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Markov Decision Process modelling of football attacks and analysis of the optimal actions

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Chapter 1

Introduction

Data analysis is becoming increasingly important in today's society and therefore also in the world of sport. The use of data began with the simple analysis of basic data to compare players and teams (such as points scored, rebounds and assists in basketball) and continued with more advanced analyses such as that of Bill James around 1980[1] Bill James invented Sabermetrics[2] which are more detailed and unbiased statistics of baseball to define a player's level. Those statistics includes the runs created[3] (the number of runs a player makes for his team) or secondary average[4] (which is the sum of the bases gained and is more precise than the simple ratio between the number of bases hit and the number of at bats). Other similar analyses to analyse the performance of a player or a team have been developed in other sports especially in football. Football is the most popular sport in the world and continues it expansion to other continents than South America and Europe. The teams are therefore always looking for ways to improve. They try to recruit the best players and coaches to achieve their objectives. Moreover, with the increasing use of data in the world and in the world of sports they also use it for different purposes. The first is the player recruitment. For example Brendford FC[5] is using algorithms to find unknown performing players and Everton FC[6] has signed a contract with the video game Football Manager to have access to their database which contained an enormous number of players and characteristics. The second purpose is the performance analysis and optimisation which is the topic of this thesis. It will be developed in this introduction.

In a football match the players are trying to score more goals than their opponents . A way to improve the results of a team is to help the players to make the best choices when they have the ball in order to increase the number of goals scored. The choice made by the players is the action they carry out.

Definition 1.1 (Action)**.** An action is what a player chooses to do when he has the ball for a specific purpose. Pass the ball and shot are actions just as dribble or carry the ball which will be use later.

However since the goals are rare events it is not the best way to evaluate the quality of a choice. In fact there are only three goals scored per match even though each team is in possession of the ball ninety-seven times per game on average. A goal can happen randomly, with an opponent's mistake or by the success of a complicated movement. Therefore the quality of a possession will be expressed by the number of goals it should have generated.

Definition 1.2 (Possession)**.** A possession is all the actions and events that occurs when a team has the control of ball and that ends when this team loses the ball or scores. It also ends when it is the end of one half.

The previous definition uses the term event which also needs to be define.

Definition 1.3 (Event)**.** An event is all that can occurs during a football match. It can be an action just like a shot, a pass, a dribble etc. It can also be a defensive fact like a ball recovery, a pressure or foul committed etc, or an information like the starting team a substitution or the end of a half.

A right choice is an action which helps the team to increase the probability of scoring during the possession. How to increase the probability of scoring for a team? The first thing we could think of when speaking about increasing the probability of scoring is the location of the striker when he shoots. In average, the closer the striker is from the goal the easier it is to score and the larger is the probability of scoring. A way to increase the probability of scoring could be to make passes in order to get closer to the goal. However, there seems that making a pass when the players are close to the goal is riskier since there are generally more defenders close to their goal. In order to verify those assumptions, a computation of the percentage of scoring shots and successful passes depending on the location on the pitch can be made.

Definition 1.4 (Scoring shot)**.** A shot is the action of kicking the ball towards the goal in an attempt to score.
A scoring shot is a shot leading to a goal namely a shot which is neither blocked by a defender nor saved by the goalkeeper. The ratio of scoring shots is the number of scoring shots divided by the number of attempted shots. The percentage of scoring shots is this ratio converted in percents.

Definition 1.5 (Successful pass)**.** A pass is the action of kicking the ball to a teammate.
A successful pass is a pass received by a teammate of the passer. A successful pass is neither intercepted by an opponent nor goes out of bounds. Similarly the ratio of successful passes is the number of successful passes divided by the number of attempted passes and the percentage of successful passes is this ratio in percents.

A shot has for now two attributes. The first one is the location where the shot starts so where the striker is when he kicks the ball. The second one is the result whether the goal is scored with this shot or not. The result can be more detailed especially on why the goal is not scored (saved by the keeper, blocked by a defender, shoot off target etc.) but it does not matter for the following plot. This plot can be done using the data from the Understat website[7]. Understat is a website collecting data from over fourteen thousand matches. The main data is about the shots taken during games in European leagues. There are more than three hundred thousand shots in the dataset. Each shot also have several features including the ones needed now: the location and the result. Other features and details about the dataset are developed at the appendix A.1. The locations of the shots are accurate to the fourteenth decimal place, so much that each location is unique. Even rounding up to a decimal would not give an interesting average for each location as there would be too few shots per location. For example if we consider that the shots are taken within thirty metres of the goal and over a width of forty metres there will be less than three shots on average per location to calculate the percentage of scoring shots. Moreover there are a little over thirty thousand scoring shots in the Understat dataset thus most of the percentage of scoring shots will be zero. Hence relying on two or three shots is not the best way to estimate the probability of scoring a goal from a given location. The pitch is divided in larger zones in which the percentage of scoring shots in each zone is computed. The figure 1.1 shows this ratio. On this plot and the others the direction of play is the right. The x-axis is in the length direction and the y-axis in the width direction.

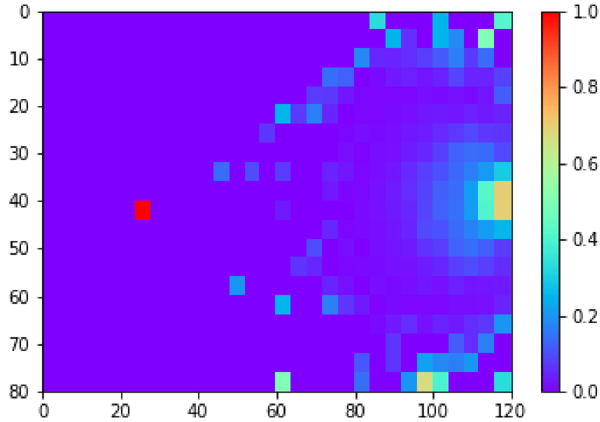


Figure 1.1: Ratio of scoring shots in zones of the pitch

A problem still occurs, it is that the ratio of scoring goals is very large in some zone distant from the goal. It is because there are only a few shots kicked in those zones and the shots are taken in suitable situation thus the results are not representative . For example, in the red zone with a ratio of scoring shots of one located in the own half of the attacking team there is only one shot taken and scored. This goal was scored by a player of SC Paderborn 07 in the last minute of the match against Hannover 96 in 2014 when the keeper of Hannover was in the penalty area of Paderborn after a corner while trying to equalize[8] (figure 1.2).

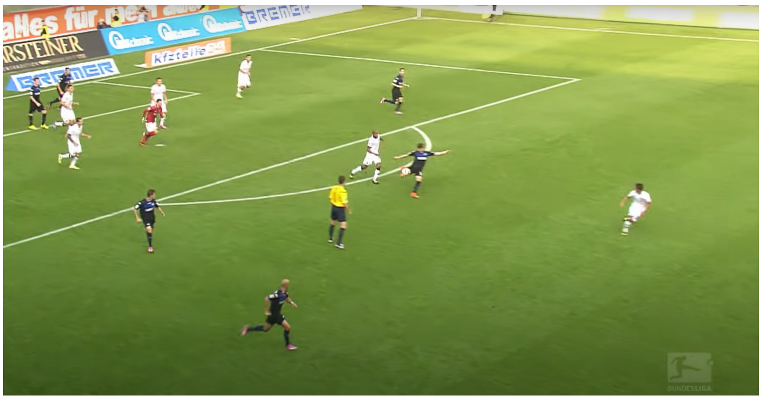


Figure 1.2: Record-breaking[9] goal of Stoppelkamp a Paderborn player

We can reasonably think that even if the ratio of scoring shots is one, the optimal solution is not always shooting at more than eighty metres away from the opposite goal. To solve this problem we will look only at the zones with a significant number of shots made from these zones. Because we consider that a shot has only two attributes for now a shot can be modelled as a Bernoulli trial. The probability of success of scoring is the ratio of scoring shots which is the mean of the number of success per trial. Therefore to have a good approximation of this ratio the confidence interval on the mean has to be small. If the number of shots is larger than thirty-five the error is smaller than 0.1. By keeping only the zone where there are more than thirty-five shots his we obtain the figure 1.3.
The intuition on the chances of scoring was true because the probability of scoring in the two zones closest to the goal is up to 0.70 compared with 0.13 twenty metres away.

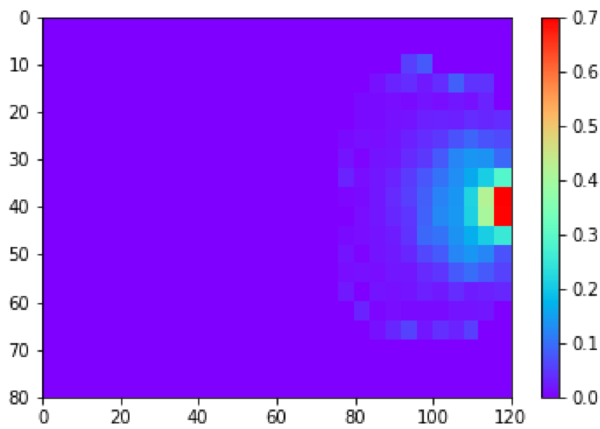
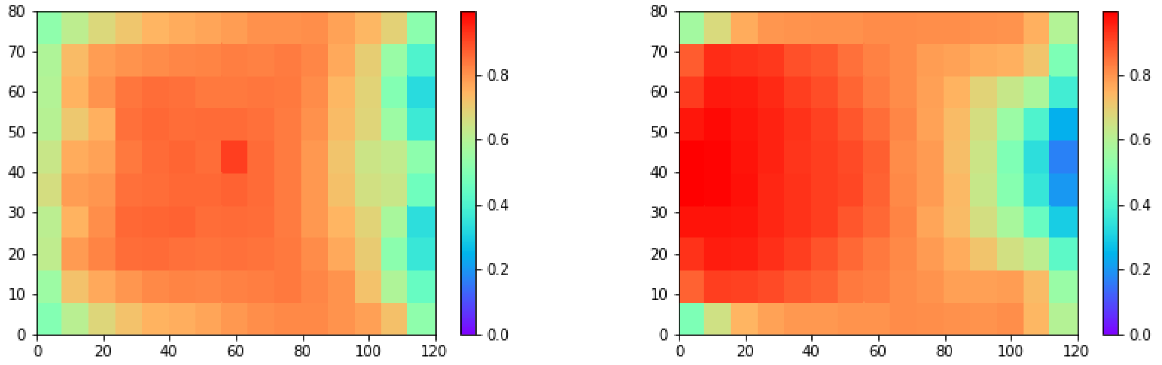


Figure 1.3: Ratio of scoring shots in significant zones of the pitch

So why the players does not just pass the ball to their striker located in one of the zones with the larger success rate ? As suggested above, making a pass to these zones is difficult, is less likely to succeed. A pass has three attributes. The first one is, as for a shot, the starting location of the pass so where the player making the pass is when he kicks the ball. Unlike the shot, there is also the ending location of the pass so where the player receiving the pass is. The last argument is the result of the pass mainly success or failure. The study of the risk of a pass can be done in two ways. Either looking at the percentage of successful pass made from a location or either to a location. As for the shots, the pitch is divided in zones to obtain a indicative value. The figure 1.4a shows the ratio between the successful passes starting from a zone and all the passes starting from this zone. The second figure (1.4b) shows the percentage of successful passes arriving in the zones.

These graphs are obtained with the open dataset of StatsBomb[10]. StatsBomb analyses matches in 90 leagues and uses camera calibration, pitch detection computer vision and human input to obtain information on the matches. Firstly there is descriptive data like the two competing teams, the final result, the league in which the match is played etc. Here it is the general analysis that will be considered and not the analysis of particular games so this sort of data will not be used. In addition to the descriptive data there are also all the events of the matches.
The free dataset of StatsBomb provides the events of 1096 matches in JSON. The matches are mainly games of FCBarcelona in the Spanish league between 2004 and 2021 (520 matches). There are also some Champions League finals (14 matches), some matches of the Invincibles of Arsenal in 2003/2004 in the English league. All the 64 matches of the 2018 World Cup and the 51 matches of the 2021 Euro are also available in the dataset. The dataset also contains matches of women's football : 52 matches of the 2019 World Cup, 36 matches of the American League and 326 matches English league. After collecting the data on the github the future analyses will be done on these events. The main events are the passes, the carries, the dribbles and the shots. There are respectively 1,100,000, 900,000, 44,000 ad 28,000 occurrences of the events in the open dataset. The attributes of the events are explained in the appendix A.2.
StatsBomb also provides 360 frames for a part of these matches : 33 matches of FCBarcelona in 2020/2021 and the 51 matches of the 2021 Euro. A 360 frame contains the locations of the players and the type of each player: if the player is the actor of the event, if the player is a teammate of the actor and/or if he is the keeper.

The figure 1.4a shows that the passes are much riskier when the passer is close to the edge



(a) Ratios when the zone considered is the one where the passer is located

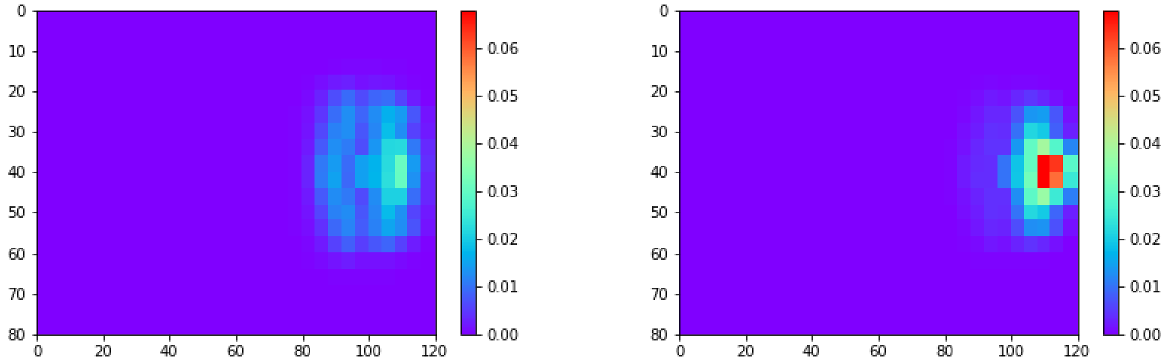
(b) Ratios when the zone considered is the one where the teammates receiving the ball is located

Figure 1.4: Ratios of successful passes depending on the zones of the pitch

but especially when close to the goal. The probability of passing the ball successfully is 0.79 on average and only 0.54 when the passer is in the last sixteen metres. The figure 1.4b confirms the difficulty to pass to a striker close to the goal with a percentage of successful passes in the last 16 metres of 0.46. Making a pass when you are in a forward position is difficult and it is even more difficult to receive a pass in this position. When a player is in a forward position there are generally more opponents than teammates and more opponents than in a position further from the goal. There are also less spaces between the defenders who are more likely to defend than sixty metres from their goal. Furthermore, for the two zones with the largest ratio of scoring shots the ratio of successful passes is 0.19. Therefore, there seems to have a inverse relation between the probability of scoring and the probability of passing the ball. There is a problem of finding the right balance between shooting and getting closer to the goal. Is this balance already found ? If so the percentage of shots taken in each zone should be similar to the percentage of goals scored in the zone. It means that the players shots when the probability of scoring is large otherwise they pass the ball. In other words the ratio between the number of shots starting from a zone and the total number of shots should be close to the same ratio but with the goals. It is because we consider that all actions are similar since the player shoots only when it is the best option i.e. making a pass or another action will decrease the probability of scoring during this possession. The figure 1.5 shows both percentages in the zones.

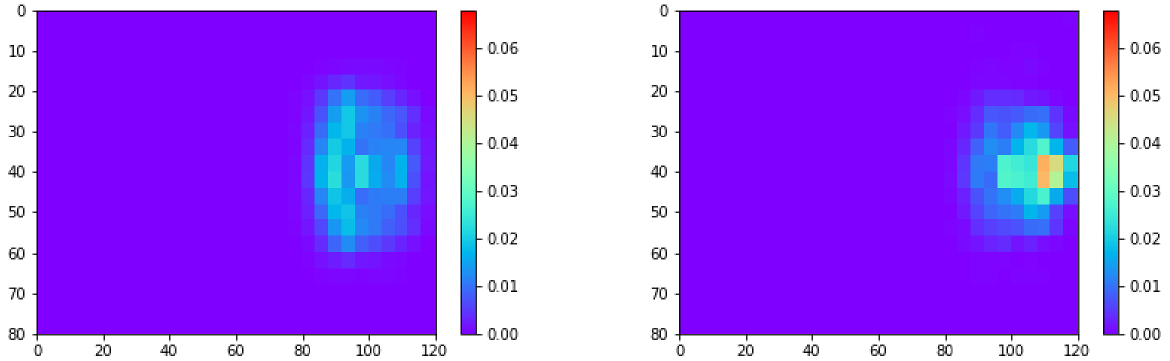
We observe that a large part of the goal are scored from the penalty area. No other conclusion can be made about this figures due to the difficulty to move forward on the field. To solve this, we compute the probability of managing a pass in the penalty area using the previous plot and computations about the percentages of success for the ending location of passes. This probability is equal to 0.36. Hence we reduce the number of scoring shots in the penalty area using this factor. The ratios of shots and goals in each zone compared to the total number of shots are goals are computed as earlier and shown at the figure 1.6.

This model is not perfect but it can give an initial idea about the use of the action "shot" by the player. Using these graphs there seems that the strikers makes attempt too far from the goal even though it is much more easier to score while closer from the goal and not that difficult to come nearer from the goal, in the penalty area.
The pitch in the previous plots is divided in zones. These zones are square to limit the distance



(a) Ratio of shot taken in each zone (b) Ratio of goal scored from each zone

Figure 1.5: Percentages of shots and goals depending on the zones of the pitch



(a) Percentage of shot taken in each zone (b) Percentages of goal scored from each zone

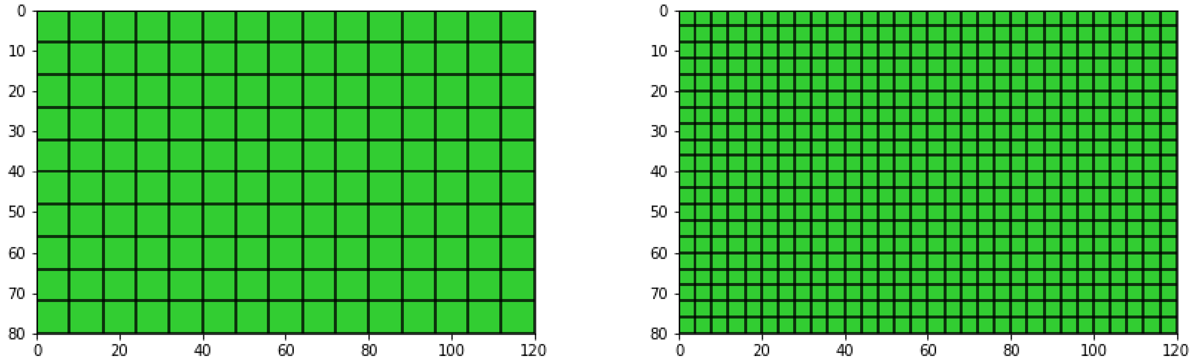
Figure 1.6: Percentages of shots and goals with a limiting factor in the penalty area

between points in the same zone. The zones are also square because it is simpler to divide the rectangular pitch in squares rather than in triangles or circles. In the StatsBomb dataset the dimensions of the pitch are one hundred and twenty metres by eighty metres. There are several possible divisions but the smaller the zones are the more accurate the results should be according to the location. Some possible divisions with rational lengths are

- 150 zones. The zones have a length of height metres. There are fifteen zone along the length and ten along the width.
- 384 zones. Here the zones have a length of five meters which corresponds to twenty-four by sixteen zones.
- 600 zones. The zones have a length of four meters. The number of zones on the length is thirty and twenty on the width.
- 1536 zones. The length of a square is two and a half metres. It is forty-height by thirty-two zones.
- And so one...

Having 1536 zones is too much because there are 900,000 carries. It means that the average number of carry per zone is 586. This number is slightly larger for the passes: 716. But this number is too small because if a player chooses to carry the ball from a zone he still has

many possibilities for its arrival location unlike the shooting action. More carries per zone will be needed to obtain the future success probabilities. Moreover choosing two divisions and comparing them may help to confirm or refute the trends found in this thesis. Choosing the division 384 and the division of 600 will not enable to have two scales because the size of the zone are too close. The lengths of the zones are four and five meters.
The first division is the one with 150 zones of height meters long shown at the figure 2.8a. The second division is 600 zones corresponding to square of length 4 metres. This division is shown at the figure 2.8b. Both divisions are symmetrical lengthwise.



(a) Division with 150 zones

(b) Division with 600 zones

Figure 1.7: Divisions of the pitch

The objective of this thesis will be to create and improve a model to obtain the best choice in all particular situation in order to increase the chance of scoring in a possession. It will start with the definition of an important concept called the expected goals. Afterwards the possessions will be modelled using the Markov Decision Process which will be explain later. The modelling will start with a simple example which will be improved with more states and actions. The model will then consider the set-pieces. The set-pieces are the penalties, the free-kicks, the corners and the throws-in. The following improvement is the consideration of the strengths of the players. The last improvement is the use of the 360 frames of StatsBomb to consider the location of the other players on the pitch. Once the model is obtained the best actions in each situation will be computed and shown. The best actions will be used for two purposes. The fist is to compare some real cases and the choices made by the players with the best actions obtained. The second is to compare the players according to how close they are to the best actions. Finally there are appendices in this thesis which add information to the model and results but are placed at the back when this would overload the main text.

Currently there are several ways to judge the quality of an action. There is a way to judge the chance of scoring during a possession depending on the location called the expected threat[11]. Looking at the increase or the decrease of the expected threat after and before and action enable to judge the quality of the action done. There is a similar work[12] which computes the difference of probabilities of scoring between two actions. It also compares the players according to how much they increases the probability of scoring of their team. Unlike those works, this master thesis will use the Markov Decision Process to obtain the optimal action in each situation. This enables to compare the actions done by the players with the optimal actions and not with an average probability of scoring.
Actually there are some articles about the Markov theory in football but most of them do not use the Markov Decision Process to obtain the optimal actions. Those articles have for subject

the modelling[13] of the set-pieces (throw-ins, free-kicks, goal kicks and corners) using 3 zones (defensive, central and offensive), obtaining[14] the strengths of a team using 9 zones on the pitch, the score or the gain or lost of the possession and obtaining[15] the quality of player considering the number of expected goal after and before an action done by the player. There is already an article[16] using a Markov model to help in decision making using the actions pass and shoot and a division of the pitch in zones. The model in this report will also use the action "carry the ball", the set pieces, the position of the other players and the strengths of the players.

Chapter 2

Model of a football attack

The goal of this chapter will be to model the possessions and the actions as a Markov Decision Process. The modelling will start from a simple example to explain what a Markov Decision Process is and how it can be used in our case. Afterwards various improvements will be added to the model. But just before the example, the concept of expected goals has to be explained.

2.1 Definition of the expected goals

While looking at the probability of scoring shots at the figure 1.1, the results were weird in some zone of the pitch and there were several problems. Those problem were not all solved by considering the significant zones just as at the figure 1.3.

- The first problem was that a shot in a very bad location was not often attempted unless the situation was favorable. The very bad location can be with a little angle like around the corner or very far away from the goal like in the example at the figure 1.2 with the goalkeeper in the opposite half. So if a shot is taken in this zone it is mostly when the condition are suitable and not representative of the average difficulty of scoring from this zone. The ratio between the goal and the shot in those zones is biased. Moreover since only a few shots are taken from this zone, the influence of a scoring shot is high. Also, if only one shot is taken and not scored from a zone, it does not mean that the probability of scoring from this zone is zero.
- The second problem is that the probability of scoring does not only depends on the position of the striker. It depends also of the positions of the defenders and of the goalkeeper, if the striker uses his strong foot or not etc. The expected goals help solving this problem.

Definition 2.1 (Expected goal)**.** An expected goal is the probability of a shot to be scored depending on different factors and based on the study of the past shots. The factors are the locations of all the players especially the striker, the type of the shot, the type of assist, the feet used by the striker and so one. The abbreviation is xG.

This statistic is more representative of the difficulty of the shots especially for the long-range shot of Paderborn at the figure 1.2. The expected goal of this shot was 0.01. It means that when the player is this far from the goal but centered in the width, with a player as close as that, with his strong foot, ... the probability of scoring is 0.01. In this situation a player only scores once in a hundred. In contrast a shot can be also be associated to a very large expected goal. Last season still in the German league, the expected goal of the shot (and goal) of Lewandowski, a Bayern Munchen striker, when he was very close to the goal[17] was equal to 0.96. The goal is shown at the figure 2.1.

Figure 2.1: Scoring shot of Robert Lewandowski against Bochum

The graph of the ratio of scoring shots of the figures 1.1 and 1.3 can now be improved using the concept of expected goals. Two shots taken at the exact same locations do not always have the same probability of goal due to the different factor affecting this value and also for two shots taken in the same zone of the divided pitch. But computing the mean of the value of the expected goals in each zone will give an approximation of the expected goal value for a shot in this zone. This computation is done at the figure 2.2. The expected goals of the shots of the Understat dataset are used for this plot and for the other plots with the shots. This dataset is used because there is are more shots in this dataset.

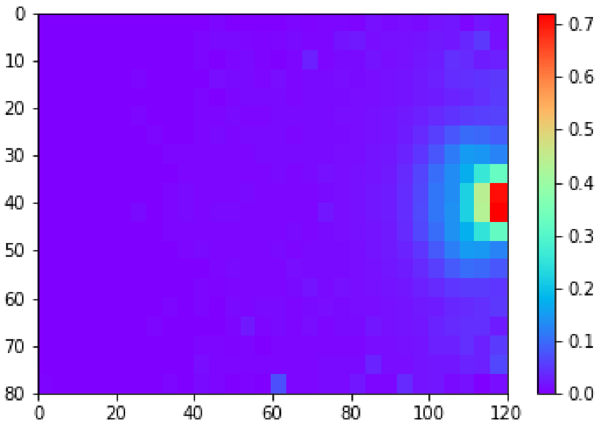
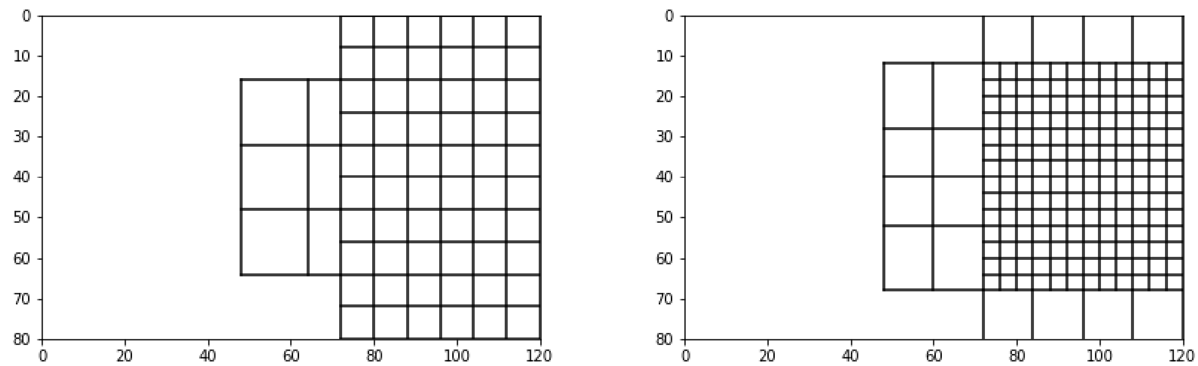


Figure 2.2: Average value of expected goals in zones of the pitch

The plot and the value are similar to the figure 1.3. Now, the probability of scoring from the two closest zones is 0.72 and 0.11 twenty metres away. The problem with this method for the zones far from the goal is that if no shot are taken from a zone then the mean of the expected goal value is zero. But the probability of scoring from this zone is not zero in the reality. Another problem is that the mean is computed on a few number of shots, sometimes one, for those distant zones. These problems are solved with larger zones to have more shots per zone. But having a lot of small zones allow for a better approximation of the expected goals due to

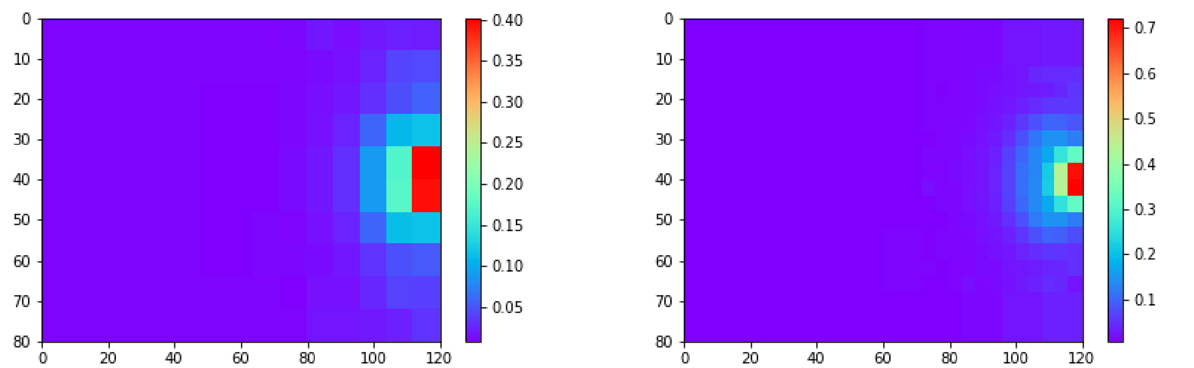
the more precise location.
The divisions of 150 and 600 zones of the figure 1.7 have to be kept for the model but other divisions may help to compute the average values of the expected goal in the zones. It will reduce the standard error and avoid having data-free zones. The division for the computation of the average values of the expected goals are shown at the figure 2.3. These divisions of the pitch combine the advantages of the small zones for when close to the goal and of the large zones when further from the goal.



(a) New division for the division of 150 zones (b) New division for the division of 600 zones

Figure 2.3: Distribution of the zones on the pitch for the computation of the mean of the expected goals

Those divisions are used only for the computation of the expected goals in each zones. For example if a possession is located in (119.0,1.0) we consider that the player is in the square zone with vertex [(120,0), (120,8), (112,8), (112,0)] for the division of 150 zones. His choices will be based on this zone. But the new division for the shots will be used only to obtained the average value of the expected goals from this zone. This value will be the same for the three zones next to it. The average values of the expected goals are shown at the figure 2.4 which allows to have values depending on enough shots in each area.



(a) Values for the division of 150 zones (b) Values for the division of 600 zones

Figure 2.4: Value of expected goals in new zones of the pitch

Now the shots are no longer consider as Bernoulli trials but as random values corresponding to the value of the expected goals. The mean of these random values can be computed in each zone. This is the previous figure 2.4. The standard error can also be computed. These new divisions help reducing the standard error from 0.018 to 0.010 for the division of 150 zones and from 0.059 to 0.018 for the division of 600 zones. Moreover this standard error is smaller with the expected goals than with the binary results.

2.2 Simple model

Before the modelling, we will start with a simple example that will help to better understand the future model. The example simplifies a football attack so that the calculations can be done by hand. Hence several assumptions will be made to simplify the attack
Let's imagine that during a football match, a team starts with a player having the ball in its own half next to the median line. The player can either shot or move the ball forward. In this example an half pitch is divided in five parts of twelve metres long. If he chooses to move the ball forward he makes a pass in the following twelve metres. When doing a pass the ball is either lost or passed to another player further up the field. If a player is in the last twelve metres he can make a pass but still in the last twelve metres. If he chooses to shot then either a goal is scored either the ball is lost. What is the best solution, the best choices to score a goal ?
If the first player starts in the twelve metres closest to the median line in its own half then he starts in the zone 0 on the figure 2.5. With the division in zones moving the ball forward is making a pass to a player in the following zone.

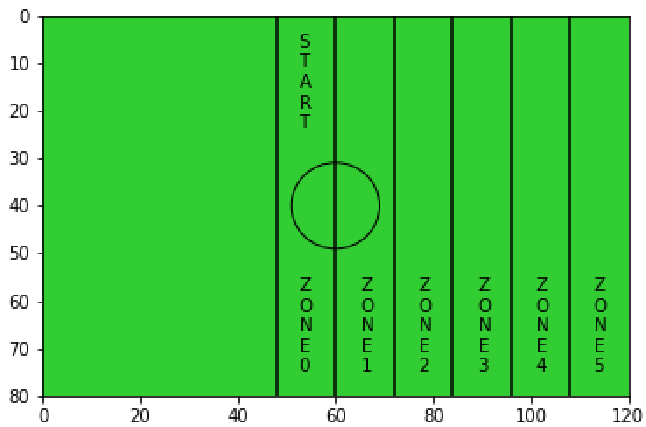


Figure 2.5: Pitch divided in simple zones

With the pitch divided in zones, it is possible to look at all the passes from a zone to the following by looking at the starting and ending locations. If a pass start in (55,40) and end in (67,40) then it is a pass from zone 0 to zone 1. In the StatsBomb dataset all passes from zone 0 to zone 1 are collected. The probability of making a passe from zone 0 to zone 1 is the ratio between the successful passes and the total number of passes collected. The other probabilities of success and also of failure of the passes from a zone to the following are computed in this manner. The values are shown at the table 2.1.

The probabilities of success are decreasing when getting closer to the goal because there is more defenders. This decrease is even greater near and in the penalty as discussed in the introduction and at the figure 1.4.

	Zone 0 to 1	Zone 1 to 2	Zone 2 to 3	Zone 3 to 4	Zone 4 to 5	Zone 5 to 5
Prob. of success	0.84	0.82	0.79	0.70	0.50	0.36
Prob. of failure	0.16	0.18	0.21	0.30	0.50	0.64

Table 2.1: Probabilities of making a pass to the next zones of the pitch

Similarly the probabilities of scoring or not from a zone can also be computed by looking at the location of the striker. These numbers are obtained by looking at the expected goals and by making the mean for all shots in the zone. The table 2.2 displays those values.

	From Zone 0	From Zone 1	From Zone 2	From Zone 3	From Zone 4	From Zone 5
Prob. of success	0.008	0.010	0.014	0.025	0.088	0.209
Prob. of failure	0.992	0.990	0.986	0.975	0.912	0.791

Table 2.2: Probabilities of scoring from each zone of the pitch

Unlike for the passes the probabilities larger near the goal also as discussed in the introduction at the figure 1.1.

Having those probabilities, the next step is to obtain the best action that is either pass or shot in each zone. The best action is the one giving the largest expected reward. Losing the ball gives no reward. Therefore the reward for scoring a goal is arbitrary, it will be equal to one. In the zone 5, the closest from the goal, the best action is shot since with a pass the ball is either lost either in the same situation as before the pass. Now, let's compute the expected rewards associated to the actions in this zone. The expected reward for the action shot in zone 5 will use the probabilities of the table 2.2. It will be written r_{5s} with r for reward, 5 for the fifth zone and s for shot.

$$r_{5s} = 0.209 \times 1 + 0.791 \times 0 = 0.209$$

The expected reward for the action shot s is the probability of scoring (0.209) multiplied by the reward associated to scoring (1) plus the probability of losing the ball (0.791) multiplied by the associated reward (0). The expected reward for the action pass, r_{5p} can not be greater than 0.209 since the only way to obtain a reward by making a pass is to hope for a successful pass and to shot in one of the following actions. The largest reward with a pass obtained when the next action is a shot and is equal to

$$r_{5p} = 0.36 \times 0.209 + 0.64 \times 0 = 0.075$$

Which is the probability of success for a pass multiplied by the reward if the following action is shot plus the probability of failure multiplied by the reward when losing the ball. Therefore shot is the best action in zone 5 and the maximum reward in the fifth zone r_5 is equal to $r_5 = 0.209$.

