

École polytechnique de Louvain

Stability Analysis of Power Electronic Converters in an Offshore Wind Farm: A Case Study of the Princess Elisabeth Energy Island

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Declaration

In the course of preparing this master’s thesis, generative AI tools were employed in a limited and supervised capacity for the following purposes:

- **Literature synthesis:** To generate concise summaries of selected scientific publications during the preliminary stages of the research, thereby facilitating an efficient initial review of the relevant background material.
- **Linguistic refinement:** To propose alternative phrasings and more precise technical terminology, thereby improving the clarity, conciseness, and overall readability of the manuscript.
- **Grammatical and stylistic review:** To identify and correct grammatical inaccuracies and punctuation errors in accordance with standard conventions of formal academic English.
- **Implementation support:** To expedite the development of MATLAB scripts used for simulation and analysis, and of CircuiTikZ code for the single-line diagrams presented in this thesis.

All AI-assisted contributions were subsequently reviewed, critically evaluated, and revised by the author. Full responsibility for the accuracy, integrity, and scientific validity of the content presented in this thesis rests solely with the undersigned.

By signing this declaration, I confirm that the work presented herein constitutes my own original research and writing, conducted with the responsible, transparent, and ethically appropriate use of artificial intelligence tools.

Muhammad Abeer

25/05/2026

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Abstract

The accelerating integration of Inverter-Based Resources (IBRs) into the European electrical grid raises new stability concerns that the classical angle/voltage/frequency framework no longer captures on its own. Interactions between grid-following power-electronic converters and the resonances of the surrounding network are a recurrent issue in modern offshore wind power plants and have been recognised in the most recent CIGRE classification as a standalone class of stability phenomena.

This thesis investigates the small-signal stability and robustness of the Princess Elisabeth Energy Island, an offshore wind project under construction in the Belgian North Sea equipped with 14 MW Type-4 wind turbines. An impedance-based framework is adopted, in which every subsystem — wind turbines, transformers, array cables, the HVAC export cable, the grid Thevenin equivalent and the planned onshore synchronous condenser — is represented as a frequency-dependent dq impedance/admittance. The analytical wind-turbine admittance is validated against a detailed EMT model in MATLAB/Simulink through perturbation-based frequency scans, and the small-signal stability and robustness of the full plant at the Point of Common Coupling are assessed over the band 100 Hz–2.5 kHz through four indicators: the Generalized Nyquist Criterion, the passivity index, the vector margin and the disk margin.

The baseline configuration is shown to be small-signal stable and globally passive, yet only marginally robust, with a worst-case vector margin of 0.10 and a disk margin of 0.096 at $f \approx 242$ Hz; the GSC's LC filter resonance, combined with its transformer leakage inductance, is identified as the dominant source of fragility. A three-step converter-side mitigation strategy — LC filter re-sizing, capacitor-current-feedback active damping and a notch filter retuned on the dominant mode of the aggregated wind-farm/grid network — raises the worst-case vector margin to 0.63 and the disk margin above 0.5, and a short-circuit-ratio sweep that includes the planned synchronous condenser confirms that the final design remains robustly stable across the full range of expected grid strengths. The thesis thereby demonstrates, on a publicly documented case study, that distinguishing between systems that are merely stable and systems that are robustly stable is both feasible and essential in the design of the next generation of offshore wind connections.

Keywords: Offshore wind power plant; Princess Elisabeth Energy Island; Type-4 wind turbine; Harmonic stability; Small-signal stability; Impedance-based analysis; Robustness Analysis; dq -frame modeling; Generalized Nyquist Criterion; Vector margin; Active damping; Notch filter.

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List of Abbreviations and Acronyms

AC	Alternating Current	PCC	Point of Common Coupling
AEMO	Australian Energy Market Operator	PED	Power Electronic Device
AVR	Automatic Voltage Regulator	PEI	Princess Elisabeth Island
CCF	Capacitor Current Feedback	PEZ	Princess Elisabeth Zone
CIGRE	International Council on Large Electric Systems	PI	Proportional–Integral (controller)
CPL	Constant Power Load	PLL	Phase-Locked Loop
DAE	Differential-Algebraic Equation	PMSG	Permanent Magnet Synchronous Generator
DC	Direct Current	POC	Point of Connection
DFIG	Doubly-Fed Induction Generator	PPM	Power Park Module
DFT	Discrete Fourier Transform	PSCAD	Power Systems Computer-Aided Design
DM	Disk Margin	PV	Photovoltaic
EMT	Electromagnetic Transient	PWM	Pulse-Width Modulation
ENTSO-E	European Network of Transmission System Operators for Electricity	RMS	Root Mean Square
EPCIC	Engineering, Procurement, Construction, Installation and Commissioning	SC	Synchronous Condenser
FACTS	Flexible AC Transmission System	SCR	Short-Circuit Ratio
FFT	Fast Fourier Transform	SDC	Supplementary Damping Controller
GCSC	Gate-Controlled Series Capacitor	SISO	Single-Input Single-Output
GNC	Generalized Nyquist Criterion	SSCI	Sub-Synchronous Control Interaction
GSC	Grid-Side Converter	SSO	Sub-Synchronous Oscillation
HF	High Frequency	SSODC	Sub-Synchronous Oscillation Damping Controller
HV	High Voltage	SSR	Sub-Synchronous Resonance
HVAC	High-Voltage Alternating Current	SSSC	Static Synchronous Series Compensator
HVDC	High-Voltage Direct Current	SSTI	Sub-Synchronous Torsional Interaction
IBR	Inverter-Based Resource	STATCOM	Static Synchronous Compensator
IEA	International Energy Agency	SVC	Static Var Compensator
IEEE	Institute of Electrical and Electronics Engineers	SVM	Space-Vector Modulation
IGE	Induction Generator Effect	TCSC	Thyristor-Controlled Series Capacitor
LCL	Inductor–Capacitor–Inductor (filter)	TL	Transmission Line
LTI	Linear Time-Invariant	TSO	Transmission System Operator
MIMO	Multi-Input Multi-Output	UPFC	Unified Power Flow Controller
MMC	Modular Multilevel Converter	VM	Vector Margin
MOG	Modular Offshore Grid	VSC	Voltage-Source Converter
MPPT	Maximum Power Point Tracking	WECS	Wind Energy Conversion System
MSC	Machine-Side Converter	WF	Wind Farm
NREL	National Renewable Energy Laboratory	WPP	Wind Power Plant
OEM	Original Equipment Manufacturer	WT	Wind Turbine
OWPP	Offshore Wind Power Plant	WTG	Wind Turbine Generator
		XLPE	Cross-Linked Polyethylene
		ZOH	Zero-Order Hold

Chapter 1

Introduction

Decarbonisation is the initiative through which Europe has committed itself to a cleaner planet. The acceleration of climate change in recent years has affected not thousands but millions of people across the continent, and the integration of renewable energy sources has emerged as one of the most decisive responses to this challenge. Whether in the form of photovoltaic panels, hydro power, batteries or wind turbines, renewable installed capacity has been growing steadily, and among all these sources wind energy continues to lead the European mix. The breakdown of the share of renewable electricity generation in 2025 is shown in Figure 1.1, where wind accounts for the largest single contribution [1].

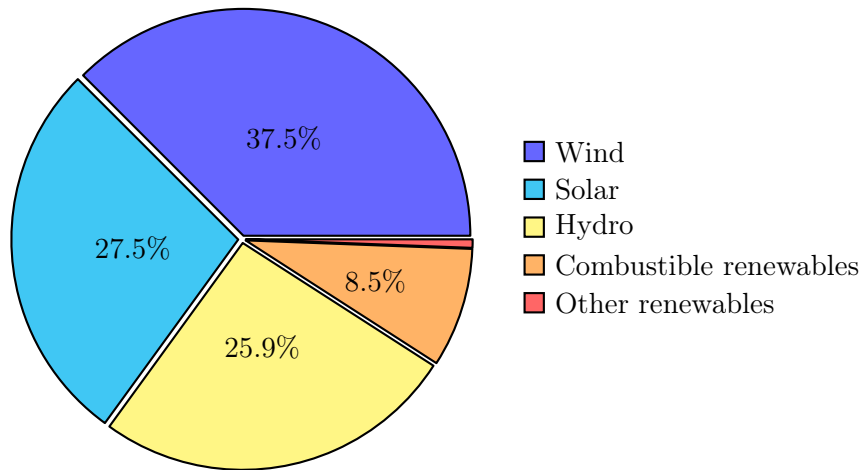


Figure 1.1: Share of renewable electricity generation by source in the European Union, 2025 [1]

With the notable exception of traditional hydropower, these resources are connected to the grid through power-electronic interfaces and are therefore referred to as *Inverter-Based Resources* (IBRs). Unlike conventional synchronous generators, IBRs do not inherently provide rotational inertia or natural damping; their interactions with the grid are shaped by the small-signal admittance of their control loops, filters and digital delays, and when this admittance fails to provide sufficient damping at the natural resonances of the surrounding cables, transformers and host grid, sustained oscillations can develop that lie at the origin of important stability concerns in modern power systems. To capture these emerging phenomena, the classical stability framework has been extended with two additional categories, namely *converter-driven stability* and *resonance stability*, as illustrated in Figure 1.2.

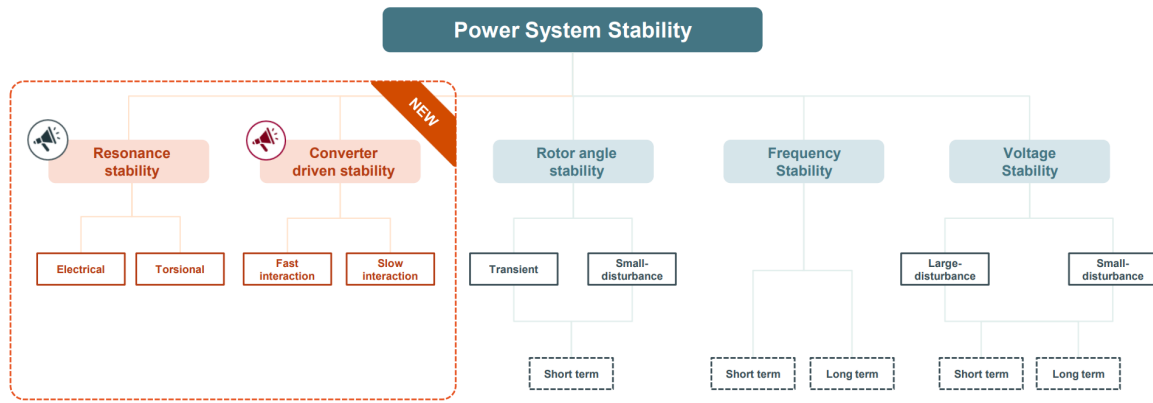


Figure 1.2: Extended classification of power system stability, including the newly introduced converter-driven and resonance stability categories [2]

These disturbances become particularly critical in offshore wind power plants, where tens of grid-following Voltage-Source Converters (VSCs) operate in parallel, are aggregated through extensive cable networks, and are evacuated to shore through long HVAC or HVDC export systems before reaching the onshore grid. The combination of converter-side dynamics with the resonances of cables, transformers, and the host grid has already led to documented converter and resonance instability events in real offshore projects, such as the BorWin 1 and BorWin 2 HVDC connections in the German North Sea and the Hornsea offshore wind farm in the United Kingdom [2], as summarised in Figure 1.3.

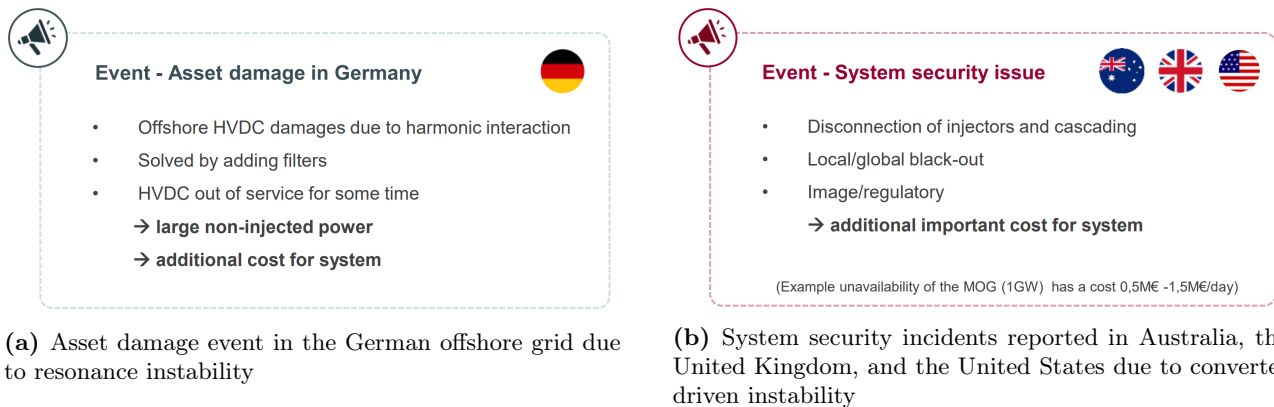


Figure 1.3: Representative real-world events linked to converter-driven and resonance stability phenomena in offshore and converter-dominated systems [2]

It is in this context that the present thesis takes the *Princess Elisabeth Energy Island*, also known as Modular Offshore Grid II (MOG II), as its case study: the world's first artificial energy island, currently under construction in the Belgian North Sea, which will serve as the offshore hub for the future expansion of Belgian offshore wind generation.

1.1 Research Question

The physical mechanism behind the new stability phenomena in converter-dominated offshore wind power plants is well understood: the small-signal admittance of grid-following converters, shaped by their filters, current controllers, outer loops, phase-locked loops and digital implementation, may insufficiently damp the resonances of the surrounding network in well-defined frequency bands. A binary

stable/unstable verdict is not sufficient: a system can be small-signal stable and globally passive and still operate so close to the stability boundary that small parametric drifts trigger sustained oscillations. The relevant engineering question is therefore not only whether the system is stable, but *how much robustness margin* the design provides, where this margin is the lowest, and which mitigation techniques are required when the margin is unacceptable.

These considerations lead to the central research question of this thesis:

Under which design and operating conditions does the HVAC connection of the Princess Elisabeth Energy Island remain not only small-signal stable, but also robustly stable, and which converter-side stability-enhancement techniques are required to restore an acceptable robustness margin where the baseline design fails to do so?

This central question is decomposed into the following sub-questions:

1. Among the small-signal stability frameworks available in the literature, which one is best suited to a converter-dominated, partially black-box offshore wind connection, and how should the wind turbine and the network be modelled within that framework?
2. Which stability and robustness indicators allow a complete and physically interpretable assessment of a converter-dominated offshore wind plant, and how should they be applied to the system?
3. For the publicly documented topology of the case study, is the wind-farm/grid interconnection at the Point of Common Coupling (PCC) small-signal stable, and if so, in which frequency bands is it closest to the stability boundary, and what is the physical origin of any fragile band?
4. Which combination of stability-enhancement techniques effectively enlarges the robustness margins in the critical bands?

To address these questions, the thesis pursues four concrete objectives: (i) to develop a complete dq -frame analytical admittance model of the Type-4 grid-side converter and to validate it against a detailed EMT model through perturbation-based frequency scans; (ii) to assemble a modular impedance model of the full offshore plant up to the onshore PCC, including the wind-turbine and substation transformers, the array cables, the HVAC export cable, the grid Thevenin equivalent and the planned synchronous condenser; (iii) to assess the small-signal stability and robustness of the baseline configuration through a coherent set of indicators and to identify the dominant source of any robustness bottleneck; and (iv) to design and tune a stability-enhancement strategy that restores the robustness margins.

The scope of the analysis is delimited as follows. The work concentrates on the HVAC export phase of the case study and on the harmonic frequency range 100 Hz–2.5 kHz, with the sub-/super-synchronous range covered analytically but not exercised at the full-network level. The wind-turbine converters are studied in grid-following mode only; grid-forming control is not considered. The analysis is small-signal: large-disturbance phenomena such as faults, protection actions and controller saturation lie outside the scope. Manufacturer-specific data that are not publicly available are replaced with values taken from the peer-reviewed literature and are documented as such.

1.2 Thesis Outline

This thesis is organised into four core chapters and a conclusion. Chapter 2 presents the technical fundamentals: it introduces the Princess Elisabeth Energy Island case study in detail, recalls the definitions of power quality and the most recent classification of power-system stability, and reviews the general framework of dq -frame modelling, state-space and impedance representations, together with the basics of wind-turbine technology.

Chapter 3 develops the analytical framework on which the rest of the work is built. It compares the state-space and impedance-based stability approaches and motivates the choice of the impedance-based formulation. It then derives the detailed dq impedance models of the Type-4 wind turbine, the transformers, the array and HVAC cables, the grid Thevenin equivalent and the synchronous condenser, introduces the four impedance-based stability and robustness indicators used throughout the thesis, and reviews the stability-enhancement techniques reported in the literature.

Chapter 4 applies this framework to the case study. It computes the simulation parameters of the Type-4 wind turbine, validates its analytical admittance against a detailed EMT model in MATLAB/Simulink through dq -frame perturbation injections, assembles the full wind-farm admittance at the PCC, and assesses the small-signal stability and robustness of the baseline configuration.

Chapter 5 implements the stability-enhancement strategy. Three converter-side techniques — LC filter re-sizing, capacitor-current-feedback active damping and a notch filter retuned on the dominant network mode of the loop gain — are applied incrementally on top of the baseline model and evaluated using the same set of indicators. The planned onshore synchronous condenser is then modelled and added to the network.

Chapter 6 synthesises the answers obtained to the research sub-questions, summarises the main contributions of the thesis, discusses the limitations of the proposed approach and outlines directions for future research.

Chapter 2

Technical Fundamentals

The research question formulated in Chapter 1 requires a technical foundation that brings together three otherwise distinct fields: the physical layout of the offshore plant under study, the modern classification of power system stability phenomena in converter-dominated grids, and the mathematical tools used to describe power-electronic systems around an operating point. This chapter establishes that foundation. It first introduces the Princess Elisabeth Energy Island case study, summarising the topology, voltage levels, equipment ratings and planned auxiliary infrastructure that are relevant for the stability analysis carried out in the rest of the thesis. It then recalls the most recent classification of power system stability proposed by CIGRE, with particular emphasis on the harmonic and sub-/super-synchronous categories that are directly addressed by the present work. Finally, it reviews the standard modelling workflow for grid-connected converters, namely the projection onto the synchronous dq reference frame and the derivation of state-space and impedance/admittance representations, before closing with a concise overview of wind-turbine technology.

2.1 Overview of the Princess Elisabeth Energy Island

The case study selected for the stability analysis developed in this thesis is the first export phase of the Princess Elisabeth Island project, also referred to as Modular Offshore Grid (MOG) II. The project, which is under construction as of 2026, is located in the Belgian North Sea, approximately 45 km off the Belgian coast. As the world's first artificial energy island, it is intended to serve as the offshore hub for the integration of large-scale offshore wind generation into the Belgian transmission system, with a target total installed capacity of 3.5 GW [3].

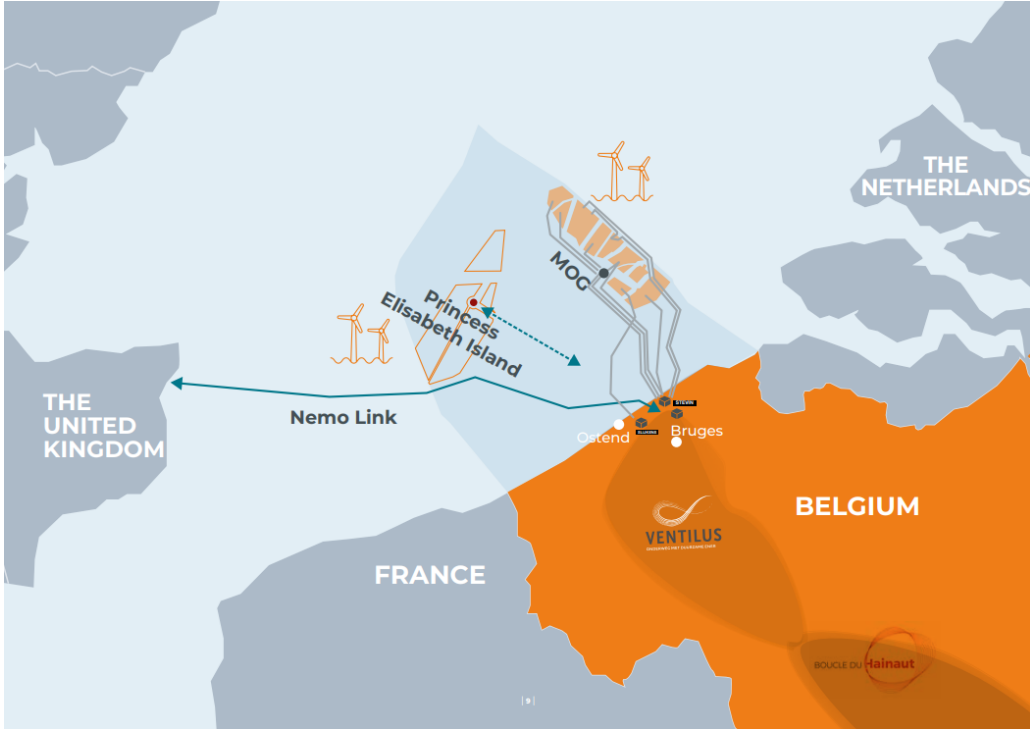


Figure 2.1: Overview of the Princess Elisabeth Zone (PEZ) [3]

The 3.5 GW of offshore wind capacity are evacuated to the onshore grid in three successive export phases. Phase 1 connects 700 MW through two 220 kV three-phase HVAC export cables of approximately 55 km [16]; Phase 2 adds 1400 MW through four further 220 kV HVAC cables, bringing the combined HVAC export capacity to 2.1 GW; Phase 3 connects an additional 1400 MW through an HVDC export system. To accommodate this layout, the island hosts four AC substation modules — two rated at 700 MW and two rated at 1050 MW — for which Elia has awarded the EPCIC (Engineering, Procurement, Construction, Installation and Commissioning) contract [3, 17]. The present thesis focuses on Phase 1, i.e. the 700 MW HVAC building block.

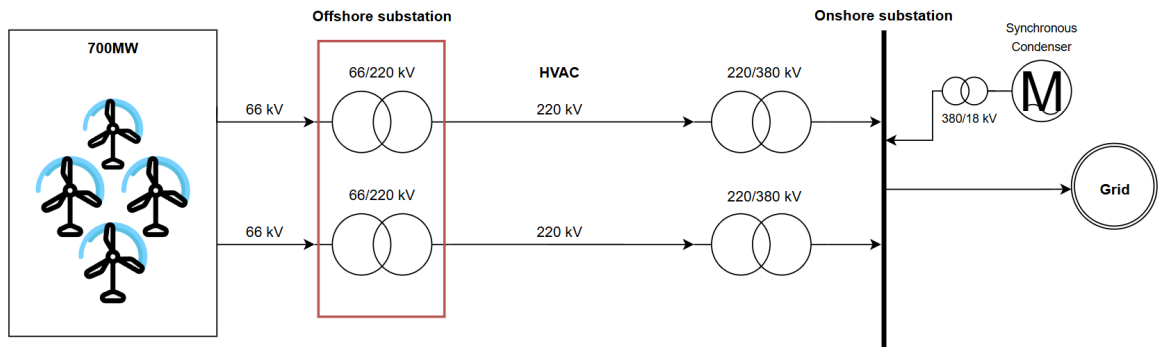


Figure 2.2: Simplified single-line diagram of the Princess Elisabeth Energy Island (Phase 1) connection to the onshore grid

On the onshore side, the HVAC export cables land at a new 380 kV substation that forms part of the Ventiluz and Boucle du Hainaut reinforcement scheme — Elia’s strategic 380 kV backbone for evacuating offshore wind toward the Belgian industrial centres and reinforcing the cross-border interconnection with France [18]. In late 2025, Elia further announced that two synchronous condensers (SCs) of 280 MVar each are planned to be installed at a coastal substation by 2030 in order to provide

additional inertia, short-circuit strength and reactive-power absorption capability [19]. The detailed electrical parameters of these machines are not publicly disclosed at the time of writing.

On the generation side, the offshore power park module is classified as a Type D power park module under the Belgian grid code, i.e. as a non-synchronous power park module. In practical terms, this corresponds to the expected use of full-converter wind turbine technology, referred to as Type-4 wind turbines, although the regulatory framework itself remains technology-neutral [20]. The currently planned Phase 1 configuration consists of 50 wind turbines rated at 14 MW each, arranged in two 350 MW sub-blocks, for a total of 700 MW per offshore building block [2].

With respect to the converter interface, large-scale Type-4 grid-side converters typically operate at medium-voltage levels such as 3.3 kV or 6.6 kV, before being stepped up to the 66 kV array collection grid by the wind-turbine transformer. Consistently with reported industrial implementations of 14–15 MW Type-4 wind turbines, the grid-side converter terminal voltage adopted in this work is of the order of 4 kV [21].

2.2 Power System Stability

The stability of a power system is the property that allows it to reach an acceptable steady-state following a disturbance. In converter-dominated power systems, the traditional classification of stability phenomena into angle stability, voltage stability, and frequency stability remains relevant, but it is no longer sufficient on its own to describe all interaction mechanisms observed in modern grids.

While the 2021 revisited classification [22] introduced the terms converter-driven stability and resonance stability to reflect the growing role of converter-interfaced generation, the more recent 2025 CIGRE classification provides a clearer and more physically grounded framework [4].

In [4], it is argued that recently introduced terms of resonance stability and converter-driven stability are too ambiguous, since they do not clearly identify the physical nature of the instability and do not provide a robust basis for model simplification, reproduction of the phenomenon, or mitigation. To overcome this limitation, the authors propose extending the classical framework by introducing frequency content as an additional classification variable and by adding two new classes: sub-/super-synchronous stability and harmonic stability as shown in the right part of Figure 2.3.

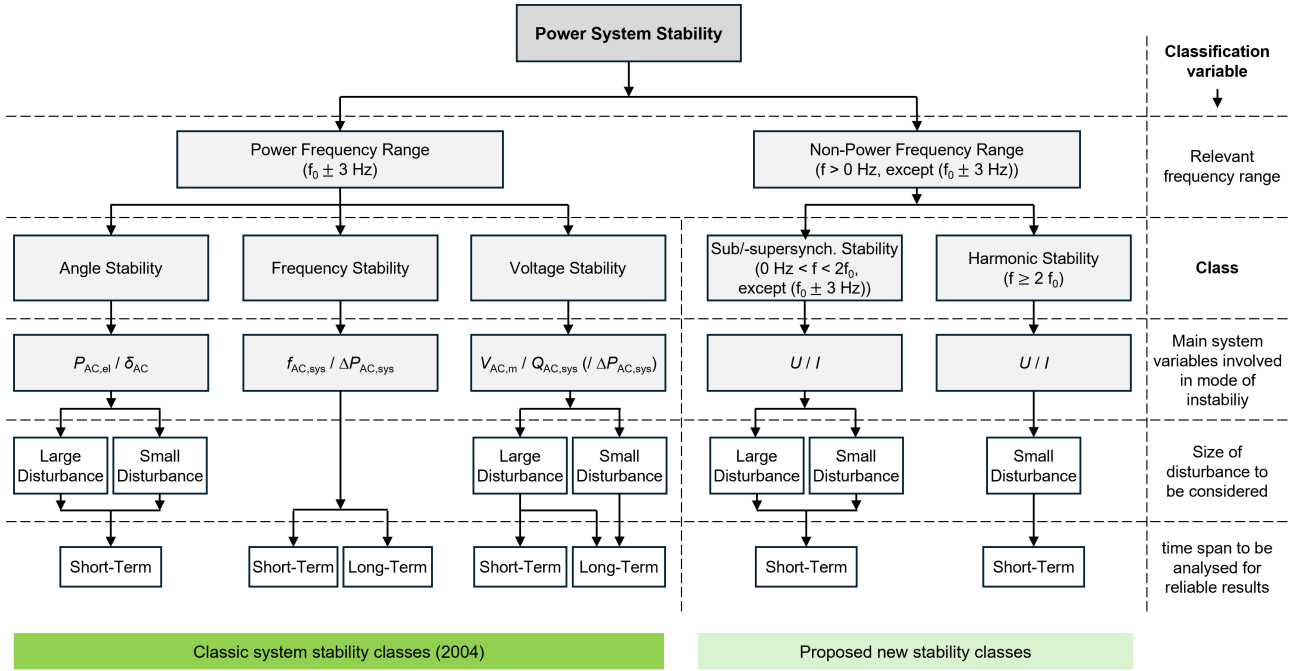


Figure 2.3: Suitable classification of Power System Stability by CIGRE [4]

2.2.1 Harmonic and Sub-/super-synchronous Stability

The property of a power system to remain stable or return to an acceptable steady-state condition when subjected to disturbances containing harmonic-frequency components, for frequencies above twice the fundamental frequency, such that these components are not amplified by interactions among converters, controls, and the network is regarded as Harmonic Stability [4].

The boundary of power-frequency range is pragmatically drawn at approximately power frequency ± 3 Hz (roughly 47 Hz to 53 Hz for Europe). Phenomena involving spectral components below this range and up to twice the fundamental frequency are classified as Sub-/super-synchronous Stability [4]. The research work in this thesis addresses harmonic and sub-/super-synchronous stability phenomena.

2.3 Small-Signal Modeling

The analysis of converter-dominated electrical systems requires mathematical models that capture both the physical energy-storage dynamics and the action of the control system. Although the original plant is inherently nonlinear, time-varying and switched because of the presence of semiconductor devices, modulation and feedback loops, the standard system-level workflow replaces it by an averaged, small-signal-linearised representation in a synchronous reference frame that retains the dynamic interactions relevant for stability and control while remaining analytically tractable [23]. The averaging and small-signal linearisation steps that produce this representation are recalled in Appendix A for completeness. The remainder of this section focuses on the three complementary forms in which the resulting model is used throughout the thesis: the synchronous dq reference frame in which it is most naturally expressed, the state-space representation, and the equivalent terminal impedance/admittance form.

2.3.1 Reference frames and dq transformation

Three-phase electrical quantities may be represented in different reference frames. In the natural abc frame, voltages and currents are physical phase variables. This frame is intuitive and exact, but sinusoidal steady-state quantities remain time-varying. In the stationary orthogonal frame $\alpha\beta$, the three-phase variables are projected onto orthogonal axes. In the synchronous rotating dq frame, the coordinates rotate with angular speed ω , so balanced sinusoidal variables appear as constant quantities in steady state. This property is one of the main reasons why the dq frame is widely used in converter modeling and control [23].

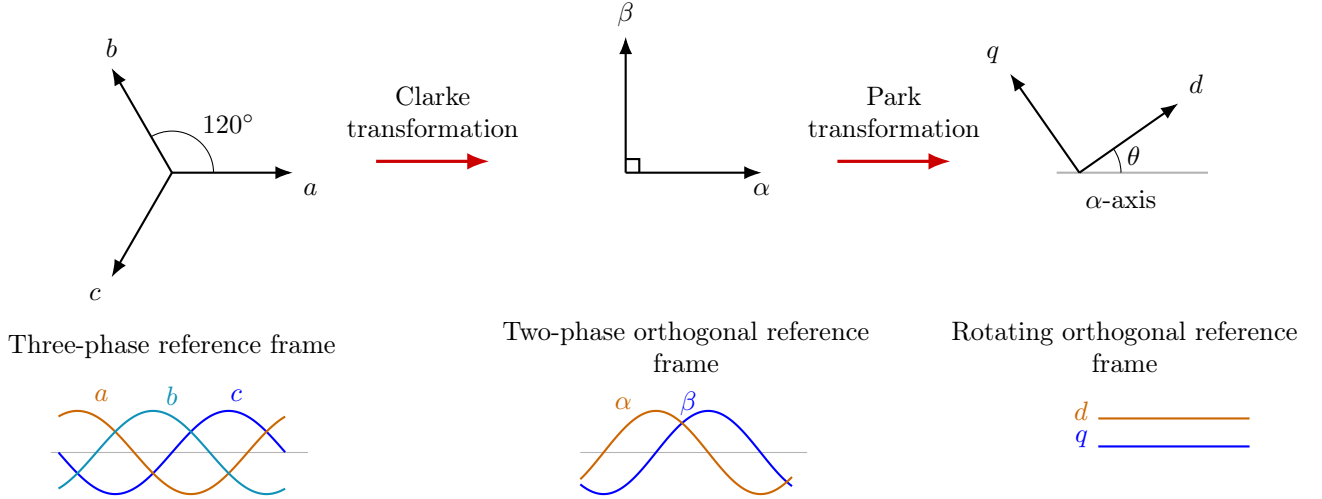


Figure 2.4: Illustration of the Clarke and Park transformations from the three-phase abc frame to the $\alpha\beta$ frame and finally to the rotating dq frame

Using a commonly adopted amplitude-invariant convention, the direct transformation from abc to dq is

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos \left(\theta - \frac{2\pi}{3} \right) & \cos \left(\theta + \frac{2\pi}{3} \right) \\ -\sin \theta & -\sin \left(\theta - \frac{2\pi}{3} \right) & -\sin \left(\theta + \frac{2\pi}{3} \right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}. \quad (2.1)$$

Here, $\theta(t)$ is the instantaneous angle of the rotating frame. For balanced three-wire systems, the zero-sequence component is zero, so the model reduces to the two-axis dq subspace [23].

The key mathematical consequence of the dq transformation is that differentiation in a rotating frame introduces coupling terms proportional to the rotation speed. Consider the phase-domain inductor equation

$$v_{abc} = R i_{abc} + L \frac{di_{abc}}{dt}. \quad (2.2)$$

After transformation to the synchronous frame, the corresponding equations become

$$v_d = R i_d + L \frac{di_d}{dt} - \omega L i_q, \quad (2.3)$$

$$v_q = R i_q + L \frac{di_q}{dt} + \omega L i_d. \quad (2.4)$$

The terms $\mp \omega L i$ are not additional physical elements; they arise from the coordinate transforma-

tion itself. Their presence is fundamental because it explains why the d and q axes are dynamically coupled and why decoupling terms are commonly introduced in current controllers [23].

The dq frame also provides a compact algebraic description of active and reactive power. For a balanced zero-sequence-free system under the convention above,

$$P = \frac{3}{2}(v_d i_d + v_q i_q), \quad Q = \frac{3}{2}(v_q i_d - v_d i_q). \quad (2.5)$$

If the reference frame is aligned with the positive-sequence grid voltage vector, then $v_q \approx 0$ in steady state, which simplifies both analysis and control interpretation. In that case, the d -axis is strongly associated with active-power transfer and the q -axis with reactive-power exchange [23].

A practical difficulty arises when several converter or network submodels are expressed in different local rotating frames. In such cases, all terminal quantities must be referred to a common frame before the models can be interconnected consistently. This issue becomes especially important in multi-converter systems and in network-level impedance assembly [24].

2.3.2 State-space modeling

The linearized small-signal model of a converter, derived in Appendix A, is most generally expressed in the state-space form

$$\dot{x} = Ax + Bu, \quad y = Cx + Du, \quad (2.6)$$

where, for simplicity, the perturbation symbol Δ has been omitted. The state vector x contains all dynamic variables required to describe the internal evolution of the system. In a grid-connected converter, a typical choice is

$$x = \begin{bmatrix} i_d & i_q & v_{dc} & \xi_{id} & \xi_{iq} & \xi_{pll} & \cdots \end{bmatrix}^T, \quad (2.7)$$

where i_d and i_q are filter currents, v_{dc} is the dc-link voltage, and ξ denotes controller integrator states. The matrices A , B , C , and D describe, respectively, the autonomous dynamics, the input-to-state coupling, the state-to-output mapping, and the direct feedthrough [25].

A central advantage of the state-space form is that it enables direct modal analysis. The eigenvalues of A , obtained from

$$\det(sI - A) = 0, \quad (2.8)$$

determine the small-signal modes of the system. If an eigenvalue is written as

$$\lambda_i = \sigma_i \pm j\omega_i, \quad (2.9)$$

then σ_i determines whether the mode decays or grows, while ω_i determines the oscillation frequency. Small-signal stability requires all eigenvalues to lie in the open left half-plane, that is,

$$\text{Re}(\lambda_i) < 0 \quad \forall i. \quad (2.10)$$

The damping ratio of an oscillatory mode is

$$\zeta_i = -\frac{\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}}. \quad (2.11)$$

Thus, the state-space model gives direct access to mode frequencies, damping, participation, and sensitivity, which makes it particularly valuable for physical interpretation and controller tuning [25].

The corresponding transfer matrix between input and output is

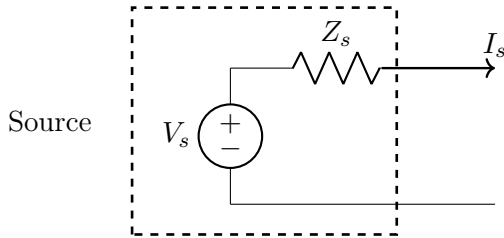
$$G(s) = C(sI - A)^{-1}B + D. \quad (2.12)$$

This relation is important because it forms the bridge between time-domain state-space models and frequency-domain models. In other words, an impedance or admittance model can be derived directly from the same linearized state-space representation by selecting terminal voltages or currents as inputs and outputs [25].

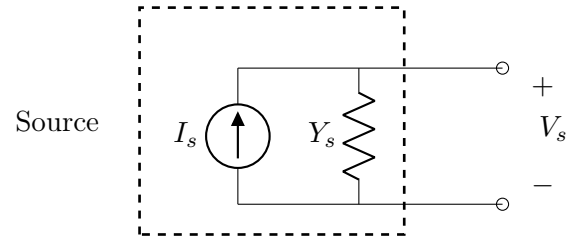
2.3.3 Impedance/admittance modeling

Whereas state-space modeling describes the full internal dynamics, impedance and admittance modeling describe the converter from its terminals. This viewpoint is especially attractive for interconnected systems because it allows each subsystem to be represented by an input-output relationship at a connection port rather than by all of its internal states. In the frequency domain, the terminal relation may be expressed either as

$$V(s) = Z(s)I(s), \quad (2.13)$$



(a) Thevenin equivalent source.



(b) Norton equivalent source.

Figure 2.5: Equivalent source representations used for network modeling.

or equivalently as

$$I(s) = Y(s)V(s), \quad Y(s) = Z^{-1}(s), \quad (2.14)$$

provided that the inverse exists.

For three-phase converters expressed in the synchronous frame, the terminal relation is generally multi-input multi-output, so the impedance or admittance becomes a 2×2 matrix [26].

Using admittance notation, the small-signal terminal model in the dq frame is

$$\begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} = \begin{bmatrix} Y_{dd}(s) & Y_{dq}(s) \\ Y_{qd}(s) & Y_{qq}(s) \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix}. \quad (2.15)$$

Similarly, the impedance form is

$$\begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} = \begin{bmatrix} Z_{dd}(s) & Z_{dq}(s) \\ Z_{qd}(s) & Z_{qq}(s) \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix}. \quad (2.16)$$

The diagonal terms represent the direct response of each axis, while the off-diagonal terms capture cross-coupling between the d and q axes. These coupling terms are especially important in systems with PLL dynamics, current-control interactions, or asymmetries in the operating point [26].

Such models follow directly from the linearized state-space equations. If the terminal voltage perturbation is chosen as input and the current perturbation as output, then

$$Y(s) = C_v(sI - A)^{-1}B_v + D_v. \quad (2.17)$$

Likewise, if the terminal current perturbation is chosen as input and the voltage perturbation as output, then

$$Z(s) = C_i(sI - A)^{-1}B_i + D_i. \quad (2.18)$$

Hence, impedance/admittance models are not conceptually separate from state-space models; rather, they are frequency-domain terminal representations derived from the same underlying small-signal dynamics [26, 25].

2.4 Wind Turbine Fundamentals

Wind turbines are the primary generation units of the case study considered in this thesis, and their electrical behaviour at the terminals is shaped both by the aerodynamic conversion process and by the converter topology adopted at the machine–grid interface. This section therefore first recalls the basic principles of wind energy conversion.

2.4.1 Wind energy conversion principles

Wind turbines convert part of the kinetic energy contained in moving air into electrical energy. The power available in the wind crossing the rotor swept area is given by

$$P_{\text{wind}} = \frac{1}{2}\rho A v^3$$

where ρ is the air density [kg/m³], A is the rotor swept area [m²], and v is the wind speed [m/s].

