

École polytechnique de Louvain

Study on the performance of robot feet on sand with varying saturation levels

Author: **Vithit GUILMAIN**

Supervisors: **Vincent LEGAT, Jonathan LAMBRECHTS**

Readers: **Michel HENRY, Hadrien RATTEZ**

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Abstract

Locomotion in granular media, such as dry or wet sand, presents significant challenges for legged robots due to the variability in the soil's response to intrusion at different saturation levels. This study aims to evaluate the performance of various foot geometries, including concave shapes, in dry and low-saturation sand, with the goal of improving robotic mobility in such environments. Experiments were conducted using controlled intrusion tests in sand at various saturation levels, supported by 2D Discrete Element Method (DEM) simulations and analytical modeling using Resistive Force Theory (RFT). The results show that convex foot geometry generates the lowest vertical resistive force, while the flat foot exhibits the highest. Concave foot geometry demonstrates improved resistive force performance at greater intrusion depths, particularly under low-saturation conditions. Implementation of cohesion in the DEM solver enables the observation of granular structures and underlying physical mechanisms that link intruder shape to resistive force in both dry and wet granular media. These findings offer design insights for engineers developing legged robots for locomotion in challenging granular terrains.

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Contents

	Abstract	iii
	Acknowledgments	v
	List of Symbols	ix
1	Introduction	1
2	Dry sand	5
	2.1 Experiments	5
	2.1.1 <i>Experimental setup</i>	5
	2.1.2 <i>Experimental results</i>	8
	2.2 Resistive force theory (RFT)	10
	2.3 Discrete Element Method (DEM)	12
	2.3.1 <i>Non-smooth contact dynamics</i>	12
	2.3.2 <i>Simulation results</i>	18
	2.4 Discussion	28
3	Wet sand	31
	3.1 Experiments	32
	3.1.1 <i>Experimental setup</i>	32
	3.1.2 <i>Experimental results</i>	33
	3.2 Extended RFT	36
	3.3 Cohesive DEM simulations	37
	3.3.1 <i>Liquid-bridge contact model</i>	37
	3.3.2 <i>Implementation in MigFlow</i>	39
	3.3.3 <i>Simulation results</i>	40
	3.4 Discussion	42
4	Conclusion	45
	Appendices	47
A	Soil Evaluation Tool	48
B	Feet technical drawings	50

C	Convex and Concave feet RFT	52
3.1	Convex case	52
3.2	Concave case	53
Note		55
Bibliography		57

List of Symbols

Symbol	Definition	Units
A	Feet frontal area	$[m^2]$
d_m	Mean particle diameter	$[m]$
$\mathbf{E}$	Explicit external forces	$[N]$
$\mathbf{F}_0$	Characteristic force for nondimensionalization	$[N]$
$\mathbf{F}_c$	Cohesive force	$[N]$
$\mathbf{F}_z$	Vertical resistive force	$[N]$
g	Inter-particle gap	$[m]$
h	Cell size for stress discretization	$[m]$
$\mathbf{l}^c$	Branch vector of the contact c	$[m]$
m_{sand}	Mass of sand in the sandbox	$[kg]$
$\mathbf{P}$	Sum of all impulses implied during a time step	$[kg \cdot m/s]$
P_r	Grains polydispersity	$[-]$
R	Cylindrical feet radii	$[m]$
$\mathbf{R}^c$	Contact force vector at the contact c	$[N]$
R_e	Radius of the hemispherical cavity	$[m]$
$\mathbf{R}_N$	Normal contact force component	$[N]$
$\mathbf{R}_T$	Tangential contact force component	$[N]$
r_{max}	Maximum particle radius	$[m]$
r_{min}	Minimum particle radius	$[m]$
S_r	Saturation level	$[-]$
t^*	Dimensionless intrusion time	$[-]$
$\mathbf{U}_c$	Additional velocity induced by cohesion	$[m/s]$
$\mathbf{U}_{free}$	Free relative velocity	$[m/s]$

Symbol	Definition	Units
$\mathbf{U}_N$	Normal relative velocity	$[m/s]$
$\mathbf{U}_T$	Tangential relative velocity	$[m/s]$
V_b	Liquid bond volume	$[m^3]$
V_{sand}	Volume of Silica Sand No. 5	$[m^3]$
V_{soil}	Bulk volume of the soil	$[m^3]$
V_{void}	Volume of void in the soil	$[m^3]$
V_{water}	Volume of water added to the dry soil	$[m^3]$
$\mathbf{v}_z$	Vertical intrusion velocity	$[m/s]$
$\mathbf{W}$	Delassus matrix	$[1/kg]$
z_f	Final intrusion depth	$[m]$
α_z	Vertical stress per unit depth	$[N/m^3]$
$\alpha_{z,max}$	Maximum vertical stress per unit depth	$[N/m^3]$
β_s	Angle of attack of a plate s	$[rad]$
γ	Liquid-vapor surface tension	$[N/m]$
γ_s	Angle of intrusion of a plate s	$[rad]$
ϵ	Soil porosity	$[-]$
θ	Particle-liquid-gas contact angle	$[rad]$
θ_{MC}	Shear band angle	$[rad]$
μ_{S-S}	Sand inter-particle friction coefficient	$[-]$
μ_{A-S}	Aluminum-sand wall friction coefficient	$[-]$
ρ_{sand}	Density of Silica Sand No. 5	$[kg/m^3]$
ρ_{soil}	Soil bulk density	$[kg/m^3]$
$\langle \sigma \rangle$	Average stress tensor	$[N/m^2]$
$\langle \sigma_{VM} \rangle$	Average von Mises stress equivalent	$[N/m^2]$
ϕ	Internal friction angle	$[rad]$

1

Introduction

Before roads existed, humans and their vehicles had to traverse natural terrains, which often posed significant challenges due to their irregularity and unpredictability. Even today, the ability to move efficiently over such landscapes remains crucial in many fields, including exploration, search and rescue, agriculture, and military operations. This ongoing need gave rise to the field of terramechanics, which focuses on understanding and optimizing the interactions between locomotion systems (such as wheels, legs, tracks, ...) and natural terrain. These environments, such as sandy deserts, snow covered landscapes, post-disaster zones, or extraterrestrial surface, often consist of granular materials that behave differently from conventional solids and fluids.

The discipline emerged formally in the 1950s, mainly through the work of M.G. Bekker, that introduced models for pressure-sinkage, shear stress and rolling resistance for wheeled and tracked vehicles on soft soils (snow, clay, dry sand, ...) [1]. Then, the beginning of planetary exploration, notably through the Apollo missions, drove the need for the development of specialized terramechanics models for lunar and Martian rover design, taking into account regolith (very fine granular soil) and low-gravity environments [2] [3] [4]. The immobilization of NASA's Spirit rover on Mars in 2009 [5] proves how challenging granular soil can be.

Nowadays, terramechanics does not only focus on wheel and tracked vehicles; there are biomimetic approach such as legged locomotion ([6], [7]) and undulatory motion ([8], [9]), which are increasingly applied to robotic operation on granular soil. A comprehensive comparison of the different locomotion systems on unstructured environments was conducted in [10]. Within this context, legged robots offer some notable advantages over wheeled systems, particularly in terms of adaptability and agility on uneven or low-cohesion terrains such as dry sand or partially saturated soil. However, their typically high energy consumption and limited maximum speed constrain their use to small-scale applications, such as search and rescue, data gathering, or even paleo-robotics [11] (the use of robots to study how the ancestors of modern land ani-

mals transitioned from swimming to walking).

The base of legged locomotion on dry non-cohesive granular soil was laid with the introduction of the Resistive Force Theory (RFT) [12]. It was validated experimentally and extended to undulatory motion in [13]. Further advancements include the extension of RFT into three dimensions (3D RFT) [14], to cohesive powder-scale media [15], where cohesion arises from Van der Waals forces, and to oversaturated granular soil [16], where the ground reaction becomes speed-dependent. A modified Archimedes's law was also proposed in [17] to explain penetration of flat and convex solids into granular media.

Terramechanics approaches also make use of numerical simulations, such as the Discrete Element Method (DEM), to model the behavior of granular soils. Granular media consist of a collection of discrete grains that ranges between $10\ \mu\text{m}$ (fine powders) to $10\ \text{cm}$ (gravel). These materials can behave like a solid (when at rest in a sand heap), a liquid (when subjected to sufficient stress), or even like a gas (when highly agitated) [18]. This versatility in behavior arises from complex grain-scale interactions and dynamics. Therefore, unlike continuum approaches, DEM simulates each particle as an individual entity, allowing for the modeling of contact interactions, friction, cohesion, and particle rearrangement.

The method was first introduced in 1979, in order to model numerically granular materials in the context of rock mechanics [19]. Then this useful tool quickly found its place in the field of terramechanics, notably given the ease of selecting one of the numerous existing and validated models of friction or cohesion between particles to suit the studied application [20]. Therefore, interaction of locomotion system with non-cohesive soil like dry sand [21] as well as cohesive soil like lunar regolith [22] were studied. A multiphase approach, combining the Discrete Element Method (DEM) with other numerical tools such as the Finite Element Method (FEM) or Smoothed Particle Hydrodynamics (SPH), has been employed to model the interaction between moving bodies and oversaturated granular soils, as demonstrated in recent studies [23] [24].

As previously discussed, legged locomotion on granular media holds significant promise but still presents numerous challenges, particularly in the design of robotic feet. Various foot shapes have been investigated, including flat and convex geometries, yet concave-shaped feet remain largely unexplored. In addition, the effect of soil saturation on locomotion performance continues to pose an open challenge in terramechanics. While the behavior of dry and fully saturated sand has been relatively well documented, the intermediate saturation state is still poorly understood. This partially saturated condition is common in many real-world environments, such as coastal zones, riverbanks, semi-flooded disaster areas, and construction sites. These terrains require robust and adaptable robots capable of navigating their moisture-variable surfaces. Therefore, a comprehensive understanding of how foot shape interacts with partially

saturated granular soils is still lacking.

This master's thesis aims to fill this gap by evaluating the performance of different foot shapes (in particular concave geometries) on granular soils under varying saturation levels, including dry and partially saturated. A pluralistic terramechanics approach is employed, with experimental testing of designed feet shape under different saturation levels, comparison with existing analytical models and finally, DEM simulations to gain insights of the physics behind the soil reaction, and particularly the vertical soil resistive force. By doing so, this thesis aims to provide guidelines for engineers who need to design robust robots that can operate efficiently on these challenging soils.

The simulation software used in this work is MigFlow (**Model for Immersed Granular Flows**) [25]. It is a mixed computational fluid dynamic - discrete element method (CFD-DEM) model that was developed to simulate flows involving both fluids and grains. The grains dynamics is solved at the grain scale using DEM, while the fluid is solved at a coarse scale using finite element method (FEM). The coupling of the 2 phases is realized by using the concept of porosity (local volume fraction of fluid), as well as an empirical drag formula. The ease with which models can be modified and implemented in MigFlow allows lots of application: pile driving in civil engineering, drug powders mixing in pharmaceutical industry, stenosis simulation, etc. However, in this work, dry and low saturated soil only require the use of MigFlow DEM solver.

This master's thesis is divided in three main part: the dry sand case, the wet sand case and the final conclusion. In the first part, the experimental setup and procedure are presented, followed by a discussion of the dry sand results. These results are compared with those obtained using RFT, after a brief presentation of this model applied to the three evaluated foot shapes. Then, the Non-Smooth Contact Dynamics method employed in the DEM MigFlow simulations is concisely explained before diving into the simulation setup and results: force-depth curves, soil stress distributions, force chain networks and velocity fields. This chapter ends with an overall comparison and discussion of the different approach. The second chapter addresses the wet sand case. It begins with an introduction to the different regimes of saturation and their underlying mechanisms. Following the experimental results, a comparison with an extended RFT models for low-saturated sand is proposed. Then, a cohesive law for low saturation conditions (pendular regime, from 0 to 30% saturation) is described, along with its implementation in MigFlow. Finally, the results of the cohesive DEM simulation under low saturation are presented and discussed. The thesis concludes with a final chapter summarizing the main findings and offering perspectives for future work.

2.1 Experiments

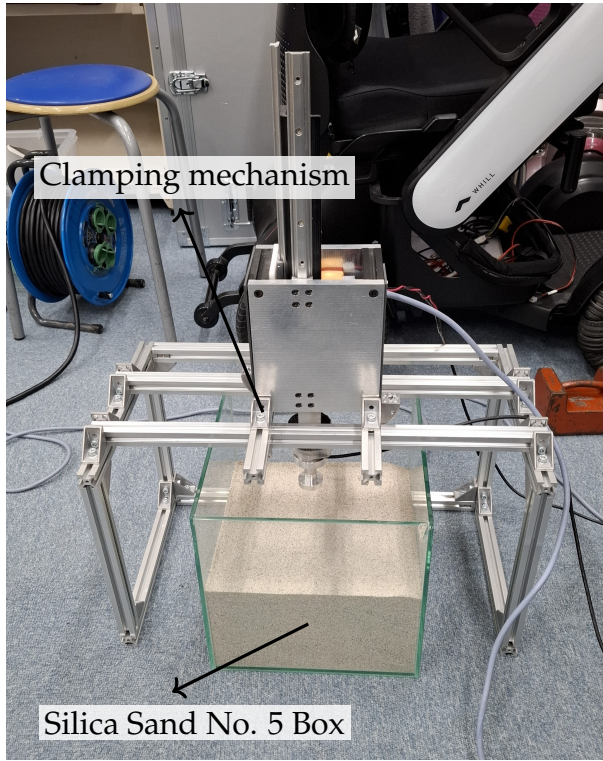
2.1.1 Experimental setup

This section focus on the study of the experimental performance of three aluminum feet shape on dry granular media. The chosen soil is Silica sand No. 5, a coarse sand from the Tohoku region in northern Japan. This sand is well-characterized and has been studied in various condition, including in artificial gravity generator at International Space Station [26]. The main properties of Silica sand No.5 are listed in table 2.1. A friction angle ϕ of 33° was assumed based on published data for loose silica sand [27].