If the player in zone 4 chooses to shot his reward will be

$$r_{4s} = 0.088 \times 1 + 0.912 \times 0 = 0.088$$

For the action pass, going to the fifth zone gives an expected reward 0.209 considering that all players and so the player in zone 5 choose the best action. And the expected reward is

$$r_{4p} = 0.5 \times 0.209 + 0.5 \times 0 = 0.105 = r_4$$

In this zone the best action is pass. In the same way, we can compute the expected rewards for both actions in the other zones

$$\begin{aligned}
r_{3s} &= 0.025 \times 1 + 0.975 \times 0 = 0.025 \\
r_{3p} &= 0.7 \times 0.105 + 0.3 \times 0 = 0.073 = r_3 \\
r_{2s} &= 0.014 \times 1 + 0.986 \times 0 = 0.014 \\
r_{2p} &= 0.79 \times 0.073 + 0.21 \times 0 = 0.058 = r_2 \\
r_{1s} &= 0.010 \times 1 + 0.990 \times 0 = 0.010 \\
r_{1p} &= 0.82 \times 0.058 + 0.18 \times 0 = 0.047 = r_1 \\
r_{0s} &= 0.008 \times 1 + 0.992 \times 0 = 0.008 \\
r_{0p} &= 0.84 \times 0.047 + 0.16 \times 0 = 0.040 = r_0
\end{aligned}$$

The expected reward decreases with distance from the goal. The best action in all zones except the last one is to pass which seems logical due to the large size of the zones.
This way of obtaining the best actions will be difficult to reproduce in other problems. Another way to obtain the best action is to look at the best actions when the team can play one action at time 1, then the second action at time 2 and see if the rewards increase, then when the team plays at the time 3 etc.
If the team plays only once at the time 1 then the best action is always to shot since a shot leads to a potential reward unlike a pass which never directly gives a reward. If an exponent is added to the notation to mean that the number of action is equal to one, it gives

$$\begin{aligned}
& r^1_{0p} = r^1_{1p} = r^1_{2p} = r^1_{3p} = r^1_{4p} = r^1_{5p} = 0 \\
& r^1_{0s} = 0.008 = r^1_0 \qquad \text{because } r_{0s} \text{ is the largest reward in zone 0} \\
& r^1_{1s} = 0.010 = r^1_1 \\
& r^1_{2s} = 0.014 = r^1_2 \\
& r^1_{3s} = 0.025 = r^1_3 \\
& r^1_{4s} = 0.088 = r^1_4 \\
& r^1_{5s} = 0.209 = r^1_5
\end{aligned}$$

Doing a second action at the time 2 may increase the reward by doing a pass in the following since we know that the probability of scoring will increase in the next zone. If the ball is lost or if a goal is scored at the first action the rewards will not change at the second because regardless of the action done a lost ball remains lost and a goal remains scored. If the player in the fifth zone shots his reward will be equal to

$$r^2_5 = 0.209 \times 1 + 0.791 \times 0 = 0.209$$

If he chooses to pass the ball he will either arrives in zone 5 where the reward is equal to r^1_{5s} with one action left either loses the ball.

$$r^2_{5p} = 0.36 \times r^1_5 + 0.64 \times 0 = 0.36 \times 0.209 = 0.075$$

The best action is still shot in zone 5 and the reward does not change when the number of action is equal to two. In the other zones there are the expected rewards

$$\begin{aligned}
&r^2_{4s} = 0.088\\
&r^2_{4p} = 0.50 \times r^1_5 + 0.50 \times 0 = 0.50 \times 0.209 = 0.105 = r^2_4\\
&r^2_{3s} = 0.025\\
&r^2_{3p} = 0.70 \times r^1_4 + 0.30 \times 0 = 0.70 \times 0.088 = 0.062 = r^2_3\\
&r^2_{2s} = 0.014\\
&r^2_{2p} = 0.79 \times r^1_3 + 0.21 \times 0 = 0.79 \times 0.025 = 0.020 = r^2_2\\
&r^2_{1s} = 0.010\\
&r^2_{1p} = 0.82 \times r^1_2 + 0.18 \times 0 = 0.82 \times 0.014 = 0.011 = r^2_1\\
&r^2_{0s} = 0.008\\
&r^2_{0p} = 0.84 \times r^1_1 + 0.16 \times 0 = 0.84 \times 0.010 = 0.0084 = r^2_0
\end{aligned}$$

In these zones the best action, when the team play twice, is to pass to take advantage of the larger expected goals in the following zone. If the team makes three actions the best action is pass in all zones except the fifth zone. The reward does not increase in this zone five. the reward is also constant in zone 4 where making a pass sends the ball to zone five where a player shots. The rewards are

$$r^3_5 = 0.075,\ r^3_4 = 0.105,\ r^3_3 = 0.073,\ r^3_2 = 0.049,\ r^3_1 = 0.016,\ r^3_{0s} = 0.009$$

After doing the same reasoning for four, five and six actions the expected rewards are

$$r^6_5 = 0.209,\ r^6_4 = 0.105,\ r^6_3 = 0.073,\ r^6_2 = 0.058,\ r^6_1 = 0.047,\ r^6_0 = 0.04$$

The best action is pass in all zones except the last zone.
And for seven actions the expected rewards are

$$r^7_5 = 0.209,\ r^7_4 = 0.105,\ r^7_3 = 0.073,\ r^7_2 = 0.058,\ r^7_1 = 0.047,\ r^7_0 = 0.04$$

No reward increases with an additional action. The reward are therefore maximum because any extra possibility of playing will not increase the expected rewards. The action leading to the rewards in each zone is the best. The best actions and the rewards are the same that found by the first method but this method is more general.

To end this small problem, let's imagine that there is only one perfect player in the team and the others are average players. Depending on the zone where the player start, what is the best action knowing that the other players will not always choose the best action ? Firstly we can start by computing the average chance of scoring in each zone. It is done by computing the ratio between the number of possession going through a zone ending by a goal and the total number of possession going through this zone. The results are shown at the table 2.3.

The values seems small especially for the last zone but due to the size of the zones in this model a possession can go into the last zone without being very dangerous. If the ball is in one of the zones (so neither lost or scored) the team will score with a probability corresponding to these ratios. The reward of going to zone 5 is therefore equal to 0.051. The reward of each action can be computed as before. In the fifth zone the reward for a shot is the same $r_{5s} = 0.209$. The reward for making a pass is different since the reward for zone 5 is different

$$r_{5p} = 0.36 \times 0.051 + 0.64 \times 0 = 0.018$$

	In Zone 0	In Zone 1	In Zone 2	In Zone 3	In Zone 4	In Zone 5
Ratio	0.010	0.015	0.015	0.019	0.038	0.051

Table 2.3: Ratio of possessions ending in a goal in each zone of the pitch

which is not better than the optimal action shot. In the fourth zone, the reward for shot and pass are

$$r_{4s} = 0.088 \times 1 + 0.912 \times 0 = 0.088$$

$$r_{4p} = 0.50 \times 0.051 + 0.50 \times 0 = 0.026$$

In zone 4 the optimal action is shot due to the small chance of scoring when letting the team plays in zone 5. In the other zones the optimal action is pass with the following rewards.

$$\begin{aligned}
r_{3s} &= 0.025 \times 1 + 0.975 \times 0 = 0.025 \\
r_{3p} &= 0.70 \times 0.038 + 0.30 \times 0 = 0.027 \\
r_{2s} &= 0.014 \times 1 + 0.986 \times 0 = 0.014 \\
r_{2p} &= 0.79 \times 0.019 + 0.21 \times 0 = 0.015 \\
r_{1s} &= 0.010 \times 1 + 0.990 \times 0 = 0.010 \\
r_{1p} &= 0.82 \times 0.015 + 0.18 \times 0 = 0.013 \\
r_{0s} &= 0.008 \times 1 + 0.992 \times 0 = 0.008 \\
r_{0p} &= 0.84 \times 0.015 + 0.16 \times 0 = 0.013
\end{aligned}$$

Considering that only the first action is the optimal will be used later.

2.3 Markov Decision Process

The previous problem can be modelled as a Markov Decision Process which is an extension of a Markov Chain.

Definition 2.2 (Markov Chain)**.** A Markov chain is a stochastic process i.e. an evolution of a random variable where the probabilities of the future event depends only on the actual state and not on the past states. In other words, it is a sequence of events the probability for each of which is dependent only on the event immediately preceding it[**?**].

For example, the weather can be simplified and modelled by a Markov chain. There are three possibilities for the weather : sunny, cloudy and rainy. If the day is sunny, then the next day will be sunny with a probability of 0.6 and cloudy with a probability of 0.4. These probabilities for tomorrow depends only on today's weather and not on the one from yesterday and the previous days. If the day is cloudy there is a fifty percent chance that the weather will remain cloudy the next day, a thirty percent chance that it will be sunny and a twenty percent chance that it will rain. Finally if the day is rainy then it will stay rainy with a probability of 0.5 and it will be cloudy with a probability of 0.5. As before these probabilities depend only on the weather of the day and not on the weather of the previous days. The probabilities are the same for two days after knowing the weather of the newt day no matter the weather of the day. When modelling this as a Markov Chain, the states are the possible situations, the possible weathers. There are three states : sunny, cloudy and rainy. There are possibilities of transfer between these states which can be written in a matrix of probabilities :

$$
P = \begin{array}{c} \\ \\ \\ \end{array}
\begin{array}{c} \begin{array}{ccc} \text{To sunny} & \text{To cloudy} & \text{To rainy} \end{array} \\ \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.3 & 0.5 & 0.2 \\ 0 & 0.5 & 0.5 \end{bmatrix} \end{array}
\begin{array}{l} \\ \text{From sunny} \\ \text{From cloudy} \\ \text{From rainy} \end{array}
$$

Note that the sum of the probabilities for each row is equal to one. The Markov chain of the weather is shown at the figure 2.6.

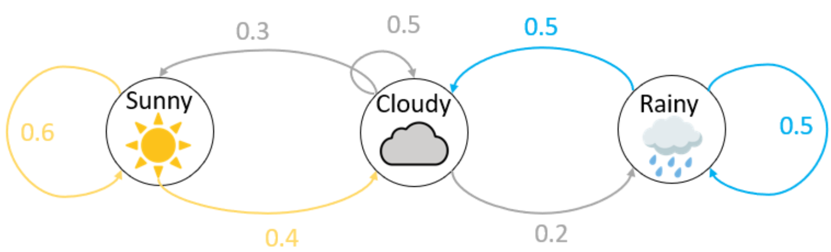


Figure 2.6: Diagram of the weather as a Markov Chain

Definition 2.3 (Markov Decision Process)**.** A Markov Decision Process abbreviated as MDP is an extension of a Markov chain with rewards obtained in the transition between states and actions influencing the probabilities of transitions between states.

A Markov decision Process is define by a 4-uple containing

- S, the state space the set of all states.
- A, the action space the set of all possible actions. It can also be written A_s for all the possible actions form state s.
- P_a the matrix of probabilities of transfer when doing the action a.
- R_a the matrix of rewards when doing the action a.

Let's go back to the simple model of a football attack of the section 2.2. In this example a player may be in different situation and can carry out actions to go from a situation to another. The actions may also lead to the same situation. The probabilities of moving from a situation to another depends only on action done and on the current situation. It verifies the Markov property about the lack of memory. The transition probabilities between the zones depends only on the actual zone no matter the previous zones. Moreover the objective is to get a reward by transiting to a Goal situation. The example can be modelled by a Markov Decision Process. The states of the MDP are the zones of the pitch, the state "Lost" when the ball is lost and the state "Goal". The probability of remaining in the two last states is one no matter the action. They are absorbing states. This is the state space. The set of actions is here Pass and Shot which can be done from all states even if it it does not change anything for the states "Lost" and "Goal". The reward is one when transiting to the state "Goal", expect from the state

"Goal" and zero otherwise. The reward matrix is so

$$R = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{cc} \begin{array}{ccccc} \text{To zone 0} & \text{To zone 1} & \dots & \text{To Lost} & \text{To Goal} \end{array} & \\ \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} & \begin{array}{c} \text{From zone 0} \\ \text{From zone 1} \\ \vdots \\ \text{From Lost} \\ \text{From Goal} \end{array} \end{array}$$

The reward for going from Lost to Goal could also be zero but it will never be used due the impossibility of moving between those zones. The matrix is the same no matter the action even if a pass have always a probability zero, for the moment, of transferring from a zone to a goal. Finally the matrices of probability are obtained using the tables 2.1 and 2.2. For the action pass the probabilities are mostly zero expect to go in the next zone or in the state Lost. This is how the matrix is constructed

$$P_p = \begin{array}{cc} \begin{array}{cccccccc} \text{To zone 0} & \text{To zone 1} & \text{To zone 2} & \text{To zone 3} & \text{To zone 4} & \text{To zone 5} & \text{To Lost} & \text{To Goal} \end{array} & \\ \begin{bmatrix} 0 & 0.84 & 0 & 0 & 0 & 0 & 0.16 & 0 \\ 0 & 0 & 0.82 & 0 & 0 & 0 & 0.18 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0.36 & 0.64 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} \text{From zone 0} \\ \text{From zone 1} \\ \vdots \\ \text{From zone 5} \\ \text{From Lost} \\ \text{From Goal} \end{array} \end{array}$$

The matrix of transition probability between states for the action shot is the following since after a shot either the ball is lost either a goal is scored.

$$P_s = \begin{array}{cc} \begin{array}{cccccccc} \text{To zone 0} & \text{To zone 1} & \text{To zone 2} & \text{To zone 3} & \text{To zone 4} & \text{To zone 5} & \text{To Lost} & \text{To Goal} \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.992 & 0.008 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.990 & 0.010 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.791 & 0.209 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} \text{From zone 0} \\ \text{From zone 1} \\ \vdots \\ \text{From zone 5} \\ \text{From Lost} \\ \text{From Goal} \end{array} \end{array}$$

To see more clearly the Markov Chain, it can be displayed as a graph with the nodes corresponding to the state and the links to the transition between the states. A Markov Decision Process can also be shown similarly but with transition depending on the actions.The figure 2.7

shows the Markov Decision Process. The states are the blue circles, the green ones correspond to the shots and the red to the passes. The rewards associated to the transition are in black. Not all probabilities are shown on the graph since it is a bit overloaded but the appendix B shows the two actions separately with all the probabilities.

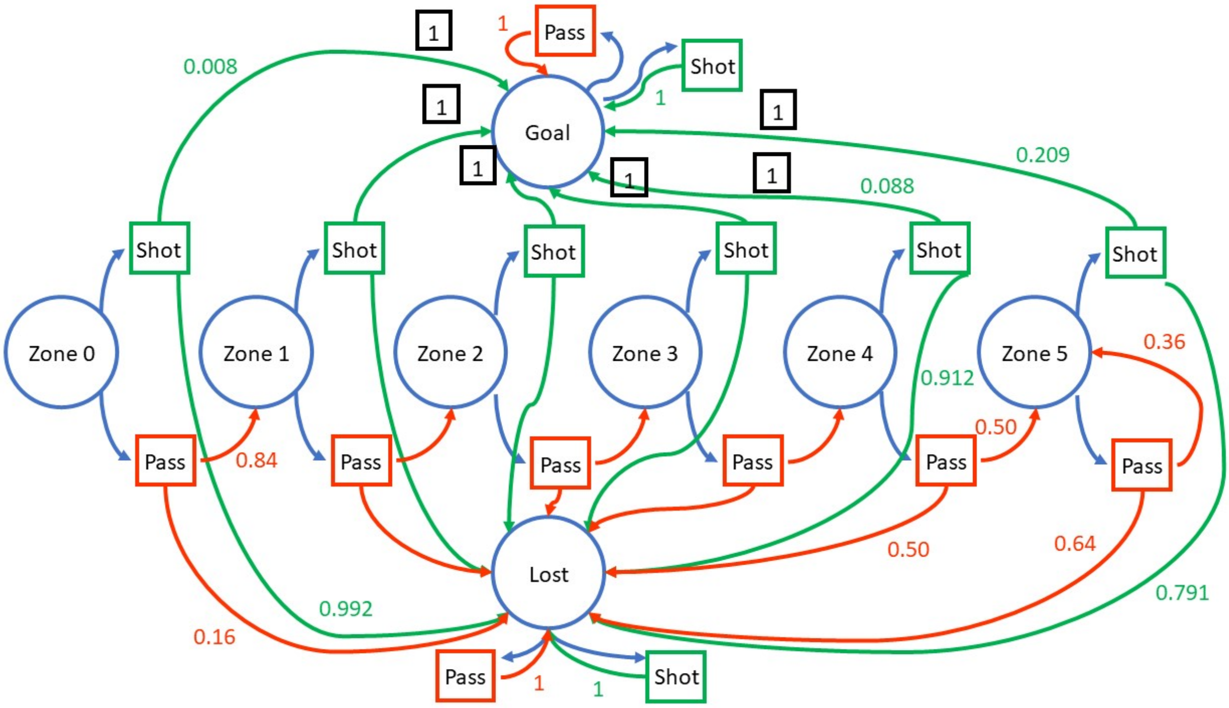


Figure 2.7: Markov Decision Process of the simple model

In the example, the objective is to score represented by a reward associated to the transition to the state Goal. To maximise the expected reward the player has to made the best choice in each zone which corresponds to the optimal policy.

Definition 2.4 (Policy)**.** A policy is a decision set. Each decision corresponds to the action done in a state.

A policy could be shot in each zone or pass in the zone 0, 1 and 2 and shot in the others. The optimal policy was to pass in each zone except in the zone 5. The action done in the state Lost and Goal did not change the expected reward and the other decision in the policy. To obtain the optimal policy an algorithm were used. The algorithm used before to obtain the best policy π is the value iteration algorithm[19] [20][21]. It starts with an arbitrary vector v(0) which represents the rewards in each state. The choice will be made on an array of zeros. At each iteration, it compares all actions to obtain the one giving the largest reward in each zone knowing the reward obtained in all zones at the previous iteration. This step is repeated while the maximum increase of the reward is greater than a threshold. The pseudo code of this algorithm is the algorithm 1
With this model and this algorithm it is the moment to upgrade the model starting with the number of actions and zones.

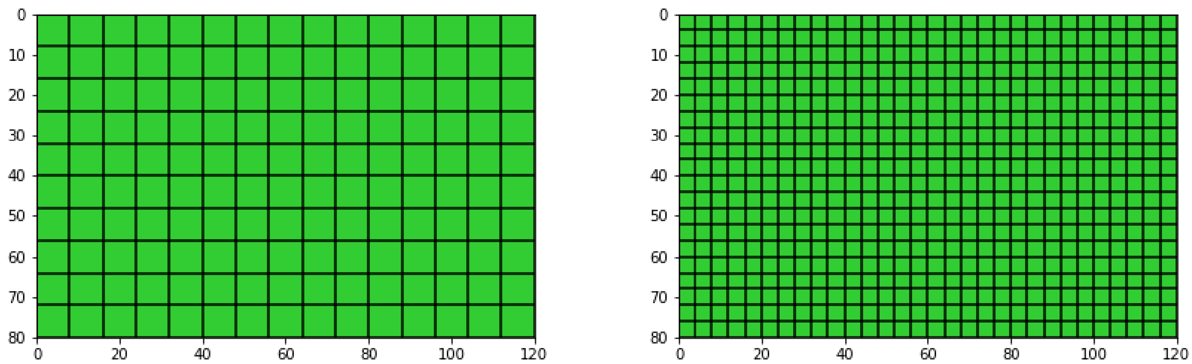
Algorithm 1 Value iteration algorithm

Require: An MDP with its 4-uple (S,A,R,P)
  A threshold ϵ
  $v(0) \leftarrow [0, 0, 0, ..., 0]$ ▷ Same size as S
  $n \leftarrow 0$
  repeat
    $n \leftarrow n + 1$
    for s in S **do**
      $v(n)_s = \max\limits_a R_{as}@P_{as} + P_{as}@v(n-1)$
      $\pi(s) = \underset{a}{argmax}\ R_{as}@P_{as} + P_{as}@v(n-1)$
    end for
  until $\|v(n) - v(n-1)\|_\infty < \epsilon$

2.4 First improvements of the model: adding states and actions

2.4.1 Adding zones

The first thing which can be improved is the number of zones. In the simple example, the probability of scoring in the fifth zone was equal to 0.209. This value is really different if the player is close to goal or in the corner as seen at the figure 2.2. Similarly, making a pass from the side line of zone 4 to the opposite line of zone 5 is more complicated than doing a short-range pass between those two zones. Increasing the number of zones will help solving this problem because the events will be closer in location than the others in the same zone. As said earlier, the number of passes is not unlimited so if the zones are too small the number of passes from a zone to another will be generally too small to be representative. The divisions used are in the introduction shown once more at the figure 2.8.



(a) Division with 150 zones

(b) Division with 600 zones

Figure 2.8: Divisions of the pitch

The number of states of the Markov Decision Process increases because each zone is a state. There are 152 or 602 states depending on the divisions because the states "Lost" and "Goal" are still used.

For the actions shot there were zones defined earlier where the mean of the values of the expected goals were computed. The zones and the corresponding probabilities of scoring were at the figures 2.3 and 2.4. As a reminder these divisions are only used for the computations of the average values of the expected goals before transferring them on the divisions of the figure 2.8.

2.4.2 Adding and modifying actions

The second way to improve the model is to add or to modify actions. Now a player can pass the ball in any zone and not only in the following one. It increases the number of possible actions because there are as much action pass as zones and so as much probability matrices as actions and zones. Each probability matrix is built in this way, with the action "Pass to zone 2" as example. This example still uses the zones of the simple example to avoid having a matrices with hundreds of entries.

$$P_{p2} = \begin{array}{c} \begin{array}{ccccccc} \text{To zone 0} & \cdots & \text{To zone 2} & \cdots & \text{To zone 5} & \text{To Lost} & \text{To Goal} \end{array} \\ \left[\begin{array}{ccccccc} 0 & \cdots & 0.54 & \cdots & 0 & 0.46 & 0 \\ 0 & \cdots & 0.82 & \cdots & 0 & 0.18 & 0 \\ \vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0.88 & \cdots & 0 & 0.12 & 0 \\ 0 & \cdots & 0 & \cdots & 0 & 1 & 0 \\ 0 & \cdots & 0 & \cdots & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \text{From zone 0} \\ \text{From zone 1} \\ \vdots \\ \text{From zone 5} \\ \text{From Lost} \\ \text{From Goal} \end{array} \end{array}$$

The probabilities are decreasing with distance from the target zone but is higher when the pass is a back pass so when the starting zone is closer than the goal than when it is a forward pass.

If an attacker is close to an opponent he can try to dribble this opponent. This event have only one location so we consider that it is the starting and ending location. Therefore this event starts and ends in the same zone. Unlike the action pass, there is no direction for the action. The probability of success of a dribble could be computed in each zone with the ratio between the number of dribbles for which the outcome is completed and the number of dribbles attempted in each zone. These probabilities are shown at the figure 2.9 with the division of 150 zones.

To chances of success decrease while going forward on pitch because it is more difficult to dribble a defender than a forward. Furthermore there are generally more opponents close to their goal which makes dribbling more difficult. A dribble is also more difficult close to the edges.

There is only one action dribble and so one probability matrix. For the previous simple example the matrix in the following. It is built in a similar way for more zones.

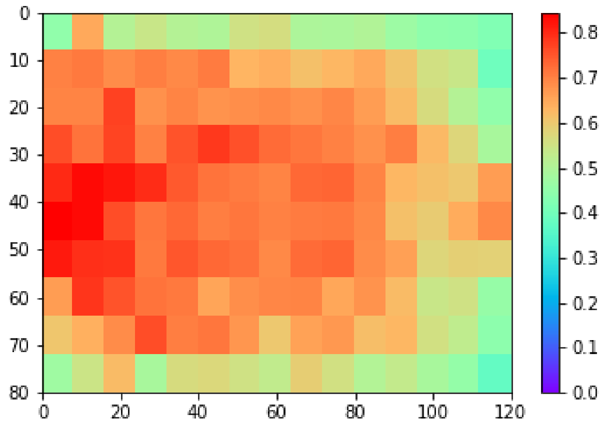


Figure 2.9: The probability of success of dribbles in each zone of the pitch

$$
P_d = \begin{array}{c}
\begin{array}{cccccccc} \text{To zone 0} & \text{To zone 1} & \text{To zone 2} & \text{To zone 3} & \text{To zone 4} & \text{To zone 5} & \text{To Lost} & \text{To Goal} \end{array} \\
\begin{bmatrix}
0.61 & 0 & 0 & 0 & 0 & 0 & 0.39 & 0 \\
0 & 0.60 & 0 & 0 & 0 & 0 & 0.40 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0.45 & 0.55 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
\begin{array}{c} \\ \text{From zone 0} \\ \text{From zone 1} \\ \vdots \\ \text{From zone 5} \\ \text{From Lost} \\ \text{From Goal} \end{array}
$$

Another action to add is "carry" when the player carries the ball, moves the ball on the pitch. If a player decides to move the ball he can therefore choose between carry and pass. In the reality, a player will choose to carry the ball to another point only if the way seems clear. A clear way means that there only a few opponents on way to the target location. The probability of success when carrying the ball is therefore very high: 0.98. The action carry is merged with the action pass to obtain the action move which corresponds to a movement of the ball the pitch. Depending on if carry the ball is possible or not the player will either pass the ball or carry it to the target location. The probability matrices of move are built in the same way as the probability matrices of pass. A high probability for the action move means that the move is easy achieve or that it is often successful. If the way is often clear from one zone to another clear then a player who want to move the ball will usually chooses to carry the ball instead of making a pass. It will lead to an high probability of success show the ease of this movements.

2.4.3 The under pressure attribute

A third improvement is to consider if the player is under pressure or not. The StatsBomb dataset has this attribute telling that an action is done while the opposite team was putting pressure or not. There is a significant part of the actions that are played under pressure: 26 percents. This proportion is more important when as progressing on the field or as getting closer to the edge of the field as plotted at the figure 2.10.

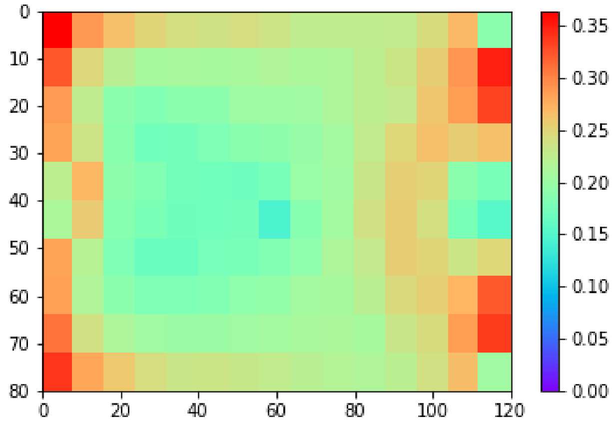
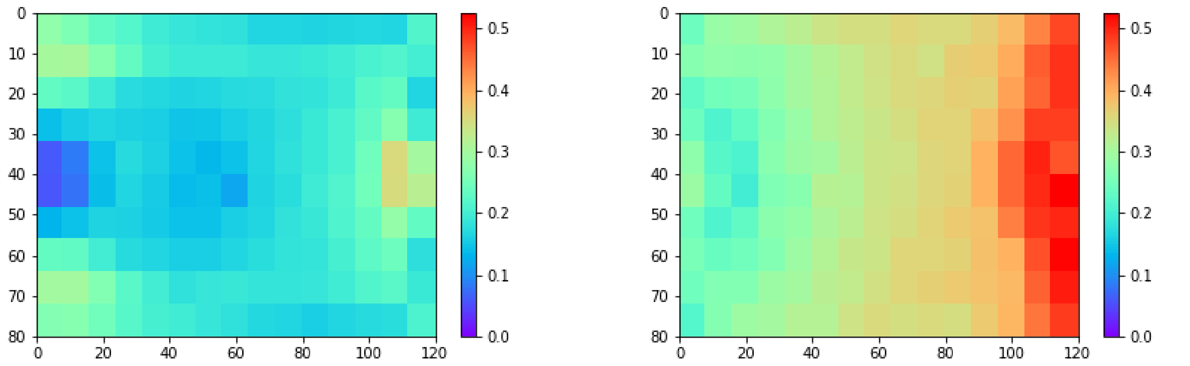


Figure 2.10: Proportion of actions done under pressure in the different zones of the pitch

A player has now two attributes his location and his state of pressure which is binary: under or without pressure. The state depends on this two attributes meaning there are two states by location, one under and one without pressure. The total number of states is now twice the number of zones plus two corresponding the states "Lost" and "Goal". An action is more difficult to accomplish while being under pressure so the probability of success and failure can not be the same when starting from a state under pressure or not. A pass is a success with a probability of 0.80 without pressure on average and only of 0.74 under pressure. The expected goals used before depends also of the attribute under pressure and so the average value for the expected goals also decreases when under pressure: 0.13 without pressure and 0.09 under pressure. For the action carry the probability of success remains very high in both situations: 0.99 without pressure and 0.97 under pressure. The action dribble can not be played without pressure because it would mean that there is no defender close to the forward and so no player to dribble so the probability of achieving a dribble without pressure is zero. Furthermore when a player did an action while under pressure, is the following state also under pressure ? An answer to this question comes by computing the proportion of actions leading to an under pressure state depending on if the action is done without or under pressure. The proportions in all zones are shown at the figure 2.11.

In both cases making an action close to the goal line have more chance to lead to an under pressure state. Furthermore it is mostly the actions done under pressure that lead to another state under pressure. The proportion in the zone in the corner is affected by corner kick and center pass taken from this zone. Finally the probability matrices can be built considering that an action can start and end in a state under or without pressure. Moreover a player deciding to move the ball does not choose the pressure attribute of the ending state so the number of actions "Move" is still equal to the number of zones. To compute the probabilities the number of actions leading to the state without pressure, under pressure and Lost has to be counted. Making the ratio between these numbers and the sum of the three numbers gives the probabilities. For example the probability matrix of the action "Move to zone 2" is built in this way



(a) Starting from a state without pressure

(b) Starting from an state under pressure

Figure 2.11: Proportion of actions leading to an under pressure state by starting zone

$$
P_{m2} = \begin{array}{c c}
\begin{array}{cccccccc}
\substack{\text{To zone 0} \\ \text{without press}} & \cdots & \substack{\text{To zone 2} \\ \text{without press}} & \cdots & \substack{\text{To zone 2} \\ \text{under press}} & \cdots & \text{To Lost} & \text{To Goal}
\end{array} & \\
\begin{bmatrix}
0 & 0 & 0.73 & 0 & 0.01 & 0 & 0.38 & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0.95 & 0 & 0.01 & 0 & 0.04 & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0.80 & 0 & 0.13 & 0 & 0.07 & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & 0 \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 1
\end{bmatrix} &
\begin{array}{c}
\substack{\text{From zone 0} \\ \text{without press}} \\ \vdots \\ \substack{\text{From zone 2} \\ \text{without press}} \\ \vdots \\ \substack{\text{From zone 2} \\ \text{under press}} \\ \vdots \\ \text{From Lost} \\ \text{From Goal}
\end{array}
\end{array}
$$

The matrix for the action dribble is built similarly to the one without considering attribute under pressure or not except that the future state can be the starting zone with or without pressure or the state Lost. And for the action "Shot" there are just more states to start in. Example of theses matrices are shown in the appendix C

In all these matrices of probability there is only a few non-zeros values. In order to limit the amount of storage required for all the probabilities it is possible to store this using the fact that they are sparse matrices. Moreover the coordinates of the non-zeros values are known in each matrix knowing the action. Therefore the sparse matrix do not need to store the coordinates. The way of storing the matrix is explain at the appendix C.

With these improvements of the model the Markov Decision Process is now difficult to represent with all the actions, states and probabilities. The figure 2.12 shows the Markov Decision Process with some states and the actions that can be done from the states "Zone i without pressure", "Goal" and "Lost". The states are in blue and each other color represents an action. The black rectangle is the reward of the transition. The probabilities are written using the notations of the appendix C.

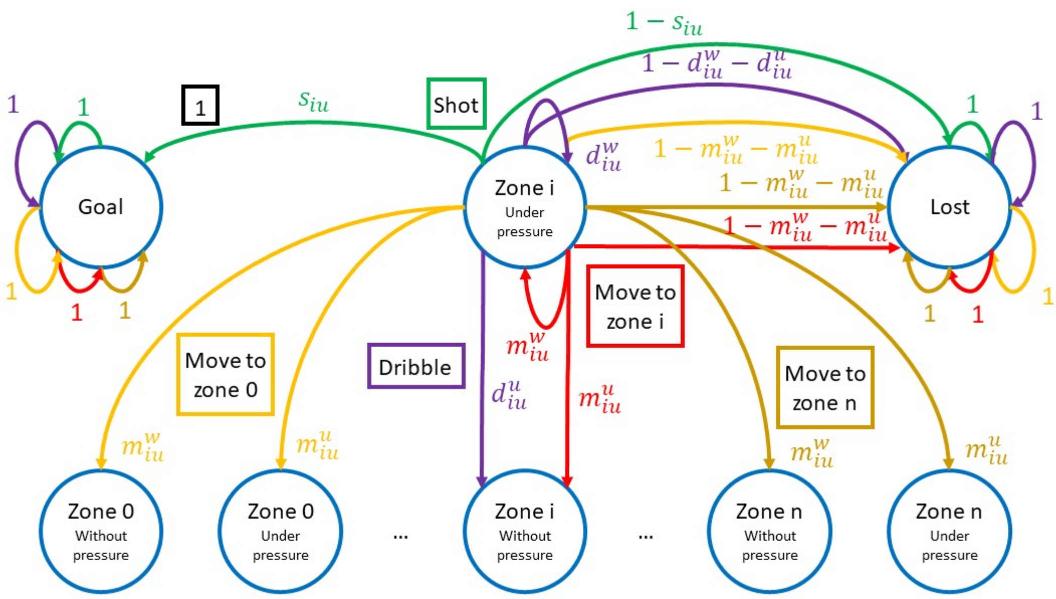


Figure 2.12: Markov Decision Process of the model with the first improvements

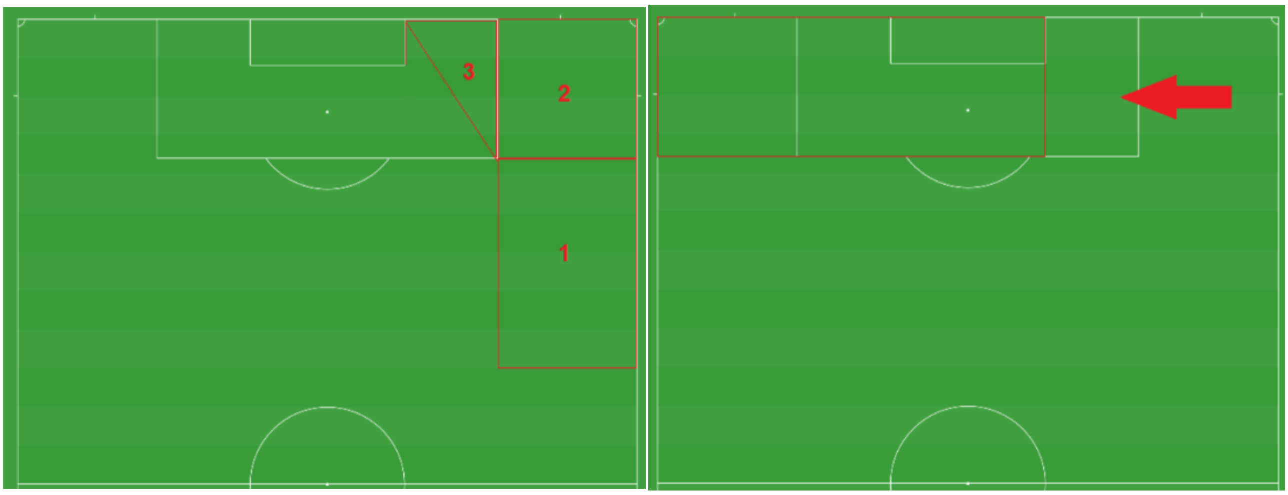
2.4.4 Expected passes and expected carries

There is a last improvement concerning the actions. At the moment the probabilities of success and failure of the actions move are computed by making the ratio between the number of success and the number of attempts. It is similar to what was done for the shots in the introduction at the figure 1.1. This was not the best way to compute the chances of success. With this way of computing the chances of success of the action move from a zone to another is not influenced by the moves from zones next to the starting zone. It is also not influenced by the moves to zones next to the target zones even if the movements are similar. The solution for the shots was the expected goals. In the same way the solution is the expected successful passes and the expected successful carries. For simplicity it will be referred as expected passes or xP and expected carries or xC.

Definition 2.5 (Expected pass)**.** The value of an expected pass is the expected chances of success of a pass. In other words it is the probability of success of this pass. The values depends on different factors just as the location, the length or the angle of the pass.

If a pass has an expected value of 0.5 it means that an average player making this pass has a chance of success of fifty percents. We expect that this pass is achieve once in two. Unlike the expected goals, the values of the expected passes are not written in the StatsBomb dataset. To obtain those values a classifier has to be trained to compute them. The classifier needs income data with different features to classify the variables. There are a lot a features in the dataset that can be used to have an accurate value for the expected passes.

- The first feature is the starting location of the pass. As seen at the figure 1.4 the passes are more difficult to achieve near the goal or the edges.
- The length of the pass. The length of a pass influences also the fulfilment of this pass. The intuition is that a short pass is easier to achieve than a long pass.
- The angle of the pass. When discussing the first improvements we saw in a transfer matrix that a backward pass was more often successful than a forward pass. The angle of the pass with respect to the x-axis is a feature used for the expected passes.
- The third spatial coordinate of a pass is the height of the pass. In the dataset the height is divided in three categories:
  1. Ground pass which is a pass staying on the ground.
  2. Low pass which a pass staying below the height of a shoulder.
  3. High pass which goes upper the height of a shoulder
- An already known feature is the pressure. It is a boolean telling that the player is under pressure or not while carrying out his action.
- The two following features are booleans about the type of pass. The first is if the pass is a cross or not. The crosses are defined by StatsBomb[22] as pass originate from any of the attack zones of the figure 2.13a on either side of the pitch and that intersects the zone of the figure 2.13b.