However, a wind turbine cannot convert all the power available in the wind. Only a fraction of it is transferred to the rotor, which is represented by the power coefficient C_p , which characterizes the aerodynamic efficiency of the rotor [7]:

$$P_{\text{rotor}} = C_p P_{\text{wind}}$$

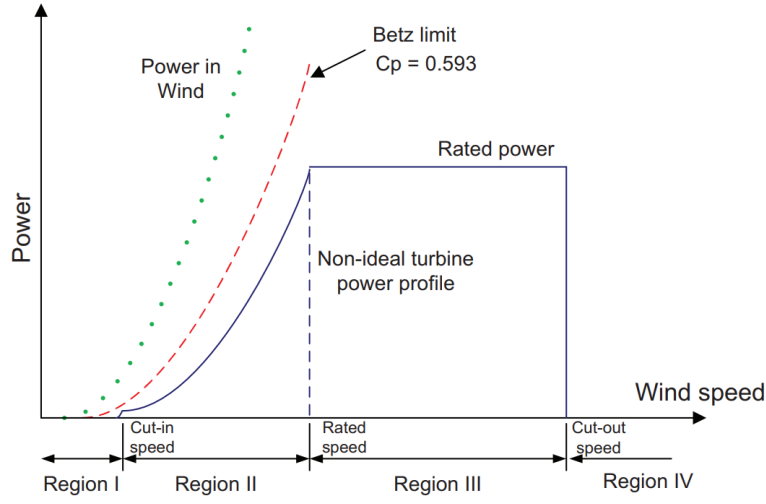


Figure 2.6: Power profile of a wind turbine [5]

The Betz limit establishes the theoretical upper bound on aerodynamic energy extraction from the wind, stating that no wind turbine can capture more than 59.3% of the kinetic power available in the incoming airflow, that is, $C_p \leq 0.593$. In practice, as seen in Figure 2.6, real wind turbines operate below this limit because of aerodynamic, mechanical, and electrical losses, as well as control constraints. Their power production is therefore characterized by distinct operating regions [5]. In practice, C_p depends mainly on the tip-speed ratio λ and the blade pitch angle β [7].

The tip-speed ratio λ is a key variable in wind turbine aerodynamics, it is defined as

$$\lambda = \frac{R\omega_R}{v}$$

where R is the blade radius, ω_R is the rotor angular speed, and v is the wind speed. For a given pitch angle, there exists an optimal value of λ that maximizes C_p [7].

The pitch angle β is the angle used to rotate the blade around its longitudinal axis. Pitch control modifies the aerodynamic operating point of the blade and is therefore used to regulate captured power, especially at high wind speeds.

Similarly, for a given rotor speed, there is only one wind speed that corresponds to optimal operation. This is the reason why variable-speed operation is advantageous in modern wind turbines.

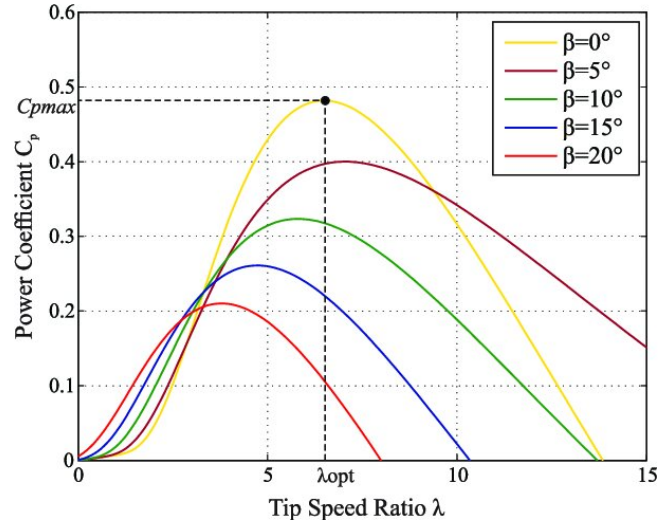


Figure 2.7: Power coefficient versus tip-speed ratio for several pitch angles [6]

Figure 2.6 present C_p curves as a function of λ for different pitch angles. These figures clearly show that:

- C_p reaches a maximum at a specific pair (λ, β) ,
- increasing pitch angle generally reduces the maximum achievable C_p ,
- variable-speed and variable-pitch operation allow the turbine to remain close to the optimal aerodynamic operating point.

The maximum power point tracking (MPPT) strategy is therefore used below rated power to adjust rotor speed so that the turbine operates near the optimal tip-speed ratio as seen in Figure 2.8.

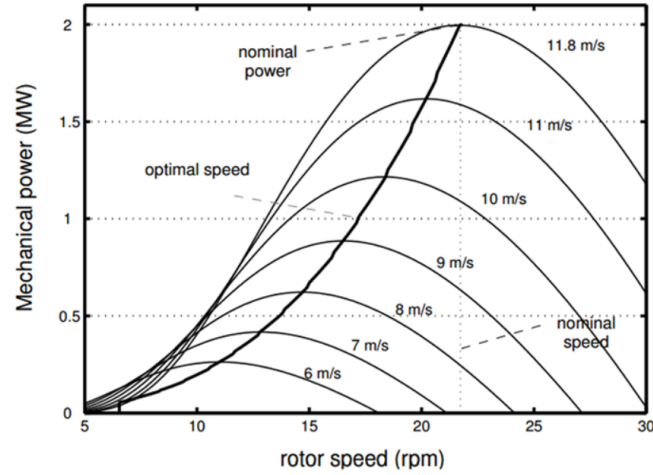


Figure 2.8: Optimal speed path for different Power output curves [7]

2.4.2 Types of Wind Turbines

Wind turbines are mainly classified into four major categories as per their configuration [27].

Type 1: Fixed-Speed Turbine with Direct Grid Connection:

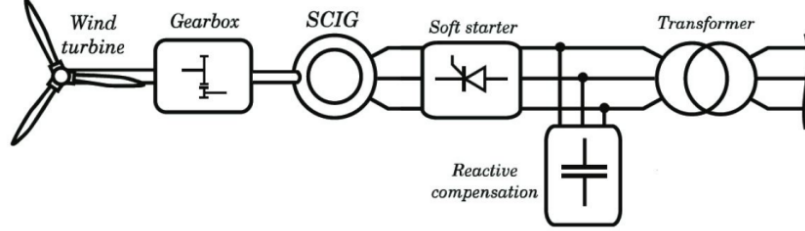


Figure 2.9: Wind Turbine Type 1 Configuration [8]

The simplest configuration is a fixed-speed wind turbine using either an induction generator or a synchronous generator directly connected to the grid, without power electronic converters [28], identified as Type 1 as seen in Figure 2.9. In practice, direct grid connection of a synchronous machine is not attractive for wind applications because mechanical power fluctuations would produce internal angle fluctuations and electromechanical oscillations [29].

For this reason, the fixed-speed solution is mainly associated with the induction generator directly connected to the grid. In such a machine, the rotor speed is close to the synchronous speed and only a limited variation is possible through slip. But since the allowable slip variation is small, this solution behaves as a constant-speed turbine [28].

Type 2: Wound-Rotor Induction Generator with Variable Rotor Resistance

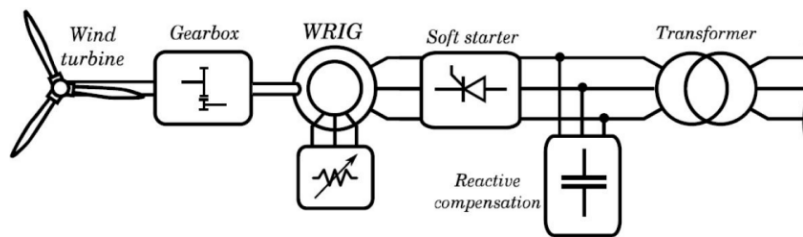


Figure 2.10: Wind Turbine Type 2 Configuration [8]

The Type 2 concept introduces a wound-rotor induction machine with variable external rotor resistance. By controlling this additional rotor resistance, the operating slip can be modified and the rotational speed can be adjusted to some extent. This offers more flexibility than Type 1, but the variable-speed range remains limited. Therefore, Type 2 can only partially exploit MPPT and does not provide the full benefits of modern variable-speed control [28].

Type 3: Doubly-Fed Induction Generator (DFIG)

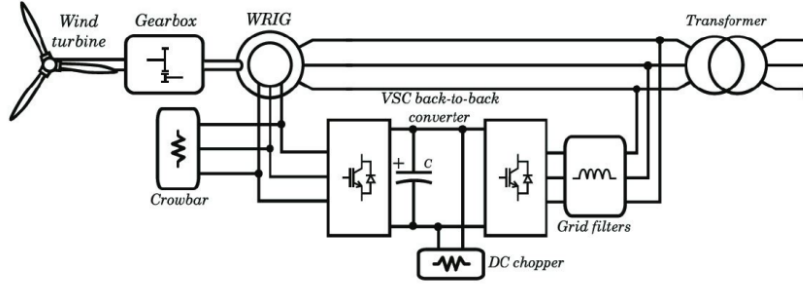


Figure 2.11: Wind Turbine Type 3 Configuration [8]

The Type 3 wind turbine is based on the doubly-fed induction generator (DFIG), which became one of the dominant solutions for variable-speed wind turbines [30]. In this configuration, the stator is directly connected to the grid, while the rotor is connected through slip rings to a back-to-back power converter.

The total electrical power exchanged by the DFIG machine is written as

$$P_{\text{total}} = P_S + P_R \quad (2.19)$$

$$P_{\text{total}} = (1 - s)P_S \quad (2.20)$$

where P_S and P_R are respectively the stator and rotor active powers.

$$P_S = \frac{1}{1 - s}P_{\text{total}} \quad P_R = \frac{s}{s - 1}P_{\text{total}} \quad (2.21)$$

These expressions highlight one of the main advantages of the DFIG topology: only the slip power flows through the rotor-side converter, while the stator remains directly connected to the grid. As a result, the converter rating can be significantly lower than the machine rated power, which reduces the overall cost and converter losses [30, 29].

Type 4: Synchronous Generator with Full-Scale Converter

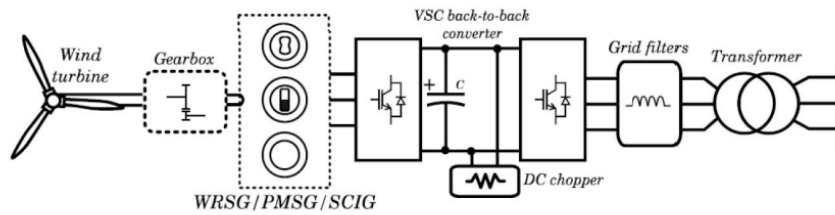


Figure 2.12: Wind Turbine Type 4 Configuration [8]

The Type 4 wind turbine is the currently most used and modern technological advancement in all the wind turbines, which connects the generator to the grid through a full-scale power converter chain.

This solution is associated with a permanent magnet synchronous generator (PMSG), although the main defining feature is the full converter interface between generator and grid. Unlike the DFIG, the full electrical power passes through the converters.

Figure 2.13 classifies Type 4 wind turbine drivetrains into high-speed, medium-speed, and low-speed direct-drive PMSG configurations. The defining feature of a Type 4 turbine is the use of a full-scale converter between the generator and the grid, allowing complete electrical decoupling from the network. This arrangement provides wide controllability of torque, active and reactive power, and DC-link voltage.

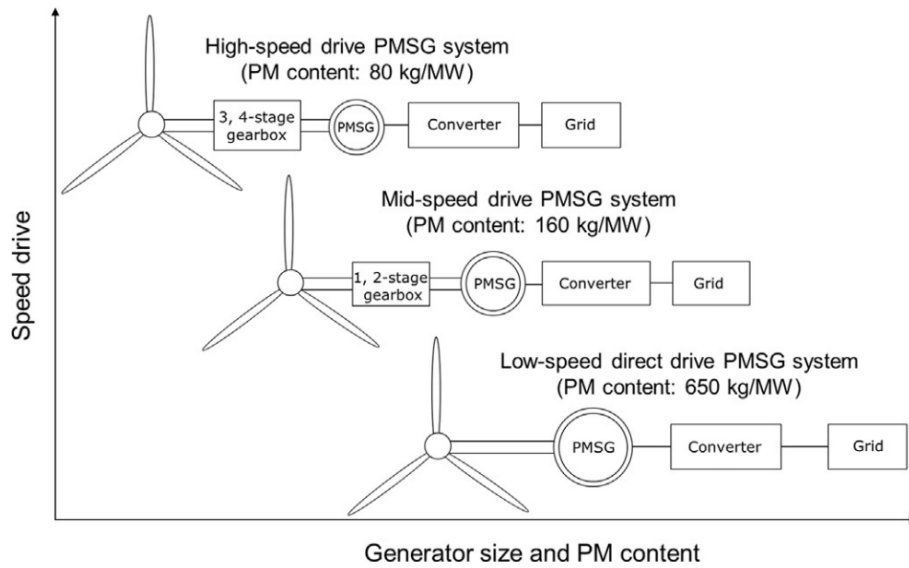


Figure 2.13: Type-IV PMSG wind turbine drivetrain configurations: high-speed, medium-speed, and direct-drive concepts. Adapted from [9]

Among these configurations, the direct-drive option removes the gearbox, thereby reducing mechanical complexity and maintenance. In contemporary offshore wind practice, direct-drive PMSG solutions are commonly adopted, as seen in several commercial Siemens Gamesa offshore turbines and in the IEA 15 MW offshore reference turbine [31].

The control structure of a Type 4 turbine is divided into machine-side and grid-side functions:

- on the machine side, field-oriented control in the rotor reference frame is used for torque control through current control
- on the grid side, synchronous-reference-frame control is used for DC bus voltage control, active power transfer, and reactive power control

This means that the Machine-Side Converter (MSC) controls the electromagnetic torque and thus rotor speed, while the Grid-Side Converter (GSC) governs the exchange of active and reactive power with the grid. Such an arrangement offers very good dynamic performance and grid-code compliance capabilities [32]. Detailed analysis on the control and modeling of the Type 4 wind turbine will be discussed in the next chapter.

Chapter 3

Framework for Impedance-Based Modeling and Stability Assessment

This chapter develops the complete analytical framework on which the small-signal stability assessment of the Princess Elisabeth Energy Island is based. It begins by comparing the two stability-analysis approaches most commonly adopted in the literature on converter-dominated systems, namely state-space eigenvalue analysis and frequency-domain impedance-based analysis, and motivates the choice of the latter as the primary framework of this thesis. On this basis, the detailed dq -frame impedance and admittance models of each component of the offshore plant are then derived: the Type 4 wind turbine is treated separately in its harmonic and sub-/super-synchronous representations; the wind-turbine and platform transformers, the inter-array cables and the HVAC export cable are modelled as frequency-dependent dq blocks; and the AC grid and the planned onshore synchronous condenser are described as parallel impedance contributions at the onshore connection node. Once these component models are available, the chapter introduces the four impedance-based indicators that will be used throughout the rest of the work, namely the Generalized Nyquist Criterion, the passivity index, the vector margin and the disk margin, and discusses how they should be interpreted in a MIMO setting. The chapter closes with a review of the stability-enhancement techniques reported in the literature for the harmonic and sub-synchronous frequency ranges, which are subsequently applied to the case study later in this Thesis.

3.1 Comparative Analysis of State-Space and Impedance-Based Stability Assessment Frameworks

The selection of the modelling and stability-analysis framework is an important step, since it directly affects the way converter-network interactions are represented and the type of stability information that can be extracted from the model. Two main approaches are commonly used in the literature described in Section 2.3.2: state-space eigenvalue analysis and frequency-domain impedance-based analysis.

In the state-space approach, the small-signal stability is assessed from the eigenvalues of the state matrix A . If all eigenvalues have negative real parts, the operating point is locally stable. The critical eigenvalue is the eigenvalue with the largest real part, and its imaginary part gives the oscillation frequency,

$$f_{\text{osc}} = \frac{|\text{Im}(\lambda_c)|}{2\pi}. \quad (3.1)$$

This makes state-space analysis powerful because it gives direct access to the system poles, damping ratios, mode shapes, and participation factors. In other words, it can identify not only whether a system is unstable, but also which internal states contribute to the unstable mode. This is one of the main strengths of state-space analysis and explains why it has traditionally been used for small-signal stability studies in conventional power systems [33].

However, the same feature also becomes a major limitation in large converter-dominated systems. A detailed state-space model of an offshore wind farm requires the internal equations and parameters of each wind turbine converter, current controller, phase-locked loop (PLL), DC-link controller, digital delay, filters, transformers, and frequency-dependent cables. As the number of wind turbines increases, the state matrix becomes very large, and the computational burden rapidly increases [34, 35]. For offshore wind power plants, a detailed stability study of all wind turbine generators is often not feasible because of the large amount of calculations; aggregation is usually required, but aggregation can also introduce errors. The same review also states that state-space modelling has low scalability, while impedance modelling treats components such as cables, transformers, and wind turbines as independent models, making the overall model easier to scale [34].

A second practical limitation is the availability of converter models. In real wind farm projects, original equipment manufacturers often do not provide complete internal control equations or parameters because of intellectual property protection. This is problematic for state-space analysis, which normally requires a detailed white-box model [36]. In contrast, impedance-based methods can use terminal impedance or admittance characteristics obtained either from analytical models, frequency scans in electromagnetic transient (EMT) tools such as PSCAD or MATLAB/Simulink, or experimental small-signal injection measurements [35].

The impedance-based method represents each subsystem by its terminal impedance or admittance. For a grid-connected converter, the converter can be represented by an output impedance $Z_i(s)$ or admittance $Y_i(s)$, while the external grid is represented by its Thevenin impedance $Z_g(s)$. In the simplest single-input single-output interpretation, the interaction is described through a minor-loop gain such as

$$L(s) = \frac{Z_g(s)}{Z_i(s)}, \quad (3.2)$$

or, equivalently, through an admittance formulation. Impedance-based stability criterion states that a grid-connected inverter remains stable if the ratio between the grid impedance and the inverter output impedance satisfies the Nyquist stability criterion [37]. In multi-phase and dq-frame converter models, the problem becomes multi-input multi-output, and the loop gain is written in matrix form. For a wind farm or multi-converter system, the Generalized Nyquist Criterion can then be applied to the eigenloci of $L(j\omega)$, or equivalently to the determinant condition [37]

$$\det(I + Z_g(j\omega)Y_i(j\omega)). \quad (3.3)$$

The main advantage of the impedance-based method for this thesis is therefore modularity. Each wind turbine, transformer, cable section, shunt element, and grid equivalent can be modelled independently and then interconnected at the network level. If one component changes, for example the cable length, transformer leakage impedance, converter current-controller bandwidth, digital delay, or

grid short-circuit strength, only the corresponding impedance block must be updated [36]. The full system does not need to be re-linearized from the beginning. This is particularly attractive for offshore wind farms because topology changes, aggregation levels, and operating points must often be studied repeatedly.

A second advantage is that impedance-based analysis directly targets the physical mechanism of harmonic instability. Frequency-domain plots such as Bode plots, Nyquist plots, eigenvalue loci, and singular-value plots provide direct information about the frequency range in which the interaction occurs [33]. This makes it possible to identify whether an instability is related to a cable resonance, an LCL/filter resonance, converter delay, current-control bandwidth, or weak-grid impedance. For the harmonic range, this frequency-localized interpretation is more useful than a large list of eigenvalues from a high-order state-space model. Moreover, harmonic stability involves wide-frequency-range oscillations tightly coupled with electrical network dynamics, which can make the state-space matrix very high order and computationally demanding [33].

Nevertheless, the impedance-based method should not be presented as universally superior in every situation. Its main limitation is that it is a small-signal, operating-point-dependent method. It assesses stability around a selected equilibrium point and cannot replace EMT time-domain simulations for large disturbances, protection actions, saturation, fault ride-through, or nonlinear controller limits. In addition, a terminal impedance model may hide internal converter modes that are weakly observable at the point of connection. Impedance-based stability prediction can be inaccurate if right-half-plane poles are hidden in measured or estimated impedances [33]. Therefore, the method must be applied carefully, especially when using measured black-box impedances.

Finally, when the impedance model is obtained by frequency scanning, the measurement process itself introduces practical challenges. The perturbation amplitude must be small enough to remain in the linear region but large enough to overcome numerical or measurement noise. The dq-frame or sequence-frame processing must also account for frequency coupling, mirror-frequency effects, and fast Fourier transform (FFT) windowing [36]. These issues are important when validating analytical impedance models using EMT simulations.

For the present thesis, the impedance-based method is therefore selected as the primary stability-analysis framework because it is modular, scalable, compatible with black-box converter data, naturally formulated in the frequency domain, and directly connected to resonance and Nyquist-based stability margins.

3.2 Type 4 Wind Turbine Detailed Impedance Modeling

Within the impedance-based framework adopted in this thesis, the Type 4 wind turbine is represented at its terminals by a Norton-equivalent admittance. Because the simplifying assumptions that keep the admittance analytically tractable differ between the two frequency ranges, the turbine admittance is conveniently split into two models in the literature: a reduced harmonic admittance, valid in the 100 Hz–2.5 kHz range, and a complete sub-/super-synchronous admittance, valid below 100 Hz. The scope of this thesis is restricted to grid-following operation.

3.2.1 Harmonic Admittance Model of Type 4 Wind Turbine

For the harmonic frequency range of 100-2.5 kHz, the impedance model can be simplified by focusing mainly on the GSC. In a full-converter Type-4 wind turbine, the PMSG and the MSC are connected to the GSC through the DC link. The oscillations between harmonic range occur too rapidly for the DC-link voltage to respond. If the small-signal oscillations of the DC-link voltage are neglected, the dynamics around and above the fundamental frequency can be predicted by modelling only the GSC; therefore, the MSC and generator-side dynamics can be neglected for the harmonic impedance model [38].

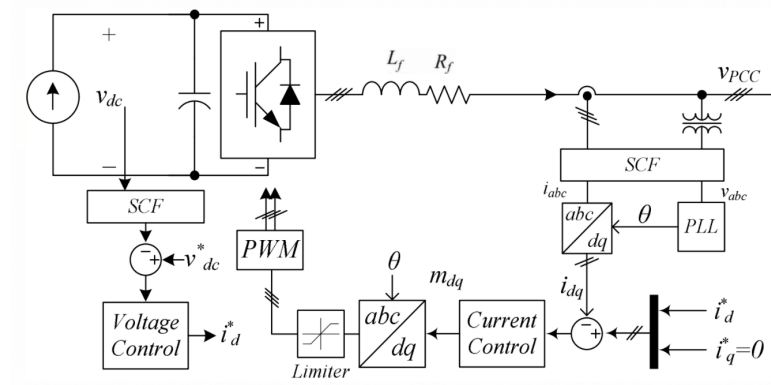


Figure 3.1: Simplified diagram of a three-phase grid-connected VSC [10]

The GSC is a voltage-source converter whose main function in grid-following operation is to regulate the current injected into the grid according to active- and reactive-power references [38]. More precisely, the GSC usually regulates the DC-link voltage through the d-axis current reference and controls reactive power or AC-voltage support through the q-axis current reference [33]. The grid or the external network primarily sets the PCC voltage, while the grid-following converter synchronizes to this voltage and controls its injected current [38].

The control system of the GSC typically consists of four main blocks: grid synchronization, DC-link voltage control, current control, and PWM/SVM modulation. The PLL estimates the phase angle of the point-of-connection voltage, and this angle is used to transform measured three-phase quantities from the abc frame into the synchronously rotating dq frame [38].

Even in the GSC, all these control blocks are not of interest at the harmonic frequency range. Figure 3.2 classifies the blocks according to frequency range where f_1 is the fundamental frequency and f_s is the switching frequency of the converter [39, 10]. DC Link voltage control and PLL are the ones that will only effect the sub-/super-synchronous range from 1 – 100 Hz [39]. Dedicated Bandwidths of both these controls are much lower than the harmonic range. Hence in that case, the harmonic impedance is mainly shaped by the current controller, PWM/digital delay and converter filter only [39, 10].

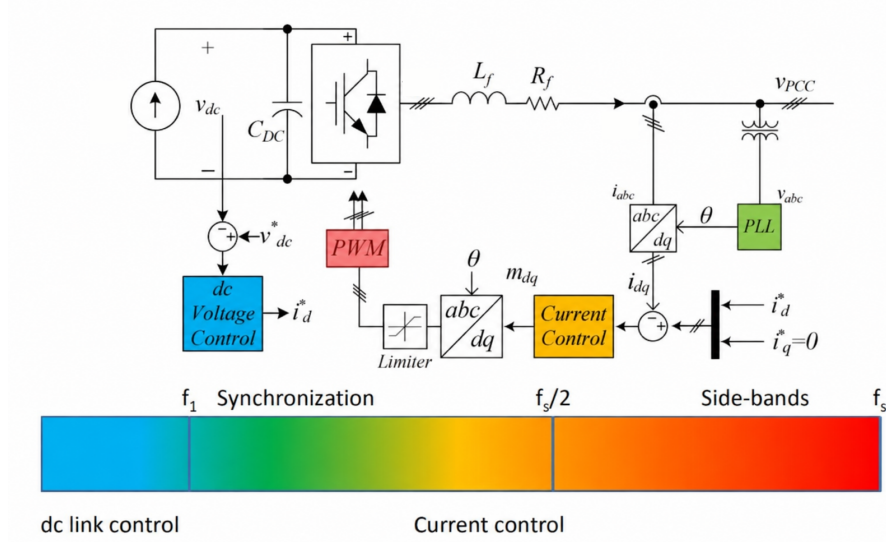


Figure 3.2: Multi-time scale stability investigation of a voltage source converter [10]

In the dq frame, sinusoidal steady-state currents become constant quantities, which allows conventional PI controllers to regulate them effectively [38]. The measured d- and q-axis currents are compared with their reference values, and the resulting errors are processed by PI current controllers. Because the d- and q-axis equations are coupled through the synchronous-frame cross-coupling terms, decoupling terms are added to improve the dynamic performance of the current loop [38]. In addition, voltage feedforward from the grid-side voltage is also included before the voltage references are sent to the PWM/SVM stage. The modulation stage then converts the dq voltage references back into three-phase switching signals for the converter [38].

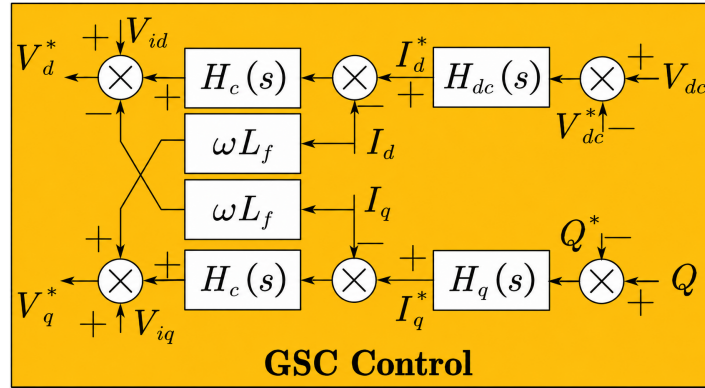


Figure 3.3: Control Loop Overview [11]

The GSC is commonly represented in the synchronous dq reference frame. In this frame, the converter-side filter is described by an impedance matrix rather than by two independent scalar impedances, because the transformation to the rotating frame introduces coupling between the d and q axes. For a simplified series RL filter, this impedance matrix is written as [38, 40].

$$\mathbf{B}_{L_f}(s) = \begin{bmatrix} R_f + sL_f & -\omega_s L_f \\ \omega_s L_f & R_f + sL_f \end{bmatrix}, \quad (3.4)$$

where R_f and L_f are the filter resistance and inductance, and ω_s is the synchronous angular frequency. The diagonal terms correspond to the physical resistance and inductance of the filter, while

the off-diagonal terms represent the synchronous-frame coupling between the two axes[40].

Assuming identical controllers on the two axes [41], its transfer-function matrix can be expressed as

$$\mathbf{B}_{PI,i}(s) = H_c(s) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H_c(s) = K_{p,i} + \frac{K_{i,i}}{s}. \quad (3.5)$$

Since the physical filter equations contain cross-coupling terms, practical GSC controllers include decoupling terms in order to improve the dynamic performance of the current loop [41]. These terms are represented by

$$\mathbf{B}_{dec} = \begin{bmatrix} 0 & -\omega_s L_f \\ \omega_s L_f & 0 \end{bmatrix}. \quad (3.6)$$

In an ideal continuous-time controller, these decoupling terms would compensate the synchronous-frame coupling introduced by the filter [41]. However, in a digitally controlled converter, the compensation is affected by sampling, computation, and PWM delay [39, 10]. Therefore, the high-frequency impedance of the converter is not only determined by the physical filter, but also by the way the control action is delayed before being applied by the converter.

Digital control and PWM effects are commonly represented by an equivalent delay transfer function [42]. A typical representation combines a computational delay with a zero-order-hold effect as

$$G_d(s) = e^{-s\lambda T_s} \frac{1 - e^{-sT_s}}{sT_s}, \quad (3.7)$$

where T_s is the sampling period and λT_s represents the computation delay. Including the PWM gain, the effective converter voltage actuation can be written as

$$G_c(s) = K_{PWM} G_d(s). \quad (3.8)$$

As the analysed frequency approaches a significant fraction of the switching frequency, the delay can reduce the effective damping provided by the current controller and may contribute to harmonic instability [42].

Voltage feedforward is another control feature that affects the converter output admittance. In many grid-following converters, the measured PCC voltage is added to the current-controller output before modulation. This improves disturbance rejection and makes the converter behave closer to an ideal current source at lower frequencies [40]. However, because the feedforward action is also affected by digital and PWM delay, its contribution becomes frequency-dependent in the harmonic range [42]. The voltage feedforward gain is denoted here by H_v .

Combining the filter dynamics, current controller, decoupling terms, voltage feedforward, and digital/PWM delay, the GSC can be represented by an equivalent dq -domain output impedance matrix:

$$\mathbf{Z}_{out,dq}(s) = \frac{\mathbf{B}_{L_f}(s) + G_c(s) [\mathbf{B}_{PI,i}(s) - \mathbf{B}_{dec}]}{1 - G_c(s)H_v}. \quad (3.9)$$

The corresponding output admittance is then obtained as

$$\mathbf{Y}_{out,dq}(s) = \mathbf{Z}_{out,dq}^{-1}(s). \quad (3.10)$$

Equation (3.10) represents the harmonic admittance of the reduced Type 4 wind turbine model as seen from the grid side [40]. It excludes the PMSG, MSC, DC-link voltage controller, and PLL dynamics, since these are mainly relevant to the sub-/super-synchronous ranges rather than the harmonic range considered in this section.

If the grid-interface filter includes a shunt capacitor, its admittance can be added in parallel to the converter-branch admittance. In the dq frame, the capacitor admittance is expressed as

$$\mathbf{B}_{C_f}(s) = \begin{bmatrix} sC_f & -\omega_s C_f \\ \omega_s C_f & sC_f \end{bmatrix}. \quad (3.11)$$

The total admittance at the converter output then becomes

$$\mathbf{Y}_{GSC,dq}(s) = \mathbf{Y}_{out,dq}(s) + \mathbf{B}_{C_f}(s). \quad (3.12)$$

An LC Filter is a standard choice for output filtering in a wind turbine converter [43]. This complete model in 3.12 will therefore be adopted for the harmonic stability analysis in the following chapters. Notably, the transformer installed at each turbine's output inherently introduces an additional series inductance, due to its leakage inductance, effectively converting the LC filter into an LCL configuration [43].

3.2.2 Sub-/super-synchronous Admittance Model of Type 4 Wind Turbine

In the sub-/super-synchronous frequency range (typically 1–100 Hz), the simplifications adopted in the harmonic model no longer hold. The phase-locked loop (PLL) is a critical component in grid-following converter-based systems because it determines the synchronization dynamics of the converter and directly affects its small-signal impedance/admittance characteristics. Several studies show that PLL dynamics can interact with grid impedance and operating conditions, reducing stability margins and potentially causing synchronization instability, particularly under weak-grid conditions [44, 45, 46, 39]. The bandwidth of the PLL fall directly within the sub-/super-synchronous range, so its dynamics actively shape the converter admittance [39, 10]. Equally important, the small-signal oscillations of the DC-link voltage can no longer be neglected: at low frequencies the DC bus is fast enough to respond, which couples the GSC to the MSC and to the PMSG through the DC capacitor [11]. As a consequence, the machine-side system must be retained as a frequency-dependent admittance in the model [11].

This subsection therefore represents a complete model of the Type 4 wind turbine, including the PMSG, the MSC with its electric-torque and current controllers, the DC-link, the GSC outer loops, and the PLL. The derivation follows the multi-port modelling approach proposed in [11], where the wind turbine is split into a DC-side admittance representing the machine-side system and an AC-side impedance representing the GSC, and both are then assembled through the DC-link.

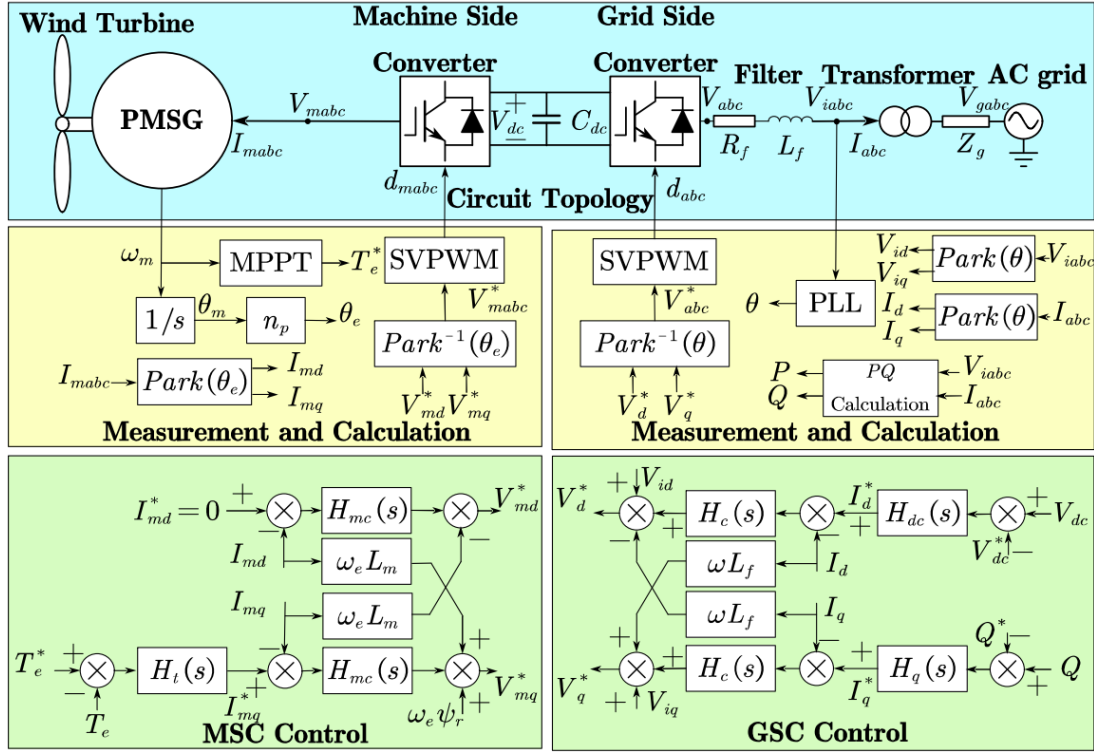


Figure 3.4: Configuration of a Type 4 wind turbine with detailed machine- and grid-side dynamics [11]

The system configuration for the sub-/super-synchronous model is shown in Figure 3.4. The PMSG is connected to the GSC via the MSC and the DC-link capacitor C_{dc} . The MSC implements zero d -axis current control together with an outer electric-torque control loop driven by the MPPT reference T_e^{ref} [11].

The derivation proceeds in two steps as shown in Figure 3.5: first, the DC-side admittance $\mathbf{Y}_{dcM}$ of the machine-side system is obtained; second, this admittance is embedded into the AC-side impedance model of the GSC to derive the complete output impedance $\mathbf{Z}_{acW}$ of the wind turbine [11].

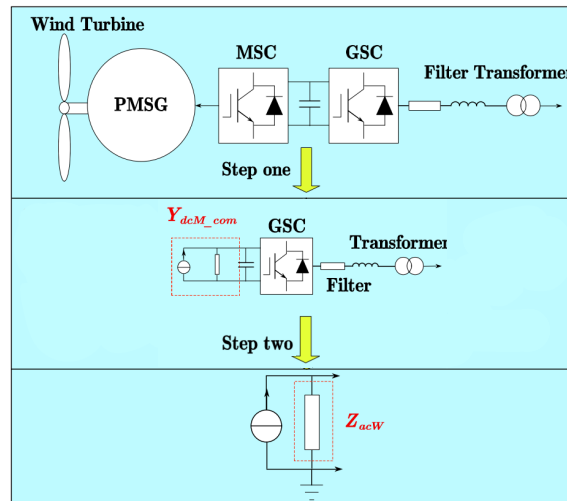


Figure 3.5: Steps of the modeling process [11]

Step 1: DC-side admittance of the machine-side system

The PMSG voltage equations in the synchronous dq frame are written as

$$\begin{bmatrix} V_{md} \\ V_{mq} \end{bmatrix} = \underbrace{\begin{bmatrix} R_m + sL_m & -\omega_e L_m \\ \omega_e L_m & R_m + sL_m \end{bmatrix}}_{\mathbf{Z}_{dq}^1} \begin{bmatrix} I_{md} \\ I_{mq} \end{bmatrix}, \quad (3.13)$$

where R_m and L_m are the stator resistance and inductance, and ω_e is the rotor electrical angular speed [11]. The power balance between the AC and DC side of the MSC is

$$I_{dcm} = \frac{3}{2} \frac{V_{md} I_{md} + V_{mq} I_{mq}}{V_{dc}}. \quad (3.14)$$

The MSC implements an outer electric-torque controller and an inner current controller [11]:

$$\begin{bmatrix} I_{md}^{ref} \\ I_{mq}^{ref} \end{bmatrix} = \begin{bmatrix} 0 \\ \underbrace{\left(K_{p,t} + \frac{K_{i,t}}{s} \right)}_{H_t} (T_e^{ref} - T_e) \end{bmatrix}, \quad (3.15)$$

$$\begin{bmatrix} V_{md} \\ V_{mq} \end{bmatrix} = \frac{V_{dc}}{2} \underbrace{\left(K_{p,im} + \frac{K_{i,im}}{s} \right)}_{H_{mc}} \begin{bmatrix} I_{md}^{ref} - I_{md} \\ I_{mq}^{ref} - I_{mq} \end{bmatrix}. \quad (3.16)$$

After small-signal linearization of (3.13)–(3.16) and elimination of the intermediate variables, the machine-side system can be represented at the DC bus by an equivalent admittance [11]:

$$\mathbf{Y}_{dcM} = -\frac{P_m}{V_{dc}^2} - \mathbf{Y}_M, \quad (3.17)$$

where the term $\mathbf{Y}_M$ accounts for the dynamics of the PMSG and of the MSC controllers, and is given by

$$\mathbf{Y}_M = \frac{3}{2} \frac{[V_{md} \ V_{mq}]}{V_{dc}} \mathbf{c} + \mathbf{dec}, \quad (3.18)$$

with the auxiliary matrices defined as

$$\mathbf{c} = - \underbrace{\begin{bmatrix} H_{mc} & 0 \\ 0 & H_{mc} H_t \left(\frac{3}{2} n_p \psi_r \right) + H_{mc} \end{bmatrix}^{-1}}_{\mathbf{Z}_{dq}^2} \begin{bmatrix} V_{md} & V_{mq} \\ V_{dc} & V_{dc} \end{bmatrix}^T, \quad (3.19)$$

$$\mathbf{d} = \frac{3}{2V_{dc}} ([I_{md} \ I_{mq}] - [V_{md} \ V_{mq}](\mathbf{Z}_{dq}^2)^{-1}), \quad (3.20)$$

$$\mathbf{e} = ((\mathbf{Z}_{dq}^1)^{-1} + (\mathbf{Z}_{dq}^2)^{-1})^{-1}, \quad (3.21)$$

where n_p is the number of pole pairs and ψ_r is the rotor flux linkage. Equation (3.17) captures the coupling between the machine- and grid-side systems through the DC-link [11].

Step 2: AC-side impedance of the wind turbine

On the grid side, the GSC implements an outer DC-voltage and reactive-power loop, and an inner current loop. The outer loop equations are

$$\begin{bmatrix} I_d^{ref} \\ I_q^{ref} \end{bmatrix} = \begin{bmatrix} \underbrace{\left(K_{p,vdc} + \frac{K_{i,vdc}}{s} \right)}_{H_{dc}} (V_{dc} - V_{dc}^{ref}) \\ \underbrace{\left(K_{p,q} + \frac{K_{i,q}}{s} \right)}_{H_q} (Q - Q^{ref}) \end{bmatrix}, \quad (3.22)$$

$$\begin{bmatrix} V_d^c \\ V_q^c \end{bmatrix} = \frac{V_{dc}}{2} \underbrace{\left(K_{p,i} + \frac{K_{i,i}}{s} \right)}_{H_c} \begin{bmatrix} I_d^{ref} - I_d^c \\ I_q^{ref} - I_q^c \end{bmatrix}, \quad (3.23)$$

where the superscript c denotes quantities expressed in the controller reference frame. The DC-link dynamics and the AC/DC power balance are described by

$$C_{dc} \frac{dV_{dc}}{dt} = -\mathbf{Y}_{dcM} V_{dc} - I_{dc}, \quad (3.24)$$

$$I_{dc} = \frac{3}{2} \frac{V_d^s I_d^s + V_q^s I_q^s}{V_{dc}}, \quad (3.25)$$

in which the superscript s denotes quantities in the system (electrical) frame. The grid-side filter is described by

$$\begin{bmatrix} V_d^s \\ V_q^s \end{bmatrix} = \begin{bmatrix} V_{id}^s \\ V_{iq}^s \end{bmatrix} + \underbrace{\begin{bmatrix} R_f + sL_f & -\omega L_f \\ \omega L_f & R_f + sL_f \end{bmatrix}}_{\mathbf{Z}_{dqf}} \begin{bmatrix} I_d^s \\ I_q^s \end{bmatrix}. \quad (3.26)$$

The PLL provides the link between system and controller frames. After small-signal linearization, the perturbations of the controller-frame quantities can be related to those of the system-frame voltage as

$$\begin{bmatrix} \tilde{V}_d^c \\ \tilde{V}_q^c \end{bmatrix} = \begin{bmatrix} \tilde{V}_d^s \\ \tilde{V}_q^s \end{bmatrix} + \begin{bmatrix} 0 & V_q^s G_{pll} \\ 0 & -V_d^s G_{pll} \end{bmatrix} \begin{bmatrix} \tilde{V}_{id}^s \\ \tilde{V}_{iq}^s \end{bmatrix}, \quad (3.27)$$

$$\begin{bmatrix} \tilde{I}_d^c \\ \tilde{I}_q^c \end{bmatrix} = \begin{bmatrix} \tilde{I}_d^s \\ \tilde{I}_q^s \end{bmatrix} + \begin{bmatrix} 0 & I_q^s G_{pll} \\ 0 & -I_d^s G_{pll} \end{bmatrix} \begin{bmatrix} \tilde{V}_{id}^s \\ \tilde{V}_{iq}^s \end{bmatrix}, \quad (3.28)$$

where the PLL transfer function is

$$G_{pll}(s) = \frac{K_{p,pll} s + K_{i,pll}}{s^2 + V_{id}^s K_{p,pll} s + V_{id}^s K_{i,pll}}, \quad (3.29)$$

and $K_{p,pll}$, $K_{i,pll}$ are the PLL proportional and integral gains [11].

