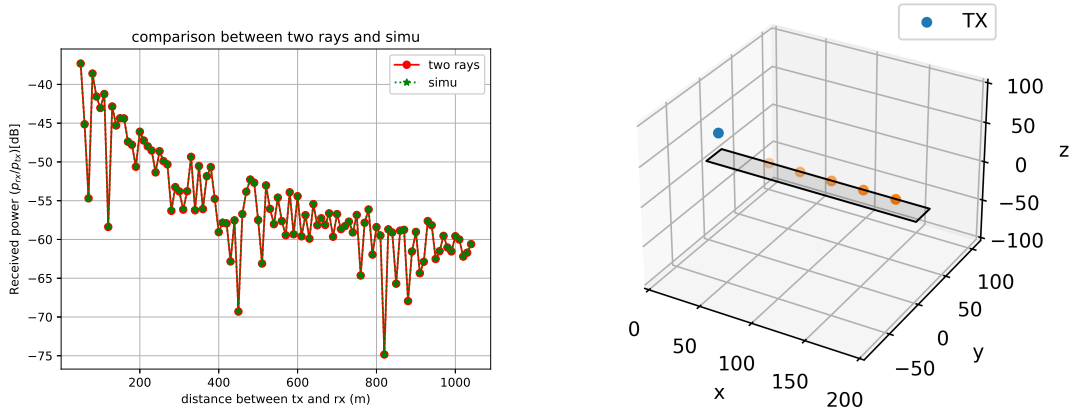


Figure 8.3: Left: Comparison between the two ray model and the results from our ray tracer, at 12.5 GHz. Right: Scenario input to the ray tracer (not to scale). Orange dots represent the receivers (antennas heights: TX=30 m, RX=10 m).

Chapter 9

Experimental validation

In this chapter, we compare the results of our ray tracer to real measurements in order to validate it. We also study the impact of taking into account street slope and higher order of interactions. We end the chapter with a study of the sensitivity of the results to the position of the TX antenna.

9.1 Environment description

The ray tracer will be validated in an outdoor suburban environment. The validation environment is chosen as “Place du Levant” in Louvain-La-Neuve Belgium and is pictured in Figures 9.1 and 9.2. This is a narrow street with a slight 3.8° incline, located between the Sainte-Barbe auditorium, and the Maxwell building, on the university campus. This particular street was chosen because [31]¹ has already performed measurements on it, which we will be using to validate our ray tracer.

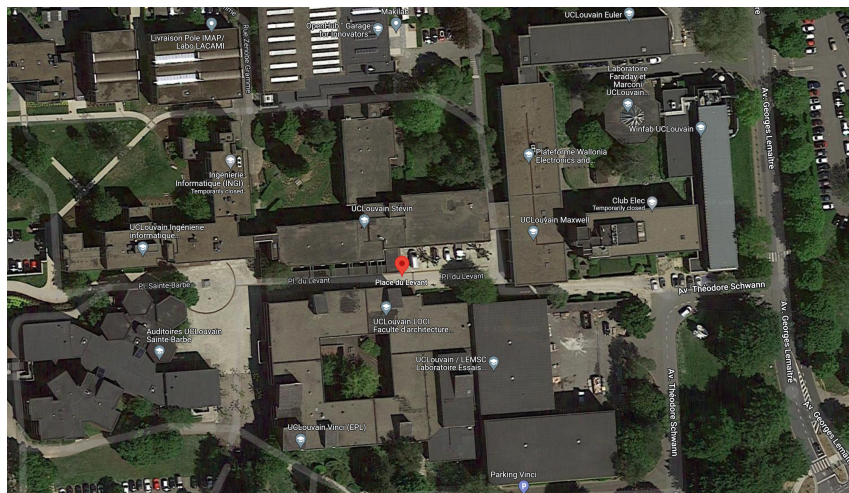


Figure 9.1: Satellite view of the Levant street and its surroundings.

¹Reference [25], written 15 years later by the same author and using the same measures is also a useful resource.



Figure 9.2: Left: Picture from the bottom of Levant street, looking at the Maxwell building. Right: April 2023 picture of the two plane trees and the hornbeam at the top of Levant street, in front of the Maxwell building.

9.2 Measurements description

The measurements are shown in Figure 9.3. They were performed at 12.5 GHz and 30 GHz under the following conditions, reported in [25]:

“The transmit antennas were two corrugated circular horns with vertical polarization and a 3 dB beamwidth of 30 degrees. The receiver was a dual-band corrugated horn with vertical polarization and a 3 dB beamwidth of 20 degrees. The transmitter was kept fixed and placed on the roof of the Maxwell building, whereas the receiver was moved away from the transmitter, stopping every meter to take a measurement, along a distance of about 56 meters (at height of 1.2 m), starting 34 m away from the Maxwell building. For each position of the mobile receiver the receiver antenna orientation was visually aligned along the boresight of the transmitter antenna using an accurate 3D positioner.”

The authors also note that the measurements were carried both in October 1998 when the trees were in full foliage, and in February 1999 when all the tree branches had been cut such that trees were reduced to their bare trunk. Their experimental environment is illustrated in Figure 9.4.

In their study they demonstrated the accuracy of their custom ray tracer against their measurements, while considering LOS rays, single reflections and a combination of reflections followed by a diffraction. The effects of scattering were neglected. They highlight the importance of considering the influence of the two plane trees shown in Figure 9.2 in their ray tracer to achieve satisfying results.

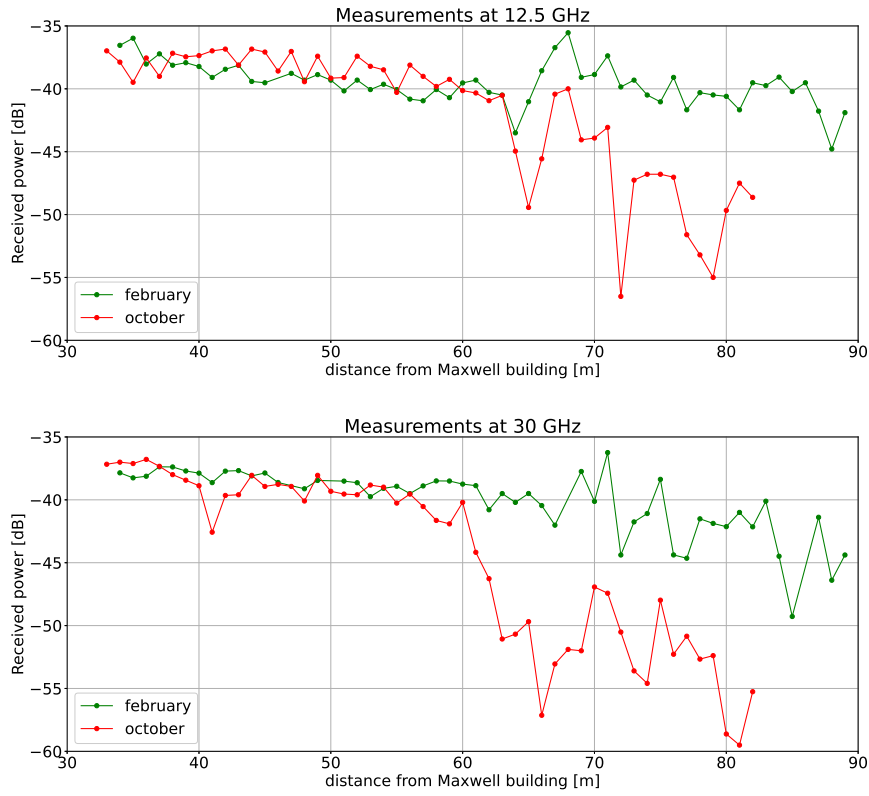


Figure 9.3: Measurements made in the Levant street, as reported in [25] at 12.5 and 30 GHz. The received power is normalised by assuming a unit transmit power.

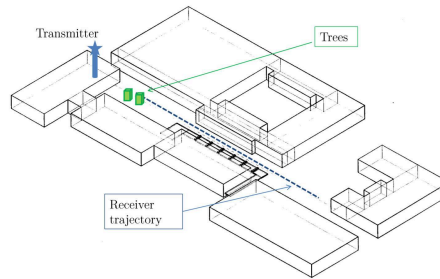


Figure 9.4: Experimental environment illustration of the measures made by the authors of [25]. The dashed line represents the path the receiving antenna followed during the measurements.

9.3 Simulation description

9.3.1 Simulated environment

Figure 9.5 shows the model of the Levant street provided to our ray tracer when the 3.8° slope of the street is ignored, and when it is accounted for. All building facades are attributed the electromagnetic properties of brick, while all building roofs, and the road, are attributed the electromagnetic properties of concrete. Details about these electromagnetic properties are provided in Appendix F.

The exact query used in overpass-turbo to generate the `.geojson` file providing the building outlines of this scenario is included in Appendix E. Since building heights were not available on OpenStreetMap for this street, we manually measured the height of each building, and the slope of the street, in order to model the scene as accurately as possible.

The TX antenna is located on the border of the roof of the Maxwell building, at a height of 1.2 m. The n simulated RX antennas are positioned in a straight line, 1.2 m above of the ground, equally spaced from each other, between 35 m and 90 m away from the base of the Maxwell building. For each RX antenna position, the RX and TX antennas are aligned to each other before carrying any computation.

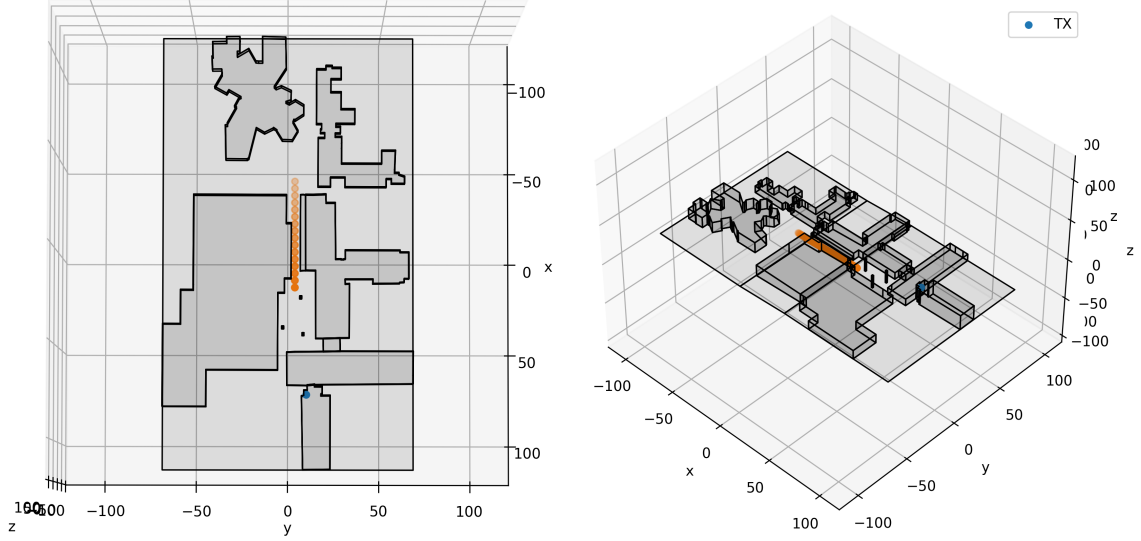


Figure 9.5: The model of the Levant street provided to our ray tracer. The blue dot represents the TX antenna, while the orange dots represent the RX antennas. Left/Right: slope of the street is ignored/accounted for.