(a) Starting zones for a cross

(b) Ending zone for a cross

Figure 2.13: Zones defining a cross according to StatsBomb

- The second boolean feature is if the pass is a cutback or not. The definition of StatsBomb for a cutback is a low or ground pass that originates in zone A (on either side of the pitch) of the figure 2.14 and end in zone B.
- Another feature used which is also a boolean is the body part. If the pass is made with the head or not. A pass made by the head seems more difficult to achieve.
- The last feature is the type of possession associated to the pass. The pass can start a possession, made to carry out a set-piece or in open play. The options are

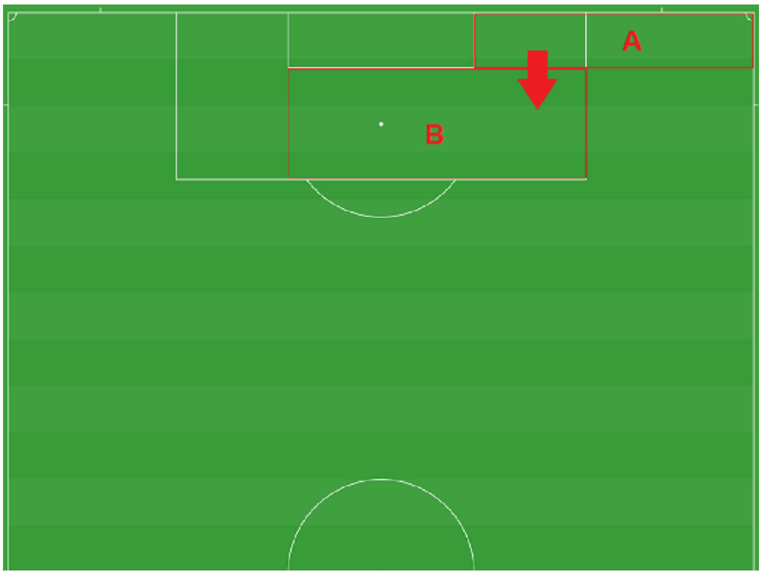


Figure 2.14: Zones defining a cutback according to StatsBomb

1. Kick-off. The pass is done at the beginning of a half or after scoring.
2. Interception. The pass is a one touch off an interception.
3. Recovery. Similarly the pass is a one touch off a recovery.
4. Goal kick. The pass is from a goal kick.
5. Corner. The pass is from a corner.
6. Throw-in. The pass puts the ball back into play.
7. Free-kick. The pass is from a free-kick.
8. Other. The pass is not done after one of the previous options. It is mainly the passes done in open play.

A pass is now written as a vector with all attributes. The boolean attributes as the cross, the cutback and the body part are written as an integer. One representing True and zero representing False. The features with different elements are not written as an integer because it classifies it. If the kick-off is associated to one, interception to two and free-kick to seven it means that a pass from a kick-off is more similar to a pass from an interception than to a pass from a free-kick. Since this is false, another way to store this feature has to be used. The way is to consider each option as a boolean. For example if the pass is from a free-kick then the first option is true and the other are false. For this feature this pass is associated to the vector [1,0,0,0,0,0,0,0]. This vector has to be merged with the vector containing the other features.
Let's imagine a ground pass starting from (80,40) with a length of 20 metres and an angle of 1 radian. This pass is done without pressure with the foot in open play and is neither a cross nor a cutback. Then the vector associated to the pass is the following

$$\left[\underbrace{80\quad 40}_{\text{Location}}\quad \overbrace{20\quad 1}^{\text{Length \& Angle}}\quad \underbrace{1\quad 0\quad 0}_{\text{Height}}\quad \overbrace{0\quad 0}^{\text{Cross \& Cutback}}\quad \underbrace{0\quad 0}_{\text{Body part \& Pressure}}\quad \overbrace{0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0}^{\text{Type of play}}\quad 1\right]$$

The classifier will use the vectors as data to fit the model. The model also needs to have the results of the pass to train the model. The result of a pass is not only success and failure. If a pass is a success it can lead to a state without or under pressure. The classifier will have to give the probabilities of the three results. There are several classifiers which can be used to predict the result of a pass. The classifiers used by other people trying to model the expected passes are mainly the linear regression and the gradient boosting classifier. I choose to use the gradient boosting even if it takes more computational time. Indeed the accuracy obtained with the gradient boosting is better than the accuracy with the linear regression. As a reminder the number of passes is equal to 1,100,000 which is the number of data. With a train set of size of 0.8 times 1,100,000 and a test set of size 0.2 times 1,100,000 the accuracy is equal to 0.84 which is larger than the 0.81 of the linear regression. Once the model is trained it returns the expected results with a vector as entry.
Let's take an example of a pass made during a match in the dataset. The first match in the dataset is the 2019 Champions League final between Tottenham and Liverpool. The observed pass is taken[23] from this match. This pass is a pass from Danny Rose to Lucas Moura. The pass is shown at the figure 2.15. Danny Rose in the vertex of the penalty area and Lucas Moura is in the penalty area close to edge of the penalty area.

Figure 2.15: A pass of Danny Rose a player of Tottenham during the Champions League final against Liverpool

This pass was a ground pass made with the foot. The pass was not very long, only sixteen meters and was not directed towards the goal. The probability of failure is equal to 0.08. To probability of success and reaching a state without pressure is equal to 0.90 and to 0.02 for reaching a state under pressure. This pass is related to an high value of expected passes since it should be successful nine times out of ten. During the match the pass was a success since it was receipt by Lucas Moura, who was not under pressure when he receives the pass as shown at the figure 2.16.

A similar work is done for the expected carries

Definition 2.6 (Expected carry)**.** The value of an expected carry is the expected chances of success of a carry. In other words it is the probability of success of this carry. The values depends on different factors just as the location or the pressure.

Unfortunately the dataset does not contains as much features when the action is carry. The only features present or calculable are the starting location, the length and the angle of the carry and the pressure. Never mind, a classifier could also be trained in order to predict the

Figure 2.16: A ball receipt of Lucas Moura a player of Tottenham during the Champions League final against Liverpool

results. The size of the train set is now 0.8 times 900,000 and the size of the test set is 0.2 times 900,000.

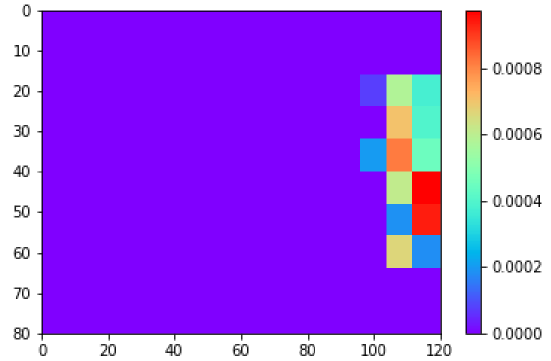
The probabilities of the transition matrix for the actions move are obtained by computing the mean of the expected passes and carries. The probability of going from a zone, with its pressure attribute, to another zone without pressure is computed using the probabilities of going to the starting to the ending zone without pressure of all moves between this states. The probabilities are predicted by the model, they are summed up and divided by the number of moves between this states. The probability to go to the same zone but under pressure and the probability to lose the ball are obtained in the same way. For example let's imagine that there are two moves starting from zone 0 without pressure to zone 1. Firstly the attributes of the moves are collected to use to model to predict the results. The gradient boosting classifier says that the first move has a probability of 0.5 to go to zone 1 without pressure, 0.2 to go to zone 1 under pressure and 0.3 to be missed. The second move has a probability of 0.6 to go to zone 1 without pressure, 0.16 to go to zone 1 under pressure and 0.24 to be missed. If the action is "Move to zone 1" the probability of moving from zone 0 without pressure to zone 1 without pressure is 0.55. The probability of moving from zone 0 without pressure to zone 1 under pressure is 0.18 and 0.27 to get lost.

2.5 Set pieces: Obtaining and results

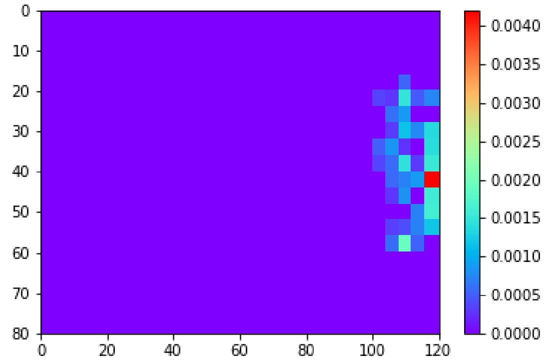
This model is binary for the moment, a shot is either missed or scored, doing another action either leads to a zone (with or without pressure) or to a lost ball. In reality an action may also end by the gain of a set piece for example a corner after a shoot, a throw-in after pass or a free-kick after a dribble. Adding the set-pieces to our model is the goal of this section. However the dataset does not always indicate which action leads to the gain of the set piece but only the zone where it is obtained. There are four possible set pieces which will be discussed in this section: Penalty, Free-kick, Corner and Throw-in.

2.5.1 Penalty

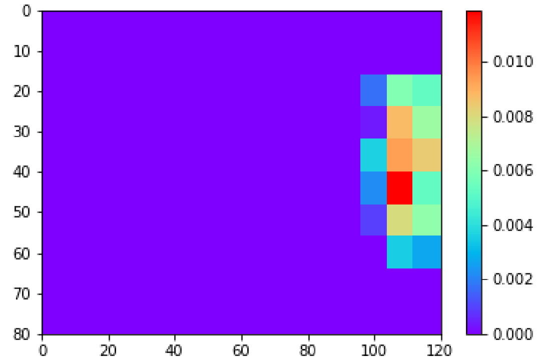
A penalty is definitely the best set piece to obtain because it transforms an action not always dangerous into a shot with high probability of scoring. Several[24] [25] websites gives 0.76 as the value of the expected goal for a penalty. This value is confirmed by the StatsBomb dataset which also gives this value for the probability of scoring a penalty. Understat gives a more precise value: 0.7612. It is this value which will be used. It remains to compute the probability of obtaining a penalty in each state. As before there is a difference between same locations but when the player is under pressure or not. The chances are obtained by computing the number of actions done by a forward in all zones and the number of actions leading to a penalty in all zones for both conditions. Computing the ratio between those values will gives an average probability of obtaining a penalty in each zones. This is displayed at the figure 2.17 for 150 and 600 zones.



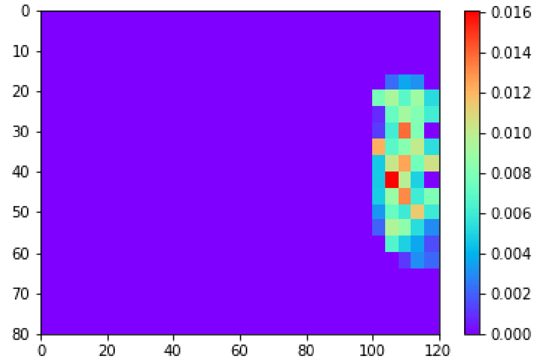
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



(c) Division of 150 zones under pressure



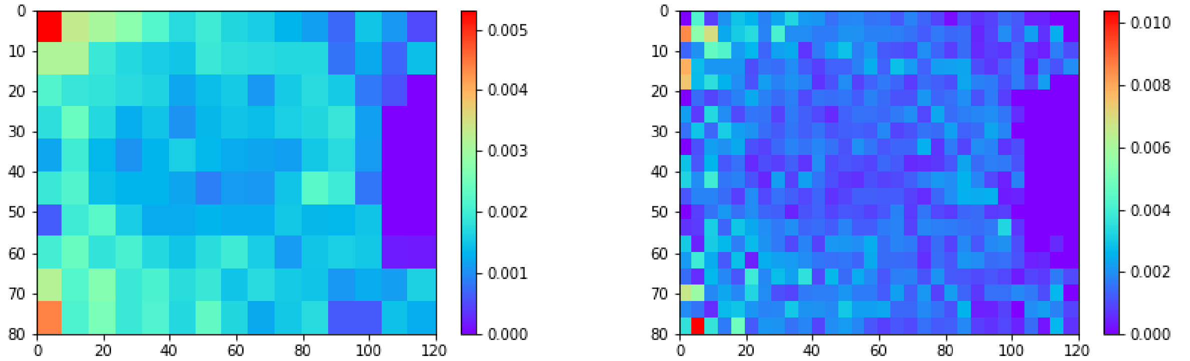
(d) Division of 600 zones under pressure

Figure 2.17: Proportion of actions leading to a penalty with both divisions

In the dataset, there is only 326 penalties conceded which can explain the non-smooth values. There are some zones which are part in the penalty area and part outside. The probability of obtaining a penalty is smaller in those zones. However obtaining a penalty when receiving the ball in the part outside the penalty are is also possible if the forward goes into the penalty with his control that is not counted as an action for example. The probability of winning a penalty is clearly higher when the player is under pressure and so when a defender is close to him . It is logical since the defender needs to be close to the forward to make a foul on him.

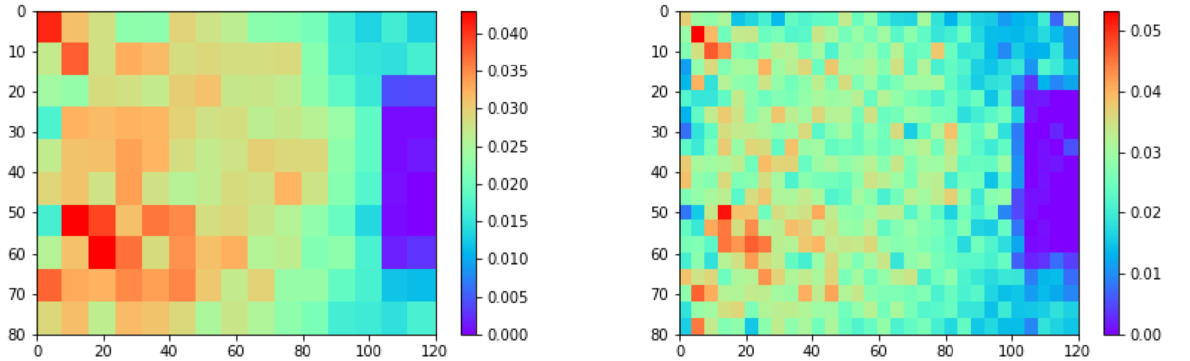
2.5.2 Free-kick

A second set-piece possible to obtain is a free-kick which is more frequent than a penalty. There were nearby 26000 free-kicks during the thousand matches studied for 552 goals so a probability of scoring from a free-kick of 0.02. As for the penalties, it is possible to compute the proportion of free-kicks won in comparison to the number of actions in each zone. As for the penalties the free-kicks can be won under or without pressure. The difference between the attributes is made to compute the probability of obtaining a free-kick in each zones. This probability is the ratio between the number of actions leading to a free-kick divided by the total number of actions. Those ratios are shown at the figure 2.18.



(a) Division of 150 zones without pressure

(b) Division of 600 zones without pressure

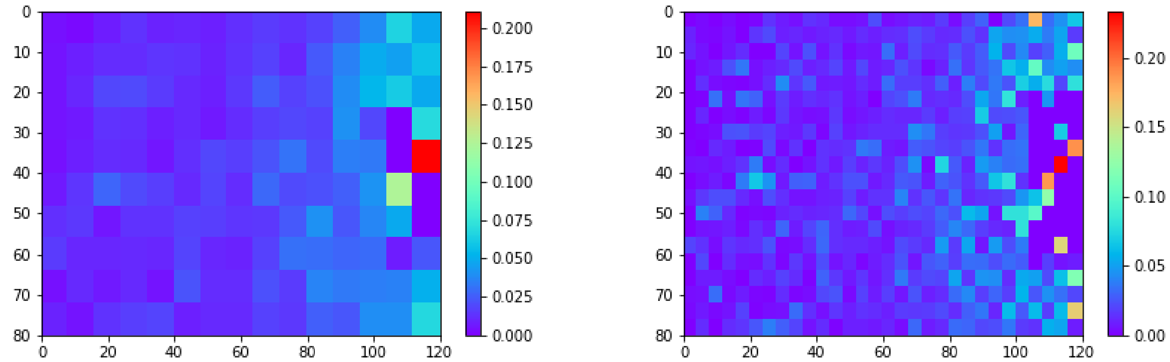


(c) Division of 150 zones under pressure

(d) Division of 600 zones under pressure

Figure 2.18: Proportion of actions under pressure leading to a free-kick with both divisions

The probability of obtaining a free-kick in the penalty area is almost zero. There are just a few indirect free-kick obtained in the penalty area. The probabilities are disparate without pressure and higher in the own half under pressure because even if being under pressure occurs more often in offensive zones gaining a foul while under pressure depends less on localisation. Moreover the forwards are weaker defensively and take more risk to recover the ball so concede more foul proportionately when they press a defender. But waiting to be under pressure in a defensive to obtain a free-kick is not the choice done by the players for two reasons. Firstly because if no free-kick is obtained there is a small probability of scoring if the zone is in the middle of the pitch. Secondly because scoring from a free-kick far away from the goal is less likely than scoring near the penalty area. Scoring from a free-kick can be done by shooting directly, after a pass or at any time in the following possession. For all possessions starting from a free-kick the zone where the free-kick was taken and the result the possession (goal or not) are known. So the probability of scoring from a free-kick in each zone is the number of possessions starting by a free-kick and ending by a goal divided by the number of possessions starting by a free-kick in this zone. But there are only 552 goals so a lucky goal scored will have a lot of influence of these ratios. To overcome this challenge the expected goals can be reused. If the possession ends by a shot then it is the value of its expected goal which is kept. Otherwise the result is considered as a lost ball without forgetting to consider the goals without shots like the own-goals. The probabilities of scoring from a free-kick in each zone are displayed at the figure 2.19.



(a) Division of 150 zones

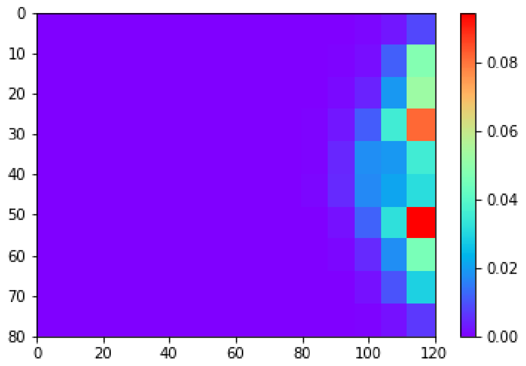
(b) Division of 600 zones

Figure 2.19: Proportion of goal scored starting from a free-kick with both divisions

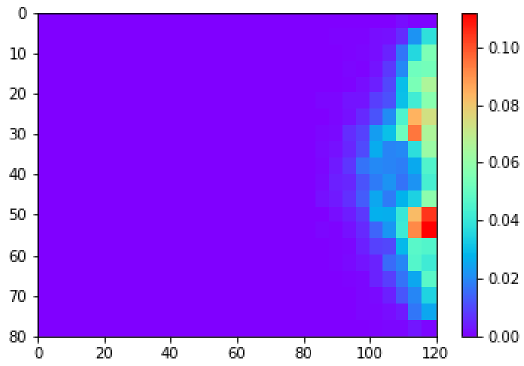
The probabilities are higher when getting closer to the goal, especially next to the corners where the probability is a bit higher to the probability of scoring from a corner (0.047). There are also some indirect free-kick in the penalty area which leads to an high probability of scoring. By multiplying the probability of obtaining a free-kick and the probability of scoring from a free-kick in all zones it gives the probability of scoring from a free-kick by going in each zone.

2.5.3 Corner

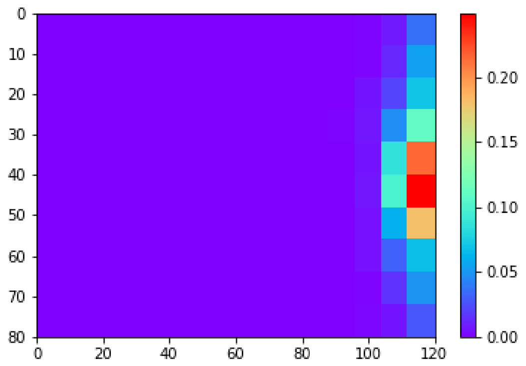
A shot may be saved by the keeper and going beyond the pitch's limits, this is the third set-piece: the corner. It can also happens when the defender block a pass, during a duel nearby the goal line etc. As for the free-kicks it is possible to compute the probability of scoring from a corner using the value of expected goal of the shot ending the action, if any, and also by considering the goals without shot. From the 9300 corners taken the total values of the expected goals generated was 435.87. As information there were 444 goals scored. The probability of scoring from a corner is 0.047 as written before.
It remains to obtain the probability of winning a corner in each zone. It is computed as before by the ratio between the actions leading to a corner and the total number of actions in all zones. 3600 corners are obtained under pressure and 5700 without so it is possible to obtain the proportion of actions without pressure leading to a corner and the same for the actions under pressure. These are the results of the figure 2.20.
The corners are mainly obtained close the goal and/or in shooting position. The probability of obtaining a corner is also higher when under pressure because it favours the duels and the blocks. The probability is lower if the striker is in the center of the goal without pressure because these actions usually end in a goal whereas the opposites players are more likely to defend if the striker is under pressure.



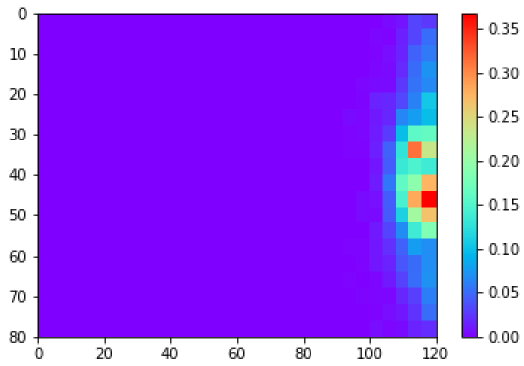
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



(c) Division of 150 zones under pressure



(d) Division of 600 zones under pressure

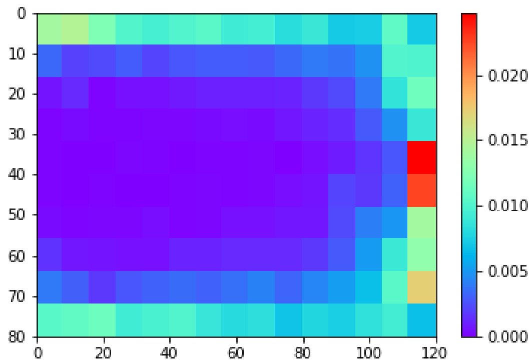
Figure 2.20: Proportion of actions leading to a corner with both divisions

2.5.4 Throw-in

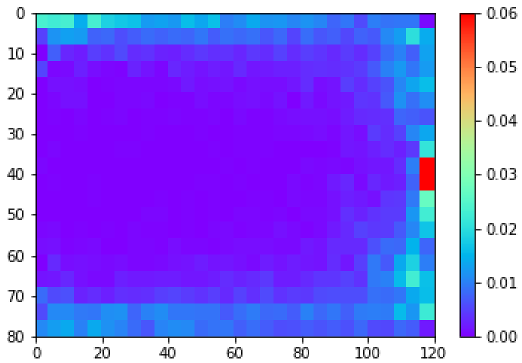
The last set piece is the throw-in happening mainly after a duel or a block by a defender. Some throws-ins are also won in defensive actions when the opponents miss a control or a pass. This way to obtain throw-ins is not taken into account since it depends on the other team or on the defensive actions of the players. An offensive action is never "The other team miss its pass". The number of throw-ins is more or less 11000 in both cases. As for all set pieces the probability of obtaining a throw-in in all zones can be computed and as for the other set-pieces a throw-in can be obtained with or without pressure. It is the point of the figure 2.21

The ratios between actions without pressure leading to a throw-in and all actions without pressure are fairly constant next to the edges. But they increase in the middle of the pitch close to the goal except in the last zone which also leads to corners. Since there is only a few actions without pressure in the two zones in the center of the goal, the clearances on the line ending in a throw-in have a strong influence on the proportion in those zones. The probability of winning a throw-in in those zone is 0.13 and 0.17 but the scale is bounded for clearness for 600 zones. For the actions under pressure the ratios increase along the pitch and the peak is also in the penalty area. The ratios are higher when under pressure because as for the corners, when the striker is under pressure there is more chances of having a duel or a block and so a throw-in. The ratios are also higher for the smaller zones since it kept only the actions really close to the line and so where is more complicated for a defender to act without conceding a throw-in and especially with the left wingers as shown on the figures.

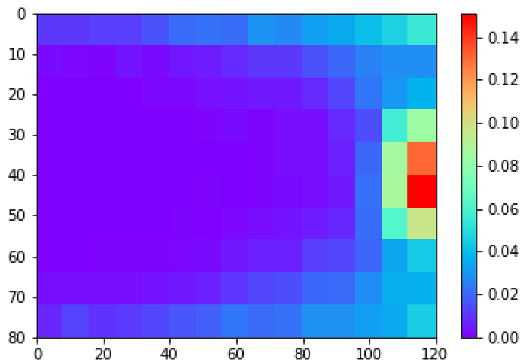
As for the free-kick the probability of scoring from a throw-in depends on the zone where it



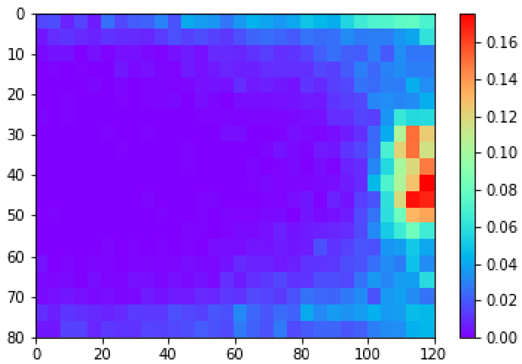
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



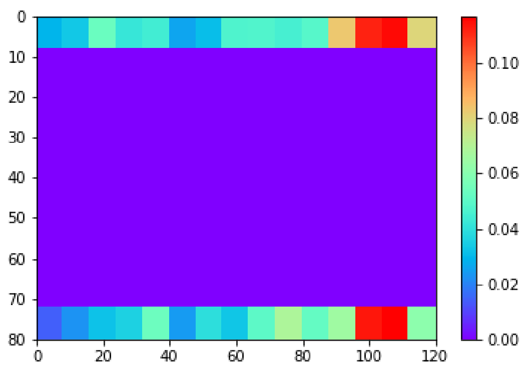
(c) Division of 150 zones under pressure



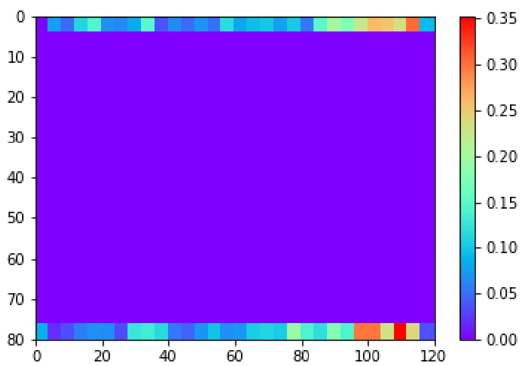
(d) Division of 600 zones under pressure

Figure 2.21: Proportion of actions leading to a throw-in with both divisions

starts. Using again the expected goals enables to obtain the probability of scoring for a throw-in starting in each zone and to plot this at the figure 2.22.



(a) Division of 150 zones



(b) Division of 600 zones

Figure 2.22: Proportion of goal scored starting from a throw-in with both divisions

The probability of scoring increases when moving forward on the pitch except in the closest zones to the corner because the possibilities to make a throw-in from this zones are limited. It is more difficult to reach a forward.

Finally unlike the free-kicks, the zone where the throw-in is played is not always the same as the zone where the throw-in is obtained, especially for the clearances. It is also the case when the throw-ins are obtained from a dual inside the pitch far from the boundaries. If a defender tackle a forward ten metres from the line then it is this location ten meters from the line which is kept for the gain of the throw-in. It is because the forward was in this position that he obtain a throw-in. But the throw-in is not played from this position. The probability of scoring does not depends on this position but on the location where the throw-in is played. So a link is needed between the zone where to throw-in is obtained and the zone where the throw-in is taken to have the probability of scoring from the throw-in. The probability of scoring from a free-kick was the probability of winning a free-kick in a zone multiplied by the probability of scoring from a free-kick starting from the same zone which is not possible for the throw-ins. We need to know the probability that the throw-in is taken in the zone B if it is obtained from the zone A and that for all zones. We need this link even if the zone B is a zone next to the line because the ball does not always go out next to the duel. After compiling all those probabilities we obtain the probability of scoring from a throw-in starting in zone B and obtained in zone A, with N_A meaning the number of actions in A, TI_A meaning the number of throw-ins obtained in A:

$$p^{ti}_{AB} = \frac{TI_A}{N_A} \times \frac{TI_A \text{ starting from B}}{TI_A} \times \text{Probability of scoring from a throw-in starting in B}$$

which is the probability of winning a throw-in in A multiplied by the probability of playing this throw-in in B and the probability of scoring from a throw-in starting in B. The TI_A can be simplified from this equation. By summing this equation for all zones B and making the difference between under pressure and without pressure we obtained the probability of scoring from a throw-in in A which can be done for all zones A.

2.5.5 All set pieces

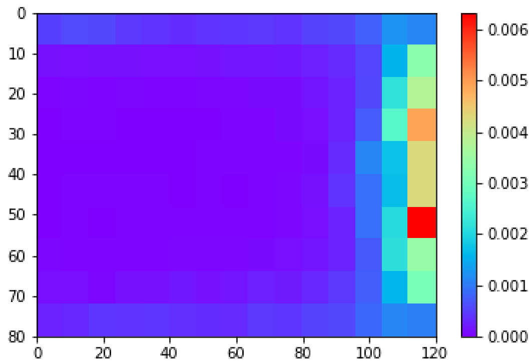
As said earlier multiplying the probability of obtaining a free-kick in a zone by the probability of scoring a free-kick in this zone gives the probability of scoring from this zone with a free-kick. For the throw-in, it is a bit more complicated but the probability of scoring from each zone with a throw-in can also be obtained. For the corners and penalties the probability of scoring from this set piece does not depend on the zone in which the set piece was obtained. The probability of scoring from some a zone with a corner or a penalty is the probability of obtaining this set piece in the zone multiplied by the probability of scoring with this set piece. The sum of the probabilities of scoring from each set-piece gives the probability of scoring from a set piece in a zone A:

$$
\begin{aligned}
p_{\text{set-piece}A} =& p_{\text{penalty}A} + p_{\text{free-kick}A} + p_{\text{corner}A} + p_{\text{throw-in}A} \\
=& \frac{\text{Number of penalties won in A}}{N_A} \times \text{probability of scoring a penalty} \\
&+ \frac{\text{Number of free-kicks won in A}}{N_A} \times \frac{\text{Number of goals scored from a free-kick starting in A}}{\text{Number of free-kicks starting from A}} \\
&+ \frac{\text{Number of corners won in A}}{N_A} \times \text{probability of scoring a corner} \\
&+ \sum_{\text{all zones B}} p^{ti}_{AB} \\
=& \frac{\text{Number of penalties won in A}}{N_A} \times 0.7612 \\
&+ \frac{1}{N_A} \times \frac{\text{Number of goals score from a free-kick starting in A}}{1} \\
&+ \frac{\text{Number of corners won in A}}{N_A} \times 0.047 \\
&+ \sum_{\text{all zones B}} p^{ti}_{AB}
\end{aligned}
$$

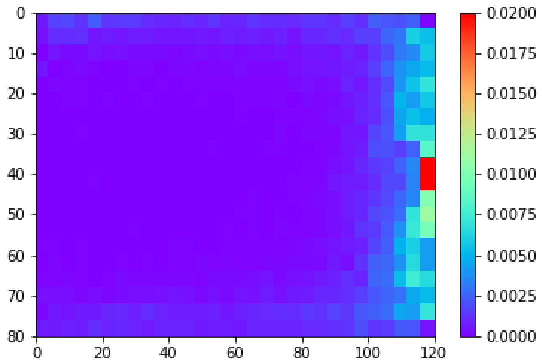
Using the previous computations and the formula, the probability of scoring from a set piece in each zone with or without pressure is shown at the figure 2.23.

Again due to the goal line clearances the values in the two zones closest to the center of the goal are really higher (0.040 and 0.035) than the values in others for the division of 600 zones without pressure so the scale is also bounded for clearness. The chances to score from a set-piece are small for the zones far from the goal and a bit higher far from the goal but next to the sideline. There are more chances in the last twenty metres and especially in the penalty area. The probabilities are higher under pressure because there are more set-pieces won under pressure as seen earlier. The results obtaining and scoring from each set-piece separately are shown at the appendix D with the probability of obtaining any set-piece in a zone which is the sum of the probabilities of obtaining each set-piece separately.

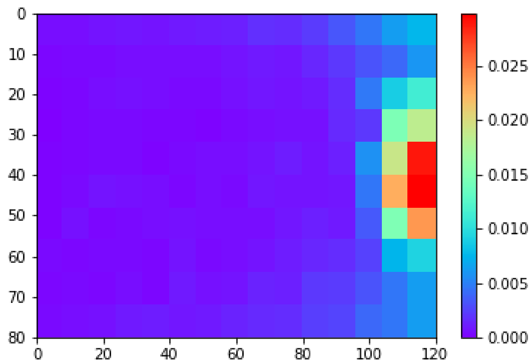
There are now several possibilities when making an action to a zone i.e. not a shot. The first one is that the action is missed and the ball is lost. Otherwise the action is successful and the ball arrives in the desired zone. If so either nothing happen and the possession continues either a set-piece is obtained and potentially scored. The probability of arriving in the zone is now the product of the probability of achieving the action by the probability of not getting a set-piece in the arrival zone. The probability of scoring (if the action is not shot) is the probability of obtaining and scoring from a set-piece multiplied by the probability of achieving the action. The probability of losing the ball is the sum of the probability of failing the action and the probability of achieving the action and obtaining a set-piece but losing the ball during the set-piece. The probability of losing the ball is also 1 minus the two first probabilities which is easier to compute. The probability matrix for the action "Move to zone 2" discussed before can be rewritten considering that there is a probability of 0.02 of winning a set-piece and of 0.001 of winning and scoring from a set-piece. Those probabilities are small since the zone 2 is far from the goal. From zone 0 the probability of achieving a pass was 0.54 but now we consider the action move to the zone 2 which has a probability of success of 0.73. The probability of success without getting a set-piece is so $0.73 \times (1 - 0.02) = 0.715$. The probability of goal is



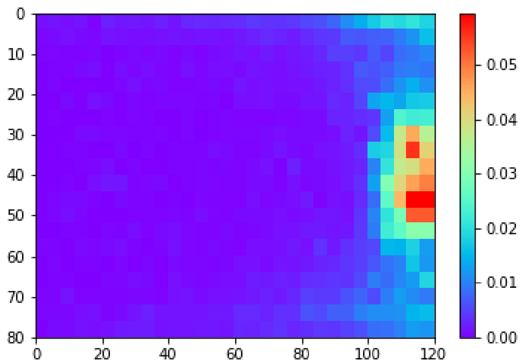
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



(c) Division of 150 zones under pressure



(d) Division of 600 zones under pressure

Figure 2.23: Probability of obtaining and scoring from a set piece with both divisions

$0.001 \times 0.73 \simeq 0.01$ and the probability of losing the ball is $1 - 0.715 - 0.001 = 0.284$. By doing this for all zones, the probability matrix of the action "Move to zone 2" becomes

$$\begin{array}{c} \\ P_{p2} = \end{array} \begin{array}{c} \begin{array}{ccccccc} \text{To zone 0} & \cdots & \text{To zone 2} & \cdots & \text{To zone 5} & \text{To Lost} & \text{To Goal} \end{array} \\ \begin{bmatrix} 0 & \cdots & 0.715 & \cdots & 0 & 0.285 & 0 \\ 0 & \cdots & 0.853 & \cdots & 0 & 0.146 & 0.001 \\ \vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0.941 & \cdots & 0 & 0.058 & 0.001 \\ 0 & \cdots & 0 & \cdots & 0 & 1 & 0 \\ 0 & \cdots & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{From zone 0} \\ \text{From zone 1} \\ \vdots \\ \text{From zone 5} \\ \text{From Lost} \\ \text{From Goal} \end{array}$$

For the action dribble the probability of a successful dribble in the second zone was 0.64. Similarly as the action move the probability of arriving in zone 2 without set-piece is the product of the probability of a successful dribble in zone 2 and the probability of not getting a set-piece in zone 2. Here it is equal to $0.64 \times 0.98 = 0.627$. The probability of scoring is $0.001 \times 0.64 \simeq 0.001$ and so the probability of losing the ball is $1 - 0.627 - 0.001 = 0.372$. A row of the probability matrix of the action dribble is built like this

$$P_{d2} = \begin{array}{c} \begin{array}{cccccc} \text{To zone 0} & \cdots & \text{To zone 2} & \cdots & \text{To zone 5} & \text{To Lost} & \text{To Goal} \end{array} \\ \begin{bmatrix} 0 & \cdots & 0.627 & \cdots & 0 & 0.372 & 0.001 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{From zone 2} \end{array}$$

Since the ball does not arrive in a zone after the action shot, the set-pieces do not change the probability matrix.
Moreover, the probability matrices with the attribute under pressure also change when considering the set piece. The case with and without pressure can be computed separately because the chances of obtaining a set-piece are known with and without pressure. Still for the zone 2, the probability of winning a set-piece is 0.037 under pressure and 0.004 without pressure. The probability that the action "Move to zone 2" is a success from zone 0 without pressure and that it arrives in zone 2 without pressure was 0.727 and 0.001 for arriving in zone 2 under pressure. With the set-pieces, the probability of arriving in zone 2 without pressure is the probability of arriving in zone 2 without pressure while getting no set-piece. Since the probability of getting a set-piece in zone 2 without pressure is 0.004 the probability of not getting a set-piece in 0.996 and the new probability of arriving in zone 2 without pressure is $0.727 \times 0.996 = 0.724$. Under pressure the probability of having a set-piece is 0.037 and so the new probability of arriving in zone 2 under pressure is $0.006 \times 0.963 = 0.006$. The probabilities of scoring are the sum of the probability of scoring from a set-piece won under pressure and without pressure. Under pressure this probability is the probability of a successful action multiplied by the probability of winning and scoring from a set-piece in zone 2 under pressure which is equal to 0.002 . Without pressure this probability is 0.0004 and so the probability of scoring is $0.006 \times 0.002 + 0.727 \times 0.0004 = 0.0003$ which is small also because the zone is far from the goal. Doing this from the other zones gives the following matrix:

$$P_{m2} = \begin{array}{c} \begin{array}{cccccccc} \begin{matrix}\text{To zone 0}\\ \text{without press}\end{matrix} & \cdots & \begin{matrix}\text{To zone 2}\\ \text{without press}\end{matrix} & \cdots & \begin{matrix}\text{To zone 2}\\ \text{under press}\end{matrix} & \cdots & \text{To Lost} & \text{To Goal} \end{array} \\ \begin{bmatrix} 0 & 0 & 0.724 & 0 & 0.006 & 0 & 0.270 & 0.0003 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0.943 & 0 & 0.007 & 0 & 0.050 & 0.0003 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0.791 & 0 & 0.129 & 0 & 0.079 & 0.0005 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \begin{matrix}\text{From zone 0}\\ \text{without press}\end{matrix} \\ \vdots \\ \begin{matrix}\text{From zone 2}\\ \text{without press}\end{matrix} \\ \vdots \\ \begin{matrix}\text{From zone 2}\\ \text{under press}\end{matrix} \\ \vdots \\ \text{From Lost} \\ \text{From Goal} \end{array}$$

The probability of scoring is higher when starting from a state under pressure which can seem weird but it is because starting from a state under pressure gives more chances to reach a state under pressure and so to obtain a set piece. Moreover it will be counterbalanced by the fact that the actions are more successful without pressure.

