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Curry's Paradox and the Inclosure Schema

Curry's Paradox, a very spicy actuality

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Table of contents

Foreword	3
Acknowledgements	4
I. Introduction.....	5
1. Structure.....	5
2. Problematic and hypothesis.....	5
3. Triviality	6
4. Theories	10
a) True theory	10
b) Set theory	11
c) Theory of properties.....	11
II. The paradox	14
5. What is Curry's Paradox?	14
6. Principles and resolutions	20
a) Non-classical logics: Curry-Completeness Responses	22
b) Classical logic: Curry-Incompleteness responses	25
7. Why is this Paradox problematic?	26
a) Paradoxes of self-reference	26
b) Triviality in our theories.....	34
8. Negation.....	35
9. C-Curry <i>vs</i> V-Curry's Paradox	37
10. Revenge paradox	38
11. What is a paradox and how to classify them?	38
a) What is a paradox?.....	38
b) Ramsey's two families.....	39

c) Priest and the principle of uniform solution	41
III. The Inclosure Schema.....	42
12. Russell's Schema	42
13. What is the Inclosure Schema?	43
14. How do paradoxes fit?.....	45
15. Priest's justification	48
a) for the Inclosure Schema.....	48
b) From the Inclosure Schema.....	49
IV. Curry's paradox and The Inclosure Schema	56
16. Context of the debate	56
17. Stakes and consequences for the supporters of the Inclosure Schema.....	58
18. The paradox does not fit the Inclosure Schema	61
a) And is of a different kind than paradoxes included in the Inclosure schema	61
b) And is of the same kind as paradoxes included in the Inclosure schema	63
19. The paradox fits the Inclosure	70
V. Conclusion	86
20. Answers to the problematic and the hypothesis.....	86
21. Opening.....	92
VI. Bibliography	94

Foreword

To complete this work, I had to make choices, despite certain contradictory recommendations. Therefore, there are choices that I know to be divisive, such as that of exposing the other paradoxes that Graham Priest includes in the Inclosure Schema without giving any further explanation. These choices were made in the light of Professor Peter Verdée's instructions to follow Doctor Florian Marion's instructions in case of doubt. I hope they will not interfere with your reading.

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I. Introduction

1. Structure

In this work, I will try to explain why we should find an interest in the Curry's Paradox, especially regarding the authority of the Inclosure Schema based on the Principle of Uniform Solution. In this introduction, I will state the main problem and my hypothesis, combined with the theories in which this problem is found. Secondly, I elaborate on the puzzling Curry's paradox and the various solutions that were offered to solve it. Given the complexity regarding paradoxes in general, I will explain this concept from a more generalist point of view. Thirdly, I will present the Inclosure Schema and the way it works with self-referential paradoxes. Finally, I will explain the issues raised by Curry's Paradox for the Inclosure Schema. By analysing different points of view, I provide more context as to whether the paradox fits this schema and explore the challenges and consequences of these findings.

2. Problematic and hypothesis

I have thus established that what's at stake in this work is to understand the impact of Curry's Paradox on the Inclosure Schema, but to be more precise the problematic of this work is: 'Does Curry's Paradox fit the Inclosure Schema? And if not, does this paradox belong to the same family as the ones that fit the schema?'. In other words, the question is to know if Curry's Paradox either fits the Inclosure Schema, or doesn't belong to the family of self-referential paradoxes (as the Liar, Burali-Forti, Grelling etc.) and if it is neither, there might be a fundamental problem to the Inclosure Schema and its justification in virtue of the Principle of Uniform Solution which is often assumed nowadays in philosophy of logic.

My central hypothesis analyses to which extent Curry's Paradox questions the relevance of the Inclosure Schema by testing how Curry's Paradox weakens the Inclosure Schema's justification of the dialetheism. The prime motivation for this approach has been that Graham Priest's argumentation in defence of dialetheism is the combination of finding a common structure for paradox that fit the Inclosure Schema and the Principle of Uniform Solution. This approach also seems relevant to me regarding the numbers of philosophers that argue that Curry's Paradox is of the family of self-referential paradoxes and that it doesn't properly fit the Inclosure Schema. I will compare my observations with this hypothesis at the end of my work, and therefore, try to conclude whether Curry's Paradox is or is not a threat to the Inclosure Schema and the Principle of Uniform solution.

3. Triviality

In order to reason on the stakes of Curry's Paradox, it is necessary to establish a definition for the notion of triviality.

To say that a theory is trivial means that all the statements in that theory are proven to be true, so a theory in which everything holds. That would then be a theory where A is true but his negation $\neg A$ too. Applied to an object, it makes it possess all the properties. If we express this trivial object as ∞ , and any object a and b , it would be the case that $A = \infty$ and $B = \infty$, even if it is not the case that $A = B$ (Priest 1998, 171-172; Shapiro & Beall 2024, §1.2).

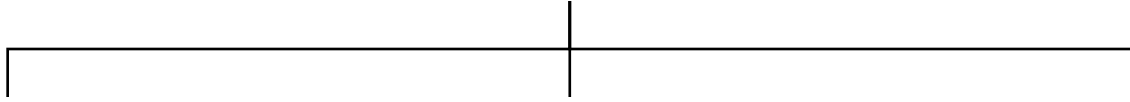
Regarding triviality a subtle distinction must be defined between triviality which is the fact that a formula entails everything and, so, renders the (set-theoretical or semantic) theory *trivial*, and Trivialism which is a metaphysical view where every sentence is true. A concept that will allow me to explain more about the difference between this and other metaphysical views is the Principle of Explosion (Priest 1998, 171-172; Kabay 2008, 29-31; Priest, Tanaka & Weber 2022, §0).

The principle of Explosion (also called *ex contradictione sequitur quodlibet*) is a principle of classical logic that became standard around the end of the nineteenth century. This principle affirms that $A, \neg A \vdash B$ with B meaning anything. It means that any contradiction entails any arbitrary conclusion, namely triviality. This view is challenged by paraconsistent logics, which denies the accuracy of the contradiction leading to triviality. This does not imply that proponents of paraconsistent logic need to claim that all contradictions are true, as trivialists do, nor that some contradictions are true as in the dialetheists' vision, it just means that they don't accept that this contradiction leads to any chosen claim (Priest, Tanaka & Weber 2022, §0).

Trivialism is a *metaphysical* thesis defined by one of its rare supporters as 'the claim that each and every proposition is true' (Kabay 2008, 29). In other words, all the sentences that you could think about are true but there negations also are (Kabay 2008, 29-31).

A way of looking at it is through the comparison between consistentism and dialetheism. Consistentism states that a contradiction $A \wedge \neg A$ cannot be true, and therefore all contradictions are false. Conversely, dialetheism claims that some (but not all) $A \wedge \neg A$ are *true*. Antagonistic towards them, trivialism declares that every single proposition, including $A \wedge \neg A$ are *true*. Consistentism is the orthodox view where most logicians argue that every contradiction is false. In fact, they put forward that contradiction is the paradigm of what is *impossible*, therefore *necessarily false*. That every contradiction is false is still the common view in philosophy. Indeed, according to 'The 2020 PhilPapers Survey' 53.6% of philosophers believe contradiction to be false. On the other hand, 26.4% of the respondents support non-classical logics (which includes dialethism). Precisely on dialetheism, 12.4% of respondents accept that there are true contradictions (Estrada-González 2012, 180-182; Bourget & Chalmers 2023, 14).

What is the truth-value of a
contradiction?



All contradiction are false.
(classical logic,
philosophical orthodoxy)

A contradiction might be true,
and some are.
(dialetheism)

All contradiction are true.
(trivialism)

The paradox that I will analyse in this work has a very special link to trivialism. Rather than asserting a true contradiction, as most paradoxes do, it implies triviality. However, in classical logic, triviality is implied by every contradiction according to Explosion. Thus, from a classical point of view, Curry's Paradox does not differ from other self-referential paradoxes. The only difference would be that the other paradoxes as the Liar Paradox entails triviality by means of Explosion. The other way around, Curry's Paradox directly entails triviality without passing by a particular contradiction and Explosion (Priest 1998, 171-172; Priest, Tanaka & Weber 2022, §0).

Some metaphysicians, the trivialists, are the only ones to endorse the conclusion of Curry's Paradox. In their regard, they consider Curry's Paradox, not as a paradox, but as a positive argument in favour of the claim that everything is true. Most metaphysicians, however, reject the metaphysical view of trivialism, and by doing so, they need to justify where the reasoning in Curry's inference doesn't hold (Kabay 2008, 54-56).

Therefore, Curry's Paradox is one of the arguments put forward by the rare trivialists such as Paul Kabay's or Carrara and Martino. Trivialism and triviality in logical theories is the subject of much debate (Kabay 2008, 54-56; Carrara & Martino 2011, 199-206).

Knowing these differences in theory, it seems worth mentioning the importance of the proof of non-triviality for paraconsistent logics.

A good illustration of the importance of having a non-trivial theory is the fact that paraconsistent theories make proofs of no-triviality, which fulfil the same role as the consistency proofs in classical theories. This has been demonstrated in Priest 1998. There he theorises why such proofs are necessary and provides one for a paraconsistent theory containing denotation predicates, naive truth and arithmetic. Others' works support this vision, namely Brady & Routley 1989, which proves that extensional dialectical set theory is not trivial. Additionally, Brady 1989 provides proof of 'The Non-Triviality of Dialectical Set Theory' (Brady 1989, 437) (Brady 1989, 437-472; Brady & Routley 1989, 415-436; Priest 1998, 171-183).

These two papers indicate the importance of triviality, and therefore of Curry's Paradox that leads to triviality, in non-classical set theory. These theories are not threatened by contradiction since they accept, for example, strange sets such as Russell's set. Conversely, they are threatened by triviality, and therefore by Curry's Paradox. Indeed, those two proofs are followed by the article Slaney 1989 which, thanks to Curry's Paradox, tackles the possibility of proving that theories are inconsistent but resist triviality. This highlights the importance of Curry's Paradox and allows one to make a sharp distinction between Curry's Paradox and the other self-referential paradoxes (Brady 1989, 437-472; Brady & Routley 1989, 415-436; Slaney 1989, 472-479).

This short explanation will allow us to tackle later the complexities of Curry's Paradox and to understand how it differs from the other paradoxes. We can already see what is at stake in implying that every formula whatever it is follows from something, and how problematic it would be to have to conclude that.

4. Theories

To finalise this introduction and prior to delving into the central theme of this work, an explanation of in which theories we find ourselves talking about Curry's Paradox seems justified. Indeed, to talk about a paradox, I must situate in which theories we can find it, to understand what is at stake. It is important to point out that Curry's Paradox is not a problem in itself. It is a problem once you try to have an intuitive theory of logic that holds. This means that the paradox is meaningless without a theory upon which it is applied.

Therefore, I will mention some of the theories where we can find it, which will be useful later in this work when I will address the reasons why this paradox is so problematic. It will allow me to explain how the paradox questions our formal theory or our logic for instance, our semantics, our mathematical set-theory and our theory of properties.

a) True theory

The formal systems in which the paradox is most often analysed are theories of truth and set theories. A lot of modern theories of truth base themselves on the Tarski's conception of truth in formalised languages. In this conception of truth, sentences are the bearers of truth. The idea is to determine what is an adequate theory of truth and what is not. To support his classification Tarski applies the convention T and the T-Schema. The convention T is a convention that says of a predicate that it is true by making it correspond to the state of the things it describes. For a statement to be true according to convention T, it must respect the T-schema, which states that $T\langle A \rangle \leftrightarrow A$ where T is the truth-predicate and $\langle A \rangle$ an appropriate name for A. In other words, T-Schema formalises the notion of truth by linking the statements of a language to the reality they designate. The theory is based on an adequacy's condition where the truth predicate of the theory can be applied to all true sentences and only the true one: every sentence that affirms something is true only if this something indeed is 'Convention T guarantees that the truth predicate given by the theory will be extensionally correct, i.e., have as its extension all

and only the true sentences of L' (Glanzberg 2003, §2) (Tarski 1944, 341-376; Tarski 1956, 152-278; Glanzberg 2018, §2; Shapiro & Beall 2024, §1.2).

b) Set theory

There are a few contemporary versions of set-theory, e.g., Zermelo-Fraenkel hierarchical theory, Russell's type-theory, Neumann-Bernays-Gödel's class-theory, and Quine's New Foundations for Mathematical Logic, those came after the intuitive first Cantorian and Fregean concept of set was put through the wringer by Russell's Paradox and other self-theoretical paradoxes such as Burali-Forti's Paradox, etc. I will explain more in detail how this paradox works in one of the following chapters. The result of Russell's (and others') discovery was the invention of several made-up conceptions of sets that use cumulative hierarchy. The common point of those theories is the characterisation by an unlimited number of sets where for any condition you state in the theory, all things included in this condition and only them form a set (Woods 2003, 155-170; Priest 2006, part. 2)

c) Theory of properties

A theory of properties is another kind of formal system where Curry's Paradox can appear. This kind of theory is characterised by an unlimited number of properties. By the comprehension principle, for each condition that can be stated, things that fulfil this condition illustrate an existing property. Formal property theories are logical systems that aim at formulating 'general noncontingent laws that deal with properties' (Bealer & Mönnich 1989, 133). To do so, they present terms that correspond to properties and variables that can have a certain range and number of properties (Bealer & Mönnich 1989, 133-251; Orilia & Paolini Paoletti 2020, §6.1; Shapiro & Beall 2024, §2).

Those intuitive theories, at the root of our logic, are some of the theories where we can find Curry's Paradox. Its presence raises questions about the relevancy of those theories and our logic. I will explain this problem in detail in point 7, namely 'Why is this Paradox problematic?' page 26.

In this work, for the sake of space I will often propose a very general version of Curry's Paradox or specifically the version that appear in theories of truth. However, this paradox appears in specific ways in each of the theories presented, and potentially in others not addressed. Different versions of the paradox can be found in the theories of truth (a), set (b) and properties (c). I will expand slightly on these different versions also in part 7, on page 26 (Moh Shaw-Kwei 1954, 37-40; Prior 1955, 177–182; Geach 1955, 71-72; Löb 1955, 115-118).

Those naively conceived theories are particularly vulnerable to Curry's Paradox. Indeed, if we combine the theories of truth (a), set (b) and properties (c) in their naive conception with classical logic, those theories are very quickly rendered trivial, if only by the Liar (which I explain in more detail on page 22). Since a trivial theory is of no use, we have three options to continue using logic (Moh Shaw-Kwei 1954, 37-40; Prior 1955, 177–182; Geach 1955, 71-72; Löb 1955, 115-118).

First, is to accept the impossibility of speaking coherently about truth, set and properties. This first solution is highly problematic, since it thwarts our use of logic, whose notions are at the foundation of mathematics, philosophy and much more.

The second option is to develop theories of truth, set and property based on other principles. The disadvantage of this is that those are no longer as intuitive. Indeed, this no longer follows the pre-theoretical conception of those notions. Historically speaking, this has been the most popular option. It is from this approach that we derive, for example, ZFC or NF set theory, or on the side of truth theories, Tarski's theory (Moh Shaw-Kwei 1954, 37-40; Prior 1955, 177–182; Geach 1955, 71-72; Löb 1955, 115-118; Woods 2003, 155-170).

The last option is to give up on some of the rules on which classical logic is based. However, it is not without its problems, since it limits deductive inference. For example, if we defend dialetheism (which I explained on pages 7 and 33), the Law of Non-Contradiction (LNC) no longer has the same force, and even if this theory does not accept most contradictions, dialetheists have to move on to a proof of non-triviality to be able to prove something, as in Brady 1989 (437-472) and Brady & Routley 1989 (415-436).

Some of the proposed solutions that follow the third option have challenged the conception of negation in classical logic. For those solutions, as for dialetheism for example, Curry's Paradox is problematic. Indeed, Curry's Paradox shows that even without negation, or even if we accept certain contradictions as true, we can trivialise those theories. Curry's Paradox is problematic because, by being paradoxical by implication and not by contradiction, it thwarts the solutions provided to previous paradoxes that attempt to keep theories intuitive (Slaney 1989, 472-479).

The fact that Curry's Paradox trivialises the theory in which it happens is puzzling because, if so, mathematics and semantics become trivial. Hence of no use at all. Indeed, a trivial theory is of no help, it is just a useless statement. However, it is very important to try to build non-trivial theories. Logic, which underpins so much of our knowledge, cannot be left to only a feeling or to the intuition that something must be true.

II. The paradox

5. What is Curry's Paradox?

The inventor of the paradox from which the name of the paradox is derived is the American logician Haskell Curry. He presented this paradox in Curry 1942b to prove that 'certain systems of formal logic are inconsistent' (Curry 1942b, 115). Its aim was to show that all formulas are provable in some theories thus, to prove triviality, but in a much simpler and less restrictive way than the paradox previously put forward (Curry 1942b, 115-117).

But a Curry-like argument had already been put forward before. For example, by the mediaeval logician Pseudo-Scotus proving God's existence or his opponent proving the opposite. It took the form 'This argument is valid, hence god does not exist.' (Read 1979, 268) which can be reformulated as Curry's Paradox: 'If this sentence is true, then God does not exist' (Read 1979, 269) where validity is replaced by condition (Read 1979, 265-274; Routley & Priest 1982, 189-199; Routley, Plumwood, Meyer & Brady 1982, 280-281)

First, a common and less formal version of the paradox would be to present in the following way. I can imagine any sentence that says of itself, if it is true, then I can affirm any chosen information. What makes it a paradox is that due to this phrasing, I can affirm any information I choose as true (Curry 1942, 115-117; Shapiro & Beall 2018, §1).

If I state the sentences 'if this sentence is true, then I am a dinosaur', this simple statement has as consequence that I am a dinosaur, that the chosen information is true. This can be proven by the following steps (Curry 1942, 115-117; Shapiro & Beall 2018, §1).

First, I can replace the sentence 'if this sentence is true, then I am a dinosaur' by 'K' which would allow me to read it as 'if K is true, then I am a dinosaur'. By the Rules of Excluded Middle, I have then two options: assume that K is false or assume that K is

true. These two options will be the two hypotheses that allow me to prove that whatever the truth-value of K is, I am a dinosaur (Curry 1942, 115-117; Shapiro & Beall 2018, §1)

In the first possibility, where I assume that K is false, the premise *K is true* is false, which makes K true. Indeed, according to the truth-table (see below this paragraph) of the material conditional in classical logic, in every case in which the antecedent is false, the conditional is true. This is coming from the *ex falso quodlibet*, which states that from a falsity everything follows. This rule has given rise to debates on the paradoxes of material implication that describe the paradoxical aspect of accepting any truth from falsity. This discussion can be found, for example, in Routley, Plumwood, Meyer & Brady 1982 (1-69).

A	B	$A \rightarrow B$
True	True	True
True	False	False
False	True	True
False	False	True

Truth-table of the material conditional in classical logic

In that way, if the antecedent *K is true* is false, the implication *if K is true, then I am a dinosaur* is true no matter the truth-value of the consequent *I am a dinosaur* is. This implication being K, it makes me a dinosaur since I am one if K is true (Curry 1942, 115-117; Routley, Plumwood, Meyer & Brady 1982, 1-69; Shapiro & Beall 2018, §1).

I can then go for the second hypothesis to see if I get another result: let K be true. Hence, it is true that *if K is true, then I am a dinosaur* (since it is just what K itself means), and it is the case that *K is true*. I can then deduce the premises and the conclusion

from this hypothesis. By propositional *Modus ponens* ($A \wedge (A \rightarrow B) \rightarrow B$) or by *Modus ponens* ($A, A \rightarrow B \vdash B$), one can argue that under this hypothesis that *K is true*, it is the case that *if K is true then I am a dinosaur*, which is actually K and thus makes it true. We then have *K true*, and *if K is true, then I am a dinosaur*. We can thus, by *Modus ponens*, affirm that *I am a dinosaur* (Curry 1942, 115-117; Shapiro & Beall 2018, §1).

Here is a more formal way to express Curry's Paradox (Rogerson & Restall 2004 and Bimbó 2006). For the next four logical deductions I present, a Curry-intersubstitutivity that follows from Curry's sentence applies. This Curry-intersubstitutivity entails that 'A' stands for ' $A \rightarrow B$ ', like 'K' for 'if K, then B', just like Curry's sentence states that if this sentence (K) is true then any statement (B) is true.

(1)	$A \equiv (A \rightarrow B)$	Curry's intersubstitutivity
(2)	$A \rightarrow A$	Law form of Identity
(3)	$A \rightarrow (A \rightarrow B)$	2, Curry-intersubstitutivity
(4)	$[A \rightarrow (A \rightarrow B)] \rightarrow (A \rightarrow B)$	Law form of Contraction Principle
(5)	$A \rightarrow B$	3, 4 <i>Modus ponens</i>
(6)	A	5, Curry-intersubstitutivity
(7)	B	5, 6, <i>Modus ponens</i>

(Rogerson & Restall 2004, 421-436; Bimbó 2006, 227-240; Shapiro & Beall 2018)

Another formal way to establish the Curry's Paradox, closer to the one originally presented by Haskell Curry, would be this one (Curry 1942, 115-117).