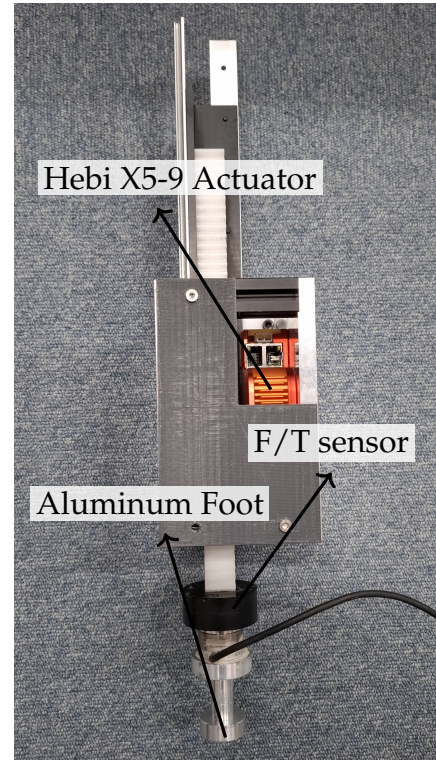
Physical characteristic	Value
Density ρ_{sand}	2648 [kg/m^3]
Maximum radius r_{max}	1.16 [mm]
Minimum radius r_{min}	0.375 [mm]
Friction angle ϕ	33 [$^\circ$]

Table 2.1 Silica sand No. 5 properties.

To conduct this study, an experimental setup was used to evaluate the performance of the 3 feet that were designed. The setup includes a sandbox with dimensions of 250 [mm^3], filled with a mass $m_{sand} = 10$ [kg] of the studied soil. A support structure holds the motion mechanism vertically in place (see 2.1a). The vertical motion is driven by a Hebi X5-9 actuator, which moves the probe at a constant speed of $v_z = 10$ [mm/s]. The actuator is coupled to a rack-and-pinion system, enabling precise linear movement. An aluminum foot is mounted to the system and connected to a force-torque (F/T) sensor, which measures the soil's reaction on the foot during its motion (see 2.1b).



(a) Sand Box and actuator support



(b) Actuator with F/T mounted sensor

Figure 2.1 Experimental Setup.

The soil was left uncompacted after mixing, to replicate the conditions of a natural sandy surface. The volume occupied by 10 kg of Silica sand No.5 is approximately $V_{soil} = 7.5 [l]$. As for the actual volume occupied by the grains, it is computed as:

$$V_{sand} = \frac{m_{sand}}{\rho_{sand}} = 3.776 [l]$$

Therefore, the volume of void in the soil can be obtained:

$$V_{void} = V_{soil} - V_{sand} = 3.724 [l]$$

This lead to the soil experimental porosity (also called void ratio):

$$\epsilon = \frac{V_{void}}{V_{soil}} = 0.497 \quad (2.1)$$

Which indeed, corresponds to loose sand conditions that can be commonly found on the surface of natural environments [28].

The feet design was a key aspect of the experimental setup. In order to evaluate the performance of concave-shaped feet on granular soil, it was decided to design and manufacture three cylindrical feet, each featuring a different bottom geometry: flat, concave (hemispherical), and convex (hemispherical cavity). The radii R of each of the cylindrical foot were chosen to be equal, so that they all present the same frontal area A , in other words, the same footprint. The concavity only influence the orientation and the size of the contact area. The dimensions of these feet must satisfy two main constraints: avoiding boundary effects caused by the box walls, and remaining in the linear soil response depth. Therefore, a radius $R = 15.0 \text{ [mm]}$ was chosen, so the intruder always remains at a distance bigger than 60 mm from the wall [12]. The height of the feet was designed so that each foot can reach an intrusion depth of at least 40 mm . Considering these constraints and design choices, the foot design followed naturally (see Fig. 2.2).

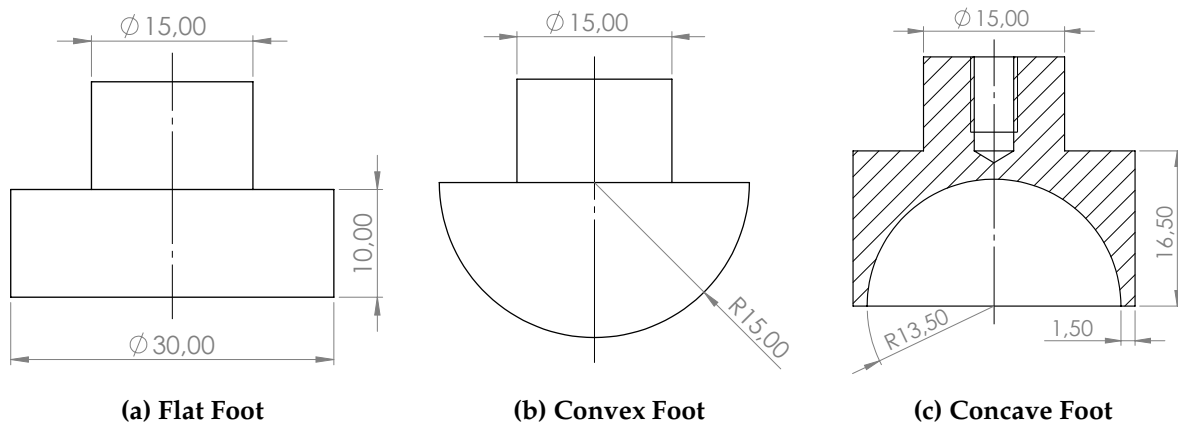


Figure 2.2 Main dimensions of the manufactured feet (in mm). The 3 feet are axisymmetric, and present different bottom curvature: flat, convex and concave. A sectional view is used in 2.2c for the third foot, to show its internal structure and related dimensions.

The experimental procedure for the dry sand case consisted of 3 to 4 trials for each foot shape. After each trial, the soil was mixed by hand, to retrieve its loose and flat surface configuration. The Torque Driver FTD5CN-S (see Appendix A) was used to check the uniformity of the soil surface characteristic. Following each mixing procedure, the torque driver was inserted 20 mm into the soil at four locations evenly spaced around the foot intrusion site, each positioned at a distance of four times the radius R . The torque driver was rotated until reaching continuous rotation speed and the maximum torque was therefore measured (this value is directly linked to the local soil shear stress).

For each trial, the foot probe was intruded with a constant speed $v_z = 10 \text{ [mm/s]}$ until reaching a maximum depth $z_f = 40 \text{ [mm]}$. The chosen vertical speed value corresponds to a low-speed regime according to [12], ensuring that inertial effects of soil particles remain negligible. The vertical resistive force of the soil on the foot was measured as a function of the intrusion depth z , defined as the distance from the initial soil surface to the lowest point of the foot (the tip of the intruder).

2.1.2 Experimental results

The dimensionless form of the vertical resistive force was defined by normalizing it with respect to the weight of the volume of soil displaced during the vertical motion of the flat foot.

The soil density used in the experiments was:

$$\rho_{soil,exp} = \frac{m_{sand}}{V_{soil}} = 1333 \text{ [kg/m}^3\text{]}$$

The frontal area of the intruder is defined as:

$$A = \pi R^2 = 706.86 \text{ [mm}^2\text{]}$$

The characteristic force used for nondimensionalization is the weight of the soil occupying a cylindrical volume of base A and height z_f :

$$F_{0,exp} = \rho_{soil,exp} g A z_f \quad (2.2)$$

where z_f is the final depth of intrusion and g is the gravitational acceleration. Therefore, we obtain $F_{0,exp} = 0.370 \text{ [N]}$. Time was non-dimensionalized as well, knowing that intruder velocity and intrusion depth are constant across the experiments, giving $t^* = \frac{t}{z_f/v_z}$. It ranges from 0 to 1, when the final intrusion z_f is reached.

The results of the dry sand experiments are presented in Fig. 2.3. Despite the considerable variance between trials (likely caused by initial packing differences resulting from the mixing technique used, as well as the inherent variability of initial force chain configurations) some consistent trends emerge. The convex foot generally performs the worst, except for $t^* < 0.25$, where it outperforms the concave foot. This range of time corresponds to the early stage of intrusion, when the concave foot is not yet fully submerged. The flat foot outperforms the two other feet, except at greater intrusion depths where its performance becomes comparable to these of the concave foot. In some cases, the concave foot even shows slightly higher force values.

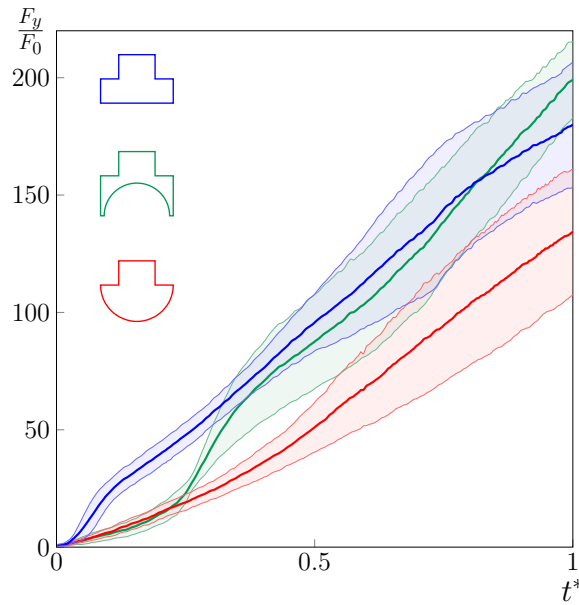


Figure 2.3 Vertical resistive force as a function of intrusion depth for the three different foot shapes. The thick lines represent the mean resistive force across trials for each shape, while the shaded regions denote one standard deviation above and below the mean, illustrating variability in the measurements.

Despite having the same frontal area, the convex foot presents a bottom area twice bigger than the flat foot ($2\pi R^2 = 1413.72 \text{ [mm}^2\text{]}$), while the concave foot has a bottom area a little smaller ($\pi(R^2 - R_c^2) + 2\pi R_c^2 = 1279.41 \text{ [mm}^2\text{]}$). These experimental results suggest that the soil resistive force is proportional to the depth of intrusion, but it is also closely linked to the intruder shape. Indeed, a clear difference in foot performance can be observed experimentally, and it seems to be linked to the difference between frontal area and total effective bottom area, which is due to the concavity orientation of the bottom of the intruder.

2.2 Resistive force theory (RFT)

The experimental results were compared to Resistive Force Theory (RFT), a simple and useful model developed in the field of terramechanics to describe the intrusion of rigid bodies into a non-cohesive granular soil. RFT works under the assumption of a relatively homogeneous and non-cohesive granular media (e.g. dry sand), for a solid intruder in low linkage ($z < 60 [mm]$) and low speed ($v < 1 [m/s]$) conditions [12]. Therefore, RFT neglects the inertial forces and assumes that the contact area and the intrusion depth are the main factors influencing the soil resisting force. It operates based on the principle of local force superposition, under the assumption of linearity.

Therefore, the intruder bottom part will be discretized into plates of area S (see Fig.2.4) and the total resistive force will be the sum of the contributions of all these plates. Each of these plates are characterized by a depth z_s , an angle of attack β , an intrusion angle γ and an intrusion velocity $\vec{v}$. The reaction of the soil onto this moving plate creates a lift and drag force, $\vec{f}_z$ and $\vec{f}_x$.

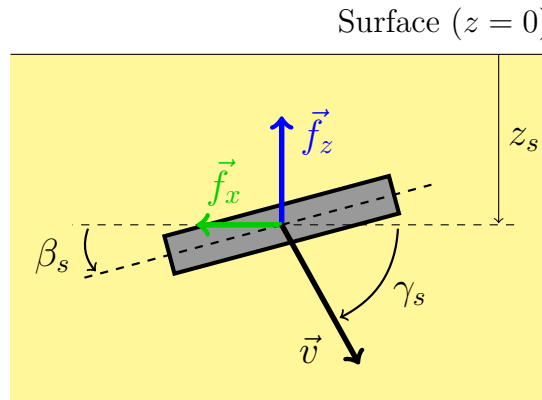


Figure 2.4 Plate element of area S .