Tree model

Each tree is modelled by a pair of rectangular prisms. The first prism represents the tree trunk where rays can reflect, while the second represents the tree crown, which rays may pass through while suffering from attenuation following the FITU-R model described in chapter 6. Figure 9.6 compares our tree model to a picture of the plane tree in front of the Maxwell building.

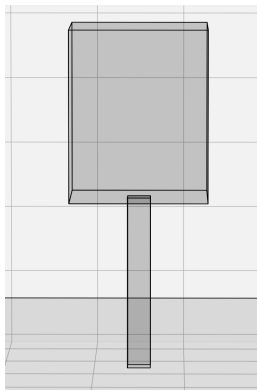


Figure 9.6: Comparison between the tree model, and a plane tree in front of the Maxwell, in full foliage.

Stairs model

The model of the covered stairs going up the slope of the Levant Street, as well as the stairs themselves, can be seen in Figure 9.7. This model allows rays to pass through the space between the stairs and their covering, effectively simulating the real-life scenario.

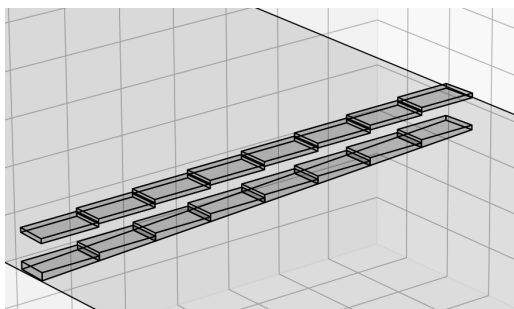


Figure 9.7: Comparison between the covered stairs model (left) and the actual covered stairs (right).

For the antenna positions and the order of interactions considered, none of the rays pass through the stairs for any receiver, as illustrated in Figure 9.8 for a single receiver. Therefore, to decrease the number of polygons in the scenario, and thus the simulation time, the stairs will be modelled by a single rectangular prism in all subsequent simulations.

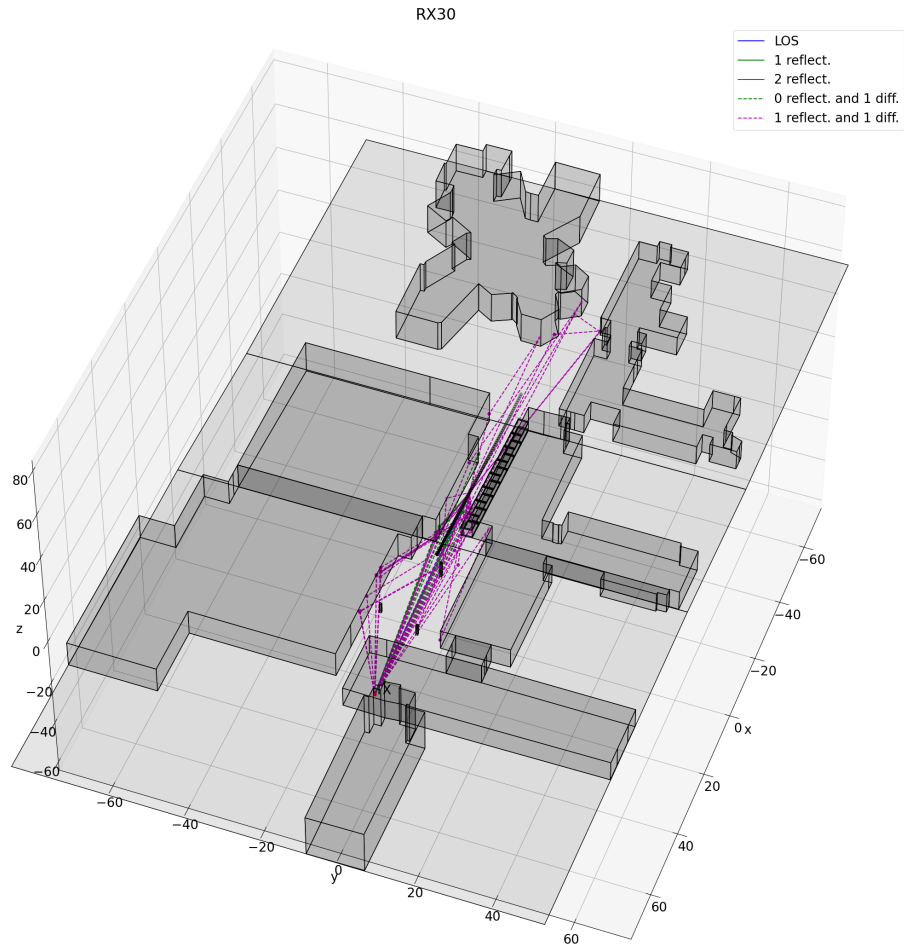


Figure 9.8: None of the rays goes through the accurate model of the stairs for the antennas and order of interaction considered.

9.3.2 Antenna radiation pattern

A radiation pattern commonly used to model antennas is the following:

$$\mathbf{F}(\theta, \phi) = \left(\frac{1 + \cos \theta}{2} \right)^\alpha \cdot \hat{\mathbf{e}}_t, \quad (9.1)$$

where α is a parameter that can be increased to increase the directivity of the antenna, and $\hat{\mathbf{e}}_t$ is the polarization of the TX antenna. In this case we will opt for $\alpha = 10$, as this value corresponds to a radiation pattern with a half power beamwidth of 30° which is consistent with the antennas used in the measurements. The corresponding radiation pattern, along with its elevation plane cross-section, are shown on Figure 9.9.

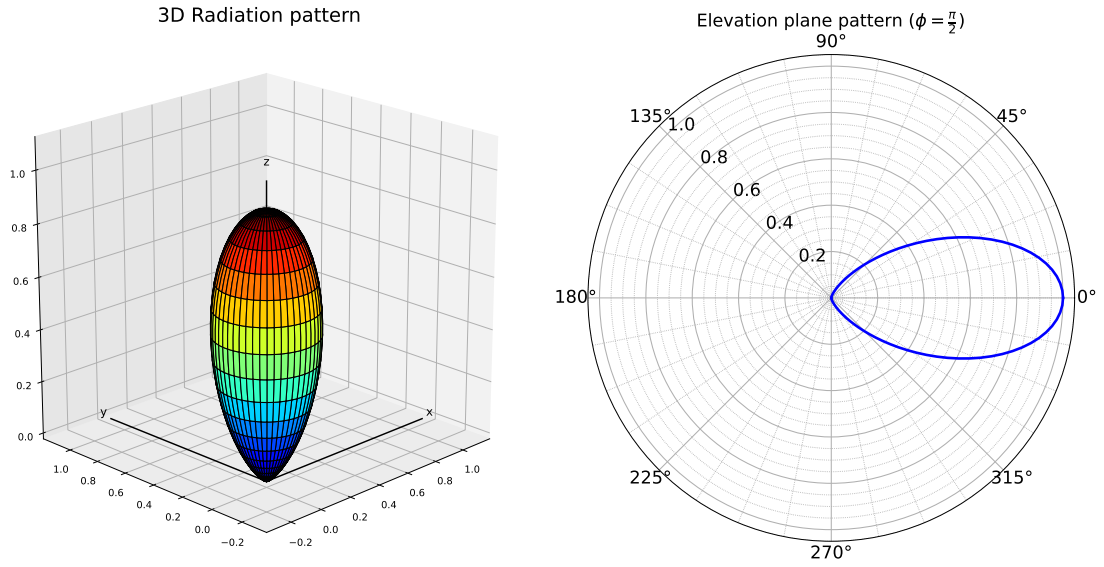


Figure 9.9: Illustration of the radiation pattern derived from Equation 9.1 with parameter $\alpha = 10$. Left: 3D view. Right: elevation plane cross-section.

9.4 Simulation results

9.4.1 Results on slanted scenario

Figure 9.10 presents the ray tracer's results considering the street slope. The ray tracer accounts for up to second order interactions². We will show in subsection 9.4.2 that it is not necessary to account for higher order interactions in this scenario, and subsection 9.4.3 will present a comparison of the results when the slope of the street is considered or neglected.