Following the procedure of [11], equations (3.22)–(3.29) are linearized and the system-frame perturbations are mapped into the controller frame using (3.27)–(3.28). Eliminating the intermediate variables and writing the result in matrix form leads to the auxiliary matrices summarized in Table 3.1, where $\mathbf{V}_{dcq}$, $\mathbf{I}_{dcq}$, $\mathbf{G}_{Ipll}$ and $\mathbf{G}_{Upll}$ collect the PLL coupling and modulation gain terms:

$$\mathbf{V}_{dcq} = \begin{bmatrix} 0 & 0 \\ \frac{3}{2}H_q V_{iq}^s & -\frac{3}{2}H_q V_{id}^s \end{bmatrix}, \quad \mathbf{I}_{dcq} = \begin{bmatrix} 0 & 0 \\ \frac{3}{2}H_q(-I_q^s) & \frac{3}{2}H_q I_d^s \end{bmatrix}, \quad (3.30)$$

$$\mathbf{G}_{I_{pll}} = \begin{bmatrix} 0 & I_q^s G_{pll} \\ 0 & -I_d^s G_{pll} \end{bmatrix}, \quad \mathbf{G}_{U_{pll}} = \begin{bmatrix} 0 & V_q^s G_{pll} \\ 0 & -V_d^s G_{pll} \end{bmatrix}. \quad (3.31)$$

Table 3.1: Auxiliary matrices used in the AC-side impedance derivation.

$\mathbf{A} =$	$H_c \mathbf{I} + \mathbf{Z}_{dqf} - H_c \mathbf{V}_{dcq}$
$\mathbf{B} =$	$\mathbf{I} + H_c \mathbf{G}_{I_{pll}} + \mathbf{G}_{U_{pll}} - H_c \mathbf{I}_{dcq}$
$\mathbf{C} =$	$-[H_c H_{dc} \ 0]^T - [V_d^s/V_{dc} \ V_q^s/V_{dc}]^T$
$\mathbf{D} =$	$\frac{3}{2V_{dc}} [I_d^s \ I_q^s]$
$\mathbf{E} =$	$\frac{3}{2V_{dc}} ([V_d^s \ V_q^s] + [I_d^s \ I_q^s] \mathbf{Z}_{dqf})$
$\mathbf{F} =$	$-P/V_{dc}^2$
$\mathbf{G} =$	$\mathbf{D} - \mathbf{E} \mathbf{A}^{-1} \mathbf{B}$
$\mathbf{H} =$	$\mathbf{F} - \mathbf{E} \mathbf{A}^{-1} \mathbf{C}$

The matrix $\mathbf{A}$ collects the inner current loop, the filter impedance and the PLL-induced voltage feedback through $\mathbf{V}_{dcq}$. The matrix $\mathbf{B}$ groups the perturbations introduced by the PLL (via $\mathbf{G}_{I_{pll}}$ and $\mathbf{G}_{U_{pll}}$) and by the outer reactive-power loop ($\mathbf{I}_{dcq}$). The matrix $\mathbf{C}$ represents how perturbations in the DC-link voltage are reflected at the converter terminals through both the outer DC-voltage loop (H_{dc}) and the modulation gain $V_{d,q}^s/V_{dc}$. The matrices $\mathbf{D}$, $\mathbf{E}$, $\mathbf{F}$ originate from the linearization of the AC/DC power balance (3.25) and the DC-link equation (3.24), while $\mathbf{G}$ and $\mathbf{H}$ are obtained by substituting the AC-side relation $\mathbf{A} \tilde{I}^s = \mathbf{B} \tilde{V}^s + \mathbf{C} \tilde{V}_{dc}$ into the DC-side equation, in order to express the DC-link dynamics in terms of $\tilde{V}^s$ only [11].

Finally, combining the AC-side equation $\tilde{I}^s = \mathbf{A}^{-1}(\mathbf{B} \tilde{V}^s + \mathbf{C} \tilde{V}_{dc})$ with the DC-link balance

$$(sC_{dc} + \mathbf{Y}_{dcM}) \tilde{V}_{dc} = -\mathbf{G} \tilde{V}^s - \mathbf{H} \tilde{V}_{dc}, \quad (3.32)$$

and eliminating $\tilde{V}_{dc}$, the complete AC-side admittance of the Type 4 wind turbine, as seen from the PCC in the dq frame, is obtained as [11]

$$\boxed{\mathbf{Y}_{acW}(s) = \mathbf{A}^{-1} \left[\mathbf{B} - \mathbf{C} (sC_{dc} + \mathbf{Y}_{dcM} + \mathbf{H})^{-1} \mathbf{G} \right]}. \quad (3.33)$$

Equation (3.33) is the central result of the sub-/super-synchronous admittance model. Compared with the harmonic admittance (3.10), it explicitly retains the DC-link capacitor C_{dc} , the machine-side admittance $\mathbf{Y}_{dcM}$ from (3.17), and the PLL and DC-voltage outer-loop dynamics through the matrices $\mathbf{B}$, $\mathbf{C}$, $\mathbf{G}$ and $\mathbf{H}$.

3.3 Transformer Impedance Modeling

Transformers are one of the few passive components whose impedance has a non-negligible influence on the harmonic and sub-/super-synchronous response of an offshore wind power plant, mainly because of their leakage path and its frequency dependence [10]. In grid-connected OWF modeling, transformers appear in three places: the wind turbine step-up transformer, the offshore platform transformer

that interfaces the strings to the export cable and the onshore HVAC step-up transformer feeding the AC grid.

The standard frequency-domain representation of a two-winding transformer used in harmonic studies is shown in Figure 3.6. It consists of two leakage branches, each composed of a leakage inductance L_l and a frequency-dependent leakage resistance $R_l(f)$, a shunt magnetizing branch with a magnetizing inductance L_M and a magnetizing resistance r_M representing iron losses (hysteresis and eddy currents), and an ideal turns ratio $N:1$ [10].

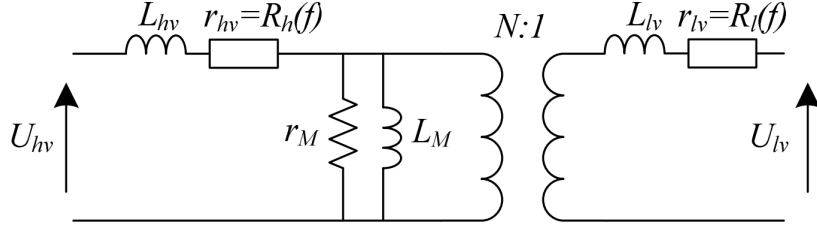


Figure 3.6: Two-winding transformer model in the frequency domain [10]

A number of simplifications are well established in the literature [47, 10]. The transformer is assumed to be balanced, i.e. all three phases share identical electrical parameters, so that the device can be analysed on a single-phase equivalent basis and then extended to the dq frame. The magnetic core is assumed to operate in the linear region, which is a reasonable assumption for a transformer rated correctly for steady-state operation around its nominal flux level; saturation effects on L_M are therefore neglected. Hysteresis is also neglected, since the small-signal perturbations injected for impedance scanning lie in a frequency band where eddy and hysteresis losses produce only a minor contribution to the overall driving-point impedance compared with the leakage path [47, 10]. With these assumptions, the magnetizing branch is treated as a constant high impedance and, for the harmonic and sub-/super-synchronous range of interest, can be considered as an open circuit [47, 10]. The transformer impedance is then dominated by the leakage branch.

The leakage inductance is assumed to be frequency-independent, which is consistent with the common practice in the literature where only the leakage resistance is updated with frequency to capture skin and proximity effects in the windings [10]. An analytical expression for this dependence is the harmonic-order based correction according to IEEE Std. 399 [48]

$$R_l(f) = R_{dc} [1 + A h^B], \quad h = \frac{f}{f_1}, \quad (3.34)$$

where R_{dc} is the DC winding resistance, f_1 is the nominal grid frequency, and A, B are dimensionless empirical coefficients that capture the skin and proximity effects in the windings [48]. As f approaches the harmonic range, $R_l(f)$ increases significantly and provides additional damping in the converter-network interaction, which can shift the location of harmonic resonances [10].

Reflecting both windings to a common reference side and lumping the two leakage branches into a single equivalent series branch yields a per-phase impedance

$$Z_T(s, f) = R_l(f) + sL_l, \quad (3.35)$$

which represents the total leakage impedance seen between the two transformer ports once the magnetizing branch has been neglected. In the synchronous dq frame, the corresponding impedance is no longer a scalar: the Park transformation introduces synchronous-frame coupling between the d - and q -axes, as it does for a series RL filter [40]. The transformer leakage impedance therefore takes the form of the 2×2 matrix

$$\mathbf{Z}_{T,dq}(s, f) = \begin{bmatrix} R_l(f) + sL_l & -\omega_s L_l \\ \omega_s L_l & R_l(f) + sL_l \end{bmatrix}, \quad (3.36)$$

where $\omega_s = 2\pi f_1$ is the synchronous angular frequency. The diagonal entries reproduce the per-phase leakage impedance of (3.35), while the off-diagonal entries account for the rotating-frame coupling. Equation (3.36) is the form in which the wind turbine, offshore and onshore transformers will enter the system-wide impedance assembly later in this thesis.

3.4 Cable Admittance Modeling

In the considered offshore wind power plant, two qualitatively different cable assets must be represented: the inter-array cables, which connect the wind turbines to the offshore substation over distances typically below a few kilometres, and the HVAC export cable, which conveys the aggregated power from the offshore platform to the onshore grid over several tens of kilometres [10, 49]. Their relative electrical lengths with respect to the analysed frequency range determine which level of modelling detail is appropriate.

3.4.1 Array cables: nominal π -section model

For the inter-array cables, a nominal π -section is sufficient, since the cable length is normally shorter than the wavelength at the highest frequency of interest. Under this condition, the distributed-parameter behaviour can be approximated by a single lumped section without introducing visible errors in the impedance characteristic over the harmonic range.

The cable therefore can be modelled as a series RL branch with the total resistance $R = R'l$ and total inductance $L = L'l$, terminated by two equal shunt capacitances $C/2 = C'l/2$ at both ends, where R' , L' and C' are the per-unit-length parameters obtained from the cable catalogue and l is the cable length [10, 49].

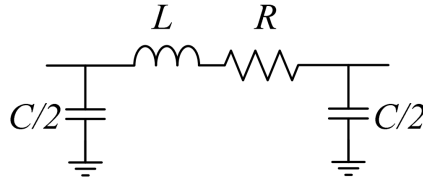


Figure 3.7: Modelling of an array cable using the nominal π -section [10].

Once the cable is transformed into the synchronous dq frame, both the series and shunt branches become 2×2 matrices because of the cross-coupling introduced by the rotating frame:

$$\mathbf{Z}_{s,dq}(s) = \begin{bmatrix} R + sL & -\omega_s L \\ \omega_s L & R + sL \end{bmatrix}, \quad \mathbf{Y}_{sh,dq}(s) = \begin{bmatrix} sC & -\omega_s C \\ \omega_s C & sC \end{bmatrix}. \quad (3.37)$$

With the series admittance $\mathbf{Y}_{s,dq} = \mathbf{Z}_{s,dq}^{-1}$ and the shunt admittance split equally between both terminals, the cable can then be assembled as a four-terminal dq two-port [10, 49]. The primitive admittance matrix takes the standard π -network form

$$\mathbf{Y}_{c,dq}^{\text{array}} = \begin{bmatrix} \mathbf{Y}_{s,dq} + \frac{1}{2}\mathbf{Y}_{sh,dq} & -\mathbf{Y}_{s,dq} \\ -\mathbf{Y}_{s,dq} & \mathbf{Y}_{s,dq} + \frac{1}{2}\mathbf{Y}_{sh,dq} \end{bmatrix}, \quad (3.38)$$

which is the 4×4 dq two-port admittance representation used in this thesis to interface the array cables with the wind turbine and the offshore platform nodes.

3.4.2 HVAC export cable: equivalent π -model from the telegrapher equations

For the HVAC export cable, the assumption underlying the nominal π -section is no longer valid. Over a length of several tens of kilometres and at frequencies up to a few kilohertz, the cable behaves as a distributed-parameter line and exhibits a series of resonances and anti-resonances within the analysed frequency range [50, 10]. Using a single nominal π would shift these resonance points or remove them entirely, while increasing the number of cascaded π -sections improves the accuracy at the cost of model complexity and still produces a deviation around the resonance frequencies [50, 10]. The accepted practice in harmonic and stability studies of long HVAC cables is therefore to use the equivalent π -model derived from the second-order telegrapher equations, which reproduces the distributed-parameter behaviour exactly at every frequency [50, 10].

Starting from the per-unit-length series impedance $Z'(f)$ and shunt admittance $Y'(f)$ of the cable, the characteristic impedance and propagation constant are defined as

$$Z_0(f) = \sqrt{\frac{Z'(f)}{Y'(f)}}, \quad \gamma(f) = \sqrt{Z'(f)Y'(f)}. \quad (3.39)$$

The series and shunt branches of the equivalent π -section, which exactly match the input–output behaviour of the distributed line, are then [50]

$$Z_s(f) = Z_0(f) \sinh(\gamma(f)l), \quad (3.40)$$

$$Y_p(f) = \frac{1}{Z_0(f)} \tanh\left(\frac{\gamma(f)l}{2}\right), \quad (3.41)$$

where l is the cable length. The equivalent π -network corresponding to (3.40)–(3.41) is shown in Figure 3.8.

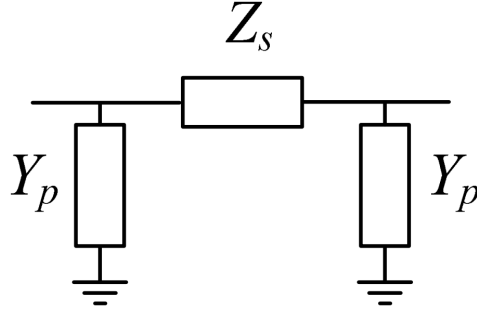


Figure 3.8: Equivalent π -model of a long HVAC cable derived from the telegrapher equations [10]

The skin and proximity effects are accounted for through the per-unit-length series impedance

$$Z'(f) = R'(f) + j\omega L'(f), \quad (3.42)$$

where the inductance is normally taken as constant and the resistance is updated with frequency. For the considered HVAC cable, the analytical expression [51]

$$R'(f) = 0.74 \left(0.267 + 1.073 \sqrt{\frac{f}{f_1}} \right) R'_1 \quad (3.43)$$

is adopted from [51], in which R'_1 is the per-unit-length DC (or nominal-frequency) resistance and $f_1 = 50$ Hz is the nominal frequency. As shown in [10], the resulting series impedance and shunt admittance of the equivalent π -section exhibit the multiple resonance/anti-resonance peaks that are characteristic of long cables and that can interact with the converter dynamics.

Because the export cable is described as a frequency-domain object, its translation into the synchronous dq frame cannot be carried out by direct substitution of $s \rightarrow s \pm j\omega_s$ inside the sinh and tanh functions, since these are nonlinear in their arguments. Instead, the standard procedure for distributed-parameter components in dq -based stability studies is followed [52]: the scalar two-port admittance is evaluated separately at the positive- and negative-sequence frequencies, $f_p = f + f_1$ and $f_n = f - f_1$, where f is the perturbation frequency in the dq frame. The scalar two-port admittance entries

$$Y_{11}(f) = Y_{22}(f) = Y_p(f) + \frac{1}{Z_s(f)}, \quad Y_{12}(f) = Y_{21}(f) = -\frac{1}{Z_s(f)}, \quad (3.44)$$

are computed at f_p and f_n to obtain the positive- and negative-sequence components Y_{ij}^+ and Y_{ij}^- for each block. The corresponding dq block is then reconstructed from the standard sequence-to- dq transformation [52]

$$\mathbf{Y}_{ij}^{dq} = \begin{bmatrix} \frac{1}{2}(Y_{ij}^+ + Y_{ij}^-) & \frac{j}{2}(Y_{ij}^+ - Y_{ij}^-) \\ -\frac{j}{2}(Y_{ij}^+ - Y_{ij}^-) & \frac{1}{2}(Y_{ij}^+ + Y_{ij}^-) \end{bmatrix}, \quad i, j \in \{1, 2\}. \quad (3.45)$$

Assembling the four 2×2 blocks gives the 4×4 dq two-port primitive admittance matrix of the HVAC export cable

$$\mathbf{Y}_{c,dq}^{\text{HVAC}} = \begin{bmatrix} \mathbf{Y}_{11}^{dq} & \mathbf{Y}_{12}^{dq} \\ \mathbf{Y}_{21}^{dq} & \mathbf{Y}_{22}^{dq} \end{bmatrix}, \quad (3.46)$$

which preserves both the distributed-parameter resonance structure of (3.40)–(3.41) and the synchronous-frame coupling required for the dq impedance-based analysis. Equation (3.46), together with the nominal π representation of the array cables in (3.38), completes the cable admittance model used in the rest of this thesis to describe the offshore collection grid and the HVAC link of the Princess Elisabeth Energy Island.

3.5 Grid and Synchronous Condenser Modeling

The last two elements required to close the impedance description of the Princess Elisabeth Energy Island system are the AC grid that receives the power exported from the offshore platform and the synchronous condenser installed at the onshore connection point to provide short-circuit capacity and dynamic reactive support. In the impedance-based stability framework, both elements appear at the same node and must be expressed as dq impedance matrices that can be added in parallel before being compared with the aggregated wind power plant admittance.

3.5.1 Grid Thevenin Equivalent in the dq Frame

For small-signal stability assessment, the external AC grid is conventionally represented by a Thevenin equivalent consisting of an ideal voltage source behind a series impedance [37, 53]. This is the standard formulation used in the impedance-based stability criterion, where the grid is described by its driving-point impedance at the point of common coupling while the inverter-based resource is described by its Norton-equivalent admittance [53].

Under the assumption of a balanced three-phase grid, the equivalent impedance can be modelled by a series resistance R_g and inductance L_g , neglecting any shunt capacitance because the equivalent shunt admittance of the high-voltage transmission network is small at the frequencies of interest [39, 10]. The transformation to the synchronous dq reference frame introduces cross-coupling between the d - and q -axes through the rotating-frame derivative, as for the converter filter and the transformer leakage path [40, 52]. The grid Thevenin impedance therefore takes the 2×2 matrix form

$$\mathbf{Z}_{g,dq}(s) = \begin{bmatrix} R_g + sL_g & -\omega_s L_g \\ \omega_s L_g & R_g + sL_g \end{bmatrix}, \quad (3.47)$$

where $\omega_s = 2\pi f_1$ is the synchronous angular frequency. The diagonal entries describe the per-phase RL behaviour of the grid, while the off-diagonal entries account for the synchronous-frame coupling. The strength of the grid is captured implicitly through the magnitude of L_g : a small L_g corresponds to a stiff (high-SCR) grid, whereas a large L_g represents a weak (low-SCR) grid in which the converter–grid interaction is more likely to produce sub-/super-synchronous instabilities [37, 39]. The resistance R_g , is the term that provides natural damping to the grid representation and therefore directly influences the margins of the converter–grid interconnection [37]. Equation (3.47) therefore provides a parametric way of stress-testing the stability margin of the offshore wind power plant against changes in the equivalent grid impedance.

3.5.2 Synchronous Condenser Impedance Model

A synchronous condenser is a synchronous machine operated without prime mover, whose role at the onshore connection point of an offshore wind power plant is to increase the short-circuit level, provide inertia and supply or absorb reactive power on a dynamic basis [54, 55]. From the point of view of the small-signal stability analysis carried out in this thesis, however, the relevant feature of the synchronous condenser is not its mechanical or excitation behaviour but the frequency-dependent impedance that it presents to the rest of the system at the connection point [54]. For this reason, the synchronous condenser is not represented as a full electromagnetic machine model with mechanical shaft dynamics, automatic voltage regulator (AVR), excitation controller and magnetic saturation, since these elements would only contribute additional dynamics at frequencies that are either below or already adequately captured by the impedance approach. Instead, the synchronous condenser is modelled as a small-signal dq impedance block, as is common in the impedance-based stability literature for synchronous machines connected to converter-dominated grids [54, 56].

Following Park's two-axis representation [12, 56], the small-signal stator equations of a balanced synchronous machine in the dq frame can be written, after eliminating the rotor circuits, as

$$\mathbf{Z}_{SC,dq}(j\omega) = \begin{bmatrix} R_s(j\omega) + j\omega L_d(j\omega) & -\omega_0 L_q(j\omega) \\ \omega_0 L_d(j\omega) & R_s(j\omega) + j\omega L_q(j\omega) \end{bmatrix}, \quad (3.48)$$

where R_s is the stator resistance, L_d and L_q are the operational inductances along the direct and quadrature axes, and ω_0 is the synchronous angular frequency [12, 56]. The off-diagonal entries are the well-known speed-voltage cross-coupling terms produced by Park's transformation, while the frequency-dependent diagonal entries $R_s(j\omega) + j\omega L_{d,q}(j\omega)$ embed the dynamics of the rotor field and damper windings into a single equivalent stator impedance.

The frequency dependence of the operational inductances follows directly from the standard machine parameters defined in IEEE Std 1110 [56, 12]. Using the open-circuit and short-circuit time constants of the field and damper circuits, the direct- and quadrature-axis operational inductances can be expressed as

$$L_d(j\omega) = L_d \frac{(1 + j\omega T'_d)(1 + j\omega T''_d)}{(1 + j\omega T'_{d0})(1 + j\omega T''_{d0})}, \quad (3.49)$$

$$L_q(j\omega) = L_q \frac{(1 + j\omega T'_q)(1 + j\omega T''_q)}{(1 + j\omega T'_{q0})(1 + j\omega T''_{q0})}, \quad (3.50)$$

where L_d , L_q are the synchronous inductances, T'_d , T'_q and T''_d , T''_q are the short-circuit transient and sub-transient time constants, and T'_{d0} , T'_{q0} , T''_{d0} , T''_{q0} are their open-circuit counterparts [12, 56]. The expressions in (3.49)–(3.50) reproduce the well-known three-region behaviour of a synchronous machine seen from the stator terminals [12]:

- at low frequency, the rotor circuits are effectively transparent and the machine reactance approaches the synchronous inductance,

$$L_d(j\omega) \approx L_d, \quad L_q(j\omega) \approx L_q, \quad (3.51)$$

which dominates the steady-state and very low-frequency behaviour;

- at intermediate frequencies, the field winding shields the air-gap flux and the effective inductance reduces to the transient inductance,

$$L_d(j\omega) \approx L'_d, \quad (3.52)$$

which is relevant for the sub-/super-synchronous range typically associated with rotor-grid interactions;

- at high frequency, the damper windings additionally shield the flux and the effective inductance further reduces to the sub-transient inductance,

$$L_d(j\omega) \approx L''_d, \quad (3.53)$$

which governs the harmonic-range response of the machine [12, 56].

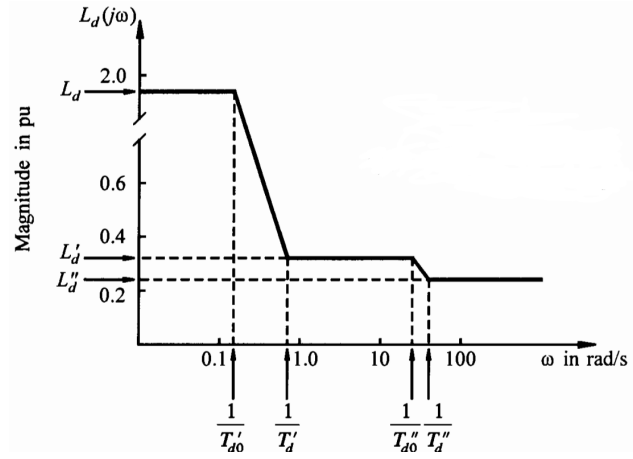


Figure 3.9: Variation of magnitude of $L_d(j\omega)$ [12]

The stator resistance is also frequency-dependent because of the skin and proximity effects in the stator winding. A widely adopted analytical expression for this dependence is the square-root law [56, 10]

$$R_s(f) = R_{s0} \left(1 + \alpha_{skin} \sqrt{\frac{f}{f_{nom}}} \right), \quad (3.54)$$

where R_{s0} is the DC (or nominal-frequency) stator resistance, f_{nom} is the nominal frequency and α_{skin} is a coefficient fitted to the manufacturer data of the specific machine. The factor in parentheses captures the additional high-frequency losses produced by the non-uniform current distribution in the conductors and by the eddy currents in the surrounding magnetic structure [56].

Substituting (3.49), (3.50) and (3.54) into (3.48) yields the complete 2×2 dq impedance of the synchronous condenser as a frequency-dependent block. This block (along with its subsequent transformer) can then be combined in parallel with the grid Thevenin impedance (3.47) at the onshore connection node to obtain the equivalent network impedance [54, 55].

3.6 Impedance-Based Stability and Robustness Indicators

The previous sections established the small-signal dq admittance and impedance models of every component of the offshore wind power plant. Once these models are aggregated (aggregation method will be discussed in the next chapter), the system seen at the Point of Common Coupling (PCC) is described by two MIMO frequency-response objects: the wind farm dq admittance $\mathbf{Y}_{WF}(j\omega)$ and the grid dq impedance $\mathbf{Z}_g(j\omega)$ derived in equation (3.47). Following the impedance-based stability framework [37, 53], the small-signal interaction between the two is captured by the open-loop gain

$$\mathbf{L}(s) = \mathbf{Z}_g(s) \mathbf{Y}_{WF}(s), \quad (3.55)$$

and the closed-loop characteristic equation of the interconnected system is

$$\det(\mathbf{I} + \mathbf{L}(s)) = 0. \quad (3.56)$$

A binary stable/unstable verdict from (3.56) alone is insufficient for engineering purposes: an offshore wind power plant is operated under continuously varying conditions, and small parametric drifts can move the system from a comfortable margin to the verge of instability. Several indicators have therefore been proposed in the literature to quantify not only whether the system is stable but also how robustly it is stable. Out of the many that exist, four complementary indicators are adopted in this thesis: the passivity index, the Generalized Nyquist Criterion (GNC), the vector margin, and the disk margin. Their definitions, the assumptions behind them, and the way they should be interpreted are reviewed in this section.

3.6.1 Passivity Index

The passivity-based stability criterion was introduced for grid-connected converters by [57, 58] and has since become a standard tool in the analysis of converter-dominated networks [52, 33]. The criterion exploits a fundamental property of linear, time-invariant systems: "the parallel/series interconnection of two passive subsystems is always stable, irrespective of the value of either subsystem's impedance" [57]. The absorbing power condition translates, in the frequency domain, into a definiteness condition on the Hermitian part of the admittance matrix. A MIMO admittance $\mathbf{Y}(j\omega)$ is said to be passive at frequency ω if the smallest eigenvalue of its Hermitian part is non-negative [58, 52]:

$$\mathcal{P}_Y(\omega) \triangleq \lambda_{\min} \left\{ \frac{1}{2} (\mathbf{Y}(j\omega) + \mathbf{Y}(j\omega)^H) \right\} \geq 0, \quad (3.57)$$

where $(\cdot)^H$ denotes the conjugate transpose. The scalar $\mathcal{P}_Y(\omega)$ is the passivity index of the admittance: positive values indicate net energy absorption at frequency ω , while negative values indicate that the device can deliver energy to the rest of the network and could therefore destabilize the interconnection.

Applied to the wind power plant, equation (3.57) provides a sufficient stability condition: if $\mathbf{Y}_{WF}(j\omega)$ is passive over the entire frequency range of interest and the network impedance $\mathbf{Z}_g(j\omega)$ is itself passive (which is automatically the case for any combination of RL branches, transformers and lossy cables [58]), then the interconnected system is small-signal stable [57, 52]. Conversely, the frequency bands in which $\mathcal{P}_{Y_{WF}}(\omega) < 0$ identify potential instability "windows": within these windows the wind farm injects negative damping, and the actual interconnection is stable only if the grid

impedance does not align with the converter response in such a way as to satisfy the Nyquist condition [58]. The passivity index therefore acts both as a robustness measure with respect to grid uncertainty and as a powerful design indicator.

3.6.2 Generalized Nyquist Criterion

The Generalized Nyquist Criterion (GNC) is the multivariable extension of the classical Nyquist stability criterion, originally formulated by [59] and adapted to power-electronic systems by [60]. The criterion states that, provided each of the two subsystems is individually stable in open loop, the interconnected system described by (3.56) is stable if and only if the eigenloci of the loop gain $\mathbf{L}(j\omega)$, i.e. the loci traced in the complex plane by the eigenvalues

$$\lambda_i(j\omega) = \text{eig}(\mathbf{L}(j\omega)), \quad i = 1, 2, \quad (3.58)$$

do not encircle the critical point $(-1 + j0)$ when ω varies from $-\infty$ to $+\infty$ [59, 60]. For a 2×2 dq system as the one considered in this thesis, two eigenloci are obtained, one for each propagation channel of the loop gain.

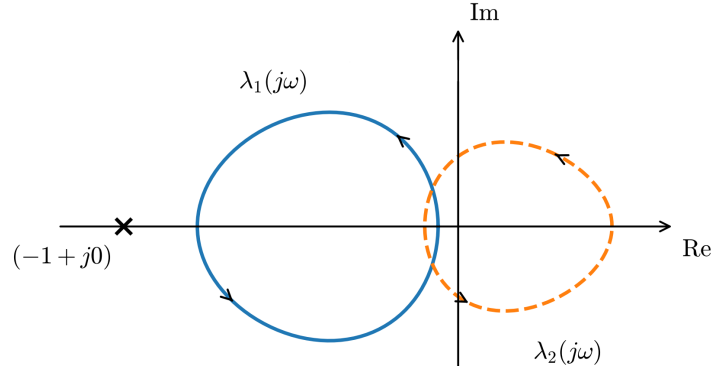


Figure 3.10: Example of Generalized Nyquist Criterion with no encirclement of $(-1 + j0)$

GNC is the natural MIMO counterpart of the impedance-based stability criterion of [37], and it is the most general necessary-and-sufficient stability test that can be derived from frequency-response data alone. However, two practical limitations should be kept in mind. First, the criterion in its strict form requires the eigenloci to be evaluated over the full positive- and negative-frequency axis, while in practice the analysis is carried out on a finite band; truncation can shift the integer encirclement count by one or more units, so GNC must be paired with a continuous robustness indicator to be reliable on a finite scan [52]. Second, GNC delivers a yes/no verdict but not a quantitative robustness measure: a system whose eigenloci graze the critical point is technically stable yet operationally fragile. For these reasons the GNC plot is, in the present work, should be complemented by the margins introduced below.

3.6.3 Vector Margin

The vector margin [61] is a continuous robustness indicator constructed directly from the GNC eigenloci. It is defined as the smallest distance between the eigenloci of $\mathbf{L}(j\omega)$ and the critical point $(-1 + j0)$ over the frequency band of interest:

$$\text{VM}(\omega) \triangleq \min_{i \in \{1,2\}} |\lambda_i(j\omega) + 1|. \quad (3.59)$$

The *worst-case vector margin* is then defined as the minimum of $\text{VM}(\omega)$ over the frequency band of interest:

$$\text{VM} \triangleq \min_{\omega} \min_{i \in \{1,2\}} |\lambda_i(j\omega) + 1|. \quad (3.60)$$

Geometrically, the worst-case vector margin is the radius of the largest circle centred at $(-1 + j0)$ that does not intersect any eigenlocus as seen in Figure 3.11. It can therefore be interpreted as the magnitude of the smallest additive perturbation of the loop gain that would drive the system to the stability boundary, which makes it a true MIMO sensitivity measure [61].

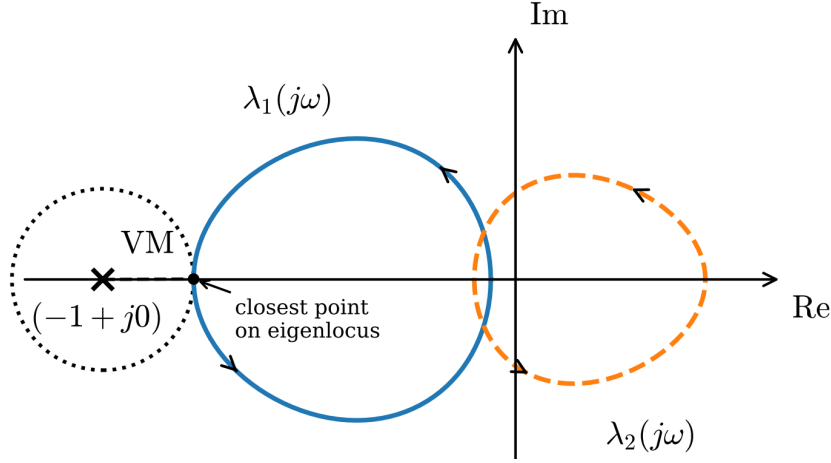


Figure 3.11: Example of Vector Margin Circle

Compared with classical gain and phase margins, the vector margin has two important advantages in the context considered here. First, it does not require the existence of a single, well-defined gain crossover frequency: in offshore wind power plants the eigenloci of $\mathbf{L}(j\omega)$ typically cross the unit circle at several frequencies because of the network – converters, transformers, the array cables and the export cable – resonances, which makes a single PM/GM number meaningless [52]. Second, it captures simultaneous magnitude and phase deficiencies, whereas classical margins consider them independently. The widely used acceptance threshold $\text{VM} \geq 0.5$ corresponds approximately to a SISO gain margin of 6 dB and a phase margin of 30° [61] and will be adopted in this thesis as the criterion for "healthy" MIMO robustness.

3.6.4 Disk Margin

The disk margin, formalized by [13], is the modern MIMO replacement for the classical, independent gain and phase margins. Whereas the vector margin quantifies the smallest additive perturbation tolerated by the loop, the disk margin quantifies the largest simultaneous gain and phase variation that the closed loop can tolerate without losing stability [13]. The set of admissible perturbations applied to the loop gain is parameterized by the bilinear map

$$f(\delta) = \frac{1 + \alpha \delta (1 - \sigma)/2}{1 - \alpha \delta (1 + \sigma)/2}, \quad |\delta| \leq 1, \quad (3.61)$$

where α is the disk-margin radius and σ is a real skew parameter. As δ varies over the unit disc in the complex plane, $f(\delta)$ traces a disc in the complex plane that captures simultaneous gain and phase variations of the loop [13]. The skew parameter σ shifts the centre of the admissible disc along the

gain axis — towards more gain increase for $\sigma > 0$ and towards more gain decrease for $\sigma < 0$ — while leaving its radius α unchanged. It is therefore a gain bias, not a trade-off between gain and phase margins as is sometimes loosely stated. The choice $\sigma = 0$ produces a balanced disc, in which the loop gain is allowed to increase and decrease by the same relative factor — i.e. the disc is symmetric on the dB scale — and is the formulation used in the present work [13].

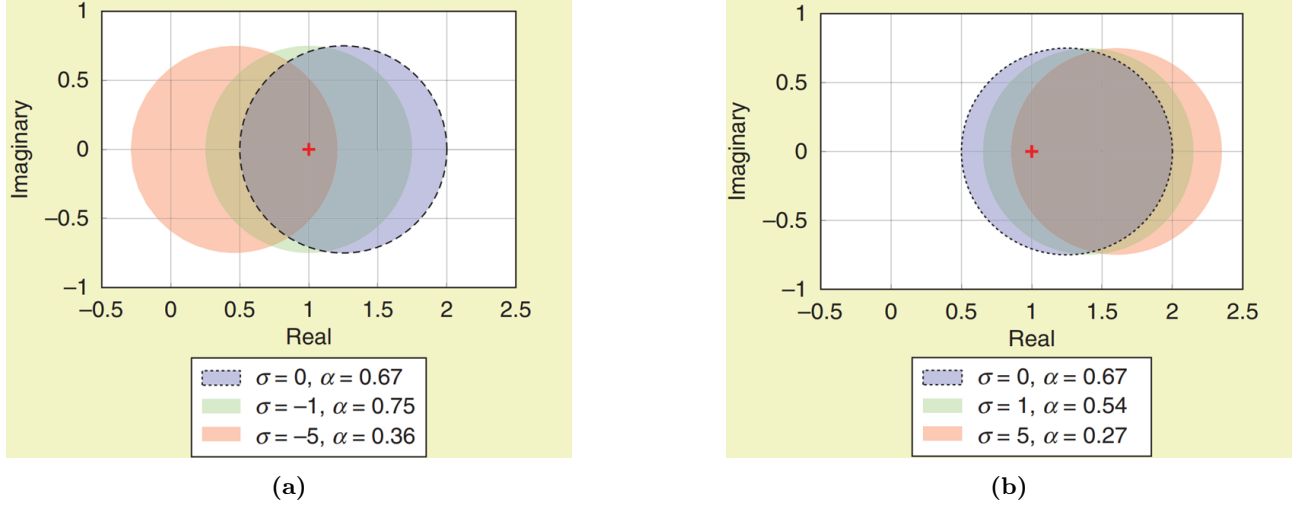


Figure 3.12: Example of Disk Margin from [13]. (a) A negative σ skews the gain variation left, toward more gain decrease. (b) A positive σ skews the gain variation right, toward more gain increase. The parameter α is selected to maintain the same radius for all disks. [13]

For a MIMO loop $\mathbf{L}(j\omega)$ in negative-feedback form, the closed loop is robustly stable for all balanced full-block perturbations of the form (3.61) provided that, for every ω ,

$$\alpha < \alpha_{mm}(j\omega) = \frac{2}{\bar{\sigma}[(\mathbf{L}(j\omega) - \mathbf{I})(\mathbf{L}(j\omega) + \mathbf{I})^{-1}]} \quad (3.62)$$

where $\bar{\sigma}[\cdot]$ denotes the maximum singular value [13]. Equation (3.62) corresponds to a full-block, unstructured uncertainty acting on the loop, and is therefore a sufficient (and conservative) multi-loop margin. The exact multi-loop disk margin, in which independent disc-shaped perturbations may be applied to each loop channel, is obtained from a structured singular value (μ) computation as detailed in [13]; in the present work the conservative full-block expression (3.62) is adopted, since it is sufficient to discriminate between robust and fragile operating points and is straightforward to evaluate from frequency-response data. The worst-case multi-loop disk margin is then the minimum of $\alpha_{mm}(j\omega)$ over the analysed band,

$$\alpha_{mm} = \min_{\omega} \alpha_{mm}(j\omega), \quad (3.63)$$

and admits a direct engineering interpretation: it is the largest scalar variation that can be applied simultaneously to the gain and phase of the loop, at the worst-case frequency, without destabilizing the closed loop.

[13] provides the analytical interpretation of α_{mm} but does not prescribe universal acceptance thresholds. In this thesis, the following practical interpretation is adopted (in line with classical 6 dB and 30° engineering use of the margin): $\alpha_{mm} \geq 0.5$ is associated with a comfortable robustness margin, $0.3 \leq \alpha_{mm} < 0.5$ with an acceptable but reduced margin, and $\alpha_{mm} < 0.3$ with a fragile design that warrants revisiting the controller or filter parameters.

Together, the four indicators introduced in this section provide complementary information. The passivity index gives a sufficient stability condition derived from energy considerations and is highly informative for controller design. The Generalized Nyquist Criterion supplies the formal stability verdict of the interconnected MIMO system. The vector margin and the disk margin then quantify, in two complementary ways, how far the system operates from the stability boundary: the former measures additive perturbations on the eigenloci, while the latter measures the largest simultaneous gain–phase variation the loop can tolerate. The combination of these four indicators forms the analytical toolset used in the rest of the thesis to assess both the stability and the robustness of the offshore wind power plant.

3.7 Stability Enhancement Techniques

The impedance models and stability indicators were introduced in the previous sections describing the impedance based stability analysis core research. When the analysis reveals that the converter admittance $\mathbf{Y}_{WF}(j\omega)$ exhibits non-passive bands, that the GNC eigenloci approach the critical point $(-1+j0)$, or that the Vector/Disk margin falls below the practical thresholds discussed in the previous section, dedicated countermeasures must be implemented to restore robust operation. Since the classification of the topic is divided into two parts, sub-/super-synchronous and harmonic range, the countermeasures, to make the system stable, should also be taken in the assessed range.