(8)	$A \equiv (A \rightarrow B)$	Curry's intersubstitutivity
(9)	$A \rightarrow A$	Law form of Identity
(10)	$A \rightarrow (A \rightarrow B)$	9, Curry-intersubstitutivity
(11)	$A \rightarrow B$	10, Contraction Principle
(12)	A	11, Curry-intersubstitutivity
(13)	B	11, 12, <i>Modus ponens</i>

In this inference, I begin by exposing that A is identical to A , by Principle of Identity ($A \rightarrow A$). From there, since I know that the sentence K is 'if this sentence is true, then I am a dinosaur', I can replace the second A , by this sentence, which means we can replace the second A by $A \rightarrow B$. The Principle of Contraction ($A \rightarrow (A \rightarrow B)$) allows me afterward to contract the two occurrences of A into one, which leaves me with $A \rightarrow B$. In the same optic, from 'if this sentence is true, then if this sentence is true then I am a dinosaur' thanks to the Principle of Contraction, I can advance that 'if this sentence is true, then I am a dinosaur'. By knowing that this sentence corresponds to K , or by replacing $A \rightarrow B$ by A , and by *Modus ponens* we can then finally conclude that I am logically a dinosaur (Curry 1942, 115-117; Shapiro & Beall 2018, §3).

The next way of formulating Curry's Paradox that I will present is the Meyer, Routley and Dunn's one (Meyer, Routley & Dunn 1979, 124-128).

(14)	$A \equiv (A \rightarrow B)$	Curry's intersubstitutivity
(15)	$((A \rightarrow B) \wedge A) \rightarrow B$	Law form of <i>Modus ponens</i>
(16)	$(A \wedge A) \rightarrow B$	15, Curry-intersubstitutivity
(17)	$A \rightarrow B$	16, Idempotency of Conjunction
(18)	A	17 Curry-intersubstitutivity
(19)	B	17, 18, <i>Modus ponens</i>

In this proof, I begin by establishing that if A implies B and that A, then B. In other terms, if 'if K is true, then I am a dinosaur' and K is true, then I am indeed a dinosaur by the Law form of *Modus ponens*. In this implication, we can then replace the first premise by K and have *If K and K, then I am a dinosaur*. The Idempotency of Conjunction ($A \wedge A \rightarrow A$) allow us then to replace $K \wedge K$ by K, which implies that we can advance that $A \rightarrow B$ or that *if K is true, then I am a dinosaur*. This last sentence being K, and $A \rightarrow B$ being A, thanks to Curry-intersubstitutivity we can operate a *Modus ponens* that implies that I am a dinosaur (Meyer, Routley and Dunn 1979, 124-128; Shapiro & Beall 2018, § 3).

The last method to expose Curry's Paradox that I will present in this work is the one of Shapiro & Beall 2018 (§ 3.1).

(20)	$A \equiv (A \rightarrow B)$	Curry's intersubstitutivity
(21)	$A \rightarrow A$	Law form of Identity
(22)	$A \rightarrow (A \rightarrow B)$	21, Curry-intersubstitutivity
(23)	$A \rightarrow B$	22, Contraction Principle
(24)	$(A \rightarrow B) \rightarrow B$	23 Curry's intersubstitutivity
(25)	B	23, 24, <i>Modus ponens</i>

Here, like all the times before, I begin by stating that A is identical to A , by the Principle of Identity ($A \rightarrow A$). From there, again, since I know that the sentence K is 'if this sentence is true, then I am a dinosaur', I can replace the second A , with this sentence, which means I can replace the second A with $A \rightarrow B$. The Contraction Principle ($A \rightarrow (A \rightarrow B)$) subsequently allows me to contract the two occurrences of A into one, leaving me with $A \rightarrow B$. I can then go from $A \rightarrow B$ to $(A \rightarrow B) \rightarrow B$ by Curry's intersubstitutivity, i.e. by swapping A with $(A \rightarrow B)$ and not touching $\rightarrow B$. Indeed, since my Curry's sentence states the sentence K 'if K is true then I am a dinosaur' I can state that *if 'if K is true then I am a dinosaur' is true then I am a dinosaur*. I then apply the *Modus ponens* of $A \rightarrow B$ and $(A \rightarrow B) \rightarrow B$ which gives me B , *I am indeed a dinosaur* (Shapiro & Beall 2018, §3.1)

6. Principles and resolutions

The four ways to demonstrate the Curry's Paradox that we have presented rely on different principles. These principles will allow us to expose different solutions to the paradox. These different principles are:

- Curry-intersubstitutivity: $A \rightarrow (A \rightarrow B)$, used in lines (1), (3), (6), (8), (10), (12), (14), (16), (18), (20), (22) and (24)
- Contraction Principle: $A \rightarrow (A \rightarrow B) \vdash A \rightarrow B$, used in line (11) and (23)
- *Modus Ponens*: $A \rightarrow B, A \vdash B$, used in lines (5), (7), (13), (19) and (25)
- Law form of Identity: $A \rightarrow A$, used in line (2), (9) and (21)
- Law form of Contraction: $(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$, used in line (4)
- Law form of *Modus ponens*: $((A \rightarrow B) \wedge A) \rightarrow B$, used in line (15)
- Idempotency of Conjunction: $A \leftrightarrow (A \wedge A)$, used in line (17)

(Curry 1942, 115-117; Meyer, Routley & Dunn 1979, 124-128; Rogerson & Restall 2004, 421-436; Bimbó 2006, 227-240; Shapiro & Beall 2018, §3)

To give you a clearer idea of which principles are used by which logical deductions, here's a summary table. This analysis will be useful on page 24 to explain one of the possible resolutions.

	1. Rogerson & Restall 2004, 421-436; Bimbó 2006, 227-240	2. Curry 1942, 115-117	3. Meyer, Routley & Dunn 1979, 124-128	4. Shapiro & Beall 2018, § 3.1
Curry-intersubstitutivity $A \rightarrow (A \rightarrow B)$	✓	✓	✓	✓
Contraction $A \rightarrow (A \rightarrow B) \vdash A \rightarrow B$	✗	✓	✗	✓
<i>Modus Ponens</i> $A \rightarrow B, A \vdash B$	✓	✓	✓	✓
Law form of Identity $A \rightarrow A$	✓	✓	✗	✓
Law form of Contraction $(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$	✓	✗	✗	✗
Law form of <i>Modus ponens</i> $((A \rightarrow B) \wedge A) \rightarrow B$	✗	✗	✓	✗
Idempotency of Conjunction $A \leftrightarrow (A \wedge A)$	✗	✗	✓	✗

a) Non-classical logics: Curry-Completeness Responses

The theories that question these principles are called ‘Curry-Completeness Responses’. They violated at least one of the logical principles used in Curry’s Paradox to find a non-trivial version of the classical logic (Shapiro & Beall 2018, §4).

The first principle that could be questioned is the Principle of Identity, it is however hardly ever the case. An exception is Nicolai & Rossi 2017, where the authors discuss the topic of logical validity and consequence. Their conception of object-linguistic consequence leads to limit the Rules of Reflexivity. Given the fact that this option is hardly defended by philosophers, I refrain from presenting this exception in more detail. An elaborate description of this option would result in a far too strong deviation of the main subject of this thesis, whilst a limited explanation on the other hand would only result in a gross simplification and therefore an injustice to this concept (Nicolai & Rossi 2017, 574-576; Shapiro & Beall 2018, §4).

The second option, which is more commonly applied, is the strategy of the contradiction-free responses. They accept *Modus ponens* but question the relevance of the Contraction Principle. To do so, one must deny the Law of Absorption ($A \rightarrow (A \rightarrow B) \vdash A \rightarrow B$), demonstrated in an example such as the Łukasiewicz 3-valued logic Ł3 (Prior 1955, 177; Priest 2008, 122-124; Shapiro & Beall 2018, §4.2).

In Ł3, there is a third truth-value that can be interpreted as *paradoxical*. Indeed, the set of truth-values is {true, false, paradoxical}. This third value makes the condition to state something as a law more complex. It adds some ‘subtleties’, when with only two truth-values in the Liar, we have a proposition that implies its own negation: we have a sentence from which the truth implies the falsity, and the falsity implies the truth. If we observe this implication with a truth table of the implication with three truth values, we observe that being limited to two truth values makes true or false some values that should be paradoxical. On the other side, with three truth-values, a *paradoxical* proposition, like one of The Liar Paradox, only has as implication that it is *paradoxical* and it solves the

paradox (Moh Shaw-Kwei 1954, 37–40; Geach 1955, 71-72; Prior 1955, 177-182; Priest 2008, 122-124; Shapiro & Beall 2018, §4.2).

A	B	$A \rightarrow B$
True	True	True
True	Paradoxical	Paradoxical
True	False	False
Paradoxical	True	True
Paradoxical	Paradoxical	True
Paradoxical	False	Paradoxical
False	True	True
False	Paradoxical	True
False	False	True

Truth-table of the <material> conditional in $\mathbb{L}3$

(Priest 2008, 122-124)

For Curry's Paradox, in a classical logic with only two truth-values (viz. *only true* and *only false*), the characteristic of classical logic that makes any implication that has a false antecedent or a true consequent true gives us two options. Either the antecedent is false then by *ex falso sequitur quodlibet* (namely, the principle that from a falsity everything follows) the consequent is true (as every sentence trivially is given *ex falso*), *I am a*

dinosaur; or the antecedent *this sentence is true* is true and *I am indeed a dinosaur*, by *Modus ponens* (Routley, Plumwood, Meyer & Brady 1982, 1-69). Curry's Paradox, through a 3-valued logic, can give to the antecedent a value that is neither *true* nor *false* but *paradoxical*, and by this way steps out of triviality. More generally, denying the Law of Absorption, not only with a 3-valued logic but Paraconsistent logics for instance, allows to counter the paradox (Geach 1955, 71-72; Prior 1955, 177-182; Moh Shaw-Kwei 1954, 37-40; Priest 2008, 122-124; Shapiro & Beall 2018, §4.2).

As mentioned, another theory that does not accept Contraction because it cannot be applied well in paradoxical situations, are paraconsistent solutions such as the one explained by Routley, Plumwood, Meyer & Brady 1982 (278-283). In it, they explain how DS (Disjunctive Syllogism: $A \& (A \supset B) \rightarrow B$), Ass (Assertion: $A \& (A \rightarrow B) \rightarrow B$) and CP or W (Contraction Principle or Weakening Principle: $A \rightarrow (A \rightarrow B) \vdash A \rightarrow B$) are all linked and play an important role in paradoxes. As can be seen in the table on page 20 of this work, which shows which rules and laws appear in the different forms of the Curry's Paradox I have described, only the V3 does not use the Contraction, neither in the form of a law or as a rule. Furthermore, that is the only Curry's Paradox's formulation that uses *Modus ponens* (MP) as a rule and as a law. This sustains the claim of Routley, Plumwood, Meyer and Brady that the Principle of Contraction and *Modus ponens* imply each other. Moreover, they argue that the Disjunctive Syllogism is equivalent to the *Modus ponens* in terms of law, but not in terms of inference. According to them, since *Modus ponens* as a law and Disjunctive Syllogism are not accepted in paraconsistent logics, there is reason not to accept Contraction Principle as relevant. If DS and MP as laws fail, so does CP (Routley, Plumwood, Meyer & Brady 1982, 278-283; Priest 2008, 122-124).

The last Curry-Completeness Responses that I will expose are the detachment-free responses that reject a certain derivation of *Modus ponens*. This form of *Modus ponens* is founded thanks to the Principle of Transitivity (Trans: $(A \vdash B \& \vdash A) \rightarrow \vdash B$) and the Converse of Single-Premise Conditional Proof (CCP: $(\vdash A \rightarrow B) \rightarrow (A \vdash B)$). Supporters of this theory must then reject one of those two principles. Therefore, there are two types

of detachment-free supporters, based on the principle they reject. By rejecting one of these two principles as the *Modus ponens*, they counteract the trivial conclusion of Curry's Paradox. On one side, strongly detachment-free response rejects single-premise conditional proof and on the other side, weakly detachment-free response denies transitivity. The second one is the weaker version of the response because they accept CCP which is a sort of Detachment Principle and therefore those supporters are not totally free of any concept of detachment (Goodship 1996, 153-161; Ripley 2013, 139-164; Beall 2015, 410-423; Shapiro & Beall 2018, §4)

b) Classical logic: Curry-Incompleteness responses

Some other solutions don't question the principles used in Curry's inferences and usually accept classical logic. This alternative, the Curry-Incompleteness responses, question the implication of Curry's Paradox; they don't believe that the paradox implies triviality for theories that present it. Some examples of it are The Revision Theory of Truth (Gupta & Belnap 1993), or The Contextualist Approaches (Burge 1979, 169-198; Simmons 1993; Glanzberg 2001, 217-251, 2004, 27-28; Shapiro & Beall 2018, §4).

Those theories don't only propose a response to Curry's Paradox but to a lot of self-referential paradoxes as the Liar and set-theoretical paradoxes (like Russell's, Burali-Forti's and others). Therefore, touching on larger solutions, I won't present them in this introduction, preferring to concentrate my research on issues specifically related to Curry's Paradox.

7. Why is this Paradox problematic?

a) Paradoxes of self-reference

We can compare Curry's Paradox with other self-referential paradoxes by interpreting it through them. Indeed, Curry's Paradox can take the shape of a set paradox but also of a semantic paradox. First, I will expose a version of the paradox that looks like Russell's Paradox. Whilst on the other hand, I will explore a version that takes the shape of the Liar and Grelling's Paradox.

Russell's Paradox occurs in a naive set theory. The idea of this paradox is to consider the set of all sets that do not contain themselves. This set is paradoxical: if it is not part of itself, it is part of itself because it is a set that does not contain itself; and if it is part of itself, it is not part of itself because it is not a set that does not contain itself, and so on. In other words, it would be a member of its own set only if it is not a member of its own set, which is a contradiction and therefore paradoxical. This set can be formulated as follows:

$$R \{x : x \notin x\}$$

where R is Russell's set and $\{x : x \notin x\}$ is the statement that it does not include himself. It becomes paradoxical because: $R \in R$ and $R \notin R$ (Irvine & Deutsch 2020, §0).

In a comparison between Curry's Paradox and other self-referential paradoxes it is possible to 'blend' Curry's Paradox into Russell's Paradox, to give them the same formal shape. It interprets Curry's sentence as a set that is part of itself only if, to keep me example, I am a dinosaur. In this context, what makes this embedding of paradoxes possible is that it is possible to interchange the set that is part of itself and the set that is part of itself *only if I am a dinosaur* (Moh 1954, 37-40; Prior 1955; Shapiro & Beall 2018, §2).

In this conception, Curry's set is $R \in R \rightarrow \pi$ being interchangeable with $R \in R$, just as $R \in R$ is interchangeable with $R \notin R$ in Russell's set. We then obtain a set-theoretic Curry's sentence, where we have the set:

$$c = \{x : x \in x \rightarrow \pi\}$$

In this formula, c is the set-theoretic Curry sentence which is *a set that is a member of itself only if I am a dinosaur* (Moh 1954, 37-40; Prior 1955, 177-182, Shapiro & Beall 2018, §2).

As mentioned, Curry's Paradox is problematic in the three kinds of theories presented: theories of truth, set and properties. The set-theoretic version of Curry's Paradox just presented is discussed in numerous works such as Routley, Plumwood, Meyer & Brady 1982, (280, 289-290), Slaney 1989 (472-479) and Pleitz 2015 (5). Forced to make choices for this work format, I devote more space in this work to the semantic version of the paradox and it is this formulation that appears most often in this work. I will next present two of the paradoxes that appear in semantic theories, and which can be linked to Curry's Paradox.

The second way to interpret Curry's Paradox through other self-referential paradoxes is to compare it to the Liar Paradox. The Liar Paradox, also often described more as a family of paradoxes, consists in saying that what you are saying is false. It can be for example the sentence 'what I am now saying is false' or 'this sentence is false'. 'Here we have a sentence [...] that is supposed to say of it-self that it is false' (Sainsbury 1995, 112). This sentence is inherently paradoxical because whether it is true or false, it implies the opposite. Indeed, if *this sentence is false* is false, it is then false that it is false, it must then be true. But if this sentence is true, it is true that *this sentence is false*, therefore this sentence is indeed false, and so on (Geach 1955, 71-72; Löb 1955, 115-118; Sainsbury 1995, 111-112; Beall, Glanzberg & Ripley 2016, §0).

A way to interpret Curry's Paradox in the theories of truth is to look at it through the Liar. This time, not by saying that a sentence is false, but by making this sentence implies any chosen statement. This is how I've expressed Curry's Paradox in most of this work: *If this sentence is true, then I am a dinosaur*. This time, the question is not to know whether Curry's sentence is part of a set, but whether it is true. Indeed, I can formalise The Liar Paradox in the following way:

$$A \dashv\vdash \neg T\langle A \rangle$$

Then, if I keep this form but transform the negation $\neg T\langle A \rangle$ by the implication of Curry's Paradox $T\langle A \rangle \rightarrow B$, I obtain:

$$A \dashv\vdash T\langle A \rangle \rightarrow B$$

A way to express Curry's Paradox as a semantic paradox that says *if this sentence (A) is true then B* (Geach 1955, 71-72; Löb 1955, 115-118; Shapiro & Beall 2018, §2; Beall, Glanzberg & Ripley 2016, §2).

Grelling's Paradox, another semantic paradox, can also be compared to Curry's Paradox and thereby can illustrate the presence of Curry's Paradox in theories of properties. Grelling's Paradox considers the propriety to *fail to exemplify the property it stands for* (that is, the property of *heteronymy*). In order to do so, it uses the *heterologicality*, this concept is a property that a word has when it does not have the property it describes. An example of this kind of word would be *monosyllabicity* because, by having more than one syllable, it does not have the property it describes. What becomes paradoxical is when we ask ourselves whether *heterologicality* fails or not to exemplify his property, whether it is *heterological* or not. If it is indeed *heterological*, it does exemplify his property and therefore does not fail to exemplify and is not *heterological*. But if it is indeed not *heterological*, it does fail to exemplify his property which makes it *heterological*, and so on (Newhard 2005, 1-3).

What makes it possible to compare Grelling's Paradox and Curry's Paradox is to interpret Curry's Paradox as observing the propriety of a sentence to 'exemplify the property it stands for *only if I am a dinosaur*'. We can give a name, for example *u*, for this property. Curry's Paradox appears then in the theory of property by being a 'sentence that says that the name *u* exemplifies the property it stands for'. Just as with Grelling's Paradox we analysed the property of exemplify itself, with Curry's Paradox we analysed the property of exemplify itself, but this time only if it implies any chosen claim. Curry's Paradox can thereby take the form of Grelling's Paradox slightly specified. This no longer leads, as with Grelling's Paradox, to a contradiction where a paradoxical object both exemplifies and does not exemplify itself, but to triviality with a paradoxical object that implies any assertion (Geach 1955, 71-72; Löb 1955, 115-118; Shapiro & Beall 2018, §2)

Although that, Curry's Paradox is essentially different from other self-referential paradoxes. Indeed, at first sight it does not involve negation. This makes some solutions to other self-referential paradoxes irrelevant for Curry's one and causes a lot of theory to question their suitability. I will discuss this subject in the next point (Priest 2002, 168-169; Shapiro & Beall 2018, §5).

We could analyse if the presence of negation is a real difference or if it is possible to find a similar general structure for those different paradoxes. This is what Prior explains in his article 'Curry's Paradox and 3-valued Logic' (Prior 1955, 177-182).

He first explains how Curry's Paradox is a sort of ontological proof of the existence of God. Indeed, if this proof is understood as the set-theory, by determining an element, the god-bearer, that, if he is a member of itself, implies an arbitrary truth: the existence of good, then we find ourselves with the same structure as Curry's Paradox. This structure is similar to the comparison between Russell's Paradox and Curry's Paradox that I have explained a few paragraphs earlier where Curry's sentence is a set that is part of itself only if I am a dinosaur. Only this time, Curry's sentence is 'if the class of God-bearers is a member of the class of God-bearers, then if the class of God-bearers is a member of the class of God-bearers, God exists' (Prior 1955, 177) which as the structure 'if A, then if A

then B' (Prior 1955, 177) just as Curry's Paradox. The same principles and steps as for Curry's Paradox can be applied to this sentence. By the Law of Absorption, the sentence leads to 'if the class of God-bearers is a member of the class of God-bearers, then God exists' (Prior 1955, 178). This cumulates to the sentence 'the class of God-bearers is a member of the class of God-bearers' (Prior 1955, 178), results by *Modus ponens* to the affirmation that God exists. We can therefore build an ontological proof of the existence of God with Curry's Paradox. This connection between the two paradoxes, through a well-known literary argument, allows him to introduce a broader connection between the different paradoxes (Prior 1955, 177-182).

More generally, by comparing the methods of Russell's and Curry's Paradoxes, Prior 1955 underlines a common structure, where negation and conditional are interchangeable. This structure can be express through the following formulation, where the symbol $\odot$ is a type of unary connective. This symbol $\odot$ can either stands for the negation so that the formula expresses Russell's Paradox, or stands for the conditional so that it expresses Curry's Paradox (Prior 1955, 177-182; Shapiro & Beall 2018, §5)

- | | | |
|-----|--------------------|---|
| (1) | $A \vdash A$ | Identity |
| (2) | $A \vdash \odot A$ | 1, Curry intersubstitutivity or Russell's set implication |
| (3) | $\vdash \odot A$ | 2, P2 |
| (4) | $\vdash A$ | 3, Curry intersubstitutivity or Russell's set implication |
| (5) | $\vdash B$ | 3, 4, P1 |

Where:

(P1) if $\vdash A$ and $\vdash \odot A$, then $\vdash B$

Which is the Principle of Explosion (if $\vdash A$ and $\vdash \neg A$, then $\vdash B$) for Russell's Schema and *Modus ponens* (if $\vdash A$ and $\vdash (A \rightarrow B)$, then $\vdash B$) for Curry's Paradox.