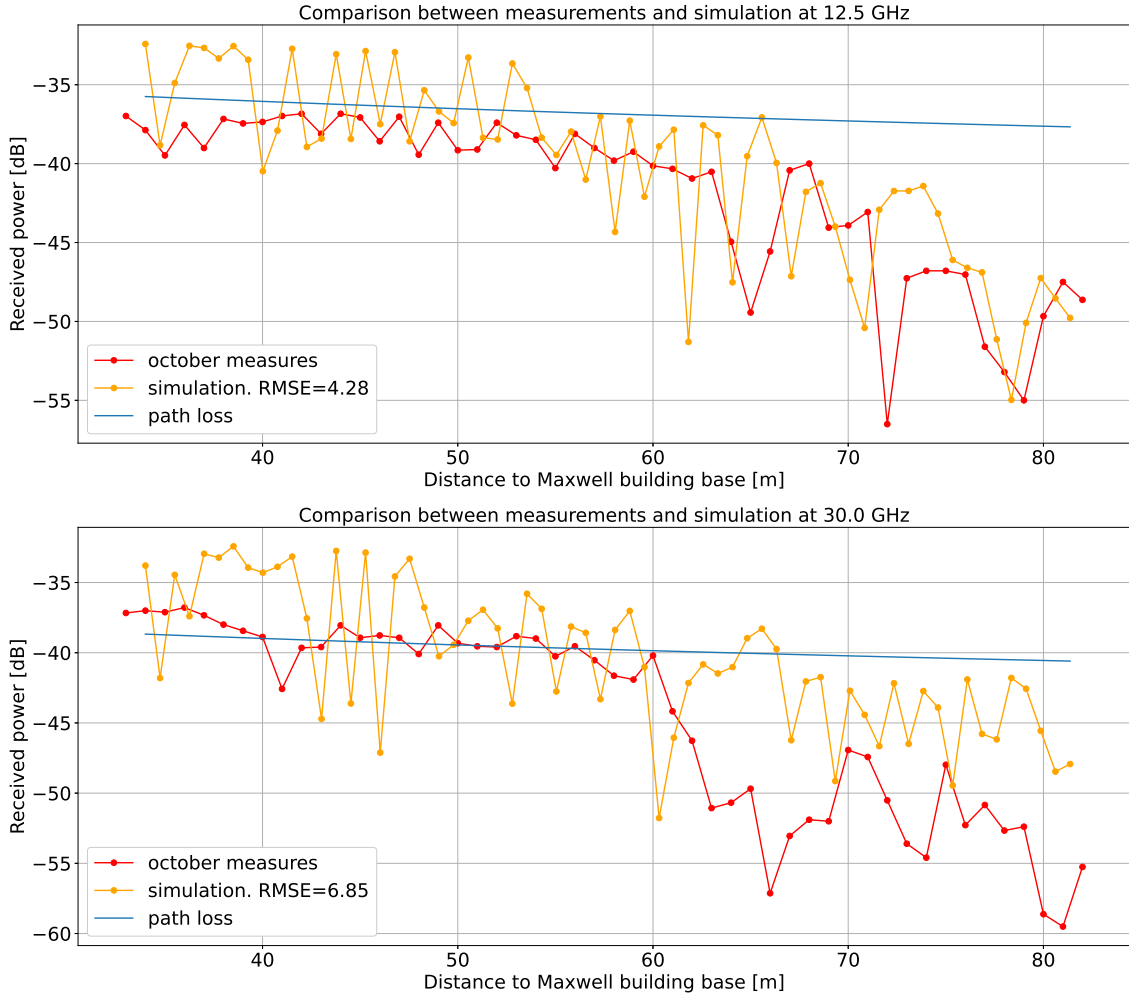


Figure 9.10: Comparison between the ray tracer's results and the October 1999 measurements at 12.5 GHz and 30 GHz. The power is normalized to a unit transmit power.

The results of our ray tracer are in reasonable agreement with the measurements. We observe that the Root Mean Square Error (RMSE) between the results of the ray tracer and the measurements is higher at 30 GHz than at 12.5 GHz. We explain

²Meaning it accounts for line of sight, simple and double reflections, simple diffraction, and simple reflections followed by diffraction.

this difference by the fact that higher frequency signals are more susceptible to interference by small environmental elements than lower frequency signals. Because we represented the same level of detail in our model at 30 GHz and 12.5 GHz, it makes sense that we obtain less accurate results at 30 GHz.

Moreover, although our poorly detailed model is responsible for one part of the error between the results of our ray tracer and the measurements, another part is due to the precise position of the antennas used in the measurements being unknown. Indeed, we will demonstrate in subsection 9.4.4 that even slight adjustments to the TX antenna position can greatly impact the results. Considering these error sources, the ray tracer's results are satisfactory and effectively validate our tool.

We can further analyze the results of the ray tracer by looking at Figure 9.11, which presents the power delay profile and an identification of the most significant path types contributing to the overall power collected by the receiver, for a receiver located near the Maxwell building, midway down the street slope, and at the bottom of the street, at 12.5 GHz. For RX30, which is in the middle of the street, the LOS path is no longer the strongest power contributor, but rather the simple reflections are. This is due to the LOS path being strongly attenuated by tree foliage. The measurements, being really close to the results of the ray tracer for that receiver, highlight the importance of modelling the vegetation.

Moreover, notice how despite the amount of paths undergoing a reflection and a diffraction before reaching any of the receivers, their contributions to the power collected at the receiver are mostly negligible compared to other sources when LOS is available.

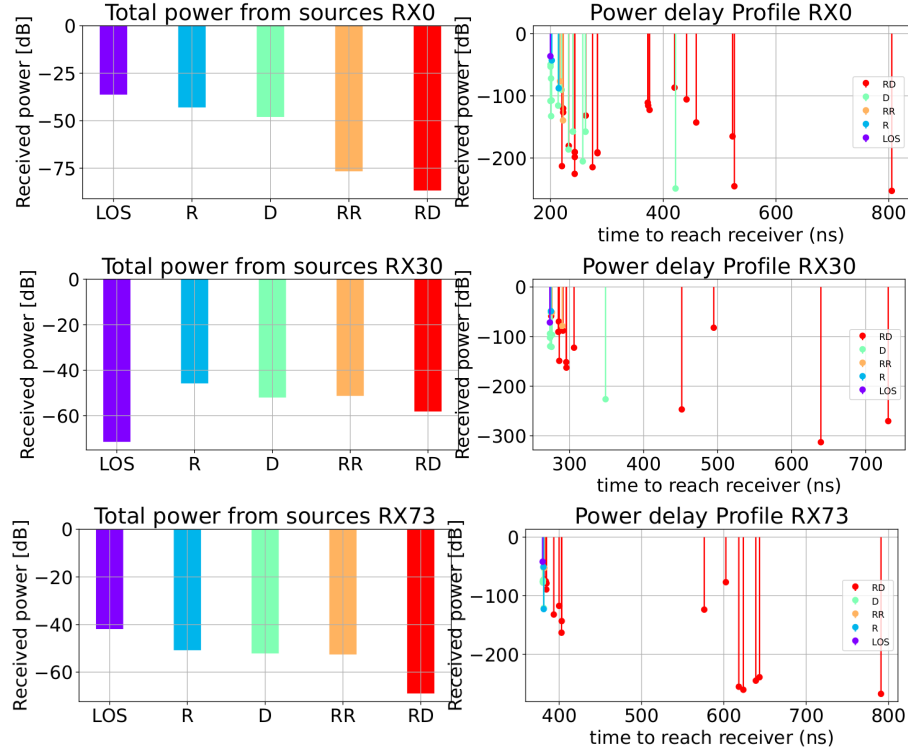


Figure 9.11: Power delay profiles, and power contributions by path type for a receiver near the Maxwell building, in the middle of the slope of the street, and at the bottom of the street respectively, at 12.5 GHz. The receiver positions are indicated by orange dots in the modelled scene shown in Figure 9.12.

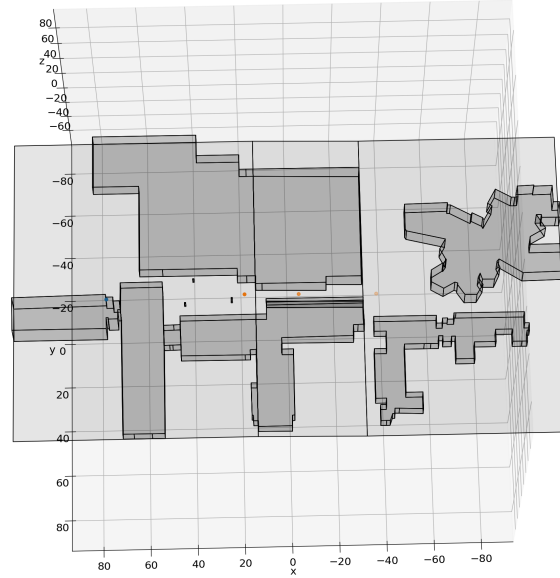


Figure 9.12: Positions of receivers 0, 30 and 73, for which we plotted the power delay profiles in Figure 9.11.

9.4.2 Simulating higher orders of interactions

To demonstrate that accounting for interactions beyond the second order is unnecessary for our scene, we conducted a simulation at 12.5 GHz on the slanted scenario, accounting for up to third order interactions. Figure 9.13 presents the power received at each receiver for different levels of interactions. Specifically, we consider zero interactions (only LOS), up to one interaction (LOS, R, D), up to two interactions (LOS, R, D, RR, RD), and up to three interactions (LOS, R, D, RR, RD, RRR, RRD). The plot reveals that the power at each receiver, whether simulating up to second or third order interactions, is the same, consequently we conclude that third order interactions are so attenuated, that they do not significantly impact the results.

The fact that third order interactions can be neglected allows us to significantly reduce the computation time of the ray tracer. Indeed, while the ray tracer completes its computations³ for 16 receiver positions in around 1 min when accounting for up to second order interactions, it takes around 22 minutes for up to third order interactions, and over 23 hours for up to fourth order interactions. It is worth noting that ray tracing is the most time-consuming part, as computing the power received at each receiver once the ray tracing is done only takes a few seconds.

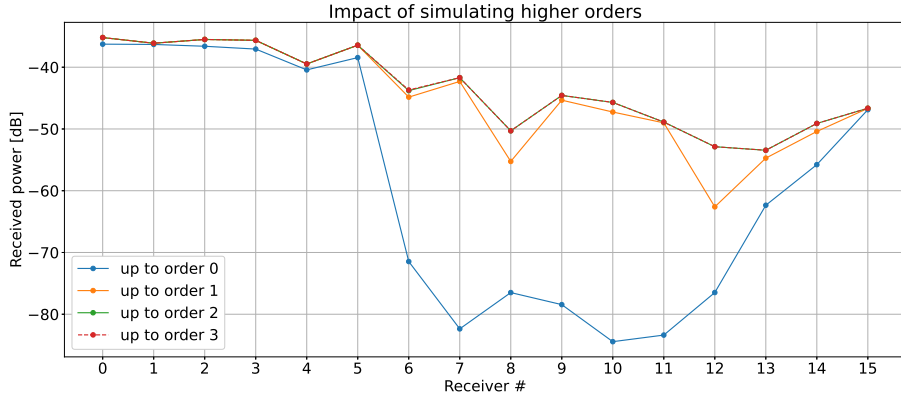


Figure 9.13: Results of the ray tracer, presenting the power received at each antenna when considering increasing orders of interactions at 12.5 GHz. Notice that the green line is merged on the red dashed line, indicating that third order interactions are so attenuated that they do not contribute significantly to the received power.

9.4.3 Importance of accounting for the slope of the street

Figure 9.14 compares the results of the ray tracer at 12.5 GHz when the slope of the street is considered, to those when it is not. From that comparison, it is clear that the fading of the signal further than 67 m away from the Maxwell building is not accurately represented when neglecting the slope of the street. Moreover, the RMSE between the measurements and the ray tracer's results on the slanted scenario is 239 % smaller than the RMSE between the measurements and the ray

³On an 8 cores desktop AMD 5800X CPU, with 16 logical processors.

tracer's results on the flat scenario. Therefore, we conclude that accounting for the sloping terrain of a scene, at high frequencies, is important to obtain accurate results. We provide an analytical proof of this in Appendix D, where we developed an analytical two ray model for a sloped ground, and compared its results to the flat ground analytical two ray model developed in the previous chapter, at 1 GHz and 12.5 GHz.