3.7.1 Harmonic Stability Enhancement Techniques

In offshore wind power plants, harmonic instabilities typically originate from one or more lightly damped resonances of the converter–network interconnection — most notably the LC filter of the GSC combined with the wind-turbine transformer leakage, and the offshore-collector-transformer leakage in series with the long HVAC export cable [62]. The closed-loop damping of these intrinsically sharp resonances depends on the converter control system, in particular on the current-controller bandwidth, the digital and PWM delays and the voltage feed-forward path; when this damping is insufficient the wind-farm admittance $\mathbf{Y}_{WF}(j\omega)$ exhibits sharp peaks that can drive the wind-farm/grid interconnection into harmonic instability [62, 33].

Two complementary families of techniques are established in the literature to suppress this resonance and reshape the converter admittance: *active damping*, which modifies the converter’s control strategy without adding hardware, and *passive filtering*, which physically dissipates the resonance energy by inserting additional R , L , C elements [63, 64]. Within the active-damping family the two most widely adopted approaches are capacitor current feedback (CCF) and digital notch filtering, both of which are reviewed below alongside passive filtering.

Capacitor Current Feedback Active Damping

Capacitor current feedback is the most extensively studied active damping technique for LCL-filtered grid-connected converters [65, 66]. The principle is to sense the capacitor current i_{C_f} through a dedicated current sensor and feed it back, multiplied by a proportional gain K_{ad} , to the modulation signal of the GSC. The damping ratio of the resonant pole pair can be tuned almost arbitrarily through

K_{ad} , and the bandwidth penalty on the inner current loop is negligible if K_{ad} is chosen correctly [65]. From the converter terminals, this feedback is mathematically equivalent to placing a *virtual resistor* in parallel with the filter capacitor [66, 52], hence damping the LCL resonance peak in the same way a physical resistor would, but without dissipating real power, as shown in Figure 3.13.

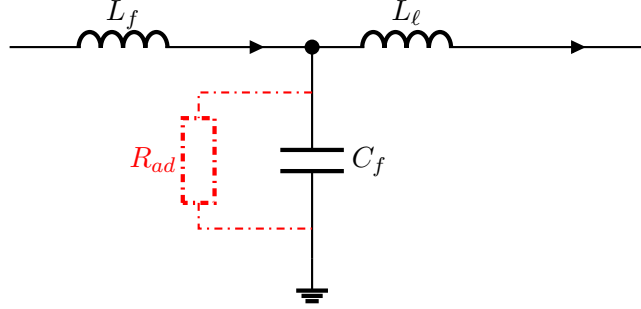


Figure 3.13: Equivalent circuit of CCF active damping illustrating the virtual parallel resistor (R_{ad})

Two practical issues, however, have been extensively reported for CCF in the harmonic-stability literature. First, in a digitally controlled converter the feedback path is delayed by sampling, computation and PWM update — typically by $1.5 T_s$ — and this delay reshapes the equivalent virtual impedance from a pure resistance into a frequency-dependent complex impedance [66, 52]. As a consequence, the virtual resistor seen at the resonance frequency may become negative for a range of resonance frequencies, corresponding to the well-known classification of "stable" and "unstable" regions of the resonance frequency relative to one sixth of the sampling frequency $f_s/6$ [66].

Second, the CCF gain interacts with the outer control loops and the PLL, so that excessive damping action can shift the non-passive region of $\mathbf{Y}_{WF}$ into the sub-/super-synchronous range [58]. As a result, CCF design is treated in the recent literature as a multi-objective problem that simultaneously considers harmonic-range damping, sub-/super-synchronous passivity, and robustness to grid impedance uncertainty [52].

Notch Filter Active Damping

Notch filter active damping suppresses the LCL resonance by inserting a digital notch — typically a second-order band-stop filter — in the current control loop, tuned at the resonance frequency f_{res} [67, 68, 69]. The notch attenuates the open-loop gain of the converter at f_{res} , so that the resonance peak no longer crosses the 0 dB line and the eigenloci of the loop gain do not approach $(-1 + j0)$ at that frequency. The transfer function of a typical second-order notch is

$$H_{notch}(s) = \frac{s^2 + \omega_n^2}{s^2 + 2\zeta_n \omega_n s + \omega_n^2}, \quad (3.64)$$

where $\omega_n = 2\pi f_{res}$ is the centre frequency of the notch and ζ_n controls its depth and bandwidth [68]. The technique requires no additional current sensor, since it operates entirely on the existing inner-loop measurement, and is therefore an attractive and effective implementation.

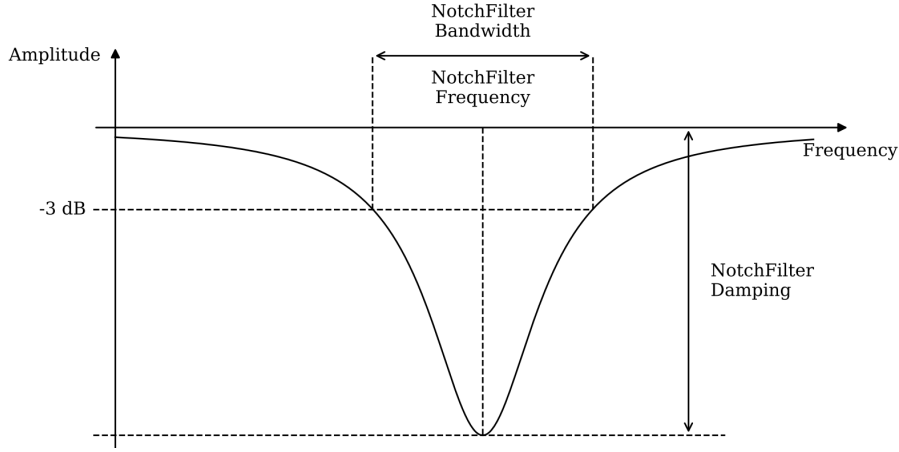


Figure 3.14: Second Order Notch Filter

The major limitation of notch filter active damping reported in the literature is its sensitivity to the actual value of the resonance frequency [67, 69]. The LCL resonance frequency depends not only on the converter-side parameters L_f and C_f , which can be considered fixed within manufacturing tolerances, but also on the grid-side inductance, which in an offshore plant includes the transformer leakage, the array cable inductance and the equivalent grid inductance at the POC. The latter varies with grid configuration and short-circuit ratio, and can drift the actual f_{res} away from the value ω_n assumed at design time. When the mismatch becomes significant, the notch loses its alignment with the resonance peak and either overdamps a non-resonant frequency or fails to damp the actual resonance, in some cases even introducing additional phase lag that worsens the stability margin [67]. Adaptive notch implementations, in which ω_n is updated online from a frequency estimator, have been proposed to mitigate this sensitivity [69].

Passive Filtering

Passive filtering achieves resonance damping and stability enhancement through the deliberate insertion of additional R , L , C elements in the filter network, rather than through control action [63, 64]. The simplest and most widely used configuration is the addition of a damping resistor R_d in series with the LCL filter capacitor C_f , which physically dissipates a fraction of the resonance energy at every cycle and lowers the Q-factor of the LCL resonance peak. More elaborate configurations exist, including $R_d C_d$ branches in parallel with C_f , $R_d L_d$ branches in parallel with the converter-side or grid-side inductor, and dedicated tuned trap filters at specific harmonic frequencies [70, 14]. A systematic comparison of these topologies in terms of damping effectiveness, harmonic attenuation, and power loss is provided in [63] and [64].

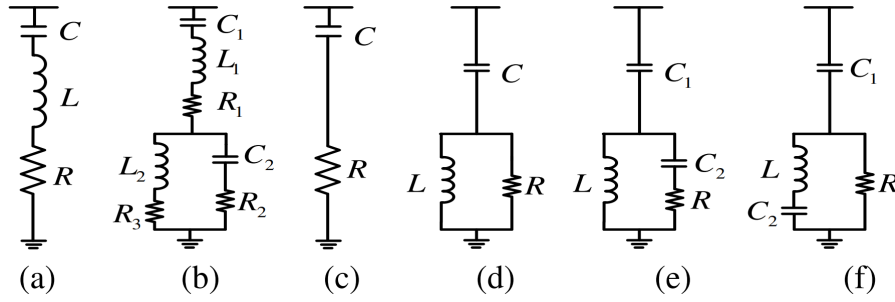


Figure 3.15: Types of passive filter circuits: (a) single-tuned filter, (b) double-tuned filter, (c) first-order high-pass filter, (d) second-order high-pass filter, (e) third-order high-pass filter, and (f) type-C filter. Reproduced from [14]

The principal advantage of passive filtering, repeatedly emphasized in the literature, is its *robustness*. Because the damping is implemented by a physical element, it is independent of the digital control implementation: it is unaffected by sampling delay, PWM update, controller bandwidth, PLL dynamics or grid-impedance variations [63]. Passive damping therefore guarantees that the converter admittance remains passive in the resonance band regardless of the operating point, which makes it especially attractive in offshore wind power plants where the equivalent grid impedance at the POC is uncertain and time-varying. The corresponding drawback is the introduction of permanent conduction losses across the damping element, which directly reduce the conversion efficiency of the wind turbine [63, 64], and the additional volume and weight of the passive components, which is a non-trivial constraint inside an offshore turbine nacelle. The choice of the damping topology and of the damping resistance value is therefore a design trade-off between damping effectiveness, harmonic attenuation, efficiency and physical footprint [64].

3.7.2 Sub-synchronous Stability Enhancement Techniques

In Type-4 wind turbine generator-based wind farms, the sub-synchronous oscillation (SSO) phenomenon is mainly associated with the interaction between converter control dynamics and the external grid impedance. Therefore, mitigation measures should not only focus on the electrical network, but also on the control structure.

Supplementary Damping Controllers (SDCs)

An effective mitigation strategy is to add supplementary damping control loops to the existing converter control system[15]. The main purpose of these loops is to detect the oscillatory component and inject a corrective control signal with suitable magnitude and phase so that positive damping is introduced at the critical sub-synchronous frequency [15].

Supplementary Damping Controllers (SDCs) are general-purpose auxiliary controllers installed in power-electronic-interfaced devices or grid-side FACTS devices to mitigate oscillations. As illustrated in Fig. 3.16, their control structure includes a gain, a washout filter, and lead-lag compensation blocks. The input signal may be selected from the active power output, line current, voltage across the series capacitor, or rotor-speed deviation [71, 72]. The output signal can then be used as a feedback signal for the GSC, the MSC, or both the GSC and MSC [73, 74, 46].

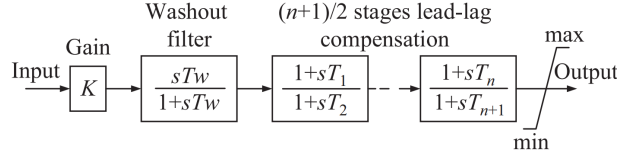


Figure 3.16: Control structure of a generic supplementary damping controller [15]

The effectiveness of an SDC depends on how its parameters are selected, and the most commonly adopted approaches are the residue-based method and the time-domain-simulation-based optimization method [75]. To handle different operating conditions during parameter tuning, researchers have further optimized the process using advanced techniques such as artificial intelligence algorithms and probability-based methods [76, 46].

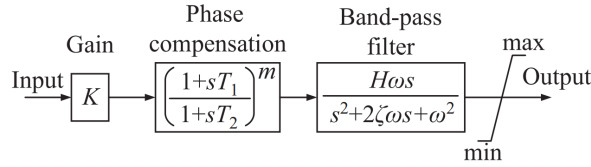


Figure 3.17: Adjusted SDC [15]

At present, most damping controllers are developed for Type-3 WTGs; however, [77] showed that damping controllers can also absorb the energy associated with Type-4 WTG-related SSO. To suit different application scenarios, SDCs are modified accordingly, as shown in Figure 3.17 [44, 78]. In particular, the control system in Figure 3.18 was specifically designed to deal with SSO related to Type-4 WTGs.

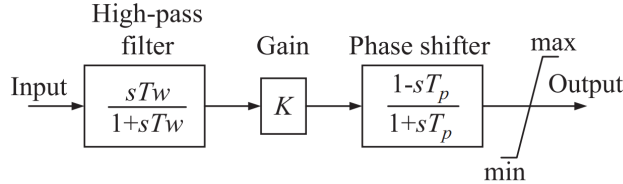


Figure 3.18: Adjusted SDC specific for Type 4 WT [15]

PLL-based Damping Control

An alternative and structurally elegant mitigation strategy for Type-4 WTG-related SSO is the Phase-Locked Loop (PLL)-based damping control, proposed by [79]. Rather than introducing external hardware or a separate parallel control branch, this approach embeds a Sub-Synchronous Oscillation Damping Controller (SSODC) directly within the existing PLL structure of the GSC, exploiting the PLL's inherent role in grid synchronisation to simultaneously provide active damping at the critical sub-synchronous frequency.

the PLL feedback loop can amplify sub-synchronous disturbances in weak-grid conditions, effectively introducing negative damping at the oscillation frequency and making the PLL itself a contributing source of sustained SSO. The PLL-based damping strategy directly addresses this by transforming

the same feedback mechanism into an active damping resource [79].

The complete control architecture is illustrated in Fig. 3.19, which shows two functional subsystems: the conventional PLL (lower path) and the superimposed SSODC (upper path).

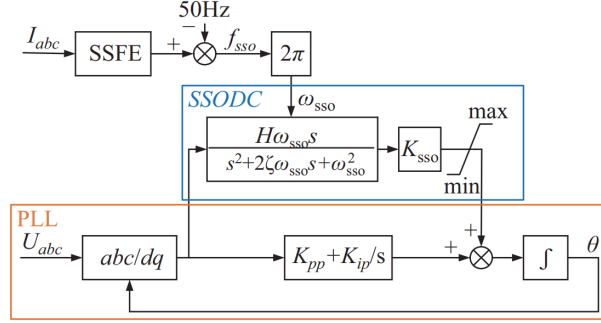


Figure 3.19: PLL-based damping control strategy [15]

The SSODC is superimposed on the PLL without modifying its internal structure. It consists of four sequential stages operating on the three-phase line current $\mathbf{I}_{abc}$:

1. **Sub-synchronous Frequency Estimation (SSFE).** The three-phase current $\mathbf{I}_{abc}$ is continuously monitored and processed by a frequency estimator, typically an adaptive algorithm such as an interpolated Discrete Fourier Transform or an Adaptive Extended Kalman Filter to identify the instantaneous SSO frequency f_{sso} [79].
2. **Angular Frequency Computation.** The estimated f_{sso} is subtracted from the nominal grid frequency of 50 Hz, and the result is converted to angular frequency:

$$\omega_{sso} = 2\pi (50 - f_{sso}). \quad (3.65)$$

This quantity represents the angular frequency of the oscillatory component as it appears in the stationary frame, and it serves as the tuning parameter for the subsequent bandpass filter.

3. **Adaptive Bandpass Filter.** The core of the SSODC is a second-order bandpass filter whose centre frequency is updated in real time by ω_{sso} :

$$H_{SSODC}(s) = \frac{H \omega_{sso} s}{s^2 + 2\zeta \omega_{sso} s + \omega_{sso}^2}, \quad (3.66)$$

where H is the filter gain and ζ is the damping ratio of the filter poles. This transfer function acts as a selective bandpass: it attenuates the nominal 50 Hz component and all other frequencies, while extracting and phase-shifting the sub-synchronous component centred at ω_{sso} . The 90° phase characteristic of a bandpass filter at its centre frequency ensures that the extracted signal is in phase quadrature with the original oscillation, which is the condition required to introduce positive damping (eliminating oscillations in a manner similar to noise-canceling headphones) [79]. Since ω_{sso} is updated continuously from the SSFE output, the filter tracks a drifting oscillation frequency without requiring manual retuning.

4. **Gain Limiting and PLL Injection.** The bandpass filter output is scaled by a single gain parameter K_{sso} and then saturated between prescribed minimum and maximum limits. The

saturation prevents the corrective signal from becoming excessively large under transient conditions, which could otherwise destabilise the outer control loops. The resulting signal is injected additively into the summation point of the PLL, directly modifying the phase angle θ produced by the integrator. Through this injection, the PLL, instead of amplifying the sub-synchronous disturbance, introduces a restoring torque that damps it [79, 15].

It should be noted, however, that this strategy remains at the research and laboratory validation stage. The PLL-based SSODC has not yet been reported as commercially deployed in operational wind farms, and the gap between simulation-validated controllers and field-commissioned solutions remains an open challenge identified by industry stakeholders [15].

Summary

The comparative discussion of state-space and impedance-based formulations has motivated the choice of the impedance-based approach as the most suitable for a converter-dominated, partially black-box offshore wind connection, given its modularity, scalability, natural treatment of digital delays and distributed-parameter cables, and direct compatibility with frequency-domain stability indicators. On this basis, detailed dq impedance and admittance models have been derived for every element of the case study: the Type 4 wind turbine in both its harmonic and sub-/super-synchronous representations, transformers, the inter-array cables, the HVAC export cable through the equivalent π -model of the telegrapher equations, the AC grid Thevenin equivalent and the planned onshore synchronous condenser. The chapter has then introduced the four impedance-based indicators, the passivity index, the Generalized Nyquist Criterion, the vector margin and the disk margin used to translate the small-signal interaction at the Point of Common Coupling into a quantitative robustness assessment. Finally, the harmonic and sub-synchronous stability-enhancement techniques reported in the literature have been reviewed and classified according to their physical mechanism and their interaction with the converter control loops. The component models, the stability indicators and the catalogue of mitigation techniques collected in this chapter therefore constitute the analytical toolset of the thesis.

Chapter 4

Princess Elisabeth Energy Island: Baseline Modeling and Stability Assessment

The framework developed in Chapter 3 is applied in this chapter to the Princess Elisabeth Energy Island case study, with the objective of assessing the small-signal stability and robustness of the baseline offshore plant configuration at the onshore Point of Common Coupling. The chapter first selects a consistent set of parameters for a representative 14 MW Type 4 wind turbine and validates its analytical harmonic and sub-/super-synchronous admittances against a detailed EMT model in MATLAB/Simulink through dq -frame perturbation injections. The remaining dq blocks introduced in Chapter 3 (transformers, array cables, HVAC export cable and grid Thevenin equivalent) are then parameterised and aggregated into the full plant admittance seen at the PCC. The resulting interconnection is finally assessed using the Generalized Nyquist Criterion, the passivity index, the vector margin and the disk margin, complemented by an impedance crossover diagnostic, in order to identify the frequency bands in which the baseline design operates closest to the stability boundary and to provide the diagnostic input on which the mitigation strategy of Chapter 5 is built.

4.1 Modeling and Validation of Wind Turbine

The stability analysis of the Princess Elisabeth Energy Island begins with the modelling and validation of the Type 4 wind turbine, following the analytical framework developed in the previous chapters. The first implementation step is the selection and calculation of the model and its associated simulation parameters.

4.1.1 Harmonic and Sub-/super-synchronous Models for Type 4 Wind Turbine

Harmonic Admittance Model

The harmonic-range admittance of the Type 4 wind turbine, $\mathbf{Y}_{GSC,dq}(s)$, is taken from equation (3.12) of Section 3.2.1 (repeated below for convenience).

$$\mathbf{Y}_{GSC,dq}(s) = \left[\frac{\mathbf{B}_{L_f}(s) + G_c(s) [\mathbf{B}_{PI,i}(s) - \mathbf{B}_{dec}]}{1 - G_c(s)H_v} \right]^{-1} + \mathbf{B}_{C_f}(s) \quad (4.1)$$

In this model the wind turbine is represented only through the grid-side converter dynamics: the inner current control loop, the LC filter, the PWM and digital delay, and the voltage feed-forward path. The PMSG, the machine-side converter, the DC-link voltage controller and the PLL are deliberately omitted, since their bandwidths lie below the harmonic range of interest.

Sub-/super-synchronous Admittance Model

For the sub-/super-synchronous range, the wind turbine is described by the complete admittance $\mathbf{Y}_{acW}(s)$ derived in equation (3.33) of Section 3.2.2. (repeated below for convenience)

$$\mathbf{Y}_{acW}(s) = \mathbf{A}^{-1} \left[\mathbf{B} - \mathbf{C}(sC_{dc} + \mathbf{Y}_{dcM} + \mathbf{H})^{-1} \mathbf{G} \right].$$

This model retains the DC-link capacitor, the machine-side admittance $\mathbf{Y}_{dcM}$ from equation (3.17), and the PLL and DC-voltage outer-loop dynamics through the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{G}$ and $\mathbf{H}$ of Table 3.1.

4.1.2 Selection and Calculation of Simulation Parameters

The impedance-based and EMT models developed in this work require a consistent set of electrical, mechanical and control parameters for a representative Type 4 offshore wind turbine. Since the complete manufacturer datasets (PMSG constants, converter parasitics, controller gains and limiters) are proprietary and not publicly available, the parameterisation adopted in this thesis is based on publicly available EMT and real-time simulation studies of the 14-15 MW offshore class.

The turbine considered is a 14 MW Type 4 unit, representative of the SG 14-222 DD class [31]. The hardware parameters of the back-to-back converter, DC-link capacitor, output filter, step-up transformer and direct-drive PMSG are taken from the detailed 15 MW model in [80], while the operating-point currents and modulation indices are recalculated for a 14 MW active-power setpoint. Detailed derivations of the parameter values are reported in the Appendix.

Table 4.1: Electrical parameters used in the 14 MW Type 4 wind turbine simulation model.

Parameter	Symbol	Value	Origin
Active power operating point	P_0	14 MW	Elia [2]
Reactive power operating point	Q_0	0 MVar	Unity power factor
Grid angular frequency	ω_s	$2\pi 50$ rad/s	50 Hz system
DC-link voltage	$V_{dc,0}$	10,000 V	From [80]
DC-link capacitance	C_{dc}	4.0×10^{-3} F	From [80]
GSC-side voltage	U_0	3266 V	Derived from 4 kV L-L RMS
Filter resistance	R_f	0.001Ω	Modelling assumption
Filter inductance	L_f	2.75×10^{-4} H	From [80]
Filter capacitance	C_f	1024×10^{-6} F	From [80]
PMSG stator resistance	R_s	0.0368Ω	From [80]
PMSG d -axis inductance	L_d	8.7×10^{-3} H	From [80]
PMSG q -axis inductance	L_q	5.8×10^{-3} H	From [80]
Average PMSG inductance (isotropic)	L_s	7.25×10^{-3} H	$(L_d + L_q)/2$
Rated mechanical speed	$\omega_{m,0}$	0.8 rad/s	From [80]
Pole pairs	n_p	162	From [80]
Electrical rotor speed	$\omega_{r,0}$	129.6 rad/s	$n_p \omega_{m,0}$
PM flux linkage	ψ_m	25.2 Wb	Estimated from $U_0/\omega_{r,0}$

Table 4.2: Grid-side converter operating-point quantities.

Quantity	Symbol	Value	Calculation
d -axis voltage	$v_{d,0}$	3266 V	U_0
q -axis voltage	$v_{q,0}$	0 V	PLL alignment
d -axis current	$I_{GSC,d0}$	2858 A	$2P_0/(3U_0)$
q -axis current	$I_{GSC,q0}$	0 A	$Q_0 = 0$
d -axis modulation index	$M_{GSC,d0}$	0.653	$2U_0/V_{dc,0}$
q -axis modulation index	$M_{GSC,q0}$	0.049	$2\omega_s L_f I_{d0}/V_{dc,0}$
Switching frequency	f_{sw}	5 kHz	Nyquist for 2.5 kHz

Table 4.3: Machine-side converter operating-point quantities.

Quantity	Symbol	Value	Calculation
d -axis current	$I_{MSC,d0}$	0 A	Assumed no field-weakening
q -axis current	$I_{MSC,q0}$	2858 A	$2T_0/(3n_p\psi_m)$
Rated torque	T_0	17.5×10^6 N m	$P_0/\omega_{m,0}$
d -axis modulation index	$M_{MSC,d0}$	-0.430	$2v_{d,0}/V_{dc,0}$
q -axis modulation index	$M_{MSC,q0}$	0.674	$2v_{q,0}/V_{dc,0}$

The converter-controller parameters are selected using standard cascade-control bandwidth principles and pole-zero cancellation tuning of the current loop.

Table 4.4: Controller quantities and PI gain definitions used in the simulation model.

Quantity	Symbol	Value	Calculation
Grid-side converter current controller			
GSC current-controller bandwidth	$f_{\text{GSC,bw}}$	400 Hz	Baseline value
GSC angular bandwidth	$\omega_{\text{GSC,bw}}$	2513.27 rad/s	$2\pi f_{\text{GSC,bw}}$
GSC proportional gain	$K_{p,\text{GSC}}$	0.691	$L_f \omega_{\text{GSC,bw}}$
GSC integral gain	$K_{i,\text{GSC}}$	2.513	$R_f \omega_{\text{GSC,bw}}$
Voltage feedforward gain	$H_v = K_{ff,v}$	1	Baseline value
DC-link voltage controller			
DC-link voltage proportional gain	$K_{p,\text{dc}}$	0.73	From [80]
DC-link voltage integral gain	$K_{i,\text{dc}}$	32.2	From [80]
Phase-locked loop			
PLL bandwidth	$f_{\text{PLL,bw}}$	15 Hz	Baseline value
PLL angular bandwidth	$\omega_{\text{PLL,bw}}$	$2\pi \cdot 15$ rad/s	$2\pi f_{\text{PLL,bw}}$
PLL damping ratio	ζ	0.707	Standard
PLL proportional gain	$K_{p,\text{PLL}}$	0.0408	$\frac{2\zeta\omega_{\text{PLL,bw}}}{U_0}$
PLL integral gain	$K_{i,\text{PLL}}$	2.72	$\frac{\omega_{\text{PLL,bw}}^2}{U_0}$
Machine-side converter current controller			
MSC current-controller bandwidth	$f_{\text{MSC,bw}}$	625 Hz	Baseline value
MSC angular bandwidth	$\omega_{\text{MSC,bw}}$	3927 rad/s	$2\pi f_{\text{MSC,bw}}$
MSC proportional gain	$K_{p,\text{MSC}}$	28.47	$L_s \omega_{\text{MSC,bw}}$
MSC integral gain	$K_{i,\text{MSC}}$	144.51	$R_s \omega_{\text{MSC,bw}}$

4.1.3 EMT-Based Validation

To confirm that the analytical admittance models reproduce the actual converter response with sufficient accuracy in the frequency range of interest, the detailed electromagnetic transient (EMT) model of the Type-4 wind turbine implemented in MATLAB/Simulink is treated as a black box, and its dq -admittance is reconstructed from time-domain perturbation experiments. The measured admittance is then compared, term by term, with the analytical admittance evaluated at the same frequencies. This procedure follows the standard EMT-based frequency-scan workflow for inverter and wind-turbine impedance validation [81, 82].

Perturbation Setup in Simulink

The validation setup of the wind-turbine admittance is illustrated in Appendix C. The PCC voltage is imposed by an ideal three-phase voltage source so that the external grid behaves as a stiff bus during the measurement.

A small sinusoidal voltage perturbation is superimposed on the nominal balanced terminal voltage. The perturbation is generated directly in the dq frame and then transformed to abc using the same reference angle adopted by the analytical model:

$$\mathbf{v}_{abc}(t) = \mathbf{v}_{abc,0}(t) + \mathcal{T}^{-1}(\theta(t)) \begin{bmatrix} \tilde{v}_d(t) \\ \tilde{v}_q(t) \end{bmatrix}, \quad (4.2)$$

where $\mathcal{T}^{-1}(\theta)$ denotes the inverse Park transformation. The terminal voltage and current are sub-

sequently sampled in abc , transformed back to dq using the same angle $\theta(t)$, and stored for post-processing.

Separate d - and q -Axis Injections

The small-signal relation between the perturbed terminal voltage and the perturbed terminal current at angular frequency ω is

$$\begin{bmatrix} \tilde{i}_d \\ \tilde{i}_q \end{bmatrix} = \begin{bmatrix} Y_{dd}(j\omega) & Y_{dq}(j\omega) \\ Y_{qd}(j\omega) & Y_{qq}(j\omega) \end{bmatrix} \begin{bmatrix} \tilde{v}_d \\ \tilde{v}_q \end{bmatrix}. \quad (4.3)$$

which shows that excitation along a single axis simultaneously generates a current response on both axes, due to the cross-coupling introduced by the rotating reference frame and by the converter controllers. Consequently, the four entries of the admittance matrix cannot be reconstructed from a single experiment. Two independent tests are performed at every injection frequency $f_{\text{inj}} = \omega/(2\pi)$:

Test A — d -axis injection. A pure d -axis perturbation is applied,

$$\tilde{v}_d(t) = A \sin(2\pi f_{\text{inj}} t), \quad \tilde{v}_q(t) = 0, \quad (4.4)$$

and both current components $\tilde{i}_d$ and $\tilde{i}_q$ are recorded. The first column of the admittance matrix follows from

$$Y_{dd}(j\omega) = \frac{\tilde{I}_d(j\omega)}{\tilde{V}_d(j\omega)}, \quad Y_{qd}(j\omega) = \frac{\tilde{I}_q(j\omega)}{\tilde{V}_d(j\omega)}. \quad (4.5)$$

Test B — q -axis injection. A pure q -axis perturbation is applied,

$$\tilde{v}_d(t) = 0, \quad \tilde{v}_q(t) = A \sin(2\pi f_{\text{inj}} t), \quad (4.6)$$

and the second column is obtained as

$$Y_{dq}(j\omega) = \frac{\tilde{I}_d(j\omega)}{\tilde{V}_q(j\omega)}, \quad Y_{qq}(j\omega) = \frac{\tilde{I}_q(j\omega)}{\tilde{V}_q(j\omega)}. \quad (4.7)$$

This sequential injection strategy is the canonical way of identifying multi-input multi-output (MIMO) admittance matrices in the dq domain, and ensures that all four entries are individually accessible.

Perturbation Amplitude

The amplitude A of the injected perturbation is a compromise between two competing requirements. On one hand, it must be small enough to keep the wind turbine operating within its linear small-signal region, so that the measured admittance is consistent with the linearised analytical model. On the other hand, it must be sufficiently large for the response to dominate over numerical noise, switching ripple and PLL jitter. In line with the values reported in EMT validation guidelines [81, 82], the perturbation amplitude is chosen as

$$A = 1 \% \cdot U_0 \quad (4.8)$$

where U_0 is the PCC peak voltage derived from 4 kV L-L RMS.

$$A = 0.01 \cdot 3266V = 32.66V \quad (4.9)$$

Phasor Extraction

For each test, the steady portion of the time-domain records of $v_d(t)$, $v_q(t)$, $i_d(t)$, $i_q(t)$ is processed by a single-bin discrete Fourier transform (DFT) tuned to the injection frequency. Given a generic signal $x(t)$ sampled at the instants $\{t_k\}_{k=1}^N$, the complex phasor at f_{inj} is computed as

$$\tilde{X}(j\omega_{\text{inj}}) = \frac{2}{N} \sum_{k=1}^N x(t_k) e^{-j2\pi f_{\text{inj}} t_k}. \quad (4.10)$$

Construction of the Measured Admittance Matrix

After the sweep is completed, the four entries are assembled into the frequency-dependent measured admittance matrix

$$\mathbf{Y}_{\text{meas}}(j\omega_k) = \begin{bmatrix} Y_{dd}(j\omega_k) & Y_{dq}(j\omega_k) \\ Y_{qd}(j\omega_k) & Y_{qq}(j\omega_k) \end{bmatrix}, \quad k = 1, \dots, K. \quad (4.11)$$

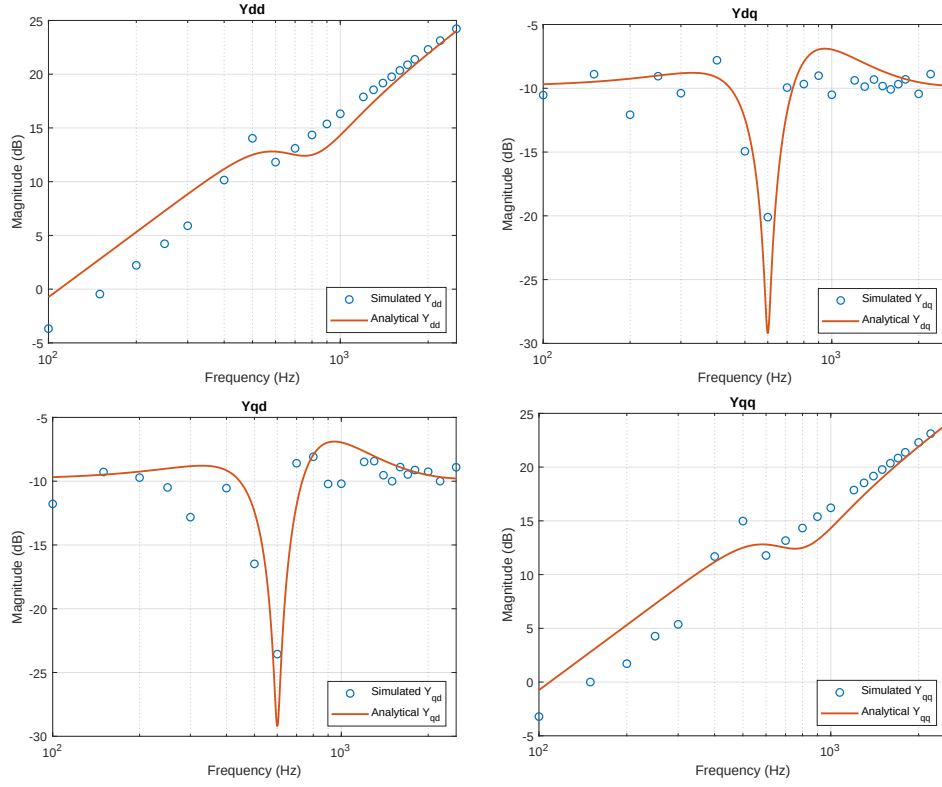


Figure 4.1: Harmonic range magnitude comparison of analytical and EMT WT model

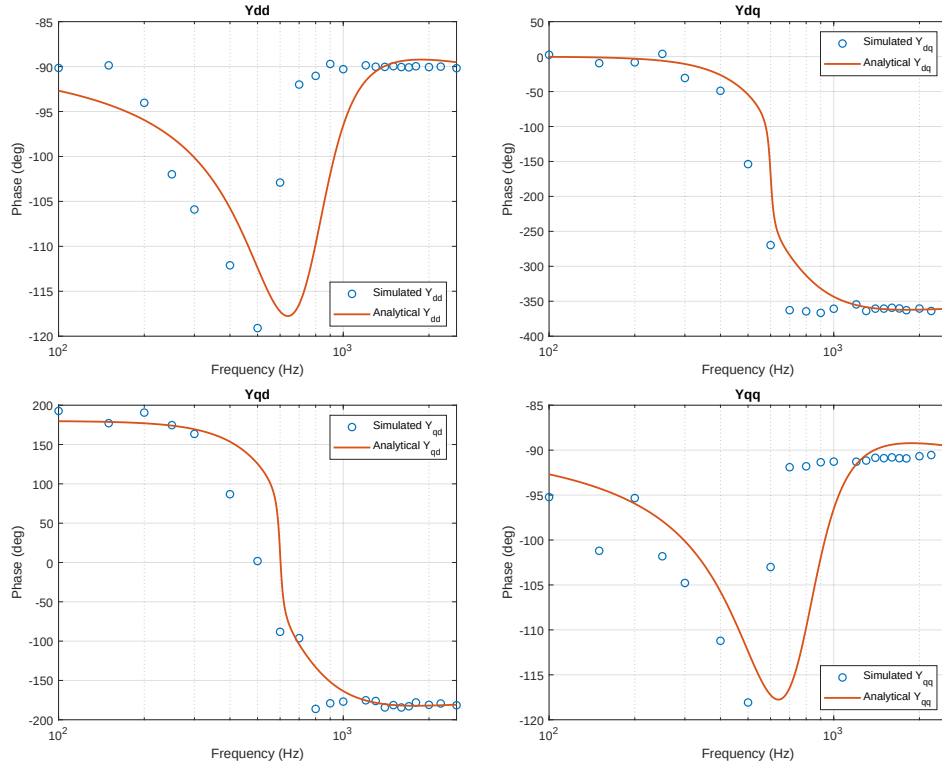


Figure 4.2: Harmonic range phase comparison of analytical and EMT WT model

The comparison between the Harmonic range analytical model and the EMT model is shown in Figure 4.1 and Figure 4.2. While most of the EMT scanned points lie around the analytical characteristic curve, small discrepancies in the validation results are expected, even when both models are implemented consistently [83]. The analytical model represents a linearized small-signal approximation around a fixed operating point, whereas the time-domain model estimates the admittance

from finite-amplitude perturbations applied to a simulated switching or averaged converter system. Therefore, the simulated admittance is affected by numerical estimation errors, finite simulation time, residual transients, discretization effects, and possible switching or sampling ripple [83]. These effects are particularly visible in frequency regions where the admittance magnitude changes rapidly or where cancellations occur between the filter dynamics, current controller, digital delay, and voltage feed-forward terms [84, 83].

The main shortcoming is that the perturbation amplitude is difficult to choose: if it is too small, the measured voltage/current response is buried in switching noise, numerical noise, and existing harmonics, so the extracted impedance becomes unreliable [83]. If it is too large, the EMT model is no longer operating in the small-signal region, meaning nonlinear effects, controller saturation, or operating-point shifts can distort the impedance result [83, 85]. Therefore, the injection method requires a compromise between good signal-to-noise ratio and preserving small-signal linearity, and this compromise may change with frequency and operating condition [83, 85].

The results can be made more accurate if the linearity is verified *a posteriori* by repeating the measurement at a few representative frequencies with $A/2$ and $2A$ and confirming that the extracted admittance entries remain essentially unchanged. If the response at the lowest frequencies is dominated by noise, the amplitude should be increased within the same band until a clean phasor can be extracted while linearity is preserved [83, 85].

Given the close agreement observed, the analytical harmonic-range admittance model is considered successfully validated and will be adopted for the impedance-based stability analysis in the remainder of this thesis.

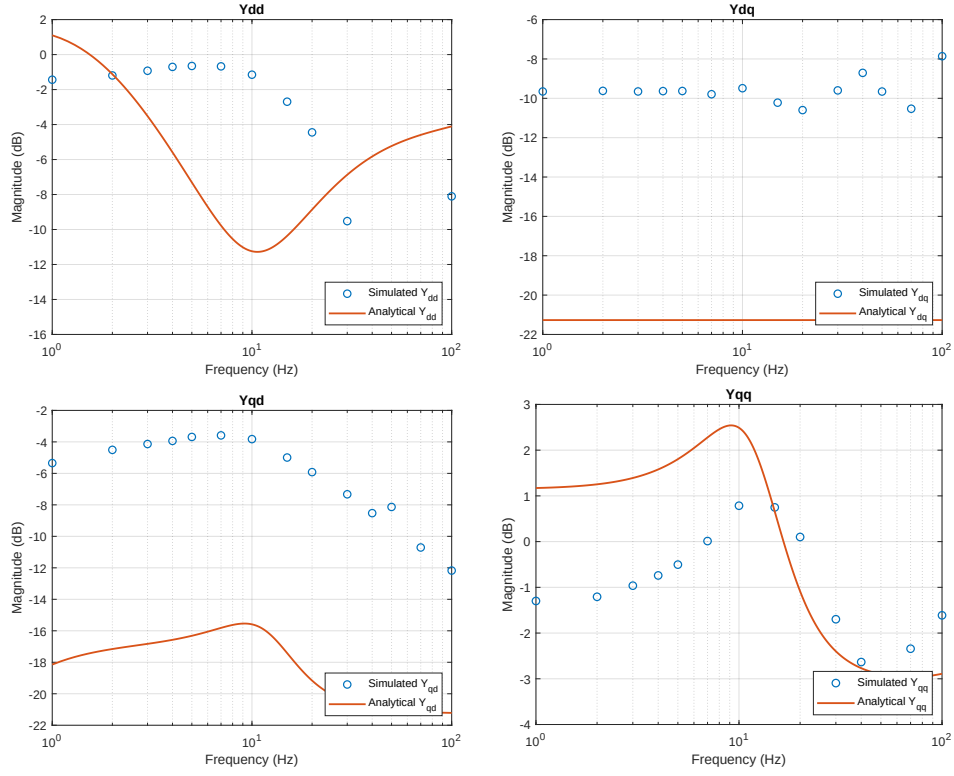


Figure 4.3: sub-/super-synchronous range magnitude comparison of analytical and EMT WT model

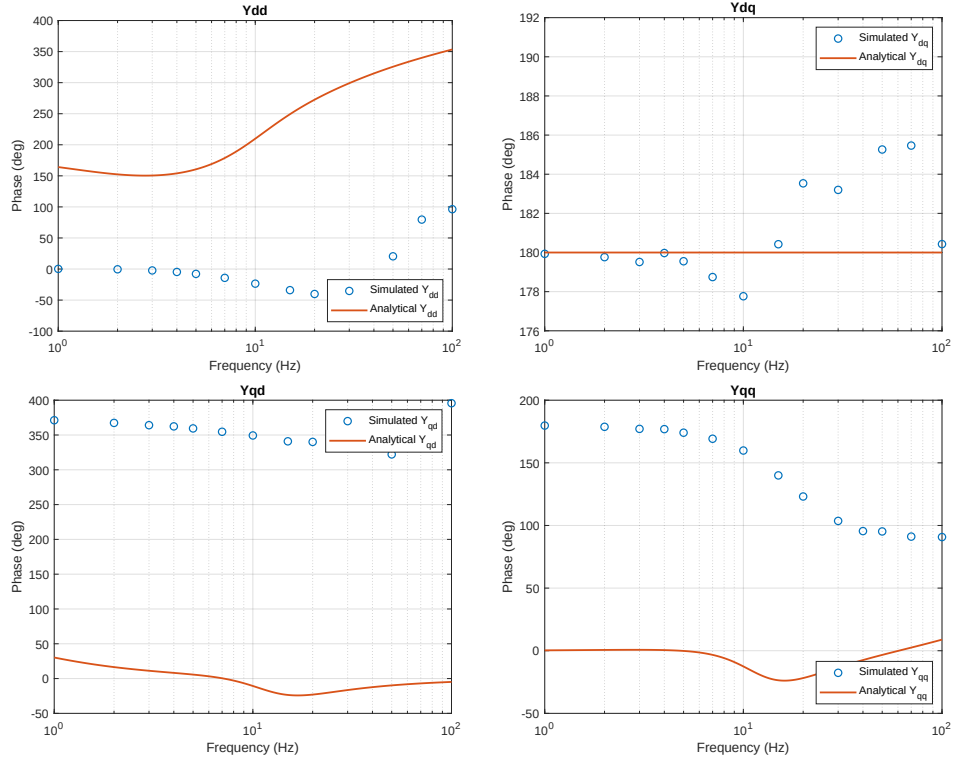


Figure 4.4: sub-/super-synchronous range phase comparison of analytical and EMT WT model

The validation in the sub-/super-synchronous range of 1–100 Hz is clearly unsuccessful, and the two models do not align. The underlying cause of this mismatch remains an open question and constitutes a possible topic for future research. To the best of the author’s knowledge, none of the existing studies on impedance-based stability analysis of Type 4 wind turbines in the sub-/super-synchronous range validates the complete analytical model (one which includes all wind-turbine components) against an

EMT model in the dq frame.