For the action dribble, if it started from the state zone 2 under pressure the next state is zone 2 without pressure with a probability of 0.01, zone 2 under pressure with a probability of 0.63 and lost otherwise. In the same way as before the probability of being in zone 2 without pressure after the dribble is reduce by the set-pieces: $0.01 \times 0.996 = 0.01$ and also under pressure: $0.63 \times 0.963 = 0.61$. The probability of scoring is also computed in the same way $0.01 \times 0.0004 + 0.63 \times 0.002 = 0.001$. And the corresponding row of the matrix is

$$P_{m2} = \begin{array}{c} \begin{array}{cccccccc} \begin{matrix}\text{To zone 0}\\ \text{without press}\end{matrix} & \cdots & \begin{matrix}\text{To zone 2}\\ \text{without press}\end{matrix} & \cdots & \begin{matrix}\text{To zone 2}\\ \text{under press}\end{matrix} & \cdots & \text{To Lost} & \text{To Goal} \end{array} \\ \begin{bmatrix} 0 & 0 & 0.01 & 0 & 0.61 & 0 & 0.38 & 0.001 \end{bmatrix} \end{array} \begin{matrix}\text{From zone 2}\\ \text{under press}\end{matrix}$$

The appendix C also shows these matrices and they array versions using the fact that they are sparse matrices.
As for the previous models the Markov Decision Process can be drawn at the figure 2.24. The states and actions are also not all drawn. The colors are the same as at the figure 2.12.

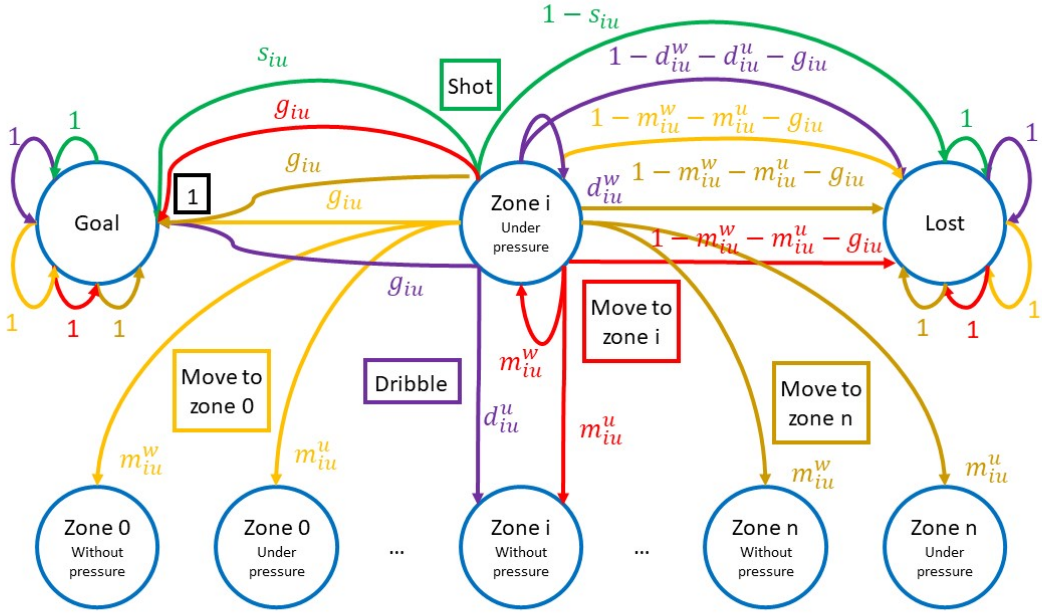


Figure 2.24: Markov Decision Process of the model with the set-pieces

2.6 Players strengths

For the moment, all the players have the same abilities. The probability of success of any action does not depends on the player performing the action. The assumption is that all players have the same chances of successfully making a pass, a long pass, a carry, a dribble a long shot a header or any other particular action. In the reality this assumption is not always verified. A simple pass, a short carry or a shot in the six-yard box (a rectangle of 18.3 metres on 5.5 metres next the goal) is usually successful for all football players. On the other hand some actions have a smaller success rate. It is by achieving these actions more often than average that some players stand out. For example Kevin De Bruyne is known for his quality of passing, especially medium and long range passes, the quality of the finishing of Erling Haaland is reputed and the same for the long-distance shots of Lionel Messi. A player does not need to be as good as those players to be above average while doing some of those actions. Is the optimal action always the same for an average player or for one of these three players ? To answer this question the transition matrices of the players have to be computed. Considering only the available actions of a player to compute his transition matrices would not be sufficiently representative due the large number of probabilities to computed compared to the number of available actions by player. The solution is to use the available actions of a player to determine if he is in a category such as long-distance strikers and to use all the actions of all players of a category to compute the corresponding transition matrix or matrices. Furthermore a player can be strong in different categories. For example Kevin De Bruyne regularly scores long-range goals in addition to his quality of pass. This section will define different categories of players, determine the players in these categories and compute the transition matrices of these categories in order to obtain the optimal actions for players in these categories in the next chapter.

2.6.1 Long-range strikers

We saw earlier, on the figure 2.4, that scoring from outside the box is difficult as witnessed by the expected goals which are smaller outside the box. It can be explained by several facts like that it requires more accuracy and strength. Moreover there are more chances that the shot is blocked by a defender and the goalkeeper has more time to react and to try to stop the shot when the shot is taken from a distant zone. However, by looking at all the shots and the goals of Kevin De Bruyne at the figure 2.25 we observe that a lot of goals are scored from outside the box. The number of shots is also larger outside the box but the expected goals will shows that it does not explain the large number of long-range goals.

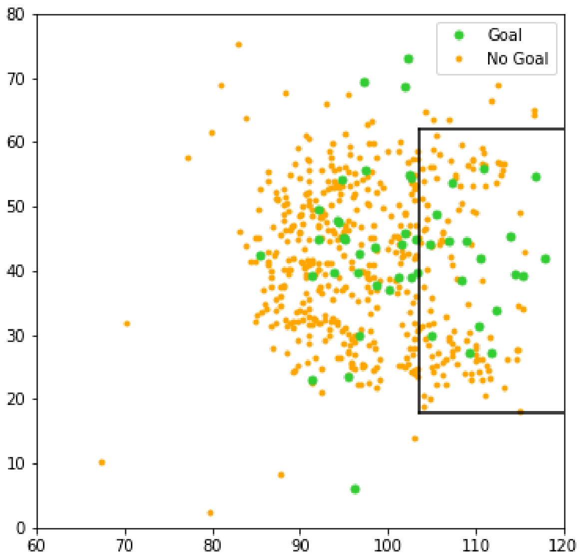


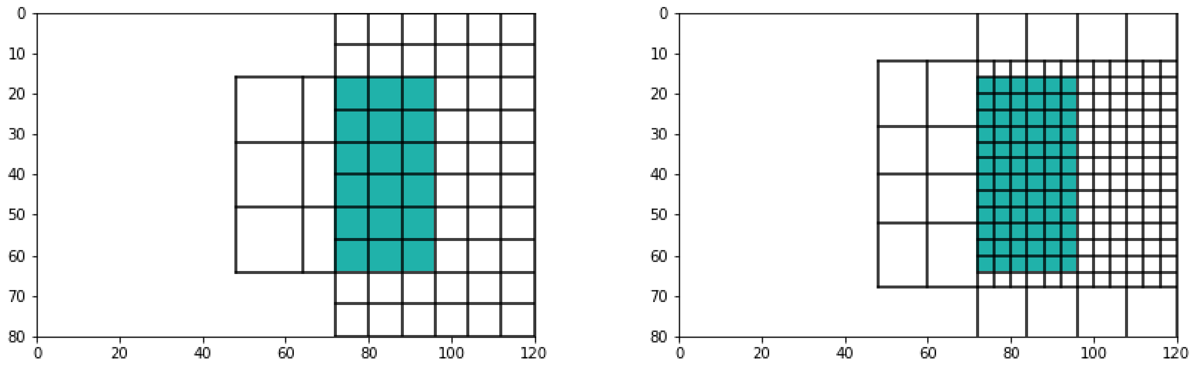
Figure 2.25: Shots of Kevin De Bruyne with their result

It is because De Bruyne is better than average for long-range shots. From the 368 shots taken by De Bruyne outside the penalty area, the total value of the expected goals is 13.88. The number of goals of an average player is similar to his number of expected goals. It means that an average player will score around fourteen goals with these shots. De Bruyne managed to score twenty-seven goals outside the box. He outperforms his xG by 13.12. He scored 194 percent of the goals that he should have scored. De Bruyne scores twice as much as he should by watching his shots. He is above average at shooting from a distance. He also outperforms his xG inside the penalty area but he scored "only" 115 percent of his xG. De Bruyne is therefore probably in the category long-range strikers. How can this category be defined ? Thresholds have to be chosen to determine if a player is a long-range striker or not. In this section we will consider the best players in their categories.

Definition 2.7 (Long-range striker)**.** A long-range striker is a player who has scored more than five goals outside the box and who has one of the best ratio between his goals and expected goals.

A threshold on the number of goal scored is necessary to eliminate the element of luck a player can have by scoring a few goal from few shots. The ration between the goals and the expected goal is here to have the players outperforming the most their expected goals. A ratio greater than one means that the player scores more goal than what the expected goals show. This ratio is equal to 1.94 for Kevin de Bruyne. With these conditions, we consider that the top players are are those with a ratio in the highest two percent. After collecting all the long-range shots with their value of expected goal and their result, the ratios are computed and sorted to obtain the best long-range strikers. In the best long-range strikers there are some well-known names such as Belgian players like De Bruyne, Mertens and Nainggolan, players known for their long shots like Pogba or Coutinho and other great players like Messi, Mbappé or Salah etc. The complete list of the eighty players is written at the appendix E.1. Once the players of the category are obtained, the transition matrix corresponding to this category can be computed. This category will only changes the transition matrix of the action shot. Moreover because the players are long-range strikers the transition matrix will be different only for shots outside the penalty area. If a player of this category shots inside the penalty area he is considered as an average player unless he also has finishing quality in the penalty area which will be discussed in the following part. To compute the probability of scoring from outside the box, the expected goals can not be used because the expected goals are the same for a player good at long shot or not. Therefore it is the proportion between the number of goals scored and the number of shots in each zones which will determine the probability of scoring from the zones. This proportion is similar but less accurate for the average strikers and is larger for long-range strikers. They scores 6.5 percents of their shots outside the box compared to the 3.5 percents for all players. However, we saw earlier that considering the proportions of scoring shots gives sometimes weird results especially in the zones far away from the goal or next to the side lines. Therefore the zones where the probability of scoring from a shot differs for the long-range striker and the other players are shown in blue on the figure 2.26 with both divisions.

The players of this category have to be determined using the Understat dataset because the StatsBomb dataset contains less shots and less players. Moreover Understat has an attribute to classify the shots in long-range shots or not. With the same parameters there are only four players in the category long-range strikers using StatsBomb: Messi, Henry, Ronaldinho and Xavi. They are four players which played for Barcelona because there is a lot of matches of FC Barcelona in the dataset so there are less different players. Counting the numbers of shots will too small to have accurate probabilities with the StatsBomb dataset. The probabilities of scoring in each zone is computed using all shots of all long-range strikers and kept for the zones of long-range shots. For the other zones the probabilities are computed with the expected goals



(a) With the division of 150 zones (b) With the division of 600 zones

Figure 2.26: Division of the pitch with zones of long-range shots

of all shots of all players as before. The chances of scoring for a player of this category with the division of 600 zones are shown at the figure 2.27.

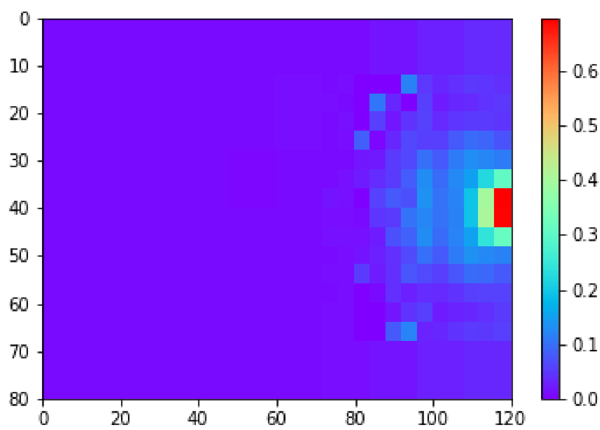


Figure 2.27: Probability of scoring for a long-range striker in each zone of the pitch

This chances should be larger in the long-range zones. To verify this we can compute the difference with the probability of scoring for the average player and plot this at the figure 2.28

The differences are indeed positive in most of the long-range zones except in some of the furthest zones which is maybe due to the fact it is the zones where the least shots are taken and so the proportions are the less accurate. The differences are nil in the other zones.

Because the dataset used in the part was Understat the attribute under pressure is not taken into account because it is only an attribute of StatsBomb. The problem is to obtain the probability of scoring with or without pressure using the probability of scoring in both cases together. To solve this, it is possible to use the StatsBomb dataset to obtain the ratios between the average chances of scoring and the chances of scoring with the attribute to transpose this to the probabilities computed before. For example in a zone if the probability of scoring with or without pressure is 0.5, 0.75 without pressure and 0.25 under pressure. It means that, in this zone, the probability of scoring under pressure is half the probability of scoring and the probability of scoring without pressure is three half the probability of scoring. If the probability of scoring

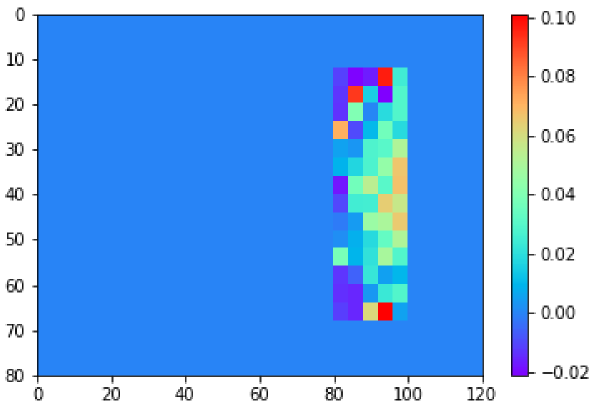
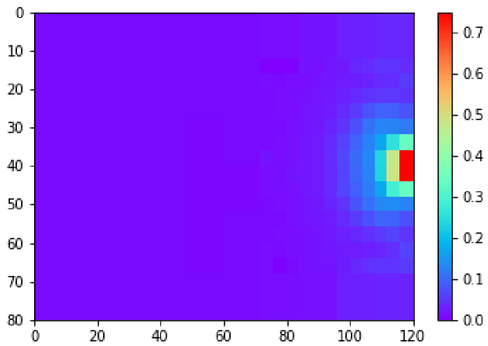
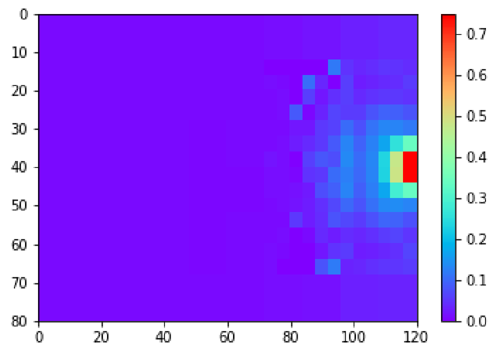


Figure 2.28: Difference between the chances of scoring of a long-range striker and an average player

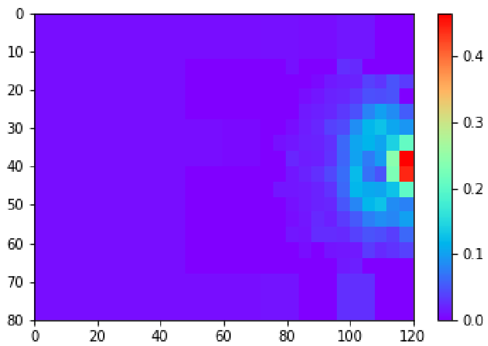
for the long-range strikers is 0.6 in this zone, the approximation will be that the probability of scoring without pressure is 0.9 and 0.3 under pressure. This will determine the new probabilities of scoring including the probabilities for the players in the category long-range strikers. All these probabilities displayed at the figure 2.29.



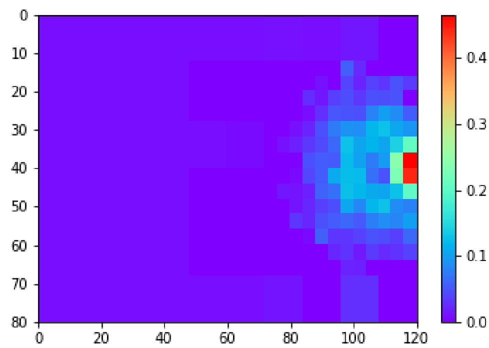
(a) Probabilities for all players without pressure



(b) Probabilities for long-range strikers without pressure



(c) Probabilities for all players under pressure



(d) Probabilities for all long-range strikers under pressure

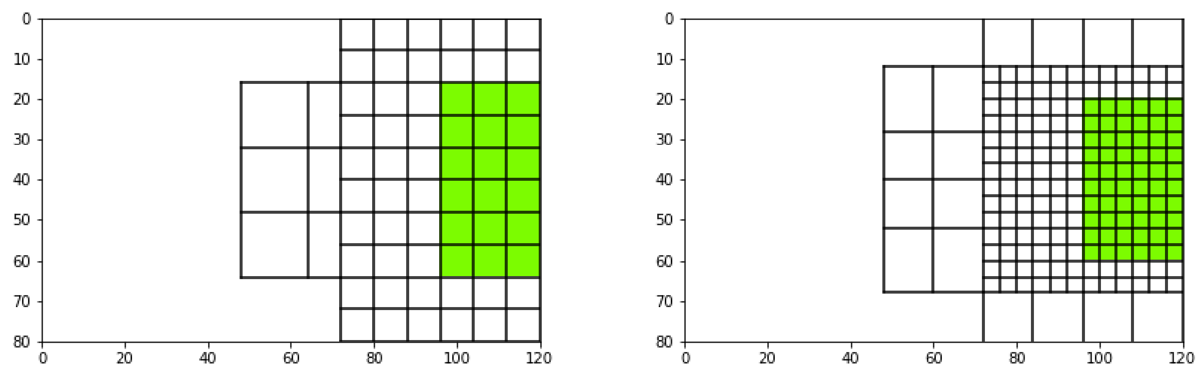
Figure 2.29: Probabilities of scoring from each zone of the pitch

2.6.2 Players with good finishing

Moving to the closest zones to the goal is difficult so it is tempting to shot earlier in the penalty area. It reduced the chances of scoring. But some players are said to be able to score even with small opportunities. They can finished a possession by a goal if the team passes the ball to them in the penalty area. Is there really players that score more often than other in the penalty area ? A study of the number of goals compared to the expected goals in the penalty area can be done using Understat. The attribute of Understat telling that a shot is taken in the penalty are will be used. It will gives the players with good finishing.

Definition 2.8 (Player with good finishing)**.** A player with good finishing is a player who score more often than the average players in the penalty area. A player who has scored more than five goal in the penalty area and who outperforms its xG as a ratio inside the penalty area the most is in this category

The threshold on the performance is the same as for the long-range strikers. The ratios are computed as before and sorted. Again the best rwo percent of players are chosen. This conditions gives the players of the category of players with good finishing. Here also there are Belgian players such as Hazard, Nainggolan and Tielemans. There are also well-known names like Haaland, Bale or Dembélé. All the other players in this category are written at the appendix E.2. As for the long-range strikers the probability of scoring will be the proportion between the shots from a zone leading to a goal and all shots in the zone. It will change the transition matrix of the action shot only in the zones of the penalty area because these players are considered as average players in the other zones except for players like Nainggolan or Messi who are players with good finishing and also a long-range striker. The zones considered as into the penalty area are the one in green on the figure 2.30



(a) With the division of 150 zones

(b) With the division of 600 zones

Figure 2.30: Division of the pitch with zones into the penalty area

The probabilities of scoring can now be computed in the penalty area using all shots of players in this category and also outside the penalty area using the expected goals of all players. The probabilities outside the box have already been computed. The probabilities of scoring for a players with good finishing are plotted at the figure 2.31.

As for the long shots, the chances are higher in the penalty area for the skilled player as evidenced by the figure 2.32 with the differences between the probability of scoring for a player

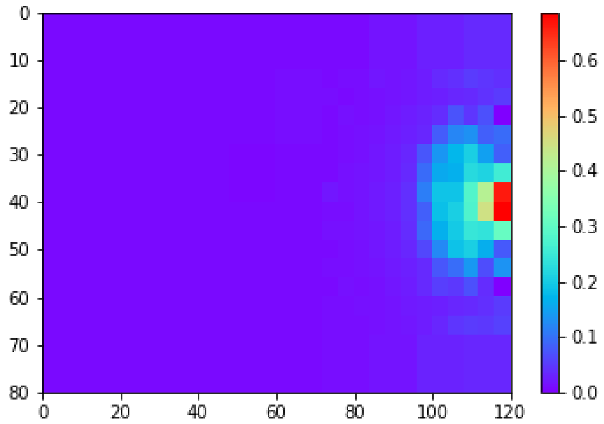


Figure 2.31: Probability of scoring for a player with good finishing in each zone of the pitch

with good finishing and the probability of scoring for other players.

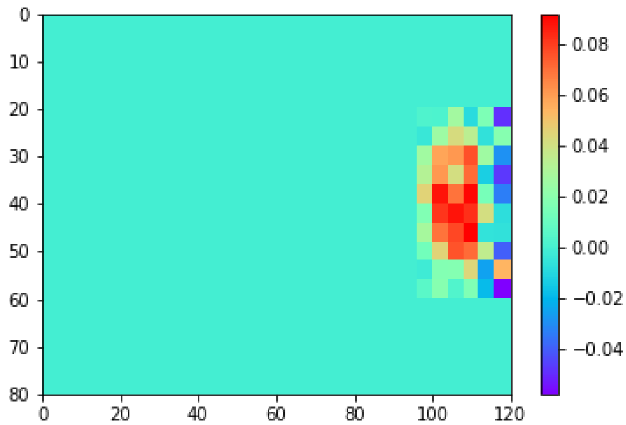
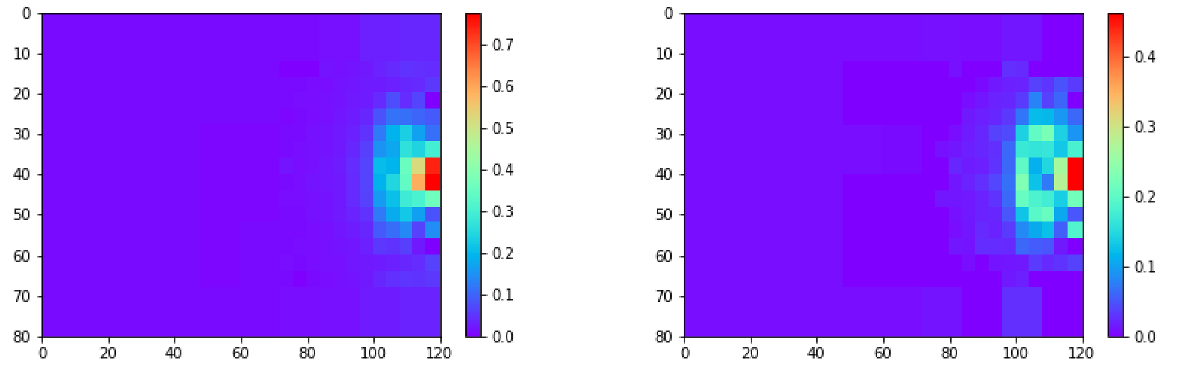


Figure 2.32: Difference between the chances of scoring of a player with good finishing and an average player

As expected, the probabilities are higher for the good strikers except also in some zones with a few shots taken from and/or in the corners of the box.
Like in the previous part the players and the probabilities are obtained with Understat. Obtaining the players with StatsBomb will gives mainly Barça players and some women's football players and not a large range of players because the number of matches is limited especially for other teams than FC Barcelona. The feminine players are Kelly, Nobbs, van de Donk, White, Kirby, Little and the Barcelona players are Messi, Rakitic, Dembele as in Understat and Xavi, Fabregas, Bojan, Sanchez and Villa whose matches are in the StatsBomb dataset but not in the Understat one.
To obtained the probabilities under and without pressure the same approximation is used in the penalty area. The probabilities outside the penalty area are once again already computed just as the chances of scoring inside the penalty area for all players. The probabilities for the players of this category are shown at the figure 2.33



(a) Probabilities of scoring without pressure

(b) Probabilities of scoring under pressure

Figure 2.33: Probabilities of scoring from each zone of the pitch for players with good finishing

2.7 Adding the location of the players

So far the location of players other than the one with the ball has not really been taken into account. The pressure attribute allows to know if an opponent is nearby to try to recover the ball. But what the other opponents are doing is unknown. Similarly, it is not known whether teammates are nearby to help the ball carrier. Furthermore, when moving to another area there could be a large number of opponents trying to get the ball back. Conversely, there may be several teammates offering solutions.
Knowing the locations of the other players may help for two purposes. The first is to add an interesting feature for the computations of the expected passes and the expected carries. The second purpose is to obtain the probabilities of successful moves for a particular frame. The frame gives the position of the other players which enables to compute the probability of moving to each state depending on the other players. The StatsBomb dataset contains the location of several players for several events. As said earlier, in addition to the location, the type of the player is written. We know if the player is a keeper, a teammate or the actor. Moreover each 360 frame is related to an event by the identifier. From an event it is possible to find the location of other players. An event could be the shot, and the goal, of Thorgan Hazard during the match Belgium-Portugal of the UEFA EURO 2020 (2021). This event is shown at the figure 2.34.

Figure 2.34: Shot of Thorgan Hazard during the match Belgium-Portugal

StatsBomb enables to move from the real match to a list of locations that can be represented. Thorgan Hazard which is the actor is the blue point. His teammates are shown in red and his opponents in white. The keeper is the black point. The figure 2.35 shows this representation.

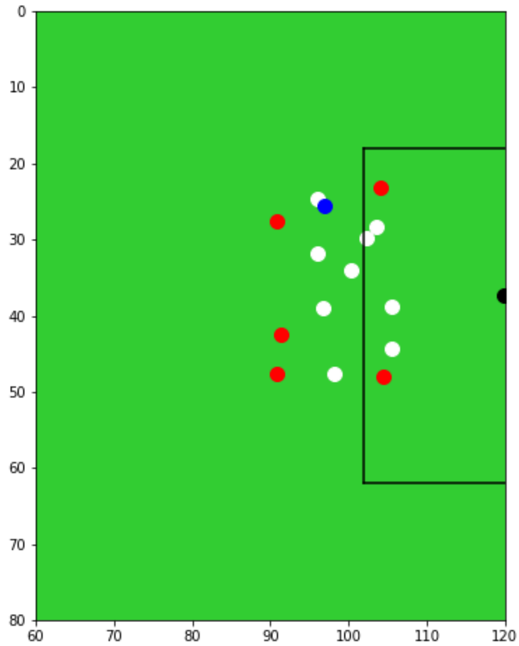


Figure 2.35: Locations of the players at the shot of Thorgan Hazard

At this moment Thorgan Hazard had to shoot quickly because an opponent was just behind him. But were the other opponents also close to him ? Were his teammates close enough to help him if he lost the ball ? StatsBomb judges that an opponent put pressure on the actor if he is in a five-yards range around the actor[26] This value corresponds to 4.57 metres. This value is used to compute the number of opponents close to the actor. Furthermore, this value is also used to obtain the number of teammates close to the actor. Having a teammates close to ball carrier helps him by giving more possibilities of playing. The 360 frames are used to compute the number of teammates and the number of players in a five-yards range around the actor.

If the action is move to a zone then the number of players located around the ending location is important. More teammates means more chance to achieve the actions and more opponents means that the action will be more difficult to achieve. The number of players in a five-yards range at the end of the move has to be computed using the 360 frames. However the locations of the players at the start and at the end of the moves are not the same. The location of the players at the start of the event has to be collected using the frame associated to the event. Because the other players could also move during the action the location of the other players are not known after a move. The players usually move in the direction of the ball. If a opponent is located at six yards of the arrival of a long pass he will not be counted inside the five-yards circle even though he has time to get closer to the target location during the time of the pass. The 360 frames are not always available at the end of a movement especially for the carries. The movements of the players during the movement of the ball must be estimated in order to know the number of teammates and opponents that can be close to the end location. The circle of

five-yards where the players are counted increases according to the time of the ball movements because is time for other players to move. To know how much it increases the duration of the movement and the distance made by the players during this time has to be computed.
To estimate the duration of the moves we have the length of the moves knowing the starting and target locations. The duration is also the length divided by the average speed of the movements. The average speed for a ball movement is obtained by doing the mean of the average speed of each movement in the dataset. Let's assume that there are three moves in the dataset. The first one is done in five seconds and is twenty metres long. The second lasts ten seconds and is sixty meters long. The last is also sixty meters long but is done in twelve seconds. The average speed is therefore 5 meters per second. If a player want to make a move of length fifty meters he knows that the other players will be able to move for about ten seconds before the move ends. Using the StatsBomb dataset, the average mean for the moves is computed and equal to $8.95[\frac{m}{s}]$.
The time during which the other players can move to get closer to the ball is known and depends on the length of the move. Now the distance covered during this time must be calculated. This will give the distance from which they should be considered as being in a near circle of the arrival. To simplify the running of a player, it will be modelled as an UARM. However, if the player reaches his maximum speed, then he will run as an URM at that maximum speed. Another assumption is that the players start with a zero initial speed because even if the players are running, they are not running directly in the direction of the target location. The acceleration and the maximal speed were estimated in a study[27] The values obtained in this study were an acceleration of $0.6[\frac{m}{s^2}]$ and a maximal speed of $8.4[\frac{m}{s}]$. These are the values that will be used. These values are approximation because it depends on the player and also on the freshness of the player. The distance covered by the player is shown at the figure 2.36.

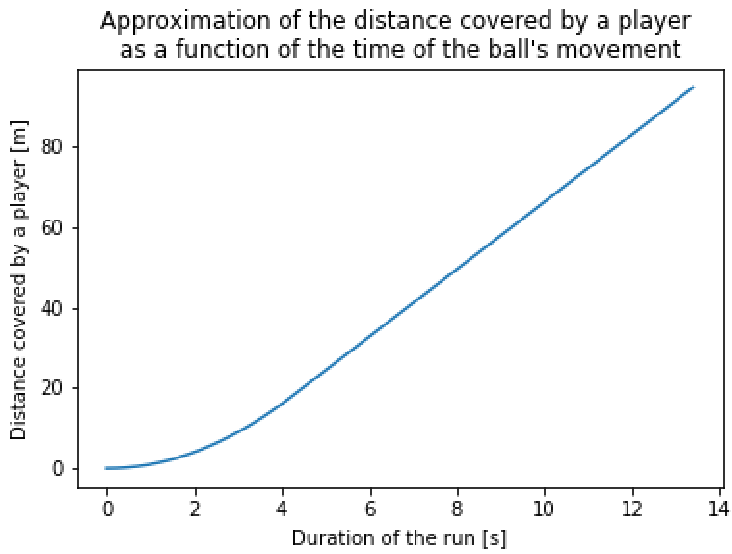


Figure 2.36: Distance covered by a player during an run

Knowing the move which will be attempted, the duration of this move can be estimated. This duration corresponds to a distance that can be covered by the players. The players counted in the five-yards range around the target location are those located at a distance from the five-yards circle smaller than the distance that can be covered. The number of teammates and opponents at the start and the end of the moves are interesting features for the expected passes and carries. Indeed these numbers have an influence on the probability of success. A pass made under pressure of two players is more difficult to achieve than a pass without pressure or under pressure of one player who must also follow a teammate nearby. The transition matrices for the action move to zone i is computed using the mean of the expected passes and carries as before but now the vectors contain the number of teammates and of opponents at the start and at the end of the move. However there are only 47,618 passes and 40,937 carries in the dataset with the 360 frames. This is not enough to compute the transition matrices due to the large number of probabilities to obtain. The matches with 360 frames are added little by little in the open dataset. The other matches have been scheduled to be added with their 360 frames. The transition matrices could be calculated with the full dataset or when the open dataset will be complete. To show the usefulness of the 360 frames without computing all the transitions matrices, we will compute probabilities for some 360 frames in the section 3.3.

Chapter 3

Optimal actions

Once the model is obtained and built, the optimal action in each state can be obtained using the algorithm 1. Similarly to the simple model, it seeks for the optimal actions, the best action which is the one giving the largest expected reward. In other words it is the one leading to more chances of scoring during the possession. The expected reward received by an action is not only the reward of the transition but it take into account the expected rewards in the other states through the several iterations of the algorithm.

3.1 First improvements and set-pieces

Firstly the optimal actions will be obtained in the case of the model with the first improvements and the set-pieces. The first improvements were

- The adding of zones : Two divisions of 150 and 600 zones.
- The adding and the modifications of actions such as move, shot and dribble.
- The consideration of the attribute under pressure.
- The computation of the expected passes and carries.

The value iteration algorithm requires a threshold ϵ which will be equal to $1e-6$. The algorithm will therefore stops when no actions increases the expected reward in a zone more than 1×10^{-6} and which corresponds to an increase of $1 \times 10^{-4}\%$ of the probability of scoring from this zone. The algorithm also requires the 4-uple of a Markov Decision Process. The state space contains 2 states per zone, a state under pressure and one without pressure. The set of actions contains the action dribble, the action shot and the actions move to zone i for all zones i. A reward of one is obtained during to the transitions to the state "Goal" if the starting state is not "Goal". The probabilities of the action dribble are obtained with the proportion of successful dribbles knowing that a dribble starts under pressure and can end either under or without pressure. The chances of scoring are computed using the expected goals of Understat and the dataset of StatsBomb to obtain the probabilities with and without pressure. For the actions move to a zone the probabilities are obtained using the expected passes and expected carries of all passes and carries from a zone to another. However there are probabilities of moving from a zone to another which are based only on the mean of the expected values of a few passes or carries especially for very long passes. It should be avoided to have these non representative probabilities by keeping small zones to stay close to the actual location. If the number of actions done to a zone i from a zone j, the probabilities depends on the mean of the actions done in the zone around the zone j. If the zone j is the zone in the center of the figure 3.1 then the probabilities depends on the zones l and k. We consider that the probabilities in the zone j depends half on the probability in zone j and half on the probabilities in the zones k and l. Moreover because the centers of the zones l are further from the center of the zone j than the centers of the zones k are the probability in the zone j will depends less on the zones l. The factor between the distances is the square root of two. The probability on zone j is therefore, if there are less than five actions in this zone:

$$P_{ij}^{\text{new}} = \frac{1}{2} P_{ij} + \frac{1}{4(\sqrt{2}+2)} \times (P_{ik_1} + P_{ik_2} + P_{ik_3} + P_{ik_4}) + \frac{1}{8(\sqrt{2}+1)} \times (P_{il_1} + P_{il_2} + P_{il_3} + P_{il_4})$$

Zone l1	Zone k1	Zone l2
Zone k2	Zone j	Zone k3
Zone l3	Zone k4	Zone l4

Figure 3.1: Zones around the zone j

The ratio between the weight of the zone j and a zone k is $\frac{1}{2(\sqrt{2}+2)}$ and $\frac{1}{4(\sqrt{2}+1)}$ with a zone l. If the zone j is on a edge or in corner, the previous formula is updated to always have the same ratio between the weight of zone j and the ones of the other zones.