In this study, the focus is exclusively on the vertical resistive force exerted by the soil on an intruder moving purely in the vertical direction. This allows to significantly simplify the integral expression of the total resistive force presented in [12]. Given that in this case, the intrusion angle is $\gamma = \frac{\pi}{2}$, Eq. 2.3 is obtained, where S denotes the total leading surface of the intruder, and dA_s , $|z_s|$, β_s , and γ_s represent, respectively, the area, depth, angle of attack, and intrusion angle of an infinitesimal surface element s .

$$F_z = \int_S \sigma_z(z_s, \beta_s, \gamma_s) dA_s = \int_S \alpha_z(\beta_s, \gamma_s) z_s dA_s \quad (2.3)$$

The quantity σ_z corresponds to the vertical stress, which behaves similarly to hydrostatic pressure due to the dominance of frictional forces in the granular media within

the RFT framework. The parameter α_z is the vertical stress per unit depth (also called vertical resistive force density), and under conditions of shallow penetration, it can be approximated as a function of the angle of attack, with a maximum value $\alpha_{z,max} = \alpha_z(0, \frac{\pi}{2})$ that must be determined experimentally.

To obtain $\alpha_{z,max}$, the measurement of the resistive force of the soil opposing the motion of a horizontal plate (corresponding to the flat case trial) is performed. Indeed, according to RFT, it corresponds to the case of maximum vertical stress, and therefore maximum force density α . From the regression fit of the flat case force-depth curve, we can easily obtain $\alpha_{z,max}$, since in this case $F_z = \alpha_{z,max} z A$, with A , the frontal area, and z , the distance between the surface and the bottom area of the foot. The slope value of the regression line allows finding $\alpha_{z,max} = 2.4176 \cdot 10^{-3} [N/mm^3]$.

From measurements of various granular soil (uniform dry granular medium composed of near-monodispersed and near-spherical particles of ~ 0.1 to ~ 1 mm in diameter), a generic model $f_z^{generic}(\beta, \gamma)$ was proposed in [12]. A discrete Fourier transform was used to obtain a fitting function:

$$f_z^{generic}(\beta, \gamma) = \sum_{m=-1}^1 \sum_{n=0}^1 \left[A_{m,n} \cos \left(2\pi \left(\frac{m\beta}{\pi} + \frac{n\gamma}{2\pi} \right) \right) + B_{m,n} \sin \left(2\pi \left(\frac{m\beta}{\pi} + \frac{n\gamma}{2\pi} \right) \right) \right]$$

This can be further simplified under the assumption of a purely vertical motion of the intruder ($\gamma = \frac{\pi}{2}$):

$$f_z^{generic}(\beta) = A_{0,0} + A_{1,0} \cos(2\beta) + B_{0,1} + B_{1,1} \sin \left(2\beta + \frac{\pi}{2} \right) + B_{-1,1} \sin \left(-2\beta + \frac{\pi}{2} \right) \quad (2.4)$$

with the following Fourier coefficients:

Coefficient	Value
$A_{0,0}$	0.206
$A_{1,0}$	0.169
$B_{0,1}$	0.358
$B_{1,1}$	0.212
$B_{-1,1}$	0.055

Table 2.2 Fourier coefficients of $f_z^{generic}(\beta, \gamma)$.

Therefore, the experimental vertical force density can be obtained as:

$$\alpha_z(\beta) = \alpha_{z,max} f_z^{generic}(\beta) \quad (2.5)$$

To obtain the resistive force for more complex intruder shape like the convex and concave feet, computations of the integral of a function over a spherical surface are needed. This surface depends on the intrusion depth z , when these feet are not fully submerged. A moving system of coordinate (2.5) is used to obtain the angle of attack β_s and the depth z_s of each plate of the contact surface. This moving frame $\mathcal{O}'$ is located at the center of the hemisphere for the convex case (see Fig. 2.5a) and at the center of the hemisphere generating the concavity for the concave case (see Fig. 2.5b). Solving the integrals detailed in Appendix C allows to obtain the resistive force F_z at each $z \in [0, z_f]$ for the convex and the concave feet.

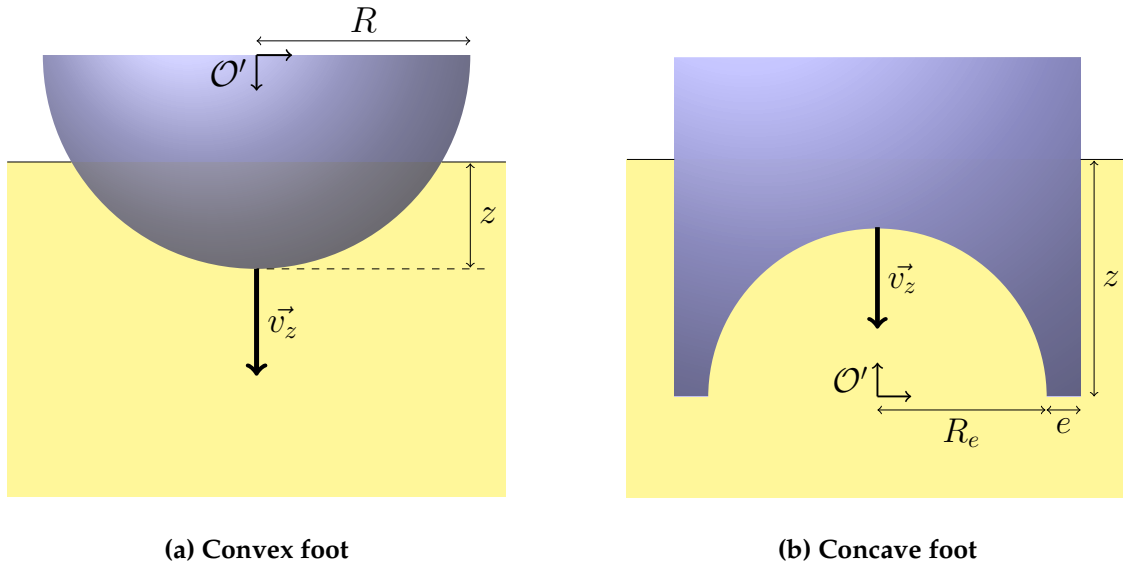


Figure 2.5 Schematic representation of the convex and concave foot geometries used in the intrusion experiments. The vertical intrusion velocity is denoted by $\vec{v}_z$, and the intrusion depth is indicated by z .

The resulting force-depth curve are presented and discussed in section 2.4.

2.3 Discrete Element Method (DEM)

2.3.1 Non-smooth contact dynamics

The MigFlow software was used to simulate 2D particles problem, without the presence of a fluid phase in the case of this study. MigFlow rely on the Discrete Element Method (DEM), implemented using the Non-Smooth Contact Dynamics (NSCD) approach [25]. This method allows the modeling of grain interactions through contact forces, including normal and tangential (frictional) components. The explanation of NSCD presented here is greatly inspired by [29].

The NSCD approach as described in [30] offers a fully implicit time integration scheme, meaning it is unconditionally stable. It is particularly suitable for simulating dense non-cohesive granular systems (granular flow or depot), since NSCD prevents grain interpenetration. This contrasts with smooth contact models (such as the linear spring-dashpot model [19] and the nonlinear Hertzian contact theory [31]) which are typically explicit and permit small overlaps between particles that can possibly create oscillations due to stiff response.

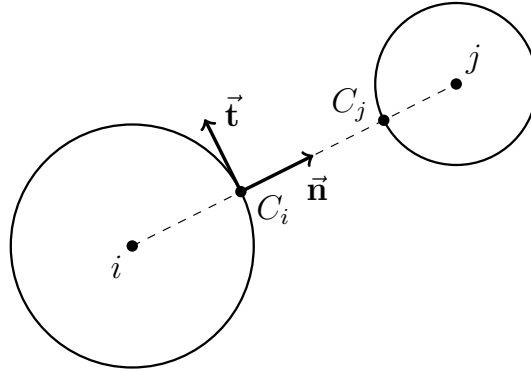


Figure 2.6 Local frame used to represent the contact involving the antagonist particle i with candidate particle j .

The granular system is seen as a set of perfectly rigid grains, characterized by geometrical and dynamics properties (radius, mass, inertia, position, velocity, acceleration, ...) at a time t . These properties determine a set of potential contacts that may occur at time $t + \Delta t$. A contact is created when the distance between potential contact points C_i and C_j (see Fig. 2.6) is smaller than an alert distance, that is fixed to the fastest grain radius.

In the absolute frame, each grains k are characterized by the coordinates of its center of gravity $\mathbf{x} = (x_k, y_k)$ and its rotation θ_k , that can be packed in the configuration vector $\mathbf{q}_k = (x_k, y_k, \theta_k)$. It is also linked to a mass matrix, that contains the mass and the inertia of the grain k :

$$\mathbf{M}_k = \begin{pmatrix} m_k & 0 & 0 \\ 0 & m_k & 0 \\ 0 & 0 & I_k \end{pmatrix}$$

This notation can be generalized to a system of N particles using:

$$\mathbf{q} = (\mathbf{q}_1 \quad \dots \quad \mathbf{q}_N)^T$$

$$\mathbf{M} = \text{diag}(\mathbf{M}_1, \dots, \mathbf{M}_N)$$

The equations of motion can be expressed using Newton's second law:

$$\mathbf{M} \frac{d\mathbf{u}}{dt} = \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{E} + \mathbf{r} \quad (2.6)$$

$$\mathbf{q}(t) = \mathbf{q}(t_0) + \int_{t_0}^t \mathbf{u} dt \quad (2.7)$$

with $\mathbf{F}$ the quadratic acceleration terms (computed using smooth variables), $\mathbf{E}$ the explicitly known forces (gravity, body or boundary forces, ...) and $\mathbf{r}$ the representative of local density of impulses exerted when contact occurs. Due to the presence of shocks, one can expect the apparition of speed discontinuities. This is the reason why the potentially non-smooth term $\mathbf{u}$ is used in the system as a generalized relative velocity, that can be modeled by discontinuous speed function. Therefore, the speed function $u(t)$ has a left limit U^- , just before the impact, and a right limit U^+ , just after the impact.

Unilateral contacts

At the scale of a potential contact (see Fig. 2.6), the following mechanical quantities are considered: the components of the relative velocity of point C_j with respect to the antagonist particle i , denoted by $\mathbf{U} = (U_T, U_N)$; the tangential and normal components of the contact force, R_T and R_N , expressed in the local frame; and the particle gap: $g(t) = \overline{C_i C_j}$.

NSCD enforces unilateral contacts, meaning that interpenetration between particles is not allowed (the distance between them must be zero or positive). As a result, particles can only push each other during a contact. In other words, since no cohesion is considered, the normal contact force is either positive when particles collide, or zero when contact ceases. This can be expressed through the Signorini condition:

$$g(t) \geq 0, \quad R_N(t) \geq 0, \quad g(t)R_N(t) = 0$$

Since $g(t)$ is continuous, we will use the bounded variation speed function to express the velocity Signorini condition (illustrated in Fig. 2.7a), since $U_N(t)$ is an image of the gap $g(t)$ in a time iterative scheme:

$$U_N(t)^+ \geq 0, \quad R_N(t) \geq 0, \quad U_N(t)^+ R_N(t) = 0$$

These relations imply an inelastic shock law. Indeed, using Newton restitution law, $U_N(t)^+ = -e U_N(t)^-$, the velocity Signorini condition implies a coefficient of restitution $e = 0$. Inelastic collisions dissipate all the kinetic energy associated with motion in the normal direction during the impact. This helps stabilize the system without needing artificial damping (like a Voigt model).

Coulomb dry friction

The tangential components of the dynamics are solved using the Coulomb dry friction law. If the friction force lies inside the friction cone (friction disk in the 2D case), then there is no tangential component to the relative speed U_T and sticking is observed. The other case is the sliding case, where $U_T \neq 0$ and a friction force $R_T = -\mu R_N$ is opposing the sliding direction. This translates into the following conditions (see Fig. 2.7b for a graphical representation):

$$\begin{aligned} U_T > 0 &\Leftrightarrow R_T = -\mu R_N \\ U_T < 0 &\Leftrightarrow R_T = \mu R_N \\ U_T = 0 &\Leftrightarrow \|R_T\| \leq \mu R_N \end{aligned}$$

In the NSCD approach, there is no distinction between static and dynamic friction coefficients, as using a single coefficient is physically acceptable for many applications. Introducing separate static and dynamic friction would require explicit handling of stick-slip transitions, which increases model complexity.

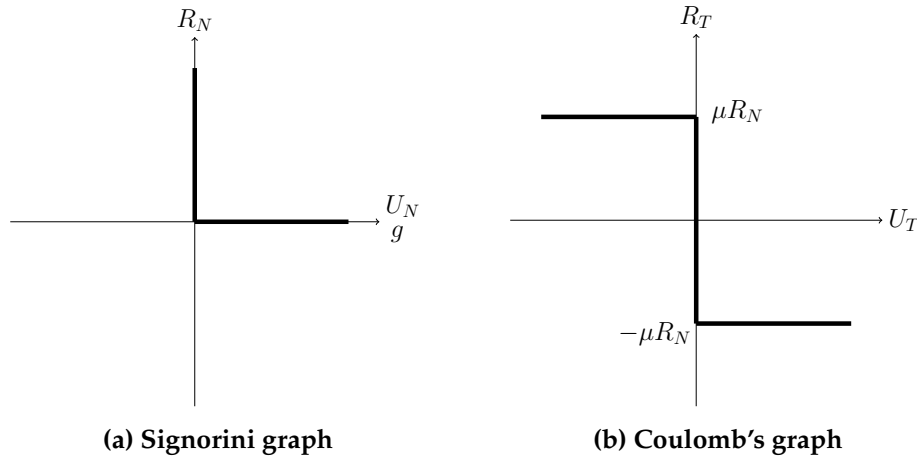


Figure 2.7 The non-smooth relations that links the contact variables are illustrated here by monotone multi-mapping graphs.