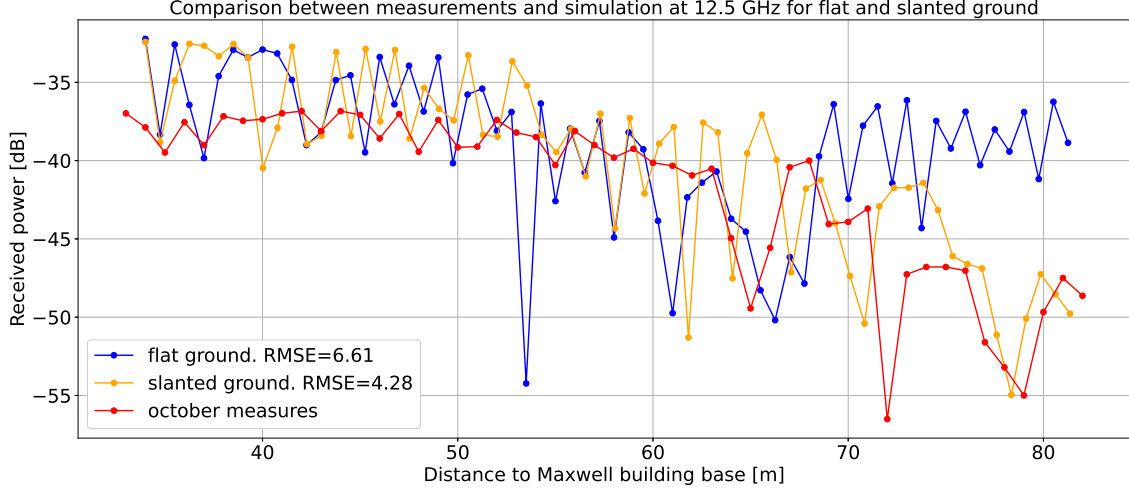


Figure 9.14: Comparison of the results of running the ray tracer at 12.5 GHz on the flat and on the slanted scenarios.

9.4.4 Sensitivity to the TX antenna position

As mentioned in subsection 9.4.3, slightly moving the TX antenna, in any direction can have a significant impact on the power received at the RX antennas. To demonstrate this we repeated the simulation at 12.5 GHz on the slanted scenario but moved the TX antenna less than 30 cm in different directions. For each movement of the TX antenna, the total power received at each receiver is compared to the baseline in Figure 9.15, and the RMSE is computed. The RMSE indicates that the power received at the RX antenna is very sensitive to the TX antenna's position relative to the RX antenna. This sensitivity is not surprising, and was already reported in [41] when working at 5.9 GHz.

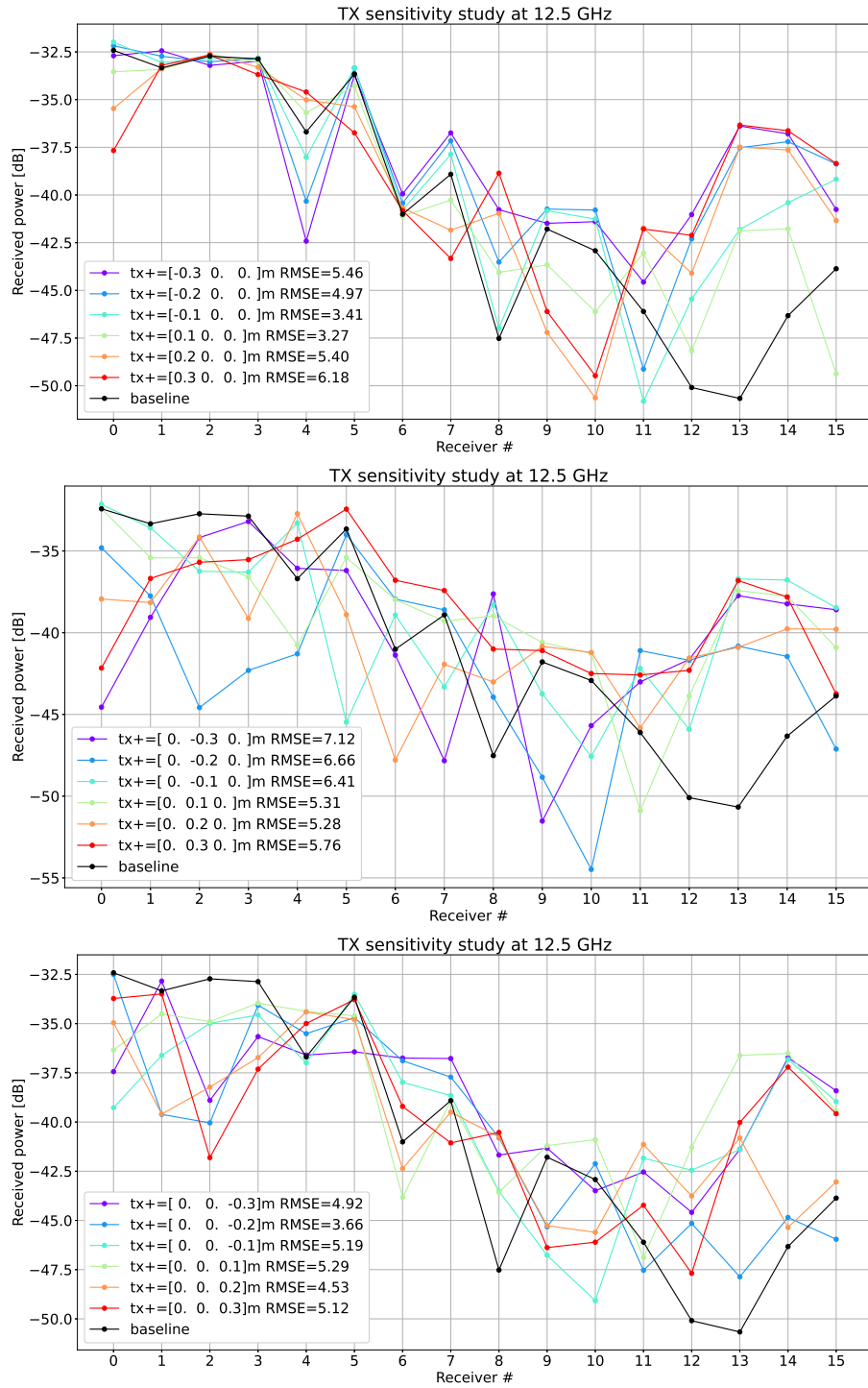


Figure 9.15: Study of the sensitivity of the power received at the RX antenna to the TX antenna's position.

Part V

Case study using the ray tracer

Chapter 10

Channel characteristics study of an urban environment

In this chapter, we demonstrate the functionality of our ray tracer at 12.5 GHz through an analysis of channel characteristics of Place de La Vieille Halle aux Blés in Brussels.

10.1 Metrics definitions

10.1.1 RMS delay spread

Root-mean-square (RMS) delay spread is a measure of the temporal dispersion of a received signal. We will adopt the definitions of RMS delay spread given in [42]. Let t_i be the arrival time in seconds of ray i , and $p(t_i)$ be the power arriving at time t_i . Let t_0 be the arrival time in seconds of the first ray reaching the receiver. Using these notations, the RMS delay spread is defined as:

$$TS_{RMS} \triangleq \sqrt{\frac{\sum_{i=0}^N (t_i - t_0 - \mu_t)^2 p(t_i)}{\sum_{i=0}^N p(t_i)}} \quad [\text{s}], \quad (10.1)$$

where N is the total number of rays reaching the receiver, and μ_t is the power weighted average delay time:

$$\mu_t \triangleq \frac{\sum_{i=0}^N t_i p(t_i)}{\sum_{i=0}^N p(t_i)} - t_0 \quad [\text{s}]. \quad (10.2)$$

Studying the temporal dispersion of the channel is important, as this characteristic limits the achievable symbol rates, and therefore data rates of the channel.

Indeed, as copies of the transmitted signal reach the receiver at different time instants, if the symbol duration chosen at the receiver is too short, intersymbol interference will occur. However, the longer the chosen symbol duration, the slower the achievable symbol rate.

10.1.2 RMS angular spread

RMS angular spread is a measure of the spatial dispersion of the channel. We will adopt the definitions of RMS angular spread given in [42]. Let α_i be either the elevation or azimuth angle of arrival of ray i , measured from the most powerful ray arriving at the receiver in radians. Let $p(\alpha_i)$ be the power arriving from angle α_i . The RMS angular spread is defined as:

$$AS_{RMS} \triangleq \sqrt{\frac{\sum_{i=0}^N (\alpha_i - \mu_\alpha)^2 p(\alpha_i)}{\sum_{i=0}^N p(\alpha_i)}} \quad [\text{rad}], \quad (10.3)$$

where N is the total number of rays reaching the receiver, and μ_α is the power weighted average angle of arrival:

$$\mu_\alpha \triangleq \frac{\sum_{i=0}^N \alpha_i p(\alpha_i)}{\sum_{i=0}^N p(\alpha_i)} \quad [\text{rad}]. \quad (10.4)$$

Studying the spatial dispersion of the channel is important to choose how directive the antennas should be.

10.2 Environment description and motivation

Now that our ray tracer has been validated, we are able to use it to examine the channel parameters of a new environment. The location we chose to study is "Place de la Vieille Halle aux Blés" in Brussels and can be observed in Figures 10.1, 10.2, and 10.3. This urban area is characterized by a Y-shaped intersection leading to another Y-shaped intersection, as well as a curved street, a sloping terrain and narrow streets. We chose this particular environment due to the scarcity of research on signal propagation in Y-shaped urban intersections and curved urban streets.