(P2) if $A \vdash \odot A$, then $\vdash \odot A$

Which is the Principle of Reduction (if $A \vdash \neg A$ then $\neg A$) for Russell's Schema and Principle of Contraction (if $A \vdash (A \rightarrow B)$ then $\vdash (A \rightarrow B)$) for Curry's Paradox.

Indeed, in the case of Curry's Paradox, $\odot$ corresponds to $A \rightarrow B$ and by the key-idea of the paradox, A can be interchanged by $A \rightarrow B$. (P1) corresponds then to the *Modus ponens* for which if $\vdash A$ and $\vdash (A \rightarrow B)$, then $\vdash B$. (P2) corresponds to the Principle of Contraction which states that if $A \vdash (A \rightarrow B)$ then $\vdash (A \rightarrow B)$. Indeed, it is the Principle of Contraction and *Modus ponens*, combined with the very principle of Curry's sentence as expressed by Curry intersubstitutivity, that allow us to conclude any chosen statement. The specificity in this Curry's version of Prior's structure is that there is no negation, that neither the unary connective $\odot$ nor the rest of the formula has any negation at all (Prior 1955, 177-182; Shapiro & Beall 2018, §5).

Alternatively, if we take Russell's Paradox, $\odot$ equals $\neg A$, (P1) corresponds to the Principle of Explosion for which if $\neg A$ and A , then B whatever B is. (P2) corresponds to the Principle of Reduction according to which if $A \vdash \neg A$, then $\neg A$ (Shapiro & Beall 2018 §5; Prior 1955, 177-182). The Explosion Principle and the Reduction Principle demonstrate the paradoxical aspect of Russell's set. Because this set is and is not included in the set of all sets that do not contain themselves, and because of the Principle of Explosion, we obtain B , which is any chosen claim and thereby, triviality (Prior 1955, 177-182; Shapiro & Beall 2018, §5).

In this way, Prior's structure with the meaning of (P1) and (P2) applied to each of the two paradoxes gives the following proofs.

	Curry's Paradox		Russell's Paradox	
(1)	$A \vdash A$	Identity	$A \vdash A$	Identity
(2)	$A \vdash A \rightarrow B$	1, Curry intersubstitutivity	$A \vdash \neg A$	1, Russell's set implication
(3)	$\vdash A \rightarrow B$	2, Principle of Contraction	$\vdash \neg A$	2, Principle of Reduction
(4)	$\vdash A$	3, Curry intersubstitutivity	$\vdash A$	3, Russell's set implication
(5)	$\vdash B$	3, 4, <i>Modus ponens</i>	$\vdash B$	3, 4, Principle of Explosion

This structure shows that Curry's Paradox has a different characteristic from other self-referential paradoxes. It is a paradox of implication, whereas the other paradoxes are paradoxes of negation. But this difference doesn't prevent it from having the same structure as the other self-referential paradoxes, so this characteristic of implication is perhaps not such an essential element in the division of paradoxes (Prior 1955, 177-182; Shapiro & Beall 2018, §5).

In Prior's way of grouping paradoxes, we can see that negation is only one element of some of the paradoxes that fit into the Schema. The fact that Curry's Paradox has another $\odot$ which does not use negation is not what makes it a sort of paradoxes, it is just the element that makes it another paradox than Russell's, for example. The way Prior summarises paradoxes shows that it is not the presence of negation rather than a trivial implication that is a decisive element in sorting paradoxes, it is their structure. Only in Curry's Paradox is negation absent; all other self-referential paradoxes use negation to achieve contradiction and thus have a paradoxical aspect. Only, in this way of presenting

paradoxes, Prior shows us that the way to achieve paradoxicality is only a variant, and that structure is the immutable element (Prior 1955, 177-182; Shapiro & Beall 2018, §5).

By this schema, Prior offers a structure that underlines the difference between both paradoxes, and that Curry's one does not depend on negation. Thereby he shows why non-classical logics that question this negation, although solving Russell's Paradox, cannot solve Curry's one (Prior 1955, 177-182; Shapiro & Beall 2018, §5).

Indeed, Curry's Paradox cannot be solved as for naive truth theory or naive set theory that uses another sort of negation. 'In short, the point is that there are non-classical logics with weak negation principles that resolve Russell's Paradox and the Liar, yet remain vulnerable to Curry's paradox' (Shapiro & Beall 2018, §5) (Geach 1955, 71-72; Shapiro & Beall 2018, §5).

There are different kinds of those theories that aren't trivial regarding paradoxes where there is negation, but trivial regarding Curry's Paradox. An example could be paraconsistent logics, the logics that don't accept the Principle of Explosion. One of the theories that respects a paraconsistent logic is the metaphysical point of view of dialetheist. Dialetheism accepts some contradictions as true. This theory can be used to solve the Liar or Russell's Paradox by accepting their contradictions. Indeed, by accepting that A and $\neg A$ can be true and not accepting the Explosion Principle, dialetheism avoids triviality. But it does not resist the triviality involved by Curry's Paradox which doesn't use contradiction to demonstrate triviality, but the implication of any chosen claim (Slaney 1989, 472-479; Goodship 1996, 153-161; Priest 2008, 122-124; Shapiro & Beall 2018, §5; Priest, Tanaka & Weber 2022, §0)

Thus, Curry's Paradox has a very similar structure to other paradoxes but is based on other principles, which makes it unsolvable by some other paradoxes' solutions (Prior 1955, 177-182; Shapiro & Beall 2018, §5).

b) Triviality in our theories

Curry's Paradox asks the question of triviality in the system. As mentioned before, triviality is a state where all declarations, and so their negation too, are true. The stakes of not having a trivial theory are enormous because having a logic which in everything is true prevents you from knowing what is true and what is not. That would keep theories from being used in our inquiries like mathematic and from 'modelling features of language or thought' (Shapiro & Beall 2018, §1.2) (Slaney 1989, 472-479; Priest 1998, 171-172; Shapiro & Beall 2018, §1.2).

With Curry's Paradox, we can advance that a system 'affirms every claim that is expressible in the language of the theory' (Shapiro & Beall 2018, §1.2), as *I am a dinosaur*, and therefore is trivial. Thus, Curry's sentence makes a system absolutely inconsistent (in classical logic) and trivial (in classical logic and non-classical logic) by proving that every sentence of a language is true and so, makes it very problematic to use. Therefore, this paradox questions the possibility of a non-trivial logic system (Slaney 1989, 472-479; Priest 1998, 171-172; Shapiro & Beall 2018, §1.2).

So, the paradox is a big problem because with our more intuitive logics: theory of truth, set theory, property theory and others we obtain triviality. That means that either our intuitive theory or our logic is problematic. However, the fact that we recognize Curry's Paradox in different theories is already giving us an argument in favour of questioning our logic. The logics that I have presented, that solve other self-referential paradoxes like paraconsistent logics and paraconsistent logics, are weakened by Curry's Paradox and by its persistence independent of the theory used. Curry's Paradox threatens our logics with triviality.

There has been a lot of attention given to Curry's Paradox and the validity that it questions over the last ten years. One of the points of attention is that some logics, that resolve most of the self-referential paradoxes, don't resist the non-trivial test which is at the core of Curry's Paradox suggestion. We can mention logics that resolve Russell's

Paradox and the Liar, like theories that modify classical negation principles, which are put in peril by Curry's Paradox. For example, some of paracomplete logics that reject the law of Excluded Middle and a lot of paraconsistent logics - which refuse that a sentence and its negation implies any chosen claim (i.e., that reject the *ex contradictione sequitur quodlibet*) - although resolving others self-referential paradoxes, don't resist Curry's one. Curry's Paradox really is philosophically alive because it questions a lot of theories that we thought had solved self-referential paradoxes and therefore Curry's Paradox plays 'a significant role in the development of non-classical logics' (Shapiro & Beall 2018, §5) (Slaney 1989, 472-479; Priest 1998, 171-172; Shapiro & Beall 2018, §5)

8. Negation

One of the complex points of this work is the concept of negation, from which emerges a certain ambiguity as to whether Curry's Paradox uses negation.

The question is whether the implication of any chosen claim can be considered as a negation or not. Indeed, what differentiates Curry's Paradox from other self-referential paradoxes is that Curry's Paradox uses implication to be paradoxical, whereas other paradoxes use negation. But if involving any element is a kind of negation, then Curry's Paradox is much closer to the other paradoxes than one might think at first sight (Pleitz 2015, 6; Ahmad 2022, 256-257)

Indeed, Pleitz 2015, to which I will return later, mentions *negation equivalence*, which would express negation as follows, where $\perp$ is an absurd sentence (Pleitz 2015, 6):

$$\neg A \leftrightarrow A \rightarrow \perp$$

In Ahmad 2022, to which I will also return later, the *negation-y* aspect of a paradox is defined by the following formulas (Ahmad 2022, 256-257).

Negative modality defined by Greg Restall

$$(A \vdash B) \rightarrow (mB \vdash mA)$$

De Morgan's Law

$$\begin{aligned} (A \vee B) \rightarrow \varphi &\dashv\vdash (A \rightarrow \varphi) \wedge (B \rightarrow \varphi) \\ (A \wedge B) \rightarrow \varphi &\dashv\vdash (A \rightarrow \varphi) \vee (B \rightarrow \varphi) \end{aligned}$$

The Triple Negation

$$((A \rightarrow \varphi) \rightarrow \varphi) \rightarrow \varphi \dashv\vdash A \rightarrow \varphi$$

The Law of Excluded Middle

$$\vdash A \vee (A \rightarrow \varphi)$$

And partially the Double Negation

$$A \vdash (A \rightarrow \varphi) \rightarrow \varphi$$

Restall 1999 suggests a comparison between classical definitions of negation, such as the dubbed Boolean ($x \models \neg A$ iff $x \not\models A$) and asks whether this connective is applicable to relevant logics. Another article worth mentioning is Sylvan 1999, in which the author explains how the orthodox conception of negation could not be more wrong. Indeed, the classical definition of negation makes it possible to use contradiction, i.e. an element and its negation, as a basis for any chosen claim. In other words, it uses *ex contradictione sequitur quodlibet*. All these references allow me to illustrate that negation is not an

operator as determined as we think. It depends on logics and points of view. The definition of negation carries within it one of the key points of differentiation between Curry's Paradox and the other self-referential paradoxes (Prior 1955, 177-182; Restall 1999, 53-76; Sylvan 1999, 299-324; Gabbay & Wansing 1999; Ahmad 2022, 256-257).

9. C-Curry vs V-Curry's Paradox

A variant of Curry's Paradox is the *Validity Curry* also called *V-Curry*. As opposed to C-Curry's Paradox which is the conditional form that was explained in more detail before. The C-Curry's Paradox uses $\rightarrow$ which means that antecedent entails consequence. This $\rightarrow$ providing *Modus ponens* and Contraction, we obtain Curry's Paradox in our theories, and logicians came with the contraction-free and detachment-free responses (Beall & Murzi 2013; Shapiro & Beall 2018, paragraph 6). The other version of Curry's Paradox, the *V-Curry's Paradox*, poses a problem because C-Curry's solutions do not necessarily resolve it. *V-Curry's Paradox* does not use conditional but validity. A way to express it could be ' $\pi \leftrightarrow \text{Val} (\ulcorner \pi \urcorner, \ulcorner \perp \urcorner)$ ' (Beall & Murzi 2013, 1-10). However, I will not say more about this variant of Curry's Paradox in this work, as the complexity of this explanation would require a lot of time and would be too far from our subject. For more details on the difference and the stakes of this variant, I refer the reader to Beall & Murzi 2013.

10. Revenge paradox

Another point raised by the paradox is *Curry's Revenge*. By solving the paradox with non-classical solutions, we therefore reject one of the principles that appears in Curry's Paradox and we obtain a coherent theory. But this process comes at a serious cost that will compel the possibility of those coherent theories and the metatheories. 'Non-classical solutions are not the 'easy way out' of the paradoxes' (Restall 2007, 273-233). Once again, this aspect is outside the scope of this work, so we won't elaborate on it any further. For more details and an analysis of this issue, I refer the reader to Restall 2007, 273-233.

11. What is a paradox and how to classify them?

To establish a common ground, a definition of the concept Paradox and its classifications is needed. This will allow me to get to the heart of my problem and to understand the different statements concerning Curry's Paradox and the Inclosure Schema. To do so I will also need to explain the Principle of Uniform Solution which is one of the main arguments of Priest but also against him (Ahmad 2022, 254).

a) What is a paradox?

Paradoxology is the comparison of paradoxes and the attempt to categorise them. Those are large subjects, and our goal in this work is not to explain all of it, but to analyse the branch that meets Curry's Paradox and the Inclosure. In a first time, in this section I will only give a large definition of paradox that allows us to go deeper and more precisely into the comprehension of the paradox that interest us. I will then present the way logicians try to sort them in next chapters.

The common popular notion of paradox describes something that seems absurd at first glance, but which is true upon further examination. On the other side, when we look at it from a philosophical point of view, a paradox is something even less true. Paradoxes can

be seen as something that creates a *conflict of reasons* inside us, that causes us to have a good reason to think that something is true and false at the same time. So, there is a correct use of reasons and yet it leads to a statement that we should have to reject: contradiction. In a paradox there is first a central element that is an argument composed of premises and a reasoning ensuing. This argument seems to not have any default, but its conclusion appears to be false. Therefore 'a paradox is an argument in which there appears to be correct reasoning from true premises to a false conclusion', or to different conclusions that are contradictory (Olin 2003, 5-7).

According to Russell, the common ground between paradoxes is self-reference or reflexivity. His paradox is a perfect example of it: it is a set that must be a member of itself which at the same time it cannot. This aspect is indeed a common denominator of paradoxes, but not only of paradoxes. It is somehow too broad because it also includes, if we keep the set example in mind, a set that is a member of itself even if it does not lead to contradiction. This definition also considers non-paradoxical assertions (Russell 1908, 222-262; Pleitz 2015, 2).

In the next chapter, I will present the schema that Russell has developed, but above all, another schema which is inspired from it and that tries to capture paradox and only paradox. This schema is the Inclosure Schema, which attempts to know what generated paradoxes.

b) Ramsey's two families

Russell thinks that all paradoxes are of the same family: the family of paradoxes that use self-references. This contradicts with a well-known way to classify Paradox which separates them between Ramsey's two families. Inspired by Peano's difference between paradoxes that belong to mathematics and the ones that belong to linguistics, Ramsey's idea is that paradox can be split in two distinct groups, namely group A and B (Russell 1908, 222-262; Priest 2002, 141-142; Pleitz 2015, 2).

Group A includes paradoxes that occur in mathematical or logical systems, they use logical or mathematical notions such as sets, numbers or infinite. Some examples of these would be Russell's Paradox (the set of all sets that do not contain themselves) or Burali-Forti's Paradox (the set of all ordinal numbers). The second group of Ramsey, group B, includes the Liar Paradox (the sentence: 'this sentence is false'), Berry's Paradox (The least integer that is not nameable in fewer than nineteen syllables) or even Grelling's paradox (Is the term 'heterological' heterological?). These paradoxes cannot be expressed with logic and mathematics alone as they need symbolic or empirical terms to be expressed through language (Ramsey 1925, 171; Priest 2002, 141-142).

This chapter is characterised by a brief overview of all the different paradoxes treated in this paper. A more detailed explanation, including a presentation of the Inclosure Schema and how paradoxes fit in it, will allow me to explain them more deeply and clearly in the following chapter.

This way of distinguishing paradoxes has been criticised, notably by Graham Priest, but it still has a huge influence on the way paradoxes are treated nowadays. Today, we call 'set-theoretic paradox' the paradoxes of group A that are related to mathematics and logic, and 'semantic paradox' those more linked to the language of group B. For Priest, the differentiation Ramsey makes is not a structural but a superficial one and solely dependent on the vocabulary. It is not a fundamental distinction and, according to Priest, Ramsey was not in a period of great mathematical advancement where he could see that paradoxes of group B do also depend on mathematics. This distinction is problematic because it leaves some paradoxes outside of the narrow-minded categorisation, such as Berkeley's paradox, even though they use the same self-referential structure. It seems like an arbitrary difference; indeed, the criteria of differentiation could just as well have been whether the paradoxes explicitly use the *notion of definability*. Priest claims that those two kinds of paradoxes are of the same kind, namely the self-referential paradox and have a similar structure. 'All the paradoxes in this group depend on a diagonaliser being applied to a limit totality' (Priest 2002, 141). I will discuss this structure, the Inclosure Schema, in the next chapter (Ramsey 1925, 171; Priest 2002, 141-142).

c) Priest and the Principle of Uniform Solution

The main argument for the Inclosure Schema that I will explain in a few lines is the Principle of Uniform Solution. This principle asserts that it is wise to use the same kind of solution for the same kind of problem. Contrarily, it makes sense to use different kinds of solutions for different kinds of problems, the problems in this case being paradoxes. This principle leads Priest to accept that paradoxes of the same families should have the same kind of solution, and a kind of solution that only solves some paradoxes of a family is not appropriate (Priest 2002, 166).

In this conception, the understanding of the concept of kind is very important. Priest defines two paradoxes as of the same kind if they meet the conditions: '(a) that there is a structure that is common to the paradoxes; and (b) that this structure is responsible for the contradictions' (Priest 2002, 166). This reading grid applied to solutions of set-theoretic paradoxes is problematic considering that semantic paradoxes are of the same kind and therefore require the same solution. The same applies for solutions to semantic paradoxes that do not solve set-theoretic paradoxes. To be precise, the problem with this reasoning is not that it does not respect the Principle of Uniform Solution, but that it has another way of determining paradoxes of the same kind (Priest 2002, 166-167). We will see how the Inclosure Schema seems to perfectly respect this principle, until the intervention of Curry's Paradox.

III. The Inclosure Schema

12. Russell's Schema

We spoke earlier about a schema that Russell made to have a better understanding of paradoxes. This schema is an observation of the structure of self-referential paradoxes. Russell's Schema inspired the Inclosure Schema, I am going to present it to make it easier to understand the Inclosure Schema later. This will then allow me to get a closer look at what is the Inclosure Schema and how self-referential paradoxes fit in (Russell 1905, 34-44, 1906, 635).

Bertrand Russell took a keen interest in the paradoxes that occur in set-theories. He found a formula that, by explaining the common point between some self-referential paradoxes, allows us to create as many paradoxes as we want (Priest 2002, 129).

This schema can be formulated as:

‘[...] given a property φ and function δ , consider the following conditions:

- (1) $\Omega = \{y; \varphi(y)\}$ exists
- (2) if x is a subset of Q : (a) $\delta(x) \notin x$
and (b) $\delta(x) \in \Omega$

(Priest 2002, 129)

This formula leads to the contradiction found in paradoxes. We obtain that $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$.

The first condition (1) entails the *existence*, or as Russell puts it '[...] then the supposition there is a class Ω of all terms having property φ and that $\delta(\Omega)$ exists [...]' (Russell 1905, 142). The second condition (2) is *transcendence* (2a) and *closure* (2b) or as express by Russell: '[...] leads to the conclusion that $\delta(\Omega)$ both has and has not the property φ .' (Russell 1905, 142) (Russell 1905, 142; Priest 2002, 129-130).

The idea behind the Inclosure Schema, and the three concepts that make it up, is that there is a single structure behind all self-referential paradoxes. This structure first establishes that a paradox must exist, which is the condition of *existence*. Then, it must be part of a certain entity, in which case it satisfies the *closure*. Finally, the Inclosure Schema describes that this paradox is also excluded from this entity and thereby has the *transcendence* (Russell 1905, 142; Priest 2002, 129-130).

13. What is the Inclosure Schema?

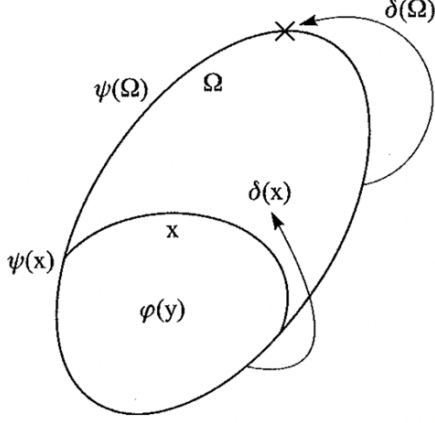
To have a schema that does not only include set-theoretic paradoxes, but more generally all self-referential paradoxes, Priest has slightly modified Russell's Schema. This broader version is the Inclosure Schema. The only difference is that 'We now require two properties φ and ψ [...]' for the rest it is quite similar to Russell's Schema: '[...] and a function δ , satisfying the following conditions:

- (1) $\Omega = \{y; \varphi(y)\}$ exists, and $\psi(\Omega)$
- (2) if x is a subset of Ω such that $\psi(x)$:
 - (a) $\delta(x) \notin x$
 - (b) $\delta(x) \in \Omega$

(Priest 2002, 134)

Priest calls the Ω an *inclosure*, the condition that Ω satisfies the *Inclosure Schema* and the paradoxes that have this framework are defined as an *inclosure contradiction* (Priest 2002, 134).