The requirement of the value iteration algorithm are fulfilled so the algorithm can be used to obtain the optimal actions of each zone. The different actions obtained by the algorithm for 150 zones are shown at the figure 3.2

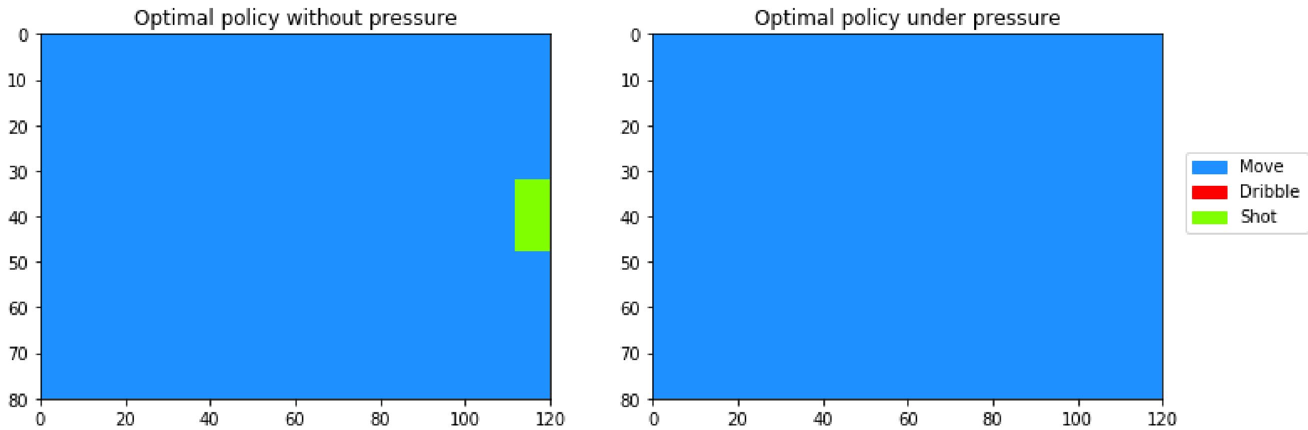


Figure 3.2: Optimal policy of all states with 150 zones

The first observation is that the optimal action without pressure is to move except in the two zones the closest to the goal where the optimal action is to shot. Under pressure the optimal

action is always to move the ball. In the two zones where the player should shot without pressure, he should not under pressure. It will gives more reward to first try to move the ball which will potentially leads to a state without pressure before attempting a shot. the action dribble is never used under pressure because a dribble leads more often to the state under pressure than a pass or a carry. If a player is under pressure he should not dribble but rather make a simple movement like a back pass to arrive in a state without pressure or try a more difficult move to arrive in a favorable zone even if this state will be under pressure. All the actions move are not done to the same zone because some moves are unlikely. A look at the best policy in the states will indicate to which zone to move to.
Starting with the case without pressure, the optimal moves from each zones for which the best action is a move are written at the figure 3.3

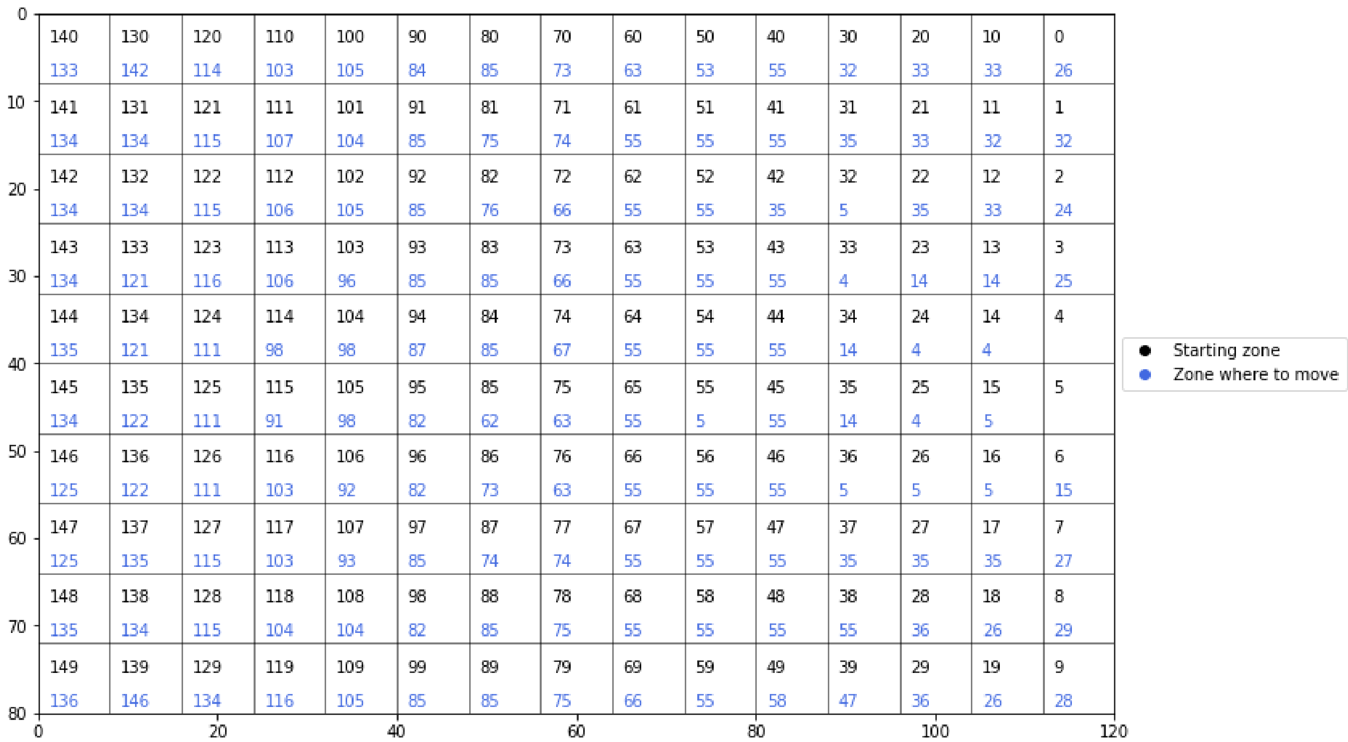


Figure 3.3: Target zones of the optimal actions without pressure

The optimal actions tends towards going in the zone 4 and 5 with are the two zones closest to the goal and the zones 14 and 15 which are the two zones before. To try to see more clearly the intentions a colour code can help. It will split the actions in different categories each associated to a color. The different categories are the following:

- Shot. The first category is the action shot.
- Move to zone 4,5. A move to either the zone 4 or 5 is in this category.
- Move to 14,15. Similar to the previous category but with the zone 14 and 15.
- A backward move to the inside of the pitch. When the move is to a zone lower on the pitch and towards to middle of pitch
- A backward move.
- A backward move to the outside of the pitch. When the move is also to a lower zone but outwards the pitch.

- When the move goes from one side of the pitch to the other.
- When the move is to a zone at approximately the same distance from the goal. It means that if the move sis clearly more along the length of the pitch than across the width then the move is in this category.
- Forwards moves. For the moves which go to a zone higher on the pitch.

The optimal actions with these categories are shown at the figure 3.4

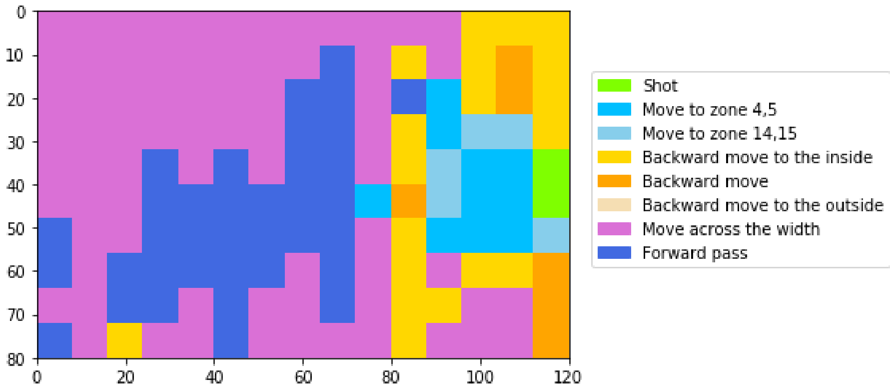


Figure 3.4: Optimal actions without pressure divided in categories

On the half of the attacking team there are mainly forward and side moves. The forward moves are more in the center of the pitch. On the zones further on pitch, the actions are mainly to make side moves or backward moves to come inside the pitch. As said when defining the category, a side move can enable to move forward on the field but less directly than a forward pass. The idea seems to move while taking small risks. In the zones close to the corners the idea is generally to move backward or also to make side moves. Finally when the players are in the centre of the pitch on the edge of or within the penalty area the aim is to reach to zone 4 and 5 where the striker can score while passing through the zones 14 and 15 or directly to the zones 4 and 5.

In the case under pressure all optimal actions are to move. The zones where the player should go are shown at the figure 3.5

There are more zones for which the actions is to move to zones 14 or 15 The figure 3.6 shows the optimal actions divided in the same categories as before.

The actions in the first two thirds are mainly forward and side moves. The side modes are mainly close the edges or the goal where the players should takes less risks and and the forward passes again in the center of the pitch. When a player is in a forward position the optimal action is to move to the zones and or 14 and 15 . This is because a pressure of a defender can free up space in the defense and the player should take risks and try to move the pass to a more favorable zone. Similarly in some zone of the penalty area a player under pressure should try to move to the zone 4 and 5 or 14 and 15. In the zones 4 and 5 a player should not shot but rather move and try to arrive in a state without pressure to shot which has a larger average value of expected goals.

These optimal actions done by all the players lead to a expected reward in each zone. The expected reward corresponds to the proportion of the possession leading to a goal if the posses-

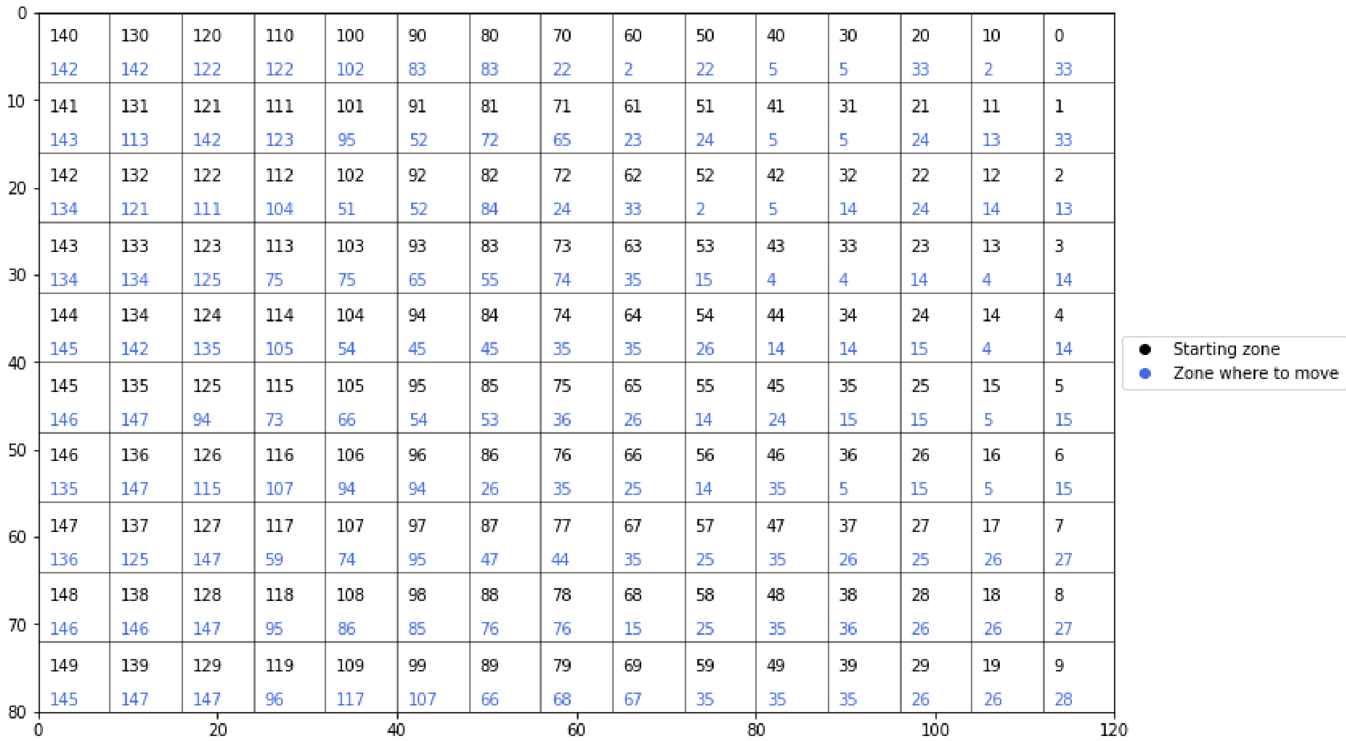


Figure 3.5: Target zones of the optimal actions under pressure

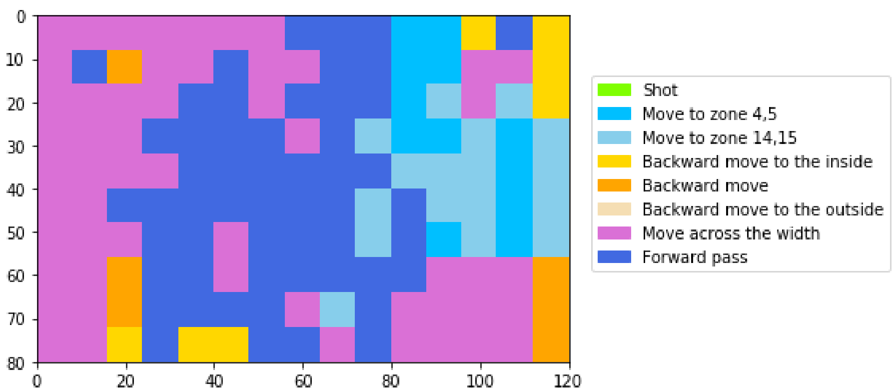
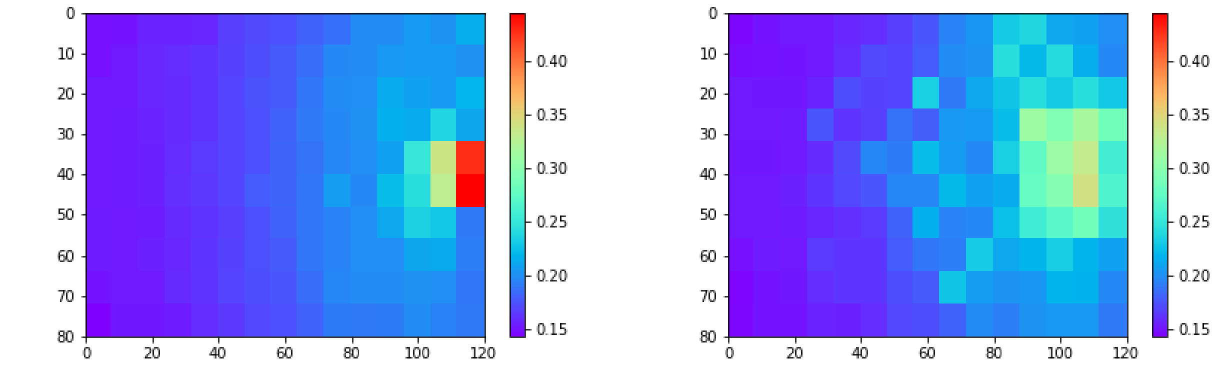


Figure 3.6: Optimal actions under pressure divided in categories

sions are played with the optimal actions . The expected rewards are zero in the state "Lost" and "Goal" as expected and the optimal action can be any action. The largest expected rewards are obtained in the zone 4 and 5 without pressure where the players shot and are equal to 0.43 and 0.45. The figure 3.7 shows the expected reward in all states.

The largest rewards without pressure are in the zone 4 and 5 followed by the rewards in the zones 14 and 15. The rewards are logically decreasing by moving away from these zones. The rewards under pressure do no decrease much in the penalty area. An explanation is that, as said earlier, when a defender press the forward it can create space behind him. Moreover the defenders press more is their own box when the possession is dangerous so when there is more possibilities for the forward which increases the probability of success of the actions in the penalty area under pressure and therefore it also increases the rewards under pressure.

An advantage of the model is that it can be used with 150 or 600 zones. Using the model with 600 zones could confirm or refute the trends obtained with 150 zones. Therefore running



(a) Expected rewards obtained without pressure in each zone

(b) Expected rewards obtained under pressure in each zone

Figure 3.7: Expected rewards obtained with and without pressure in all 150 zones

the algorithm with 600 zones provides a different perspective. The Markov Decision Process is very similar with 600 zones. There are just more states and actions and the probability matrices are different but all is done in the same way. The optimal action in each state is obtained with the value iteration algorithm and shown at the figure 3.8

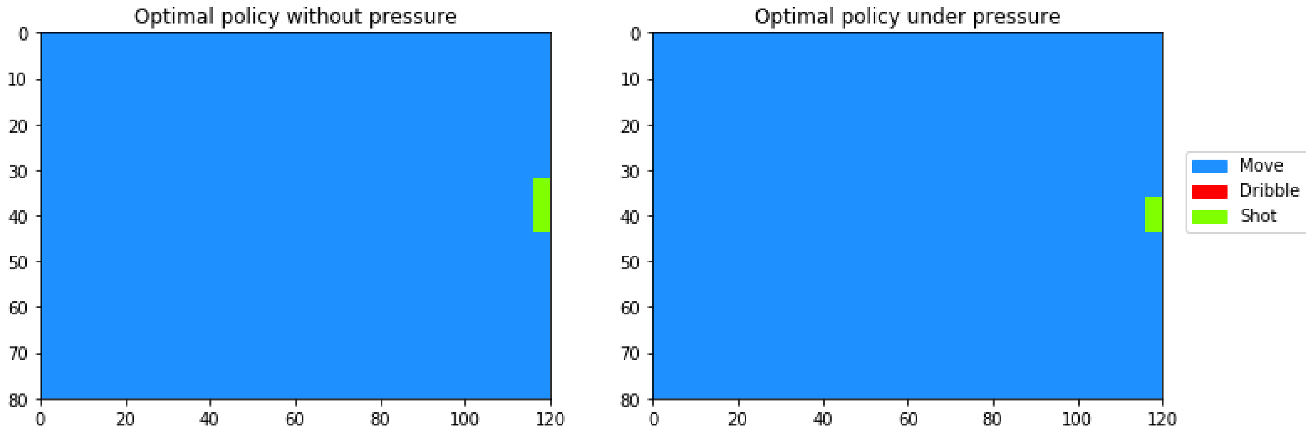


Figure 3.8: Optimal policy of all states with 600 zones

Without pressure the results seems similar since the player should shot only when he is very close to the goal and move otherwise. Under pressure the action is always to move except in the two zone closest to the goal. This makes it possible to specify the actions obtained with 150 zones. The action dribble is never the optimal action as for 150 zones. To keep on verifying the results, the target zone of all actions move can be shown. The optimal actions divided in the same categories as earlier are shown at the figure 3.9
. The category move to zone 4 and 5 keeps the same name but now it corresponds to a move to the zones 8,9,10,11,28,28,30,31 which are the zones with this division corresponding to the zone 4 and with the division of 150 zones. This category is therefore still making a move to the zones closest to the goal. The category move to zone 14, 15 also keeps the same name and still corresponds to the zone just before the closest zones to the goal even if these zones are no longer named 14 and 15. The plot with the detailed zones and actions corresponding is shown in the appendix F.1

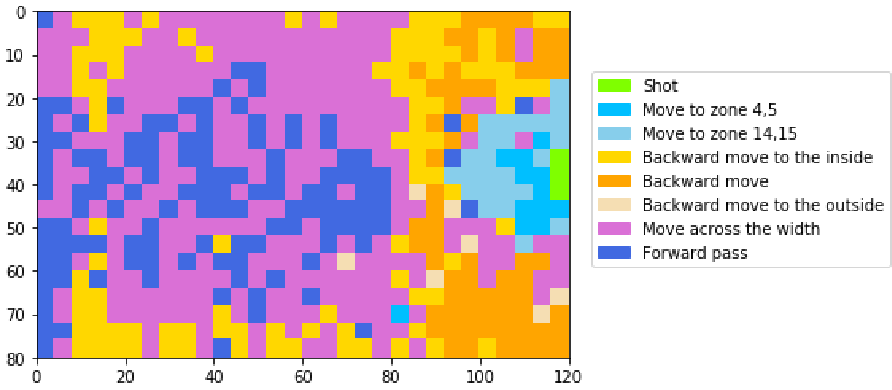


Figure 3.9: Optimal actions without pressure divided in categories

The actions in the two first thirds of the pitch are similar to the ones with 150 zones with forward moves in the center of the pitch and side moves close the edges. This division sows that there are zones where the optimal action is a backward move. The backward moves are on the edges of pitch. It is because most of the actions done in these zones or next to the defensive penalty area. These actions are often done just after a defensive phase so when the opponents are around and where a lost ball may lead to a goal for the opposite team. Most importantly the results on the most offensive zones are verified since there are the sames with both division. The general idea is still to move to the zones closest to the goal to shot from these zones. The moves are sometimes done with a step in the zones just before. If the player is not in the middle of the pitch the optimal actions is still to do a backward move, if possible to the inside, to get closer to the middle of the penalty area.

The optimal actions with 600 zones may also help to confirm the results under pressure. As for the case without pressure the optimal actions divided in categories are shown at the figure 3.10 and the detailed results at the appendix F.1

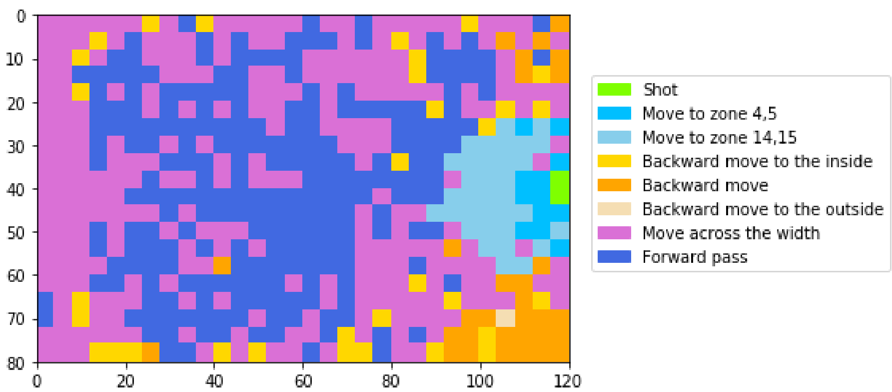
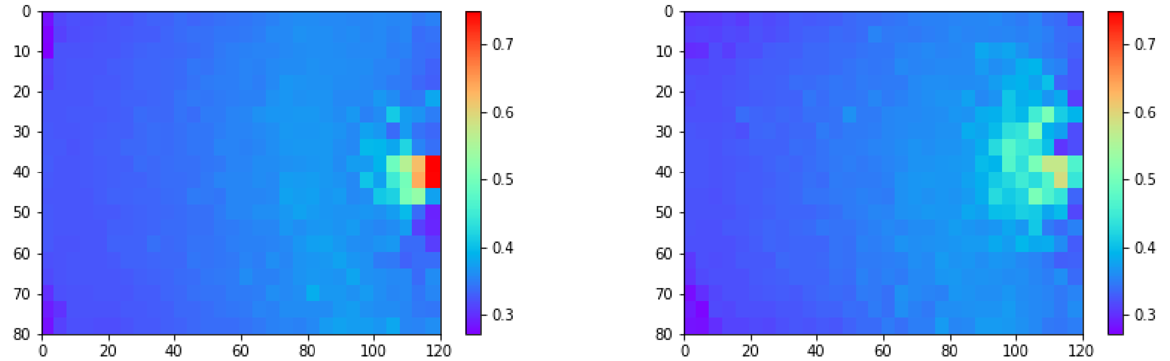


Figure 3.10: Optimal actions under pressure divided in categories

This figure confirms the results obtained with 150 zones. The action in the first two third are backward moves, mainly to the inside, on the sides, some forward moves in the middle and a lot of sides moves. The optimal actions in the offensive zones on the sides on the pitch are forward or side move to get closer to the zone 14 and 15. Inside and around the penalty area the objective is to reach the zones corresponding to the zones 14 and 15 of the division of 150 zones and the zones corresponding to the zones 4 and 5. Once the ball is in the zones closest to the goal either a move is done to stay is these zones but to try being without pressure or either

shot if the player is very close to the goal. It differs from the case with 150 zones because those two zones are closer to the goal than the zones of the case 150 zones so the mean values of the expected goals are very large even under pressure.
Hence, if the player shots from this zone his expected reward is large on average. This reward is equal to 0.75 without pressure and about 0.45 under pressure. Since shot is the optimal action, this is the expected rewards obtained in these zones. The expected rewards obtained in the other states are shown at the figure 3.11



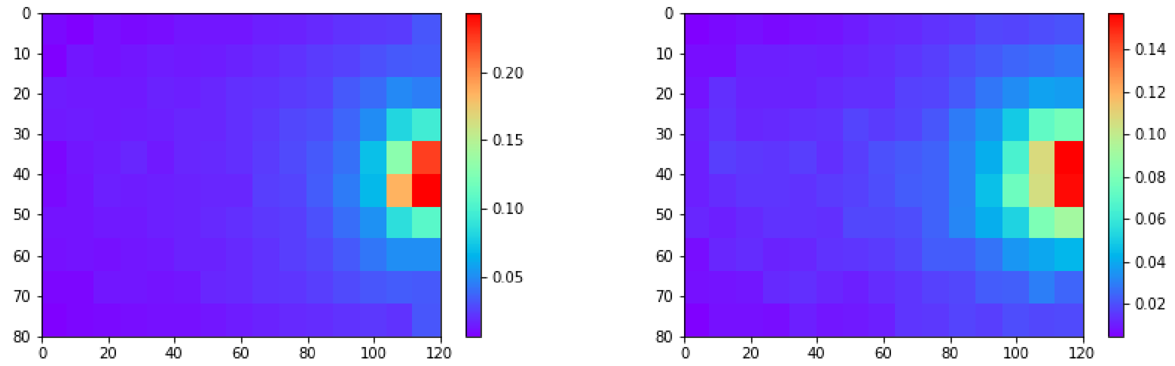
(a) Expected rewards obtained without pressure in each zone

(b) Expected rewards obtained under pressure in each zone

Figure 3.11: Expected rewards obtained with and without pressure in all 600 zones

The expected rewards are larger than the expected rewards with the division of 150 zones because there are more states and actions so more choices for the players and so more chances to have possibilities leading to a very favorable state. The results are also large in some zones inside the penalty area without pressure and the rewards are also more homogeneous in the penalty area under pressure as before.

Now let's find out what happens when only the first action is optimal. It means that the player with the ball in a state chooses an action and has to maximize the average reward during the possession knowing that all others actions will be average. Firstly we need to obtain the average reward in all states. The average reward in a state could be computed using all the possessions passing through this state and ending by a goal and all the possessions passing through this state. However, as for the set-pieces, the expected goals values will be used. The average reward in a zone is the sum of the expected goals of all the possessions passing through a state divided by the number of the possessions passing through a state. The goals without shots must be added to the sum of the expected goals. For example there is three possessions pass through the zone i. The first leads to a shot related to an expected goal of 0.6. The ball is lost before a shot is taken in the other. The average reward in this zone is 0.2. For the case with 150 the rewards are shown at the figure 3.12
The reward are logically maximum in the two zones closest to the goal and decrease with distance from the goal.
In each zones, we have to look at all actions to obtained the one leading the largest reward which is similar to one iteration of the algorithm 1 with v_0 containing the average rewards. We consider that if the ball is in a state different from "Lost" and "Goal" then the team plays averagely and so gains the average reward of the zones. For example if the average reward in



(a) Average rewards obtained without pressure in each zone

(b) Average rewards obtained under pressure in each zone

Figure 3.12: Average rewards obtained by the team with and without pressure in all 150 zones

the zone 4 is 0.22 without pressure and 0.16 under pressure. If the action leads to the zone 4 without pressure with a probability of 0.4 and 0.3 to the zone 4 under pressure, the reward with this action will be equal to:

$$r = 0.4 \times 0.22 + 0.3 \times 0.16 = 0.136$$

The optimal actions with this condition are shown at the figure 3.13.

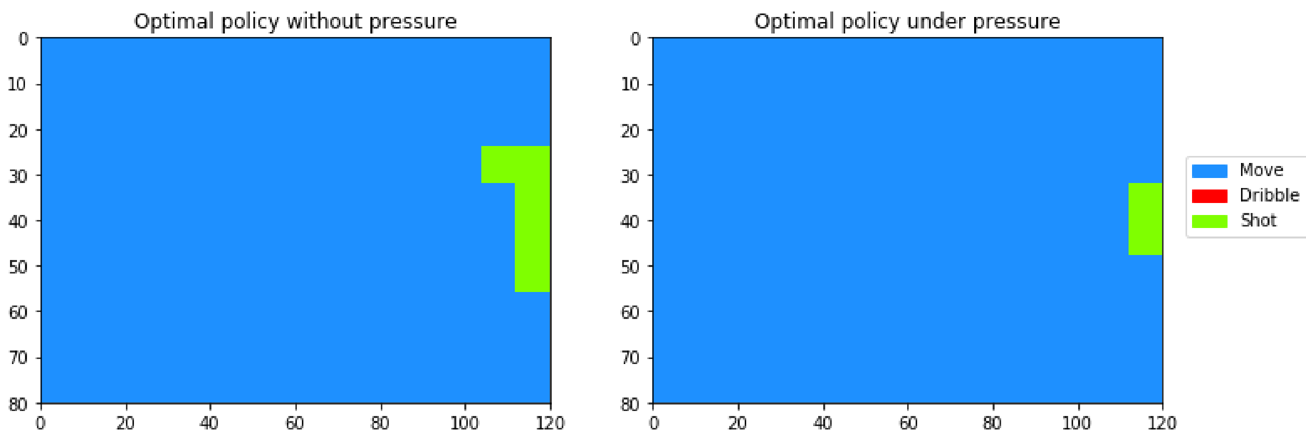


Figure 3.13: Optimal first actions in all states with 150 zones

Firstly the optimal action is more often to shot. In some zones without pressure where the optimal action was to pass to the zone 4,5 or 14,15 the optimal action is now to shot because making a pass to zones 14, 15 is difficult so if the ball arrives in these zones the next action has to be optimal or nearby which is not the case usually. If a move is made to the zone 14 and the player in zone 14 takes a bad decision then the risks to move the ball in zone 14 is not worthy. The optimal action under pressure is now to shot in the two zones the closest to the goal. Secondly the other actions are moves to a zone. The target zones of these actions are shown divided in categories at the figure 3.14 without pressure, at the figure 3.15 under pressure and with the exact target zone at the appendix F.2

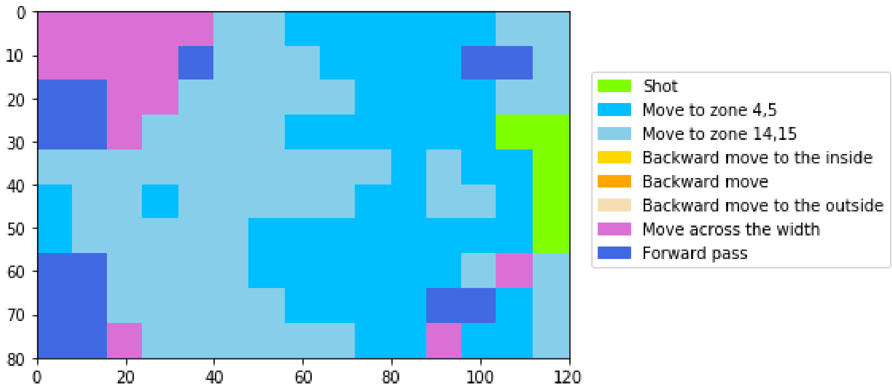


Figure 3.14: Optimal actions without pressure divided in categories when the first action is optimal with 150 zones

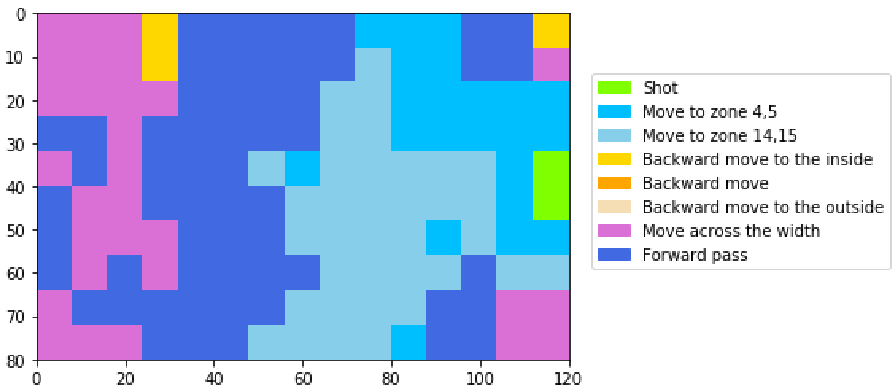
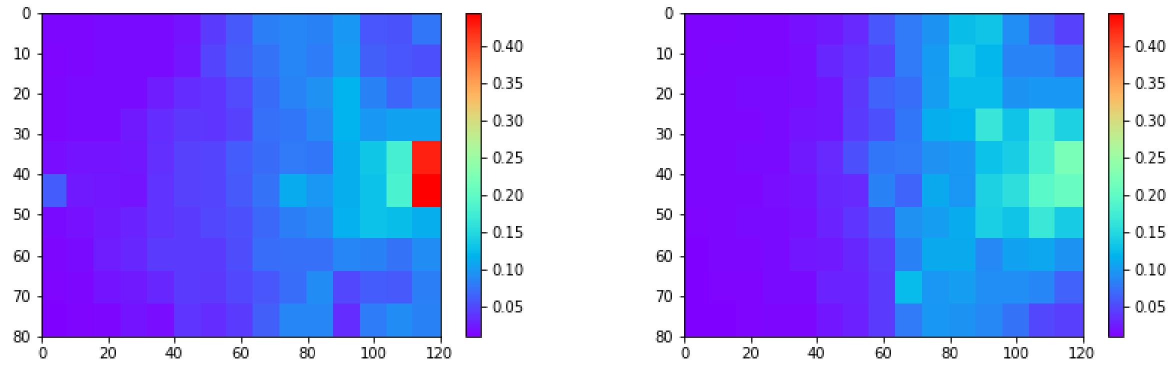


Figure 3.15: Optimal actions under pressure divided in categories when the first action is optimal with 150 zones

When only the first action is optimal, the players should take more risks and try to put the ball closer to the goal. The objective is either to go forward or in the same four zones as before. Since the average reward is significantly higher in the penalty area then the optimal action is to move to penalty area because a simple move without risk as before which leads less often to a goal because after the move the actions will not be the best. The idea of taking more risks is understandable but often trying to go in the penalty area event from its own half is more weird. We will look at the division of 600 zones to see it in a different way.
Before looking at the other division, the expected rewards when the first action is optimal can be computed and displayed at the figure 3.16.

The expected rewards are larger than the average rewards because the first actions is optimal. The expected rewards are smaller than the expected rewards when all the actions are optimal. As for the other plot of rewards, the largest rewards are in the two zones closest to the goal, decrease with distance from the goal and are larger without pressure.
The optimal policy when only the first action is optimal is computed in the way as the one with 150 zones. It is shown at the figure 3.17.