Implicit Euler formulation

NSCD resolves contacts through increments over a time step, meaning that velocity $\mathbf{u}$ is updated using impulses and constraint forces rather than integrating smooth contact forces over time. Therefore, time discretization of the Newton's second law for the set of particles can be expressed as followed, assuming that there is no quadratic acceleration terms:

$$\begin{aligned} \mathbf{M} \left(\mathbf{u}^{(i+1)} - \mathbf{u}^{(i)} \right) &= \mathbf{E}^{(i)} + \mathbf{p}^{(i+1)} \\ \mathbf{q}^{(i+1)} &= \mathbf{q}^{(i)} + \Delta t \mathbf{u}^{(i+1)} \end{aligned}$$

The vector $\mathbf{E}^i$, which represents external forces as gravity or intruder wall forces, is applied before resolution of the contacts (to obtain the new velocity vector after a time step Δt). The impulsion $\mathbf{p}^{(i+1)}$ is introduced as the sum of all impulses experienced during the time step Δt . This constitutes an implicit Euler scheme, with the primary unknowns being the impulsion and the approximation of the relative velocity after impact.

Resolution of the contacts

Resolution of one contact in the local frame was described in the previous sections. Solving the contacts in a global frame will require the linear mappings $\mathbf{H}(\mathbf{q})$ and its transpose $\mathbf{H}^*(\mathbf{q})$. Therefore, for each potential contact, the local relative velocity and local impulse are linked to their global counterparts through:

$$\mathbf{U} = \mathbf{H}^*(\mathbf{q})\mathbf{u} \quad \mathbf{R} = \mathbf{H}^*(\mathbf{q})\mathbf{r}$$

The Delassus matrix, useful for the resolution of the contacts, can be obtained as:

$$\mathbf{W} = \mathbf{H}^*\mathbf{M}^{-1}\mathbf{H} = \begin{pmatrix} \mathbf{W}_{NN} & \mathbf{W}_{TN} \\ \mathbf{W}_{NT} & \mathbf{W}_{TT} \end{pmatrix}$$

For a contact as described in Fig. 2.6, we can obtain the components of this matrix, knowing that $\mathbf{W}_{TN} = \mathbf{W}_{NT} = \mathbf{0}$ in the case, since particles are described as disks:

$$\begin{aligned} W_{NN,ij} &= m_i^{-1} + m_j^{-1} \\ W_{TT,ij} &= m_i^{-1} + m_j^{-1} + I_i^{-1}r_i^2 + I_j^{-1}r_j^2 \end{aligned}$$

If two particles are detected as a contact but are either separating or just touching (relative velocity close to zero), no contact impulse is needed. The case of a collision between two particles is described locally by:

$$g = 0, \quad R_N \geq 0$$

Since the known external force are applied before the resolution of the contacts (to compute the new velocity vector after a time step Δt), the contact dynamics can be expressed in the global frame as:

$$\begin{aligned} \mathbf{H}^*\mathbf{u}^+ &= \mathbf{H}^*\mathbf{u}^- + \mathbf{H}^*\mathbf{M}^{-1}\mathbf{H}\mathbf{P} \\ \Leftrightarrow \mathbf{U}^+ &= \mathbf{U}^- + \mathbf{W}\mathbf{P} \end{aligned}$$

with $\mathbf{P}$ the sum of all impulses that are implied in the contact during the time step. Using the discrete formulation, and by introducing the contact free velocity $\mathbf{U}_{free}$ (velocity if there is no contact), the expression becomes:

$$\begin{aligned}\mathbf{U}_N^{i+1} &= \mathbf{U}_{free,N} + \mathbf{W}_{NN}\mathbf{P}_N^{i+1} \\ \mathbf{U}_T^{i+1} &= \mathbf{U}_{free,T} + \mathbf{W}_{TT}\mathbf{P}_T^{i+1}\end{aligned}$$

Knowing that $U_N^{i+1} = 0$ since restitution coefficient is imposed to zero, the expression of the overall force impulses that will prevent interpenetration of the two particles and lead to an inelastic collision is obtained:

$$\begin{aligned}\mathbf{P}_N^{i+1} &= -\mathbf{U}_{free,N}/\mathbf{W}_{NN} \\ \mathbf{P}_T^{i+1} &= \begin{cases} -\mathbf{U}_{free,T}/\mathbf{W}_{TT} & \text{if stick} \\ -\mu\mathbf{P}_N^{i+1} \frac{\mathbf{U}_{free,T}}{\|\mathbf{U}_{free,T}\|} & \text{if slip} \end{cases}\end{aligned}$$

At the scale of a simple 2D contact implying only two particles, the normal and tangential reaction that will prevent interpenetration in an inelastic way during the time step can therefore be expressed as:

$$\begin{aligned}\mathbf{R}_N^{i+1} &= \begin{cases} \frac{\mathbf{U}_{free,N} - g/\Delta t}{\mathbf{W}_{NN}\Delta t} & \text{if } \mathbf{U}_{free,N} > g/\Delta t \\ 0 & \text{if } \mathbf{U}_{free,N} \leq g/\Delta t \end{cases} \\ \mathbf{R}_T^{i+1} &= \min\left(\mu\mathbf{R}_N^{i+1}, \|\mathbf{U}_{free,T}\|/(\mathbf{W}_{TT}\Delta t)\right) \frac{\mathbf{U}_{free,T}}{\|\mathbf{U}_{free,T}\|}\end{aligned}$$

To solve all the contacts at each time steps, an Improved NonLinear Gauss-Seidel method (INLGS) was implemented in the nonsmooth DEM solver of MigFlow [25]. Using a queue containing all the potential contacts, each active contact is treated one by one, using the NSCD formalism. If the contact is resolved, it is removed from the queue. Otherwise, if it is not resolved, it is reinserted in the queue along with all contacts involving the two particles concerned. This method allows to account the new contact that can appear through the change of velocity of the resolved grains. When the queue is empty, the overall solved state without any overlapping grains is reached for this time step.

In conclusion, the solving scheme of the nonsmooth discrete element method can be resumed as followed: at each time step, the external forces are used to compute the free motion of the particles. The algorithm estimates how many substeps are needed to ensure stable contact resolution. For each substep, the external forces are applied to bodies and particles, and INLGS is invoked to solve the contacts. If contact resolution fails, the time step is recursively subdivided. Otherwise, body states are updated.

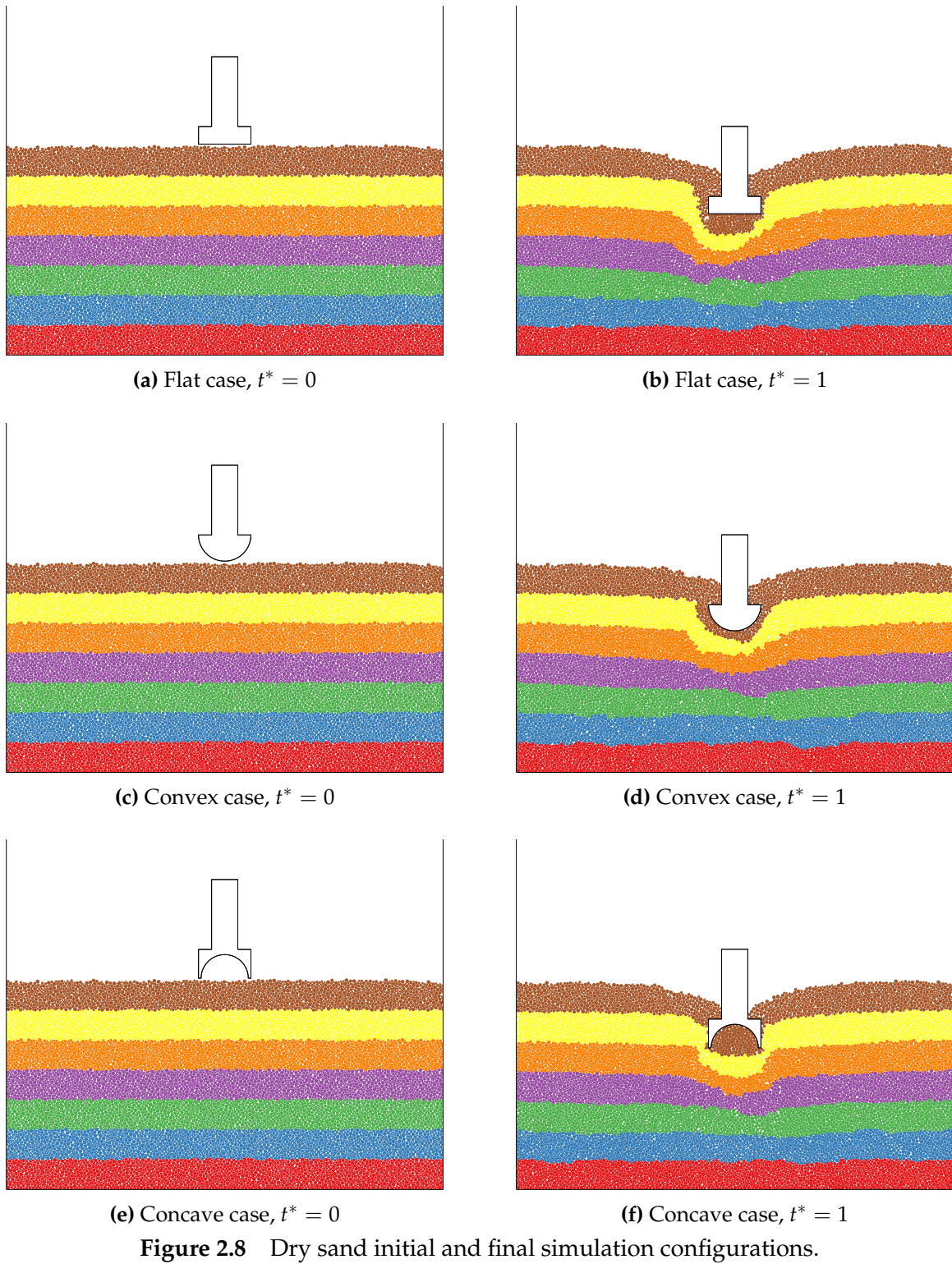
2.3.2 Simulation results

To simulate the experiments numerically, the friction coefficients between the aluminum intruder body and sand grains μ_{A-S} , as well as between grains μ_{S-S} , were determined from 3D DEM simulations fitting with experimental results from Cone Penetration Tests (CPT), as reported in [24]. For Silica Sand No.5, it was shown that inter-particle friction coefficient $\mu_{S-S} = 0.491$ and wall friction coefficient $\mu_{A-S} = 0.22$. The simulation presents therefore an internal friction angle $\phi_{sim} = \arctan(\mu_{S-S}) = 26.15^\circ$, which is smaller than the real value. Finally, the polydispersity of the granular media is defined as the ratio between the maximum and minimal radii: $P_r = \frac{r_{max}}{r_{min}} = 3.093$.

A 2D depot is initially generated in a domain of width $W = 0.25 [m]$ and height $H = 0.6 [m]$. This sandbox is modeled by boundaries, which are segments of infinite mass. The depot has a height of about $0.12 [m]$ and is constituted of 13136 particles characterized by the properties in Table 2.1. Particles were generated using a uniform distribution of grain sizes between the minimum and maximum radii. The stabilized state of the depot (after two seconds of simulation) was imported in each simulation case to ensure consistent initial conditions (see Fig. 2.8a, 2.8c and 2.8e). The intruder is a moving rigid body with an imposed constant speed $v_z = 0.01 [m/s]$ (meaning its mass is infinite with respect to the grain particles). The foot is positioned just above the depot. Due to 2D assumption and simplification of real grains into disk particles, soil density of the simulation is greater than the experimental one: $\rho_{soil, sim} = 2269.22 [kg/m^3]$. Therefore, this leads to a greater reference force $F_{0, sim} = 0.629 [N]$.

The dry sand simulations consist in moving the intruders from the surface to the maximum depth $z_f = 0.4 [m]$. The final states are reached after four seconds. They are represented in Fig. 2.8b, 2.8d and 2.8f. In these configurations, the flat and convex feet remain surrounded by particles from the $2 cm$ surface layer. The flat and concave feet entrain a group of surface particles beneath their bottom surfaces. For the concave foot, these trapped particles fill its cavity, making it appear similar to a flat foot. Overall, the flat foot appears to destabilize the most layers, followed by the concave foot, while the convex foot causes the least disturbance.