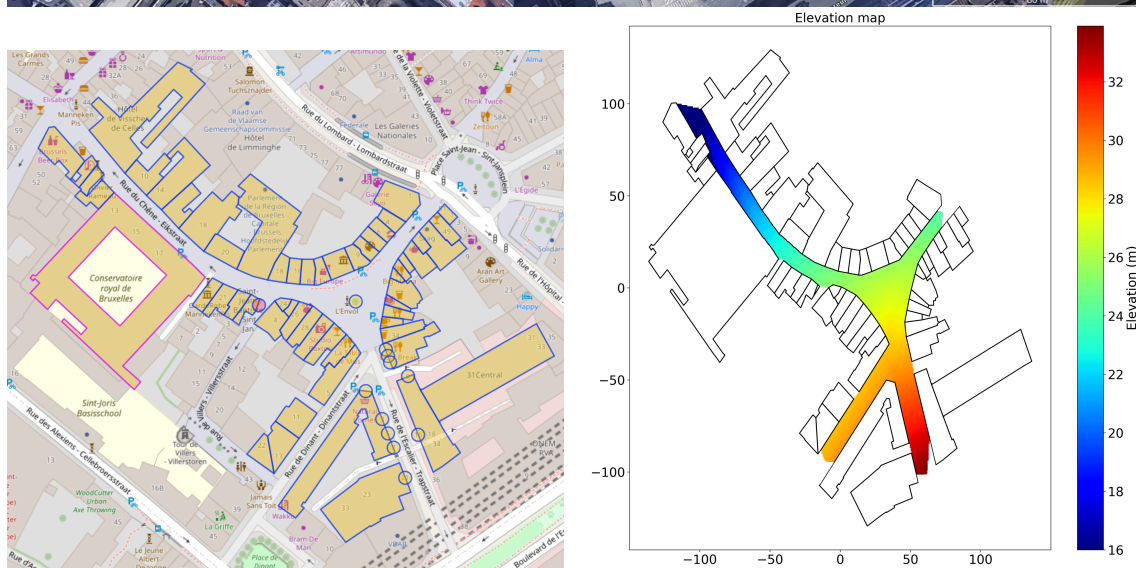


Figure 10.1: Satellite top view of Place de la Vieille Halle aux Blés, buildings to be modelled, and corresponding elevation map.



Figure 10.2: Street view of the street leading to Place de la Vieille Halle aux Blés (top), and of the Place (bottom).



Figure 10.3: Street view from the place towards Rue du Chêne (top), and towards the intersection of Rue de l'Escalier and Rue de Dinant (bottom.)

10.3 Simulated environment

Figure 10.4 shows the modelled environment. Building heights were acquired with Google Earth and are therefore not very accurate, neither relative to each other, or relative to the ground [43, 44, 45].

For all the following simulations the RX antennas will be placed at a height of 1.2 m relative to ground, the reason being that it is representative of a cellphone in a user's pocket, or hands. Meanwhile, the TX antennas will be placed 3 m above the surface they are placed on, to represent antennas mounted on building facades, or poles. We will assume a unitary gain for the TX and RX antennas, and a unitary radiation power.

To prevent the computational cost of the ray tracer from becoming prohibitively expensive the environment is modelled coarsely. This means modelling building facades as flat brick¹ walls, rather than incorporating recesses such as those found in door frames or window frames. Adding these recesses would require the inclusion of at least 5 polygons for each recess, which would be too computationally expensive. Moreover, many street features such as bollards, street signs, bike racks, flower pots and lampposts are neglected. Cars are not modelled either. Since all these potential scatterers are not modelled, the spatial and temporal dispersion of the channel will be underestimated. Note that the slope of the terrain is taken into account, as we have shown in the previous part that this was an important factor when working with high frequencies. Building roofs are considered as flat concrete surfaces, and the road is considered as concrete.

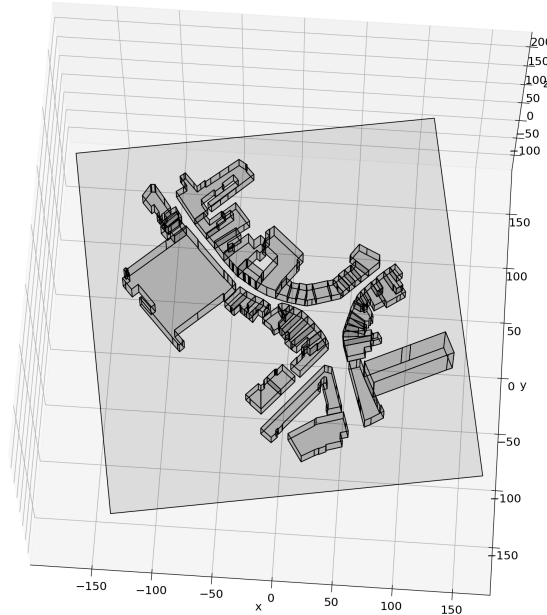


Figure 10.4: Modelled place.

¹The specific electromagnetic properties of every material mentioned are available in Appendix F.

10.4 Signal propagation under ideal conditions

Figure 10.5 present radiomaps for an isotropic 12.5 GHz TX antenna placed at the top of Rue de l'Escalier, and at the bottom of Rue du Chêne. The TX antennas are placed 3 m above ground, on the side of the street as if they were mounted on building facades. The radiomaps are created by placing 160 isotropic RX antennas in the streets, at least one meter away from the buildings and four meters away from each other and linearly interpolating in between the receivers. Due to computational complexity, only up to second order interactions are computed, although rays undergoing higher order interactions are not expected to affect the results significantly.

These two radiomaps illustrate that the transmitted signal has difficulty propagating upwards or downwards a curved street. This raises the question of where the TX antenna should be placed in order to provide coverage to streets around Place de la Vieille Halle aux Blés. In the following section, we will study whether positioning the TX antenna at street level in the middle of the intersection, or atop a nearby high rise building would be better suited.

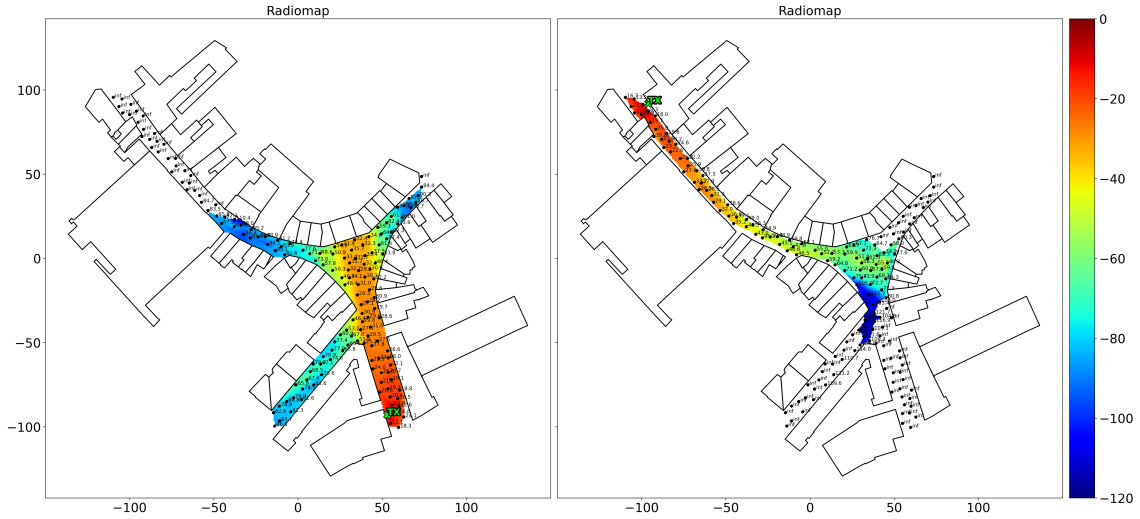


Figure 10.5: Radiomaps at 12.5 GHz for different TX antenna positions (indicated by a green cross).

10.5 Channel characteristics for a TX antenna at street/roof level

10.5.1 Signal strength and propagation

Figure 10.6 presents radiomaps of the place when the TX antenna is placed 3 m above ground in the middle of the intersection and when it is placed 3 m above the roof of the tallest nearby building, which at 48 m high is around 25 m higher than surrounding buildings. As before, all TX and RX antennas are isotropic, with a

unitary gain and radiation power.

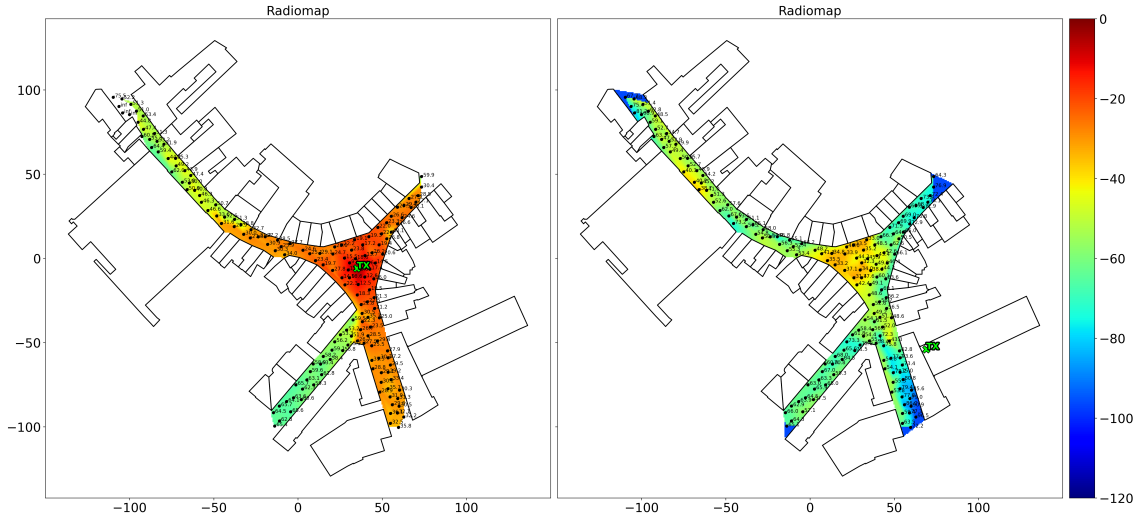


Figure 10.6: Radiomaps for the TX antenna placed in the middle of the intersection (left), and on the roof of a 48 m building (right).

These radiomaps indicate that placing the TX antenna in the middle of the intersection provides better signal strength in the surrounding streets compared to placing the same antenna on the highest nearby building. This is because the signal emitted from the antenna on top of the building suffers from significant path loss due to the distance it must travel to reach the ground.

Trying to counteract this effect by reducing the height of the building to 30 m such that it is only slightly taller than the other buildings, we observe on Figure 10.7 that no significant signal strength improvement is made compared to when the building was 48 m high. Although at this reduced TX antenna height, the signal strength at the base of the building and in the intersection has slightly improved, shadowing effects now prevent the signal from reaching the bottom of Rue du Chêne.