He also provides a more visual version of the Inclosure Schema, which takes this form:



(Priest 2002, 156)

Just like Russell's Schema, there is the contradiction that $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$ (Priest 2002, 134). Just like Russell's Schema, we have three elements: existence, transcendence and closure, but they don't have the same expression.

$$\text{(Existence)} \quad \exists \Omega (\Omega = \{x \mid \varphi(x)\} \wedge \psi(\Omega))$$

$$\text{(Transcendence)} \quad \forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow \delta(x) \notin x)$$

$$\text{(Closure)} \quad \forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow \delta(x) \in \Omega)$$

(Pleitz 2015, 3)

What differentiates the Inclosure Schema from Russell's Schema is that 'the diagonaliser is guaranteed to function only on a sub-family of the power set of Ω , viz., those sets that satisfy ψ ' (Priest 2002, 134). The diagonaliser comes from the diagonalization method that enabled mathematician David Hilbert to prove the difference between the infinity of real numbers and that of natural numbers. It was used to show that it is not possible to count all natural numbers. This method of argumentation was then extensively used in mathematical and logical proofs, notably to show that an element

is not in a set. Here, in the Inclosure Schema, it is reused through the diagonaliser to show the contradiction (Priest 2002, 130; Cattabriga 2019, 1).

14. How do paradoxes fit?

Now that I have explained how this schema works, we can test its application to different self-referential paradoxes.

A first example could be The Liar Paradox, which is the paradox follows from the sentences that says of itself that it is false, that is, the self-referential sentence ‘this sentence is false’ (Priest 2002, 144-145; Marion 2023, 95-96)

Inclosure Schema	$\delta(x)$	$\psi(y)$	$\varphi(z)$	Ω
Liar	l , i.e., $l \notin x$	y is definable	z is true	Tr

(Priest 2002, 144-145; Pleitz 2015, 4)

To convert the Liar into the Inclosure Schema, we must make Ω the set of all truths (Tr). δ is ‘a function that takes a subset X of Ω to a sentence that says of itself that it is not in X ’ (Ahmad 2022, 255) and $\delta(x)$ is in x . $\delta(x)$ is a Liar sentence, that says of itself that it is not true, like the sentence ‘this sentence is not true’. We then have ψ as the property of being properly defined and φ as the property of being true. We then have a sentence that is and is not in x , since $\delta(x)$ is in x and at the same time says of itself that it is not in x . So, we have closure since $\delta(x)$ is in X and transcendence since $\delta(x)$ states about itself that it is not included in X . $\delta(x)$ both is and is not in Ω . In other words, Ω is the set of all the things that have the property φ , the property of being true. In this Ω , there is a subset x that satisfies ψ by being definable. If we take the function δ and apply it to x which is true, definable and in Ω , we obtain l , i.e., $l \notin x$. This is a liar sentence that says about itself (the function of the subset x) that it is not in x . We obtain the Liar which is both inside and outside the set of all truths, Ω (Priest 2002, 144-145; Beall 2014, 839-840; Ahmad 2022, 254-255).

The same thing can be applied on Russell's Paradox, the paradox that follows from considering the set that contains all sets that do not contain themselves.

Inclosure Schema	$\delta(x)$	$\Psi(y)$	$\varphi(z)$	Ω
Russell	$\{x : x \notin x\}$	$\lambda y y = y$	z is a set	V

(Priest 2002, 130-131; Pleitz 2015, 4)

In this interpretation of Russell's Paradox, the Ω is V which corresponds to the universe of sets. $\varphi(z)$ means that z is a set and that φ is the property of being a set. $\delta(x)$ is Russell's set that contains all sets that do not contain themselves. Indeed $\delta(x)$ is $\{x : x \notin x\}$, so that δ is a function which leads to a x that is in the subset $\{x : x \notin x\}$ only if he does not contain himself ($x \notin x$). This is problematic since x is in Ω and thus is a set. That leads $\delta(x)$ to be Russell's set, and the resulting contradiction. We then have a set x which, by applying the function δ , is and is not part of itself. We have closure: $x \in x$, we have a set that belongs to sets that do not contain themselves. Resulting, we get transcendence because this set is a set that contains himself and therefore does not contain himself since he only contains sets that do not contain themselves: $x \notin x$. This set is both in and not in Ω (Priest 2002, 130-131; Pleitz 2015, 4).

Thereby we have one of each kind of Ramsey's families, a semantic and a set paradox, namely, the Liar Paradox and Russell's Paradox. We can then apply the Inclosure Schema to a less well-known paradox (than the Liar and Russell's Paradox): Berry's Paradox. This paradox results from the statement 'the smaller integer not nameable in less than 99 words'.

Inclosure Schema	$\delta(x)$	$\Psi(y)$	$\varphi(z)$	Ω
Berry	$\mu w w \notin x$	$y \in DN_{99}$	$z \in DN_{99}$	DN_{99}

(Priest 2002, 134; Pleitz 2015, 4)

In this representation of the paradox Ω is the set of all numbers that we can define in less than 99 words. So, x being a subset of Ω , it is also definable in 99 words or less. The subset $\delta(x)$ is a *Berry Description* : ‘not definable in less than 99 words’ formulated as $\mu w w \notin x$. In this formulation, μw means ‘the smallest number w such that’. So, $\mu w w \notin x$ can be read as ‘the smallest number w such that w is not a number definable in less than 99 words’. But x being a subset of Ω , it is definable in less than 99 words and so is $\delta(x)$. The contradiction appears because we have a $\delta(x) \notin x$ and $\delta(x) \in \Omega$. Thereby we have transcendence because $\delta(x) \notin x$. Indeed, $\delta(x)$ by its definition $\mu w w \notin x$ which means ‘the least number w such that w is not definable in less than 99 words’ is not in the set of all numbers that we can define in less than 99 words. Conversely, we have closure by $\delta(x) \in \Omega$, because something not definable in less than 99 words is part of ‘the set of numbers definable in 99 words’ in Ω , as it is just done (Priest 2002, 130-134; Pleitz 2015, 4).

To finish with a last example from the other family regarding Ramsey's division, I will apply the Inclosure Schema to Burali-Forti's Paradox.

Inclosure Schema	$\delta(x)$	$\psi(y)$	$\varphi(z)$	Ω
Burali-Forti	$\log(x)$	$\lambda y y = y$	z is an ordinal	ON

(Priest 2002, 131; Pleitz 2015, 4)

In this reading, Ω is ON, which means ‘the set that includes all ordinals’. δ , expressed $\log$, means ‘the smaller ordinal bigger than’. $\varphi(z)$ means that z is an ordinal. Since x is a subset of Ω and satisfies φ , it is an ordinal. Therefore, the function δ applied to x , $\delta(x)$, is the smaller ordinal bigger than all ordinals. Because the set of all ordinals cannot be less than itself, the closure holds. We then have the von Neumann convention (that claims that $\Omega < \Omega$ is equivalent to say that Ω is an element of Ω) and by the reasoning of the paradox (‘In modern set-theory R is called the cumulative hierarchy (since every member of the sequence can be shown to be a proper subset of any later member)’ (Priest 2002, 120)), those two elements lead to $\Omega < \Omega$ where transcendence holds. In other words, on one hand we have closure since we have that the smaller ordinal bigger than all ordinals

belongs to the set that includes all ordinals, expressed by $\delta(x) \in \Omega$. And on the other hand, we have transcendence since the paradox combined with von Neumann convention proves that the ordinal is less than itself by $\Omega < \Omega$, and so it proves that $\delta(x) \notin x$. Those two elements lead to the contradiction (Priest 2002, 130-134; Pleitz 2015, 4).

Priest gives us a lot of other examples of paradoxes that fit the Inclosure Schema (Priest 2002, 131). For (almost) every self-referential paradox, we have the corresponding element in the Inclosure Schema. Here I give you some of the Paradoxes and how they fit into the Inclosure Schema. For reasons of space, I will not be able to present them all. For further details see Priest 2002 (131-135).

Inclosure Schema	$\delta(x)$	$\psi(y)$	$\varphi(z)$	Ω
Köinig	$\mu y y \notin x$	y is definable	$z \in DOn$	DOn
Richard	diag(x)	y is definable	$z \in DR$	DR
Berkeley	$\varepsilon y y \notin x$	x is conceivable	τy	C

(Priest 2002, 134)

15. Priest's justification

a) for the Inclosure Schema

A first element of Priest's justification is that Russell's Schema is not large enough. Even though it looks like it can explain the limit we meet when analysing paradoxes, Priest generalises it in order to explain much more paradoxes. That is how he creates the Inclosure Schema. Russell's Schema is actually just a *limit case* of the more general Inclosure Schema. Russell's Schema is one example, but what Priest is after, is a law, a generalisation, a pattern that does not only display the structure of a kind of paradox but explains why it is paradoxical (Priest 2002, 133-136).

But making a schema is not enough to explain why self-referential paradoxes occur. Finding a pattern that appears in self-referential paradoxes does not explain why this contradiction appears. The Inclosure Schema supplies more than this pattern, it provides a reason why a totality is affected by a diagonaliser that makes it 'lever itself out'. For Priest, the Inclosure Schema, in its use of the diagonaliser, shows us how to obtain an object that is both part of a whole and outside it. The Inclosure Schema shows how the diagonaliser brings out the contradiction at the limit. (Priest 2002, 136).

Ramsey's two kinds of self-referential paradoxes is the framework on which today's works are aligned. Priest shows by the Inclosure Schema and the way it can be applied to (almost) any self-referential paradox that they are in fact of the same family, as shown in the tables on how paradoxes fit into the Inclosure Schema on pages 45 to 47. They have the same structure, which is the Inclosure Schema. 'All the paradoxes in this group depend on a diagonaliser being applied to a limit totality' (Priest 2002, 143) and therefore should be considered of the same kind (Priest 2002, 143).

b) From the Inclosure Schema

To show why his solution is the best, Priest gives a presentation and a criticism of different solutions brought to solve the paradoxes.

Ramsey's solutions to solve self-referential paradoxes are guided by his division. Therefore, he has two totally different approaches to solve it, depending on if the paradox is of the group A or B. His solution for group A is quite similar to Russell's one. (Priest 2002, 148)

Russell's solution, unlike Ramsey's solution and the solutions of those who followed, was designed to solve all self-referential paradoxes. His way of solving it is by denying the existence of Ω . He offers a theory of orders in which, 'Since variables range over only one order, it follows that any set can contain things of only one order, and hence that there is no totality of all propositions, sets, ordinals, or any other inclosure.' (Priest 2002, 137). One of the problems with this solution is the impossibility of making certain statements, like for example the Law of Excluded Middle, which states a law about all propositions

and therefore cannot be stated. What's even worse, according to Priest, is that these theories cannot be explained without generalising. This theory quantifies all functions, to say that this cannot be done. By expressing that all propositional functions have a given order, this theory is doing precisely what it is trying to prevent from doing, and thereby refutes itself (Priest 2002, 137).

As mentioned before, Ramsey's way of solving those paradoxes depends on their kind. He gave a solution for set-theoretic paradoxes and another for semantic paradoxes. The way of solving paradoxes of group A is a simplified version of Russell's solution and therefore can be criticised for the same reasons. Paradoxes of group B on the other hand, are solved in a totally different way. Ramsey states that all those paradoxes use some words as 'means' and that making those words unambiguous solves the paradoxes. Some of the criticisms that can be levelled at this solution are that, first, words of this type may not appear in all semantic paradoxes. Secondly, this leads some contentious hypotheses to argue that satisfaction must be analysed in terms of properties Thirdly, that the ambiguity of words similar to 'means' must be proven (Priest 2002, 149-150).

In general, the solution found to solve a self-referential paradox must deny at least one condition of the Inclosure Schema. For example, one modern set theory like Zermelo's solution denies that Ω exists and thereby denies the existence in the Inclosure Schema. In this conception, not every set from any arbitrary condition can exist. 'In general, we can show only the existence of the subset of any given set, comprising objects satisfying that condition.' (Priest 2002, 157). This solution is countered by the Domain Principle according to which we can infer the existence of a domain of variation for each variable, because in this theory we do have those variables which assume the collection of all sets (Hallet 1984, 12-32; Cantor 1886, 9; Priest 2002, 157-158).

To be able to use a solution that questions the existence without being restricted by the Domain Principle, some philosophers came up with different theories. An example is the *Intuitionist ZF* in which infinity is considered as a potential infinity. In this conception the infinite totality is not completed. Unfortunately, this theory doesn't stand up to the Domain Principle either. Indeed, the Domain Principle applies equally to this theory because this way of conceiving infinities presupposes a set of all sets. So, there is indeed a conception of infinity that is not just a potential infinity, a complete one (Priest 2002, 159).

Another theory that tries to avoid the Domain Principle suggests an *Indefinitely extensible totalities* of some concepts. The argument is that, just as for a vague concept like a heap of sand, it is not possible to give a determinate meaning to predicates used in statements about absolute infinities. The predicate 'is an ordinal', for example, is a vague concept, so we cannot give it a determinate value. Priest counter argues that those kinds of concepts like 'is an ordinal' are different from a heap or other vague concepts, because they don't come by degrees. Furthermore, for a concept like 'an absolute whole' to make sense, you must have some idea of it, so it cannot have such a vague meaning (Priest 2002, 161-162).

Another solution presented to solve self-referential paradoxes, but which does not seek to circumvent the Domain Principle, is that of Von Neumann. In this theory, collections are divided into two different categories. On one side are the I-objects, which, in the statement $x \in y$, appear on the left and are the x . On the other, the II-objects are the y , and appear on the right of the statement $x \in y$. The whole point is that II-objects are *proper classes* that are not in the domain of quantification of I-objects which are sets. As a proper class cannot be a member of something, paradoxes are foiled. The difference between the two is that proper classes are so large that they cannot be used in the same way as sets, and this subterfuge thwarts the contradiction. The problem with this theory is that it fails in the exact same way as what it is trying to avoid (Priest 2002, 163).

However, the same problem arises as with Zermelo's theory: the Domain Principle is not respected. This is because von Neumann's theory applies to all collections, not just set or class collections, but to the totality of all collections which is the totality of all classes and sets. Yet this collection cannot exist according to the theory, and so the theory itself, by stating variables that cover all collections, makes itself incoherent (Priest 2002, 165).

So, for Priest, the standard solutions exposed do not find a right way to solve paradoxes. But on top of that, Priest criticises them for not respecting the Inclosure Schema. Indeed, even if those theories were still holding it together, they would still be threatened by the common resolution that all inclosure paradoxes required by the Inclosure Schema combined with the Principle of Uniform Solution. Thus, in addition to being problematic as a theory, for Priest when you take a few steps back, you realise that they only resolve certain paradoxes that are part of a much larger family of paradoxes. Indeed, for Priest, those solutions only try to solve some kind of self-referential paradox, like set-theoretical paradoxes for example. This is troublesome for those theories since, by the Principle of Uniform Solution, a solution that only solves some of the problems in a family of problems does not seem to solve them in any fundamental way. Priest thus criticises the way these theories determine a kind of paradox, even if he explains that those theories have a problem with the Principle of Uniform Solution, they just don't have the same way of categorising paradoxes, and they use the PUS just like him. If we take, for example, Ramsey's way of solving paradoxes, he solves them according to whether they belong to group A or B. Ramsey follows the Principle of Uniform Solution just as Priest. The only difference is that Priest determines the kind of paradox differently. He distinguishes paradoxes such as Curry's Paradox from Inclosure Schema paradoxes while Ramsey distinguishes group A from group B (Priest 2002, 166).

As we saw earlier, this principle is based on the concept of kind, since it states that for the same kind of paradox, we have to provide the same kind of solution. We saw that we can consider paradoxes of the same kind when there is a shared structure among paradoxes

and when this structure is what is causing contradiction. By showing how, but more importantly, by explaining why paradoxes appear through the Inclosure Schema, Priest offers a way to categorise self-referential paradoxes that makes perfect sense with his explanation of kind. Showing, through the Inclosure Schema and the description of kind, that all those paradoxes are of the same kind allows him to claim that to respect the Principle of Uniform Solution, a good solution for any self-referential paradox must be one that solves all self-referential paradoxes. That, rather than, on the one hand, PUS providing a solution for the family A and, on the other hand, another for the family B from Ramsey (Priest 2002, 166).

Thus, the justification of the Inclosure strategy is based on the Principle of Uniform Solution, but what differentiates, it is the 'same kind of paradox' part of this method. Both Ramsey and Priest advocate the Principle of Uniform Solution, but it is the way they determine the kind of paradoxes that sets them apart. Because the Principle of Uniform Solution states that the same kind of problem must have the same kind of solution, it constitutes, combined with the Inclosure Schema which captures the structure of self-referential paradoxes, a perfect argument for solving them all with the same method (Beall 2014a, 840).

Priest also explains how the differentiation between Ramsey's two groups of paradoxes has influenced the solutions that came after him. Almost all of those focused on either group A or group B paradoxes. Yet Russell's vision of a unified solution for all self-referential paradoxes was the right one. For Priest, since Ramsey's group A and group B paradoxes are of the same kind, and by the Principle of Uniform Solution, we must solve them in the same way. It is the same approach as Ramsey's, just with a different way of determining the kind of paradox, namely, for Priest, the Inclosure Schema (Priest 2002, 167).

I will go into more detail on Priest's view of Curry's Paradox and the Inclosure Schema in the next chapter. But this reasoning is similar to his reasoning on Curry's Paradox. Indeed, if the Liar and other self-referential paradoxes and Curry's Paradox are not unified by the Inclosure Schema, then the Liar and other self-referential paradoxes and Curry's Paradox are not of the same type. So, by the Principle of Uniform Solution, the Liar, the other self-referential paradoxes and Curry's Paradox do not necessarily have the same type of solution (Priest 2002, 167).

One way of summarising the difference between Priest's and Rasmey's point of view might be:

- (1) If the paradoxes X and Y are not unified by the paradoxical pattern Z, then X and Y are not of the same family.
- (2) X and Y are not unified by the paradoxical pattern Z.
- (3) X and Y are not of the same family (1, 2, *Modus ponens*).
- (4) If X and Y are not of the same family, then PUS does not require X and Y to be solved in the same way.
- (5) So, X and Y need not be solved in the same way (3, 4, *Modus ponens*)

Where X,Y and Z are :

	X	Y	Z
For Ramsey	Family A	Family B	
For Priest	Liar and other self-referential paradoxes	Curry's Paradox	The Inclosure Schema

Since almost all theories that try to resolve paradoxes limit themselves to one of Ramsey's groups of paradoxes and the solution that they offer for this group is not applicable to the other group, they are threatened by the combination of the Inclosure Schema and the Principle of Uniform Solution (Priest 2002, 167).

Priest uses this line of reasoning to justify a theory that respects this Principle of Uniform Solution and the common structure of the self-referential paradoxes (what differentiates him from Ramsey): the dialetheic approach defends the existence of true contradictions, and in this way is a good metaphysical response to Priest's observation of the limits of thought, which he generalises thanks to the Inclosure Schema. This theory makes acceptable some of the contradictions that the Inclosure Schema captures. Not all dialetheists are defending the Inclosure Schema. Routley, for example, has a different way of defending dialetheism. I will talk about this theory and its different types of supporters in the next point, which addresses the context of the debate (Priest 2002, 169).

IV. Curry's paradox and The Inclosure Schema

16. Context of the debate

As I have explained, Priest uses the Inclosure Schema as a justification for considering all self-referential paradoxes of the same kind. He then uses the Principle of Uniform Solution, combined with this analysis of all paradoxes being of the same family, as an argument for dialetheism (the metaphysical thesis that some contradictions are true). This does not mean that all dialetheists use those same arguments, and that is actually where Curry's Paradox and the Inclosure Schema meet. Priest defending that all inclosure paradoxes must be solved by the dialetheist strategy does not mean that all supporters of the Inclosure Schema are dialetheists and that all dialetheists recognize the Inclosure Schema as a relevant generalisation for paradoxes (Beall 2014a, 829; Weber, Ripley, Hyde, Colyvan & Priest 2014, 813-828).

Curry's Paradox was brought up as an argument in an exchange between two kinds of dialetheists arguing about whether the Sorites paradox should or should not be solved with the glut theory. The glut theory argues that there is a glut truth-value, this value being both true and false. Thus, some sentences are both true and false. Dialetheism is a view that accepts those gluts. Dialetheists believe that there are at least some true contradictions, where contradiction means that you have a proposition and its negation (Beall 2014a, 829; Priest, Berto & Weber 2024, § 0).

Another kind of dialetheists, that do not defend their theory with the Inclosure Schema, are the ones that share Routley's point of view.

Routley defends dialetheism but not in the same way as Priest. For Routley, dialetheism is a simple solution for both logical and semantic paradoxes that must be considered and taken more seriously than classical logic dictates. We can see that, in some situations,

there are overdetermined assertions that are both true and false, and in others there are underdetermined assertions that are neither true nor false. This is the case for paradoxes in which the world can be considered inconsistent, 'there is nothing impossible or absurd about this.' (Routley 2019, 28). What Routley is questioning is the way classical logic works, so that the contradiction of paradoxes has an impact on all the rest of the discourse. He explains that thinking the paradoxical does not lead, as classical logic would have us believe, to a cataclysm. It is a contradiction that remains quite isolated (Routley 2019, 27-39).