At this point, a discussion on the wall contribution to the vertical resistive force can be made, since concave geometry implies a higher wall length than the flat foot. In Fig. 2.9, the bottom part of the foot contributes the most to the total force (in black), with a negligible negative contribution of the wall and the foot upper parts, due to the weight of the particles that goes in the same direction as the intruder's motion. Similar results can be observed for the convex and concave feet.



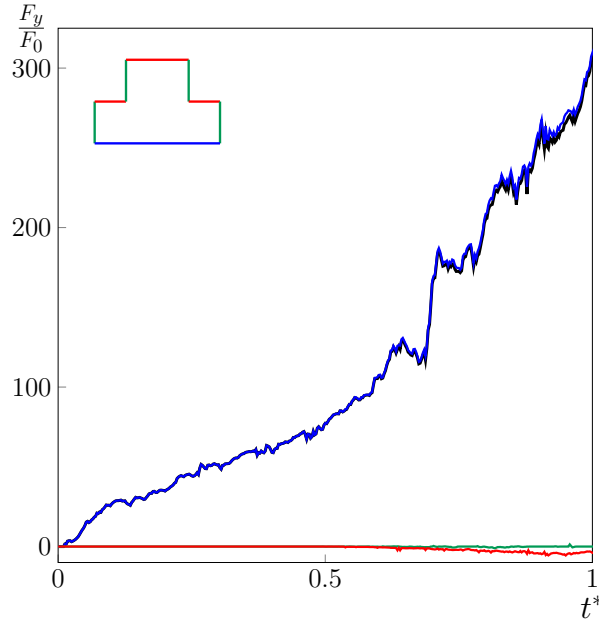


Figure 2.9 Dimensionless vertical resistive force contributions of the different sections of the flat foot during its intrusion in the dry sand. The total resistive force is in black, the bottom contribution in blue, the lateral contribution in green, and the upper part contribution in red. There is no noise filtering on these curves.

Stress field

A way to gain more insights in the soil mechanical response to intrusion is to analyze the stress field. A simple approach was described in [32]. The Love-Weber formula provides the average stress in a granular medium within a control volume V . Under the assumption of the absence of inertial effects, the components of this matrix can be expressed as:

$$\langle \sigma \rangle = \frac{1}{V} \sum_{c=1}^{N_c} \mathbf{R}^c \mathbf{l}^c \quad (2.8)$$

for N_c contacts within the control volume, with $\mathbf{R}^c$ being the contact force vector and $\mathbf{l}^c$ the branch vector of the contact c , both expressed in the same frame.

The simulation domain was discretized into square cells of size h . For each cell (i, j) in the mesh, the average plane stress tensor was computed as follows:

$$\langle \sigma_{ij} \rangle = \frac{1}{h^2} \sum_{c=1}^{N_c} \mathbf{R}^c \mathbf{l}^c \Big|_{ij} \quad (2.9)$$

Finally, the 2D von Mises equivalent stress is computed at each cell (see Eq. 2.10), and the result for time $t^* = 0$ is presented in Fig. 2.10. The hydrostatic-like granular soil

internal stress can be clearly seen on the depot without intruder contact. The maximum planar stress ranges around 10000 [Pa], mainly at the bottom of the sandbox. The stress field is relatively inhomogeneous, since the higher internal stresses are concentrated in the main branches of the force network.

$$\langle \sigma_{VM} \rangle = \sqrt{\sigma_{xx}^2 - \sigma_{xx}\sigma_{yy} + \sigma_{yy}^2 + 3\sigma_{xy}^2} \quad (2.10)$$

The results for $t^* = 0.5$ and $t^* = 1$, can be seen in Fig. 2.11. The propagation of the high stress zone under the foot can be seen for the three cases, by comparing the stress field at $t^* = 0.5$ and $t^* = 1$. For the three cases, it is observed that the stress concentration zone is far from the wall, confirming that the setup implies no wall friction or boundary-induced compaction. For the convex foot, the stress concentration zone is almost insignificant at $t^* = 0.5$, and its final form is a very thin cylinder directly beneath the foot, characterized by value reaching maximum values similar to the initial depot. But for both the flat and concave cases, the stress concentration zone already appears at $t^* = 0.5$ and presents a more conic shape that eventually reaches the high stress zone of the bottom of the sandbox. They both present an important inhomogeneous concentration of average stress between 10000 [Pa] and 20000 [Pa], but they are more pronounced for the flat case, with a maximum value $\sigma_{VM} = 20826$ [Pa], bigger than the concave one $\sigma_{VM} = 17373$ [Pa]. All of this indicates that the soil's response to vertical intrusion is not merely a surface-level reaction, but a multilayered physical phenomenon primarily governed by how the force network organizes itself in relation to the intruder's frontal geometry, as it is demonstrated in the next section.

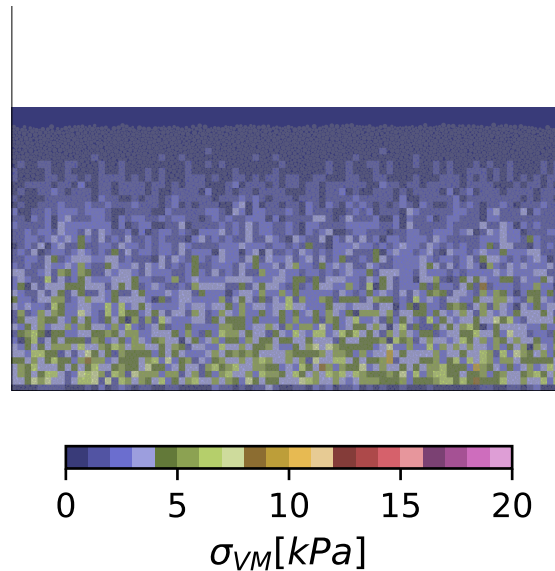


Figure 2.10 Discrete von Mises stress field in the depot at rest. The size of the cells was set to $h = 3$ [mm], approximately three times the maximum particle radius. This stress scale is also used in Fig. 2.11 and in Fig. 3.7.

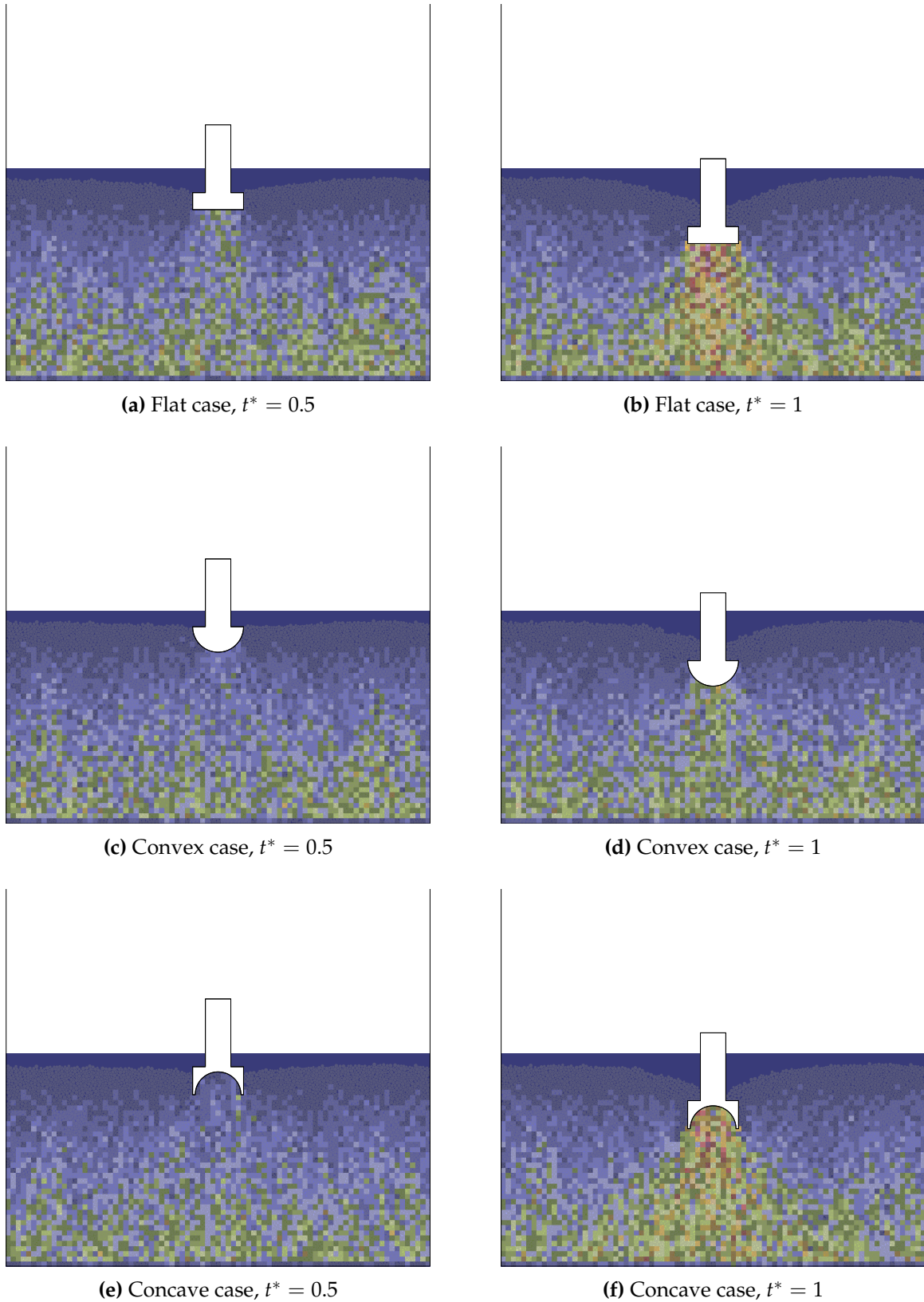


Figure 2.11 Discrete von Mises stress field in the granular medium at mid-intrusion (two seconds) and at the final state (four seconds) for the different foot geometries. The same stress scale as in Fig. 2.10 is used for comparison. Grains appear behind the field using a semi-transparent overlay.

Force network

Another approach to analyze the soil response to intrusion is to study the force network, which is a representation of the contact forces between particles in a granular medium. Unlike the stress field approach, force network preserve the grain-scale resolution. Therefore, it reveals force chains, providing insights into how forces are transmitted through the material and how this tree-like structure evolves with the intruder motion.

As it can be seen in Fig. 2.12a, 2.12c and 2.12e, for $t^* = 0.5$, the principal normal contact forces are located near the bottom of the sandbox, and forms ramifications that follows the hydrostatic-like stress field. In this low intrusion depth configuration, the stress concentration zone is not very pronounced in terms of shape an intensity, confirming the average von Mises stress field results. At $t^* = 1$, the force network is well-developed for both the flat and concave feet, forming a conical shape. The main branches (corresponding to higher normal reactions) are concentrated around the axis of the cone. Despite the similar overall geometry, the internal stress fields differ slightly: in the flat foot case, high compressive stresses are concentrated near the central axis and drop off sharply toward the cone walls. In contrast, the concave foot exhibits a smaller high-compression region with a more gradual decrease in stress toward the walls.

The force network observed in the flat foot case can be compared to that reported in [33], where the authors used photo-elastic experiment and DEM simulations to study the force transmission beneath a piston with a geometry and size very similar to the flat foot used in this study. The particle diameters were also in a comparable range. However, a key difference lies in the boundary conditions: their setup included a ceiling constraining the surface of the depot and had lateral walls positioned closer to the piston, indicating a stronger influence of wall boundary effects.

Despite these differences, both studies show a dominant force chain emanating from the center of the piston at low intrusion depths (see Fig. 2.12a), which branches diagonally toward the walls. In contrast to the results of these authors, who observed the highest contact forces near the piston in the main force chain segments, our results show that these high-force contacts are located in the stress concentration zone, both near and slightly farther below the flat foot. This difference is likely due to the absence of a ceiling in our free-surface setup, which prevents the formation of upward force branches originating from the surface.