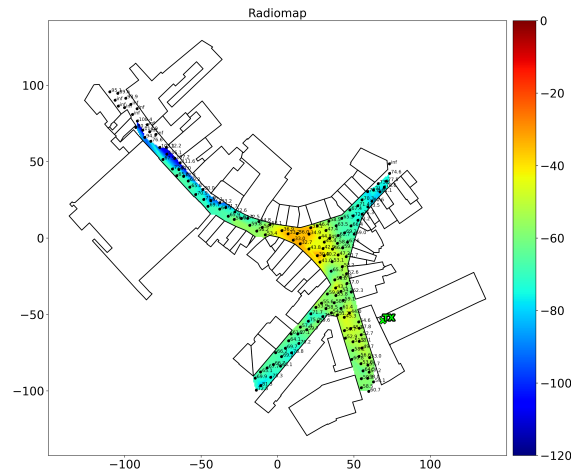


Figure 10.7: Radiomap when the height of the building is reduced from 48 m to 30 m.

10.5.2 Channel dispersion

Spatial dispersion

Figure 10.8 compares the RMS angular spreads cumulative distribution function (CDF) of the various situations. It shows that the azimuthal spread does not significantly change between the different TX antenna positions and is quite broad. About the elevation spread, as it can be expected the higher the TX antenna relative to the RX antennas, the greater the elevation spread.

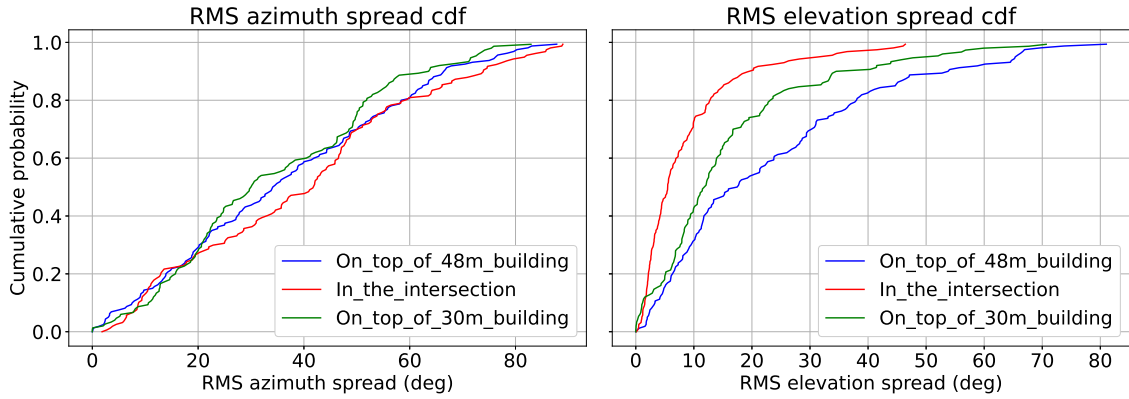


Figure 10.8: Comparison of the CDF of RMS angular spread for different TX antenna positions.

Temporal dispersion

Figure 10.9 compares the RMS delay spread CDF of the various situations. The plot indicates that temporal dispersion is greater when the TX antenna is on the 30 m building's roof compared to when it is on the 48 m building's roof. This is because the higher antenna has more receivers in line of sight, than the lower antenna. Overall, placing the TX antenna at street level in the intersection provides a much lower temporal channel dispersion: for 80 % of receivers all the most powerful multipaths reach the receiver within 25 nanoseconds, compared to only 58 % when the antenna is on the roof of the 48 m building.

What we conclude at the end of this section is that at 12.5 GHz, placing a TX antenna at street level in the middle of an intersection will in theory allow for faster data rates in the surrounding streets than those reached when placing the antenna on the roof of a nearby building, due to a lower temporal dispersion, and stronger signal strength.

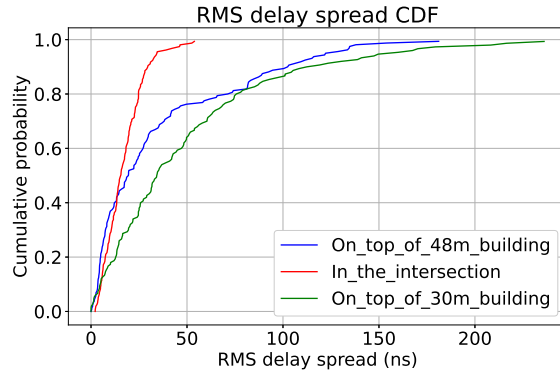


Figure 10.9: Comparison of the CDF of RMS delay spread for different TX antenna positions.

10.6 Evaluation of the accuracy of a n ray model in an urban environment

n ray models are often used in LOS scenarios to quickly estimate the amount of power received at the receiver, by considering only n paths between the TX and RX antenna. Depending on the amount of rays considered these models can often be solved analytically. It can be interesting to evaluate the precision of such models in an urban environment. To this end, we placed a 12.5 GHz isotropic TX antenna in the middle of Place de la Vieille Halle aux Blés, ignored all RX antennas that were not in LOS, and computed for each isotropic RX antenna the proportion of power reaching it from the n most dominant rays.

The results are presented in Figure 10.10 and demonstrate that only considering a single ray (equivalent to only considering free space path loss) in an urban environment is not enough to accurately estimate the power received at a receiver, as for 40 % of the receivers it represented less than 61 % of the power that reached them.

Obviously the more rays are considered, the more accurate the estimation will be, but the results show that considering two rays provides a significant accuracy improvement compared to a single ray. Specifically, for every receiver considered the two most dominant rays account for at least 50 % of the total power received. Moreover, they accounted for at least 85 % of the total power received for 80 % of the receivers.

Finally, the results show that considering additional rays can still provide marginal accuracy improvements.

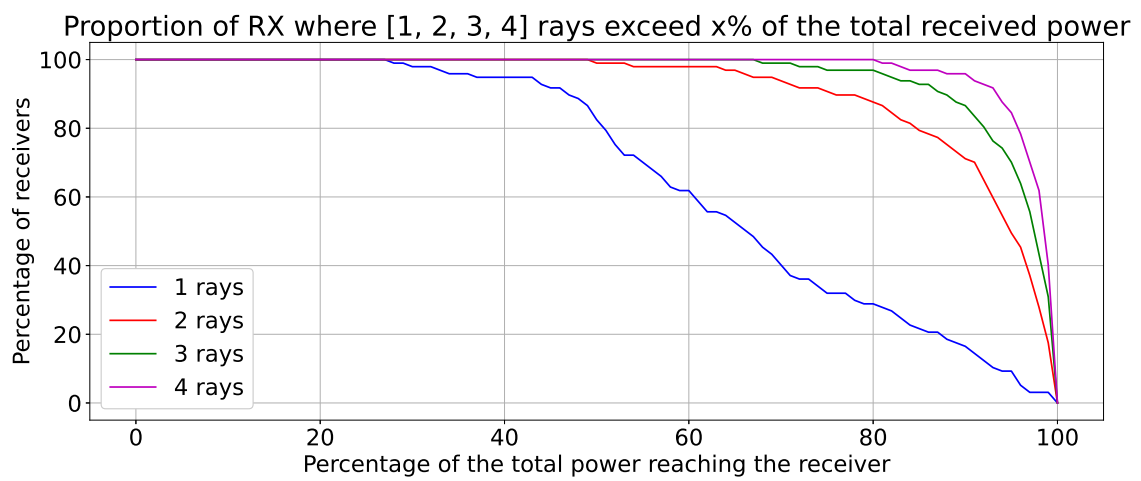


Figure 10.10: Comparison of the percentage of power coming from the n most powerful rays reaching the receiver.

Chapter 11

Conclusion

We started this work with Part I, where we dived into the establishment of the far field approximation, starting from Maxwell's equations, and reviewed some essential antenna theory. This theoretical foundation allowed us to find out how to combine the electric fields propagated along each path, identified through the min path tracing method, to determine the total power received at the receiver.

Part I did not address the propagation of electromagnetic waves interacting with the environment through reflections and diffraction. This gap was filled in Part II which introduced the concept of rays, and linked it to Maxwell's equations. A model to deal with diffuse reflections was also presented. Because Geometrical Optics is not able to deal with diffractions, we presented the Uniform Theory of Diffraction, an extension of GO which deals with this limitation. The part ended with a comparison of popular vegetation attenuation models, which justified the one chosen in the implementation of our ray tracer.

Relying on the theory presented in the previous parts we built a ray tracer. More specifically, we multithreaded the min path tracing algorithm developed in [14] and built the electromagnetism framework around it. Part IV validated the ray tracer first theoretically, against an analytical two ray ground reflection model, and then experimentally, against real measurements at 12.5 GHz and 30 GHz. This validation highlighted the sensitivity of the power received at the receivers, to the transmitting antenna position when working with high frequencies. It also emphasized both the importance of modelling the sloping terrain, and the vegetation of the environment, to obtain accurate results at these frequencies. Furthermore, we found that computing third order and above interactions was not necessary to achieve accurate results, greatly reducing computation time.

Finally, Part V demonstrated the capabilities of the ray tracer by computing channel dispersion in an urban intersection, for a transmitting antenna placed at street level or atop a building. We further highlighted that when there is Line Of Sight between an emitter and a receiver in an urban environment, merely considering the LOS ray for computing the power at the receiver is inadequate, and that considering a second ray would significantly increase accuracy. Indeed, for 40 % of the receiver positions

considered, the LOS ray constituted less than 61 % of the total power received.

Let us conclude this thesis, by recognizing that although min path tracing seamlessly provides the ability to deal with curved geometries, and multiple diffractions, its advantages compared to the image method are not easily demonstrated at the macro modelling scale, and high frequencies we worked at. On one hand, this stems from the difficulty of accurately modelling a 3D environment, especially curved surfaces, from limited environmental data, all without making the computational cost of the path tracing algorithm explode. On the other hand, paths undergoing multiple diffractions only make a significant power contribution at the receiver in very specific cases, which are hard to highlight within our coarsely modelled environments.

Future work should be dedicated to finding methods to improve the realism of model environments, all while keeping the computational cost of the path tracing algorithm reasonable. Further efforts should also be focused on accelerating the path tracing algorithm, through optimization and GPU acceleration, in order to enable more detailed environmental models to be studied.

Part VI

Appendices

Appendix A

Reference frame transformations

Figure A.1 illustrates the various transformations of reference frames.

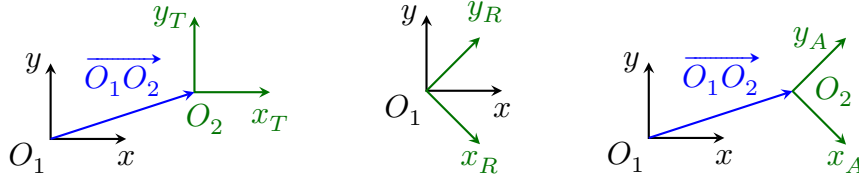


Figure A.1: 2D illustration of translational, rotational and rigid transformations respectively.