Routley explains how many people have already shared this dialetheist vision, and it is anything but new. In Aristotle's day, not everyone agreed with his unacceptance of contradiction; Heraclitus and Antisthenes, for example, were opposed to it. Later, among those who accepted contradictions, we can mention Hegel and the way he inspired Marxist theory. It is classical logic that makes itself trivial by involving disasters. As Routley puts it 'The consistency of the world, when one thinks about it, is not at all easy, and perhaps impossible, to establish in a non-question begging way.' (Routley 2019, 38). Therefore, dialetheism cannot be dismissed out of hand and it is actually hard to find good arguments for Consistencism when you're not a dogmatist. So, when taken seriously, dialetheism is not as absurd as common opinion would have us believe. Perhaps it is the classical solution that is the most absurd. Routley suggests that we could rename the classical vision the *catastrophe vision*. Indeed, classical logic imagines that everything will collapse as soon as we accept a contradiction, but this is not the case! Moreover, it is not because we are faced with a paradoxical situation that logical reasoning stops, so these situations must be considered. It is also possible to reason within paradoxical situations (Routley, Plumwood, Meyer & Brady 1982, 58-67; Routley 2019, 27-39; Priest 2002).

The exchange where Curry's Paradox and the Inclosure Schema met appears because some gluts theorists have advanced that Sorites Paradox should be treated as other paradoxes that fit the Inclosure Schema. Making the Sorites Paradox fit the Inclosure Schema is the argument of Zach Weber, David Ripley, Graham Priest, Dominic Hyde and

Mark Colyvan - called WRPHC by Jc Beall - in defence of treating the Sorites Paradox *along gluttony lines*. One part of Beall's strategy to prove that the Sorites cannot be treated this way is to attack the Inclosure Schema. It is in this context that the question of knowing whether or not Curry's Paradox fits the Inclosure Schema came up (Beall 2014a, 829; Weber, Ripley, Hyde, Colyvan & Priest 2014, 813-828).

Beall's critique towards supporters of the Inclosure Schema is the beginning of the critique of the Inclosure Schema through Curry's Paradox and it presents itself as follows. WRPHC uses an Inclosure strategy; this strategy argues that all inclosure paradoxes should be solved the same way. Indeed, the Principle of Uniform Solution can be applied to all paradoxes that have the same structure captured by the Inclosure Schema, and therefore they are of the same kind and should have the same kind of solution. The problem for Beall, and subsequent logicians, is that Curry's Paradox is of the same kind as paradoxes included in the Inclosure Schema, but it cannot be solved in the same way. If Curry's Paradox is indeed also an Inclosure Paradox (or of the same kind as the one included in it), but cannot be solved in the same way, either the Inclosure Schema does not adequately capture a kind of paradox, or the Principle of Uniform Solution is inconclusive. I will go on to explain Beall's argument in more detail later, but his reasoning gives us an overview of the issues at stake (Beall 2014a, 829-840).

17. Stakes and consequences for the supporters of the Inclosure Schema

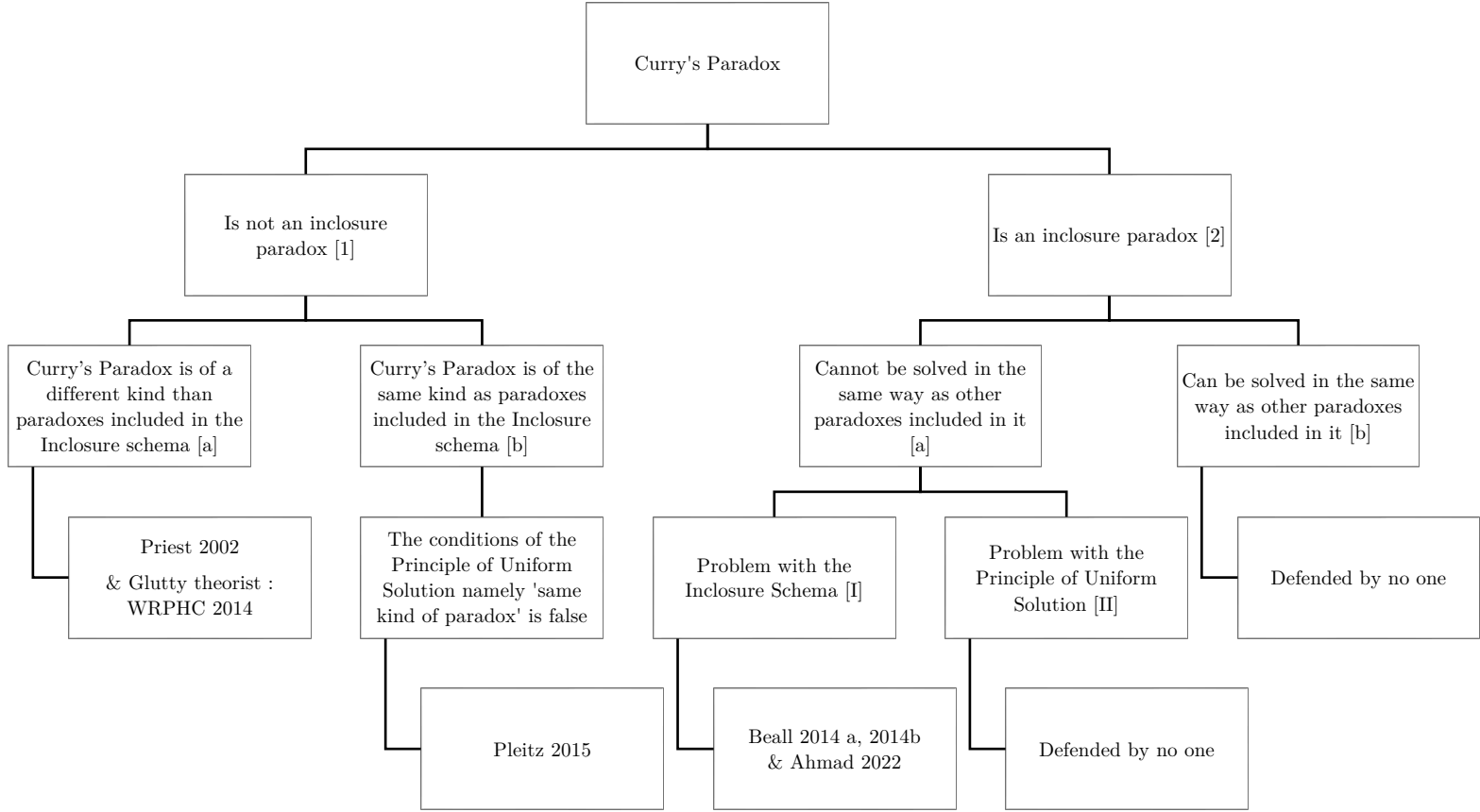
For some dialetheists, what is at stake is to defend the relevance of their theory, without applying it beyond what it can do. But what interests me in this work is the challenge for dialetheists that support this theory through the Inclosure Schema. Moreover, the supporters of the Inclosure Schema do not all have to be dialetheists. The problem is that if Curry's Paradox is of the same family as paradoxes included in the Inclosure Schema, the Inclosure Schema is not a good schema for grouping paradoxes of the same kind. It is just a way of categorising them, but not a very pertinent one when it comes to knowing

how to solve them. Furthermore, if Curry's Paradox is a self-referential paradox, which means of a family of paradoxes from which all other paradoxes are included in the Inclosure Schema, the assumption that this schema brings together all paradoxes of the same kind is wrong, since it leaves one of it on the side.

To be more precise, the stakes here are double, the first question is whether [2] or not [1] the paradox is an inclosure paradox.

A first option is that Curry's Paradox is not an inclosure paradox [1] and that all the paradoxes included in the Inclosure Schema are not of the same kind as Curry's Paradox [a]. Another is that Curry's Paradox is not an inclosure paradox [1] and that all the paradoxes of the same kind as Curry's Paradox are included in the Inclosure Schema [b]. This would mean that one of the conditions of the Principle of Uniform Solution, namely 'same kind of paradox', is false, since the Inclosure Schema would not be a good analytical criterion for grouping paradoxes of the same kind.

On the other hand, if Curry's Paradox is an inclosure paradox [2], and it cannot be solved in the same way as other paradoxes included in it [a], there is a problem with the Inclosure Schema which does not give a good selection of paradoxes of the same kind [I]. By the Principle of Uniform Solution, the same kind of paradoxes must be solved by the same kind of solutions. However, this group of paradoxes could not be solved in the same way, so it would not be the right kind. We could also argue that if Curry's Paradox is of the same family as paradoxes included in the Inclosure Schema and is not solvable the same way, it makes no sense to use the Principle of Uniform Solution with it [II]. But this consideration of Curry's Paradox fitting the Inclosure Schema, which would call into question the PUS, is not defended. Indeed, it seems more likely that the Inclosure Schema is not a good reading grid for paradoxes of the same type, rather than that the Principle of Uniform Solution is not a good principle. The other option for this first hypothesis, namely that Curry's Paradox is an inclosure paradox [1], and that it can be solved in the same way as other paradoxes included in it [b], is also not defended. Indeed, accepting contradiction does not solve the triviality to which Curry's Paradox leads.



Now that I have shown the stakes of understanding whether Curry's Paradox fits or not the Inclosure Schema, I can expose the different points of view. To do so, I will differentiate two points of view. The first one is the standpoint that the paradox does not fit the Inclosure Schema. In this perspective, I will make the primordial difference between those who argue that Curry's Paradox is of the same kind as other paradoxes included in the Inclosure Schema and those who argue that it is of a different kind. To those opinions that Curry's Paradox does not fit the Schema, I will oppose the opinion that Curry's Paradox does fit the Inclosure Schema.

18. The paradox does not fit the Inclosure Schema

a) And is of a different kind than paradoxes included in the Inclosure Schema

Priest

Priest argues that Curry's Paradox does not fit the Inclosure Schema, but contrary to Pleitz, whose theory we shall see later, he argues that it is fundamentally different from other self-referential paradoxes. For Priest, Curry's Paradox is only related to paradoxes included in the Inclosure Schema, but not of the same kind as them. It differs from other self-referential paradoxes by not having A that involves $\neg A$ but A that involve everything. According to Priest, this makes Curry's Paradox an entirely different paradox: a paradox of implication, whereas the other paradoxes that fit into the Inclosure Schema are paradoxes of contradiction. Curry's Paradox does not lead to contradiction (which itself leads to triviality according to classical logic, but not according to Priest's logic), but directly to triviality. For Priest, it is a completely different kind of paradox, and it needs to be solved differently (Priest 2002, 168).

For Priest, Curry's Paradox does indeed sometimes fit the Inclosure Schema. It is the case when $\rightarrow$ is a material conditional, which makes $A \rightarrow \perp$ equivalent to $\neg A$ because the curried version of the paradox is the same as the uncurried (as in classical logic where 'not-A' is equivalent to 'if A then falsum/absurdity'). On the other hand, Curry's Paradox does not fit the Inclosure Schema if $\rightarrow$ is a non-material conditional. The equivalence between $A \rightarrow \perp$ and $A \rightarrow \neg A$ does not hold anymore, and the two notions are quite different. 'In this case, the curried versions of the paradoxes belong to a quite different family. Such paradoxes do not involve negation and, a fortiori contradiction.' (Priest 2002, 169) (Priest 2002, 169; Priest 2017, 64).

To be more precise, the material conditional is usual in classical logic: it is defined by the truth table, and has the property of being *detachable*, that is to say, of validating the *Modus ponens* (whether in the form of a rule or a law: if ((if A then B) & A) then B). That is an important point for Priest, since the conditional in the relevant logics is not *detachable*: that is to say it does not validate the *Modus ponens* in the form of a law (because the *Modus ponens* in the form of a law is logically equivalent to the Disjunctive Dyllogism (if (A or B) & not-A) then B), which is not valid in the relevant logics). This point is related to the analysis of Routley, Meyer, Plumwood & Brady 1982 (278-283) that I explained on page 24. In it, the Principle of Contraction is also equivalent to *Modus ponens* in the form of a law. Therefore, there would be a kind of equivalence between the Laws of *Modus ponens*, Contraction and the Disjunctive Syllogism (Routley, Meyer, Plumwood & Brady 1982, 278-283; Priest 2002, 169; Priest 2017, 64).

Glutty theorist

Zach Weber, David Ripley, Dominic Hyde and Mark Colyvan share Priest's point of view. They all defend that Curry's Paradox does indeed not fit the Inclosure Schema. They oppose Beall's opinion (that I will explain in more detail later) that Curry's Paradox fits the Inclosure Schema but cannot have a dialetheical solution. In Weber, Ripley, Hyde, Colyvan & Priest 2014, the authors take issue with the assertion that Curry's Paradox fits the Inclosure Schema. They argue that the problem is that a reasoning such as Curry's Paradox proves not only false assertions, but any arbitrary argument. They claim that the paradox does not fit the Inclosure Schema since it does not prove contradiction but any arbitrary claim, no matter whether it is true or false. There are indeed forms of Curry's Paradox that allow us to prove $\perp$, in which we find the contradiction. These forms fit into the Inclosure Schema. But Curry's Paradox can also take the shape of a paradox without contradiction, which proves any chosen claim. That shape of a paradox leads to triviality even for theories that accept contradictions and does not fit into the Inclosure Schema. Indeed, if we choose a claim that is true, transcendence does not hold anymore since it lacks contraposition. It is only when the chosen claim in Curry's sentence corresponds to

a contradiction that Curry's Paradox fits the Inclosure Schema. But an example does not make it an inclosure paradox. Just as some rectangles being squares does not prove that rectangles are squares, taking a special case for a general conclusion is not correct reasoning. (Weber, Ripley, Hyde, Colyvan & Priest 2014, 823).

In addition, they refer to the Principle of Uniform Solution that Beall uses in his argument. Whatever the chosen claim is, all versions of Curry's Paradox are of the same kind, and some versions of the paradox do not fit the Inclosure Schema. Those two elements, combined with the Principle of Uniform Solution, impose on all those versions of Curry's Paradox the same treatment, a different one than the one of inclosure paradoxes (Weber, Ripley, Hyde, Colyvan & Priest 2014, 823).

Furthermore, Curry's Paradox is basically devoid of any negation, 'It therefore has nothing to do with contradictions that occur at various boundaries (limits), which is what inclosures are all about' (Weber, Ripley, Hyde, Colyvan & Priest 2014, 823).

b) And is of the same kind as paradoxes included in the Inclosure Schema

Pleitz

Not in the Inclosure

Another logician who argues that Curry's Paradox is not included in the Inclosure Schema is Martin Pleitz. His article, Pleitz 2015, suggests a larger definition of self-referential paradoxes, stating that they are not all encapsulated by the Inclosure Schema. This article first explains how Curry's Paradox is part of this category of paradoxes. It then explains the Inclosure Schema's limits regarding his impossibility to explain Curry and offers a new schema for it. Lastly, Pleitz formulates an even larger schema that joins together the Inclosure Schema and the Curry Schema and explains the consequences for Priest's works (Pleitz 2015, 1-3).

To do so, Pleitz first explains how self-referential paradoxes - that he prefers to call logical paradoxes - work. There is first the assertion that a sentence, a predicate or a set exists. We then use an *unrestricted bridge principle*, like a truth theory or a set theory, to join them together. Finally, those become paradoxical because of classical logic, for which contradiction becomes triviality. From these three components, we have three ways to solve the paradox: we can first question the metaphysical claim by asking if those sets or predicates do indeed exist, a second option is to question those bridges, so for example revise our set theories and thirdly, we can question our classical logic (Pleitz 2015, 1-3). For more details, I have outlined the various possibilities on pages 12 to 13.

After quoting some of the most common solutions that use such theoretical moves, Pleitz draws our attention to the fact that these more orthodox solutions are not uniform. Indeed, common solutions make a metaphysical adjustment for set theoretic paradoxes, whereas for semantic paradoxes, they make a logical or semantic adjustment. There, he questions Ramsey's separation between set-theoretical paradoxes (Burali-Forti, Cantor, Russell) and semantic paradoxes (Liar, Grelling, etc.), which seem to be closer than what common opinion lets us believe (Pleitz 2015, 2-3).

Like Russell's review of metaphysics with the vicious circle, Priest and some other dialetheists offer a uniform solution by rethinking logic, which will apply to semantic and set paradoxes in the same way. As explained, Priest claims to offer a general solution that applies to all members of a kind of paradoxes, and by the Principle of Uniform Solution, if we have similar sorts of paradoxes, we must find a similar kind of solution. He considers set and semantic paradoxes of different groups but from the same family and claims to solve them with a unique solution that, because it respects the Principle of Uniform Solution, is more relevant than other common solutions (Priest 2002, 166-167; Pleitz 2015, 3).

To explain how Curry is not included in the Inclosure Schema, Pleitz shows us how the Liar Paradox, Russell's Paradox, Burali-forti's Paradox and Berry's Paradox can be interpreted through the Inclosure Schema, and how they can be summarised by the three principles of the Inclosure Schema: existence, transcendence, and closure. Although those other self-referential paradoxes are correctly characterised by the Inclosure Schema, Curry's Paradox does not fit into it. To show that this is problematic for Priest's schema, Pleitz explains how Curry's Paradox is indeed part of this family of self-referential paradoxes. He shows that if we want the Inclosure Schema to characterise the first part of the Principle of Uniform Solution (same kind of paradox), then Curry's Paradox should also be included in the Inclosure and solved by dialetheists (Priest 2002, 166; Pleitz 2015, 3).

Indeed, if Curry's Paradox is of the same kind as paradoxes included in the Inclosure Schema, this harms the Inclosure Schema. That would mean that the Inclosure fails to characterise all paradoxes of the same kind and, therefore, cannot be used to apply the Principle of Uniform Solution. Moreover, Priest has a second problem over there, since the paradox leads to triviality by allowing to claim any chosen sentence but not by *ex contradictione sequitur quodlibet*. It raises questions for his dialetheist resolution of the paradoxes (Pleitz 2015, 3-5).

Pleitz argues that Curry's Paradox is indeed a self-referential paradox, and therefore a problem for the inclosure strategy, for the following reasons. First, Curry's Paradox is rooted in the same existential affirmation as The Liar Paradox and Russell's Paradox. This concurs with our explanation in the paragraph that compares the differences between Curry's Paradox and those two other self-referential paradoxes pages 26 to 33. Even though what I tried to do in this paragraph was to differentiate those paradoxes, Prior's explanation has forced me to note that they have a lot in common, and even more, to see that the paradox appears in similar theories. Indeed, we can match the presence of the Curry sentence with a Liar sentence, and the presence of the Curry set with the Russell set (Pleitz 2015, 6).

Secondly, as mentioned in the explanation of what constitutes the paradoxes of self-reference, they use bridges; those are, as for the Liar Paradox and Russell's Paradox, the naive set abstraction or Tarski's truth schema. Curry's Paradox uses the same bridges. It appears in the same theories and uses the same concepts, regardless of whether or not they resolve the other paradoxes. (Pleitz 2015, 6).

Thirdly, Curry's Paradox is not fundamentally separate from other self-referential paradoxes, but rather overlaps with some of them. Indeed, we must pay particular attention to the absence of negation in Curry's Paradox, and not interpret it directly as a differentiating element. As mentioned in the second part of our paragraph about the difference between Curry's Paradox and the two other self-referential ones, we can find a common structure between Curry's Paradox and Russell's Paradox. But more than that, Pleitz mentions that with the *negation equivalence* $\neg A \leftrightarrow A \rightarrow \perp$, which is held in many theories and says that the negation comes down to say that it implies absurd, we can say that 'Russell's paradox is a limit case of Set Curry and the Liar is a limit case of sentence Curry' (Pleitz 2015, 6). By this *negation equivalence*, we can observe that Curry's Paradox overlaps those other paradoxes by making the implication of the absurd and the negation equivalent. (Pleitz 2015, 6)

Pleitz opposes this last argument to Priest's point of view. In fact, he uses what seems to be the same argument as Priest uses against people who use Curry's Paradox against his Inclosure Schema: that he is only looking at one part of the whole picture. As Priest insists that not all versions of Curry's Paradox fit into the Inclosure Schema, Pleitz observes that Priest does not look at Curry's Paradox as a whole. In fact, he is only interested in Curry's sentences that have a contradictory conclusion. Furthermore, he elides a whole section of them in the absence of *negation equivalence*. For him, when a Curry's instance does not involve negation, it belongs to another family of paradoxes. For Pleitz, to mention that some of Curry's instances are the same as the uncurried one puts the overlap wrong. Priest should not be looking for a generalisation in a schema that only

considers some of Curry's instances. According to Pleitz, Russell's Paradox and the Liar are limit cases of Curry's Paradox, and unlike those two paradoxes, Curry's Paradox also has some not absurd conclusions. So, since Curry's Paradox overlaps with Russell's Paradox and the Liar Paradox and has many more possible conclusions, it is strictly more general than these two. Therefore, it seems rather odd that Priest is only interested in some of Curry's cases. (Pleitz 2015, 7).