(a) Reward obtained without pressure in each zone (b) Reward obtained under pressure in each zone

Figure 3.16: Reward obtained when the first action is optimal in all states with 150 zones

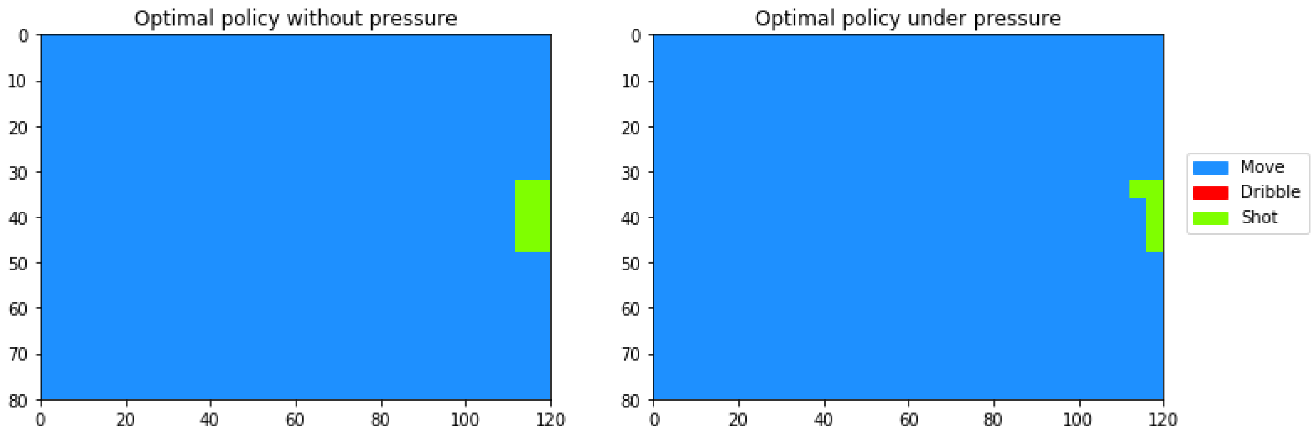


Figure 3.17: Optimal first actions in all states with 600 zones

As for the division in 150 zones when the first action is the only one optimal then there are more states were the optimal action is shot. As before, to know where the action move is directed the result are shown at the figures 3.18 and 3.19 without and with pressure. The exact target zones are plotted at the appendix F.3.

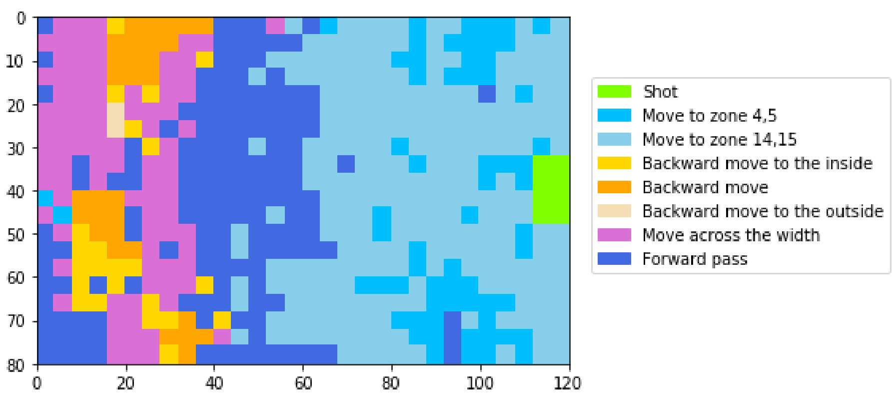


Figure 3.18: Optimal actions without pressure divided in categories when the first action is optimal with 600 zones

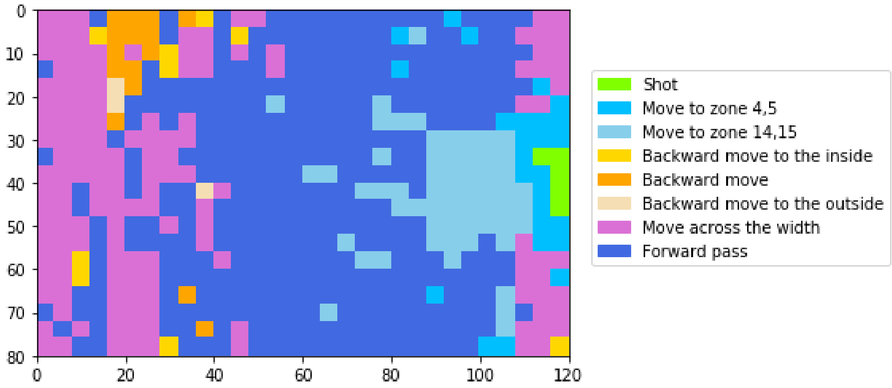
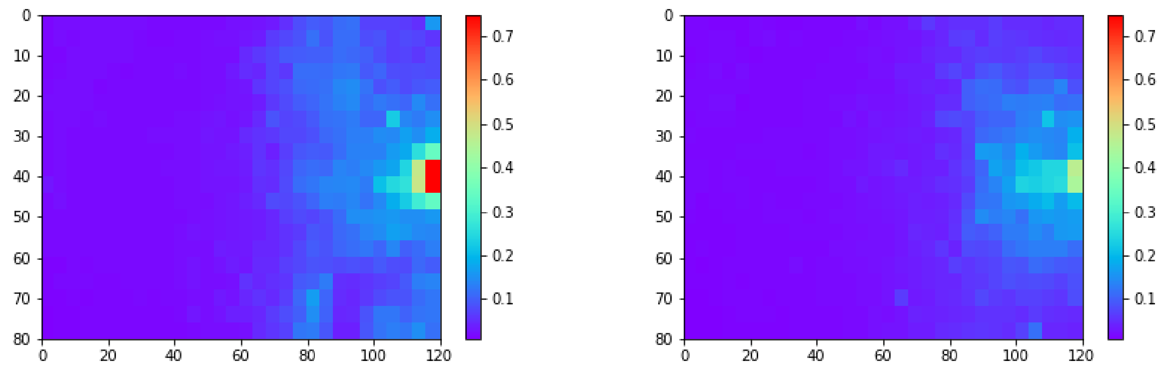


Figure 3.19: Optimal actions under pressure divided in categories when the first action is optimal with 600 zones

These figures make it possible to moderate those obtained with 150 zones, the figures 3.14 and 3.15. There are more backward moves in defensive zones to avoid losing the ball. The objective in most zones stays to quickly reach the penalty area and the zones where the average is high. The figure 3.20 presents the expected rewards associated to this policy.



(a) Reward obtained without pressure in each zone (b) Reward obtained under pressure in each zone

Figure 3.20: Reward obtained when the first action is optimal in all states with 600 zones

The plots shows that the expected rewards are largest close to the goal and decrease with distance to the goal.
For now we have the optimal actions in all states in different cases. The goal of this thesis is to transpose these policies and plots to real cases. It will be done in the chapter 4 This plots of the optimal actions will be used in the next chapters to transpose them in real cases.

3.2 Players strengths

In this section the best policies will we computed considering the strengths of the players.

3.2.1 Long-range strikers

The first categories of players studied in the section 2.6.1 was the long-range strikers. There are the best players to shoot from distant zones. What is the best policy for a long-range strikers

? If a player is in this category it does not change the quality of the other players in his team. The other players keeps the same transition matrices for all actions. Thus if another player has the ball the optimal policy and the expected rewards are the ones obtained in the section 3.1. So apart from the actions done by the long-range striker the others are already found. If a player in this category does one action leading to another states, the reward obtained in this state is the optimal expected reward. Since there is no longer any difference between carry and pass the long-range striker will play once before being a player without a good quality of shot. One iteration of the algorithm 1 using the rewards obtained before as v_0 will give the optimal policy considering that the other actions are optimal. The optimal policy with 150 and 600 zones are shown at the figures 3.21 and 3.22.

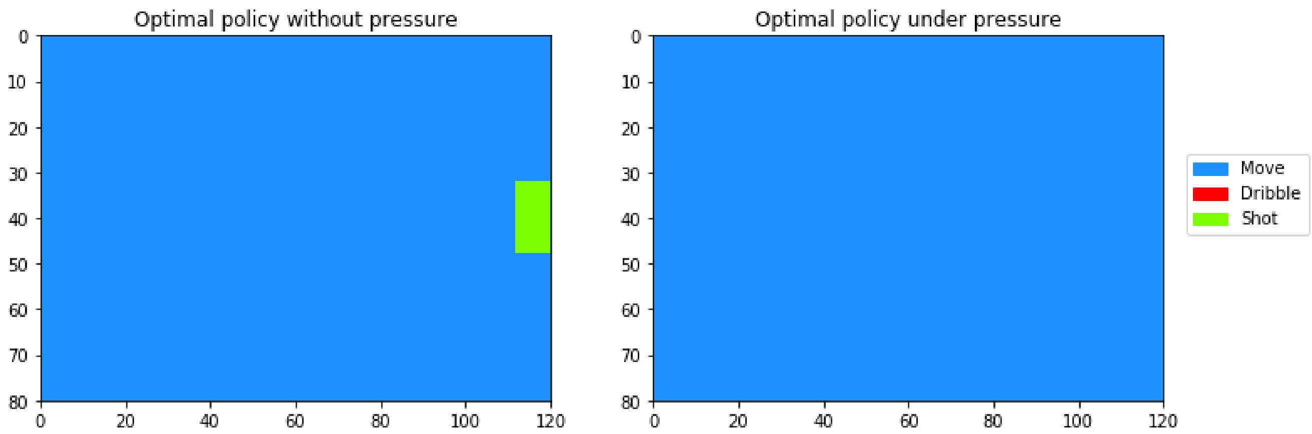


Figure 3.21: Optimal actions in all states for long-range strikers with 150 zones

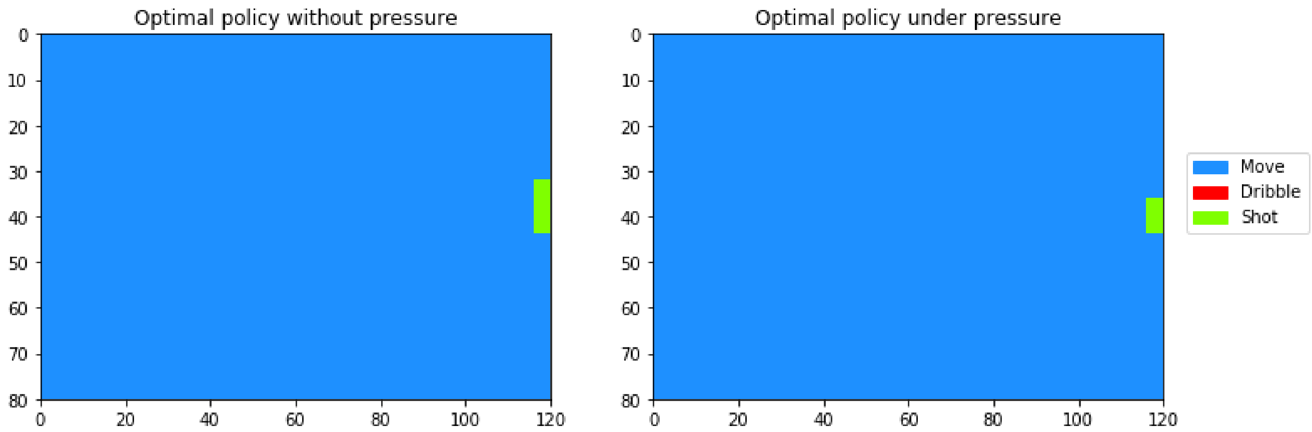


Figure 3.22: Optimal actions in all states for long-range strikers with 600 zones

The optimal actions are the same as for the players which are not in this category. The optimal action is never for a player far from the goal is never to shot even if the player is one of the best long-range strikers. It confirms the point made in the introduction that shooting from a distance is never a good idea. The actions moves in the other states are the same as for a other player because the transition matrices for the actions move are the same. The expected rewards are also the same. For a long-range striker the probabilities of scoring from outside the penalty area are larger than those for an average players but stays small. A goal from outside the box is rare. Thus if the other players chose the best action when they have the ball it is better for a striker to work with his team to get closer to the goal where the probabilities of scoring are high. However if the other players do not choose the best actions and play in an average, the striker may be tempted to take his chance from afar. He may shot if he has the opportunity rather than pass the ball which could be more easily lost by his teammates. The best policy when only the first action is optimal and made by a long range striker could also be obtained. It is computed using one iteration of the value iteration algorithm with the transition matrices of a long-range strikers. As a reminder these matrices are the same than the usual matrices except for the probabilities of scoring from afar zones. The figures 3.23 and 3.24 shows those results.

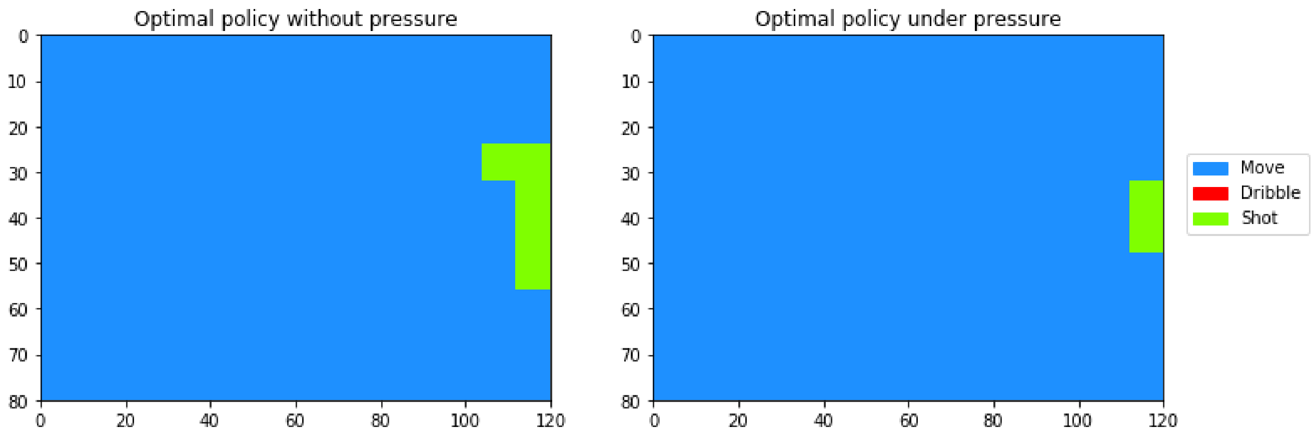


Figure 3.23: Optimal first action in all states for long-range strikers with 150 zones

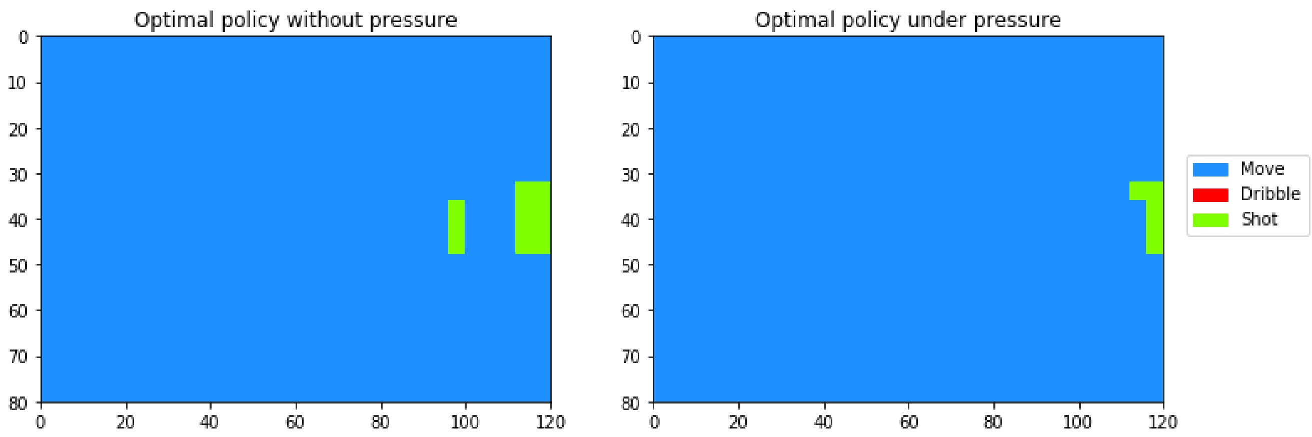


Figure 3.24: Optimal first action in all states for long-range strikers with 600 zones

The division of 600 zones shows that if the striker is close to the penalty area without pressure he should shoot. If a player is one of the best long-range strikers then the optimal policy is the same as previously bar in the vicinity of the penalty area where he should use his quality to shot. Being a long-range strikers can change the choices to be made in an offensive position. The expected rewards increases in these zones comparing to the expected rewards for the other players. In the three zones where the long-range strikers shoot the mean of the expected rewards increases from 0.13 to 0.14.

3.2.2 Players with good finishing

The second category of players is the players with a good finishing. They are the best players for scoring in the penalty area. As for the long-range strikers the value iteration algorithm can be used to obtain the optimal actions in all states if the player has good finishing skills. As before the actions after the one done by this player are those done by the other players. The rewards associated to the zones are the optimal expected rewards computed before as the figure 3.7 and 3.11. The optimal actions when the players has good finishing skills are shown at the figures 3.25 and 3.26.

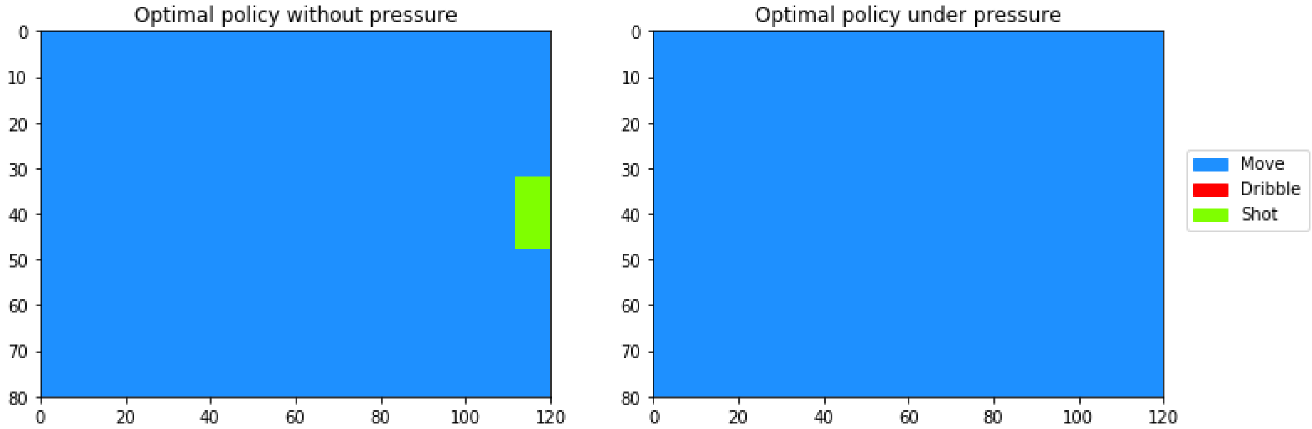


Figure 3.25: Optimal actions in all states for finishers with 150 zones

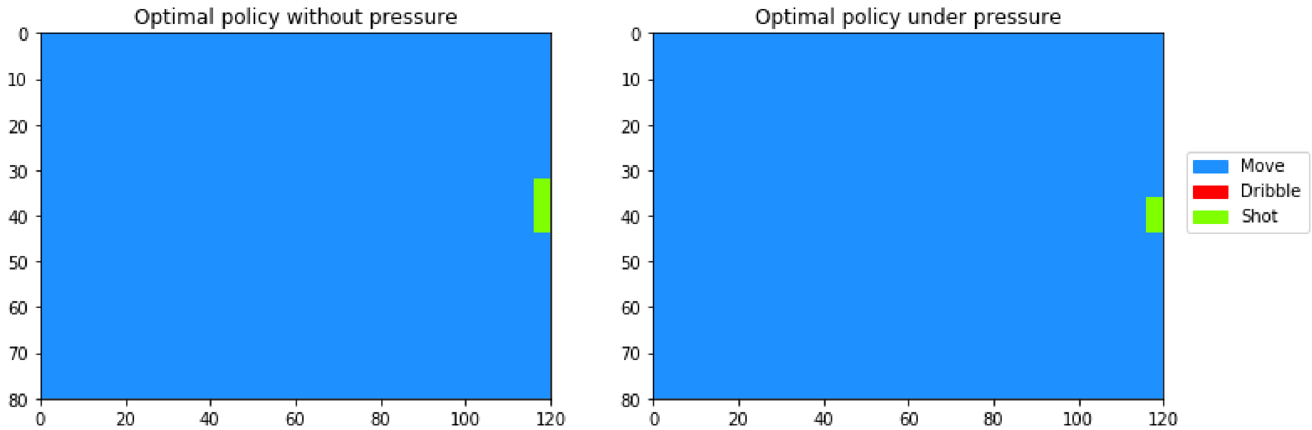


Figure 3.26: Optimal actions in all states for finishers with 600 zones

As for the long-range strikers the optimal actions are the same for the finishers and the other players. This is due to the quality of the other players who carry out the best action in each state. The increase of the probability of scoring in the penalty area is not enough to change the optimal policy. But if the other actions are not optimal the policy could change for the players in this category. The best actions if the other actions are done by average players are shown at the figures 3.27 and 3.28.

Here the policy changes. The finishers should shoot more often in the penalty area. Because these payers have more chances to score there are more zones in the penalty area where they should shots to maximise their expected reward instead of trying to reach a zone closer to the goal. The expected reward is different in some zones for two reasons. Either the action is

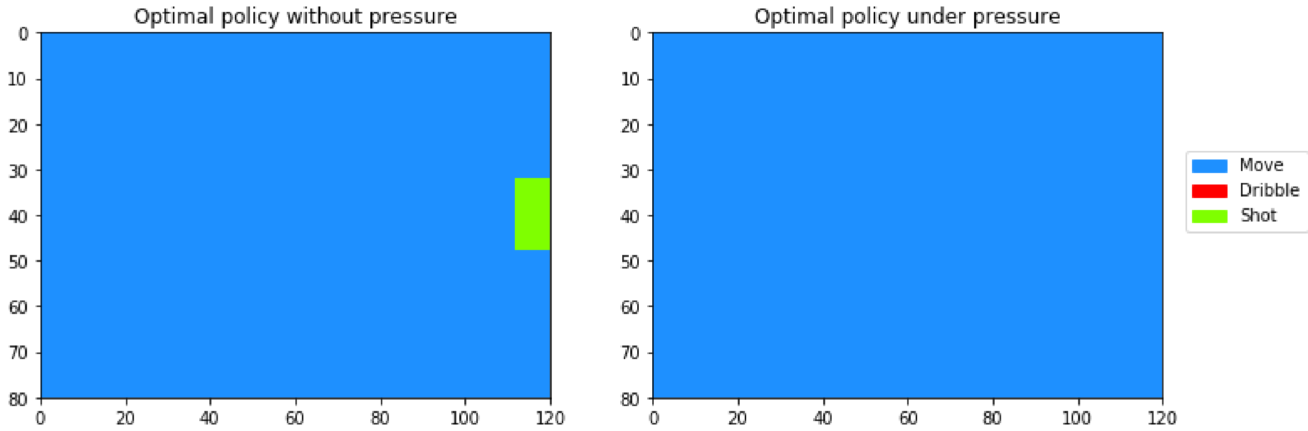


Figure 3.27: Optimal first action in all states for finishers with 150 zones

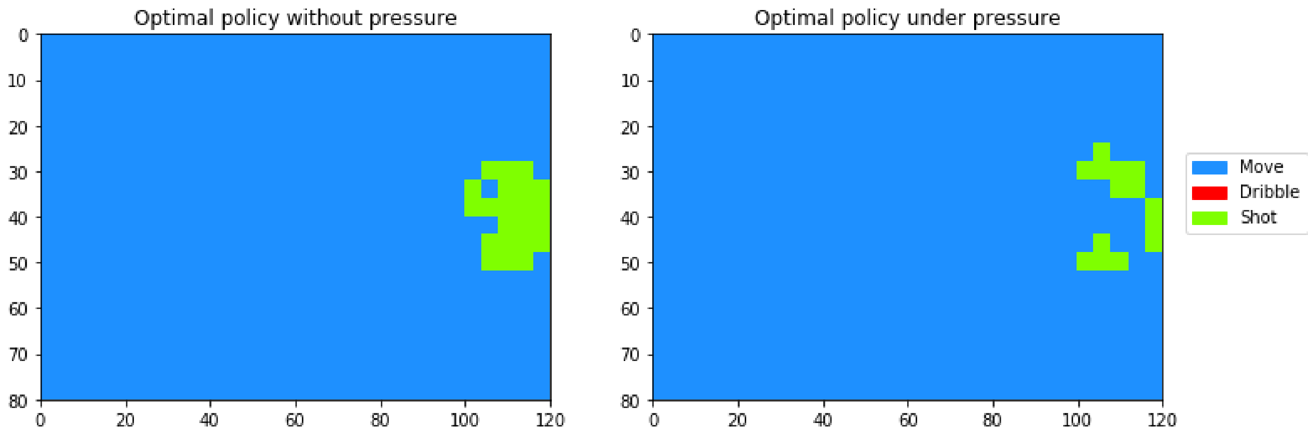


Figure 3.28: Optimal first action in all states for finishers with 600 zones

different also the reward either the action is shot in both cases but the probability of scoring is different. A comparison between the expected reward in these zones can be computed. The mean of the expected rewards for average players is 0.27. For the finishers this mean is equal to 0.29. Being a players with good finishing increases the expected reward.

3.3 Adding the location of the players

As said earlier in the section 2.7 the number of events of 360 frames is too low to obtain the transition matrices. Instead we consider that only the first action takes the freeze frames into account. The others actions are the optimal actions found at the section 3.1. It is possible to obtain the probabilities of transition if the starting location is in the center of the pitch where the most events are and there are 150 zones. Moreover to simplify the problem, the attribute under pressure is not used but is replaced with the number of teammates and opponents in a five-yards range. Let's look at the possession of the figure 3.29.

The player with the ball is Kevin De Bruyne during the match Finland-Belgium at the UEFA EURO 2020 (2021). The freeze frame corresponding to this event is shown at the figure 3.30.

Figure 3.29: Possession lead by Kevin De Bruyne, a Belgian player

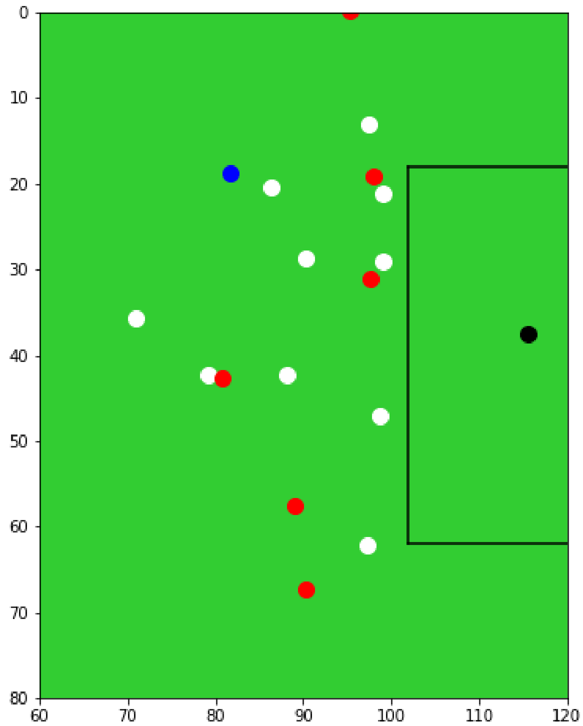


Figure 3.30: Freeze frame of the figure 3.29

The probabilities of moving to the other zones are obtained by looking at the distance and the angle to the zones. Moreover it depends also on the number of teammates and opponents who can reach the end zone. The probabilities are shown at the figure 3.31

The expected rewards obtained in each state are the optimal expected rewards found earlier in the section 3.1. Multiplying the expected reward obtained in zone i by the probability of making a successful moves to zone i gives the expected reward obtained by moving to zone i. These expected rewards are shown at the figure 3.32.
The actions leading to the highest expected rewards are those sending the ball into the box.

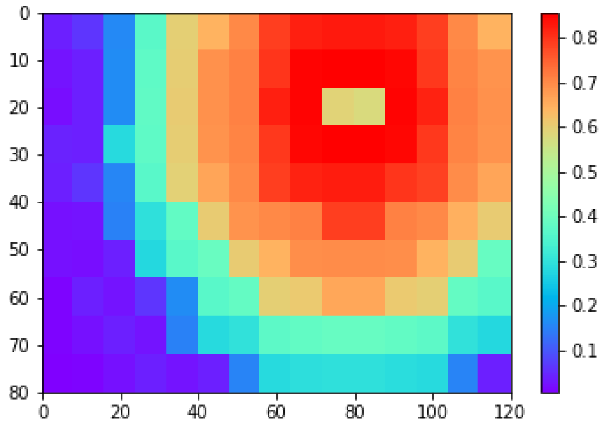


Figure 3.31: Probabilities of successful moves starting from the zone of De Bruyne

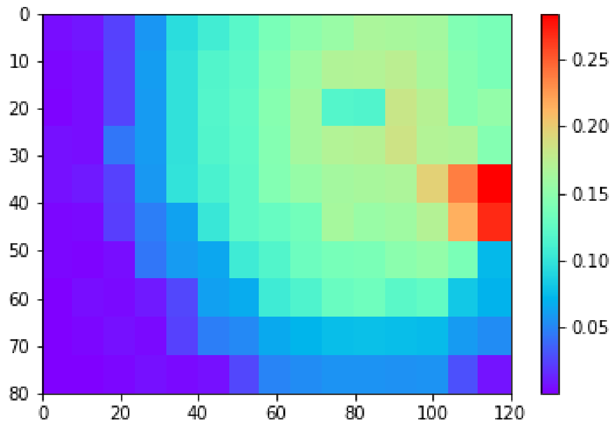


Figure 3.32: Expected rewards obtained for each action move starting from the zone of De Bruyne

Moreover the expected rewards obtained by the action dribble (0.13) and shot (0.02) are lower. In this possession Kevin De Bruyne made a good choice by passing the ball to Lukaku who were close to the goal as shown at the figure 3.33.

Figure 3.33: Shot of Lukaku after a pass of Kevin De Bruyne, two Belgian players

Chapter 4

Real cases

This chapter will use the results obtained before by using the model to compare the choices made by players during matches and the optimal choices. The Spanish league, La Liga, provides the video of the full match for some games. We will use one of this match in this chapter. This match is FCBarcelona-Celta Vigo[28] played during the 2017-2018 season. The events of this match are in the dataset. It enables to have the exact location of the events. The first event we will look at is the shot of Luis Suarez at the seven minute. The shot is shown at the figure 4.1.

Figure 4.1: Event of Luis Suarez a player of FCBarcelona

A this location Suarez is in the zone 12 for the division of 150 and 45 for the division of 600 zones. He is also under pressure. The optimal policy in both division in to make a move towards the center of the penalty area to be in front of the goal. The best policy is similar when only the first action is optimal. The zones where he should move depend on the division and if all actions are optimal. But the aim is always to recenter rather than to shoot. Luis Suarez did not take the right decision. It is confirmed by the keeper's save.
The second event observed is an event of Jordi Alba during the same match. The event is shown at the figure 4.2.

In this position the optimal action is to move also to get closer to the goal. Both divisions agree on the fact that it is better to move to be in a favorable position instead of shooting. However if only the first action is optimal, the best action is to shoot according to the division of 150 zones. However, it seems that Jordi Alba trust Luis Suarez since he passes the ball to Luis Suarez which scores.

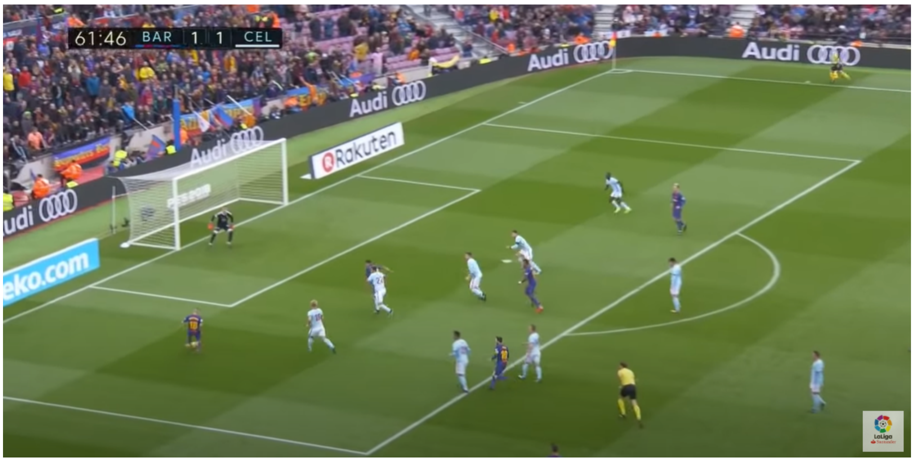


Figure 4.2: Event of Jordi Alba another player of FCBarcelona

The third events is one of Ivan Rakitic shown at the figure 4.3

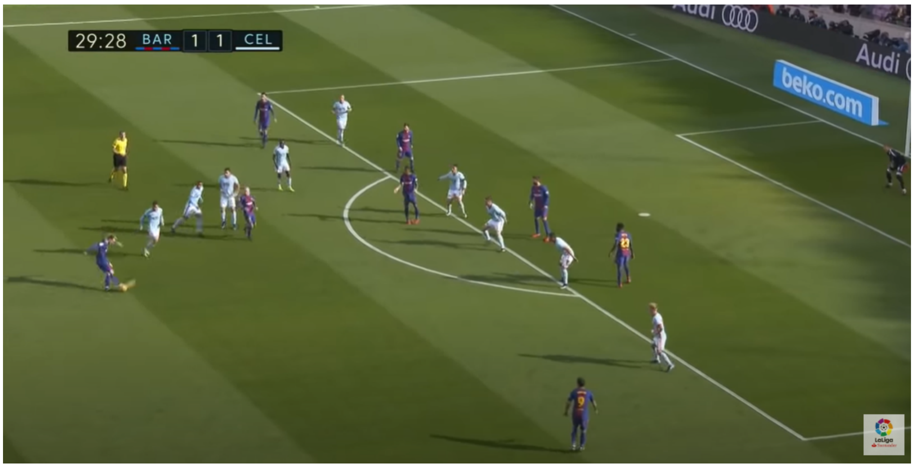


Figure 4.3: Event of Ivan Rakitic another player of FCBarcelona

In this situation the optimal solution is to move the ball forward and neither dribbles nor shoots. A move forward is the best solution for both division no matter if only the first action is optimal or all actions. Instead Rakitic shoots off target. Moreover Rakitic is not a long-range striker. To increase the expected reward of his team he should not have tried to shoot in this situation.
The last event is the event of Lionel Messi in the penalty area shown at the figure 4.4.

The best action is this situation was to first get closer to the goal before shooting. However in the case where the player has good finishing skills and only the first action is optimal the optimal action was to shoot. Lionel Messi is a player with good finishing as seen in the section 2.6.2 so he makes the right choice by shooting. In addition, a goal is scored with this shot.

It is possible to continue to watch particular actions in order to correct or approve the players' choices. However, it would also be interesting to make a study of each action to obtain the players who perform the actions the closest to the optimal actions on average.

Figure 4.4: Event of Lionel Messi a player of FCBarcelona

Chapter 5

Comparison of the players

This last chapter will look at the ability of the players to make the right choices but also to succeed when doing the actions associated to these choices. To do this we will compare the states after the action done by a player and the best state that he could have reached. There are several steps in this process.

- The first step is to find each events where the player moves, shoots or dribbles.
- The second step is to collect the data from theses events. The data gives the states where the player was before his action. The state depends on his location and on the pressure attribute.
- The third step is to look at the optimal expected reward in this state. It is the initial reward.
- Afterwards, it is to find the state after the action done by the players. It can be either a other state associated to a zone and a pressure attribute or one of the states Lost and Goal.
- The next step is to look whether the transition has been positive for the reward. If the state after the actions is associated to a larger reward than the initial reward then the player has made a good choice. The expected reward before the next action is larger than his initial expected reward. If the player score then he has been very effective and the difference between the goal (the reward associated to this transition is one) and his expected initial reward is necessarily positive. However if the player lost the ball then the difference between a lost ball (a reward of zero) and his initial reward is negative.
- Finally the mean of the differences between the rewards is computed for each player.

These differences will certainly be negative as it is based on an initial expected reward corresponding to the best policy. Comparing all the players in the datasets is not the best thing to do because the male and female players have played a different number of matches, in different competitions at different times. It is more interesting to compare all the players in one competition. A recent competition we have is the season 2020/2021 of FCBarcelona. Here the players played a closer number of matches in the same competition against the same opponents. The results are sorted by the position of the players on the pitch, the defenders the midfielders and the forwards. The goalkeepers are not included due to their special position. The average difference corresponds to the difference per action. The differences for the twenty-two players of FCBarcelona is shown at the figure 5.1.