A.1 Translational transformation

If the reference frame is being translated without rotation, the transformation can be done using simple vector addition. For example, transformation of a vector $\mathbf{v}$ from frame O_1 to frame O_2 is achieved via:

$$\mathbf{v}_T = \mathbf{v} + \overrightarrow{O_1 O_2}. \quad (\text{A.1})$$

The inverse transformation is achieved by vector subtraction:

$$\mathbf{v} = \mathbf{v}_T - \overrightarrow{O_1 O_2}. \quad (\text{A.2})$$

A.2 Rotational transformation

If the reference frame is being rotated, without translation, such that the origins of both reference frames coincide, rotation of the reference frame around the origin is achieved using matrix multiplication.

Indeed, consider the original reference frame O_1 with basis $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ and the rotated frame O_2 with basis $(\hat{\mathbf{x}}_R, \hat{\mathbf{y}}_R, \hat{\mathbf{z}}_R)$. Also consider (v_1, v_2, v_3) the components of a

vector $\mathbf{v}$ in the original reference frame and (v_1^R, v_2^R, v_3^R) its components in the rotated reference frame. Then the vector $\mathbf{v}$ can be written:

$$\mathbf{v} = v_1 \hat{\mathbf{x}} + v_2 \hat{\mathbf{y}} + v_3 \hat{\mathbf{z}} = v_1^R \hat{\mathbf{x}}_R + v_2^R \hat{\mathbf{y}}_R + v_3^R \hat{\mathbf{z}}_R. \quad (\text{A.3})$$

Notice that given $\mathbf{v} = v_1 \hat{\mathbf{x}} + v_2 \hat{\mathbf{y}} + v_3 \hat{\mathbf{z}}$, and knowing the basis of both reference frames, one can directly find its components in the rotated reference frame using a few dot products:

$$\begin{aligned} v_1^R &= (v_1 \hat{\mathbf{x}} + v_2 \hat{\mathbf{y}} + v_3 \hat{\mathbf{z}}) \cdot \hat{\mathbf{x}}_R, \\ v_2^R &= (v_1 \hat{\mathbf{x}} + v_2 \hat{\mathbf{y}} + v_3 \hat{\mathbf{z}}) \cdot \hat{\mathbf{y}}_R, \\ v_3^R &= (v_1 \hat{\mathbf{x}} + v_2 \hat{\mathbf{y}} + v_3 \hat{\mathbf{z}}) \cdot \hat{\mathbf{z}}_R. \end{aligned} \quad (\text{A.4})$$

Which written in matrix form gives:

$$\begin{pmatrix} v_1^R & v_2^R & v_3^R \end{pmatrix} = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix} \overline{\overline{\boldsymbol{\lambda}}}, \quad (\text{A.5})$$

where $\overline{\overline{\boldsymbol{\lambda}}}$ is the rotation matrix:

$$\overline{\overline{\boldsymbol{\lambda}}} \triangleq \begin{pmatrix} \hat{\mathbf{x}} \cdot \hat{\mathbf{x}}_R & \hat{\mathbf{x}} \cdot \hat{\mathbf{y}}_R & \hat{\mathbf{x}} \cdot \hat{\mathbf{z}}_R \\ \hat{\mathbf{y}} \cdot \hat{\mathbf{x}}_R & \hat{\mathbf{y}} \cdot \hat{\mathbf{y}}_R & \hat{\mathbf{y}} \cdot \hat{\mathbf{z}}_R \\ \hat{\mathbf{z}} \cdot \hat{\mathbf{x}}_R & \hat{\mathbf{z}} \cdot \hat{\mathbf{y}}_R & \hat{\mathbf{z}} \cdot \hat{\mathbf{z}}_R \end{pmatrix}. \quad (\text{A.6})$$

Because the rotation matrix is orthogonal ($\overline{\overline{\boldsymbol{\lambda}}}^T = \overline{\overline{\boldsymbol{\lambda}}}^{-1}$), the inverse transformation is achieved simply by transposing the rotation matrix:

$$\begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix} = \begin{pmatrix} v_1^R & v_2^R & v_3^R \end{pmatrix} \overline{\overline{\boldsymbol{\lambda}}}^T. \quad (\text{A.7})$$

A.3 Rigid transformation

A rigid transformation combines translation and rotation of the axes. Note that the order in which the transformations are applied can matter, depending on the basis the translation vector is expressed in. Indeed, if the $\overrightarrow{O_1 O_2}$ vector linking the origins of the two reference frames is expressed in the basis of O_1 , translation should be done first, and then rotation:

$$\mathbf{v}^A = (\mathbf{v} + \overrightarrow{O_1 O_2}) \overline{\overline{\boldsymbol{\lambda}}}. \quad (\text{A.8})$$

On the other hand, if $\overrightarrow{O_1 O_2}$ is expressed in the basis of O_2 , then rotation should be done first, and then translation:

$$\mathbf{v}^A = \mathbf{v} \overline{\overline{\boldsymbol{\lambda}}} + \overrightarrow{O_1 O_2}. \quad (\text{A.9})$$

Appendix B

Computation of the transition function

The Fresnel integrals $S(x)$ and $C(x)$ are well known functions for which scientific libraries (in particular `scipy.special.fresnel`) provide solvers:

$$S(x) = \int_0^x \sin\left(\frac{1}{2}\pi u^2\right) du, \quad C(x) = \int_0^x \cos\left(\frac{1}{2}\pi u^2\right) du. \quad (\text{B.1})$$

From Euler's formula the following is easily deduced: $C(x) + jS(x) = \int_0^x e^{j\pi u^2/2} du$. We can rewrite the transition function $F(x)$ in another way, such as to make the Fresnel integrals appear [46]:

$$\begin{aligned} F(x) &= 2j\sqrt{x}e^{jx} \cdot \int_{\sqrt{x}}^{+\infty} e^{-ju^2} du \\ &= 2j\sqrt{x}e^{jx} \cdot \left(\int_0^{+\infty} e^{-ju^2} du - \int_0^{\sqrt{x}} e^{-ju^2} du \right) \\ &= 2j\sqrt{x}e^{jx} \cdot \left(\int_0^{+\infty} e^{-ju^2} du - \int_0^{\sqrt{x}} \cos(u^2) du + j \int_0^{\sqrt{x}} \sin(u^2) du \right) \\ &\quad \downarrow \text{ Let } u = \sqrt{\frac{\pi}{2}}v, \quad \text{thus} \quad du = \sqrt{\frac{\pi}{2}} dv, \\ &= 2j\sqrt{x}e^{jx} \cdot \sqrt{\frac{\pi}{2}} \cdot \left(\int_0^{+\infty} e^{-j\frac{\pi}{2}v^2} dv - \int_0^{\sqrt{x}} \cos\left(\frac{1}{2}\pi v^2\right) dv + j \int_0^{\sqrt{x}} \sin\left(\frac{1}{2}\pi v^2\right) dv \right) \\ &= 2j\sqrt{x}e^{jx} \cdot \sqrt{\frac{\pi}{2}} \cdot \left(\underbrace{C(+\infty)}_{\frac{1}{2}} - j \underbrace{S(+\infty)}_{\frac{1}{2}} - C\left(\sqrt{\frac{x}{\pi/2}}\right) + jS\left(\sqrt{\frac{x}{\pi/2}}\right) \right). \end{aligned} \quad (\text{B.2})$$

Which is easily computed.

Appendix C

Simplified two ray model

For all subsequent calculations we will use the notations introduced in subsection 8.1.1.

Two waves arrive at the receiver: the LOS and the reflected wave. They can be expressed as:

$$E_{los} = E_0 \text{Re} \left(\frac{e^{-jk d_{los}}}{d_{los}} e^{j\omega t} \right), \quad (\text{C.1})$$

$$E_{ref} = E_0 \text{Re} \left(\frac{e^{-jk d_{ref}}}{d_{ref}} e^{j\omega t} \right) \cdot \Gamma. \quad (\text{C.2})$$

The total field at the receiver is:

$$E_{tot} = E_{los} + E_{ref}. \quad (\text{C.3})$$

The reflected ray is longer than the LOS ray by a distance Δd , which will lead to a phase difference $\Delta\phi = \frac{2\pi}{\lambda} \Delta d$ between the two waves:

$$\begin{aligned} \Delta\phi &= \frac{2\pi}{\lambda} (d_{ref} - d_{los}) \\ &= \frac{2\pi}{\lambda} \left(\sqrt{(z_{tx} + z_{rx})^2 + L^2} - \sqrt{(z_{tx} - z_{rx})^2 + L^2} \right) \\ &= \frac{2\pi}{\lambda} \cdot L \left(\sqrt{\left(\frac{z_{tx} + z_{rx}}{L} \right)^2 + 1} - \sqrt{\left(\frac{z_{tx} - z_{rx}}{L} \right)^2 + 1} \right). \end{aligned} \quad (\text{C.4})$$

This expression can be simplified when $L \gg z_{tx} + z_{rx}$, using the first order Taylor series expansion of $\sqrt{1 + \epsilon}$:

$$\sqrt{1 + \epsilon} \approx 1 + \frac{\epsilon}{2}, \quad \text{when } \epsilon \rightarrow 0.$$

Therefore the phase difference is:

$$\Delta\phi \approx \frac{2\pi}{\lambda} \cdot \frac{2z_{tx}z_{rx}}{L}. \quad (\text{C.5})$$

Moreover, as L becomes large:

- $\theta \approx 0$ therefore the Fresnel reflection coefficient Γ tends to -1, that is $\Gamma \approx -1$.
- $d_{ref} \approx d_{los} \approx L$.
- $\theta_{tx}, \theta_{rx} \approx 0$ therefore the gains attributed to the LOS ray and the reflected rays are approximately the same: $G_{los} \approx G_{ref}$.

Thus, the total field E_{tot} at the receiver, is given by:

$$E_{tot} \approx \frac{E_0}{d_{los}} \cos(\omega t - kd_{los}) - \frac{E_0}{d_{ref}} \cos(\omega t - kd_{ref}). \quad (C.6)$$

And if we evaluate it at the time the ground reflected ray reaches the receiver, that is at $t = \frac{d_{ref}}{c}$ we get:

$$\begin{aligned} E_{tot} &= \frac{E_0}{d_{los}} \cos(\Delta\phi) - \frac{E_0}{d_{ref}} \cos(0) \\ &\approx \frac{E_0}{L} (\cos(\Delta\phi) - 1), \quad \text{as } d_{ref} \approx d_{los} \approx L. \end{aligned} \quad (C.7)$$

Using phasor addition we compute the norm of E_{tot} :

$$|E_{tot}| = \frac{E_0}{L} \cdot 2 \sin\left(\frac{\Delta\phi}{2}\right). \quad (C.8)$$

Finally, the power at the receiver is given by:

$$\begin{aligned} \frac{P_{rx}}{P_{tx}} &= \frac{|E_{tot}|^2}{Z_0} \cdot A_{e,rx} \cdot G_{tx} \\ &= \frac{1}{Z_0} \left[\frac{E_0}{L} \cdot 2 \sin\left(k \frac{z_{tx} z_{rx}}{L}\right) \right]^2 \cdot \frac{\lambda^2}{4\pi} G_{rx} \cdot G_{tx}. \end{aligned} \quad (C.9)$$

For any additional information, see reference: [47].