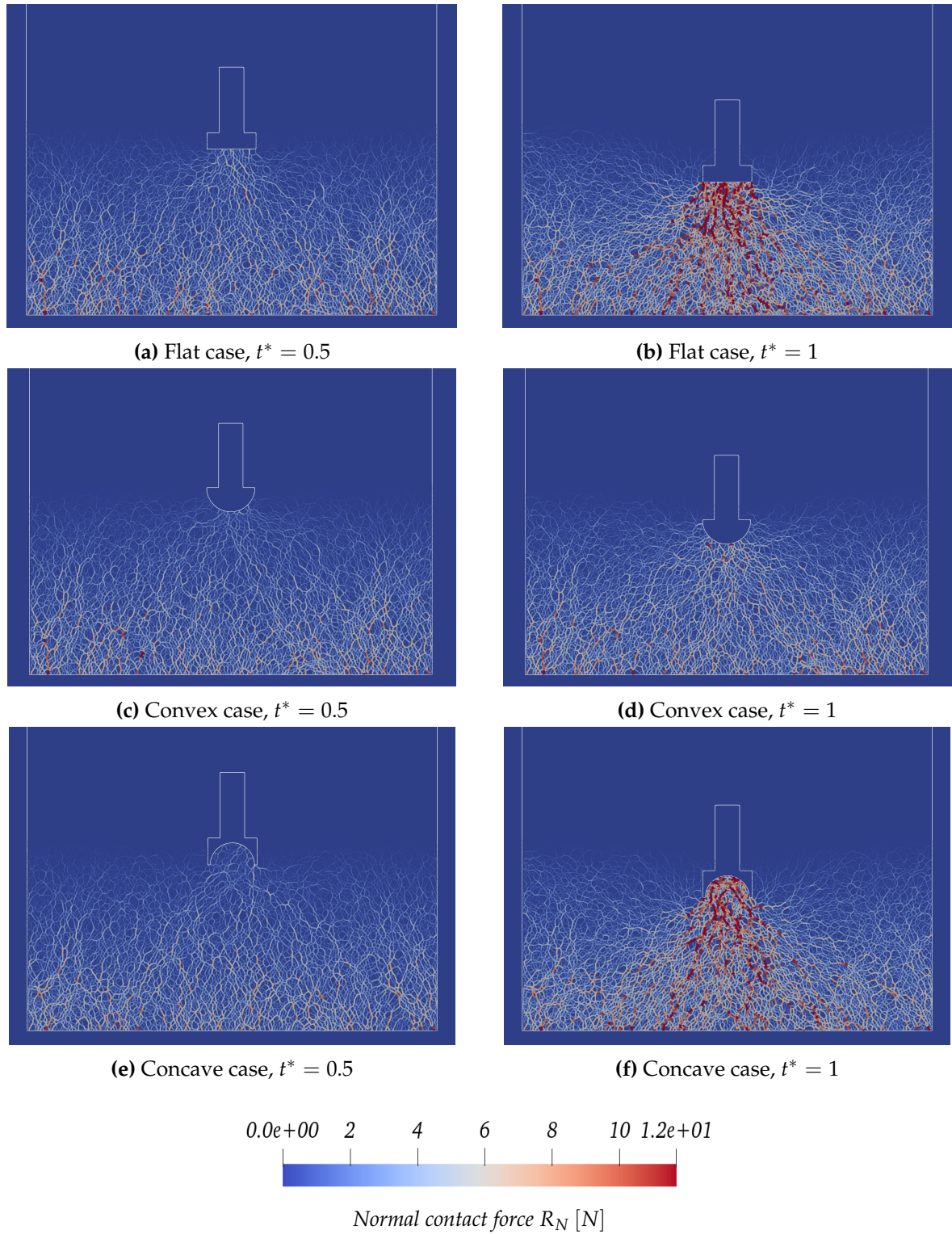


Figure 2.12 Force network graph visualization of the three simulations, for $t^* = 0.5$ and $t^* = 1$. Particles are represented as nodes and normal contact forces as edges. The color and the thickness of the edges represents the magnitude of the normal contact force R_N [N]. Forces that exceed 12 [N] are displayed in red.

Velocity field

Finally, the velocity field of the particles can be studied to understand how the intruder motion affects the surrounding grains. In Fig. 2.13, it is observed that the flat bottom surface induces a vertical downward motion of the particles located below the bottom surface, generating a stream of particles moving at velocities close to that of the intruder, which is called stagnation zone in [34]. The authors used NSCD simulations and observe that the stagnation zone forms an inverted-conic shape, which is also evident in our results. This phenomenon can lead to the formation of small vortex at the bottom of the sandbox, that draw the particles away from the main pushing stream, following some small stream, as it can be seen in Fig. 2.13a. Another prominent velocity structure in the flat foot case is the circulation of particles from the bottom of the sandbox to the top, forming large and slow vortices driven by the main particles stream (see Fig. 2.13b). In this figure, we also observe the rapid descent of surface grains into the conical cavity excavated by the intruder, indicating that, for slopes of this steepness, friction is insufficient to maintain their stability within the dynamic system.

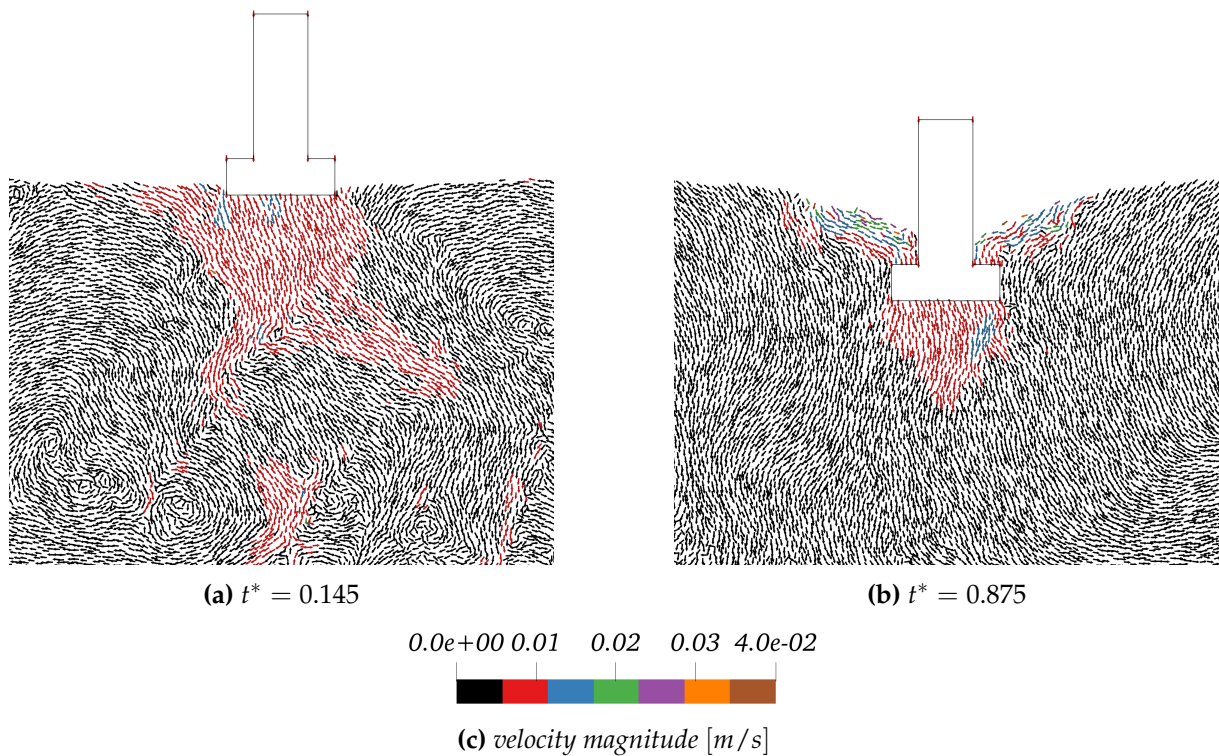


Figure 2.13 Velocity field of particles, for the flat foot case, at selected dimensionless time steps t^* . The color of the particle velocity vectors represents their magnitude. Virtual particles located at the foot's vertices generate velocity vectors in the figure, but should not be considered, as they have zero radius and mass.

In the case of the convex foot, the motion of the particles is less constrained by the intruder motion, as it can be seen in Fig. 2.14. Indeed, when it starts to intrude, the surrounding particles can easily flow outward from the intruder's vertical axis and even flow upward to the surface (see Fig. 2.14a). As for greater intrusion depth, a number of small vortices form near the foot, facilitating the diversion of particles away from the intruder's trajectory (see Fig. 2.14b). This suggests that the convex intruder shape promotes more effective fluidization of the granular medium, due to improved particle circulation around the intruder and reduced vertical resistance.

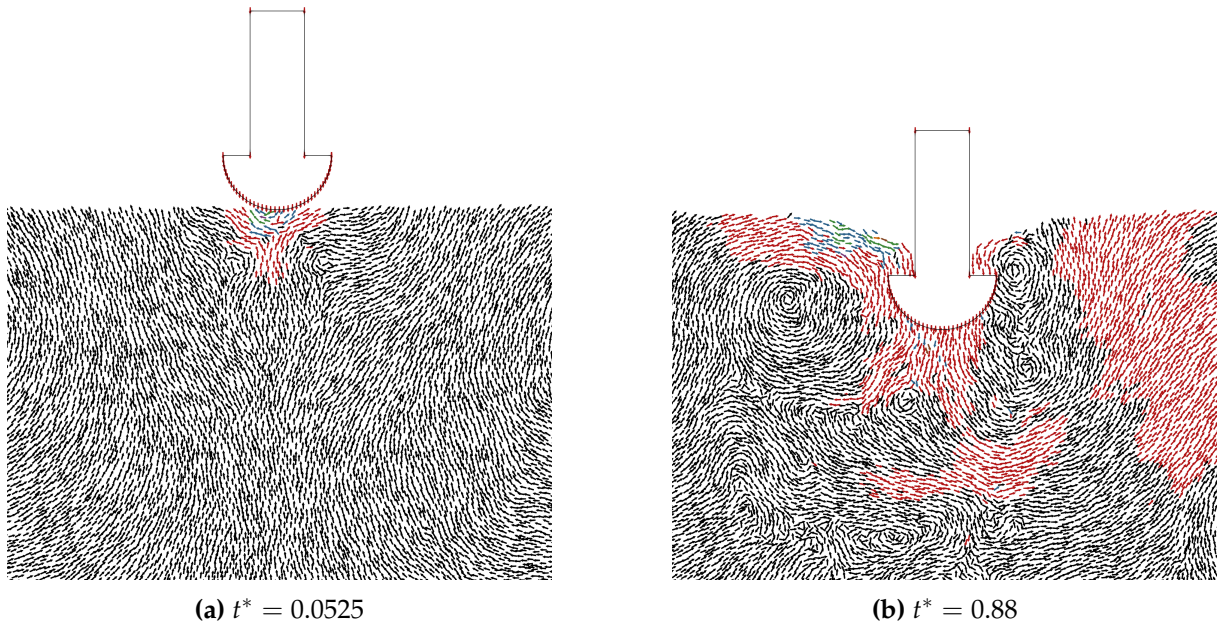


Figure 2.14 Velocity field of particles, for the convex foot case, at selected dimensionless time steps t^* . The velocity magnitude is the same as the one used in 2.13c.

Finally, the concave foot leads to more complex grains motion patterns, as can be seen in Fig. 2.15. At low depth, the particles are pushed downward, but they are also drawn inward the foot cavity. This interaction generates vortices near the tip of the cavity (see Fig. 2.15a), which effectively fluidize the particles inside. As a result, the particles fill the cavity at a speed greater than that of the intruder, encountering minimal resistance. While the concave foot is not fully intruded, there is also apparition of diagonal shear bands in the media, as it can be seen on the left side of Fig. 2.15b. These are regions of high vorticity, where the particles are pushed upward into the foot cavity on one side and downward on the other. The angle of the observed shear band with the horizontal plane is about 60° , which is close to the value $\theta_{MC} = 45^\circ + \phi_{sim}/2 = 58.08^\circ$ predicted by the Mohr-Coulomb theory [35]. Indeed, according to this theory, failure occurs when the shear stress on a plane reaches a critical value determined by the normal stress and the friction angle. The orientation that maximizes this condition determines the failure plane (shear band direction in 2D).