Pleitz's schemata

After this critique of Priest's point of view, Pleitz offers a schema to understand Curry's Paradox. For him, Curry's Paradox is actually more of a schema 'that delivers a multitude of paradoxical arguments' (Pleitz 2015, 6). Compared to the Inclosure Schema, which underlines the contradiction in paradoxes, which can be solved by a paraconsistent theory, Curry's argument can put forward any conclusion. Curry's Paradox is not included in the Inclosure Schema and therefore cannot be *inclosed*, but Pleitz goes the other way around and 'it turns out that the Inclosure Schema can be curried!' (Pleitz 2015, 8). To do so, he reworks the Inclosure Schema to create a Curry Schema that includes all of Curry's variants. He only needs to modify the condition of transcendence of the Inclosure Schema. He replaces this condition with the *p-Spiciness* (Pleitz 2015, 8-9). The Schema is as follows:

- (Existence) $\exists \Omega (\Omega = \{x \mid \varphi(x)\} \wedge \psi(\Omega))$
- (P-Spiciness) $\forall x(x \subseteq \Omega \wedge \psi(x) \rightarrow (\delta(x) \in x \rightarrow p))$
- (Closure) $\forall x(x \subseteq \Omega \wedge \psi(x) \rightarrow \delta(x) \in \Omega)$
- (Pleitz 2015, 9)

He then shows how special cases of Curry's Paradoxes, as the Set Curry or the Sentence Curry, are included in this schema:

Paradox	$\delta(x)$	$\psi(y)$	$\varphi(z)$	Ω
Set Curry	$\{w \in x \mid w \in w \rightarrow p\}$	$\lambda y y = y$	z is a set	V
Sentence Curry	c , i.e., $c \in x \rightarrow p$	y is definable	z is true	Tr

(Pleitz 2015, 9)

Pleitz then offers then to compare the Inclosure Schema with his Curry Schema. This allows us to have a clear view of the difference between those two in the second line.

(Inclosure Schema)

$$\begin{aligned} \exists \Omega (\Omega = \{x \mid \varphi(x)\} \wedge \psi(\Omega)) \\ \forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow \delta(x) \notin x) \\ \forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow \delta(x) \in \Omega) \end{aligned}$$

(Pleitz 2015, 10)

(Curry Schema)

$$\begin{aligned} \exists \Omega (\Omega = \{x \mid \varphi(x)\} \wedge \psi(\Omega)) \\ \forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow (\delta(x) \in x \rightarrow p)) \\ \forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow \delta(x) \in \Omega) \end{aligned}$$

Here, we almost have twice the same schema. The first column consists of existence, transcendence and closure, which form the Inclosure Schema. The second column is made up of existence, *p-Spiciness* and closure. To be even more precise, the only difference is in the second part of the formula of the *p-Spiciness*. The difference lies between the implication of $\delta(x) \notin x$ for the Inclosure Schema and the implication of $(\delta(x) \in x \rightarrow p)$ for the Curry Schema. This Curry Schema is therefore different in that it does not keep the transcendence of the Inclosure Schema, which indicates that the paradoxical object emerges from an entity, which brings the contradiction. Instead, it states, thanks to the *p-Spiciness*, that the paradoxical object involves an arbitrary element p , and this is what leads to paradoxicality (Pleitz 2015, 10).

When we look at the structure, we notice that these two schemata are very similar, which leads Pleitz to propose a new schema. This allows him to modulate a generalisation that looks as follows:

(The Most General Schema) Let φ and ψ be predicates and δ a function.

Then the following threefold condition holds:

- (Existence) $\exists \Omega (\Omega = \{x \mid \varphi(x)\} \wedge \psi(\Omega))$
- (Gamma) $\forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow \Gamma)$
- (Closure) $\forall x (x \subseteq \Omega \wedge \psi(x) \rightarrow \delta(x) \in \Omega)$

(Pleitz 2015, 12)

In this broader Schema, the Inclosure Schema and the Curry Schema are two special cases where Γ has a specific signification: ‘Set $\Gamma = \langle \delta(x) \notin x \rangle$ to get the Inclosure Schema, set $\Gamma = \langle \delta(\Omega) \in \Omega \rightarrow p \rangle$ to get the Curry Schema’ (Pleitz, 2015, p. 12). For the schema to illustrate a paradox, all you need is not to have a Γ that corresponds to ‘ $\delta(x) \in x$ ’. As we can see, this Most General Schema encompasses Curry's Paradox and the paradoxes of the Inclosure Schema. Indeed, by involving Γ , the Most General Schema makes it possible to involve both the contradiction and the implication of any chosen claim (Pleitz 2015, 12).

By all these means, Pleitz has shown an even more general schema that includes the Inclosure Schema and the Curry Schema but also the Set Curry and Sentence Curry, since they fit the Curry Schema. This shows that all those schemata have a common structure and are special cases. This allows Pleitz to reinforce his argument that the Inclosure Schema is not sufficient. Indeed, this schema ties Curry's Paradox even more closely to the other self-referential paradoxes and reinforces the lack of relevance of the Inclosure Schema as a basis for the Principle of Uniform Solution. This even calls into question the

use of this principle. Indeed, Pleitz finds reasons to count Curry's Paradox among the other self-referential paradoxes and by expanding the number of paradoxes included in this family, the logical revision seems less convincing. Pleitz argues that his schema, which generalises self-referential Paradoxes, makes the Priest's solution that revisits classical logic less convincing. Logical resolution is challenged by the expansion of the number of paradoxes in the same kind, which makes it seem less uniform. And if we think back to the way Pleitz presents the different paradox resolutions, he argues that we should perhaps turn to a metaphysical rather than a logical solution (Pleitz 2015, 13).

19. The paradox fits the Inclosure

Beall

Finding tolerance without Gluts

I have mentioned Beall's opinion earlier, where he opposes Weber, Ripley, Hyde, Colyvan and Priest, by advancing that Curry's Paradox fits the Inclosure Schema but cannot have a dialetheic solution. This directly questions the Principle of Uniform Solution because, if this can be verified, the same kind of paradox cannot be solved by the same kind of solution.

In Beall 2014 b, Beall mentions paradoxes that seem to belong to the Inclosure Schema. He first mentions the Barber Paradox (Does a barber who trims the beards of all the people who do not trim their own beards trim his own beard?) that does have an inclosure structure but does not require a dialetheist resolution. That is what he identified as 'plausibility' which allows the supporters of the Inclosure Schema to sort out that this paradox is not an inclosure paradox, even though it has the same structure. 'This is where plausibility becomes critical in the target strategy: to be an inclosure paradox—to be a phenomenon of the sort that properly triggers the uniform-solution principle for gluts—there needs to be a plausible inclosure-structure argument.' (Beall 2014b, 799-800).

In other words, Beall states that what makes a paradox fit into the Inclosure Schema is the plausibility of an inclosure argument, not just the fact that it has this inclosure structure or not. For example, the Barber Paradox might fit into the Inclosure Schema, but it is not plausible to solve it by using the inclosure argument. The Barber Paradox is much better resolved by saying that there is no such barber, and thus the argument of plausibility prevents it from being an inclosure paradox (Beall 2014b, 799-800).

So as not to dwell on the Barber Paradox, which is not the argument that interests us most, we can look at the second paradox Beall mentions as having an Inclosure structure but not being resolvable by the dialetheist: Curry's Paradox. He mentions that for Weber, Ripley, Hyde, Colyvan and Priest, the material-conditional form of Curry's Paradox fits the Inclosure Schema, and he asks about the other version, the detachable form. For Beall, all that is at stake is the concept of plausibility, which distinguishes between paradoxes that are indeed inclosure paradoxes and those that just look like they are. Not considering Curry's Paradox as an inclosure paradox is not based on the Principle of Contraction but on the concept of plausibility. Beall argues that some Curry's Paradox instances use Contraction and can be considered plausible. According to him, the Contraction Principle makes it at least as plausible as the Sorites Paradox that Curry's Paradox is an inclosure structured argument. This does not mean that the Contraction Principle is valid, it just means that it is plausible enough to make Curry's Paradox an inclosure argument. Furthermore, it is certainly not less plausible than making the Sorites Paradox an inclosure structured argument, since it relies on detachable tolerance. So, if Russell's Paradox, the Liar Paradox and, even more, Sorites Paradox are inclosure paradoxes by plausibility, then so is Curry's Paradox (Beall 2014b, 799-801).

For Beall, this notion of plausibility has too much importance and should be completed by soundness. This would rule out Curry's Paradox but also the Sorites. Telling that Curry's Paradox does not satisfy the Inclosure Schema makes no sense when we accept that the Sorites does. (Beall 2014b, 801)

End of inclosure

To this argument, Weber, Ripley, Priest, Hyde and Colyvan have replied with what I explained earlier. But Beall, arguing that he had not been properly understood, reformulates his argument as follows. In Beall 2014a, Beall explains the strategy used by the supporters of the Inclosure Schema. He then attacks their argument by explaining his previous article, their counterargument and he answer to it (Beall 2014a, 839).

More precisely, he first explains the Inclosure Schema, then presents a bit more of the Inclosure strategy. As seen before, WRPHC use the Inclosure Schema as what determines the same kind of paradoxes in the Principle of Uniform Solution. This Inclosure Schema determining the same kind of Paradox together with the Principle of Uniform Solution provide an argument to solve all these paradoxes in the same way, with dialetheism (Beall 2014a, 840).

Beall then gets to the heart of his criticism, stating that since the inclosure strategy is based on the Principle of Uniform Solution, having a different resolution for inclosure paradoxes would devalue the Inclosure Schema. And, according to Beall, Curry's Paradox is precisely an inclosure paradox that gets a different treatment. Curry's Paradox is a paradox in the same way as the Liar or Russell's Paradox. He illustrates this by quoting the way it was understood by his opponents (Beall 2014a, 840-841):

'Curry's paradox is said [by Beall] to fit the inclosure conditions where these are as follows (and T is the truth predicate):

$$\varphi(y) := \text{'Ty'}$$

$$\psi(x) := \text{'x is definable'}$$

$$\delta(x) := s$$

where s is a sentence of the form $\langle s \in x \rightarrow \perp \rangle$, and x is a name for x. Let $\Omega = \{y : y \text{ is true}\}$.

The function $\delta(x)$ is clearly defined when $\psi(x)$ holds. The set Ω exists since it is a set of sentences of some (countable) language, and it is obvious that $\psi(\Omega)$. Take an x such that $x \subseteq \Omega$ and $\psi(x)$. Suppose that $\delta(x) = s$ and $s \in x$. Then, since $x \subseteq \Omega$, it follows that s is true, and so $s \in x \rightarrow \perp$, which gives $\perp$. Discharging the supposition, $s \in x \rightarrow \perp$; that is, Ts ; that is $s \in \Omega$ (closure). But since $s \in x \rightarrow \perp$ is true, if $s \in x$ then $\perp \in \Omega$, which it is not. Hence, by contraposition, $s \notin x$ (transcendence). The limit contradiction arises when x is Ω . Then $s \in \Omega$ and $s \notin \Omega$, where $\delta(\Omega) = s$.' (Weber, Ripley, Hyde, Colyvan & Priest 2014, 822-823).

In this explanation, Beall's opponents present how Beall fits Curry's Paradox into the Inclosure Schema. He does so by using a version of Curry's Paradox that implies something false. This implication of the false, namely $\rightarrow \perp$, is contradictory and therefore, allows transcendence and Curry's Paradox to fit into the Inclosure Schema (Weber, Ripley, Hyde, Colyvan & Priest 2014, 822-823).

In this application of the Inclosure Schema to Curry's Paradox, we see that Curry's Paradox is as *plausible* as other paradoxes that fit into the Inclosure Schema. Beall is not saying that all Curry's instances fit the Inclosure Schema, but that the reasons that push supporters of the Inclosure not to consider Curry's Paradox are not that strong. Some cases of Curry's Paradox can fit the Inclosure Schema and, therefore, it cannot be rejected as other non-inclosure paradoxes. If some instances of Curry's Paradox are indeed an inclosure paradox, supporters of the Inclosure Schema are left with two options: answer to the paradox with a dialetheist theory, which leads to triviality and does not solve the paradox, or solve it any other way, which makes pointless the strategy of solving all inclosure paradoxes the same way (Beall 2014a, 840-841).

Beall then exposes the counter-argument of Weber, Ripley, Hyde, Colyvan and Priest that I have explained earlier, according to which only the applications of Curry's Paradox for which we find contraposition fit the Inclosure Schema, but one special case does not make Curry's Paradox an inclosure paradox in itself. Therefore, Beall would not have

properly identified Curry's Paradox, and it would not be plausible for it to fit the Inclosure Schema (Beall 2014a, 841-842).

The heart of the argument comes next when Beall exposes a new argument in Beall 2014a. For him, the objection of Weber, Ripley, Hyde, Colyvan and Priest is problematic. 'The problem can be seen as a dilemma; either the inclosure strategy captures too much (on pain of absurdity) or too little (it misses target phenomena/paradoxes).' (Beall 2014a, 842). To prove this, Beall comes up with two questions, each of which can be answered in two ways (Beall 2014a, 842).

‘(Q1) What, then, is an inclosure paradox?’

(Q2) What is it for a sentential scheme to be (inclosure-) paradoxical?’

(Beall 2014a, 842)

There are two possible answers to Q1. The first one is to argue that inclosure paradoxes are individual sentences. If not, then the second option is to argue that they are sentential schemes, an example of which is wrphc's sentential scheme (Beall 2014a, 842).

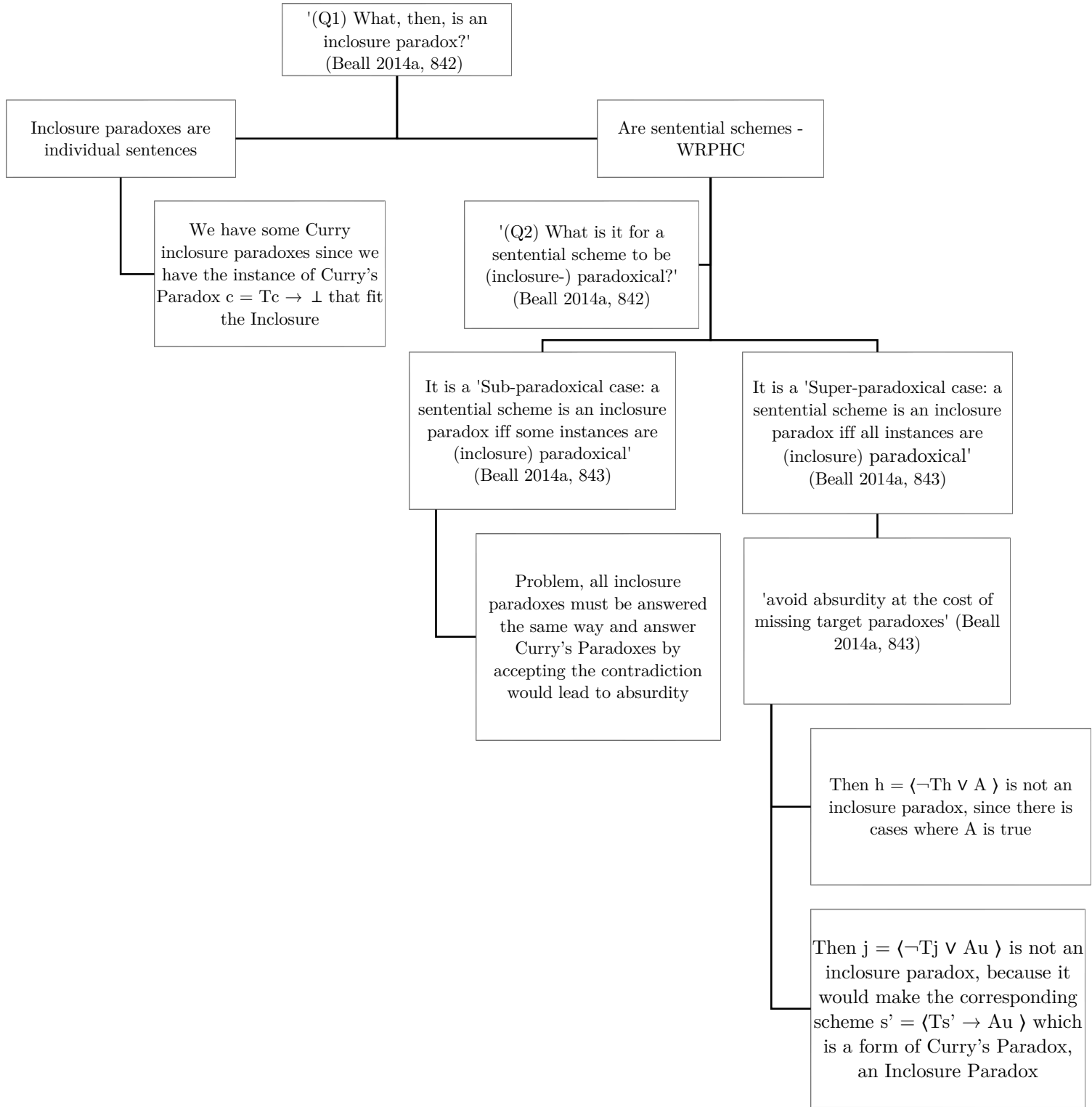
If it is the first option, then we have some Curry inclosure paradoxes, since we have the instance of Curry's Paradox $c = Tc \rightarrow \perp$ that fits the Inclosure. What Weber, Ripley, Priest, Hyde, and Colyvan seem to have in mind is the second option: the inclosure paradoxes are *sentential schemes* (Beall 2014a, 842).

This second option leads us to the second question: 'What is it for a sentential scheme to be (inclosure-)paradoxical?' (Beall 2014a, 842) which also leaves us two ways of answering. Either this sentential scheme is a 'Sub-paradoxical case: a sentential scheme is an inclosure paradox iff some instances are (inclosure) paradoxical' (Beall 2014a, 843), or it is a 'Super-paradoxical case: a sentential scheme is an inclosure paradox iff all instances are (inclosure) paradoxical' (Beall 2014a, 843).

We can opt for the first answer, arguing that you only need some instances of Curry's Paradox to fit the Inclosure Schema to make Curry's Paradox an inclosure paradox. But we will face a problem: since all inclosure paradoxes must be answered the same way, we would answer all instances of Curry's Paradox by accepting the contradiction and this would lead to absurdity (Beall 2014a, 843).

The other option is to affirm the *Super-paradoxical* case where you need all instances of a paradox to fit the Inclosure Schema to consider it an inclosure paradox. This allows us to 'avoid absurdity at the cost of missing target paradoxes' (Beall 2014a, 843) like $g = \langle \neg Tg \vee \perp \rangle$. This option would leave us with $h = \langle \neg Th \vee A \rangle$, which isn't an inclosure paradox either, since in the cases where A is true, it doesn't fit the Inclosure Schema (Beall 2014a, 843). We could take the more precise $j = \langle \neg Tj \vee A_u \rangle$ 'where A_u is schematic over gently or otherwise untrue sentences' (2014 b, 843). This schema would be a *super-paradoxical* case and therefore fit the Inclosure Schema. But to consider it as an Inclosure Paradox dismantle the argument of the supporters of the Inclosure Schema, because it would make the corresponding scheme $s' = \langle Ts' \rightarrow A_u \rangle$, which is a form of Curry's Paradox, an Inclosure Paradox (Beall 2014a, 843-844).

So, we have the following options open to us:



Beall then brings a new version of the paradox to the debate: the Neo-Curry, which can be expressed as $j = \langle Tj \rightarrow (A \ \& \ \neg A) \rangle$. This version is a variant of $s' = \langle Ts' \rightarrow Au \rangle$ mentioned just before. This is a response to the argument that Curry's Paradox does not use negation and that it makes it fundamentally different from the other inclosure paradoxes. Neo-Curry threatens WRPHC's argument since it uses negation. Indeed, this version of the paradox entails $A \ \& \ \neg A$, which, whatever the value of A , leads to a contradiction since it implies an element and its negation. Beall anticipate WRPHC's response to describe this version of Curry's Paradox as just one example and to say that the 'real Curry' does not allow contraposition and therefore does not fit the inclosure (Beall 2014a, 844-845).

To this, Beall answers that even if we consider that the general scheme is the 'real Curry', Neo-Curry is a kind of specific case, but which constitutes a paradox on its own, which is surely an inclosure paradox. This is not just an example of Curry's Paradox, it is a variant. It is, in fact, a general formula of paradoxes that groups together several instances. For Weber, Ripley, Priest, Hyde and Colyvan, all Curry's Paradoxes are of one kind. But Beall disagrees and points out that WRPHC also contradicts themselves: they mention that certain kinds of Curry's Paradox, like Neo-Curry, fit into the Inclosure Schema through contraposition. (Beall 2014a, 845). We are facing a problem for the Inclosure Schema since if it is supposed to capture a kind of paradox, it is strange that it captures the family of Curry's Paradox instances, which is the Neo-Curry but not all of them (Beall 2014a, 844-845).

For Beall, we must consider the possibility that there are different kinds of paradoxes in the wider family of Curry's Paradox. But this way of seeing things does not solve the problems that it brings for the Inclosure strategy. The inclosure strategy does not seem, as WRPHC hope, to explain why all self-referential paradoxes are paradoxical, since a large proportion of them cannot be explained by, or even be included in, this schema. Indeed, the Inclosure Schema does not include, for example, 'either I am not true or Tyrannosaurus is herbivorous' formulated by Beall as $g = (\neg Tg \vee \bot)$ or $r = (\neg Tr \ \& \ T)$.