Appendix D

Sloped two ray model

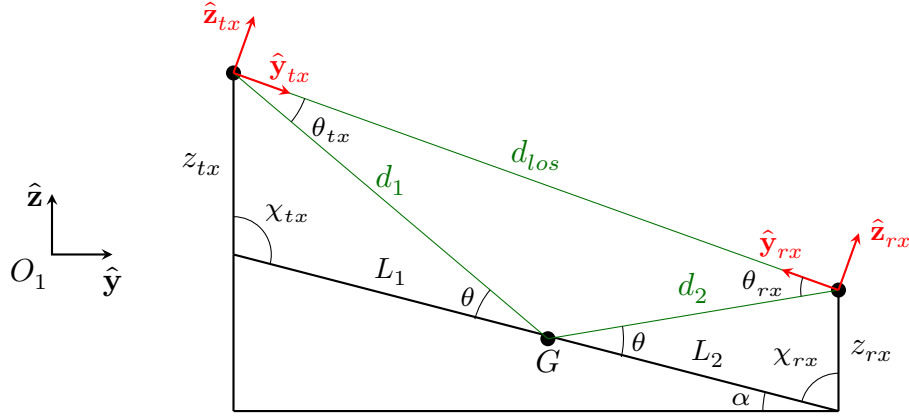


Figure D.1: Sloped two ray model

D.1 Geometry

Given the angle of the slope α , the distance $L \triangleq L_1 + L_2$ between the bases of the antennas, the heights z_{tx} and z_{rx} of the antennas, and assuming a smooth ground, all the other quantities shown in Figure D.1 can be found by solving the following system of equations:

$$\begin{cases} \frac{\sin \theta}{z_{tx}} = \frac{\sin \chi_{tx}}{d_1} \\ \frac{\sin \theta}{z_{rx}} = \frac{\sin \chi_{rx}}{d_2} \\ L = L_1 + L_2 \\ d_1^2 = z_{tx}^2 + L_1^2 - 2z_{tx}L_1 \cos \chi_{tx} \\ d_2^2 = z_{rx}^2 + L_2^2 - 2z_{rx}L_2 \cos \chi_{rx} \end{cases}, \quad (\text{D.1})$$

where $\chi_{tx} = \frac{\pi}{2} + \alpha$ and $\chi_{rx} = \frac{\pi}{2} - \alpha$.

By substitution, we find the following second order equation where only L_2 is unknown:

$$L_2^2(z_{tx}^2 - z_{rx}^2) + L_2(2z_{rx}^2 L + 2z_{rx}^2 z_{tx} \sin \alpha - 2z_{tx}^2 z_{rx} \sin \alpha) - z_{rx}^2 L^2 - 2z_{rx}^2 z_{tx} L \sin \alpha = 0. \quad (\text{D.2})$$

Once L_2 is found solving the rest of the system is trivial and gives us the values of the angle of reflection θ , the lengths d_1 and d_2 of the reflected ray, and the distance L_1 and L_2 from the base of the antennas to the reflection point. The length d_{los} of the line of sight path is given by:

$$d_{los} = \sqrt{d_1^2 + d_2^2 + 2d_1 d_2 \cos(2\theta)}. \quad (\text{D.3})$$

The emission θ_{tx} and reception θ_{rx} angles of the reflected path are found with:

$$\theta_{tx} = \sin^{-1} \left(\frac{d_2}{d_{los}} \sin(2\theta) \right), \quad \theta_{rx} = 2\theta - \theta_{tx}. \quad (\text{D.4})$$

Once all these geometrical quantities are found, the same reasoning developed in subsection 8.1.2 can be followed to find the total power at the receiver.

D.2 Comparison to flat two ray model

Figure D.2 compares the results of the flat and analytical two ray model at 12.5 GHz for a 3° slope, for transmit and receive antenna heights of 10 m and 2 m respectively, which are increasingly far apart from each other. The root mean square error of 7.62 dB between the two curves shows that at 12.5 GHz, even a 3° slope can significantly change the results of the analytical model. Figure D.3 repeats the experiment at 1 GHz, and shows that at this lower frequency the effect of the slope of the street is significantly less important, with a RMSE of only 1.88 dB.

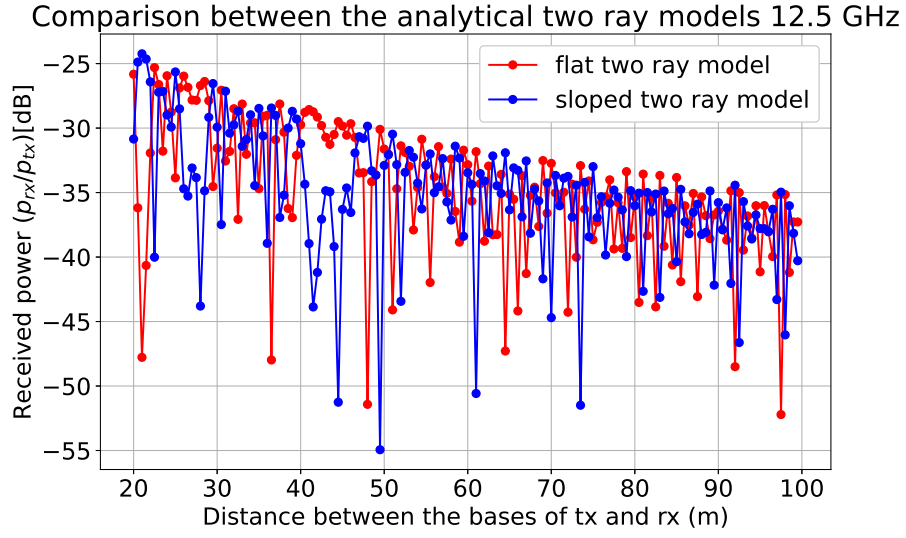


Figure D.2: Analytical comparison between the flat and sloped two ray models at 12.5 GHz. The RMSE of 7.62 dB between two curves shows that even a slight slope can significantly change the results of the models.

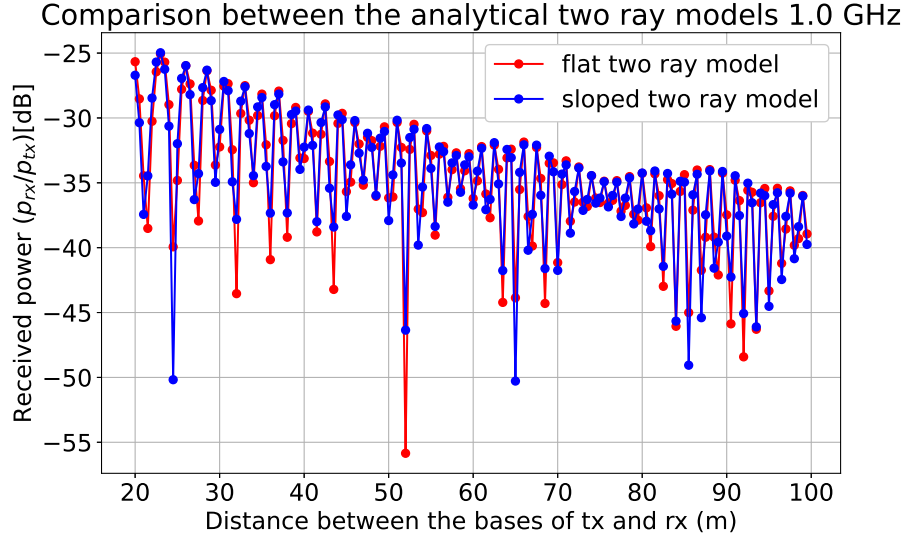


Figure D.3: Analytical comparison between the flat and sloped two ray models at 1 GHz. Taking into account the slope of the street barely affects the results, with a RMSE of only 1.89 dB.

Appendix E

Query used to generate the Levant street

Figure E.1 provide the exact query that creates the `.geojson` file used to generate our validation scenario.

```
1 [out:json][timeout:25];
2 (
3   way(162923960); //stevin
4   way(143236237); //barb
5   way(480739936); //between stevin/maxwell
6   way(162923962); //maxwell
7   way(162876893); //stairs
8   way(38771965); //reaumur
9   relation(192842); //vinci
10  node(5710653392); //tree near stevin
11  node(5710653391); //tree near maxwell
12  node(5710653390); //tree near vinci
13 );
14 (._;>);
15 out body;
```

Figure E.1: Overpass-turbo query used to export the building shapes and positions that generated our validation scenario.

Appendix F

Material properties

Material	ϵ_r	σ	μ_r	Validity [GHz]	Δh_0 [m]
Air	1	0	1	0.001-100	-
Concrete	5.24	$0.0462f^{0.7822}$	1	1-100	10^{-4}
Brick	3.91	$0.0238f^{0.16}$	1	1-40	10^{-3}
Wood	1.99	$0.0047f^{1.0718}$	1	0.001-100	10^{-4}
Glass	6.31	$0.0036f^{1.3394}$	1	0.1-100	10^{-6}
Metal	1	10^7	1	1-100	10^{-6}
Medium-dry ground	$15f^{-0.1}$	$0.035f^{1.63}$	1	1-10	10^{-2}

Table F.1: Electromagnetic properties of various materials in function of frequency (f is the frequency in GHz), with the corresponding frequency range validity. Source: [48]. Surface roughness is approximated.

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About the use of AI

I would like to acknowledge the assistance provided by ChatGPT, throughout the writing process of this thesis.

Although the contents of this thesis are solely the result of my own efforts, ChatGPT was used as a language helper, aiding in the reformulation of sentences, to enhance their clarity and improve the overall linguistic quality of the text. Much like a spell checker, it provided alternative phrasings, and grammatical corrections which contributed to the readability and coherence of this thesis. ChatGPT was also used to optimize some Python functions, and write their docstrings.

It is important to note that ChatGPT's involvement was limited to *enhancing* existing self written content, and code, rather than being directly responsible for the creation of the content, and code itself.

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