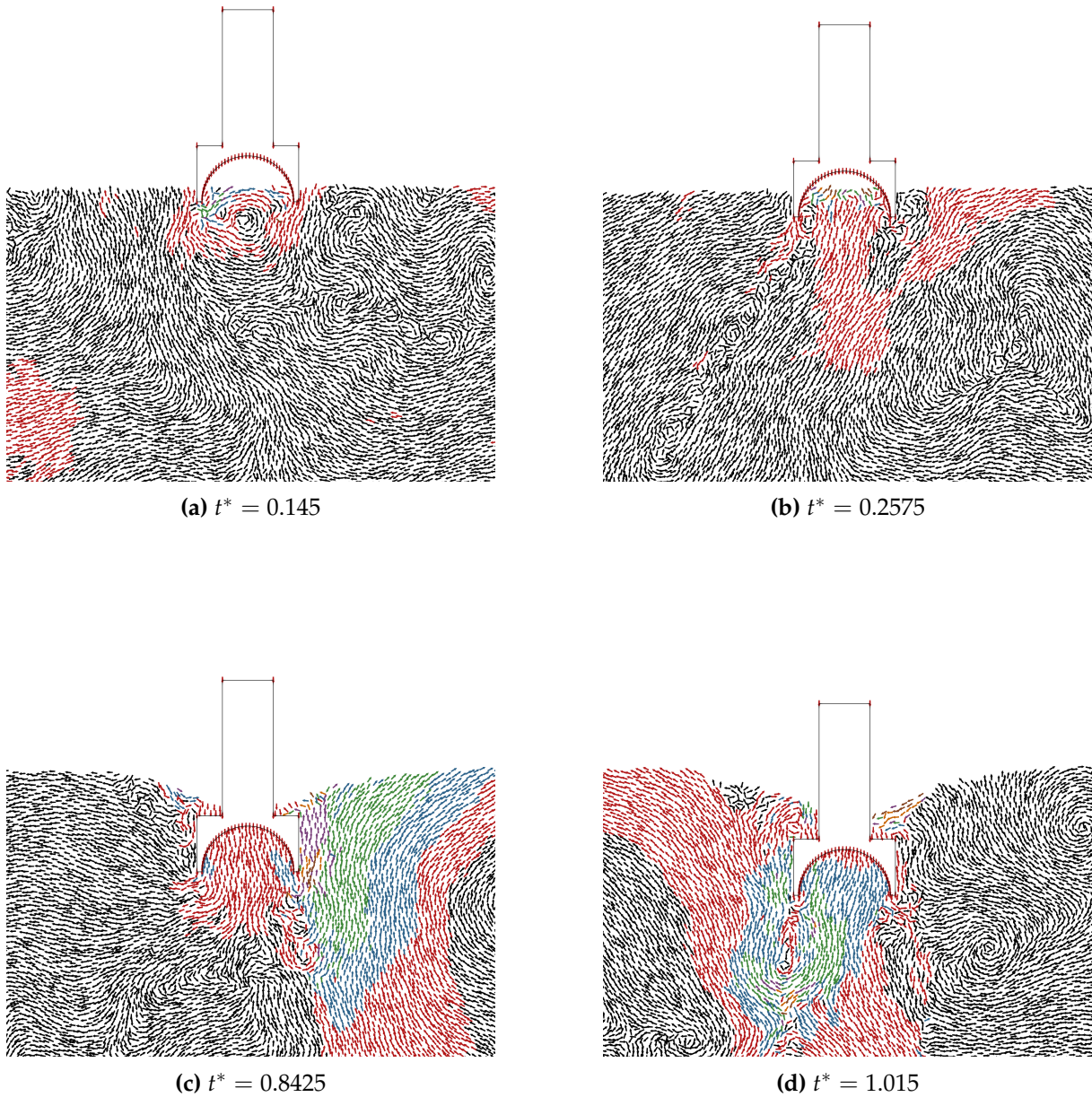


Figure 2.15 Velocity field of particles, for the concave foot case, at selected dimensionless time steps t^* . The velocity magnitude is the same as the one used in 2.13c.

At greater intrusion depths, the foot carries a cluster of particles trapped within the cavity, moving downward at almost the same speed as the intruder. The accumulation of particles within the concave foot causes it to behave similarly to a flat foot. Indeed, the motion patterns are quite similar, with apparition of large vortices that transport grains from the bottom of the sandbox to the surface. In Fig. 2.15c, a vortex that rotate in the opposite direction and exhibit a steeper velocity gradient compared to the flat foot's one can be observed on the right. Moreover, burst of particles momentum fol-

lowing streamlines from the cavity to the surface can be observed (see Fig. 2.15d). This indicates that, even though the particles are trapped and adhere to the cavity, they can still transmit their momentum to the surface particles.

2.4 Discussion

The comparison of the feet's performance prediction from the different approaches is mainly focused on the vertical soil resistive force. It is observed that each approach to model the soil reaction to the intrusion of a foot has its own advantages and limitations.

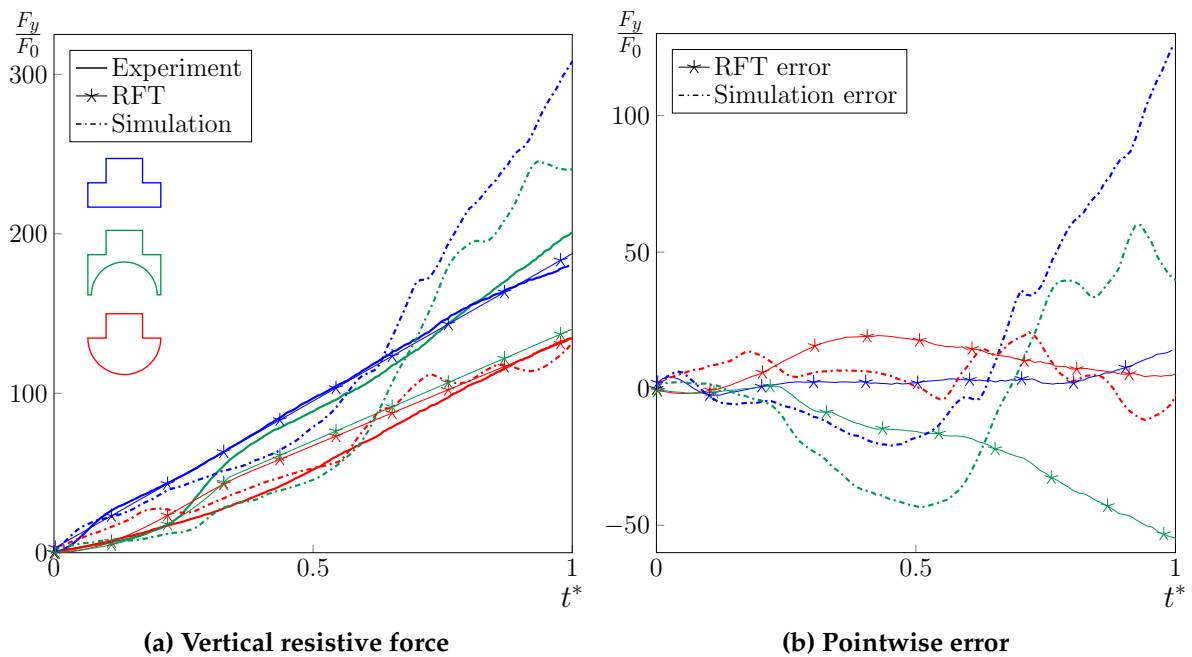


Figure 2.16 Figure 2.16a shows the dimensionless vertical resistive force as a function of dimensionless time for various foot shapes and modeling approaches. The DEM curves were smoothed using a moving average filter with a window size corresponding to 5% of the total number of data points. Figure 2.16b presents the pointwise error as a function of dimensionless time, defined as the difference between the RFT or simulation results and the experimental data. The corresponding Root Mean Square Errors (RMSE) are listed in Table 2.3.

In Fig. 2.16a, the vertical resistive force is plotted in function of the time for the three foot shapes. Similarly to the experimental results and the previous studies [12] [17], the vertical resistive force increases with the intrusion depth for both RFT and DEM simulations. The increase is mainly linear for the RFT model, with the foot shape having a minor effect on the curve's curvature at shallow intrusion depths. In this regime ($t^* < 0.3$), the concave foot exhibits lower resistive force compared to the convex foot. This is because, when not fully intruded, the concave shape presents less resistance

due to its smaller contact surface area and the less favorable orientation of its plate elements. For higher intrusion depth, both concave and convex cases present a similar slope of linear increase of the vertical resistive force, which concave case being slightly higher than the convex case. The RFT model for dry sand always predict that the vertical resistive force performance of the flat foot is the best compared to the two other shapes.

As for the DEM simulations, the vertical resistive force shows less linearity than the RFT model, even after curve smoothing, particularly for the flat and concave foot shapes. They both present a strong increase of resistive force around $t^* = 0.5$, a behavior not observed in either the RFT or experimental results. While the RFT and DEM predictions align well for the convex foot, the DEM results for the flat and concave feet fall below the RFT predictions before exceeding them sharply around $t^* \simeq 0.6$. Unlike RFT, the DEM simulations capture the transition at higher intrusion depths, where the behavior of the concave foot becomes similar to that of the flat foot.

Using the pointwise errors of the model and simulation relative to the experimental results in Fig. 2.16b, a discussion over the performance of these approaches can be made. Both the RFT and DEM slightly overestimate the vertical resistive force for the convex foot compared to the experimental data. In the flat foot case, the RFT error is minimal, as $\alpha_{z, max}$ was calibrated using the flat experimental results. At low intrusion depths, DEM slightly underestimates the resistive force for the flat foot, but significantly overestimates it at greater depths. A similar trend is observed for the concave foot DEM results, although it is less pronounced. However, for $t^* \in [0.2, 0.7]$, the DEM underestimates the experimental resistive force. This behavior is also seen in the RFT results for the concave case, where the model consistently underestimates the experimental values across the entire time range.

By comparing the overall Root Mean Square Errors (RMSE) presented in Table 2.3, the RFT model appears to provide accurate quantitative predictions of the resistive force for flat and convex foot designs. In contrast, DEM simulations capture the qualitative behavior of the resistive forces for all three foot shapes, despite having nonlinear responses for geometries that induce high stress gradients, such as the concave and flat cases. For the convex geometry, which generates lower stress gradients, DEM also yields good quantitative predictions. Additionally, unlike RFT, DEM simulations offer insights into the soil's response to intrusion, enabling to visualize granular dynamics.

Case	RMSE [N/N]
RFT flat	4.475
RFT convex	11.711
RFT concave	28.843
DEM flat	44.019
DEM convex	8.850
DEM concave	32.405

Table 2.3 Root Mean Square Error (RMSE) of the analytical model and simulations results relative to the experimental results for each foot shape.

3

Wet sand

In this chapter, the influence of saturation on the soil's resistive force against the intrusion of a solid body is studied. Unlike the dry case, where the voids are filled with air, which has a negligible effect on the soil's mechanical response, the wet case introduces greater complexity. In partially saturated soils, the voids contain both air and water, resulting in different behaviors depending on the water content. This water content is quantified by the saturation ratio S_r , defined as the ratio of the volume of water to the volume of voids in the soil. Saturation ranges from 0% (completely dry soil) to 100% (fully saturated soil), passing through multiple intermediate regimes, as illustrated in Fig. 3.1. The model and simulations specifically focus on the pendular regime of saturation, while the experiments cover the whole range.

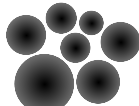
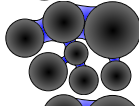
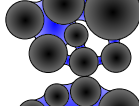
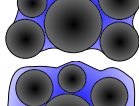
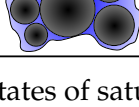
Liquid Content	State	Schematic Diagram	Physical Description
None	Dry		Cohesion is negligible.
Minimal	Pendular		Cohesion acts through liquid-bridges.
Transitional	Fenicular		Cohesion acts through liquid-bridges and liquid filled pores.
Saturated	Capillary		All voids are filled with liquid. The surface liquid is drawn into the pores due to capillary action, which gives rise to particle cohesion.
Over Saturated	Droplet/Slurry		No cohesion occurs as liquid pressure is equal to or greater than that of the air.

Figure 3.1 States of saturation of a granular media, from [36].

3.1 Experiments

3.1.1 Experimental setup

The experimental setup is identical to that used in the dry case (same sandbox dimensions, intrusion speed, foot shapes, etc.). The water addition procedure was carried out using water at room temperature. Starting from the dry case, the appropriate volume of water was added to reach the desired saturation level S_r . It was computed using the saturation equation 3.1, knowing the volume of void V_{void} and the volume of sand V_{sand} for the dry case.

$$S_r = \frac{V_{water}}{V_{soil} - V_{sand}} = \frac{V_{water}}{V_{void}} \quad (3.1)$$

The required water volumes for each saturation level are listed in Table 3.1. The range of the experiments covers saturation levels from 0% to 100%, even though the focus is on the low saturation case. For each foot shape, 3 to 4 experimental trials were conducted. Between trials, the soil was mixed manually, followed by a resting phase to allow for settling. Saturation levels were checked at several points near the intrusion epicenter to ensure consistency. The soil Moisture Meter DM-18R was used for these tests (see Appendix A).

S_r [%]	V_{water} [l]
0	0
10	0.372
30	1.117
60	2.234
70	2.607
80	2.979
90	3.352
100	3.724

Table 3.1 Soil saturation level studied experimentally, with the corresponding volume of water added to the dry soil.

3.1.2 Experimental results

Force-depth curves were measured for each foot shape and studied saturation level. The results are presented in Fig. 3.2. Similarly to [37], the vertical resistive force increases with saturation in the pendular regime (saturation level up to $S_r = 30\%$), due to the emergence of cohesion between particles caused by the formation of liquid bridges. This trend remains valid in the funicular regime ($S_r \in [30\%, 60\%]$), as the coexistence of liquid bridges and partially filled pores continues to enhance interparticle cohesion.

However, the curve corresponding to 10% saturation falls below the dry case locally. This behavior is observed for the convex foot at greater intrusion depths and for the concave foot within the range $t^* \in [0.3, 0.6]$. Since it is not very pronounced, it is likely attributed to measurement uncertainties and difficulties in maintaining uniform saturation in the whole sandbox at low saturation levels.

The sudden drop of the resistive force between 60% and 70% saturation is due to the transition from the funicular state to the capillary state, where the liquid bridges start to overlap and form a continuous liquid phase, leading to a decrease of the cohesion force. Indeed, the cohesion that acts through liquid bridges is bigger than the cohesion that acts through capillary action only. In this fluidized regime, the resistive force decreases with increasing saturation and becomes velocity-dependent, resembling the behavior of a fluid ([16]).

The convex foot performs the worst across the whole range of soil's saturation, confirming that its hemispherical shape offers the least resistance to the soil. As for the concave foot, for low saturation ($S_r = 0$ and 10%) and shallow intrusion depths, the resistive force is lower than that of the flat foot. However, as the intrusion depth increases, the resistive force becomes comparable and eventually exceeds that of the flat foot. As saturation increases, the concave foot appears to outperform the flat foot across the entire range of intrusion depths.