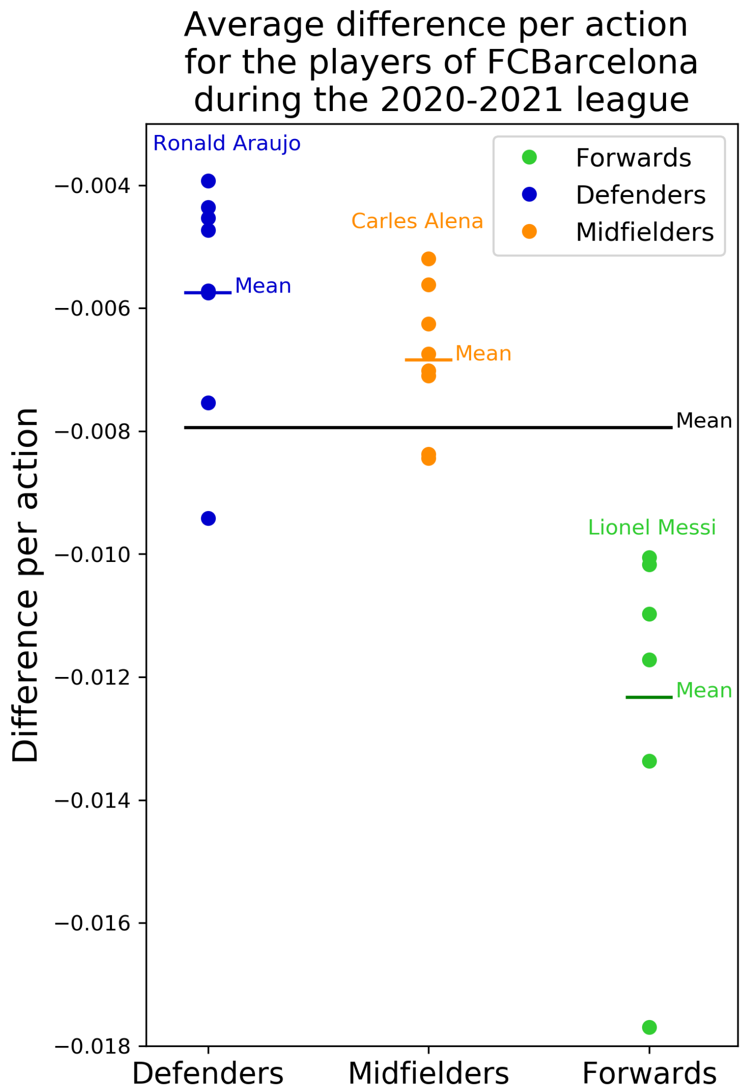


Figure 5.1: Average difference between the reward after the action and the initial reward for the players of FCBarcelona

The values for all players are written at the appendix G. The average differences are indeed negative. The mean is equal to -0.008. It means that the players deviate from the optimal expected rewards with a value of 0.008 on average. The forwards have the worst differences. It does not means that the forwards decrease the reward of the possession. It means that the actions done by the forwards gives an expected reward smaller than the expected rewards obtained if all the choices were the best. The choices for a defender are easier to take because the mean idea is to move forward on the pitch. Moreover the optimal expected rewards are similar in the defensive zones. Hence making a bad choice will not decrease much the optimal expected reward. The mean for the defender is equal to -0.0057 while it is equal to -0.0068 for the midfielders and to -0.0123 for the forwards. The player making the best choices and who carrying his actions with success is Ronald Araujo with a average difference of -0.039. The forward making the best choices is Lionel Messi and the players and forward making the worst is Martin Braithwaite. It corresponds to the quality of the players Messi being recognized as one of the best players, which is not the case for Braithwaite. Finally the midfielder making the best choices is surprisingly Carles Alena as he was sold by the FCBarcelona at the end of the season.

This thesis ends by this comparison between the players about the reward loss. The results obtained can be completed by analysing new matches or other players. The model can also be improved to be more realistic. It can be enhanced by adding new data or new improvements. The use of 360 frames can also be improved with more data or with a different model. Finally it is possible to add other categories of player strength such as cross passers or long passers. The previous categories can also be refined to obtain different levels of strikers.
The next pages contain appendices that complete what has been seen before.

Appendix A

Datasets

A.1 Underststat

The website Understat, `understat.com`, which has accepted the use of the data contains data about more than 14000 matches of six leagues since 2014:

- The Premier League, the English league
- La Liga, the Spanish league
- Il Serie A, the Italian league
- La Ligue 1, the French league
- Die Bundesliga, the German league
- The Russian Premier League

Which are, except for the Russian league, the biggest leagues in Europe. For each match there are some pieces of information like the date, the teams, the league and all the players who played. There are also some statistics like the number of shots, goals, etc. But the useful data on the website are the shots. There are several useful details on each shot:

- The player who takes the shot.
- The location where the shot was taken. The locations are also sorted by location as "Out of box" or "Penalty area".
- The result of the shot, if it was scored, saved by the keeper, blocked by a defender or missed.
- The shot type, if it was done with the left foot, the right foot or the head.
- The situation of the shot. If it was a penalty, a free-kick, from a corner or in open play.
- The expected goals value of the shot.

These details will be use in this thesis.
Moreover for the more than 10000 players the shots are gathered per player. Each player has statistics on his shots, goals and expected goals depending on the situation, the zone or the type.

A.2 StatsBomb

The second dataset is the open dataset of StatsBomb `github.com/statsbomb/open-data` which contains over a thousand matches. For each match there is information like the teams and the players. Above all there are all the events of the match detailed in the dataset. There are to one hundred and fifty attributes per event. Not all attributes are useful but the most important ones are:

- The identifier and the index
- The type of events : Pass, shot, dribble, carry and others
- The team in possession of the ball
- The player concerned by the event and the secondary players of the event.
- The pattern, if the events come from a free-kick, a corner, in regular play, etc.
- The location of the events
- The attribute under pressure

There are also features used for some type of events. A pass with its interesting features is for example:

Feature	Value	Feature	Value	Feature	Value
Type.name	Pass	Identifier	68f1458f-6ddb...	Match_id	22912
Period	1	Timestamp	00:00:00.208	Possession	1
Duration	1.666205	Location	(61, 41)	Under_pressure	NA
Play_pattern.name	From Kick Off	Length	26.360956	Angle	3.0007687
End_location	(34.9, 44.7)	Cross	NA	Cutback	NA
Name	Ground Pass	Body_part	Right foot	Outcome	NA

The useful features for the event carry the starting and ending location, the pressure attribute and the outcome. For the event dribble the features are the location, the outcome and the pressure attribute which is always equal to under pressure. The used features of the event dribble are the location the pressure attribute and the result as for the others. The event shot also need the value of the expected goals.

Moreover there are attributes relative to a particular event. There is the end location for the events carry and pass. The outcome of a dribble and the result of a shot. There is also the expected goals value of a shot. There are a lot of other attributes but they are not used in this thesis.
Moreover, as said in the introduction, there are the 360 frames associated to the events. It contains the locations of the players and the type of each player.

Appendix B

Markov Decision Process

The figure B.1 shows the Markov Chain of the action pass for the Markov Decision Process of the simple model.

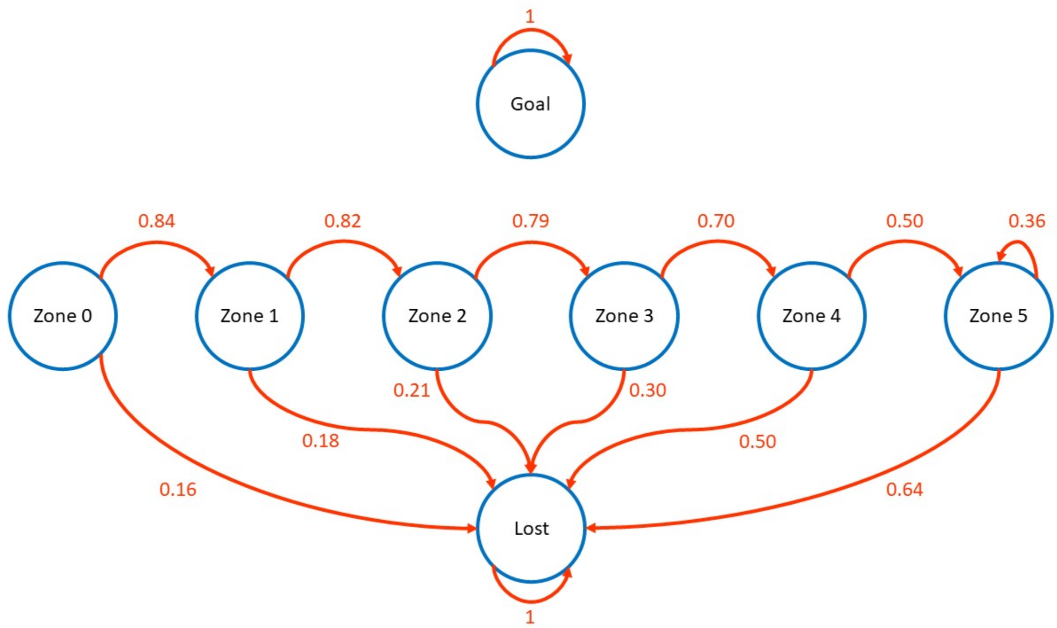


Figure B.1: Markov Chain which is the Markov Decision Process of the simple model limited to action pass

The figure B.2 shows the Markov chain of the same MDP but for the action shot.

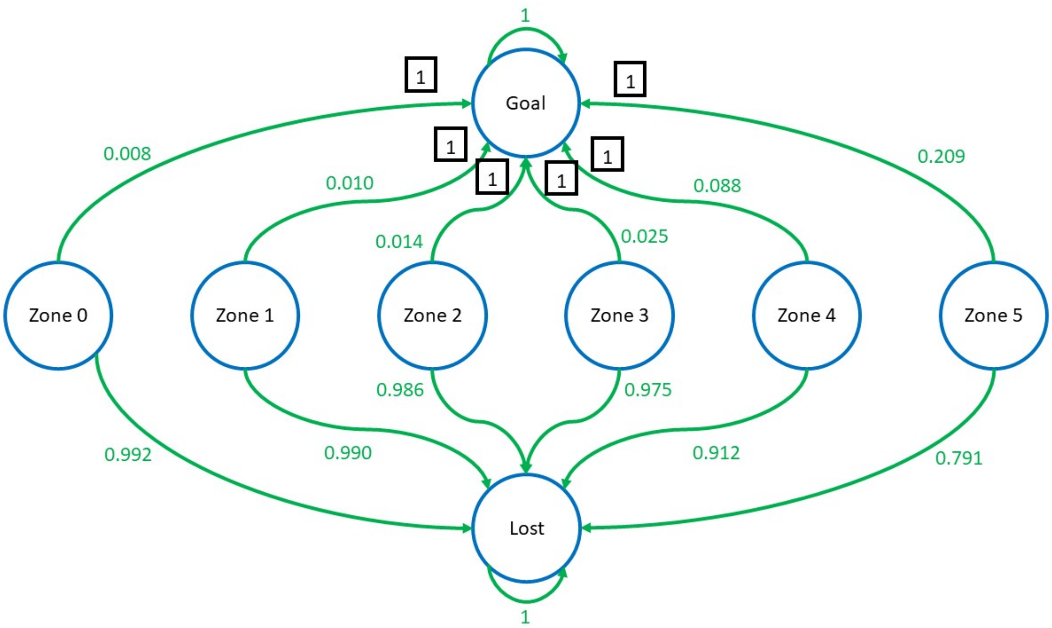


Figure B.2: Markov Chain which is the Markov Decision Process of the simple model limited to action shot

Appendix C

Sparse matrices

The transition matrices of this Markov Decision Process most of the elements are zero. For example the transition matrices of all actions move for the first improvement of the model have only two non-zero elements per row: the probability of moving to the desired zone and the probability of losing the ball. Moreover the coordinates are known, if the action is move to zone 2 then only the third column (the column "To zone 2") and the penultimate column (the column "To Lost") have the non-zeros elements. One value per row may not be stored since the sum of the probabilities of each row is equal to one but all non-zero values are necessary so store all values avoid some future computations. The only exceptions are for the two last rows for which the probability of staying in these zones are always one. This section shows how the different transition matrices are stored for the different models.

C.1 First improvements of the model (without considering the pressure)

This model, discussed in the section 2.4, has the action move to, dribble and shot. The action move to can be done to any zone and the model does not take into account the pressure for now.

C.1.1 Action "Move to zone i"

The rows of the action move have only two (or less) non-zero elements since it is built like this, with n zones and m_j the probability of a successful action if it start from zone j.

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} \text{To zone 0} & \cdots & \text{To zone i} & \cdots & \text{To zone n} & \text{To Lost} & \text{To Goal} \end{array} \\
P_{mi} = & \begin{bmatrix}
0 & \cdots & m_0 & \cdots & 0 & 1-m_0 & 0 \\
0 & \cdots & m_1 & \cdots & 0 & 1-m_1 & 0 \\
\vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & m_n & \cdots & 0 & 1-m_n & 0 \\
0 & \cdots & 0 & \cdots & 0 & 1 & 0 \\
0 & \cdots & 0 & \cdots & 0 & 0 & 1
\end{bmatrix}
\begin{array}{c} \text{From zone 0} \\ \text{From zone 1} \\ \vdots \\ \text{From zone n} \\ \text{From Lost} \\ \text{From Goal} \end{array}
\end{array}
$$

Therefore it is possible to rewrite this matrix in an array form keeping only the non-zeros elements. All elements that can be non-zeros are considered as non-zeros elements. It means that if m_0 is zero it is kept in the new array because it enables to know the true coordinates of the elements and it will be done in all arrays. The array for "Move to" is the following

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} \text{To zone i} & \text{To Lost} & \text{To zone i} & \text{To Lost} & \cdots & \text{To zone i} & \text{To Lost} \end{array} \\
A_{mi} = & \begin{bmatrix} \underbrace{m_0 \quad 1-m_0}_{\text{From Zone 0}} & \underbrace{m_1 \quad 1-m_1}_{\text{From Zone 1}} & \cdots & \underbrace{m_n \quad 1-m_n}_{\text{From Zone n}} \end{bmatrix}
\end{array}
$$

The value from the state "Lost" and "Goal" are not kept because it is always one. From the $(n+2)\times(n+2)$ values of the transition matrix only $2n$ values are kept. Moreover there are n transition matrices because it is the number of zone and so of action "Move to" and the number of values to keep is $2n \times n$ instead of $(n+2)\times(n+2)\times n$ which reduces a lot the number of values to store especially if n is large.

C.1.2 Action "Dribble"

As the action "Move to zone i", the transition matrix of action dribble has two non-zero elements per row. These elements are either in the column "To Lost" either on the diagonal as show on the following matrix. Let d_i be the probability of success of a dribble in zone i.

$$
\begin{array}{c}
\begin{array}{ccccccc} \text{To zone 0} & \cdots & \text{To zone i} & \cdots & \text{To zone n} & \text{To Lost} & \text{To Goal} \end{array} \\
P_d = \begin{bmatrix}
d_0 & \cdots & 0 & \cdots & 0 & 1-d_0 & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & d_i & \cdots & 0 & 1-d_i & 0 \\
\vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & d_n & 1-d_n & 0 \\
0 & \cdots & 0 & \cdots & 0 & 1 & 0 \\
0 & \cdots & 0 & \cdots & 0 & 0 & 1
\end{bmatrix}
\begin{array}{c} \text{From zone 0} \\ \vdots \\ \text{From zone i} \\ \vdots \\ \text{From zone n} \\ \text{From Lost} \\ \text{From Goal} \end{array}
\end{array}
$$

Which is rewritten like this

$$
\begin{array}{c}
\begin{array}{ccccccc} \text{To zone 0} & \text{To Lost} & \text{To zone i} & \text{To Lost} & \cdots & \text{To zone n} & \text{To Lost} \end{array} \\
A_d = \begin{bmatrix} \underbrace{d_0 \quad 1-d_0}_{\text{From Zone 0}} & \underbrace{d_1 \quad 1-d_1}_{\text{From Zone 1}} & \cdots & \underbrace{d_n \quad 1-d_n}_{\text{From Zone n}} \end{bmatrix}
\end{array}
$$

It also enables to keep $2n$ values from the $(n+2) \times (n+2)$ initial values of the transition matrix.

C.1.3 Action "Shot"

The last action is shot which still have two non-zero elements but in the columns "Lost" and "Shot":

$$
\begin{array}{c}
\begin{array}{ccccccc} \text{To zone 0} & \cdots & \text{To zone i} & \cdots & \text{To zone n} & \text{To Lost} & \text{To Goal} \end{array} \\
P_s = \begin{bmatrix}
0 & \cdots & 0 & \cdots & 0 & 1-s_0 & s_0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & 1-s_i & s_i \\
\vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & 1-s_n & s_n \\
0 & \cdots & 0 & \cdots & 0 & 1 & 0 \\
0 & \cdots & 0 & \cdots & 0 & 0 & 1
\end{bmatrix}
\begin{array}{c} \text{From zone 0} \\ \vdots \\ \text{From zone i} \\ \vdots \\ \text{From zone n} \\ \text{From Lost} \\ \text{From Goal} \end{array}
\end{array}
$$

As for the other action it is rewritten like this

$$
\begin{array}{c}
\begin{array}{ccccccc} \text{To zone 0} & \text{To Lost} & \text{To zone i} & \text{To Lost} & \cdots & \text{To zone n} & \text{To Lost} \end{array} \\
A_s = \begin{bmatrix} \underbrace{s_0 \quad 1-s_0}_{\text{From Zone 0}} & \underbrace{s_1 \quad 1-s_1}_{\text{From Zone 1}} & \cdots & \underbrace{s_n \quad 1-s_n}_{\text{From Zone n}} \end{bmatrix}
\end{array}
$$

Like in the previous arrays only $2n$ elements are kept instead of the $(n+2) \times (n+2)$. For all transition matrices $2n \times (n+2)$ elements are kept from the $(n+2) \times (n+2) \times (n+2)$ elements. The space complexity is now $\mathcal{O}(n^2)$ instead of $\mathcal{O}(n^3)$

C.2 Model with pressure

Another improvement of the model was considering the attribute under pressure or not. This model changes the number of states from $n+2$ to $2n+2$. For the actions "Move to zone i" and "Dribble" there are three non-zero elements per row: to the zone without pressure, to the zone under pressure and to the state "Lost".

C.2.1 Action "Move to zone i"

When doing the action "Move to zone i" the transition can be done to three states and so three column: Zone i without pressure, Zone i under pressure and Lost as shows on the following transition matrix. m^u_{0w} is the probability of success stating from zone 0 without pressure to zone i under pressure.

$$
P_{mi} = \begin{array}{c}
\begin{array}{cccccccc}
\text{To zone 0 without press} & \cdots & \text{To zone i without press} & \cdots & \text{To zone i under press} & \cdots & \text{To Lost} & \text{To Goal}
\end{array} \\
\left[\begin{array}{cccccccc}
0 & 0 & m^w_{0w} & 0 & m^u_{0w} & 0 & 1-m^w_{0w}-m^u_{0w} & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & m^w_{iw} & 0 & m^u_{iw} & 0 & 1-m^w_{iw}-m^u_{iw} & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & m^w_{0u} & 0 & m^u_{0u} & 0 & 1-m^w_{0u}-m^u_{0u} & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & 0 \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 1
\end{array}\right]
\end{array}
\begin{array}{c}
\\
\text{From zone 0 without press} \\
\vdots \\
\text{From zone i without press} \\
\vdots \\
\text{From zone 0 under press} \\
\vdots \\
\text{From Lost} \\
\text{From Goal}
\end{array}
$$

Which is rewritten as

$$
A_{mi} = \begin{array}{c}
\begin{array}{ccccccc}
\text{To zone i without press} & \text{To zone i under press} & \text{To Lost} & \cdots & \text{To zone i without press} & \text{To zone i under press} & \text{To Lost} \quad \cdots
\end{array} \\
\left[\begin{array}{ccccc}
\underbrace{m^w_{0w} \quad m^u_{0w} \quad 1-m^w_{0w}-m^u_{0w}}_{\text{From zone 0 without press}} & \cdots & \underbrace{m^w_{0u} \quad m^u_{0u} \quad 1-m^w_{0u}-m^u_{0u}}_{\text{From zone 0 under press}} & \cdots
\end{array}\right]
\end{array}
$$

With $3 \times 2 \times n$ values instead of $(2n+2) \times (2n+2)$ elements for each of the n transition matrices.

C.2.2 Action "Dribble"

The transition matrix of the action dribble is the following, with also three non-zeros elements per row

$$P_d = \begin{array}{c} \begin{array}{cccccccc} \substack{\text{To zone 0}\\ \text{without press}} & \cdots & \substack{\text{To zone i}\\ \text{without press}} & \cdots & \substack{\text{To zone i}\\ \text{under press}} & \cdots & \text{To Lost} & \text{To Goal} \end{array} \\ \begin{bmatrix} d^w_{0w} & 0 & 0 & 0 & 0 & 0 & 1-d^w_{0w}-d^u_{0w} & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & d^w_{iw} & 0 & d^u_{iw} & 0 & 1-d^w_{iw}-d^u_{iw} & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & d^w_{iu} & 0 & d^u_{iu} & 0 & 1-d^w_{iu}-d^u_{iu} & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \substack{\text{From zone 0}\\ \text{without press}} \\ \vdots \\ \substack{\text{From zone i}\\ \text{without press}} \\ \vdots \\ \substack{\text{From zone i}\\ \text{under press}} \\ \vdots \\ \text{From Lost} \\ \text{From Goal} \end{array}$$

Corresponding to the following array

$$A_d = \begin{array}{c} \begin{array}{cccccccc} \substack{\text{To zone i}\\ \text{without press}} & \substack{\text{To zone i}\\ \text{under press}} & \text{To Lost} & \cdots & \substack{\text{To zone i}\\ \text{without press}} & \substack{\text{To zone i}\\ \text{under press}} & \text{To Lost} & \cdots \end{array} \\ \begin{bmatrix} \underbrace{d^w_{iw} \qquad d^u_{iw}}_{\substack{\text{From zone i}\\ \text{without press}}} & 1-d^w_{iw}-d^u_{iw} & \cdots & \underbrace{d^w_{iu} \qquad d^u_{iu}}_{\substack{\text{From zone i}\\ \text{under press}}} & 1-d^w_{iu}-d^u_{iu} & \cdots \end{bmatrix} \end{array}$$

It could be improved with the fact that the probability of achieving a dribble without pressure is zero because a dribble starts never under pressure. It also kept $3 \times 2 \times n$ values from the $(2n+2) \times (2n+2)$ elements of the transition matrix.

C.2.3 Action "Shot"

The action "Shot" still has two column with non-zero elements but with more rows in this model

$$P_s = \begin{array}{c} \begin{array}{cccccccc} \substack{\text{To zone 0}\\ \text{without press}} & \cdots & \substack{\text{To zone i}\\ \text{without press}} & \cdots & \substack{\text{To zone i}\\ \text{under press}} & \cdots & \text{To Lost} & \text{To Goal} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1-s_{0w} & s_{0w} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-s_{iw} & s_{iw} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-s_{0u} & s_{0u} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \substack{\text{From zone 0}\\ \text{without press}} \\ \vdots \\ \substack{\text{From zone i}\\ \text{without press}} \\ \vdots \\ \substack{\text{From zone 0}\\ \text{under press}} \\ \vdots \\ \text{From Lost} \\ \text{From Goal} \end{array}$$

With these values kept

$$A_s = \begin{array}{c} \begin{array}{ccccccccc} \text{To Goal} & \text{To Lost} & \cdots & \text{To Goal} & \text{To Lost} & \cdots & \text{To Goal} & \text{To Lost} & \cdots \end{array} \\ \left[\begin{array}{ccccccccc} \underbrace{s_{0w} \quad 1 - s_{0w}}_{\substack{\text{From zone 0} \\ \text{without press}}} & \cdots & \underbrace{s_{iw} \quad 1 - s_{iw}}_{\substack{\text{From zone i} \\ \text{without press}}} & \cdots & \underbrace{s_{0u} \quad 1 - s_{0u}}_{\substack{\text{From zone 0} \\ \text{under press}}} & \cdots \end{array} \right] \end{array}$$

There are $2 \times 2 \times n$ values kept instead of $(2n+2) \times (2n+2)$ for the transition matrix of shot. For all transitions matrix we kept $6n \times n + 6n + 4n = 6n^2 + 10n$ elements ($\mathcal{O}(n^2)$) instead of $(2n+2)^2 \times (n+2)$ elements ($\mathcal{O}(n^3)$).

C.3 Model with set-pieces

This model add a value per row since doing an action other than shot can also lead to a goal except in the last two rows, the states "Lost" and "Goal".

C.3.1 Action "Move to zone i"

The transition matrix is built in a similar way as before but now m_{iw}^u takes into account the fact that a set-piece can be obtained and g_i is the probability of winning and scoring from a set-piece in zone i.

$$P_{mi} = \begin{array}{c} \begin{array}{cccccccc} \substack{\text{To zone 0} \\ \text{without press}} & \cdots & \substack{\text{To zone i} \\ \text{without press}} & \cdots & \substack{\text{To zone i} \\ \text{under press}} & \cdots & \text{To Lost} & \text{To Goal} \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & m_{0w}^w & 0 & m_{0w}^u & 0 & 1 - m_{0w}^w - m_{0w}^u - g_i & g_i \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & m_{iw}^w & 0 & m_{iw}^u & 0 & 1 - m_{iw}^w - m_{iw}^u - g_i & g_i \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & m_{0u}^w & 0 & m_{0u}^u & 0 & 1 - m_{0u}^w - m_{0u}^u - g_i & g_i \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 1 \end{array} \right] \end{array} \begin{array}{c} \\ \substack{\text{From zone 0} \\ \text{without press}} \\ \vdots \\ \substack{\text{From zone i} \\ \text{without press}} \\ \vdots \\ \substack{\text{From zone 0} \\ \text{under press}} \\ \vdots \\ \text{From Lost} \\ \text{From Goal} \end{array}$$

Which is rewritten as

$$A_{mi} = \begin{array}{c} \begin{array}{ccccccccccc} \substack{\text{To zone i} \\ \text{without press}} & \substack{\text{To zone i} \\ \text{under press}} & \text{To Goal} & \text{To Lost} & \cdots & \substack{\text{To zone i} \\ \text{without press}} & \substack{\text{To zone i} \\ \text{under press}} & \text{To Lost} & \text{To Goal} & \cdots \end{array} \\ \left[\begin{array}{ccccccc} m_{0w}^w & \underbrace{m_{0w}^u \quad g_i \quad \substack{1 - m_{0w}^w \\ - m_{0w}^u - g_i}}_{\substack{\text{From zone 0} \\ \text{without press}}} & \cdots & m_{0u}^w & \underbrace{m_{0u}^u \quad g_i \quad \substack{1 - m_{0u}^w \\ - m_{0u}^u - g_i}}_{\substack{\text{From zone 0} \\ \text{under press}}} & \cdots \end{array} \right] \end{array}$$

Which kept $4 \times 2 \times n$ values instead of $(2n+2) \times (2n+2)$ elements for each of the n transition matrices which can be again be decrease since g_i is stored more than once.

C.3.2 Action "Dribble"

There are now also four non-zero element per row in the transition matrix. d^w_{iu} is now the probability of success of dribble in zone i under pressure which leads to a state without pressure (still zone i) without winning a set-piece.

$$
P_d = \begin{array}{c}
\begin{array}{cccccccc}
\text{To zone 0 without press} & \cdots & \text{To zone i without press} & \cdots & \text{To zone i under press} & \cdots & \text{To Lost} & \text{To Goal}
\end{array} \\
\left[\begin{array}{cccccccc}
d^w_{0w} & 0 & 0 & 0 & 0 & 0 & 1 - d^w_{0w} - d^u_{0w} - g_0 & g_0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & d^w_{iw} & 0 & d^u_{iw} & 0 & 1 - d^w_{iw} - d^u_{iw} - g_i & g_i \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & d^w_{iu} & 0 & d^u_{iu} & 0 & 1 - d^w_{iu} - d^u_{iu} - g_i & g_i \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & 0 \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 1
\end{array}\right]
\end{array}
\begin{array}{c}
\text{From zone 0 without press} \\ \vdots \\ \text{From zone i without press} \\ \vdots \\ \text{From zone i under press} \\ \vdots \\ \text{From Lost} \\ \text{From Goal}
\end{array}
$$

Which is stored in the following array

$$
A_d = \begin{array}{c}
\begin{array}{ccccccccccc}
\text{To zone i without press} & \text{To zone i under press} & \text{To Goal} & \text{To Lost} & \cdots & \text{To zone i without press} & \text{To zone i under press} & \text{To Lost} & \text{To Goal} & \cdots
\end{array} \\
\left[\begin{array}{ccccccccccc}
d^w_{iw} & \underbrace{m^u_{0w} \quad g_i}_{\text{From zone i without press}} & 1-d^w_{iw}-d^u_{iw}-g_i & \cdots & d^w_{iu} & \underbrace{d^u_{iu} \quad g_i}_{\text{From zone i under press}} & 1-d^w_{iu}-d^u_{iu}-g_i & \cdots
\end{array}\right]
\end{array}
$$

As for the action pass it kept only $4 \times 2 \times n$ values.

C.3.3 Action "Shot"

Considering the set-pieces does not change the transition matrix of the action shot. Therefore the array for the action shot is still, with $4nvalues$

$$
A_s = \begin{array}{c}
\begin{array}{ccccccccc}
\text{To Goal} & \text{To Lost} & \cdots & \text{To Goal} & \text{To Lost} & \cdots & \text{To Goal} & \text{To Lost} & \cdots
\end{array} \\
\left[\begin{array}{cccccc}
\underbrace{s_{0w} \quad 1 - s_{0w}}_{\text{From zone 0 without press}} & \cdots & \underbrace{s_{iw} \quad 1 - s_{iw}}_{\text{From zone i without press}} & \cdots & \underbrace{s_{0u} \quad 1 - s_{0u}}_{\text{From zone 0 under press}} & \cdots
\end{array}\right]
\end{array}
$$

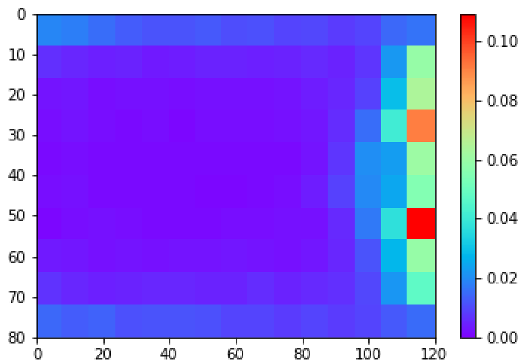
The number of total values kept increases. It is now equal to $8n \times n + 8n + 4n = 8n^2 + 12n$. But it stays clearly smaller to the number of values stored before $((2n+2)^2 \times (n+2))$ if n is large. The space complexity is also smaller $\mathcal{O}(n^2)$ instead of $\mathcal{O}(n^3)$.

Appendix D

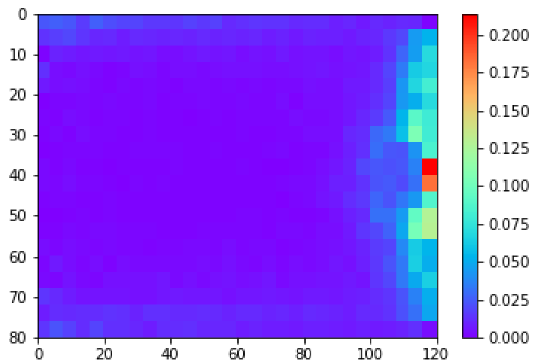
Set-pieces

D.1 Probabilities of obtaining any set-piece

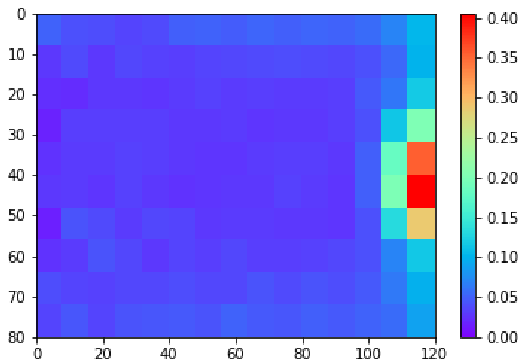
The following graph shows the sum of the probability of obtaining one of the set-pieces.



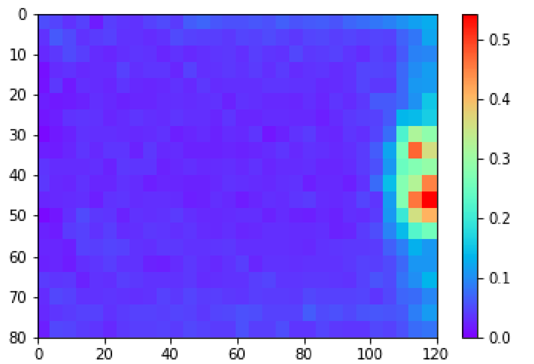
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



(c) Division of 150 zones under pressure



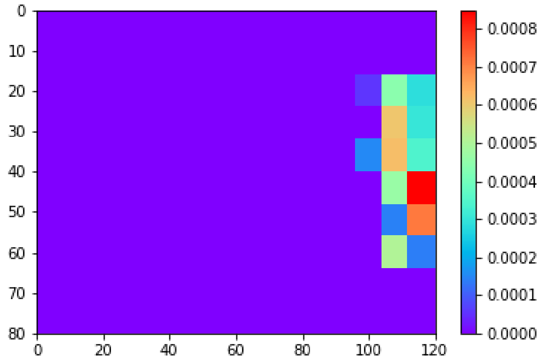
(d) Division of 600 zones under pressure

Figure D.1: Probability of obtaining any set-piece with both divisions

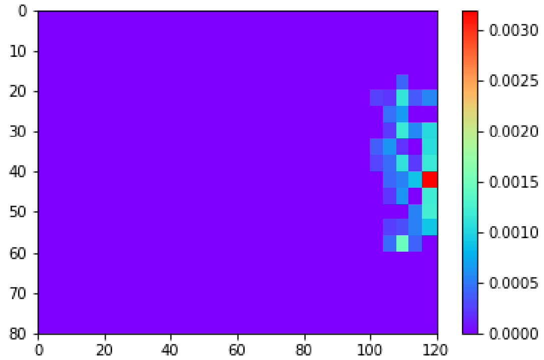
D.2 Probabilities of obtaining and scoring a set-piece

The following parts shows the probability of obtaining a particular set piece and score it in each zone with and without pressure.