The authors of [11], from which the analytical model used here is constructed, also perform the validation by converting the 2×2 dq admittance matrix into a diagonal pn -sequence matrix, thereby eliminating the off-diagonal entries. However, discarding the off-diagonal terms in the sub-/super-synchronous range renders the resulting stability analysis incomplete and, for the present purposes, unreliable.

The work in [86], which also models the complete impedance of a Type 4 wind turbine, validates its model from the DC side rather than from the AC terminals at the GSC output. The perturbations are injected at the DC side, and the resulting currents and voltages flowing into the MSC and into the GSC are measured in order to extract $Y_{L,dc}$ — the parallel combination of the PMSG/MSC admittance and the DC-link capacitor, $Y_{L,dc}(s) = Y_{dc}^{1,2}(s) + sC_{cap}$ — and the source impedance $Z_{s,dc}(s) = Z_{dc}^{3,4}(s)$, as shown in Figure 4.5.

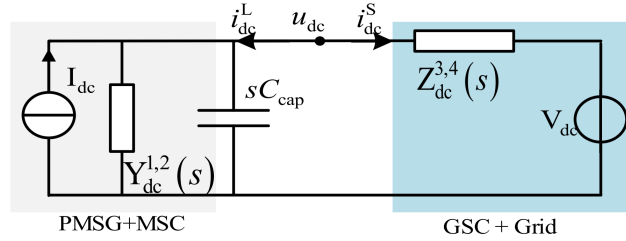


Figure 4.5: Equivalent MSC admittance and GSC+Grid Impedance seen from the DC Link

Applying the same method, the validation was carried out from the DC side. The results are shown in Figure 4.6

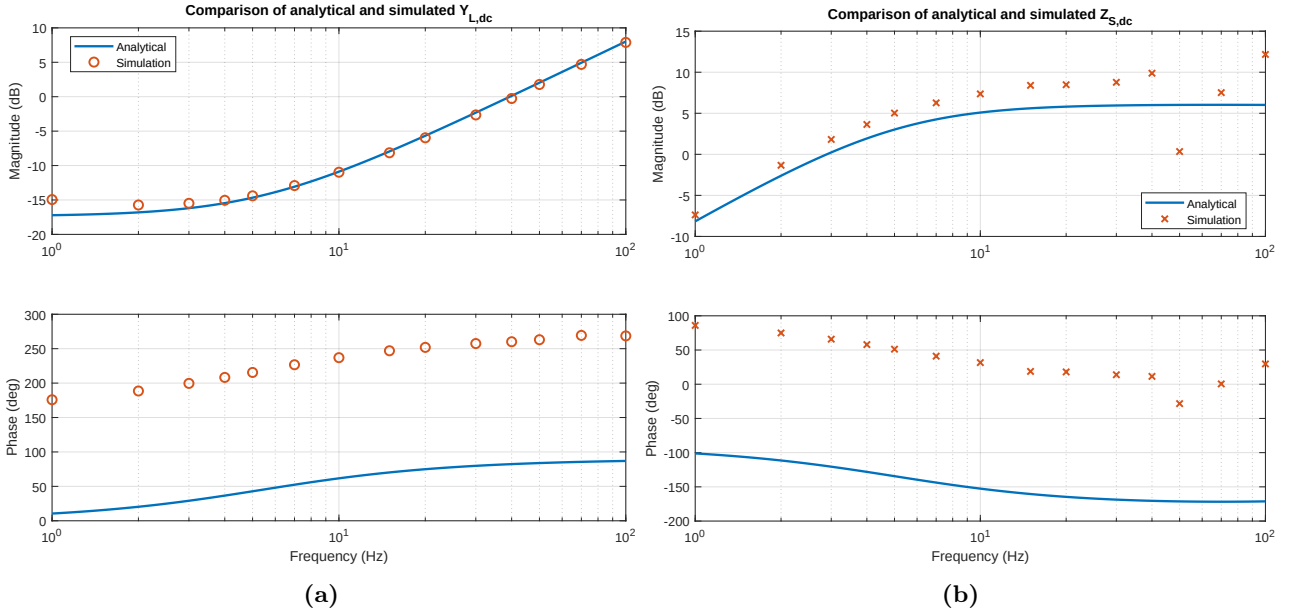


Figure 4.6: sub-/super-synchronous range comparison of $Y_{L,dc}$ and $Z_{s,dc}$

The analytical and simulated models show very close agreement, and the DC-side admittance validation is successful. The 180° phase difference observed between the two curves originates from the sign convention adopted by the Simulink implementations and does not affect the validity of the comparison. The alignment confirms that the analytical model is correctly derived in accordance with the

literature; nevertheless, no further conclusions can be drawn directly from this DC-side result since it does *not* in anyway automatically validates the the AC-terminal admittance $\mathbf{Y}_{acW}$. As shown in [86], under marginal stability conditions, the DC-side analysis is equivalent to the AC-side analysis, which can only be performed on a single wind turbine. However, since the objective of this work is to perform analysis of the entire wind farm, a DC-side analysis of an individual turbine is not useful for the present study.

For this reason, the analysis carried out in the remainder of this thesis is limited to the harmonic frequency range. Within this range, The 2×2 dq admittance matrices exhibit a symmetric structure $Y_{dd} = Y_{qq}$ and $Y_{dq} = -Y_{qd}$. As a consequence, when the four entries are plotted, the only difference between Y_{dq} and Y_{qd} is a 180° phase shift, while Y_{dd} and Y_{qq} are identical. For this reason, only Y_{dd} and Y_{dq} are shown in the remainder of this thesis whenever a 2×2 dq matrix with this structure is reported.

4.2 Transformers

As mentioned in Section 2.1, transformers appear at three locations in the system: the wind turbine step-up transformer, the offshore platform transformer that connects the array grid to the export cable, and the onshore HVAC step-down transformer that feeds the AC grid.

Consistent with the literature review, each transformer is represented by the 2×2 dq impedance introduced in Section 3.3, equation (3.36) (repeated below for convenience):

$$\mathbf{Z}_{T,dq}(s, f) = \begin{bmatrix} R_l(f) + sL_l & -\omega_s L_l \\ \omega_s L_l & R_l(f) + sL_l \end{bmatrix},$$

with

$$R_l(f) = R_{dc} [1 + A h^B], \quad h = \frac{f}{f_1}.$$

The coefficients of the frequency-dependent resistance are set to $A = 0.1$ and $B = 1.5$ for all three transformers since they belong to the same broad class of multi-MVA power transformers operating at high voltage [2, 87].

Table 4.5 lists the parameter values used for each transformer, and Figure 4.7 shows the corresponding admittance Bode plots. As seen in Figure 4.7, the only noticeable difference between the three transformer plots is the increase in admittance magnitude with the transformer rating.

Table 4.5: Transformers Parameters

Transformer	Voltage Ratio (kV)	Rated Power (MVA)	L_l (H)	R_{dc} (Ω)	Source
Wind Turbine	4/66	15	0.25×10^{-3}	3.8788×10^{-3}	[2, 87]
Offshore substation	66/220	400	4.16×10^{-3}	0.0198	[2, 87]
Onshore substation	220/380	450	132.8×10^{-3}	0.6419	[2, 87]

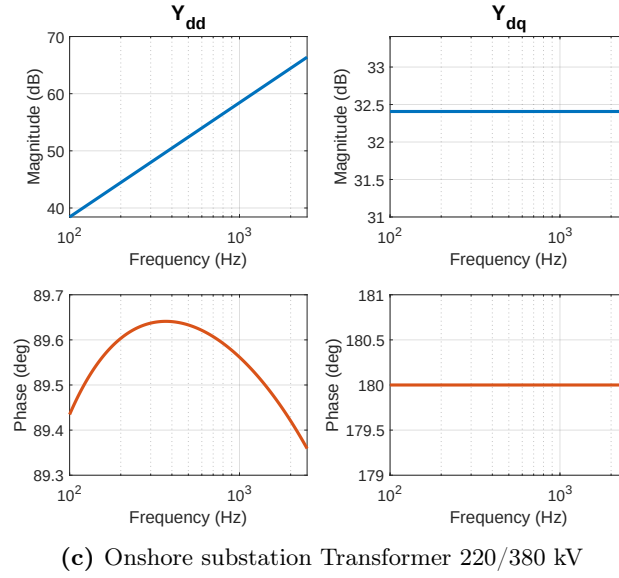
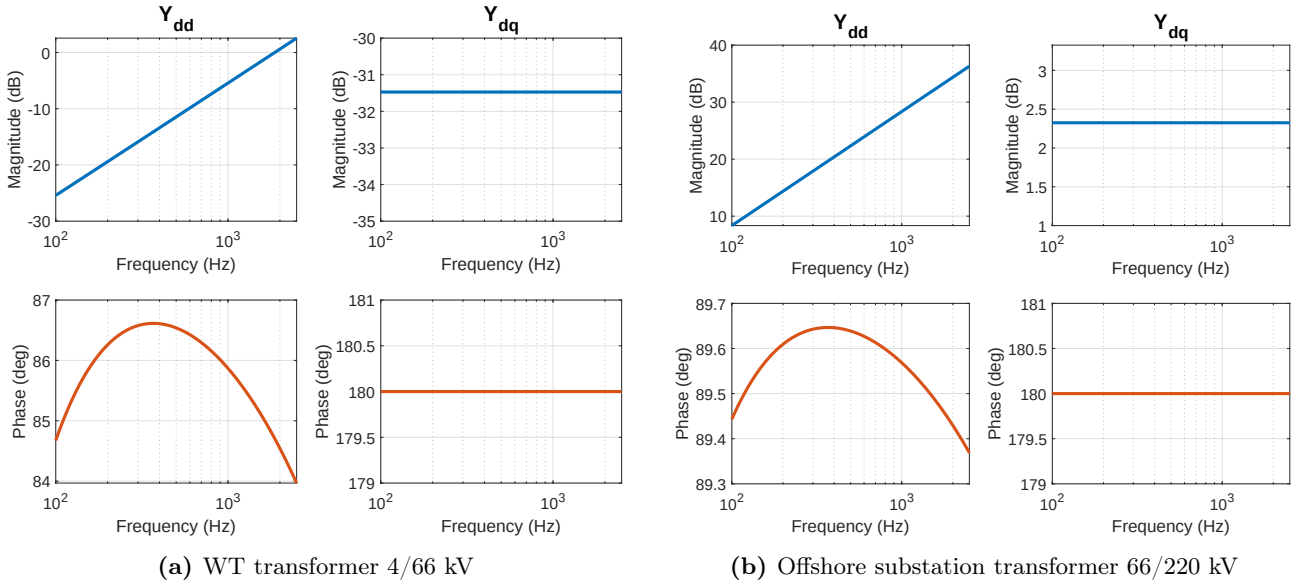


Figure 4.7: Admittance plots of Modelled Transformers

4.3 Inter-array cables

The inter-array cables of the offshore wind farm are short enough that wave-propagation effects can be neglected (average length ≈ 2 km [88]), so the nominal π -section model introduced in Section 3.4.1 suffices. Expressed in the dq frame, the model relates sending- and receiving-end currents and voltages through the 4×4 admittance matrix

$$\mathbf{Y}_{c,dq}^{\text{array}} = \begin{bmatrix} \mathbf{Y}_{s,dq} + \frac{1}{2}\mathbf{Y}_{sh,dq} & -\mathbf{Y}_{s,dq} \\ -\mathbf{Y}_{s,dq} & \mathbf{Y}_{s,dq} + \frac{1}{2}\mathbf{Y}_{sh,dq} \end{bmatrix}, \quad (4.12)$$

with series and shunt blocks

$$\mathbf{Y}_{s,dq}(s) = \begin{bmatrix} R + sL & -\omega_s L \\ \omega_s L & R + sL \end{bmatrix}^{-1}, \quad \mathbf{Y}_{sh,dq}(s) = \begin{bmatrix} sC & -\omega_s C \\ \omega_s C & sC \end{bmatrix}. \quad (4.13)$$

The R , L , and C values used in this work are taken from [89] and correspond to a three-core

armoured XLPE submarine cable at 66 kV typical of offshore wind-farm collection systems:

$$R = 0.04 \, \Omega, \quad L = 5.6 \times 10^{-4} \, \text{H}, \quad C = 2.38 \times 10^{-7} \, \text{F}.$$

The sixteen entries of $\mathbf{Y}_{c,dq}^{\text{array}}$ are not independent. The dq rotational symmetry fixes the internal structure of each 2×2 block, while the reciprocal, terminally symmetric nature of the π -section enforces $\mathbf{Y}_{22} = \mathbf{Y}_{11}$ and $\mathbf{Y}_{21} = \mathbf{Y}_{12}$. Introducing the four scalars

$$\mathbf{Y}_{11} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}, \quad \mathbf{Y}_{12} = \begin{bmatrix} c & d \\ -d & c \end{bmatrix},$$

the full admittance matrix reduces to

$$\mathbf{Y}_{c,dq}^{\text{array}} = \begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ c & d & a & b \\ -d & c & -b & a \end{bmatrix}. \quad (4.14)$$

The frequency response of the cable is therefore completely characterised by the four transfer functions a , b , c , and d , identified respectively with $\mathbf{Y}_{c,dq}^{\text{array}}(1,1)_{dd}$, $\mathbf{Y}_{c,dq}^{\text{array}}(1,1)_{dq}$, $\mathbf{Y}_{c,dq}^{\text{array}}(1,2)_{dd}$, and $\mathbf{Y}_{c,dq}^{\text{array}}(1,2)_{dq}$. Only these elements are reported in Figure 4.8; the remaining twelve follow by sign changes and block repetition and carry no additional information.

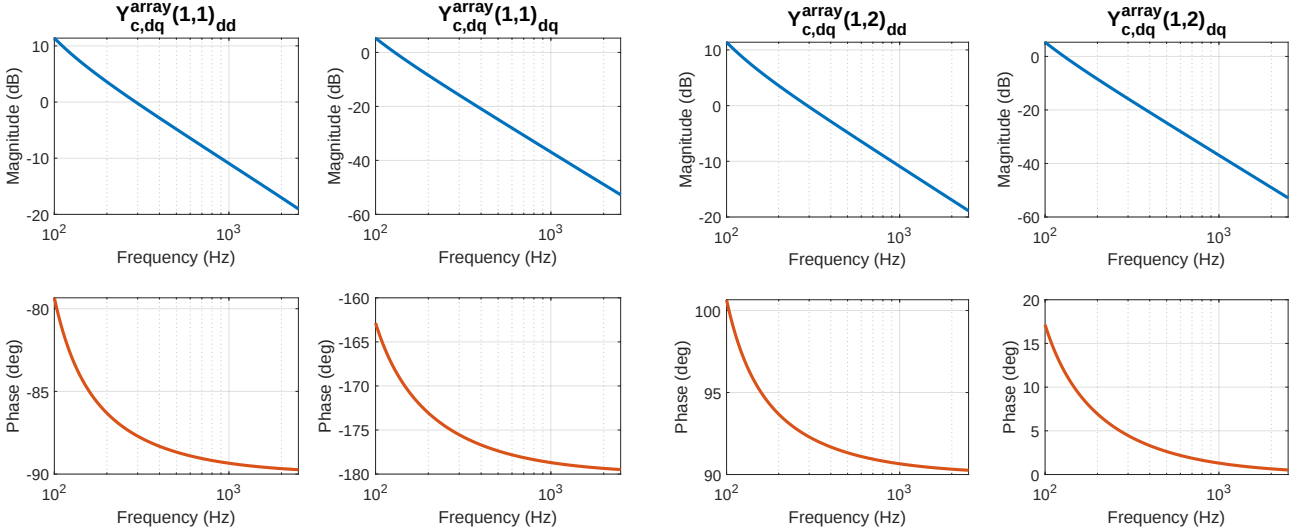


Figure 4.8: Admittance bode plot of $\mathbf{Y}_{c,dq}^{\text{array}}(1,1)_{dd}$, $\mathbf{Y}_{c,dq}^{\text{array}}(1,1)_{dq}$, $\mathbf{Y}_{c,dq}^{\text{array}}(1,2)_{dd}$, and $\mathbf{Y}_{c,dq}^{\text{array}}(1,2)_{dq}$

As seen in Figure 4.8, across the band 100 Hz – 2.5 kHz, the cable is well below its π -section resonance, so the response is dominated by the series inductor. The effect of these cable blocks once they are connected to the corresponding wind turbines is examined in Section 4.6.

4.4 HVAC cable

As mentioned in Section 2.1, the Energy Island contains a 55 km, 220 kV HVAC export cable. The modeling of long HVAC cables is discussed in Section 3.4.2. Because of its length, the cable behaves as a distributed-parameter line and, owing to wave-propagation effects, exhibits a series of

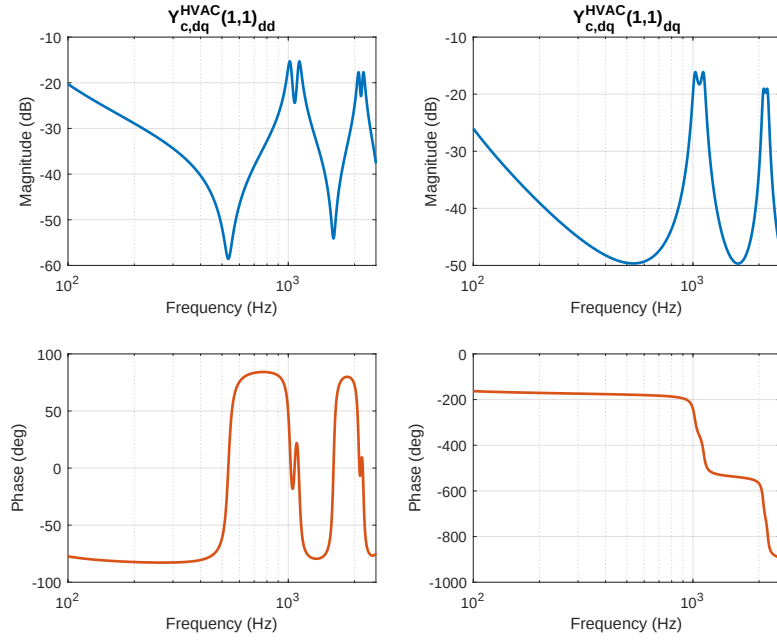
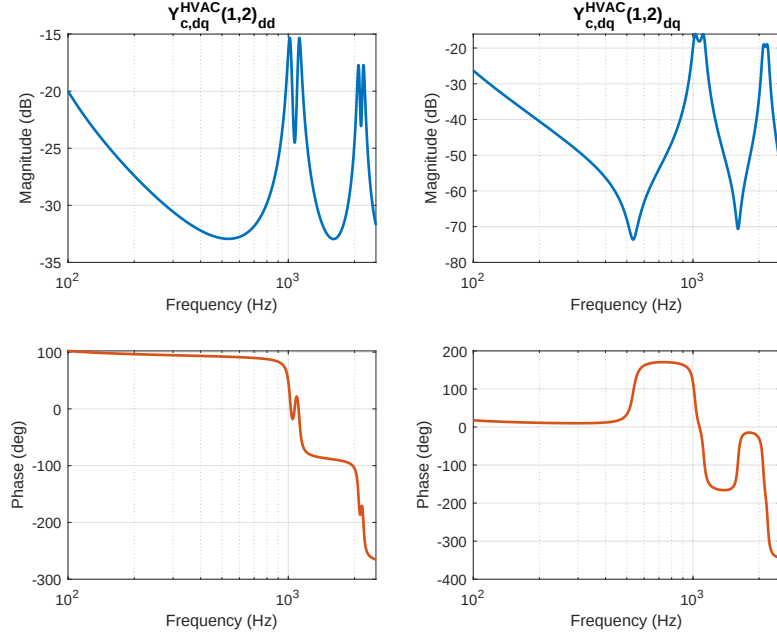
resonances and anti-resonances within the harmonic frequency range [10]. For such a system, the model of Equation 3.45 is used, which is again a 4×4 matrix:

$$\mathbf{Y}_{ij}^{dq} = \begin{bmatrix} \frac{1}{2}(Y_{ij}^+ + Y_{ij}^-) & \frac{j}{2}(Y_{ij}^+ - Y_{ij}^-) \\ -\frac{j}{2}(Y_{ij}^+ - Y_{ij}^-) & \frac{1}{2}(Y_{ij}^+ + Y_{ij}^-) \end{bmatrix}, \quad i, j \in \{1, 2\}.$$

The details of the elements of this equation are given in Section 3.4.2. The values of R' , L' , and C' used in this equation are taken from [10] and are listed in Table 4.6. This 4×4 matrix follows the same symmetry as the array-cable matrix in Equation 4.14, so only four entries carry independent information. Figure 4.9 shows the admittance plots of these four elements of $\mathbf{Y}_{c,dq}^{\text{HVAC}}$.

Symbol	Description	Value
R'	Series resistance per unit length	$0.03 \, \Omega/\text{km}$
L'	Series inductance per unit length	$0.38 \, \text{mH}/\text{km}$
C'	Shunt capacitance per unit length	$0.19 \, \mu\text{F}/\text{km}$
l	Length	$55 \, \text{km}$

Table 4.6: 220 kV HVAC cable parameters


 (a) $\mathbf{Y}_{c,dq}^{\text{HVAC}}(1,1)_{dd}$ and $\mathbf{Y}_{c,dq}^{\text{HVAC}}(1,1)_{dq}$

 (b) $\mathbf{Y}_{c,dq}^{\text{HVAC}}(1,2)_{dd}$ and $\mathbf{Y}_{c,dq}^{\text{HVAC}}(1,2)_{dq}$
Figure 4.9: Admittance Bode plots of $\mathbf{Y}_{c,dq}^{\text{HVAC}}$

As seen in Figure 4.9, all four entries of $\mathbf{Y}_{c,dq}^{\text{HVAC}}$ exhibit a series of regularly spaced resonances and anti-resonances, with the first notch appearing around 500 – 600 Hz, the first peak near 1 kHz, and additional peaks above 2 kHz. This comb-like pattern is the characteristic signature of a distributed-parameter line: the 55 km cable is electrically long in the harmonic range, so standing-wave reflections between its two terminals produce a sequence of resonances whose frequencies are set by the cable length and the wave propagation velocity.

4.5 Grid Thevenin Equivalent

The last block required to close the modeling of the individual components is the external AC grid. Following the impedance-based stability framework reviewed in Section 3.5.1, the grid is represented by its Thevenin equivalent in the dq frame, which yields the 2×2 matrix of equation (3.47), repeated here for convenience:

$$\mathbf{Z}_{g,380}(s) = \begin{bmatrix} R_g + sL_g & -\omega_s L_g \\ \omega_s L_g & R_g + sL_g \end{bmatrix},$$

where $\mathbf{Z}_{g,380}(s)$ denotes the Thevenin impedance referred to the 380 kV onshore bus. The grid is modelled as a strong and inductive equivalent with $X/R \approx 10$, and the corresponding resistance and inductance, derived from the short-circuit power published for the 380 kV onshore bus by Elia [90], are

$$R_g = 0.72 \, \Omega, \quad L_g = 22.9 \times 10^{-3} \, \text{H}.$$

The PCC adopted in this thesis lies, however, at the 220 kV side of the onshore substation rather than at the 380 kV bus. The onshore 220/380 kV substation transformer modelled in Section 4.2 must therefore be placed in series with $\mathbf{Z}_{g,380}(s)$ in order to obtain the equivalent impedance actually seen from the PCC:

$$\mathbf{Z}_g(s) = \mathbf{Z}_{T,\text{onshore}}(s) + \mathbf{Z}_{g,380}(s). \quad (4.15)$$

For simplicity, the combined equivalent in equation (4.15) is referred to as the grid impedance throughout the remainder of this thesis. Figure 4.10 reports its Bode plot.

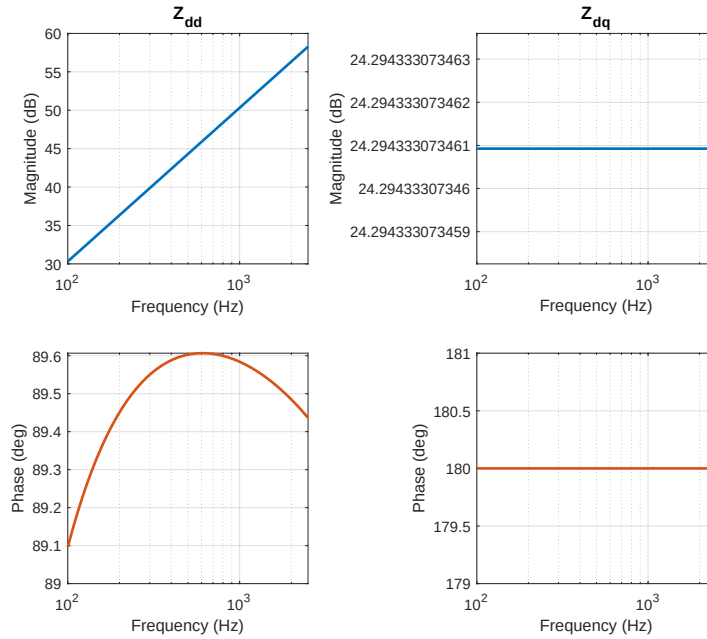


Figure 4.10: Bode plot of the equivalent grid impedance $\mathbf{Z}_g(s)$ seen from the PCC

4.6 Network Modeling

The full wind farm of MOG II Phase 1, as described in Section 2.1, has a rated power of 700 MW. This 700 MW block is arranged into two 350 MW sub-blocks, with five strings per sub-block. Each string consists of five 14 MW wind turbines. An overview of one sub-block and its connection to the onshore grid is shown in Figure 4.11.

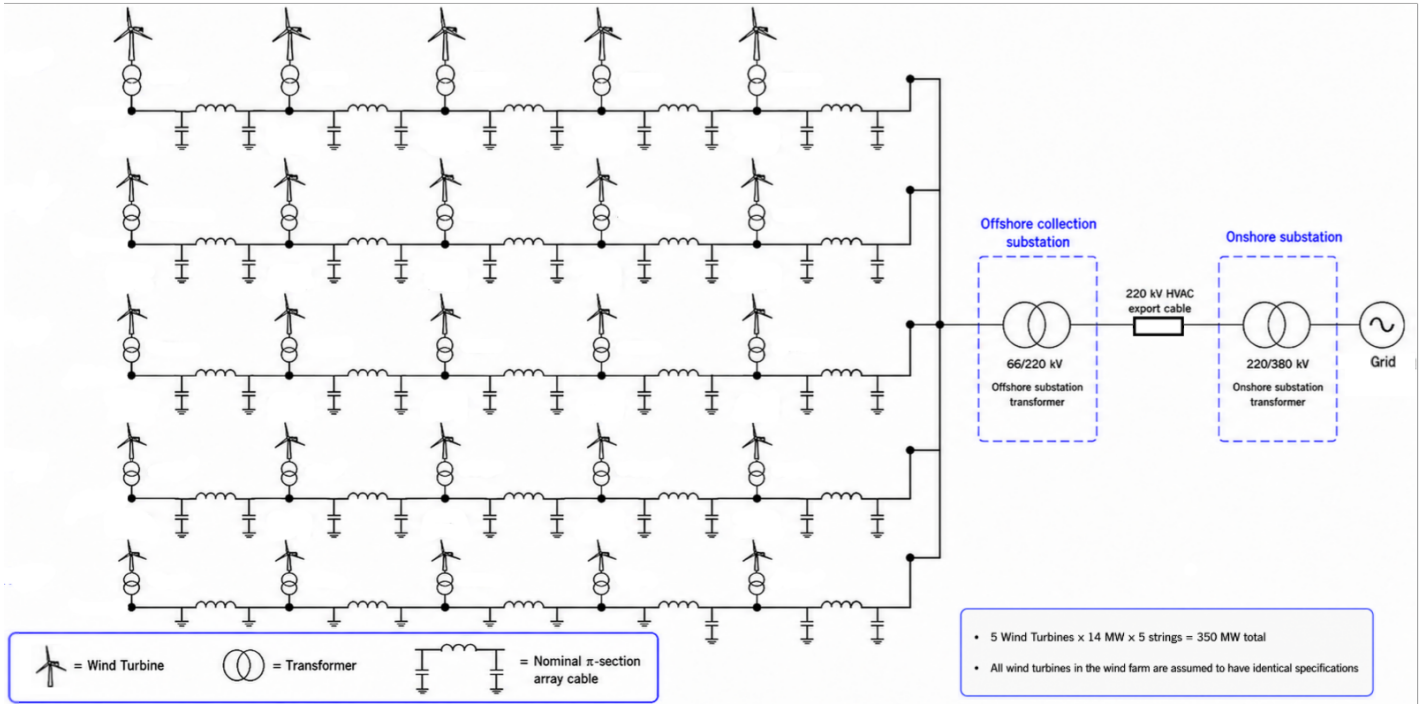


Figure 4.11: Overview of the 350 MW sub-block

The following assumptions are made for the wind farm:

- the wind farm operates in grid-following mode;
- every wind turbine has the same specifications and parameters;
- the wind speed is assumed constant (no variable wind speed);
- all 5 strings are connected in parallel at a single offshore collector bus.

The POC (Point of Connection), is taken as the point right after the offshore collection substation transformer while the PCC (Point of Common Coupling) lies right before the onshore substation transformer.

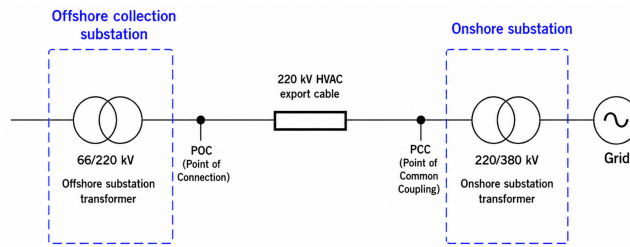


Figure 4.12: POC and PCC visualisation

4.6.1 Wind Turbine with Step-Up Transformer

The first step is to connect each wind turbine to its step-up transformer in series. This is done by adding the transformer impedance in series with the wind turbine impedance. Figure 4.13 shows the resulting admittance. Since the transformer model includes the leakage inductance, the WT filter is no longer an LC filter but an LCL filter.

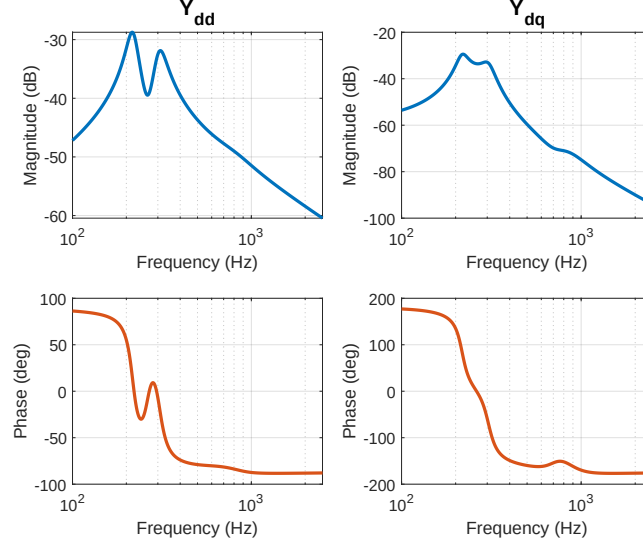


Figure 4.13: Admittance of the wind turbine together with its transformer

As seen in Figure 4.13, the resonant behaviour of the LCL filter is visible in the frequency range from 150 Hz to 450 Hz. These resonances are of particular importance for the stability analysis carried out later in this thesis.

4.6.2 String Connection

The string is assembled from its individual components using the nodal admittance method, and the internal buses are then eliminated by Kron reduction to obtain the equivalent 2×2 dq admittance seen from the string's external terminal. The full procedure — the bus stamping rules for the inter-array cables and wind turbines, and the Schur-complement form of the Kron reduction — is detailed in Appendix D.

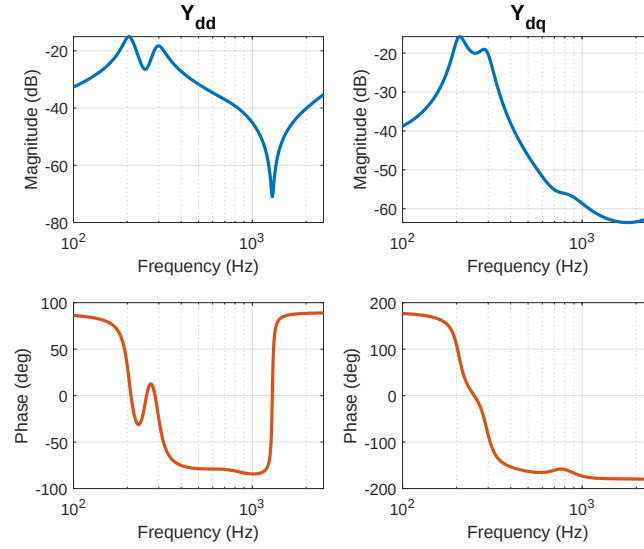


Figure 4.14: Admittance of one wind turbine string

Applied to one string of five wind turbines and five nominal π -sections, the procedure yields the equivalent dq admittance shown in Figure 4.14. The combination not only increases the admittance magnitude but also creates an anti-resonance around 1200 Hz in Y_{dd} .

4.6.3 Wind Farm Aggregation

The procedure of combining all strings into a single equivalent admittance seen from the POC is referred to as wind farm aggregation. The following standard assumptions are adopted:

- the five strings are electrically identical, i.e. same number of turbines, same inter-array cable lengths, and same operating points;
- the strings are electrically decoupled, so that any mutual coupling or unbalance between them is neglected.

Under these assumptions, the admittance of one string is multiplied by the number of strings, $N_s = 5$, which is equivalent to the parallel connection of five identical strings at the offshore collector bus. The resulting block is then placed in series with the 66/220 kV offshore substation transformer. Figure 4.15 shows the resulting dq admittance $\mathbf{Y}_{fc}$, which represents the entire 350 MW offshore sub-block seen from the POC. Compared with the single-string admittance of Figure 4.14, the aggregated admittance in Figure 4.15 shows the dominant low-frequency resonance shifting, while an additional resonance appears around 1.5 - 1.7 kHz. Both effects are introduced by the offshore collector transformer, whose leakage inductance is now placed in series with the parallel combination of the five strings and forms a new LC interaction with the capacitive part of the aggregated network.

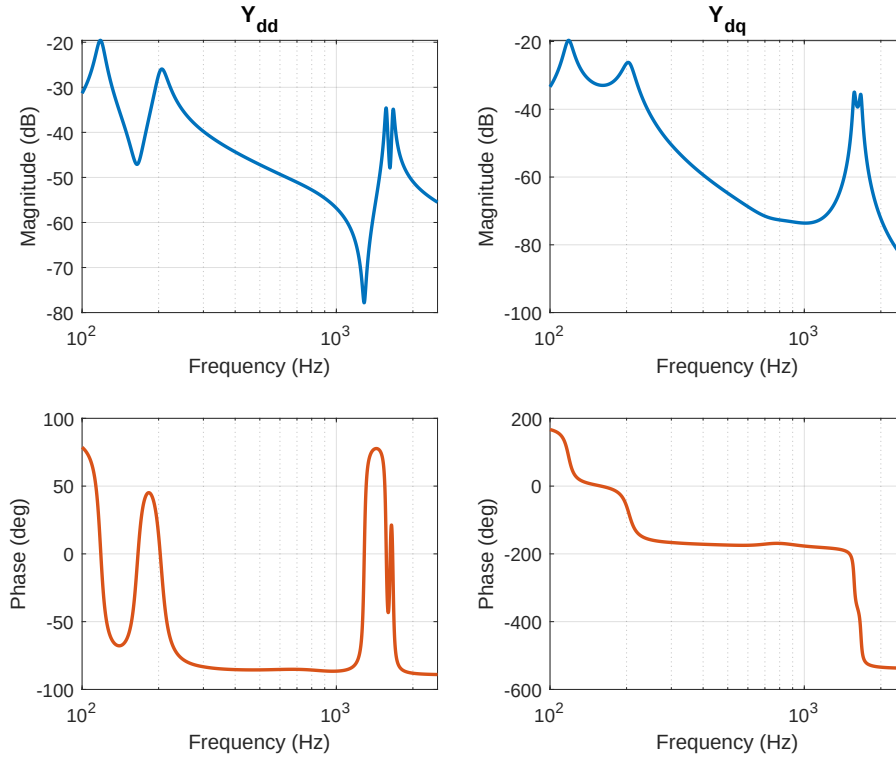


Figure 4.15: Aggregated admittance of the five offshore strings combined with the offshore collector substation transformer, seen from the POC

4.6.4 PCC Equivalent Admittance of the Wind Farm

The HVAC export cable is a two-port element described by the 4×4 dq admittance matrix derived in Section 3.4.2. The cable is stamped between two buses: the aggregated wind farm admittance of Section 4.6.3 (POC) and the onshore bus, which is taken here as the PCC where the offshore wind farm meets the onshore substation.

Partitioning the HVAC cable admittance into its 2×2 blocks,

$$\mathbf{Y}_{c,dq}^{\text{HVAC}} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix},$$

where Port 1 denotes the onshore (PCC) side and Port 2 the offshore (POC) side, the bus admittance matrix obtained after stamping is

$$\mathbf{Y}_{bus} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} + \mathbf{Y}_{fc} \end{bmatrix}.$$

Reducing the bus by Kron reduction [91] yields the equivalent admittance of the full wind farm seen from the PCC:

$$\mathbf{Y}_{WF,dq} = \mathbf{Y}_{11} - \mathbf{Y}_{12}(\mathbf{Y}_{22} + \mathbf{Y}_{fc})^{-1}\mathbf{Y}_{21}. \quad (4.16)$$

Figure 4.16 shows the resulting admittance.

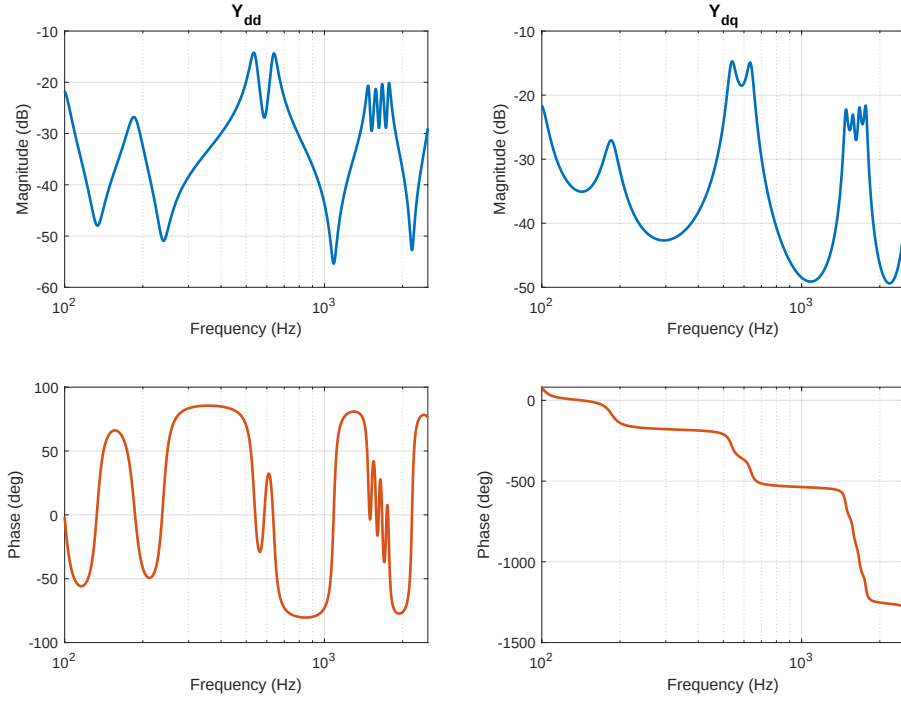


Figure 4.16: Admittance of the offshore wind farm combined with the HVAC export cable, seen from the PCC

Compared with the offshore admittance of Figure 4.15, the admittance seen at the PCC in Figure 4.16 inherits the comb of resonances and anti-resonances already identified in Figure 4.9, now superimposed on the lumped offshore-side dynamics. The exact locations of the peaks shift because the cable's offshore terminal is no longer measured in isolation but is loaded by the wind-farm equivalent admittance, whose admittance differs from the cable's characteristic admittance and therefore redistributes the standing-wave pattern.