Therefore, for Beall the supporters of the Inclosure Schema do not bring an argument strong enough to solve all the inclosure paradoxes the same way, namely by adopting a dialetheist theory (Beall 2014a, 846).

Ahmad

Another author that gives his point of view on Curry's Paradox fitting the Inclosure Schema is Ahmad. In Ahmad 2022, he offers an argument against the Inclosure Schema. To achieve this, he highlights the links between the Liar and Curry's Paradox, then forces Curry's Paradox to fit the Inclosure Schema and finally shows how other paradoxes can also fit in the Inclosure Schema and that it comes at a high cost. To do so, he explains how the Inclosure Schema is both too broad and too narrow and brings together the wrong paradoxes. He then provides a *recipe* for semantic paradoxes and exposes the kind of solution it offers when combined with the Principle of Uniform Solution (Ahmad 2022, 254).

In his article, Ahmad first offers a comparison between Russell's and Priest's analysis of self-referential paradoxes and how Priest's Inclosure Schema follows from Russell's Schema. He then offers an explanation of the Inclosure Schema and applies the example of the Liar to it. Then comes the question of Curry's Paradox (Ahmad 2022, 254-256).

Too Narrow

For his analysis, Ahmad questions the Inclosure Schema and the paradoxes that it covers. For him, there is a problem in the selection of paradoxes gathered by the Inclosure Schema: it 'is both too narrow and too broad' (Ahmad 2022, 254). He first explains how the Inclosure Schema is too narrow.

To make progress on whether Curry's Paradox fits or not the Inclosure Schema, Ahmad suggests comparing the Liar and Curry's Paradox. Those two paradoxes are 'self-referential, employ the truth predicate, rely on the T-schema, and crucially involve ungroundedness.' (Ahmad 2022, 256). The thing that could strongly differentiate them is that Curry's Paradox seems negation-free. But it is only an appearance, and Curry is much closer to the Liar than we might think. As Prior's argument, he shows the common structure between Curry's Paradox and a classic self-referential paradox. The comparison with Curry's Paradox is made by Ahmad with the Liar Paradox and by Prior with Russell's Paradox (Ahmad 2022, 256).

Furthermore, Ahmad adds a comparison between the two elements that makes them different for Prior: the unary connective, which stands for the negation in the Liar and the conditional in Curry's Paradox. Indeed, Ahmad tells us about the negative modality of Curry's Paradox and shows that the paradox, even though it seems like it, is not negation-free. The specificity of Curry's Paradox to implicate any chosen claim is in fact very *negation-y* which means it uses negation much more than at first glance. If we consider the negative modality defined by Greg Restall such as $(A \vdash B) \rightarrow (mB \vdash mA)$, we can prove that negation is a negative modality but also that $\rightarrow B$ (where B stands for any chosen claim) is a negative modality. We could defend that it is also what Prior suggests and demonstrates with his diagram that brings together Curry's Paradox and Russell's Paradox. Perhaps Prior was right and Restall's formula goes in his direction. Ahmad would then allow us to see that Prior's comparison is even more relevant if we have this conception of negation in mind (Prior 1955, 177-182; Restall 1999, 53-76; Sylvan 1999, 299-324; Ahmad 2022, 256-257).

Ahmad shows then how $\rightarrow B$ is negation-y by using

De Morgan's Law

$$(A \vee B) \rightarrow \varphi \dashv\vdash (A \rightarrow \varphi) \wedge (B \rightarrow \varphi)$$

$$(A \wedge B) \rightarrow \varphi \dashv\vdash (A \rightarrow \varphi) \vee (B \rightarrow \varphi)$$

The Triple Negation

$$((A \rightarrow \varphi) \rightarrow \varphi) \rightarrow \varphi \dashv\vdash A \rightarrow \varphi$$

The Law of Excluded Middle

$$\vdash A \vee (A \rightarrow \varphi)$$

And partially The Double Negation

$$A \vdash (A \rightarrow \varphi) \rightarrow \varphi$$

The only exception is the double negation elimination, but not all negations have it. All of those comparisons between the implication of anything and the negation are, for Ahmad, a way to show that Curry's Paradox is actually not that far at all from paradoxes that use negation. Not that far since Curry's implication and the negation do actually share a lot of properties, even all of them for the negations that do not use double negation elimination (Prior 1955, 177-182; Restall 1999, 53-76; Sylvan 1999, 299-324; Ahmad 2022, 256-257).

Priest's response to this way of looking at negation is that it is not at all clear that there is a correspondence between these two modes of negation, since one of them uses conditionality. He adds that even if this were the case, the fact that the same element occurs at the end of two arguments does not make the Liar and Curry's Paradox the same argument (Priest 2017, 119).

Like Pleitz, Ahmad argues that Curry's Paradox is way closer to the Liar than we could think as they share the same pattern. That is also what Prior suggests and demonstrates with his schema that brings together Curry's Paradox and Russell's Paradox. As mentioned just before, the negation is more present in Curry's Paradox than what can be

seen at first sight, and seeing negation through the negative modality makes the comparison way easier, not to say evident, according to Ahmad. Since the Liar and Curry's Paradox are so close, Curry's Paradox should be included in the Inclosure Schema. Even more so, given that that Priest argues that the Inclosure Schema encompasses all paradoxes of the same kind, namely all self-referential paradoxes. Curry's Paradox is a self-referential paradox and the only thing that could differentiate it is that it does not use negation, but it is actually not the case (Ahmad 2022, 256-257).

Moreover, Ahmad shows us how Curry's Paradox does actually fit the Inclosure. He wants to show that the argument of Priest does in fact allow Curry's Paradox to fit the Inclosure, both its forms with negative and positive implication. But this comes at a certain cost. The supporters of the Inclosure Schema advance that Curry's Paradox fits the schema only when the consequence of the sentence is false. According to Ahmad, this comes down to saying that we need external empirical information. He shows that we can fit all of Curry's Paradox into the Inclosure Schema by beginning the demonstration with the version of Curry's Paradox with a false consequence and by using, as Priest, some empirical information (Ahmad 2022, 258-259).

To do so, he first shows how a Curry's Paradox with a false consequence can fit the Inclosure Schema. He then mentions that what is missing in the version with a true consequence is the contraposition, and that it cannot provide transcendence. To make this missing part of Curry's Paradox with true consequence appear, he brings some outside empirical information coming from a Curry's Paradox with false consequence version. At the same time, he states that this is the real problem, the need to bring in an empirical element to make all instances of Curry's Paradox fit the Inclosure Schema (Ahmad 2022, 258).

So, to prove that the Curry's Paradox version with true consequences fits the Inclosure Schema if we bring empirical information, he suggests this demonstration:

'Let s_2 be $\langle s_2 \in X \rightarrow A \rangle$ ("If s_2 is true, then snow is white"). We start by carrying out the Inclosure Schema for the Curry sentence with a negated consequent of s_2 , namely, s_1 as displayed above. From the reasoning about s_1 , we have closure for s_1 , namely $s_1 \in \Omega$, and we also have 'if $s_1 \in \Omega$ then $\neg A \in \Omega$ '. By *Modus ponens*, we get $\neg A \in \Omega$, and hence $A \notin \Omega$, and thus we can use this in s_2 (along with "if $s_2 \in X$ then $A \in \Omega$ ") in order to make the contraposition move and get transcendence - $s_2 \notin X$.' (Ahmad 2022, 258).

In this demonstration, Ahmad uses two Curry's sentences to force one of them into the Inclosure Schema. He begins the demonstration by taking a Curry's sentence, whose consequence is true. He then uses the negation of this consequence to obtain closure. Finally, he uses the Curry's sentence with a true consequence to obtain transcendence. In this way, he proves that, despite what Priest and the Glutty theorists argue, by first using a Curry's sentence with a false implication, one can fit a Curry's sentence with a true consequence into the Inclosure Schema. All we need to do is bring in some outside information, which is exactly what Priest does in order to show that Curry's Paradox does not fit the Inclosure Schema. Indeed, Priest takes a Curry's sentence whose consequence is true as a counter-example. He shows that the lack of contradiction in some of Curry's instances prevents the general Curry's Paradox from fitting into the Inclosure Schema (Ahmad 2022, 258).

So, for Ahmad, the fact that Priest expresses that only the instances of Curry's Paradox with a false consequence fit the Inclosure Schema uses as much empirical information as he does when he forces this version of Curry's Paradox into the Inclosure Schema. In other words, since Priest already uses empirical information, there is no problem using it to make more versions of the paradox (Ahmad 2022, 258).

All of this allows Ahmad to claim that if we are to do justice to our intuition that Curry's Paradox is in the same family as the Liar and therefore fits the Inclosure Schema, we must force it to fit in by adding empirical information. This is problematic for Ahmad. Indeed 'empirical truths have no place in logic, or in a theory of paradox. Neither a logical system nor a theory of paradox should be able to distinguish whether a sentence, or a consequent of a conditional in this case, is true or false.' (Ahmad 2022, 259). I will later show that this opinion is not shared by everyone (Ahmad 2022, 259).

Ahmad then exposes three options for the supporters of the Inclosure Schema:

- Argue that Curry's Paradox does not fit into the Inclosure Schema
- Argue that Curry's Paradox fits into the Inclosure Schema by adding an empirical information
- Dismiss the Inclosure Schema

(Ahmad 2022, 259)

Ahmad's solution is the third one, to dismiss the Inclosure Schema. Indeed, the two other options are either a miscalibration of the Inclosure Schema, since Curry's Paradox is of the same family as the Liar but does not fit into it, or the statement that the versions of Curry's Paradox with false consequences fit the Inclosure Schema, and so do those with true consequences. But a fourth option is conceivable, the one mentioned earlier in Beall's theory: the sub-paradoxical account (Beall 2014a, 843). As mentioned, sub-paradoxical cases apply when 'a sentential scheme is an inclosure paradox iff some instances are (inclosure) paradoxical' (Beall 2014a, 843). But, for Ahmad, this solution is basically not different from the second, which is to argue that Curry's Paradox fits the Inclosure Schema by adding empirical information. For the version of the paradox with a false consequence to fit into the Inclosure Schema, an empirical element is needed, and the version with a true consequence fits in the same way (Ahmad 2022, 259-260).

Ahmad then mentions the Neo-Curry that could be a solution to the need of empirical information. It would be a version of Curry's Paradox that fits the Inclosure Schema and could make it possible without all other versions with empirical input. This would make Curry's Paradox fit into the Inclosure Schema sub-paradoxically and without empirical information. The problem this time, however, is that we're bringing in useless information. Indeed, this reasoning requires us to bring in extra information that should not be relevant to just know whether Curry's Paradox fits into the Inclosure Schema or not. Those elements are not used in order to express the paradox. We do not express that something and its negation are implied, we express that a chosen claim is implied if this sentence is true. Indeed, at no point in this work have I expressed that if this sentence is true then I am dinosaur and I am not a dinosaur. That is not what we were covering, but rather Curry's Paradox in general. We are concerned by the one that implies anything, not by the one that implies anything and its negation (Ahmad 2022, 260).

Too broad

In a second phase, Ahmad explains why the Inclosure Schema is also too broad. He does so by comparing the Sorites and Curry's Paradox and by showing two trivialising cases. The Sorites Paradox allows us to take a closer look at the lack of limit that characterises vagueness, in other words, the non-existence of a positive case immediately followed by a negative case. The paradox is as follows: one grain of sand is not a heap of sand, 10,000 grains of sand are indeed a heap, but it is complex to determine a quantity n of grains of sand such that n is a heap and $n-1$ is not a heap. It is complex to know from which grain of sand, and not the one before, the grains become a heap of sand. So, if we add each grain of sand one by one until we reach 10,000 grains, having no grain that takes us from 'non-heap' to heap, we end up with 10,000 grains that do not form a heap, which is paradoxical (Oms & Zardini, 2019).

The Sorites fits into the Inclosure Schema and, unlike Curry's Paradox, seems to have a completely different structure than the Liar and the other self-referential paradoxes. The Sorites raises questions since it makes it possible to prove any statement by starting from

the right assertion, which is exactly what Curry's Paradox is accused of. But Ahmad insists on the difference between those two paradoxes. Indeed, he mentions that 'What is paradoxical about a Curry with true consequent is that we can prove the consequent out of thin air. On the other hand, what is paradoxical about the Sorites is reaching a contradiction or an unacceptable conclusion.' (Ahmad 2022, 262). Even if the two paradoxes are different, Ahmad maintains that making the Sorites fit the Inclosure Schema is problematic. Indeed, the Sorites is not based on *ungrounded sentences* or self-referential. By including the Sorites, which is so far removed from the other paradoxes included in the Inclosure Schema, the schema becomes too broad (Ahmad 2022, 260-262).

Ahmad then exposes the two cases that trivialise the Inclosure Schema. The first one is that any contradiction of the form $Pa \wedge \neg Pa$ is included in the Inclosure Schema. Thereby, he shows that the Inclosure Schema is not a good way to identify what makes that paradoxes are paradoxical, since any contradiction can be expressed in a way that fit the Inclosure Schema (Ahmad 2022, 263).

The second way to trivialise the Inclosure Schema is by showing that the Inclosure Schema can include much broader arguments and is therefore too broad. To do this, the author takes Zeno's Paradox example. This paradox shows that it is impossible to reach a certain endpoint, since we always have to cover half the distance we've yet to cover, and this ad infinitum. Ahmad then opposes it to our common sense, which leads us to think that things are in motion. Those two elements can be formulated to make a contradiction in the shape of the Inclosure Schema and thereby it can be shown that the Inclosure Schema is too broad and captures more than a *specific structure of paradox*. It captures way more contradictions (Ahmad 2022, 263-264; Marion 2023: 125-149).

In the last part of his article, Ahmad offers a new recipe for paradoxes, but not for all of them, only for the semantic ones. I will not present it in this work since it does not offer a general solution for semantic and set-theoretic paradoxes and thereby does not challenge the Inclosure Schema as Prior's Schema for example (Ahmad 2022, 265-276).

V. Conclusion

20. Answers to the problematic and the hypothesis

Through this work, I have first tried to give an overview of the extent of Curry's paradox. To do so, I have presented the different theories in which it appears, several versions within the same theory and an overview of the different types of resolution. I have then attempted to formulate an explanation of the Inclosure Schema, of how it encompasses most self-referential paradoxes, and of how it is defended. I then linked this schema to the Principle of Uniform Solution to explain how, once combined together, it constitutes one of many ways to defend dialetheism.

Those explanations enabled me to tackle the central problem of this work: whether or not Curry's Paradox fits into the Inclosure Schema. At the beginning of this work, I have expressed that the problematic would be: 'Does Curry's Paradox fit the Inclosure Schema? And if not, does this paradox belong to the same family as the ones that fit the schema?'. I have also established that my central hypothesis would be that 'Curry's Paradox questions the relevance of the Inclosure Schema' and thereby diminishes the relevance of a defence of dialetheism through the Inclosure Schema (which tells us nothing about the other justifications for dialetheism).

Having set out the different points of view as to whether or not Curry's Paradox is an inclosure paradox and whether or not it fits into the Inclosure Schema, I can now test my hypothesis and attempt to answer my problematic.

Problematic: Does Curry's Paradox fit the Inclosure Schema? And if not, does this paradox belong to the same family as the ones that fit the schema?

I have divided the answers into two points of view ($[1]$, $[2]$), each of which has two sub-answers ($[a]$, $[b]$).

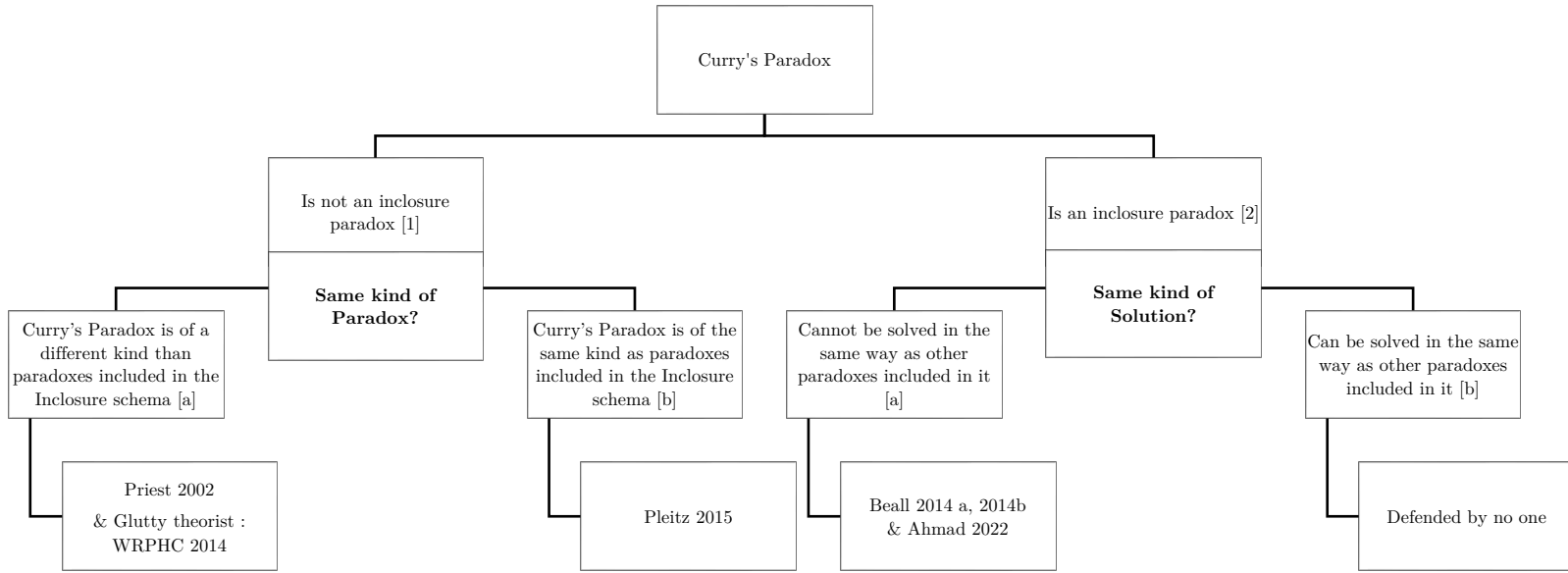
[1] First, I looked at whether the authors were arguing that Curry's Paradox is an inclosure paradox. For those who argued that it is not, I then analysed whether these authors thought Curry's Paradox is of a different kind than paradoxes included in the Inclosure Schema.

[a] To this, the Glutty theorists (Weber, Ripley, Hyde, Colyvan and Priest) reply that, yes, it is of a different kind. They emphasize the difference between Curry's Paradox and the other paradoxes included in the Inclosure Schema. According to them, only certain versions of Curry's Paradox fit into the Schema. Besides, it does not use negation therefore it is fundamentally different (Priest 2002, Weber, Ripley, Hyde, Colyvan & Priest 2014, 813-828).

[b] Pleitz opposes them. To the question of whether Curry's Paradox is of the same kind as the paradoxes included in the Inclosure Schema, he replies that Curry's Paradox is indeed of the same kind: it is a self-referential paradox. For Pleitz, Curry's Paradox has many elements in common with inclosure paradoxes, and it makes no sense to only consider Curry's instances that have a false consequence, as Priest does. Thus, Pleitz puts his advice into practice by proposing a larger schema that brings together the Inclosure Schema and Curry's Paradox. Thereby, all paradoxes of the same kind, those included in the Inclosure Schema and instances of Curry's Paradox are grouped together in the *Most General Schema*, which gives us a broader categorization of paradox's kind (Pleitz 2015, 1-15).

[2] I then went to the second type of answers to my first question: the answer that Curry's Paradox is an inclosure paradox. The question underlying this question is whether Curry's Paradox can be solved in the same way as the other paradoxes included in the Inclosure Schema. Indeed, by sub-questioning in the other first answer whether Curry's Paradox is of the same kind as self-referential paradoxes, I was questioning the kind of paradox. Thereby, within the second general question, I could tackle the kind of solution. This line of questioning enabled me to challenge Inclosure Schema from every angle of the Principle of Uniform Solution. The two answers to the first question [1] allowed me to

know whether the Inclosure Schema proposes the right categorisation of kind of paradox. Then the two answers to the second question [2] allowed me to know whether it is the right determinant for the same kind of solution.



[a] Two authors argue that Curry's Paradox is an inclosure paradox. However, they both agree that Curry's Paradox cannot be solved in the same way as the other paradoxes included in the Inclosure Schema. One of them is Beall. The latter shows several paradoxes which, although they fit into the Inclosure Schema, are not solved in the same way as the other inclosure paradoxes. One of those paradoxes is Curry's Paradox. To show that Curry's Paradox fits into the Inclosure Schema, Beall asks, among other things, 'What is it for a sentential scheme to be (inclosure-) paradoxical?' (Beall 2014a, 842). He shows that no answer is satisfactory. Either we solve Curry's Paradox wrongly, since we include certain paradoxes that cannot be solved by dialetheism in the Inclosure Schema, which

leads to triviality; or we do not include into the schema some versions of Curry's Paradox that are just other ways of expressing some of the paradoxes included in the Inclosure Schema, and so we miss targets. For a more detailed outline of this dilemma, see the diagram on page 76 (Beall 2014a 829–849; Beall 2014b 791–811).