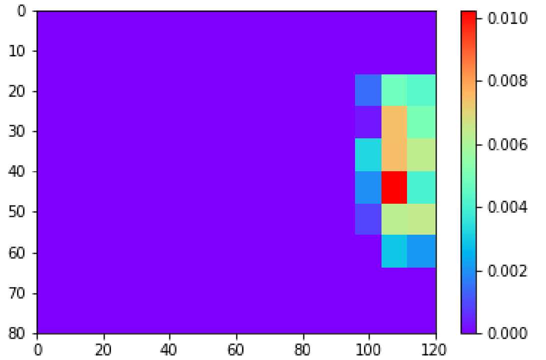
D.2.1 Penalty



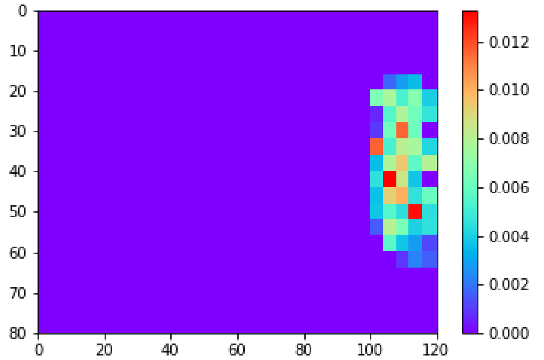
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



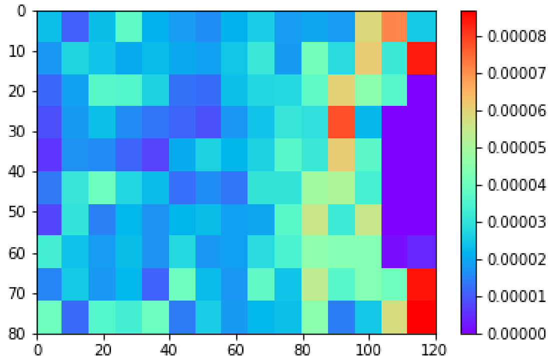
(c) Division of 150 zones under pressure



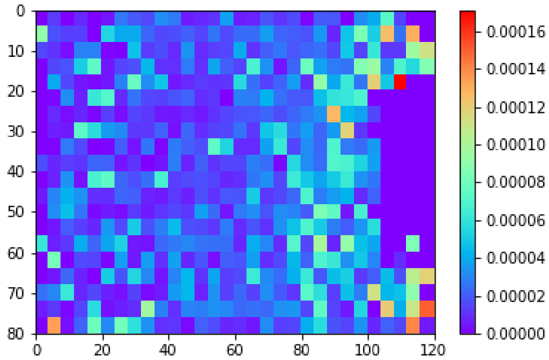
(d) Division of 600 zones under pressure

Figure D.2: Probability of obtaining and scoring from a penalty with both divisions

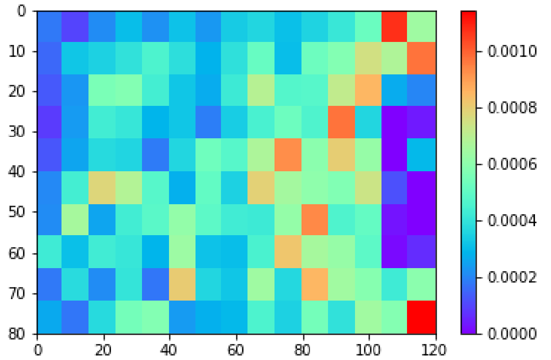
D.2.2 Free-kick



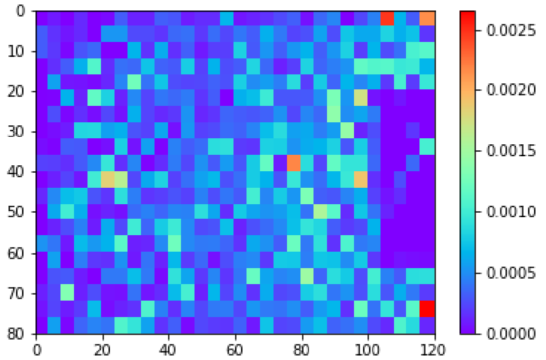
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



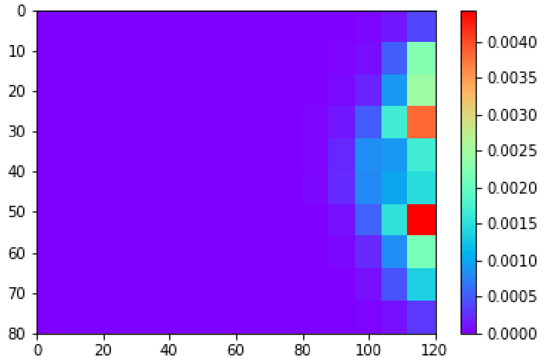
(c) Division of 150 zones under pressure



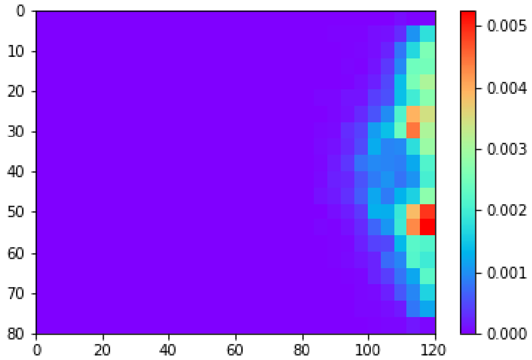
(d) Division of 600 zones under pressure

Figure D.3: Probability of obtaining and scoring from a free-kick with both divisions

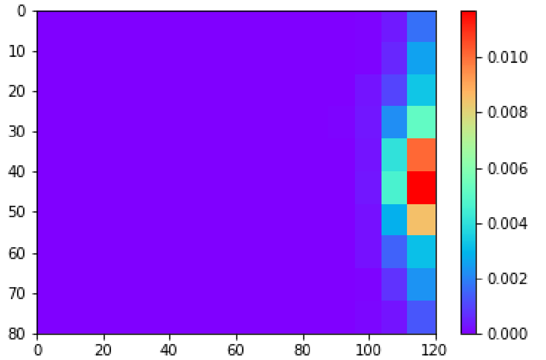
D.2.3 Corner



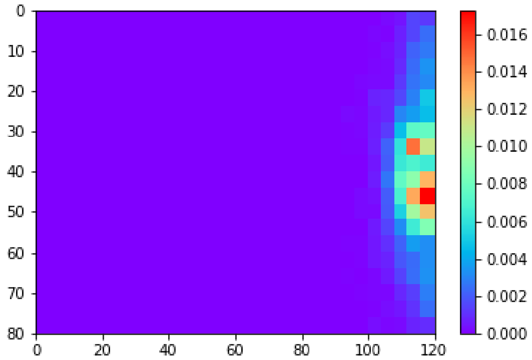
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



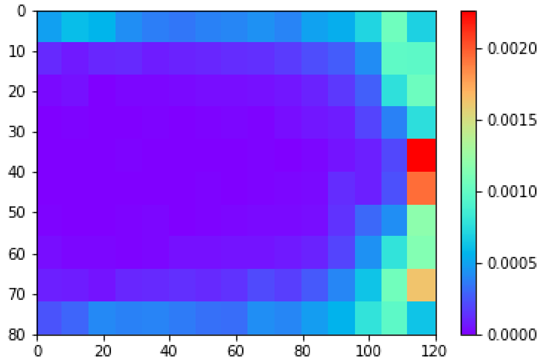
(c) Division of 150 zones under pressure



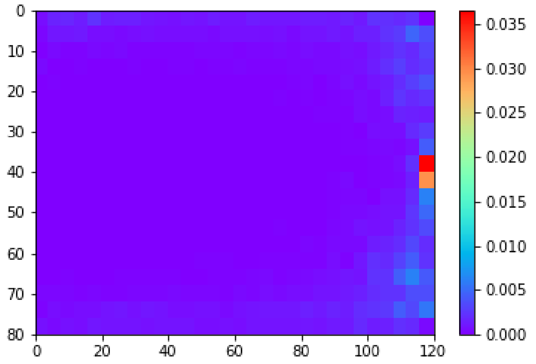
(d) Division of 600 zones under pressure

Figure D.4: Probability of obtaining and scoring from a corner with both divisions

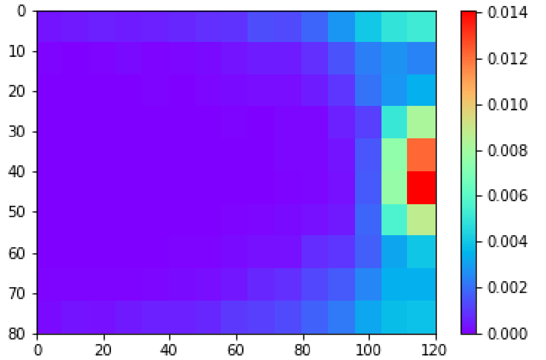
D.2.4 Throw-in



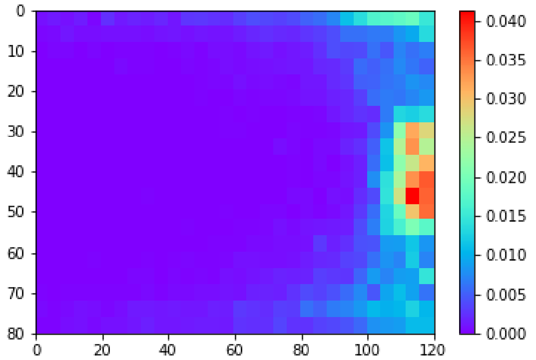
(a) Division of 150 zones without pressure



(b) Division of 600 zones without pressure



(c) Division of 150 zones under pressure



(d) Division of 600 zones under pressure

Figure D.5: Probability of obtaining and scoring from a throw-in with both divisions

Appendix E

Player strengths

E.1 Long-range strikers

The list of all players in the category long-range strikers define at the section 2.6.1 is
Aaron Cresswell, Adem Ljajic, Adrien Rabiot, Alassane Pléa, Alejandro Gomez, Aleksandar Kolarov, Aleksandr Golovin, Alessandro Florenzi, Alexandre Lacazette, Alfred Duncan, Álvaro Morata, Andreas Pereira, Andrej Kramaric, Andros Townsend, André Hahn, Ángel Correa, Ángel Di María, Ante Rebic, Anthony Martial, Anthony Modeste, Antoine Griezmann, Antonin Bobichon, Arjen Robben, Arkadiusz Milik, Arsen Zakharyan, Arturo Vidal, Benjamin Bourigeaud, Bernardo Silva, Blerim Dzemaili, Cengiz Ünder, Christian Eriksen, Christopher Nkunku, "Clinton NJie", Cristian Tello, Cristiano Biraghi, Daniel Didavi, Daniel Parejo, Daniele Baselli, Daniil Utkin, Danilo Cataldi, Danny Ings, David Alaba, David Timor, Davide Zappacosta, Denis Glushakov, Divock Origi, Douglas Costa, Dries Mertens, Eden Hazard, Éder, Edinson Cavani, Emil Forsberg, Emre Can, Erick Pulgar, Ezequiel Ávila, Fabián, Farid Boulaya, Federico Viviani, Fedor Smolov, Felipe Anderson, Fernando, Flavien Tait, Florent Mollet, Florian Thauvin, Floyd Ayité, Fredy Guarín, Gerard Deulofeu, Gerson Acevedo, Giacomo Bonaventura, Giannelli Imbula, Giovanni Simeone, Gonçalo Guedes, Granit Xhaka, Gregoire Defrel, Grzegorz Krychowiak, Gylfi Sigurdsson, Harry Kane, Hatem Ben Arfa, Henrikh Mkhitaryan, Hernanes, Hulk, Hwang Ui-Jo, Iago Aspas, Iago Falque, Idrissa Gueye, Ivan Oblyakov, James Maddison, James Rodríguez, James Ward-Prowse, Jason Puncheon, Jean Michael Seri, Jean-Kevin Augustin, Jean-Paul Boëtius, Jhon Córdoba, Joan Jordán, John McGinn, Jonathan Clauss, Jonathan Schmid, Jordan Ayew, Jordan Henderson, Jorge Molina, Joselu, Joshua Kimmich, Joshua King, Josip Brekalo, João Moutinho, Juan Mata, Julian Brandt, Kelechi Iheanacho, Kevin De Bruyne, Kevin-Prince Boateng, Khouma Babacar, Kingsley Coman, Konstantinos Stafylidis, Kylian Mbappe-Lottin, Lars Stindl, Lautaro Martínez, Leandro Paredes, Leon Bailey, Leon Goretzka, Lionel Messi, Lorenzo Pellegrini, Lucas Biglia, Lucas Digne, Luis Muriel, Luka Modric, Malcom, Manolo Gabbiadini, Manuel Fernandes, Manuel Lanzini, Marcel Risse, Marcel Sabitzer, Marco Asensio, Marco Parolo, Mariano, Mark Uth, Martin Terrier, Mason Greenwood, Matheus Pereira, Matteo Politano, Mauricio Pereyra, Mauro Zárate, Max Gradel, Maximilian Arnold, Maximilian Philipp, Maxwel Cornet, Memphis Depay, Milan Badelj, Milos Jojic, Miralem Pjanic, Mohamed Salah, Moi Gómez, Morgan Sanson, Muniain, Naby Keita, Nicolas de Preville, Nikola Vlasic, Oleg Shatov, Óscar Rodríguez, Paco Alcácer, Patrik Schick, Paul Pogba, Paulo Dybala, Philippe Coutinho, Pierre Lees-Melou, Pierre-Emile Højbjerg, Piotr Zielinski, Quincy Promes, Rachid Ghezzal, Radja Nainggolan, Raphinha, Raúl de Tomás, Reziuan Mirzov, Rifat Zhemaletdinov, Riyad Mahrez, Robert Andrich, Robert Lewandowski, Robert Snodgrass, Roberto Soriano, Robin Quaison, Rodri, Rodrigo, Rodrigo de Paul, Romain Hamouma, Roman Eremenko, Rubén Alcaraz, Rubén Rochina, Ruslan Malinovskiy, Rémy Ca-

bella, Sandro Ramírez, Scott McTominay, Serge Gnabry, Sergej Milinkovic-Savic, Sergi Darder, Sergio Agüero, Simone Verdi, Sofiane Boufal, Son Heung-Min, Stuart Armstrong, Stuart Dallas, Suso, Thomas Partey, Timo Werner, Tomás Rincón, Téji Savanier, Valter Birsa, Vitinho, Wahbi Khazri, Wanderson, Wilfried Zaha, Willian, Xherdan Shaqiri, Yaroslav Rakitskiy, Yaya Touré, Yves Bissouma, Zlatan Ibrahimovic]

E.2 Finishing players

The next list is the list of all players with a good finishing talked in the part 2.6.2.
Achraf Hakimi, Adam Ounas, Admir Mehmedi, Adrien Thomasson, Aleksandr Samedov, Aleksey Sutormin, Alexander Meier, Amine Harit, Ander Herrera, Andrey Mostovoy, André Hahn, André-Pierre Gignac, Andy Carroll, Angelo Fulgini, Anssumane Fati, Anthony Martial, Anthony Mounier, Antonio Di Natale, Arjen Robben, Arnaud Kalimuendo Muinga, Artem Dzyuba, Artur Ionita, Artur Yusupov, Ayoze Pérez, Benito Raman, Benjamin Bourigeaud, Benjamin Moukandjo, Bibras Natcho, Blaise Matuidi, Bojan, Bruno Peres, Carlos Tévez, Casimir Ninga, Cesc Fàbregas, Cheick Diabaté, Christian Noboa, Cristhian Stuani, Cédric Bakambu, Daniel Ciofani, Daniel Didavi, Daniele Verde, Daniil Lesovoy, Dejan Kulusevski, Denis Glushakov, Dimitri Payet, Dmitriy Poloz, Eden Hazard, Éder, Edgar Méndez, Emre Can, Enis Bardhi, Eric Bicfalvi, Erling Haaland, Evans Kangwa, Éver Banega, Evgeni Lutsenko, Evgeni Markov, Ezequiel Ávila, Fabinho, Federico Bonazzoli, Federico Fazio, Fedor Smolov, Felipe Caicedo, Fin Bartels, Florian Wirtz, Floyd Ayité, Francesco Totti, Franck Honorat, Franck Ribéry, Gamid Agalarov, Gareth Bale, Georginio Wijnaldum, Gerson, Giacomo Raspadori, Gianluca Scamacca, Gonzalo Escalante, Gonzalo Melero, Gonçalo Guedes, Gregoire Defrel, Gökdeniz Karadeniz, Habib Diallo, Harry Kane, Ignatius Ganago, Imanol Agirretxe, Ismaila Sarr, Issa Diop, Ivan Rakitic, Jack Harrison, Jadon Sancho, Jairo Samperio, Jamal Musiala, James Maddison, James Rodríguez, Janik Haberer, Jesé, Joaquín, Joia Nuno Da Costa, Jonathan Christian David, Jonathas, Jordan Henderson, Jordan Veretout, Josip Drmic, Josip Ilicic, Juanmi, Julian Brandt, Julien Féret, Junior Stanislas, Justin Kluivert, Jérémy Menez, Jérôme Roussillon, Kamil Glik, Keita, Kevin Mirallas, Kevin Volland, Khoren Bayramyan, Kwon Chang-Hoon, Lebo Mothiba, Lionel Messi, Loïc Remy, Luis Alberto, Luis Muriel, Luiz Araujo, "MBala Nzola", Maciej Rybus, Maksim Glushenkov, Manolo Gabbiadini, Manuel Fernandes, Manuel Lanzini, Marc Albrighton, Marcel Halstenberg, Marcelo, Marcelo Brozovic, Marco Benassi, Marco Borriello, Marco Sau, Marcos Llorente, Marcus Ingvartsen, Marek Hamsik, Marius Wolf, Mark Noble, Marten de Roon, Martin Terrier, Mason Greenwood, Massimo Maccarone, Mathieu Debuchy, Mathieu Valbuena, Max Kruse, Maxi Gómez, Maximilian Arnold, Maximilian Philipp, Maxwel Cornet, Mesut Özil, Morales, Nabil Bentaleb, Naby Keita, Niclas Füllkrug, Nicolas Benezet, Nicolas Pepe, Nikola Vlasic, Nils Petersen, Nolito, Odise Roshi, Olivier Giroud, Oscar Wendt, Othman El Kabir, Ousmane Dembélé, Patrik Schick, Peter Crouch, Philipp Max, Piotr Zielinski, Portu, Quincy Promes, Radja Nainggolan, Rafael Leão, Ramy Bensebaini, Raphael Guerreiro, Remo Freuler, Roberto Gagliardini, Roman Eremenko, Sadio Mané, Sam Vokes, Sami Khedira, Sasa Kalajdzic, Seko Fofana, Senad Lulic, Serge Gnabry, Sergey Tkachev, Sergio Ramos, Shkodran Mustafi, Simon Zoller, Simy, Son Heung-Min, Stefan Aigner, Stephan El Shaarawy, Stephy Mavididi, Suso, Vincenzo Grifo, Virgil van Dijk, Vyacheslav Grulev, Wesley Said, Willi Orban, Willian José, Wylan Cyprien, Xabi Prieto, Yannis Salibur, Youri Tielemans, Yuri, Yusuf Yazici

Appendix F

Optimal actions with first improvements and set-pieces

F.1 Optimal actions with 600 zones

This section shows the detailed actions for the division of 600 zones without and under pressure.

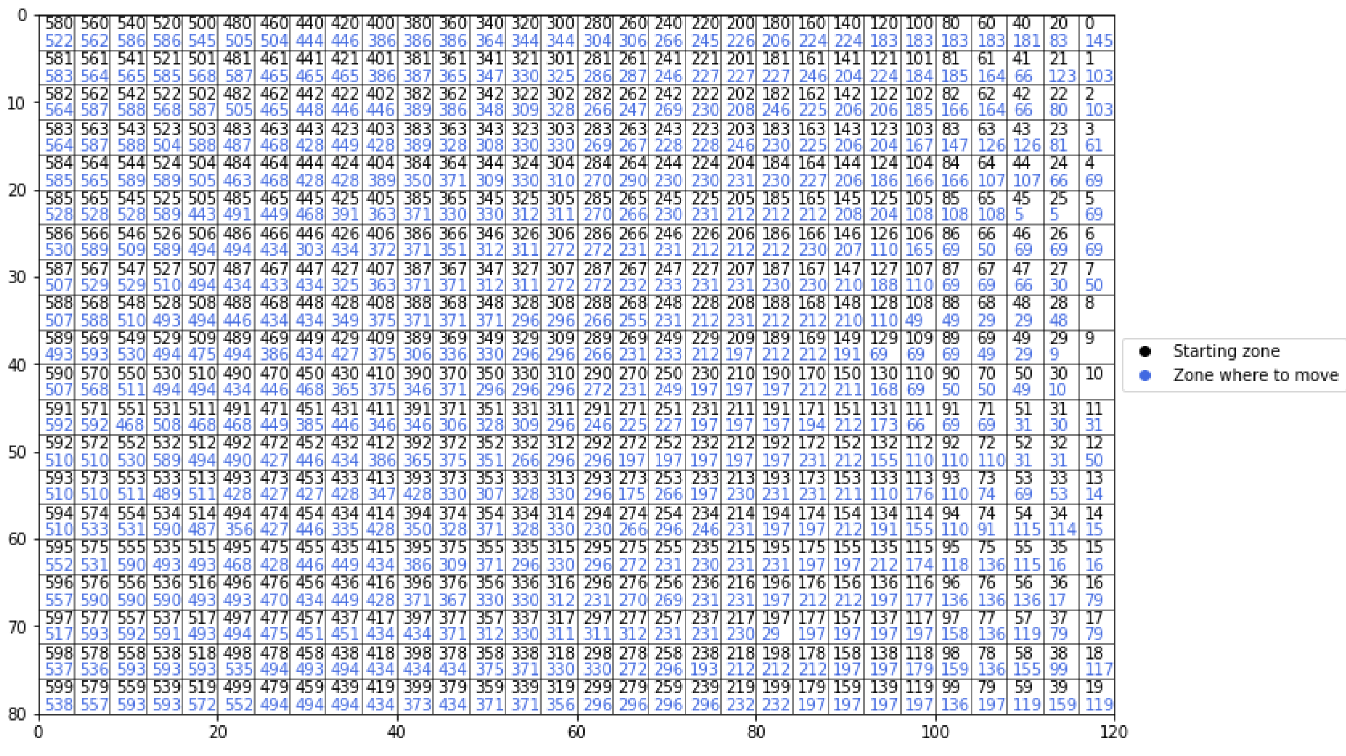


Figure F.1: Target zones of the optimal actions without pressure

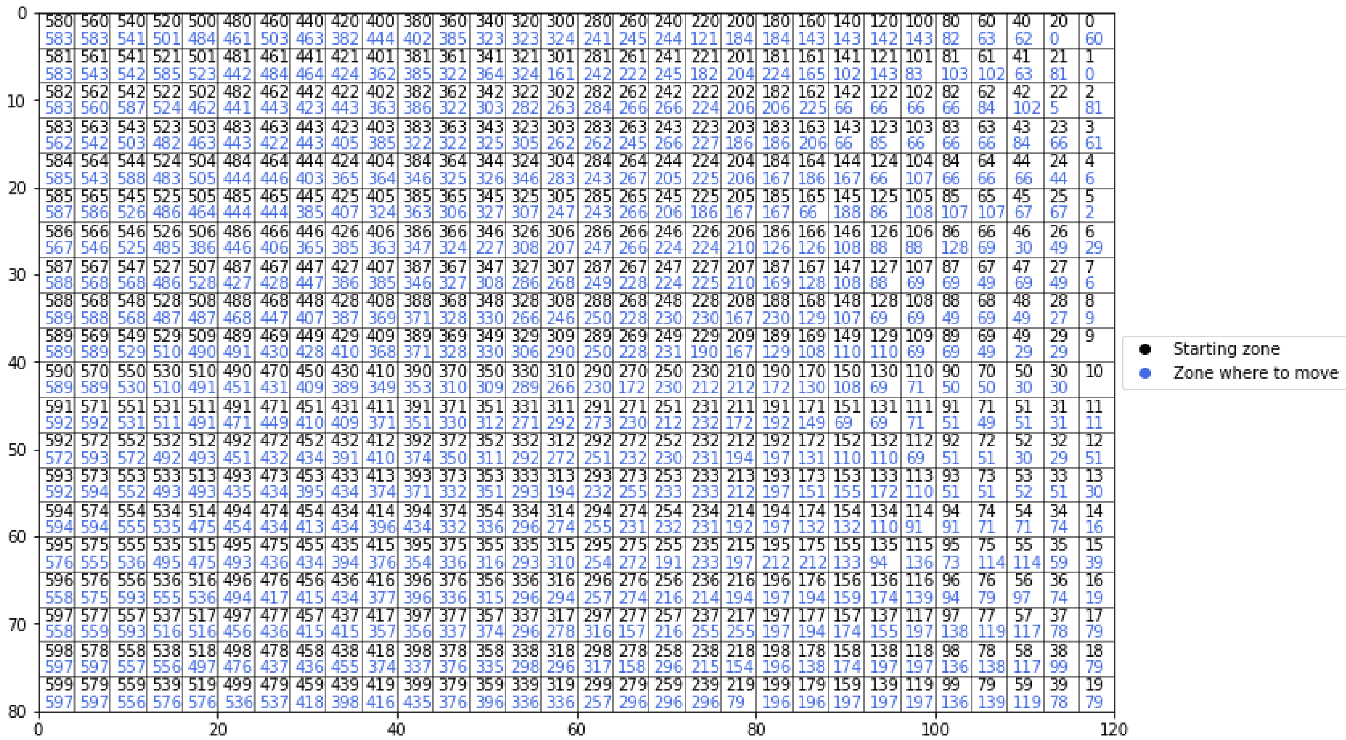


Figure F.2: Target zones of the optimal actions under pressure

F.2 Optimal actions when only the first is optimal with 150 zones

This section shows the detailed actions when only the first actions is optimal and the others are average for the division of 150 zones without and under pressure. This graphs correspond to the color plots of the section 3.1 when the optimal action is shown as a color plot.

F.3 Optimal actions when only the first is optimal with 600 zones

This section shows the detailed actions when only the first actions is optimal and the others are average but now for the second division, with 600 zones. This plots are used in the section 3.1 where only the color plots are shown.

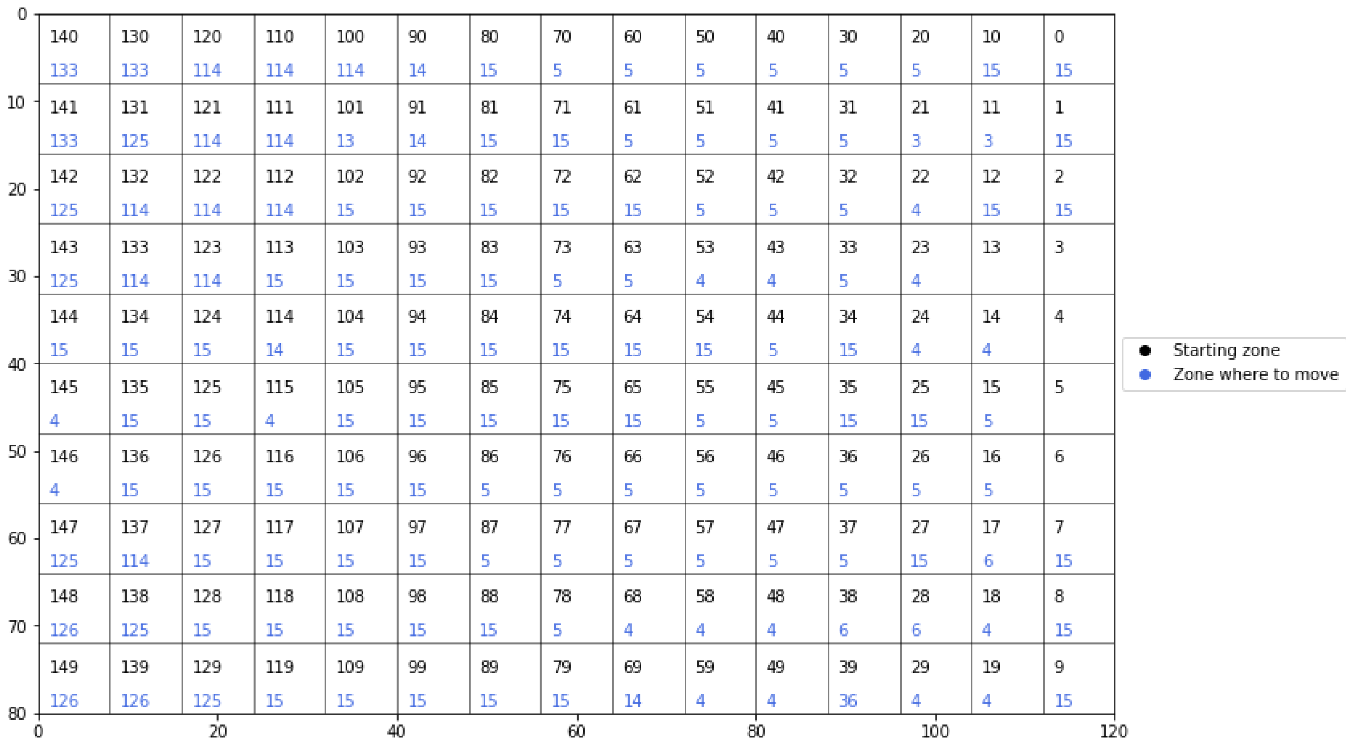


Figure F.3: Target zones of the optimal actions without pressure

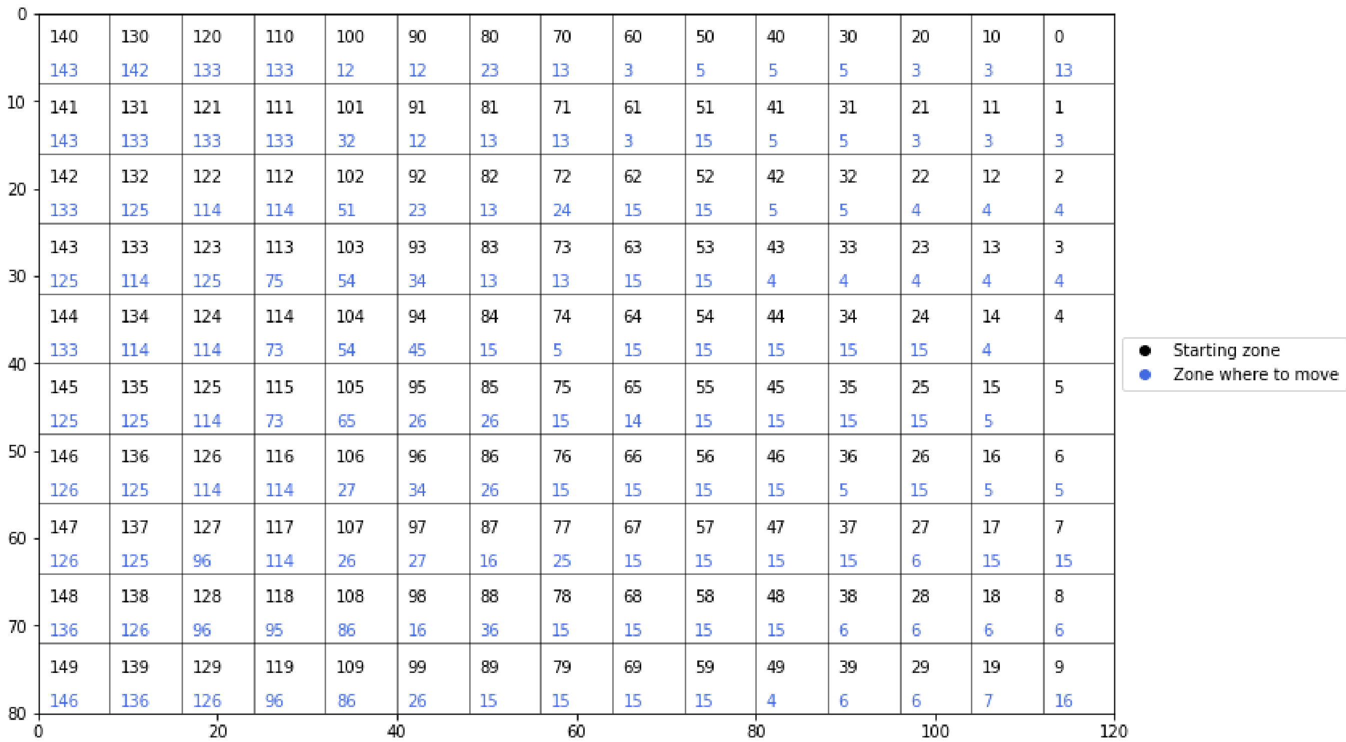


Figure F.4: Target zones of the optimal actions under pressure

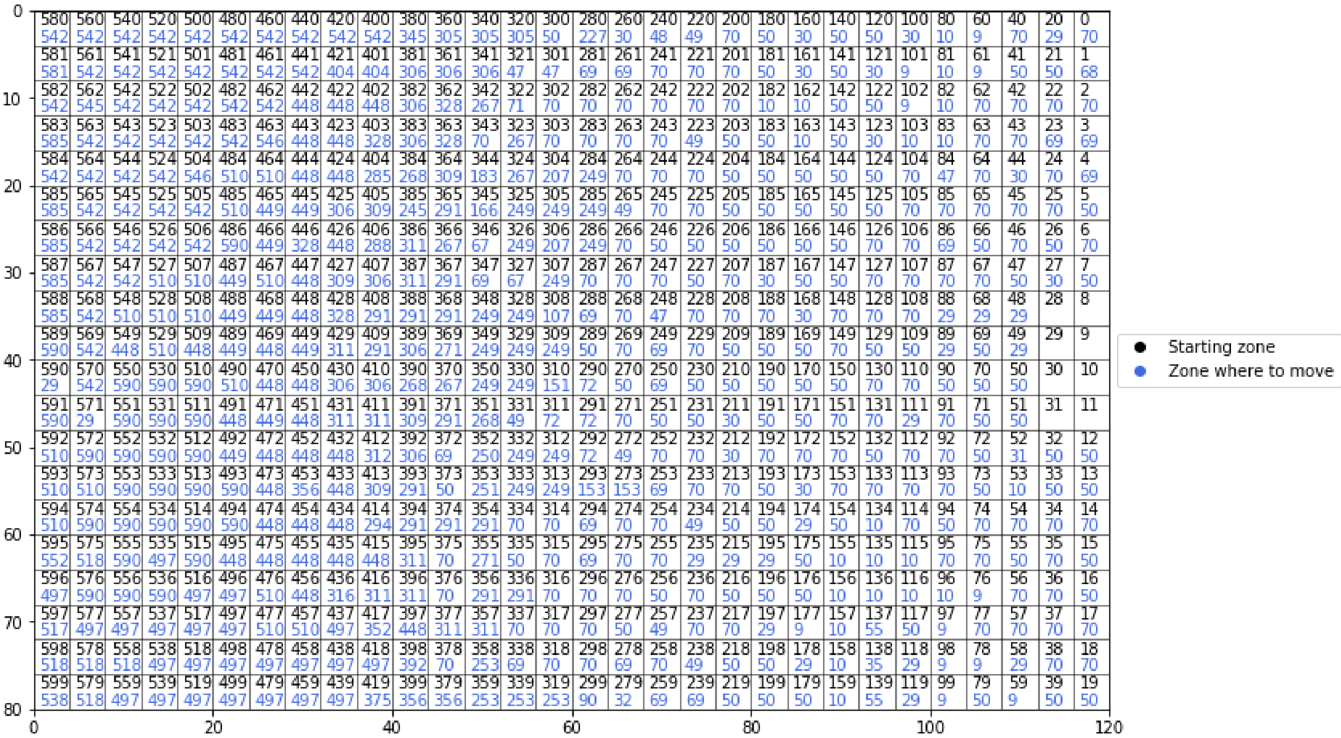


Figure F.5: Target zones of the optimal actions without pressure with 600 zones

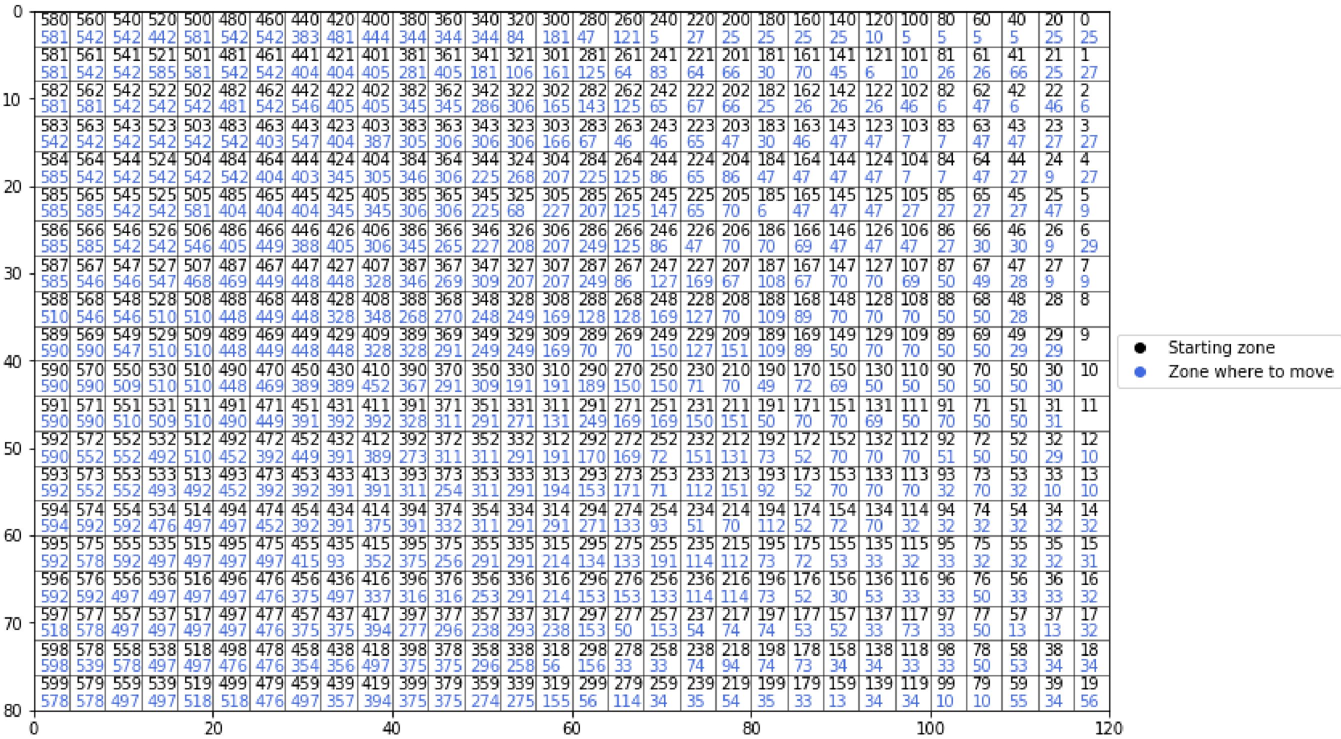


Figure F.6: Target zones of the optimal actions under pressure with 600 zones

Appendix G

Comparison between the players

The figure 5.1 shows the average difference between the optimal expected reward after the action done by the players and the initial optimal expected reward. The exact value for each player of FCBarcelona during the Spanish league 2020-2021 is written here.

Name of the player	Average difference of rewards
Anssumane Fati	-0.0101
Antoine Griezmann	-0.0125
Carles Aleña Castillo	-0.0052
Clément Lenglet	-0.0047
Francisco António Machado Mota de Castro Trincão	-0.0117
Frenkie de Jong	-0.0056
Gerard Piqué Bernabéu	-0.0057
Héctor Junior Firpo Adames	-0.0057
Jordi Alba Ramos	-0.0094
Lionel Andrés Messi Cuccittini	-0.0102
Marc-André ter Stegen	-0.0089
Martin Braithwaite Christensen	-0.0177
Miralem Pjanic	-0.0084
Moriba Kourouma Kourouma	-0.0067
Norberto Murara Neto	-0.0118
Óscar Mingueza García	-0.0044
Ousmane Dembélé	-0.0134
Pedro González López	-0.0084
Philippe Coutinho Correia	-0.011
Ricard Puig Martí	-0.007
Ronald Federico Araújo da Silva	-0.0039
Samuel Yves Umtiti	-0.0045
Sergi Roberto Carnicer	-0.0071
Sergino Dest	-0.0075
Sergio Busquets i Burgos	-0.0063

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