The admittance $\mathbf{Y}_{WF,dq}$ obtained in (4.16) is the final dq admittance of the full offshore wind farm seen from the PCC, and is the quantity on which the stability analysis will be based.

4.7 Stability and Robustness Assessment of the Baseline Model

The full network model assembled in the previous sections, with its default parameter set, is referred to hereafter as the *baseline model*. The objective of this section is to assess its small-signal stability and robustness at the Point of Common Coupling (PCC) by applying the four indicators introduced in Sections 3.6.1, 3.6.2, 3.6.3 and 3.6.4, namely the passivity index, the Generalized Nyquist Criterion, the vector margin and the disk margin. An impedance-crossover plot is also included as a preliminary diagnostic. All indicators are evaluated on the same harmonic band $100 \text{ Hz} \leq f \leq 2.5 \text{ kHz}$, which is the range over which the analytical wind-turbine admittance was validated against the EMT model in Section 4.1.3. The minor-loop gain analysed throughout this section is

$$\mathbf{L}(s) = \mathbf{Z}_g(s) \mathbf{Y}_{WF}(s), \quad (4.17)$$

with $\mathbf{Y}_{WF}(s)$ the wind-farm admittance at the PCC derived in equation (4.16) and $\mathbf{Z}_g(s)$ the grid Thevenin impedance defined in Section 3.5.1.

4.7.1 Generalized Nyquist Criterion

The most fundamental stability indicator employed in this work is the Generalized Nyquist Criterion (GNC) reviewed in Section 3.6.2. The two eigenloci $\lambda_{1,2}(j\omega)$ of the minor-loop gain $\mathbf{L}(j\omega)$ in equation (4.17) are traced in the complex plane in Figure 4.17, together with a zoom on the critical point $(-1 + j0)$.

Neither eigenlocus encircles the critical point, so according to the GNC the baseline system is small-signal stable. The zoom in Figure 4.17, however, shows that both eigenloci pass in close proximity to $(-1 + j0)$, which is a first indication that the system, although nominally stable, may be only weakly damped at certain frequencies and therefore lack the robustness margin required for reliable operation in the presence of the unavoidable parametric drifts of an offshore network. As emphasised in Section 3.6.2, the GNC yields a binary stable/unstable verdict only and must be complemented by continuous robustness indicators when, as is the case here, the eigenloci graze the critical point.

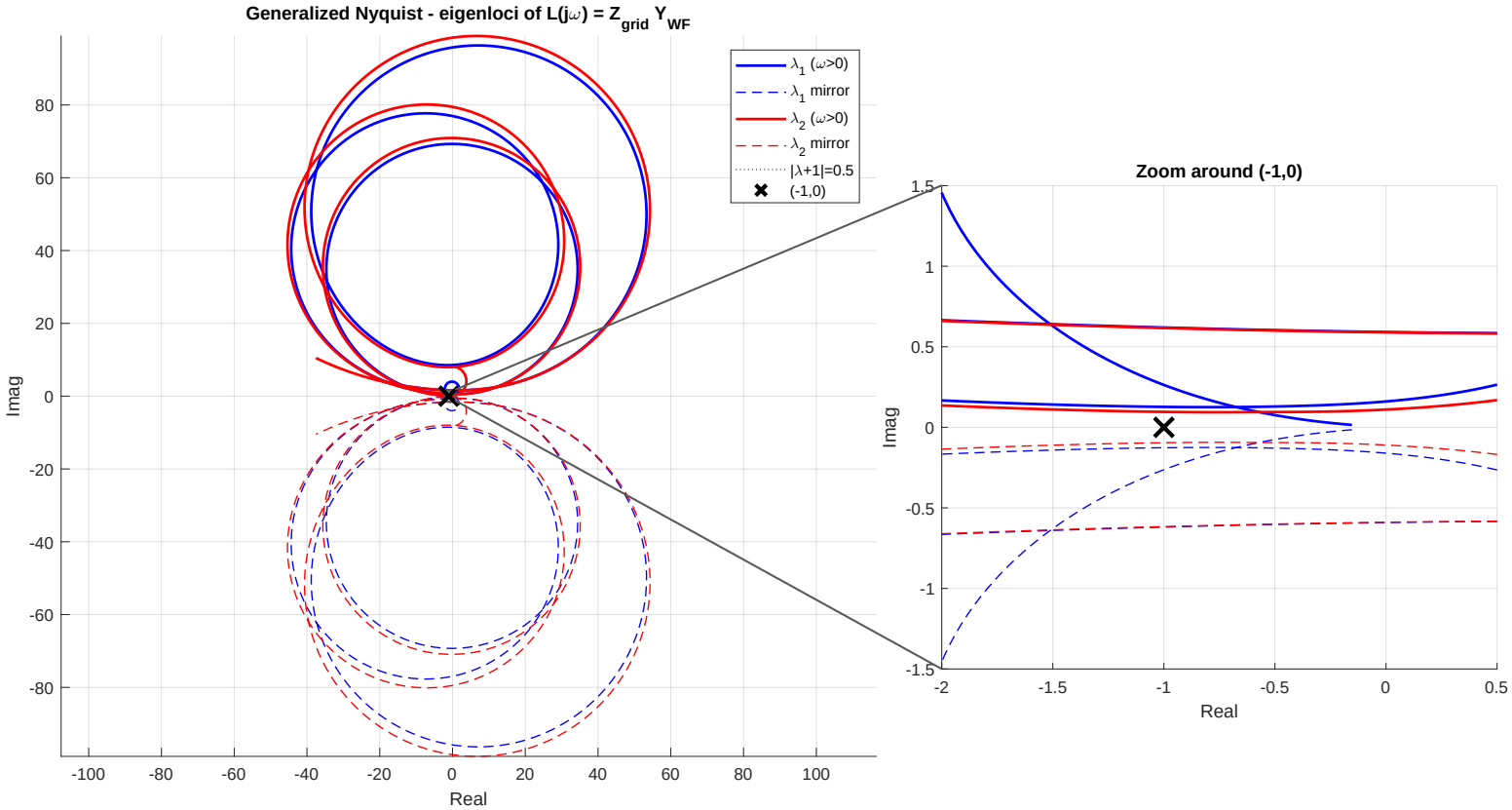


Figure 4.17: Eigenloci of the minor-loop gain $\mathbf{L}(j\omega)$ of the baseline model in the complex plane, with a zoom on the critical point $(-1 + j0)$.

4.7.2 Impedance Crossover Diagnostic

A first qualitative indication of the frequency regions in which the wind farm and the grid may interact can be obtained by plotting the wind-farm impedance $\mathbf{Z}_{WF}(j\omega) = \mathbf{Y}_{WF}^{-1}(j\omega)$ and the grid impedance $\mathbf{Z}_g(j\omega)$ on the same Bode diagram. In SISO impedance-based stability analysis [37], the intersections between the source and load impedance magnitudes correspond to the gain-crossover frequencies of the minor-loop gain and therefore identify the bands in which the relative magnitude of the two subsystems becomes comparable.

In the present work the system is represented as a dq MIMO problem, and intersections observed on individual entries Z_{dd} or Z_{dq} do not, on their own, support a rigorous stability conclusion. They nevertheless remain useful as a preliminary diagnostic, since they highlight the frequency bands in which $|\mathbf{Z}_{WF}|$ and $|\mathbf{Z}_g|$ are of the same order of magnitude and where stronger dynamic interaction can therefore be expected.

Figure 4.18 shows the dd and dq entries of $\mathbf{Z}_{WF}(j\omega)$ together with the corresponding entries of $\mathbf{Z}_g(j\omega)$. For the dd entry, the two magnitudes become comparable mainly in two regions: the low-frequency band approximately between 100 Hz and 310 Hz, and the resonance cluster located around 1 kHz. For the dq entry, the crossings appear lower in magnitude, in the intermediate range around 370 and 850 Hz and again in the cluster above 1.2 kHz, suggesting in addition the presence of cross-coupling between the direct and quadrature axes. The two bands, around 100–400 Hz and around 1 kHz, will reappear in every indicator presented in the remainder of this section, which confirms the diagnostic value of the impedance-crossover plot in the present setting.

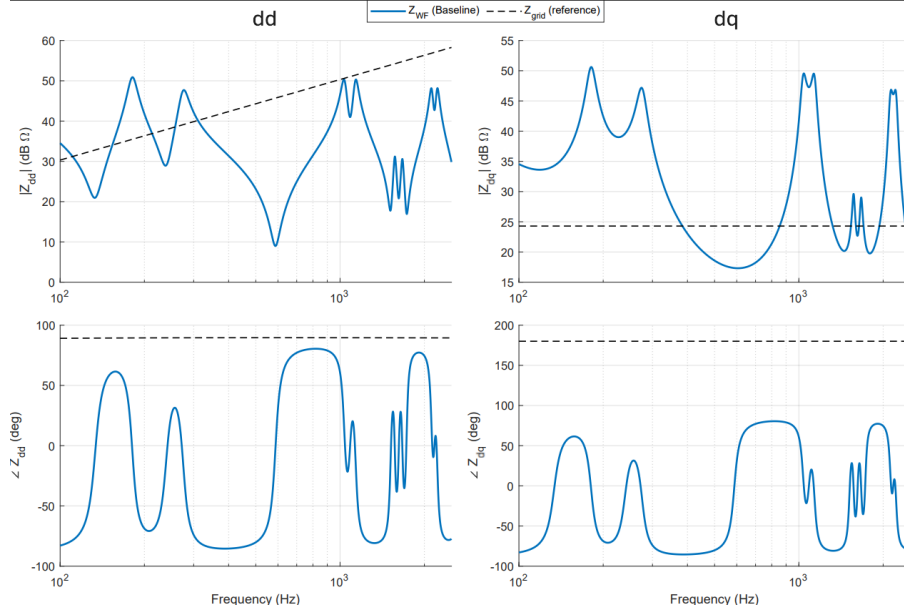


Figure 4.18: Bode plots of the wind-farm impedance $\mathbf{Z}_{WF}(j\omega)$ and the grid impedance $\mathbf{Z}_g(j\omega)$ for the dd and dq entries.

4.7.3 Passivity Analysis

The passivity-based stability criterion introduced in Section 3.6.1 is applied to the wind-farm admittance $\mathbf{Y}_{WF}(j\omega)$ obtained in equation (4.16). Following equation (3.57), the passivity index of the wind farm is computed as the minimum eigenvalue of the Hermitian part of the admittance,

$$\mathcal{P}_{Y_{WF}}(\omega) \triangleq \lambda_{\min} \left\{ \frac{1}{2} (\mathbf{Y}_{WF}(j\omega) + \mathbf{Y}_{WF}(j\omega)^H) \right\}. \quad (4.18)$$

Figure 4.19 shows $\mathcal{P}_{Y_{WF}}(\omega)$ evaluated over the harmonic range. The passivity index remains strictly positive across the whole band, which proves that $\mathbf{Y}_{WF}(j\omega)$ is passive at every frequency in the scan. Because the grid impedance $\mathbf{Z}_g(j\omega)$ is itself passive (an RL Thevenin equivalent in series with a passive transformer is passive by construction [58]), the passivity theorem guarantees that the baseline interconnection is small-signal stable, in agreement with the GNC verdict of Section 4.7.1.

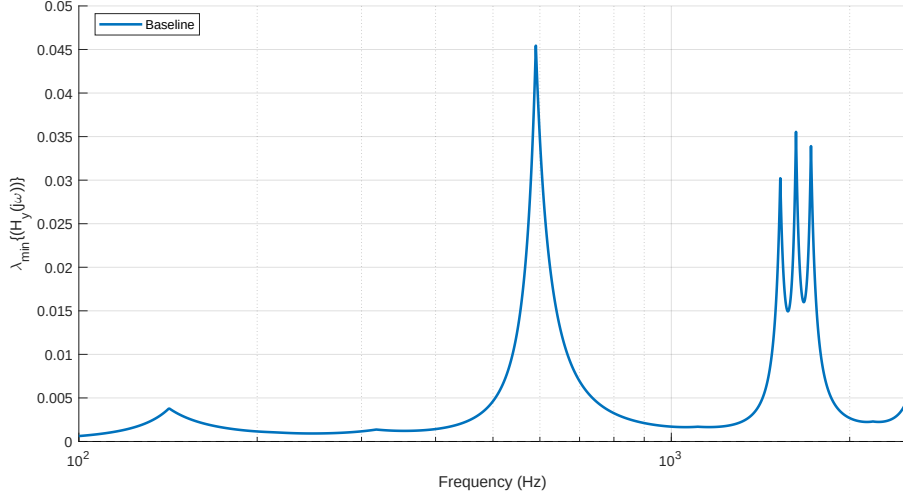


Figure 4.19: Passivity index $\mathcal{P}_{Y_{WF}}(\omega)$ of the baseline wind-farm admittance over the harmonic range.

Two bands, however, deserve particular attention. In the range 100–125 Hz and in the range 170–350 Hz, the passivity index lies very close to zero, meaning that the Hermitian part of $\mathbf{Y}_{WF}(j\omega)$ is only marginally positive definite. In these "warning" regions, a small modification of the model — a parametric perturbation of the GSC current controller or change in the output setpoints of the converter, etc — can break the passivity condition in these bands.

These two regions almost coincide with the impedance-crossing bands identified in Section 4.7.2, which confirms that the low-frequency window dominated by the converter and LCL dynamics is where the system is most exposed to converter-grid interaction.

4.7.4 Vector Margin Analysis

Both the passivity index and the GNC confirm that the baseline system is small-signal stable, but neither indicator quantifies *how close* the system is to losing stability. For this purpose, the vector margin (VM) introduced in Section 3.6.3 and defined in equation (3.60) is computed for the baseline model. Geometrically, $\text{VM}(\omega)$ is the distance, at frequency ω , between the closest eigenlocus of $\mathbf{L}(j\omega)$ and the critical point $(-1 + j0)$, and the worst-case vector margin

$$\text{VM} = \min_{\omega} \min_{i \in \{1,2\}} |\lambda_i(j\omega) + 1| \quad (4.19)$$

is the radius of the largest disc centred at $(-1 + j0)$ that does not intersect either eigenlocus. Following the convention adopted in Section 3.6.3, a value $\text{VM} \geq 0.5$ is considered a healthy MIMO robustness margin (approximately equivalent to a 6 dB gain margin and a 30° phase margin in the SISO sense [61]), whereas lower values indicate progressively poorer robustness.

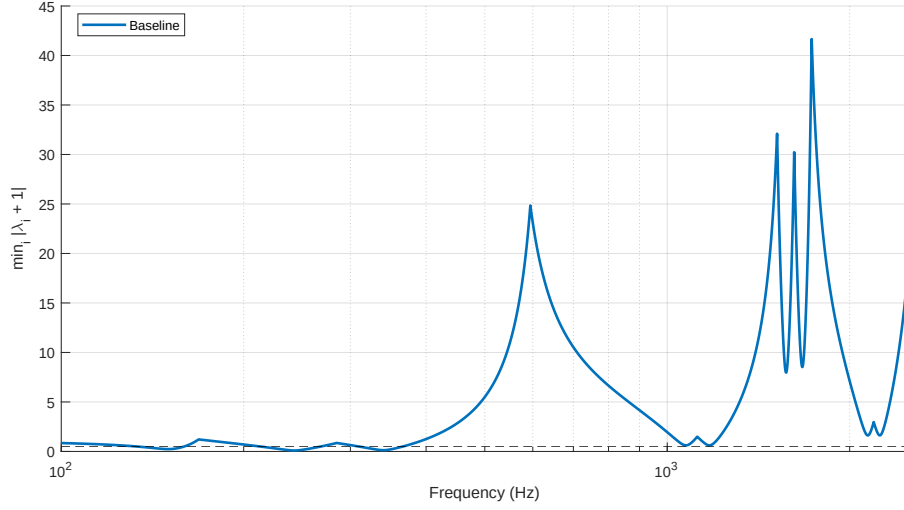


Figure 4.20: Frequency-dependent vector margin $VM(\omega)$ of the baseline wind-farm/grid interconnection, with the $VM = 0.5$ robustness threshold indicated.

The frequency-dependent margin $VM(\omega)$ is plotted in Figure 4.20. The global minimum of $VM(\omega)$ over the analysed band is $VM = 0.1$, attained at $f \approx 242.6$ Hz, i.e. well below the 0.5 robustness threshold. The baseline system is therefore stable but *poorly robust*: an additive perturbation of the loop gain of magnitude ~ 0.1 at this frequency is sufficient to drive the closed loop to the stability boundary.

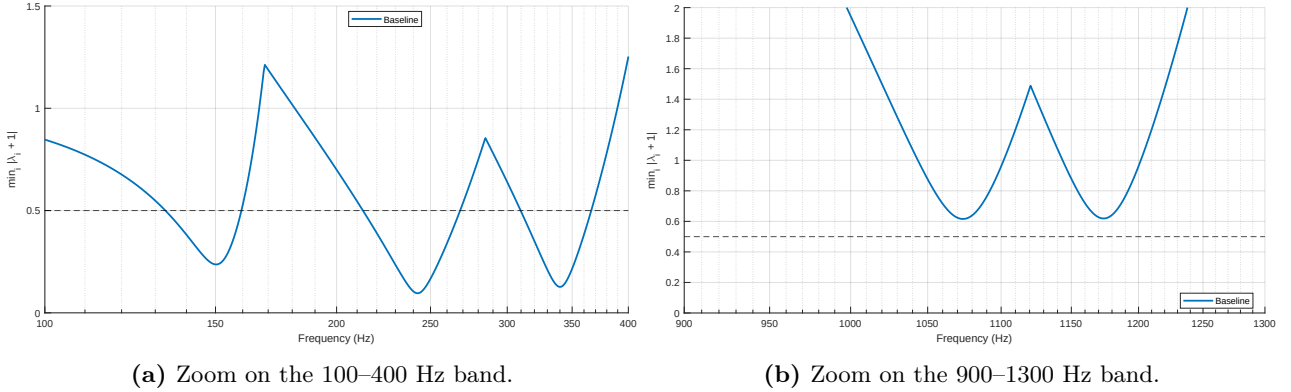


Figure 4.21: Vector margin $VM(\omega)$ of the baseline model zoomed on the two regions

Figure 4.21a zooms on the band 100–400 Hz: three local minima of $VM(\omega)$ are visible, at approximately 150 Hz, 242 Hz and 340 Hz, all of them below the 0.5 threshold, with the worst case at around 242 Hz. Figure 4.21b zooms on the band 900–1300 Hz, where two further local minima exist around 1 kHz to 1.1 kHz; in this band, however, the lowest value remains above 0.5, so the resonance cluster around 1 kHz is not the dominant robustness bottleneck even though it appears prominently in the impedance intersection diagnostics.

The finding of Vector Margin in lower frequency region falling below 0.5 coincide with the findings of impedance crossings of Section 4.7.2 and with the passivity warning bands of Section 4.7.3. The indicators are therefore mutually consistent and point to the result that the oscillations are coming from the power electronic converters. The lightly damped filter resonance of the GSC interacts with the network and the grid and produces oscillations in this lower frequency band. Although the wind-farm admittance remains globally passive, these peaks are sufficient to bring the eigenloci of the loop gain

very close to $(-1 + j0)$.

4.7.5 Disk Margin Analysis

Whereas the vector margin quantifies the smallest *additive* perturbation of the loop gain that destabilises the system, the disk margin reviewed in Section 3.6.4 quantifies the largest *simultaneous* gain and phase variation that the loop can absorb without losing stability. The frequency-dependent disk margin $\alpha_{mm}(j\omega)$ is computed from equation (3.62) with the balanced choice $\sigma = 0$, so that the admissible disc is symmetric on the dB scale and gain increase and gain decrease are penalised equally.

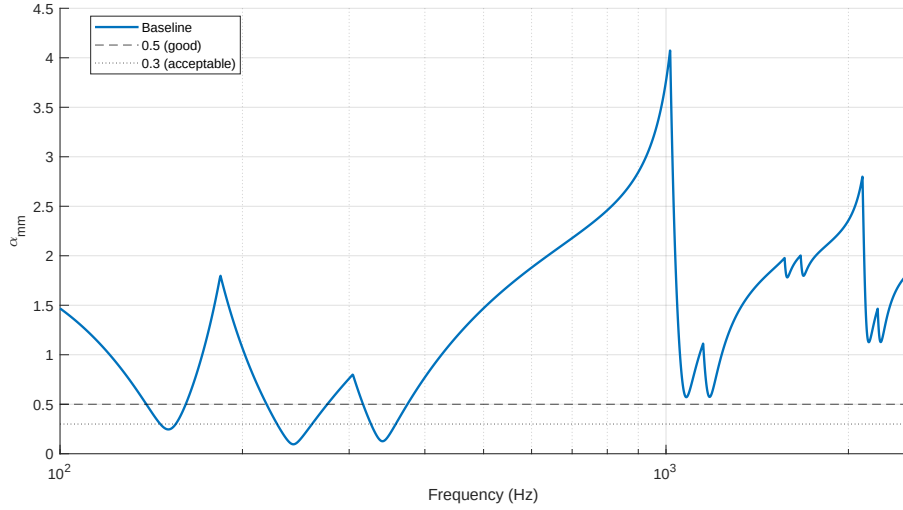


Figure 4.22: Frequency-dependent disk margin $\alpha_{mm}(j\omega)$ of the baseline model, with the robust (0.5) and acceptable (0.3) thresholds indicated.

The result is plotted in Figure 4.22. Consistently with the vector-margin and passivity results, the disk margin falls below the robustness threshold $\alpha_{mm} = 0.5$ in the same low-frequency region 100–400 Hz, and at the most critical frequencies, it lies even below the acceptable threshold $\alpha_{mm} = 0.3$, with the lowest of 0.096 at 242 Hz. The corresponding loop therefore tolerates only a very limited amount of combined gain and phase uncertainty: in practical terms, normal variations of the grid Thevenin impedance, of the LC component values, of the controller gains, or of the digital-delay model can move the operating point of the wind farm out of the safe region. In contrast, the local minimum of $\alpha_{mm}(j\omega)$ associated with the resonance cluster around 1 kHz remains well above the 0.5 threshold, confirming once more that the bottleneck of the baseline design is the *LCL filter resonance of the GSC* in the lower part of the harmonic range.

4.7.6 Synthesis of the Baseline Assessment

The four indicators applied to the baseline model give a coherent and physically consistent picture, summarised in Table 4.7.

Table 4.7: Synthesis of the stability and robustness indicators for the baseline model in the band 100 Hz–2.5 kHz.

Indicator	Worst-case value	Verdict
GNC encirclements of $(-1 + j0)$	N/A	Stable
Passivity index $\mathcal{P}_{Y_{WF}}(\omega)$	> 0 over the whole band	Passive (sufficient stability)
Vector margin VM	0.1 at ≈ 242 Hz	Below 0.5: poor robustness
Disk margin α_{mm}	0.096 at ≈ 242 Hz	Below 0.3: fragile

The Generalized Nyquist Criterion confirms that the system is small-signal stable, since neither eigenlocus encircles $(-1 + j0)$. The passivity index is positive over the entire harmonic range and therefore provides a sufficient stability condition with respect to any passive grid impedance. However, the vector and disk margins show that this stability is achieved with very little robustness in the band 100–400 Hz while the impedance-crossover plot and the passivity index identify the same band as the region in which the wind farm and the grid have comparable impedances and the wind-farm passivity index is closest to zero.

The root cause of this fragility is the oscillations coming from the power electronic converters. The lightly damped filter resonances of the GSCs interact with the network and the grid in the 100–400 Hz region bringing the eigenloci of the loop gain $\mathbf{L}(j\omega)$ very close to the critical point, which translates into the poor vector and disk margins observed here.

The baseline design is therefore *stable but not robust*. Dedicated mitigation measures are required to reshape $\mathbf{Y}_{WF}(j\omega)$ in the critical low-frequency band and to bring the vector and disk margins above the practical thresholds defined in Sections 3.6.3 and 3.6.4. The harmonic-stability enhancement techniques are applied to the baseline model in the next chapter, and their effect on the four indicators presented here is assessed using exactly the same methodology, so that any improvement in stability and robustness can be quantified on a like-for-like basis.

Chapter 5

Implementation of Stability Enhancement Techniques

The baseline assessment of Section 4.7 established that the offshore wind power plant is small-signal stable but exhibits poor robustness in the band 100–400 Hz, with a worst-case vector margin $VM = 0.1$ and a disk margin $\alpha_{mm} = 0.096$, below the acceptable thresholds and at the same frequency $f \approx 242$ Hz. The four indicators applied in that section all pointed to the lightly damped LC resonance of the GSC, modified by the WT step-up transformer leakage, and combined with the rest of the network and the grid impedance, as the dominant source of the fragility. The present chapter implements, on top of the baseline model, the harmonic-stability enhancement techniques and quantifies their effect on the indicators using exactly the same methodology, so that any improvement can be assessed on a like-for-like basis.

Three complementary measures are applied in sequence: a re-sizing of the LC filter (Section 5.1); capacitor-current-feedback (CCF) active damping (Section 5.2); and the notch-filter-based active damping on the system’s critical mode (Section 5.3). Each measure is incrementally applied to the model resulting from the previous step, so that the indicators reported after each technique reflect the combined effect of all measures introduced up to that point. After these three mitigation steps, the synchronous condenser of the onshore connection is modelled and added to the network in Section 5.4, where the robustness of the final design to variations in grid strength is also verified through a short-circuit-ratio sweep. The chapter closes with a synthesis of the achieved improvements and an outlook on additional measures for the residual frequency bands (Section 5.5).

5.1 LC Filter Re-sizing

The first mitigation step is a re-design of the GSC output filter. The LCL resonance was identified in Section 4.7 as the dominant origin of the poor robustness margin, and the classical LCL design rule is [92],

$$10 f_1 < f_{res} < 0.5 f_{sw}, \quad (5.1)$$

indicates that, for a 50 Hz grid and a 5 kHz switching frequency, the LCL resonance should sit in the band $500 \text{ Hz} \leq f_{res} \leq 2.5 \text{ kHz}$ [92]. The baseline value $f_{res} \approx 434$ Hz lies just below the lower bound of this interval, which is consistent with the strong converter–grid interaction observed at low frequency in the baseline analysis. Pushing f_{res} above 500 Hz is therefore the primary design objective of this

section.

The natural way to move f_{res} upward is to reduce the filter capacitance C_f and/or to increase the filter inductance L_f . Two independent one-dimensional parametric sweeps are performed, with the worst-case vector margin

$$VM = \min_{\omega} \min_{i \in \{1,2\}} |\lambda_i(j\omega) + 1|$$

adopted as the objective function. In each sweep the remaining parameters of the converter are kept fixed; in particular, the GSC current-controller gains $K_{p,GSC}$ and $K_{i,GSC}$ are recomputed at every step according to the design relations of Table 4.4, so that the closed-loop bandwidth of the inner current loop remains at its nominal value $\omega_{GSC,bw}$.

5.1.1 Filter Inductance Sweep

Figure 5.1 reports the worst-case vector margin as a function of L_f . The margin is essentially insensitive to the variation of L_f within the explored range. This result is a direct consequence of the design choice $K_{p,GSC} = L_f \omega_{GSC,bw}$ adopted in Table 4.4: any modification of the filter inductance is automatically compensated by a proportional re-tuning of the current controller, so that the closed-loop bandwidth of the inner loop and the resulting shape of the converter admittance $\mathbf{Y}_{GSC,dq}(j\omega)$ around the resonance remain practically unchanged. From a small-signal stability perspective, L_f is therefore a degree of freedom of the design but not a robustness lever for the baseline controller tuning.

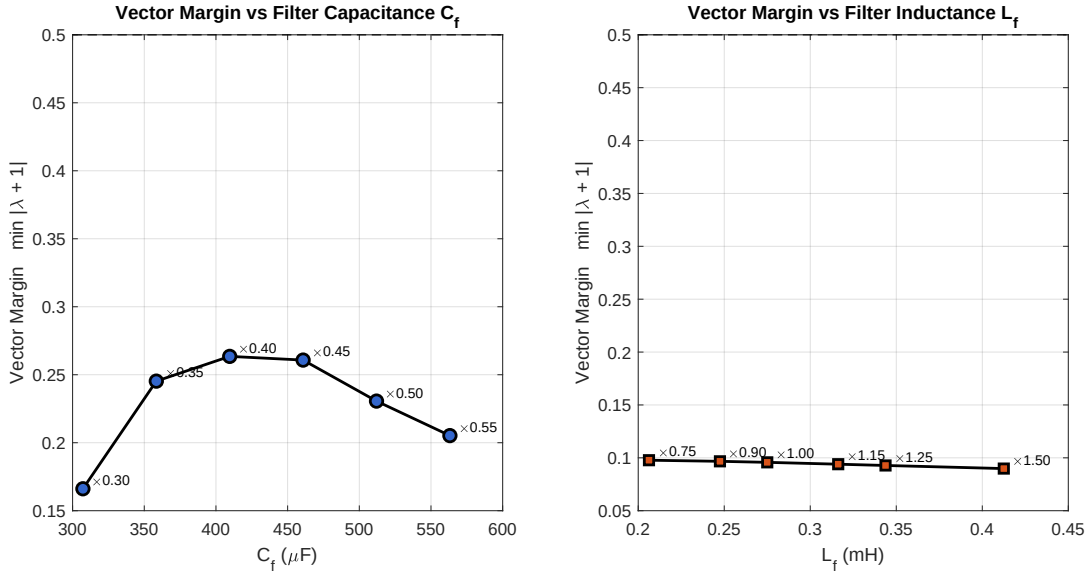


Figure 5.1: Worst-case vector margin as a function of the filter inductance L_f and C_f

5.1.2 Filter Capacitance Sweep

The sweep on C_f is also reported in Figure 5.1. Reducing C_f pushes the LCL resonance upward in frequency and the worst-case vector margin increases monotonically until $C_f \approx 0.4 C_{f,0}$, beyond which a new minimum appears around 180 Hz and VM starts to decrease again. The emergence of this second minimum reflects the intrinsic limit of pure-resizing: once the LCL resonance has been pushed sufficiently high in frequency, the worst-case mode of the loop gain is no longer dictated by the filter itself but by a lower-frequency mode of the aggregated wind-farm-grid network. The optimum filter capacitance is therefore selected at the value that brings VM to its maximum before this transition

occurs,

$$C'_f = 0.4 C_{f,0} = 0.4 \times 1024 \mu\text{F} = 409.6 \mu\text{F}, \quad (5.2)$$

which corresponds to a redesigned LCL resonance

$$f'_{res} = \frac{1}{2\pi} \sqrt{\frac{L_f + L_\ell}{L_f L_\ell C'_f}} \approx 687 \text{ Hz},$$

comfortably above the 500 Hz lower bound prescribed by (5.1) [92].

5.1.3 Effect on the Stability Indicators

The indicators are recomputed with the redesigned capacitance and compared with the baseline model in Figures 5.2 to 5.4. The passivity index in the critical band 100–400 Hz moves further away from zero, indicating that the wind-farm admittance becomes more strongly passive at the frequencies where it was previously almost marginal. The worst-case vector margin improves from $\text{VM} = 0.10$ in the baseline to $\text{VM} = 0.23$ after the re-sizing, with the new minimum located at $f \approx 180$ Hz. The disk margin is correspondingly shifted into the acceptable region $\alpha_{mm} \geq 0.3$.

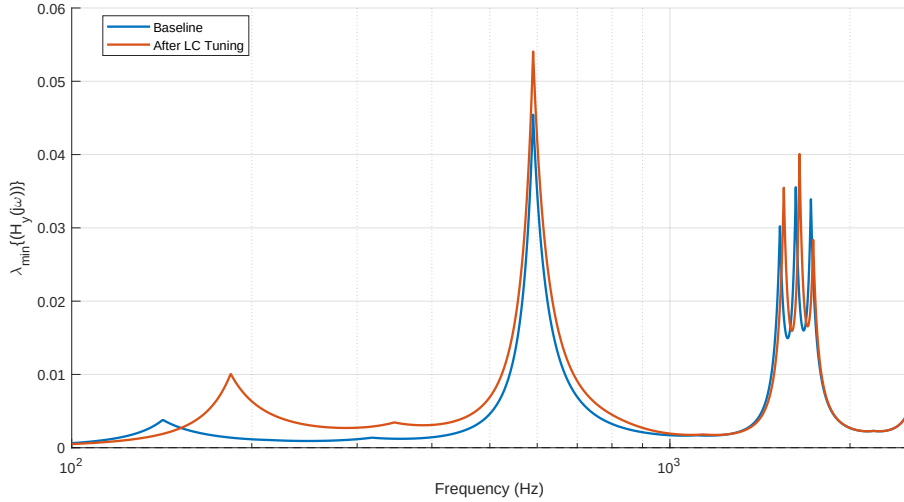


Figure 5.2: Passivity index $\mathcal{P}_{Y_{WF}}(\omega)$ before and after the LC filter re-sizing.

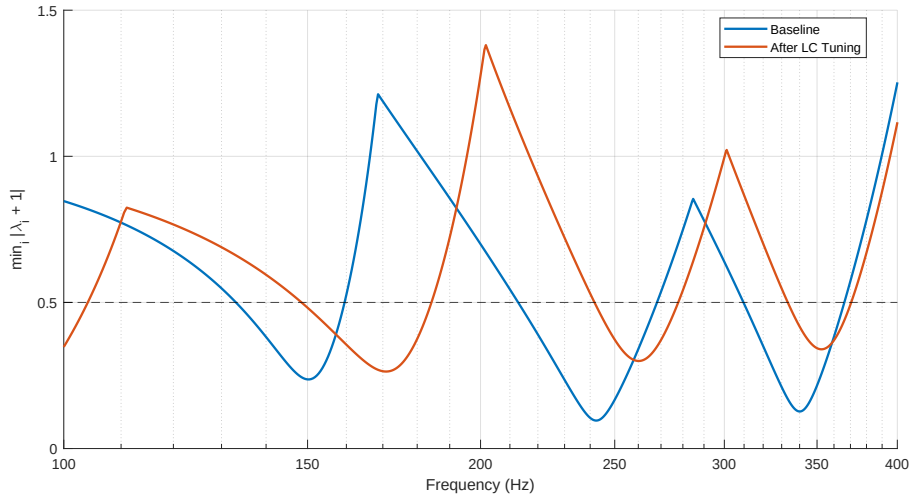


Figure 5.3: Vector margin $\text{VM}(\omega)$ (zoomed at 100–400 Hz) before and after the LC filter re-sizing.

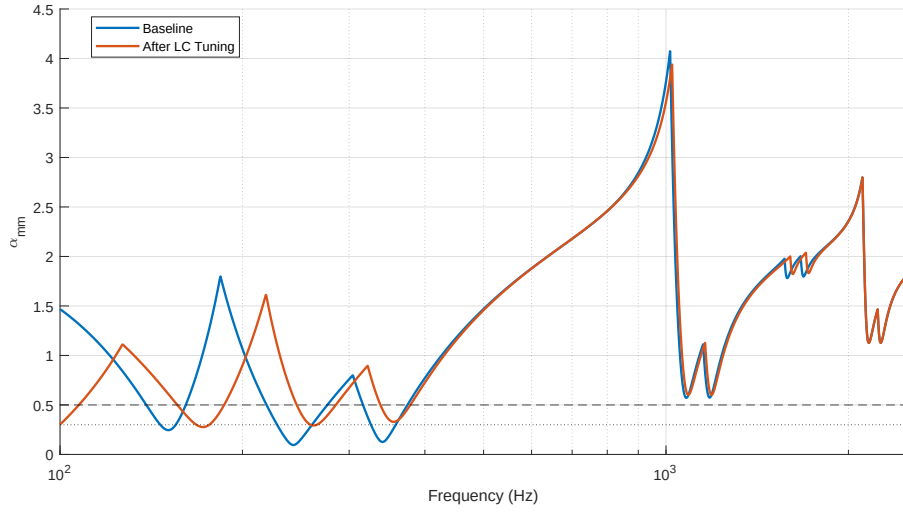


Figure 5.4: Disk margin $\alpha_{mm}(j\omega)$ before and after the LC filter re-sizing.

The system is therefore noticeably less fragile than the baseline configuration, but it has not yet reached the comfortable robustness threshold $VM \geq 0.5$. Additional measures are required to achieve this goal.

5.2 Capacitor Current Feedback Active Damping

The second mitigation step is the implementation of capacitor current feedback (CCF) active damping, reviewed in Section 3.7.1. The capacitor current i_{C_f} is sensed, multiplied by a proportional gain K_{ad} , and added to the modulation signal of the GSC as shown in Figure 5.5.

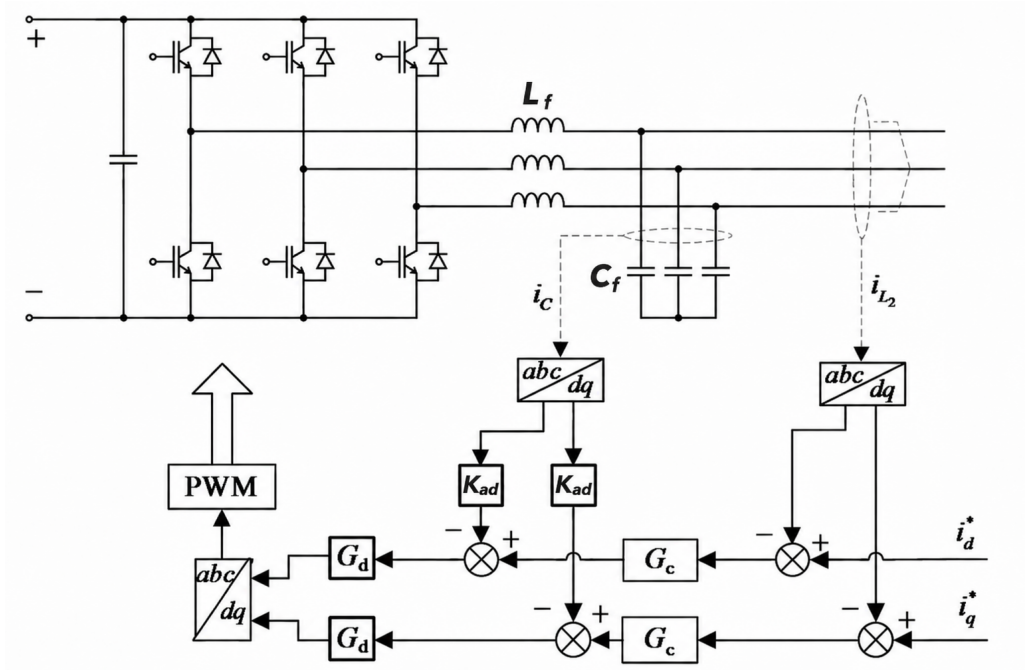


Figure 5.5: Capacitor-current-feedback active damping implementation for the grid-side converter

Mathematically, the CCF action enters through the voltage feed-forward path $G_c(s)H_v$, and the gain K_{ad} is the only additional design parameter. As detailed in Section 3.2.1 and repeated in Section 4.1.1 in equation 4.1, final GSC output admittance equation now becomes:

$$\mathbf{Y}_{GSC,dq}(s) = \left[\frac{\mathbf{B}_{L_f}(s) + G_c(s) (\mathbf{B}_{PI,i}(s) - \mathbf{B}_{dec})}{(1 - G_c(s)H_v) \mathbf{I} + G_c(s)K_{ad}\mathbf{B}_{C_f}(s)} \right]^{-1} + \mathbf{B}_{C_f}(s) \quad (5.3)$$

where $\mathbf{I}$ is a 2×2 identity matrix.

As shown in Figure 3.13 of Section 3.7.1, this feedback is electromagnetically equivalent to placing a virtual resistor in parallel with the filter capacitor, and therefore damps the LCL resonance without dissipating real power.

The optimum value of K_{ad} is selected through a one-dimensional sweep on the worst-case vector margin, applied on top of the re-sized filter of Section 5.1.2. The result is reported in Figure 5.6. The optimum is found at

$$K_{ad}^* \approx 0.15, \quad (5.4)$$

beyond which the additional feedback starts to interact unfavourably with the inner current loop and the vector margin decreases again, in line with the limitation of CCF discussed in Section 3.7.1.