The other author that defends this point of view is Ahmad. He shows that Curry's Paradox, despite what we might think, uses negation. He also shows that if we use the same reasoning as Priest, i.e. we involve empirical information, any instance of Curry's Paradox fits into the Inclosure Schema. Ahmad then exposes two cases that trivialise the Inclosure Schema, these cases showing that the schema includes every contradiction and does not explain where the paradoxical aspect of the paradoxes comes from (Ahmad 2022, 254–281)

[b] The position according to which Curry's Paradox is included in the Inclosure Schema and can be solved in the same way as other paradoxes is not defended since it leads to triviality.

We can observe several elements of these 'answers' to my problematic.

First of all, I have to say that I am rather surprised. When I started this work, I expected the main criticism to be that the Inclosure Schema was not wide enough. Indeed, a special paradox seemed to be different from paradoxes included in the schema, but at the same time very similar to them. I expected most opponents of the Inclosure Schema to argue that Curry's Paradox is of the same kind as paradoxes in the Inclosure Schema, not to argue that Curry's Paradox actually fits into the schema. I expected most opponents of the Inclosure Schema to argue that the Inclosure Schema is not wide enough, as Plietz does.

But two of the authors, Ahmad and Beall, agree that the Inclosure Schema is far too broad. Beall argues this by showing two examples of paradoxes that fit into the schema

even though they have a different kind of solution, namely the Barber and Curry's Paradox. Ahmad does the same, but even more broadly. He shows that not only Curry's Paradox, but any contradiction can fit into the Inclosure Schema, whereas not all contradictions can be solved with the same kind of solution. If we respect the Principle of Uniform Solution, this shows that the Inclosure Schema is not a good way of determining which paradoxes are of the same kind or that the dialectical resolution is not the right one (Beall 2014a, 841-847, Ahmad 2022, 261-265).

Furthermore, it seems to me that the critics of the Inclosure Schema have the opposite reasoning to that which Priest and the Glutty theorists attribute to them. Indeed, they are telling people who think Curry's Paradox fits into the Inclosure Schema to take a broader view and to look at all the other versions of Curry's Paradox that do not fit into the Inclosure Schema. They argue that you should not take particular cases as generalisations. It seems, on the contrary, that the opponents of the Inclosure Schema manage to have a much more general point of view than the Glutty theorists. Firstly, for the reason I have just stated: because Beall and Ahmad see how the Inclosure Schema is too broad to be a good indicator of a kind of Paradox (Weber, Ripley, Hyde, Colyvan & Priest 2014, 821-824).

Secondly, Plietz and Prior propose other schematics that incorporate Curry's Paradox into the sort of paradoxes included in the Inclosure Schema. On one hand, Plietz offers a schema that is simply broader, which include the Inclosure Schema and Curry's Paradox. It will not be criticised for being too narrow, but it risks however to suffer from the same criticism as the Inclosure Schema of being too broad. On the other hand, Prior proposes a precise, all-encompassing schema that includes Curry's Paradox and self-referential paradoxes. This schema proposes a generalisation of the various stages leading to either contradiction or triviality. It makes it possible to identify what differentiates Curry's Paradox from other self-referential paradoxes, but also what they have in common. Perhaps they can be criticised, but certainly not for not having a general enough vision (Prior 1955, 177-182; Plietz 2015, 11-12).

The last element that convinces me that the opponents of the Inclosure Schema have a very general point of view is that they are not satisfied with certain variants of Curry's Paradox. They use those variants to reason and understand whether the Inclosure Schema satisfies them, but what they are really looking for is a general categorization of Curry's Paradox, not just one specific case, as the Glutty theorists would have us believe. Indeed, I have mentioned that for Pleitz, Curry's Paradox has many elements in common with inclosure paradoxes, and it makes no sense to only consider Curry's instances that have a false consequence. Similarly, Ahmad mentions Neo-Curry, but it requires us to bring in extra information that should not be relevant to just know whether Curry's Paradox fits into the Inclosure Schema (Pleitz 2015, 1-7; Ahmad 2022, 260).

Hypothesis: Curry's Paradox questions the relevance of the Inclosure Schema

I therefore suggest that our hypothesis is confirmed. Even if, among the logicians quoted in this work, there remains a larger number who defend the Inclosure Schema, namely Weber, Ripley, Hyde, Colyvan and Priest, there are still two very different types of point of view defended that threaten the Inclosure Schema. Indeed, both the view that Curry's Paradox does not fit into the Inclosure Schema but is of the same kind as those included in it, and the view that Curry's Paradox is an inclosure paradox, threaten the Inclosure Schema.

In the end, only the logicians who believe that Curry's Paradox is not an inclosure paradox and who also believe that it is of a different kind from paradoxes included in the Inclosure Schema, and only them, can continue to support the inclosure schema as a defence of dialetheism. These are two very specific and well-challenged conditions. Indeed, there are many arguments against them. Thereby, I am not saying that Curry's Paradox makes the Inclosure Schema ineffective, but rather that it threatens it, and that the responses to these threats have not yet seemed sufficient.

21. Opening

I would like to finish this work with a few opening possibilities which, if space and time had allowed, would have greatly interested me.

First, there seems to be a lot to say about Prior's schema. In general, all authors who advocate a strategy other than the inclosure strategy emphasize the similarities between self-referential paradoxes and Curry's Paradox. Indeed, Ahmad argues that Curry's Paradox is way closer to the Liar than we could think, as they share the same pattern. Furthermore, Pleitz proposes a comparison between the elements that make up a self-referential paradox and identifies the same ones in Curry's Paradox. Like these two authors, Prior identifies the similarities between Curry's Paradox and other self-referential paradoxes. The difference is that he goes further. Prior's schema is not widely discussed in the literature, yet it offers what would seem to be a perfect alternative to the Inclosure Schema. It is a schema that is complete enough to account for all self-referential paradoxes, but it is not so broad that you can fit any contradiction into it (Prior 1955, 177-182).

Then, with more time, I would go a step further in Ahmad's critique of Priest's empirical input. I would look at the extent to which empirical information is not an inherent empirical element of paradoxes. To do this, I would use Kripke 1975 (690-716), which seems to point in this direction. This would then allow me to analyse the suitability of Ahmad's answer to Priest, in which he proves that Curry's instances with true consequences can fit into the Inclosure Schema if another Curry's instance with false consequence is brought (Ahmad 2022, 258-260).

Another thing I would like to spend more time on is the sentence Ahmad enunciates about Priest's defence, which, to me, seems to misrepresent the context in which Priest uttered the words Ahmad quotes. Ahmad says that 'Priest responds to this trivializing case as follows' and then quotes something that does not seem to be an answer to this trivializing case but to the pertinence of Russell's Schema (Priest 2002, 135; Ahmad 2022, 263).

The last opening that I would like to propose is to investigate the vague limits of the concept of kind. This would perhaps allow us, for example, to formulate a contextualist categorization of the kind of paradox, where, depending on the situation in which we find ourselves, the reading of self-referential paradoxes seems more relevant through the Inclosure Schema or Prior's schema. Or perhaps a many-valued categorization of the kind of paradox, where the Liar is more of a self-referential paradox than Curry's Paradox, which is still partly one (Oms & Zardini, 2019).

VI. Bibliography

AHMAD, R., « A Recipe for Paradox », in *Australasian Journal of Logic*, XIX, n°5 (2022), pp. 254-281.

BEALER, G., and MÖNNICH, U., « Property Theories », in *Handbook of Philosophical Logic*, IV (1989), pp. 133-251.

BEALL, J.C., « End of Inclosure », in *Mind*, CXXIII, n° 491 (2014a), pp. 829-849.

BEALL, J.C., « Finding Tolerance Without Gluts », in *Mind*, CXXIII, n° 491 (2014b), pp. 791-811.

BEALL, J.C., « Free of Detachment: Logic, Rationality, and Gluts », in *Noûs*, XLIX, n°2 (2015), pp. 410-423.

BEALL, J.C., and MURZI, J., « Two flavors of Curry's paradox », in *The journal of Philosophy*, CX, n°3 (2013), pp. 143-165.

BEALL, J.C., GLANZBERG, M., and RIPLEY, D., «Liar Paradox », in *The Stanford Encyclopedia of Philosophy* (Winter 2023 Edition), eds. ZALTA, N. E., & NODELMAN, U., 2016, URL = <<https://plato.stanford.edu/archives/win2023/entries/liar-paradox/>>, consulted on 10/08/2024 at 01h05.

BIMBO, K., « Curry-Type Paradoxes », in *Logique & Analyse*, XLIX, n°195 (2006), pp. 227-240.

BOURGET, D., and CHALMERS, D. J., « Philosophers on Philosophy: The 2020 PhilPapers Survey », in *Philosophers' Imprint*, XXIII, n°11 (2023).

BURGE, T., « Semantical Paradox », in *Journal of Philosophy*, LXXVI, n°4 (1979), pp. 169-198.

BURGIS, B., and BUENO, O., « Liars with Curry: Dialetheism and the Prospects for a Uniform » in RIEGER, A., and YOUNG, G., (ed) *Dialetheism and its Applications*, Dordrecht, Springer (coll. « Trends in Logic » LII), 2019.

CANTOR, G., « Über die verschiedenen Ansichten in Bezug auf die actualunendlichen Zahlen », in *Bihang Till Koniglen Svenska Vetenskaps Akademiens Handligar*, XI, n°19 (1886), pp. 1-10.

CARRARA, M., and MARTINO, E., « Curry's Paradox. A new Argument for Trivialism » in *Logic and Philosophy of Science*, IX, n°1 (2011), pp. 199-206.

CATTABRIGA, P., « Diagonalization As A Definition », 2019, URL = <https://www.researchgate.net/publication/334942524_DIAGONALIZATION_AS_A_DEFINITION> consulted on 02/08/2024 at 18h58.

CURRY, H., « The Combinatory Foundations of Mathematical Logic », in *Journal of Symbolic Logic*, VII n°2 (1942a), pp. 49-64.

CURRY, H., « The Inconsistency of Certain Formal Logics », in *Journal of Symbolic Logic*, XVII, n°3 (1942b), pp. 115-117.

ESTRADA-GONZÁLEZ, L., « Models of Possibilism and Trivialism », in *Logic and Logical Philosophy*, n ° 21 (2012), pp. 175-205.

GABBAY, D.M. and WANSING, H. (eds), « What is Negation? », Dordrecht, Springer, 1999.

GEACH, P.T., « On Insolubilia », in *Analysis*, XV, n°3 (1955), pp. 71-72.

GLANZBERG, M., « A Contextual-Hierarchical Approach to Truth and the Liar Paradox », in *Journal of Philosophical Logic*, XXXIII, n°1 (2004), pp. 27-88.

GLANZBERG, M., « The Liar in Context », in *Philosophical Studies*, CIII, n°3 (2001), pp. 217-251.

GLANZBERG, M., « Truth », in *The Stanford Encyclopedia of Philosophy* (Fall 2023 Edition), eds. ZALTA, N. E., & NODELMAN, U., 2018, URL = <<https://plato.stanford.edu/archives/fall2023/entries/truth/>>, consulted on 10/08/2024 at 01h16.

GOODSHIP, L., « On Dialethism », in *Australasian Journal of Philosophy*, LXXIV, n°1 (1996), pp. 153-161.

GRELLING, K., and NELSON, L., « Bemerkungen zu den Paradoxien von Russell und Burali-Forti », in *Abhandlungen der Fries'schen Schule*, II (1908), pp. 301-334.

GUPTA, A., and BELNAP, N., *The Revision Theory of Truth*, Cambridge, MIT Press, 1993.

HALLET, M., *Cantorian Set Theory and Limitation of Size*, Oxford, Oxford Clarendon Press, 1984.

IRVINE, A.D., and DEUTSCH, H., « Russell's Paradox », in *The Stanford Encyclopedia of Philosophy* (Spring 2021 Edition), ed. ZALTA, N. E., & NODELMAN, U., 2020, URL = <<https://plato.stanford.edu/archives/spr2021/entries/russell-paradox/>>, consulted on 10/08/2024 at 01h21.

KABAY, P., *A defense of trivialism*, PhD thesis, Melbourne, University of Melbourne, 2008.

KRIPKE, N.A., « Outline of a Theory of Truth », in *The Journal of Philosophy*, LXXII, n° 19 (1975), pp. 690-716.

LÖB, M. H., « Solution of a Problem of Leon Henkin », in *Journal of Symbolic Logic*, XX, n°2 (1955), pp. 115-118.

MARES, E., « Belief revision, probabilism, and logic choice » in *Review of Symbolic Logic*, VII (2014), pp. 647-670.

MARION, F., *Modalité et changement. δύναμις et cinétique aristotélicienne*, PhD Thesis, Louvain-la-Neuve, Université Catholique de Louvain, 2023, pp. 95-149.

MEYER, K.R., ROUTLEY, R., and DUNN, J. M., “Curry’s paradox”, in *Analysis*, XXXIX, n°3 (1979), pp.124-128.

MOH SHAW-KWEI, « Logical Paradoxes for Many-Valued Systems », in *Journal of Symbolic Logic*, XIX, n°1 (1954), pp. 37-40.

NEWHARD, J., « Grelling’s Paradox » in *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition*, CXXVI, n°1 (2005), pp. 1-27.

NICOLAI, C., and ROSSI, L., « Principles for Object-Linguistic Consequence: from Logical to Irreflexive », in *Journal of Philosophical Logic*, XLVII (2017), pp. 549–577.

ØGAARD, T.F., « Paths to Triviality » in *Journal of Philosophical Logic*, XLIII, n° 3, (2016), pp. 237-276.

OLIN, D., *Paradox*, Stocksfield, Acumen Publishing, 2002.

OMS, S., (éd.) ZARDINI, E., (éd.) ASGEIRSSON, H., BONES, I., COBREROS, P., ÉGRÉ, P., EKLUND, M., KENNEDY, C., KURTSAL, I., MAGIDOR, O., PAOLI, F., PRIEST, G., RIPLEY, D., SANTOS, R., SOAMES, S., TRANCHINI, L., VERHEYEN, S., and WRIGHT, C., *The Sorites Paradox*, Cambridge, Cambridge University Press, 2019, pp. 4-173.

ORILIA, F., and PAOLINI PAOLETTI, M., « Properties », in *The Stanford Encyclopedia of Philosophy* (Spring 2022 Edition), ed. ZALTA, N. E., 2020, URL =

<<https://plato.stanford.edu/archives/spr2022/entries/properties/>>, consulted on 10/08/2024 at 01h28.

PLEITZ, M., « Curry's Paradox and the Inclosure Schema », in *Logica Yearbook 2014*, ed. ARAZIM, P., and DANČÁK, M., 2015, pp. 233-248.

PRIEST, G., *An Introduction to Non-Classical Logic. From if to is, Second Edition*, Cambridge, Cambridge University Press (coll. « Cambridge Introduction to Philosophy»), 2008.

PRIEST, G., *Beyond the Limits of Thought*, Second Edition, Oxford, Oxford University Press, 2002.

PRIEST, G., *In Contradiction: A Study of the Transconsistent*, Second Edition, Oxford, Oxford University Press, 2006.

PRIEST, G., « The Trivial Object and the Non-Triviality of a Semantically Closed Theory with Descriptions », in *Journal of Applied Non-Classical Logics*, VIII (1998), pp. 171-183.

PRIEST, G., « What If ? The Exploration of an Idea » in *Australasian Journal of Logic*, XIV, n°1 (2017), pp. 57-123.

PRIEST, G., and ROUTLEY, R., « Lessons from Pseudo Scotus », in *Philosophical Studies*, XLII (1982), pp. 189-199.

PRIEST, G., BERTO, F., and WEBER, Z., "Dialetheism", *The Stanford Encyclopedia of Philosophy* (Summer 2024 Edition), eds. ZALTA, N. E., & NODELMAN, U., 2024, URL = <<https://plato.stanford.edu/archives/sum2024/entries/dialetheism/>>, consulted on 10/08/2024 at 01h29.

PRIEST, G., ROUTLEY, R., and NORMAN, J. (eds.), *Paraconsistent Logic. Essays on the Inconsistent*, München, Philosophia, 1989.

PRIEST, G., TANAKA, K., and WEBER, Z., « Paraconsistent Logic », in *The Stanford Encyclopedia of Philosophy* (Spring 2022 Edition), ed. ZALTA, N. E., 2022, URL = <<https://plato.stanford.edu/archives/spr2022/entries/logic-paraconsistent/>>, consulted on 10/08/2024 at 01h31.

PRIOR, A.N., « Curry's Paradox and 3-valued Logic », in *Australasian Journal of Philosophy*, XXXIII, n°3 (1955), pp. 177-182.

RAMSEY, F., « The Foundations of Mathematics », in *the London Mathematics Society*, XXV, n°2 (1925), pp. 338-384.

READ, S., "Self-Reference and Validity", in *Synthese*, XLII, n° 2 (1979), pp. 269-270.

READ, S., *Thinking About Logic. An Introduction to the Philosophy of Logic*, Oxford, Oxford University Press, 1994.

RESTALL, G., « Curry's Revenge: The Costs of Non-classical Solutions to the Paradoxes of Self-reference » in BEALL, J.C. (ed.), *Revenge of the Liar. New Essays on the Paradox*, Oxford, Oxford University Press, 2007, pp. 262-271.

RESTALL, G., « Negation in Relevant Logics (How I Stopped Worrying and Learned to Love the Routley Star) », in GABBAY, D.M., WANSING, H. (eds) *What is Negation?*, Dordrecht, Springer, (Coll. « Applied Logic Series » XIII), 1999, pp. 53-76.

RIPLEY, D., « Paradoxes and Failures of Cut », in *Australasian Journal of Philosophy*, XCI (2013), pp. 139-164.

ROGERSON, S., and RESTALL, G., « Routes to Triviality », in *Journal of Philosophical Logic*, XXXIII, n°4 (2004), pp. 421-436.

ROUTLEY, R., *Exploring Meinong's Jungle and Beyond. An Investigation of Noneism and the Theory of Items*, Canberra, Australian National University, 1980.

ROUTLEY, R., and PRIEST, G., « Lessons from Pseudo-Scotus » in *Philosophical Studies*, XLII (1982), pp. 189-199.

ROUTLEY, R. and ROUTLEY, V., « Negation and Contradiction » in *Revista Colombiana de Matematicas*, IXX (1985), pp. 201-231.

ROUTLEY, R., and WEBER, Z. (ed.), *Ultralogic as Universal?*, Dordrecht, Springer (coll. « The Sylvan Jungle », IV), 2019.

ROUTLEY, R., PLUMWOOD, V., MEYER, R. K. and BRADY, R. T., *Relevant Logics and Their Rivals. Part. 1. The Basic Philosophical and Semantical Theory*, Atascadero, Ridgeview, 1982.

RUSSELL, B., « Les paradoxes de la logique », in *Revue de Métaphysique et de Morale*, XIV, n°5 (1906), pp. 627-650.

RUSSELL, B., « Mathematical Logic as Based on the Theory of Types », in *American Journal of Mathematics*, XXX, n°3 (1908), pp. 222-262.

RUSSELL, B., « On Some Difficulties in the Theory of Transfinite Numbers and Order Types », in *Proceedings of the London Mathematical Society*, II, N° 4 (1905), pp. 29-53.

RUSSELL, B., *Principles of Mathematics*, Cambridge University Press, Cambridge, 1903.

SAINSBURY, R.M., *Paradoxes*, Cambridge, Cambridge University Press, 1995, pp. 111-112.

SHAPIRO, L., and BEALL, J.C., « Curry's Paradox », in The Stanford Encyclopedia of Philosophy (Winter 2021 Edition), ed. ZALTA, N. E., 2018, URL = <<https://plato.stanford.edu/archives/win2021/entries/curry-paradox/>>, consulted on 10/08/2024 at 01h02.

SIMMONS, K., *Universality and the Liar: An Essay on Truth and the Diagonal Argument*, Cambridge, Cambridge University Press, 1993.

SYLVAN, R., « What is that Item Designated Negation? », in Gabbay, D.M., Wansing, H. (eds) *What is Negation?*, Dordrecht, Springer, (Coll. « Applied Logic Series » XIII), 1999, pp. 299-324.

TAPIA-NAVARRO, M.E., and ESTRADA-GONZÁLEZ, L., « When Curry met Abel », in *Logic Journal of the IGPL*, XXVIII, n° 6 (2020), pp. 1233-1242.

TARSKI, A., « The Concept of Truth in Formalized Languages », in *Logic, semantics, metamathematics*, Oxford, Clarendon Press, 1956, pp. 152-278.

TARSKI, A., « The Semantic Conception of Truth and the Foundations of Semantics », in *Philosophy and Phenomenological Research*, IV, n°3 (1944), pp. 341-376.

WHITEHEAD, A.N., and RUSSELL, B., *Principia Mathematica*, 3 vols, Cambridge, Cambridge University Press, 1910, 1912 and 1913.

WOODS, M., *Conditionals*, Oxford, Oxford University Press, 1997.

WOODS, J., *Paradox and Paraconsistency. Conflict Resolution in the Abstract Sciences*, Cambridge, Cambridge University Press, 2003.