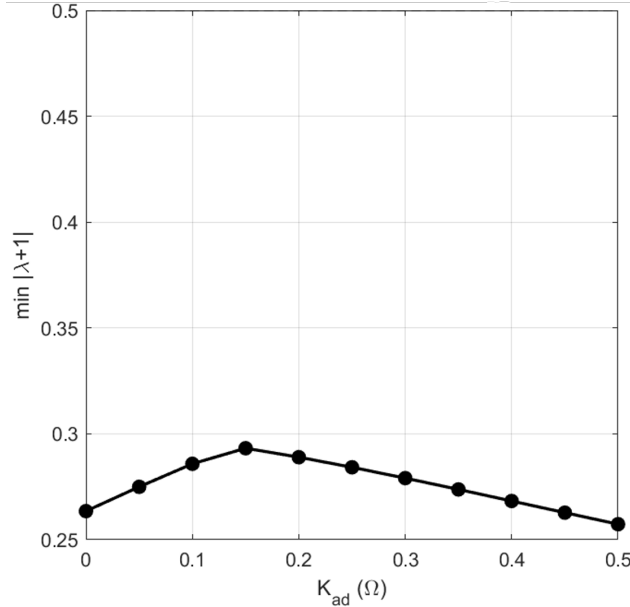


Figure 5.6: Worst-case vector margin as a function of the CCF active-damping gain K_{ad} .

5.2.1 Effect on the Stability Indicators

When applied to the re-sized model, CCF yields only a modest additional improvement of the four indicators (Figures 5.7–5.9). The reason is that, after the LCL re-sizing of Section 5.1, the resonance frequency has already been pushed above 500 Hz and the worst-case frequency of the loop gain has migrated to ~ 180 Hz, where CCF acts only indirectly through the virtual resistance seen by the LCL capacitor. CCF is most effective when the critical mode coincides with the LCL resonance, which is no longer the case here. The indicators nevertheless show a small but consistent improvement in the passivity index and in the vector and disk margins, and CCF is retained in the model as a built-in active-damping layer that protects the converter against any operating condition in which the LCL resonance could re-emerge as the dominant mode.

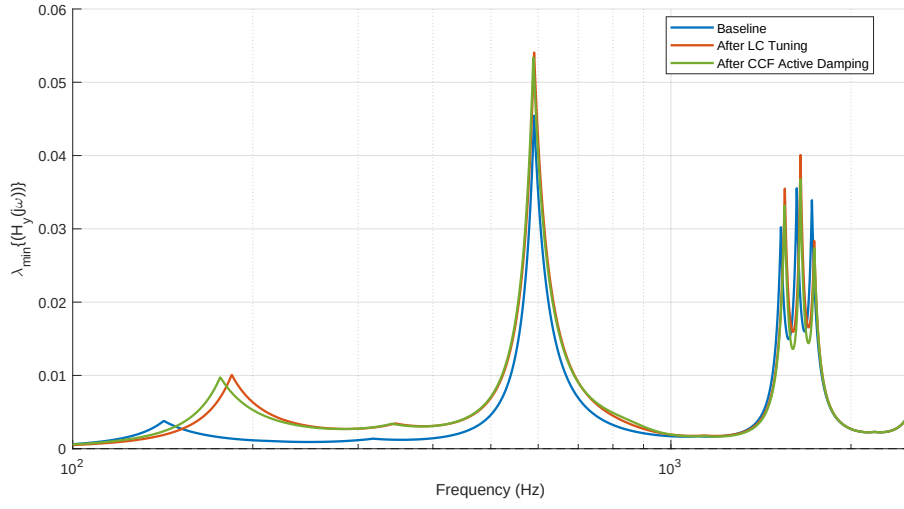


Figure 5.7: Passivity index $\mathcal{P}_{Y_{WF}}(\omega)$ before and after the addition of CCF active damping.

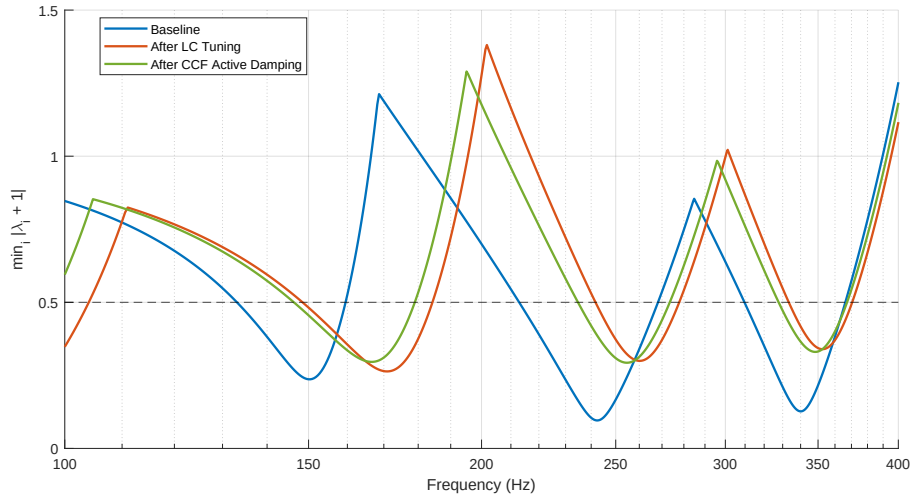


Figure 5.8: Vector margin $VM(\omega)$ (zoomed at 100–400 Hz) before and after the addition of CCF active damping.

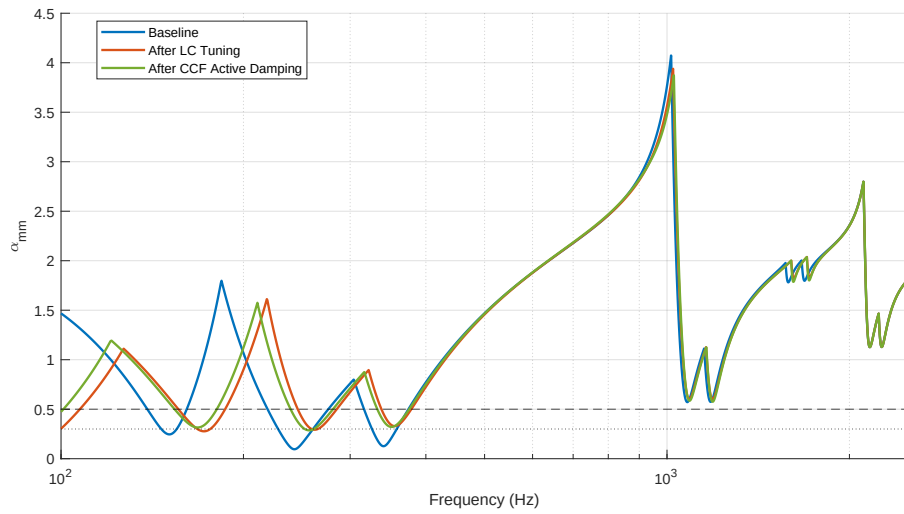


Figure 5.9: Disk margin $\alpha_{mm}(j\omega)$ before and after the addition of CCF active damping.

The next step must therefore act not on the LCL resonance itself but on the network-driven mode that now sets the worst-case frequency of the loop gain. This mode sits well below the GSC current-

controller bandwidth of 400 Hz, where the converter behaves as a near-ideal current source and provides essentially no damping to it. Hence a control-side measure that selectively introduces damping at this frequency is required, which motivates the critical-mode notch filter damping.

5.3 Critical-Mode Notch-Filter Damping

After the LCL re-sizing and the CCF action, the residual robustness bottleneck of the wind farm is located around 180 Hz, in the network-driven mode of the aggregated wind-farm/grid loop rather than at the LCL resonance. To target this mode directly, a notch is applied to the voltage feed-forward path of the GSC, with its centre frequency aligned *not* on the LCL resonance — as is customary in the classical notch-damping scheme reviewed in the previous chapter — but on the actual critical mode of the loop gain $\mathbf{L}(j\omega)$. This re-targeting is the key conceptual step: the converter is no longer asked to damp its own filter resonance (which has already been displaced by the re-sizing of Section 5.1), but to react to the dominant mode of the surrounding network, so that the GSC actively participates in damping the combined converters–cables–transformers–grid interaction.

The notch is implemented as the depth-controllable second-order band-stop filter. Hence H_v in Equation 5.3 becomes

$$H_v = H_{notch}(s) = \frac{s^2 + 2\zeta_n \omega_n s + \omega_n^2}{s^2 + 2\zeta_d \omega_n s + \omega_n^2}, \quad (5.5)$$

which generalises the simple second-order notch of equation (3.64) by introducing two independent damping coefficients. The coefficient ζ_n in the numerator controls the depth of the attenuation at the centre frequency, the coefficient ζ_d in the denominator controls the bandwidth of the notch and therefore the rate at which the gain is restored away from ω_n , and the ratio ζ_n/ζ_d fixes the notch depth at ω_n while leaving the out-of-band gain unchanged at unity [68].

5.3.1 Parameter Selection

The three tuning parameters $(\omega_n, \zeta_n, \zeta_d)$ are selected through a coarse three-dimensional grid search with the worst-case vector margin as the objective function as shown in Figure 5.10. The search range have a starting reference centered on the frequency $\omega_n \approx 2\pi \cdot 250$ rad/s, and explores depth ratios ζ_n/ζ_d between 0 and 0.5 and denominator dampings ζ_d between 0 and 2.

The optimum returned by the grid search is

$$f_n^* = \frac{\omega_n^*}{2\pi} \approx 180 \text{ Hz}, \quad \zeta_n^* \approx 0.2, \quad \zeta_d^* \approx 0.7, \quad (5.6)$$

and the corresponding notch profile is shown in Figure 5.11. The notch is sufficiently narrow and deep to selectively decouple the GSC current controller from the network resonance, while leaving the loop gain practically unaffected at all other frequencies — in particular at the LCL resonance and around the cluster at 1 kHz.

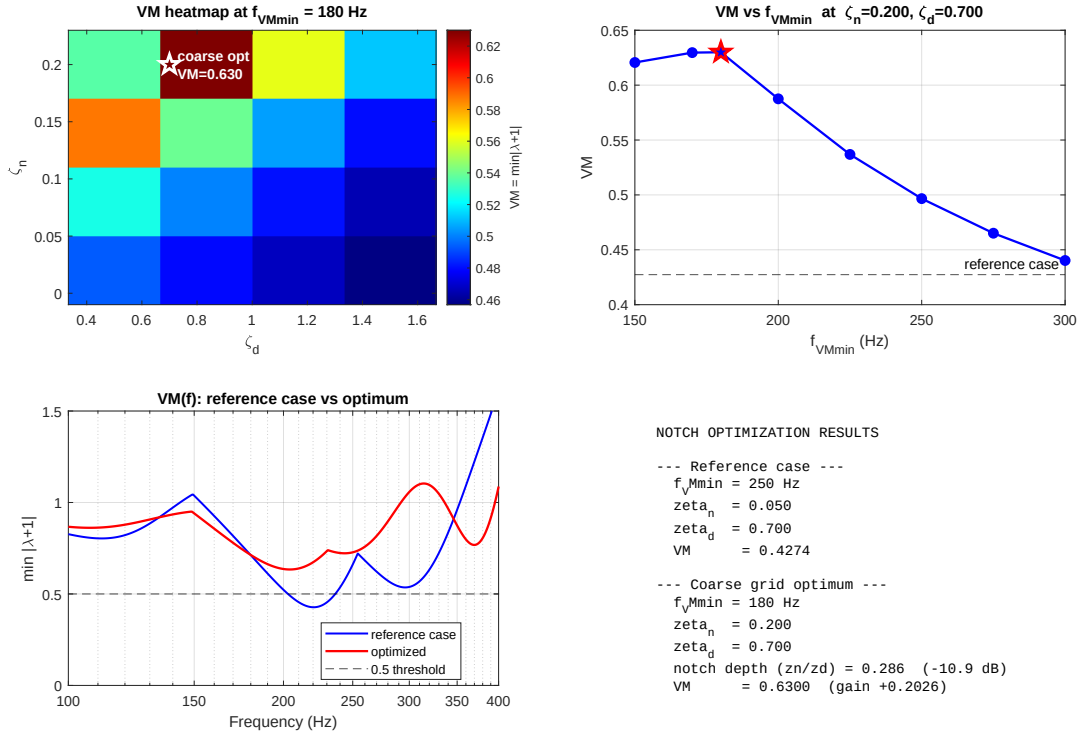


Figure 5.10: Coarse three-dimensional grid search of the notch parameters $(\omega_n, \zeta_n, \zeta_d)$.

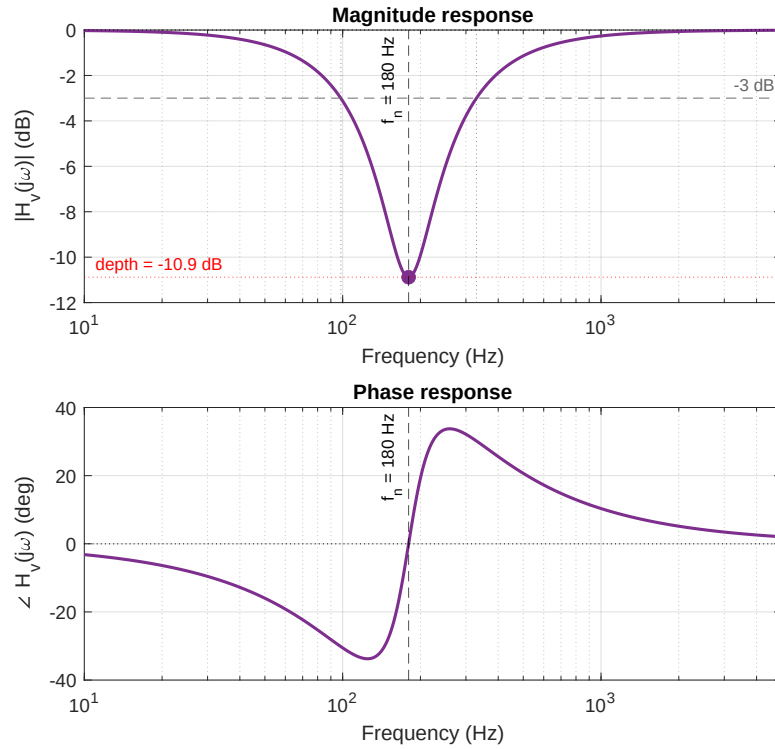


Figure 5.11: Frequency response of the notch filter $H_{notch}(j\omega)$ tuned on the critical mode of the loop gain.

5.3.2 Effect on the Stability Indicators

All the indicators recomputed after the application of the critical-mode notch filter are reported in Figures 5.12–5.16. Compared with the previous step, the passivity index increases substantially in the 100–400 Hz region, which becomes strongly passive. The worst-case vector margin reaches

$$VM = 0.63,$$

well above the 0.5 robustness threshold defined in Section 3.6.3. Zooming in on the critical $(-1 + j0)$ point on GNC plot in Figure 5.12, a major difference can be seen compared to the loci of baseline model GNC in Figure 4.17. The disk margin also moves above 0.5. From a small-signal point of view, the wind farm has therefore evolved from a stable-but-fragile baseline to a robustly stable system in the harmonic range.

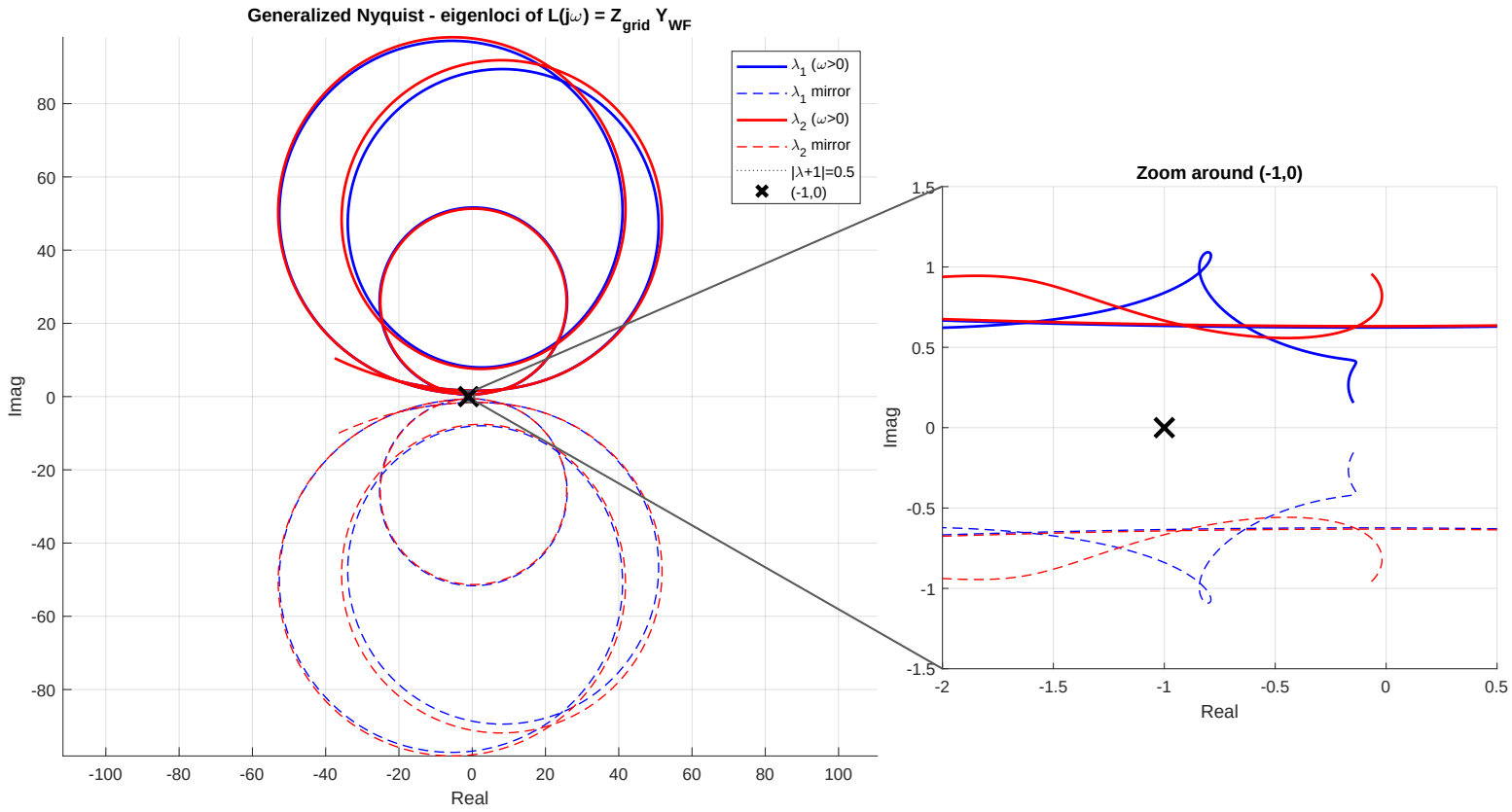


Figure 5.12: Eigenloci of the minor-loop gain $L(j\omega)$ in the complex plane after LC tuning, CCF Active Damping and Critical-Mode Notch Filter Damping, with a zoom on the critical point $(-1 + j0)$.

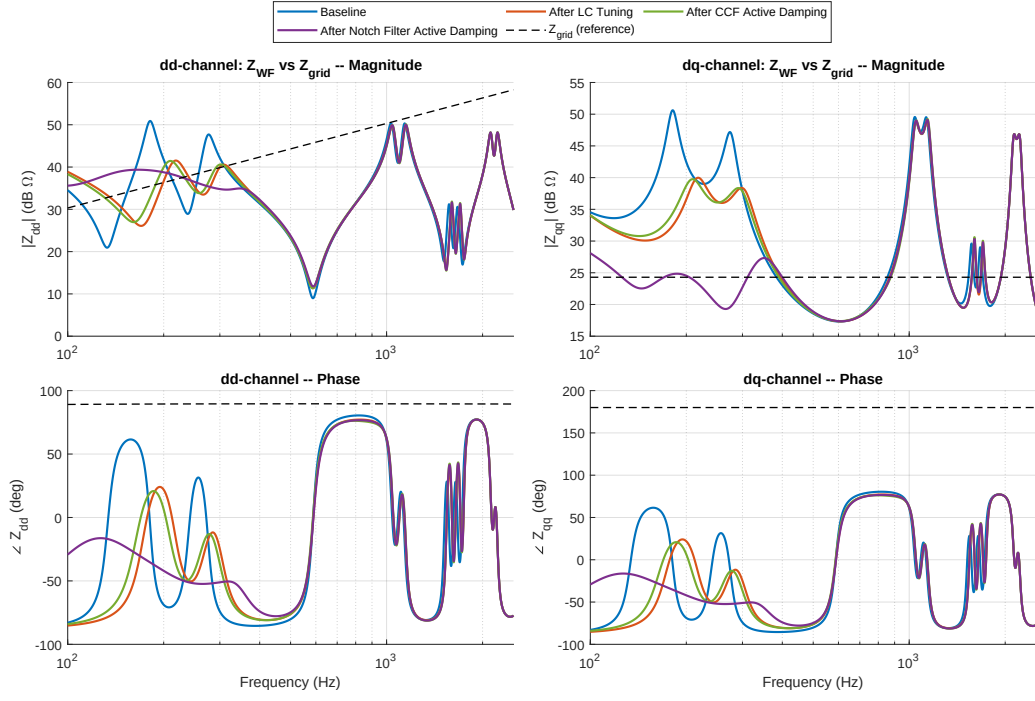


Figure 5.13: Bode plots of the wind-farm impedance $\mathbf{Z}_{WF}(j\omega)$ and the grid impedance $\mathbf{Z}_g(j\omega)$ for the dd and dq entries for each implementation

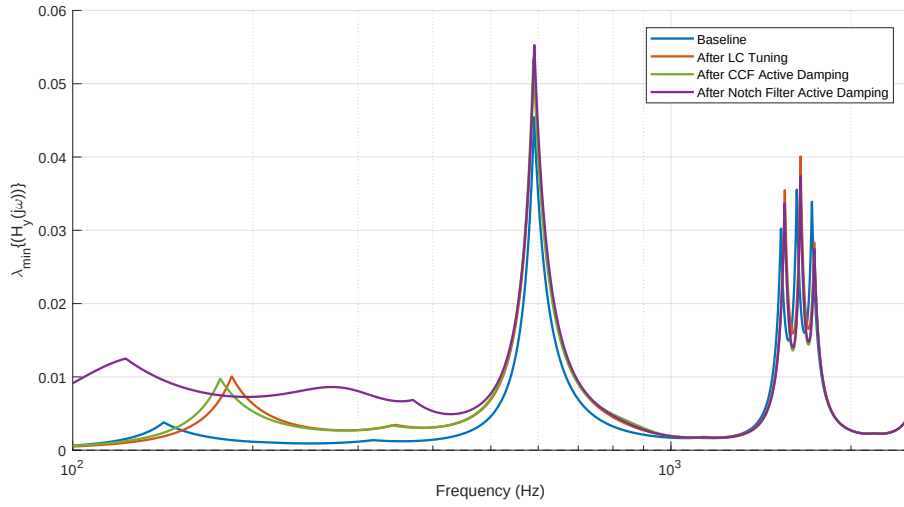


Figure 5.14: Passivity index $\mathcal{P}_{Y_{WF}}(\omega)$ before and after the critical-mode notch filter.

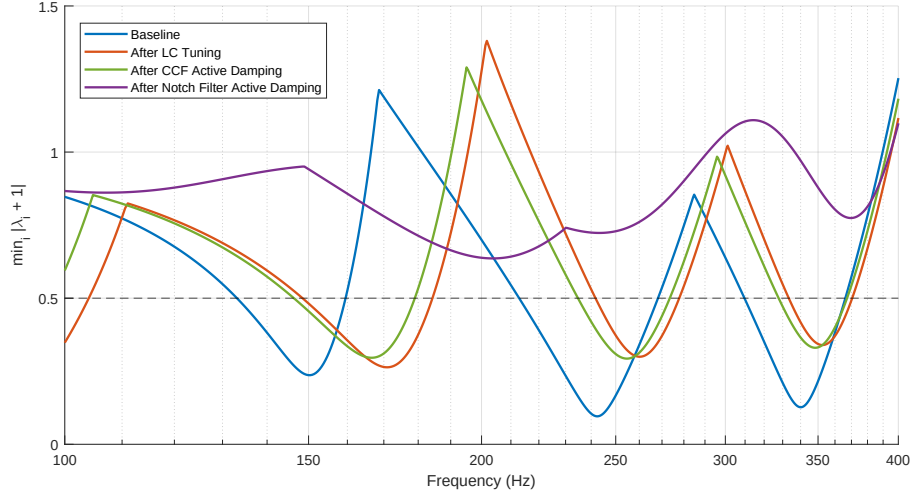


Figure 5.15: Vector margin $VM(\omega)$ (zoomed at 100–400 Hz) before and after the critical-mode notch filter.

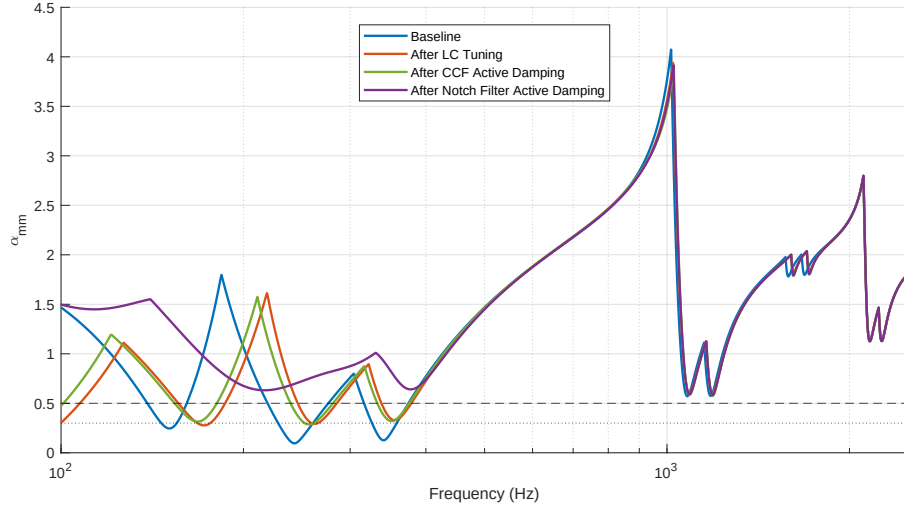


Figure 5.16: Disk margin $\alpha_{mm}(j\omega)$ before and after the critical-mode notch filter.

The mechanism behind this improvement is physically transparent. The notch reshapes the converter admittance $\mathbf{Y}_{GSC,dq}(j\omega)$ to add damping selectively at the dominant network mode of the aggregated system, so that, once the GSC is interconnected with the rest of the wind farm and with the grid, the loop gain $\mathbf{L}(j\omega)$ no longer comes close to the critical point $(-1 + j0)$ around ω_n . Equivalently, the converter actively *reacts* to the resonance of the surrounding network — array cables, transformers and grid Thevenin equivalent — no longer behaving as an ideal current source at around 180 Hz but presenting a finite damping admittance to that mode.

5.4 Synchronous Condenser Modelling and Connection

The last element required to close the model of the Princess Elisabeth Energy Island system is the synchronous condenser (SC) installed at the onshore connection point, whose role in the impedance-based stability framework was already discussed in the literature-review chapter. The analytical 2×2 dq impedance model of the synchronous condenser was derived there from Park’s two-axis representation;

the model is repeated here for convenience:

$$\mathbf{Z}_{SC,dq}(j\omega) = \begin{bmatrix} R_s(j\omega) + j\omega L_d(j\omega) & -\omega_0 L_q(j\omega) \\ \omega_0 L_d(j\omega) & R_s(j\omega) + j\omega L_q(j\omega) \end{bmatrix}, \quad (5.7)$$

with the frequency-dependent operational inductances $L_d(j\omega)$ and $L_q(j\omega)$ given by equations (3.49)–(3.50) and the frequency-dependent stator resistance $R_s(j\omega)$ given by equation (3.54).

5.4.1 Parameter Selection

The synchronous condenser of the Energy Island will have a nameplate reactive-power rating of 280 MVar [19], which is the only machine quantity that is publicly disclosed by Elia. The remaining electrical parameters of the machine — stator resistance, synchronous, transient and sub-transient inductances along the d - and q -axes, together with the associated open- and short-circuit time constants — are not provided in the public documentation. They are therefore taken from the optimisation-based identification of a large solid-rotor synchronous condenser reported in [93] and are summarised in Table 5.1. The skin- and proximity-effect coefficient α_{skin} entering equation (3.54) and the open- and short-circuit time constants $T'_{d,q}$, $T''_{d,q}$, $T'_{d0,q0}$, $T''_{d0,q0}$ entering equations (3.49)–(3.50) are taken from the same reference and are not repeated here for compactness.

Table 5.1: Electrical parameters of the synchronous-condenser model.

Parameter	Symbol	Per-unit	SI value
Rated apparent power	S_{base}	—	280 MVA
Terminal line-to-line voltage (RMS)	$V_{base,LL}$	—	18 kV
Nominal frequency	f_{nom}	—	50 Hz
Base impedance	$Z_{base} = V_{base,LL}^2 / S_{base}$	—	1.157 Ω
Base inductance	$L_{base} = Z_{base} / \omega_0$	—	3.68 mH
Stator resistance	R_s	0.003	3.47 m Ω
d -axis synchronous inductance	L_d	1.80	6.63 mH
d -axis transient inductance	L'_d	0.30	1.10 mH
d -axis sub-transient inductance	L''_d	0.20	0.74 mH
q -axis synchronous inductance	L_q	1.70	6.26 mH
q -axis transient inductance	L'_q	0.55	2.03 mH
q -axis sub-transient inductance	L''_q	0.22	0.81 mH

Figure 5.17 reports the resulting impedance of the synchronous condenser computed from equation (5.7) over the harmonic range 100 Hz–2.5 kHz.

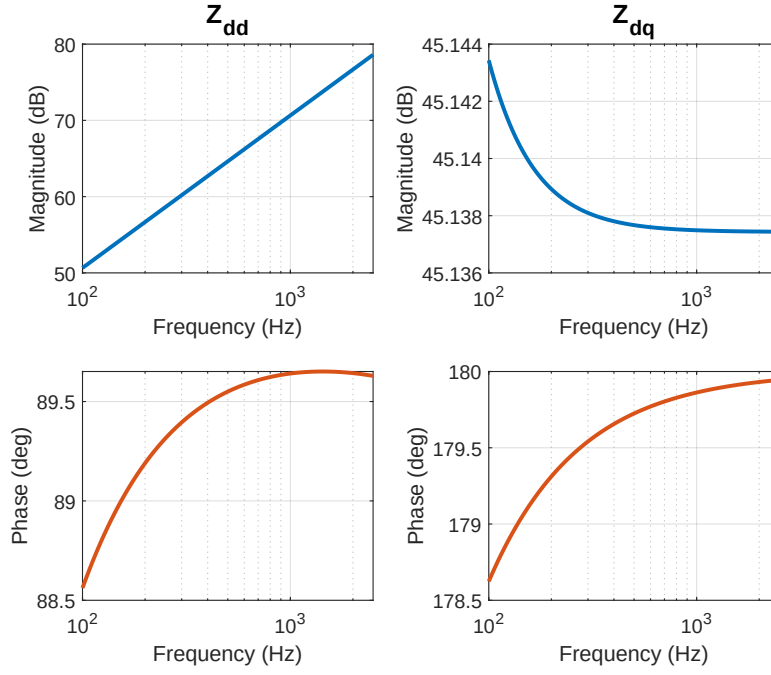


Figure 5.17: Impedance Bode plot of the synchronous-condenser model in the harmonic range.

The diagonal entry Z_{dd} is essentially inductive: its magnitude increases linearly with frequency on the Bode plot and its phase remains close to 90° over the whole band. The off-diagonal entry Z_{dq} is, in contrast, almost constant in magnitude and exhibits a phase close to 180° , which is the expected signature of the synchronous-frame cross-coupling term $-\omega_0 L_q(j\omega)$. The dq cross-coupling introduced by the rotating-frame representation is therefore present but does not produce any resonance or anti-resonance in the harmonic band. This behaviour is consistent with the fact that, well above the sub-transient corner frequency $1/T''_{d0}$, the operational inductances of a large synchronous machine settle on their sub-transient values L''_d and L''_q [12, 56], so that the machine appears from its terminals as a simple RL branch.

5.4.2 Step-up Transformer of the Synchronous Condenser

The synchronous condenser is connected to the 380 kV onshore bus through a dedicated step-up transformer. The transformer is modelled with the same 2×2 dq impedance representation of equation (3.36) adopted for the WT, offshore-substation and onshore-substation transformers in Section 3.3, with the parameters listed in Table 5.2. The single-line diagram of the connection is shown in Figure 5.18.

Table 5.2: Parameters of the synchronous-condenser step-up transformer.

Transformer	Voltage ratio (kV)	Rated power (MVA)	L_l (H)	R_{dc} (Ω)	Source
SC step-up	18/380	280	0.230	1.289	[93, 19]

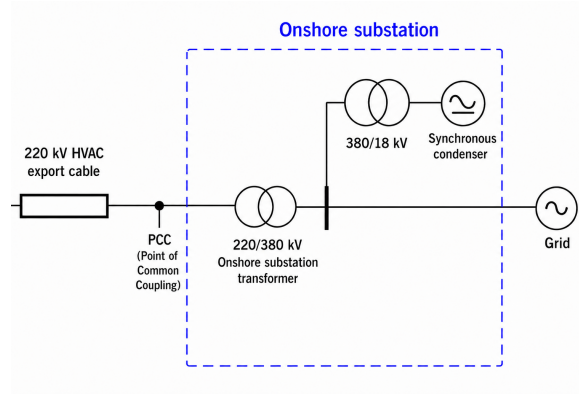


Figure 5.18: Connection of the synchronous condenser to the 380 kV onshore bus through its step-up transformer.

5.4.3 Effect on the Equivalent Grid Impedance at the PCC

The synchronous condenser and its step-up transformer are introduced into the network model in parallel with the grid Thevenin equivalent at the onshore 380 kV bus. The equivalent network impedance seen from the PCC is therefore the onshore 220/380 kV substation transformer placed in series with the parallel combination of the 380 kV grid Thevenin equivalent and the synchronous-condenser branch (machine plus its step-up transformer).

$$\mathbf{Z}_{net}(j\omega) = \left[\mathbf{Z}_{g,380}(j\omega)^{-1} + (\mathbf{Z}_{T,SC}(j\omega) + \mathbf{Z}_{SC,dq}(j\omega))^{-1} \right]^{-1} + \mathbf{Z}_{T,onshore}(j\omega), \quad (5.8)$$

Figure 5.19 compares the equivalent network impedance with and without the synchronous condenser, for the weak-grid scenario at the PCC.

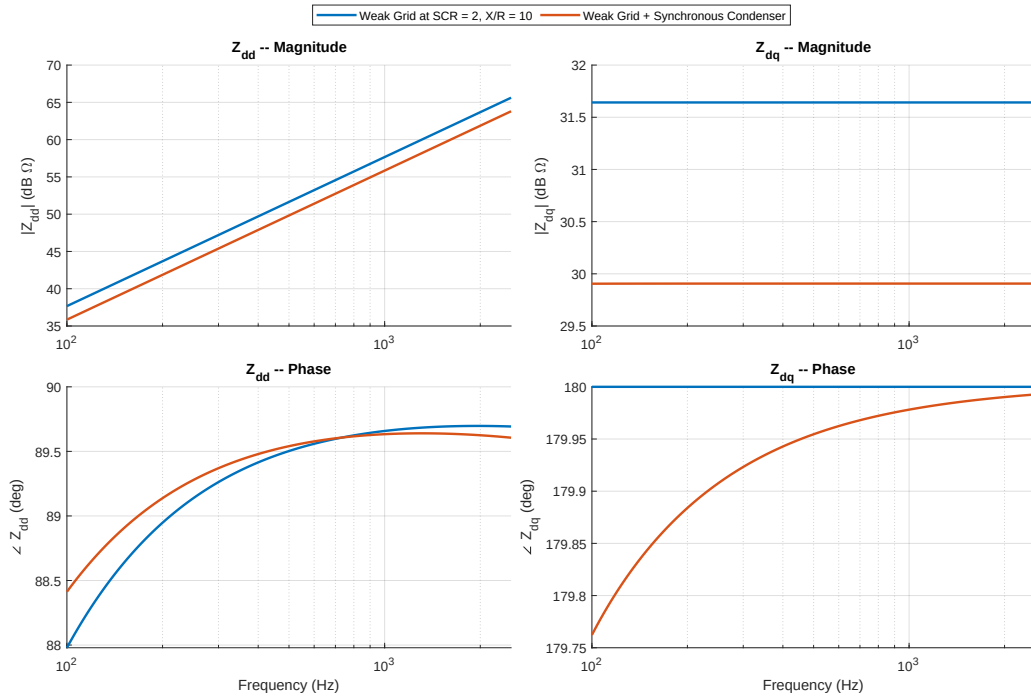


Figure 5.19: Equivalent network impedance seen from the PCC, with and without the synchronous condenser in parallel.

The inclusion of the synchronous condenser produces only a minor modification of the equivalent impedance in the investigated frequency range: the magnitude of $\mathbf{Z}_{net}(j\omega)$ is slightly decreased, which

is the expected consequence of placing an additional inductive branch in parallel with the grid, while the phase response have a minor change. The harmonic-range behaviour of the network equivalent is therefore not significantly altered. As a consequence, if the weak-grid case has already been shown to be small-signal stable with sufficient margin, then the inclusion of the synchronous condenser is not expected to change the stability verdict in the harmonic range. Conversely, if the weak-grid case operated close to the critical point in the GNC or with a low vector margin, the synchronous-condenser case would have to be re-checked explicitly, since even small impedance variations can shift a marginal stability condition.

5.4.4 Robustness to Grid Strength

Since the synchronous condenser does not introduce any resonance or anti-resonance in the harmonic range and only marginally modifies the equivalent network impedance at the PCC, its main contribution to the impedance-based stability analysis is to raise the short-circuit ratio (SCR) of the grid equivalent. The role of the SC is therefore best assessed indirectly, by sweeping the SCR of the grid Thevenin equivalent and recomputing the worst-case vector margin at every operating point. This sweep captures, on a single curve, the combined effect of any system element — including the SC — whose net contribution is to modify the effective Thevenin inductance seen from the PCC. The result is reported in Figure 5.20.

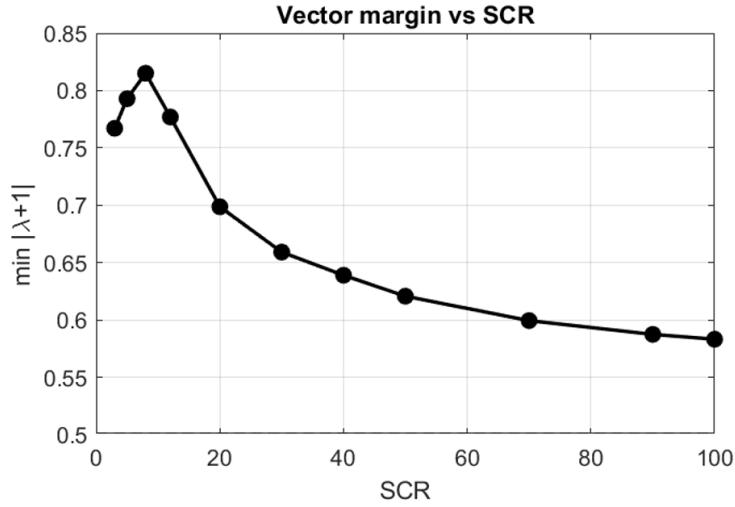


Figure 5.20: Worst-case vector margin as a function of the short-circuit ratio of the grid Thevenin equivalent.

The dependence of VM on the SCR is clearly non-monotonic, with a maximum at an intermediate value $\text{SCR} \approx 9$ and a smooth decay on both sides. The sweep is deliberately extended into an unrealistically high SCR range so that this asymptotic lower bound becomes visible on the plot. This behaviour is a direct consequence of the fact that the onshore 220/380 kV substation transformer is a fixed hardware element placed in series with the variable Thevenin grid impedance, and can be interpreted as the balance between two competing effects. At low SCR, the grid resistance R_g is large in absolute terms and provides natural damping of the lightly damped wind-farm resonances, which keeps the vector margin reasonably high. At high SCR, on the contrary, R_g vanishes asymptotically but the leakage impedance of the substation transformer remains, so that the path between the wind farm and the infinite bus becomes an essentially undamped inductive branch and the vector margin tends to a lower bound set by the transformer alone.

For every value of SCR scanned in Figure 5.20, VM remains comfortably above the 0.5 robustness threshold defined in Section 3.6.3, which shows that the final design — including the LC re-sizing, the CCF active damping and the critical-mode notch filter damping of Sections 5.1–5.3 — is robust with respect to grid-strength variation. The role of the synchronous condenser in this study can therefore be summarised as follows: in the harmonic range, the SC does not, by itself, shift the stability boundary or alter the worst-case vector margin in any significant way; its function is to raise the effective SCR of the onshore grid and to place the operating point of the wind farm further away from the low-SCR region in which VM may begin to increase or decrease depending on the initial values of SCR, onshore substation transformer and the grid while being robust in the entire range.

5.5 Synthesis and Outlook

The combined effect of the three mitigation measures is summarised in Table 5.3. The wind farm has been progressively moved from the baseline configuration, which was stable but fragile, to a final configuration with $VM = 0.63$ and a disk margin above the robust threshold $\alpha_{mm} = 0.5$, both indicators being well within the comfortable region defined in Sections 3.6.3 and 3.6.4.

The grid-strength sweep of Section 5.4.4 further confirms that the final design remains above the 0.5 vector-margin threshold across the entire SCR range scanned, and that the synchronous condenser at the onshore connection acts essentially as an SCR booster without introducing any new harmonic-range dynamics.

Table 5.3: Worst-case stability and robustness indicators after each mitigation step.

Configuration	$\mathcal{P}_{Y_{WF}}^{\min}$	VM	α_{mm}	f_{critical}
Baseline	≥ 0 (marginal)	0.10	< 0.3	~ 242 Hz
+ LC filter re-sizing	≥ 0	0.23	≥ 0.3	~ 180 Hz
+ CCF active damping	≥ 0	0.29	≥ 0.3	~ 180 Hz
+ Critical-mode notch	≥ 0 (strong)	0.63	> 0.5	~ 1100 Hz

Once the notch filter was applied, the global minimum of $VM(\omega)$ migrated to the cluster around 1.1 kHz. The dominant contribution to $\mathbf{Y}_{WF}(j\omega)$ in this band comes from the offshore-collector-transformer leakage in series with the HVAC export cable, whose distributed-parameter resonance pattern was identified in Section 4.9. In the present configuration the corresponding vector margin remains comfortably above 0.5, so no further measure is required for the studied case.

If the same residual resonance approach the stability boundary in a higher frequency region — for example for a different HVAC cable parameters, or a different array-cable layout — two complementary measures can be considered.

The first option is a passive damping branch installed at the POC and tuned at the residual frequency, which dissipates the corresponding resonance energy independently of any control implementation and is therefore inherently robust to controller-design uncertainty. The trade-off, as already discussed in the previous chapter, is the introduction of permanent conduction losses and the additional volume and weight of the passive components, which is non-trivial inside an offshore platform.

The second option is a wind-farm-level coordination of the active damping action, in which the notch contribution at f_n is distributed unevenly across the turbines or strings instead of being applied identically at every GSC. By assigning different damping shares to different turbines, the risk of common-mode interaction between converters acting with identical dynamics is reduced, and the damping effort can be allocated where it is most effective according to each turbine's contribution to the critical mode. The cost is the introduction of a centralised dispatch layer at wind-farm level and of a supervisory communication channel between the dispatch and the turbine controllers.

Chapter 6

Conclusion and Future Work

This thesis investigated the small-signal stability and robustness of an offshore wind power plant connected to the high-voltage grid through power-electronic converters, with the HVAC connection of the Princess Elisabeth Energy Island adopted as a representative case study. Starting from the analytical dq admittance of a Type-4 wind turbine, the analysis was progressively extended to the full offshore plant up to the Point of Common Coupling (PCC), the small-signal stability and robustness of the baseline design were quantified through a coherent set of four indicators, and a converter-side stability-enhancement strategy was designed, tuned and shown to restore an acceptable robustness.

6.1 Answers to the Research Sub-Questions

The four sub-questions of Section 1.1 can now be answered as follows.

(i) Modelling framework. The comparison carried out in Chapter 3 motivated the choice of the impedance-based framework over the state-space approach for OWF stability studies, on the basis of its modularity, its compatibility with both simulated and experimentally measured data. Within this framework, each subsystem of the Phase 1 connection was represented as a frequency-dependent dq admittance or impedance matrix: the Type-4 wind turbine through a reduced grid-side-converter admittance for the harmonic range and a complete machine-side, DC-link and PLL model for the sub-/super-synchronous range; the wind-turbine, offshore-substation and onshore-substation transformers through their leakage impedance with synchronous-frame coupling; the array cables through a lumped nominal π -section, and the 55 km HVAC export cable through an equivalent π -model derived from the telegrapher equations; the grid through a Thevenin equivalent with parameterised short-circuit ratio; and the synchronous condenser through its dq impedance with frequency-dependent operational inductances. These component models were then assembled by series and parallel combination and reduced through Kron reduction to obtain the aggregated wind-farm admittance $\mathbf{Y}_{WF}(j\omega)$ at the PCC, on which the system-level stability and robustness assessment is built.

(ii) Stability and robustness indicators. Chapter 3 identified four indicators that are natively expressed in the impedance-based framework and that, when applied jointly, provide a complete picture of the small-signal behaviour of the plant. The Generalized Nyquist Criterion (Section 3.6.2), applied to the eigenloci of the minor-loop gain $\mathbf{L}(j\omega) = \mathbf{Z}_g(j\omega) \mathbf{Y}_{WF}(j\omega)$, yields a binary stable/unstable verdict through the encirclements of the critical point $(-1 + j0)$. The passivity index (Section 3.6.1), defined as the minimum eigenvalue of the Hermitian part of the wind-farm admittance $\mathbf{Y}_{WF}(j\omega)$,

provides a sufficient stability condition with respect to any passive grid impedance. The vector margin (Section 3.6.3) measures the radius of the largest disc centred at $(-1 + j0)$ that does not intersect any eigenlocus of the loop gain — i.e., the smallest additive perturbation that drives the closed loop to the stability boundary — while the disk margin (Section 3.6.4) quantifies the largest simultaneous gain and phase variation that the loop can absorb without losing stability. This four-indicator framework is applied consistently throughout Chapters 4 and 5, so that the effect of every design and mitigation choice can be assessed on a like-for-like basis.

(iii) Baseline stability and robustness. The full network model assembled in Chapter 4 is small-signal stable in the Generalized Nyquist sense and globally passive, but the vector margin reaches a worst-case value $VM = 0.10$ and the disk margin drops to $\alpha_{mm} = 0.096$ at $f \approx 242$ Hz, both well below the acceptable thresholds. The baseline design is therefore *stable but fragile*, and the four indicators consistently identify the oscillations from lightly damped resonance coming from the GSC’s LCL filter (the converter LC combined with the WT transformer leakage) as the dominant source of fragility through its interaction with the rest of the collection network and with the grid in the 100–400 Hz band.

(iv) Stability-enhancement techniques. Three converter-side mitigation techniques were applied incrementally in Chapter 5. The LC filter was first re-sized: the inductance L_f proved not to be a robustness lever because the GSC current-controller gain is automatically re-tuned with it, while reducing the filter capacitance to $C'_f = 0.4 C_{f,0}$ pushed the LCL resonance from ~ 434 Hz to ~ 687 Hz and raised the worst-case vector margin from 0.10 to 0.23. Capacitor-Current-Feedback active damping with an optimal gain $K_{ad}^* \approx 0.15$ provided a modest additional improvement, because after the re-sizing, the worst-case mode of the loop gain had migrated to a network-driven mode around 180 Hz sitting well below the GSC current-controller bandwidth of 400 Hz, where the converter behaves as a near-ideal current source and provides essentially no damping to it. The decisive step was a depth-controllable second-order notch filter tuned on the critical mode of the loop gain ($f_n^* \approx 180$ Hz, $\zeta_n^* \approx 0.2$, $\zeta_d^* \approx 0.7$), so that the GSC actively reacts to the dominant mode of the aggregated wind-farm/grid network, which raised the worst-case vector margin to $VM = 0.63$ and the disk margin above 0.5, both well within the comfortable robustness region. The planned onshore synchronous condenser was then modelled as a 2×2 frequency-dependent dq impedance: it introduces no resonance in the harmonic range and acts essentially as an SCR booster, and a dedicated SCR sweep confirmed that the final design remains above the 0.5 vector-margin threshold across the entire scanned range of grid strengths.

6.2 Limitations and Future Work

The conclusions of this work hold within a well-defined domain of validity. The analysis is small-signal and linear by construction, and therefore does not address large-disturbance phenomena such as faults, fault-ride-through, protection actions or controller saturation, for which EMT time-domain studies remain complementary. The EMT validation was carried out at the level of a single Type-4 wind turbine; an explicit EMT-based validation of the full wind-farm admittance at the PCC was beyond the scope of this thesis. The wind-turbine converters were studied in grid-following mode only, and several parameters not publicly disclosed were replaced with values from the peer-reviewed literature and should be revisited when the corresponding manufacturer data become available. Finally, the system-level stability assessment was restricted to the harmonic range 100 Hz–2.5 kHz; the complete

sub-/super-synchronous wind-turbine model derived in Section 3.2.2 was not exercised at the network level.

These limitations naturally suggest the directions for further research. The first is the extension of the system-level assessment to the sub-/super-synchronous range using the complete Type-4 model, in order to address PLL- and DC-link-driven synchronisation phenomena. The second is the explicit EMT-based validation of the full wind-farm admittance and the time-domain verification of the proposed mitigation strategy under large-disturbance events. The third is the wind-farm-level coordination of the active-damping action. The fourth is the inclusion of grid-forming wind-turbine control, which reshapes the converter admittance fundamentally and is a natural candidate for future low-SCR offshore connections. The fifth is the extension to the HVDC export phase planned for the third building block of the Princess Elisabeth Zone, whose offshore and onshore VSC-HVDC converters introduce additional dynamic interfaces that fit directly into the present framework.

Beyond the specific case of the Princess Elisabeth Energy Island, the framework developed in this thesis is generic and modular, and the systematic assessment of harmonic stability and robustness is expected to become a routine step in the design of the next generation of offshore wind connections.

Appendices

A Small-signal linearization

The averaged dq -domain model of a converter is still nonlinear in general, because it includes products of states and inputs, nonlinear trigonometric terms from synchronization, saturation or limiter effects, and modulation-dependent relationships. For stability analysis, the usual next step is to linearize the nonlinear model around an operating point. Let the nonlinear system be written as

$$\dot{x} = f(x, u), \quad y = g(x, u). \quad (1)$$

Assume that (x^*, u^*) is an equilibrium point satisfying

$$f(x^*, u^*) = 0. \quad (2)$$

Then the state, input, and output are decomposed into steady-state values and small perturbations:

$$x = x^* + \Delta x, \quad u = u^* + \Delta u, \quad y = y^* + \Delta y. \quad (3)$$

Applying a first-order Taylor expansion around the equilibrium gives

$$\Delta \dot{x} = \left. \frac{\partial f}{\partial x} \right|_{\star} \Delta x + \left. \frac{\partial f}{\partial u} \right|_{\star} \Delta u + \mathcal{O}(\|\Delta\|^2), \quad (4)$$

$$\Delta y = \left. \frac{\partial g}{\partial x} \right|_{\star} \Delta x + \left. \frac{\partial g}{\partial u} \right|_{\star} \Delta u + \mathcal{O}(\|\Delta\|^2). \quad (5)$$

Neglecting the second-order terms yields the linear small-signal model

$$\Delta \dot{x} = A \Delta x + B \Delta u, \quad \Delta y = C \Delta x + D \Delta u, \quad (6)$$

with Jacobian matrices

$$A = \left. \frac{\partial f}{\partial x} \right|_{\star}, \quad B = \left. \frac{\partial f}{\partial u} \right|_{\star}, \quad C = \left. \frac{\partial g}{\partial x} \right|_{\star}, \quad D = \left. \frac{\partial g}{\partial u} \right|_{\star}. \quad (7)$$

B Detailed Derivation of the 14 MW Type-4 Wind Turbine Simulation Parameters

Operating-Point Assumptions and dq Reference Frame

The operating point used in the simulation corresponds to rated active-power export at unity power factor. The main assumptions are:

- rated active power: $P_0 = 14$ MW;
- reactive power setpoint: $Q_0 = 0$ MVar;
- grid frequency: $f_0 = 50$ Hz;
- low-voltage converter-side line-to-line RMS voltage: $V_{LL,RMS} = 4$ kV;

- DC-link voltage: $V_{dc,0} = 10$ kV;
- rated mechanical speed: $\omega_{m,0} = 0.8$ rad/s.

A grid-voltage-oriented synchronous dq reference frame is used. The PLL aligns the d -axis with the PCC voltage vector, such that

$$v_{q,0} \approx 0, \quad v_{d,0} = U_0. \quad (8)$$

For a balanced three-phase system, the d -axis voltage magnitude is equal to the phase peak voltage. Therefore,

$$U_0 = v_{d,0} = \sqrt{\frac{2}{3}} V_{LL,RMS}. \quad (9)$$

Using $V_{LL,RMS} = 4$ kV gives

$$U_0 = \sqrt{\frac{2}{3}} \times 4000 \approx 3266 \text{ V}. \quad (10)$$

The dq power convention used in this thesis is

$$P = \frac{3}{2} (v_d i_d + v_q i_q), \quad (11)$$

$$Q = \frac{3}{2} (v_q i_d - v_d i_q), \quad (12)$$

where positive Q denotes reactive power absorbed by the converter (load convention).

Since $v_{q,0} = 0$ and $Q_0 = 0$, the steady-state q -axis current on the grid side is approximately zero. The d -axis current is then obtained from

$$P_0 = \frac{3}{2} U_0 I_{GSC,d0}, \quad (13)$$

which gives

$$I_{GSC,d0} = \frac{2P_0}{3U_0} = \frac{2 \times 14 \times 10^6}{3 \times 3266} \approx 2858 \text{ A}. \quad (14)$$

The electrical rotor speed of the PMSG is obtained from the mechanical speed and the number of pole pairs:

$$\omega_{r,0} = n_p \omega_{m,0}. \quad (15)$$

With $n_p = 162$ and $\omega_{m,0} = 0.8$ rad/s,

$$\omega_{r,0} = 162 \times 0.8 = 129.6 \text{ rad/s}. \quad (16)$$

The magnet flux linkage ψ_m is not directly provided in the published parameter set. It is therefore estimated from the steady-state PMSG voltage relation under the no-load assumption. With $i_{md,0} = i_{mq,0} = 0$ and neglecting the stator resistance term, the terminal voltage reduces to

$$v_{mq,0} \approx \omega_{r,0} \psi_m. \quad (17)$$

Taking the generator-side rated voltage consistent with the 4 kV converter-side voltage gives

$$\psi_m \approx \frac{U_0}{\omega_{r,0}} = \frac{3266}{129.6} \approx 25.2 \text{ Wb}. \quad (18)$$

This no-load estimate slightly overestimates ψ_m once the loaded v_{md} component is non-zero, but the

error is small in the analytical model.

Derived Electrical Parameters for the Simulation Model

The electrical parameters used in the analytical and EMT simulations are summarised in Table 4.1. The table distinguishes between directly adopted values and derived operating-point quantities.

Grid-Side Converter Operating Point

For the grid-side converter, the PLL-oriented reference frame gives $v_{q,0} = 0$ and $v_{d,0} = U_0$. At unity power factor, the grid-side current operating point is therefore

$$I_{\text{GSC},d0} = 2858 \text{ A}, \quad I_{\text{GSC},q0} = 0 \text{ A}. \quad (19)$$

The converter modulation index depends on the normalisation used in the average converter model. In this work the average converter voltage is represented as

$$v_{\text{conv},dq} = \frac{V_{\text{dc}}}{2} m_{dq}, \quad (20)$$

so that

$$m_{dq} = \frac{2 v_{\text{conv},dq}}{V_{\text{dc}}}. \quad (21)$$

At steady state, assuming a small filter resistance and $i_{q,0} \approx 0$, the d -axis converter voltage is approximately equal to the PCC voltage:

$$v_{\text{conv},d0} \approx U_0. \quad (22)$$

Therefore,

$$M_{\text{GSC},d0} = \frac{2U_0}{V_{\text{dc},0}} = \frac{2 \times 3266}{10\,000} \approx 0.653. \quad (23)$$

The q -axis converter voltage contains the synchronous-frame cross-coupling term of the filter:

$$v_{\text{conv},q0} \approx \omega_s L_f I_{\text{GSC},d0}. \quad (24)$$

Thus,

$$M_{\text{GSC},q0} = \frac{2 \omega_s L_f I_{\text{GSC},d0}}{V_{\text{dc},0}} \approx 0.049. \quad (25)$$

The resulting GSC operating-point quantities are listed in Table 4.2.

Machine-Side Converter Operating Point

On the machine side, the turbine is represented by a direct-drive PMSG. For the rated operating point considered here, the d -axis stator current is assumed to be zero,

$$I_{\text{MSC},d0} = 0, \quad (26)$$

which is a common simplifying assumption when field-weakening is not considered. Under the non-salient approximation — i.e. neglecting the reluctance torque $\frac{3}{2}n_p(L_d - L_q)i_d i_q$, which is identically

zero here because $i_{d,0} = 0$ — the electromagnetic torque is

$$T_e = \frac{3}{2} n_p \psi_m i_q. \quad (27)$$

The mechanical torque corresponding to rated power is

$$T_0 = \frac{P_0}{\omega_{m,0}} = \frac{14 \times 10^6}{0.8} = 17.5 \times 10^6 \text{ N m}. \quad (28)$$

The corresponding q -axis machine current is therefore

$$I_{\text{MSC},q0} = \frac{2T_0}{3 n_p \psi_m} \approx 2858 \text{ A}. \quad (29)$$

The machine-side converter modulation indices are obtained from the steady-state PMSG dq voltage equations. With $i_{md,0} = 0$, the d -axis machine voltage is

$$v_{md,0} \approx -\omega_{r,0} L_q I_{\text{MSC},q0}, \quad (30)$$

and the q -axis machine voltage is

$$v_{mq,0} \approx R_s I_{\text{MSC},q0} + \omega_{r,0} \psi_m. \quad (31)$$

Using the same average converter normalisation,

$$M_{\text{MSC},d0} = \frac{2 v_{md,0}}{V_{\text{dc},0}} \approx -0.430, \quad (32)$$

$$M_{\text{MSC},q0} = \frac{2 v_{mq,0}}{V_{\text{dc},0}} \approx 0.674. \quad (33)$$

The negative sign of $M_{\text{MSC},d0}$ reflects the cross-coupling EMF $-\omega_{r,0} L_q I_{\text{MSC},q0}$ produced by the q -axis current through the q -axis inductance under zero- d -axis current control; it is not field-weakening, which would require $I_{\text{MSC},d0} < 0$.

The MSC operating-point quantities are summarised in Table 4.3.

Controller Transfer Functions and Gain Selection

The controller structures used in this work are conventional PI controllers of the form

$$H(s) = K_p + \frac{K_i}{s}. \quad (34)$$

Grid-side current controller. The current-loop bandwidth is selected as

$$\omega_{ci,\text{GSC}} = 2\pi \times 400 \text{ rad/s}, \quad (35)$$

sufficiently lower than the 5 kHz switching frequency to preserve the usual separation between switching frequency and current-loop bandwidth. For an RL filter, the PI gains are selected by pole-zero cancellation,

$$K_{p,\text{GSC}} = L_f \omega_{ci,\text{GSC}}, \quad K_{i,\text{GSC}} = R_f \omega_{ci,\text{GSC}}. \quad (36)$$

Using $L_f = 2.75 \times 10^{-4}$ H and $R_f = 0.001 \Omega$ gives

$$K_{p,\text{GSC}} \approx 0.691, \quad K_{i,\text{GSC}} \approx 2.513. \quad (37)$$

DC-link voltage controller. The small-signal relation between the DC-link voltage and the d -axis current can be approximated using the DC capacitor energy balance. Around the operating point, the plant can be written as an integrator with gain

$$G_{v_{\text{dc}} i_d}(s) \approx \frac{1.5 U_0}{C_{\text{dc}} V_{\text{dc},0}} \frac{1}{s}. \quad (38)$$

A second-order design with $\omega_n = 2\pi \times 10$ rad/s and damping ratio $\zeta = 0.707$ gives

$$K_{p,\text{dc}} = \frac{2\zeta\omega_n}{1.5 U_0 / (C_{\text{dc}} V_{\text{dc},0})}, \quad K_{i,\text{dc}} = \frac{\omega_n^2}{1.5 U_0 / (C_{\text{dc}} V_{\text{dc},0})}. \quad (39)$$

Substituting the numerical values gives

$$K_{p,\text{dc}} \approx 0.73, \quad K_{i,\text{dc}} \approx 32.2. \quad (40)$$

Phase-locked loop. The synchronous-reference-frame PLL is represented as a PI controller acting on the raw q -axis voltage V_q^c (in volts). The small-angle phase-detector linearisation $\tilde{V}_q^c \approx V_{id}^s(\tilde{\theta}_g - \tilde{\theta}_{\text{pll}})$, with $V_{id}^s \approx U_0$, introduces a voltage scaling factor into the loop. The PI gains are designed to absorb this factor at design time, so that the closed loop recovers the standard second-order PLL form (consistent with the formulation of $G_{\text{pll}}(s)$ in Section 3.2.2):

$$K_{p,\text{PLL}} = \frac{2\zeta\omega_n}{U_0}, \quad K_{i,\text{PLL}} = \frac{\omega_n^2}{U_0}. \quad (41)$$

Using $\omega_n = 2\pi \times 15$ rad/s, $\zeta = 0.707$, and $U_0 = 3266$ V gives

$$K_{p,\text{PLL}} \approx 0.0408, \quad K_{i,\text{PLL}} \approx 2.72. \quad (42)$$

Machine-side current controller. The MSC current loop is tuned with the bandwidth

$$\omega_{ci,\text{MSC}} = 2\pi \times 625 \text{ rad/s}. \quad (43)$$

Because the PMSG is mildly salient ($L_d \neq L_q$), a single isotropic inductance

$$L_s = \frac{L_d + L_q}{2} = \frac{8.7 + 5.8}{2} \times 10^{-3} = 7.25 \times 10^{-3} \text{ H} \quad (44)$$

is adopted for controller tuning, so that a common pair of gains applies on both axes. The PI gains are then

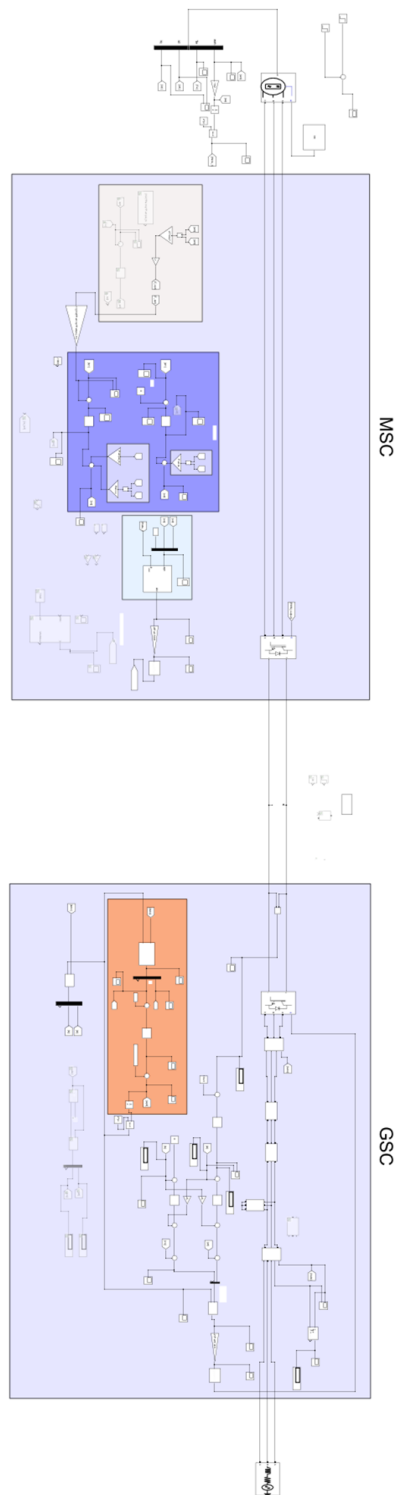
$$K_{p,\text{MSC}} = L_s \omega_{ci,\text{MSC}}, \quad K_{i,\text{MSC}} = R_s \omega_{ci,\text{MSC}}. \quad (45)$$

With $L_s = 7.25 \times 10^{-3}$ H and $R_s = 0.0368 \Omega$, this yields

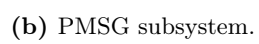
$$K_{p,\text{MSC}} \approx 28.47, \quad K_{i,\text{MSC}} \approx 144.51. \quad (46)$$

The adopted controller gains are summarised in Table 4.4.

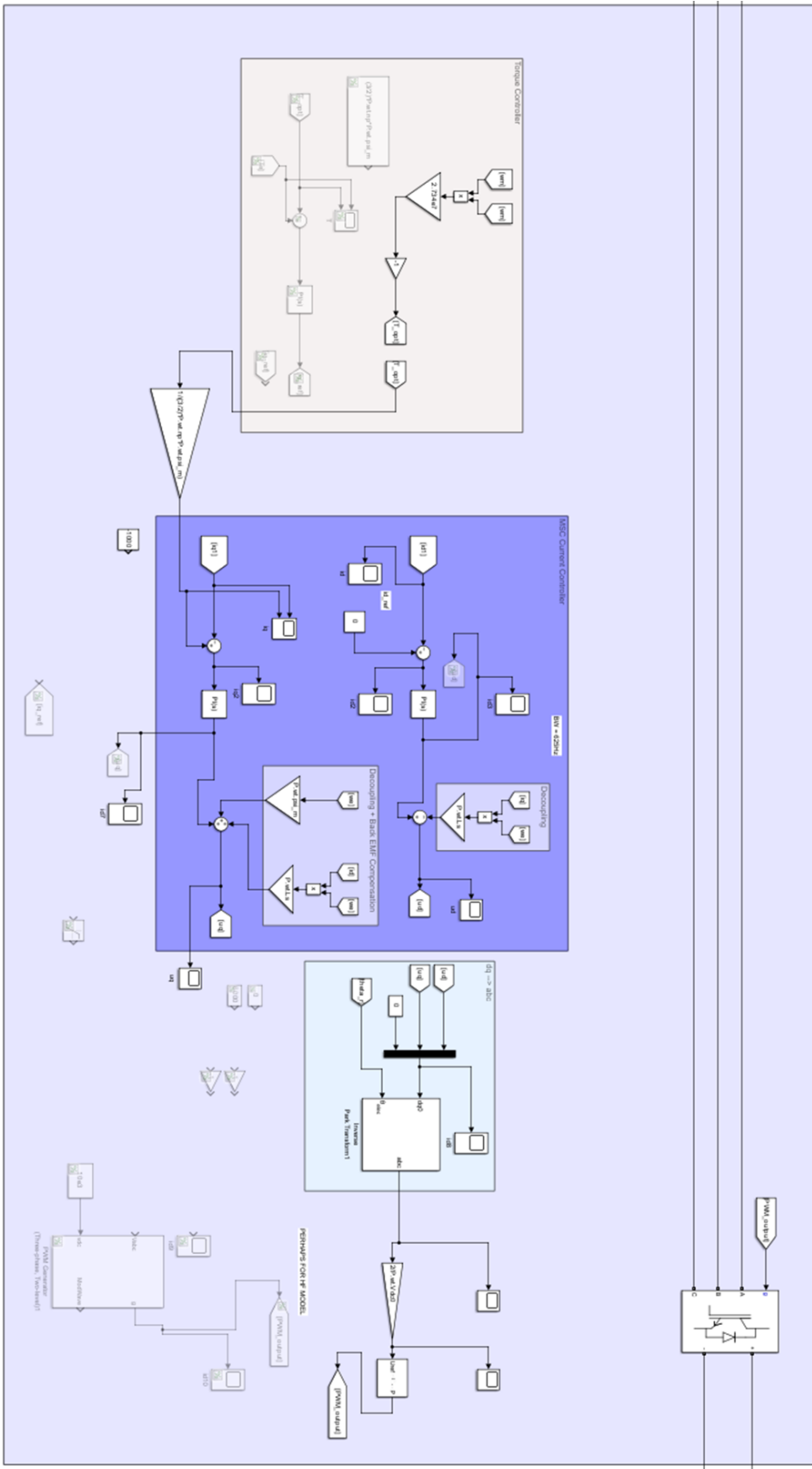
C EMT-Based Validation Setup in Simulink



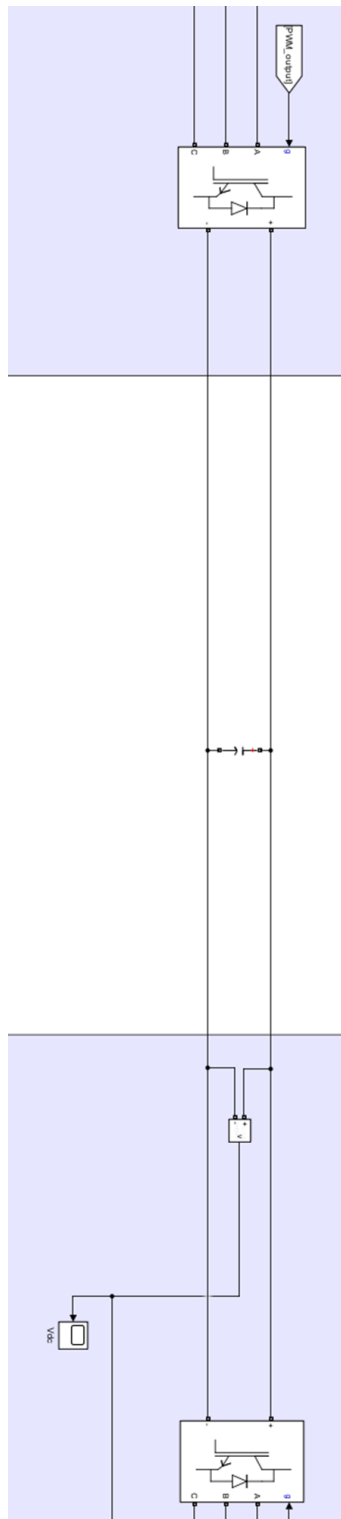
(a) Overview of the grid-connected Type-4 wind turbine EMT model.



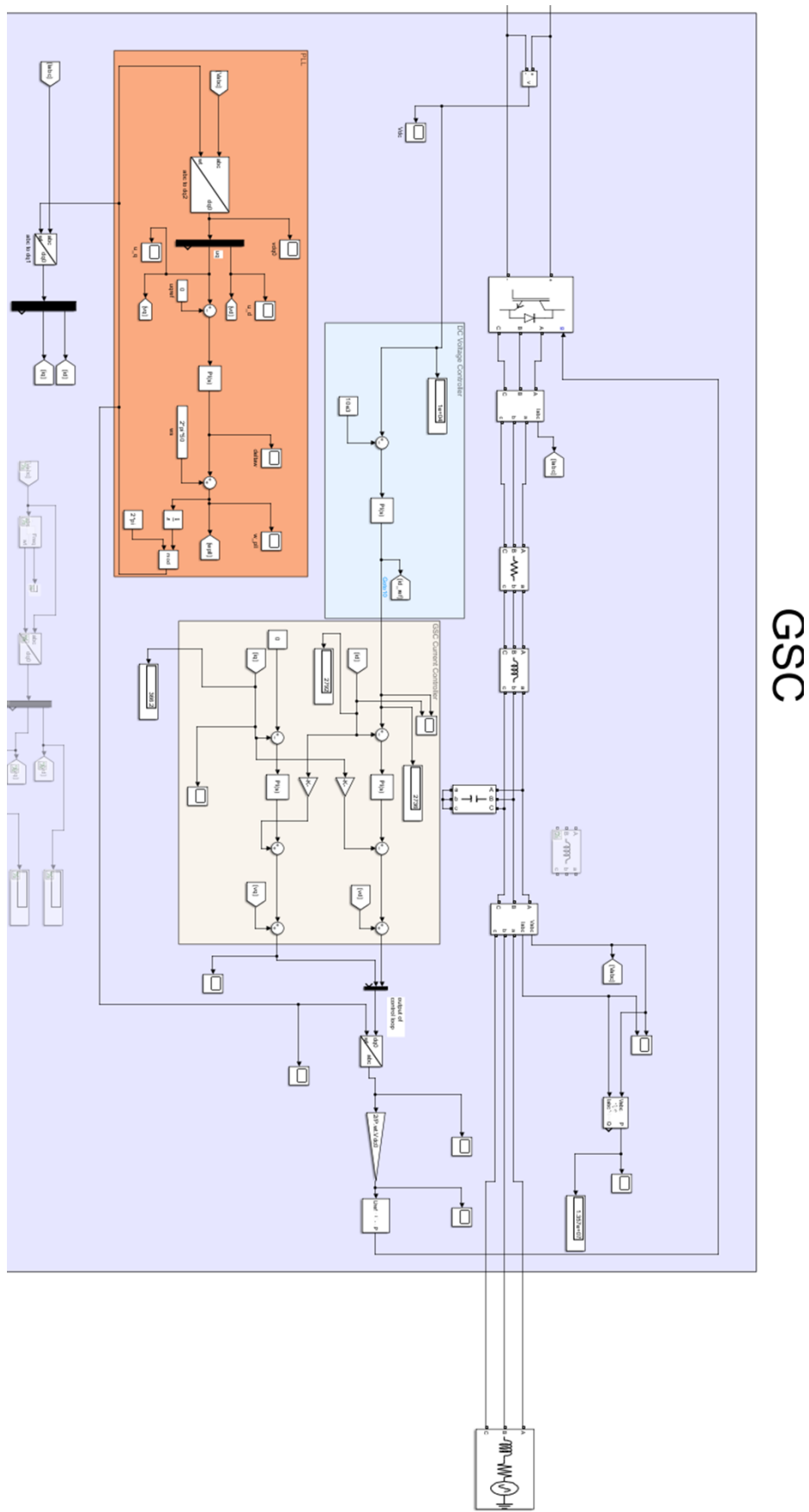
MSC



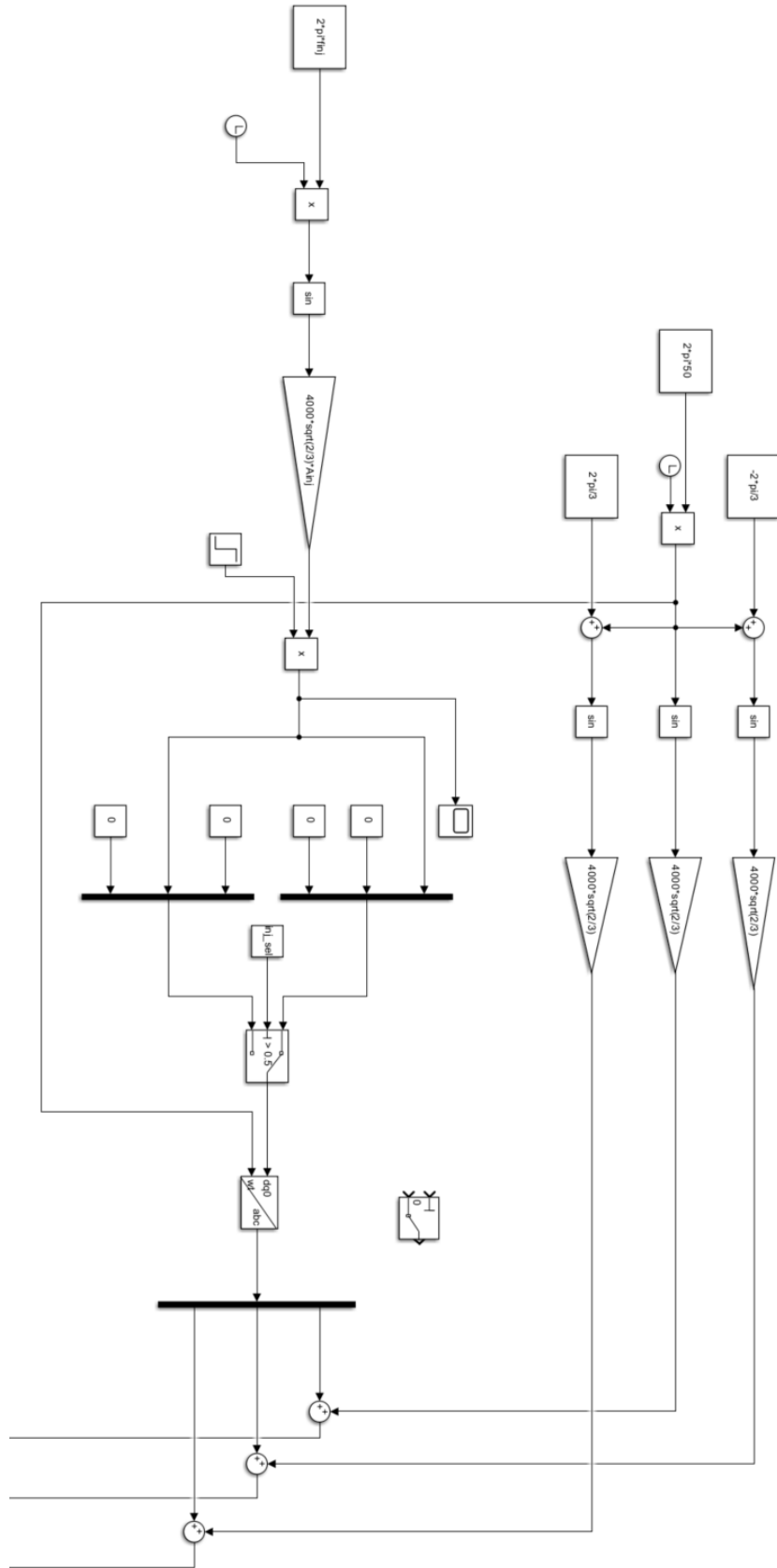
(c) MSC subsystem.



(d) DC-link subsystem.



(e) GSC-grid subsystem.



(f) Perturbation-injection setup.

D Nodal Admittance Assembly and Kron Reduction of a Wind-Turbine String

This appendix details the nodal admittance assembly and Kron reduction used in Section 4.6.2 to obtain the equivalent dq admittance of one wind-turbine string seen from its external terminal.

String Topology and Bus Indexing

Each 350 MW sub-block of the case-study wind farm contains five strings of five 14 MW wind turbines interconnected by inter-array cables. A single string therefore comprises $N = 6$ buses: five *internal* buses, one at the output of each wind-turbine step-up transformer, and one *external* bus at the string's connection point to the offshore collector. Each bus carries the two dq components, so the string is described by a bus admittance matrix $\mathbf{Y}_{bus}$ of dimension $2N \times 2N$.

Nodal Admittance Stamping

The string is assembled from its individual components using the nodal admittance method: every component contributes (or *stamps*) its admittance matrix into $\mathbf{Y}_{bus}$ at the bus indices to which it is connected. Two stamping rules are required — one for the inter-array cables, one for the wind turbines together with their step-up transformer.

Cable stamp. A cable connecting any two buses α and β contributes the 4×4 dq admittance derived in Section 3.4.1, written in block form as

$$\mathbf{Y}_{cable,dq} = \begin{bmatrix} \mathbf{Y}_{\alpha\alpha} & \mathbf{Y}_{\alpha\beta} \\ \mathbf{Y}_{\beta\alpha} & \mathbf{Y}_{\beta\beta} \end{bmatrix}, \quad \mathbf{Y}_{\alpha\alpha}, \mathbf{Y}_{\alpha\beta}, \mathbf{Y}_{\beta\alpha}, \mathbf{Y}_{\beta\beta} \in \mathbb{C}^{2 \times 2}. \quad (47)$$

These four blocks are added to the corresponding 2×2 entries of $\mathbf{Y}_{bus}$:

$$\begin{aligned} \mathbf{Y}_{bus,\alpha\alpha} &\leftarrow \mathbf{Y}_{bus,\alpha\alpha} + \mathbf{Y}_{\alpha\alpha}, & \mathbf{Y}_{bus,\alpha\beta} &\leftarrow \mathbf{Y}_{bus,\alpha\beta} + \mathbf{Y}_{\alpha\beta}, \\ \mathbf{Y}_{bus,\beta\alpha} &\leftarrow \mathbf{Y}_{bus,\beta\alpha} + \mathbf{Y}_{\beta\alpha}, & \mathbf{Y}_{bus,\beta\beta} &\leftarrow \mathbf{Y}_{bus,\beta\beta} + \mathbf{Y}_{\beta\beta}. \end{aligned}$$

Wind-turbine stamp. A wind turbine combined with its step-up transformer is a shunt element, so its 2×2 admittance $\mathbf{Y}_{WT,dq}$ contributes only to the diagonal block of the bus α at which it is connected:

$$\mathbf{Y}_{bus,\alpha\alpha} \leftarrow \mathbf{Y}_{bus,\alpha\alpha} + \mathbf{Y}_{WT,dq}.$$

Applying these two rules to every inter-array cable and every wind turbine of the string completes the assembly of $\mathbf{Y}_{bus}$.

Kron Reduction

For the stability analysis, only the terminal behaviour of the string at its external bus is of interest; the internal buses carry no externally injected current and can therefore be removed without loss of information [91]. The bus set is partitioned into two disjoint subsets: the set $\mathcal{C}$ of *external* (kept) buses — here reduced to the single external bus — and the set $\mathcal{I}$ of *internal* buses to be eliminated. Reordering $\mathbf{Y}_{bus}$ and the injected-current vector with this partition,

$$\begin{bmatrix} \mathbf{I}_c \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{cc} & \mathbf{Y}_{ci} \\ \mathbf{Y}_{ic} & \mathbf{Y}_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{V}_c \\ \mathbf{V}_i \end{bmatrix}, \quad (48)$$

where $\mathbf{Y}_{cc}$ and $\mathbf{Y}_{ii}$ are the self-admittance blocks of the external and internal sets, $\mathbf{Y}_{ci}$ and $\mathbf{Y}_{ic}$ the cross-coupling blocks between them, and the zero on the left-hand side reflects the absence of current injection at internal nodes. The lower block gives $\mathbf{V}_i = -\mathbf{Y}_{ii}^{-1}\mathbf{Y}_{ic}\mathbf{V}_c$, which substituted into the upper block yields the equivalent admittance seen at the external terminal — the Schur complement of $\mathbf{Y}_{ii}$ in $\mathbf{Y}_{bus}$ [91]:

$$\mathbf{Y}_{eq} = \mathbf{Y}_{cc} - \mathbf{Y}_{ci} \mathbf{Y}_{ii}^{-1} \mathbf{Y}_{ic}. \quad (49)$$

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