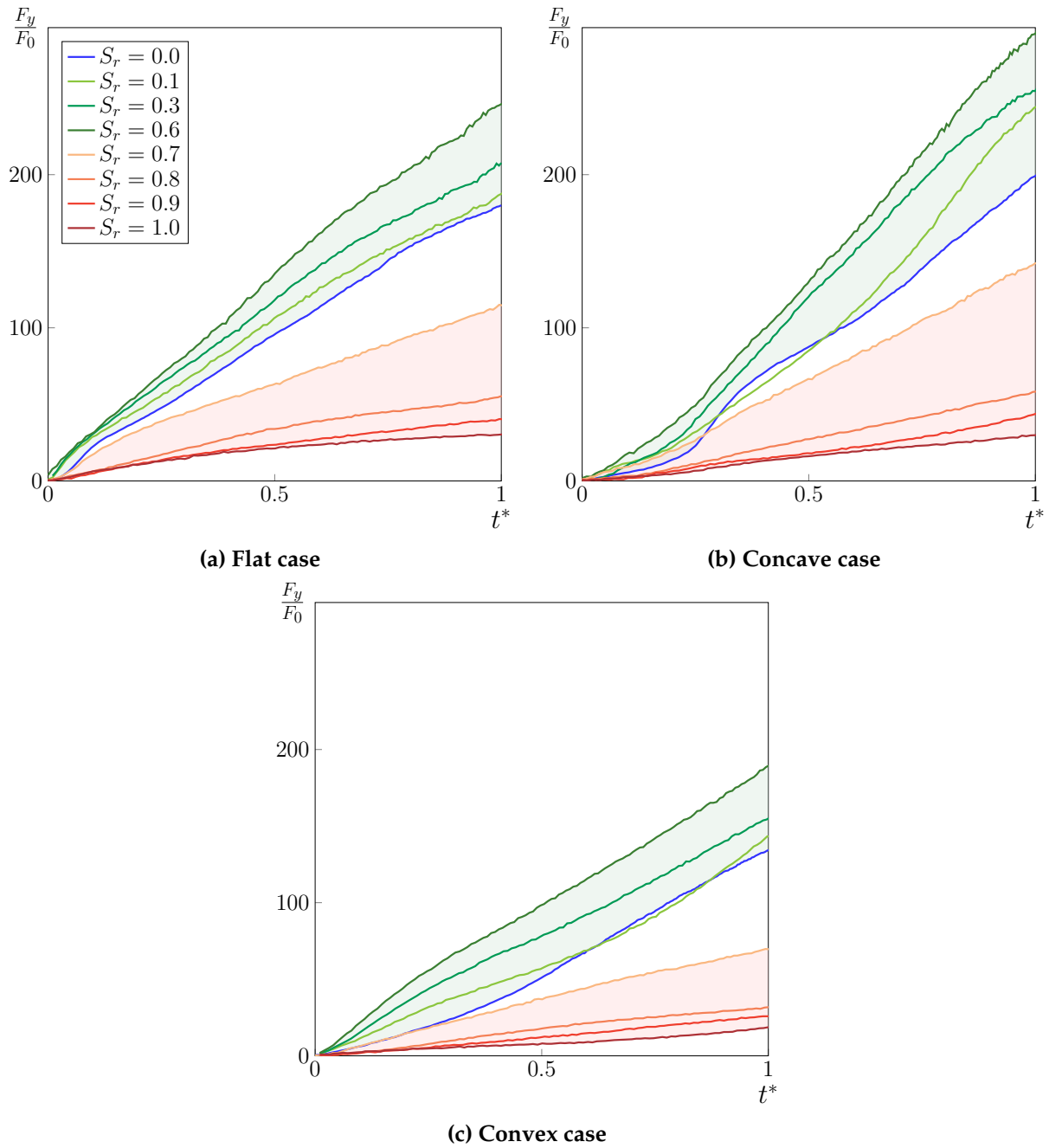


Figure 3.2 Force-depth curves for the different foot shapes and saturation levels. The green zone (from 0 to 60%) corresponds to the zone where an increase of saturation leads to an increase of the reaction force, while the red zone (from 70 to 100%) corresponds to a decrease of the reaction force. There is a sudden transition between the 2 regimes between 60% and 70% saturation. The dry case is represented by the blue curve, for all graphs. The curves are averaged over 3 to 4 trials.

(a) $S_r = 10\%$ (b) $S_r = 70\%$ **Figure 3.3** Footprint in the sand for low saturation (left) and high saturation (right).

Another interesting observation across the different saturation regimes is the soil's response after the intrusion. In the dry case, the cavity dug by the foot in the sand collapses almost instantly once the foot is removed, leaving only a faint footprint due to the lack of cohesion. For saturation levels between 10% to 60%, the footprint remains visible and doesn't collapse, due to the presence of liquid bridges (see Fig. 3.3a). However, at saturation levels above 70%, the footprint disappears again, with a rate that increases with the saturation. Indeed, in this regime, the liquid bridges are no longer strong enough to hold the sand particles together, and the soil begins to behave more like a fluid (see Fig. 3.3b).

3.2 Extended RFT

An extended RFT model is proposed in this study to account for the influence of saturation on the soil reaction force in the pendular regime (saturation until 30%, as explained in section 3.3.1). Indeed, for low saturation levels, experiments show that the flat foot force-depth curve still presents a linear behavior, with a slope that increases with the saturation level, as shown in Fig. 3.2a.

Therefore, the maximum vertical stress per unit volume $\alpha_{z,max}$ is computed from the slope of the experimental flat foot force-depth curve of each pendular regime saturation levels. It is presented in Table 3.2.

S_r	$\alpha_{z,max} [\text{N}/\text{mm}^3]$
0	$2.4176 \cdot 10^{-3}$
10%	$2.6198 \cdot 10^{-3}$
30%	$2.9233 \cdot 10^{-3}$

Table 3.2 Evolution of the maximum vertical stress per unit volume $\alpha_{z,max}$ with the saturation level S_r . It was evaluated from the slope of the flat case experimental force-depth curves.

These experimental fits can be applied in Eq. 2.5 to calculate the vertical stress per unit volume as a function of the angle of attack for these low saturation levels. This approach ultimately allows to obtain the force depth-curves for the other geometries (see Fig. 3.9).

It comes naturally to extend the RFT model to pendular regime level of saturation by assuming that the maximum vertical force density $\alpha_{z,max}$ is a function of the saturation level S_r . We performed here a least squares fit of this parameter calibrated with the low saturation experiments. This leads to this linear relation:

$$\alpha_{z,max}(S_r) = mS_r + \alpha_{z0,max} \quad (3.2)$$

with $\alpha_{z0,max} = 2.4176 \cdot 10^{-3}$ and $m = 1.7193 \cdot 10^{-3}$. This gives an error of 1.15% and 0.34% respectively, compared to the experimental data from $S_r = 0.1$ and $S_r = 0.3$.

3.3 Cohesive DEM simulations

3.3.1 Liquid-bridge contact model

The numerical simulations for the wet sand case were limited to the low saturation regime, as validated DEM models for higher intermediate saturation levels are currently unavailable. Even within the low saturation regime studied here, the pendular regime, characterized by independent liquid bridges linking the particles, there is no clear consensus.

Indeed, in [38], it is said to correspond to a saturation level around 30%, whereas in [39], the authors used a geometrical approach to model capillary bridges, and found out that menisci overlappings are negligible until $S_r = 15\%$ (but they remain cautious by affirming that this value depends greatly on the microstructural arrangement and porosity of the medium). Moreover, in this reference article [40], the authors showed that for medium-sized sand (particle size ranging from 0.2 to 0.9 [mm]), the tensile strength is maximum at $S_r = 30\%$ within the pendular regime, for porosity of around 40%. Since the sand in our experiments has similar grain size and porosity, we consider our tests at 10% and 30% saturation to fall within the pendular regime.

In pendular regime, there is also a difference between the drying and wetting scenario due to hydraulic hysteresis [39], which is not taken into account in the present study.

Therefore, DEM simulations are employed to study low saturation cases, using a cohesive law implemented within the NSCD method to account for the capillary bridges formed by the presence of water. The capillary cohesion force was modeled using an approximate solution of the Laplace-Young equation, presented in [41].

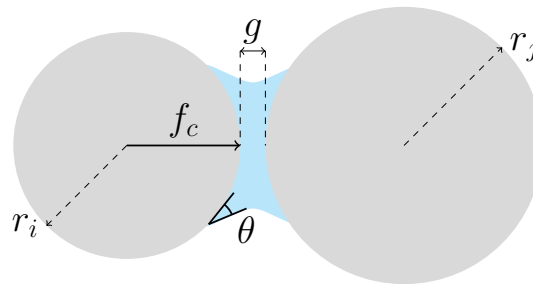


Figure 3.4 Capillary force between two disk particles, due to the presence of a liquid bridge.

This force depends on the liquid bond parameters (see Fig. 3.4), which are the gap g , the liquid bond volume V_b , the liquid surface tension γ , and the particle-liquid-gas contact angle θ . Finally, this force is given by:

$$f_c = \begin{cases} -\kappa r_g e^{-g/\lambda} & \text{for } 0 \leq g \leq d_{rupt} \\ 0 & \text{for } g \leq d_{rupt} \end{cases} \quad (3.3)$$

where $r_g = \sqrt{r_i r_j}$ is the geometrical mean of particle radii and the prefactor κ is expressed as:

$$\kappa = 2\pi\gamma \cos \theta$$

The debonding distance d_{rupt} is defined as:

$$d_{rupt} = \left(1 + \frac{\theta}{2}\right) V_b^{1/3}$$

Finally, the length scale λ , that controls the exponential decrease of the capillary force, can be expressed by:

$$\lambda = c h(r_i, r_j) \left(\frac{V_b}{r_h}\right)^{1/2}$$

with $c \simeq 0.9$, a fitting parameter, $h(r_i, r_j) = \sqrt{\min\left(\frac{r_i}{r_j}, \frac{r_j}{r_i}\right)}$, a function of the particle radii, and $r_h = \frac{2r_i r_j}{r_i + r_j}$, the harmonic mean of the particle radii.

In the pendular regime, the liquid bond volume can be linked to the saturation level and the soil bulk porosity [38]:

$$V_b = \frac{\epsilon^2 d_m^3 S_r}{3(1 - \epsilon)} \quad (3.4)$$

where ϵ is the soil porosity (see Eq. 2.1) and $d_m = r_{min} + r_{max}$ is the mean particle diameter.

The assumption that particles are perfectly wettable (meaning the binding liquid covers the particles) implies that the contact angle θ always equals zero, independently of the liquid viscosity, as in [41] [42].

The physical parameters used in our simulations are presented in Table 3.3.

Parameter	Value
Contact angle θ	0 [°]
Liquid-vapor surface tension γ (at 20°C)	0.0728 [N/m]
Soil porosity ϵ	0.497 [–]
Mean particle diameter d_m	$1.535 \cdot 10^{-3}$ [m]

Table 3.3 Capillary-related parameters used in the study.

This approach yields liquid bond volumes of $5.90 \cdot 10^{-11}$ and $1.77 \cdot 10^{-10}$ [m³], for saturation S_r of 10% and 30%, respectively. These values are consistent with previously reported liquid bridge volumes for particles in a similar size range. For instance, in [43], for smaller sand particles with diameters between 0.212 and 0.5 [mm], the liquid bond volume at a saturation of $S_r = 10\%$ is estimated to be approximately equal to $8.58 \cdot 10^{-12}$ [m³]. In [44], a volume $V_b = 7.5 \cdot 10^{-11}$ [m³] was used for characterizing a wet granular media of saturation $S_r = 8\%$ and mean particle diameter $d_m = 2.2$ [mm].

3.3.2 Implementation in MigFlow

Regarding the implementation of this cohesive law in the NSCD method, it involves adding a negative cohesive force $\mathbf{F}_c$ to the contact. When the contact gap is below the debonding distance, the capillary bridge effect is represented as an additional externally induced velocity $\mathbf{U}_c$, oriented in the positive normal direction of the local contact frame of reference (see Fig. 2.6). Over the time step Δt , the norm of this velocity $\mathbf{U}_c$ is given by:

$$\|\mathbf{U}_c^{i+1}\| = \|\mathbf{W}_{\mathbf{NN}} \cdot \mathbf{F}_c^{i+1}\| \Delta t$$

An adapted version of the Signorini condition can be expressed as following, knowing that the normal contact reaction R_N is considered positive since it is a repulsive force, while the capillary force F_c is negative since it is an attractive force:

$$\begin{array}{ll} g \leq 0 & R_N \leq 0 \\ \text{if } g(t) > 0 & \Rightarrow F_{\text{tot}} = F_c \leq 0 \\ \text{if } g(t) = 0 & \Rightarrow F_{\text{tot}} = F_c + R_N \geq 0 \end{array}$$

This set of conditions is illustrated in Fig. 3.5, which shows the exponential decay of the capillary force with the gap g between particles, as well as the jump in total force when there is contact ($g = 0$) and when the debonding distance is reached ($g = d_{rupt}$).

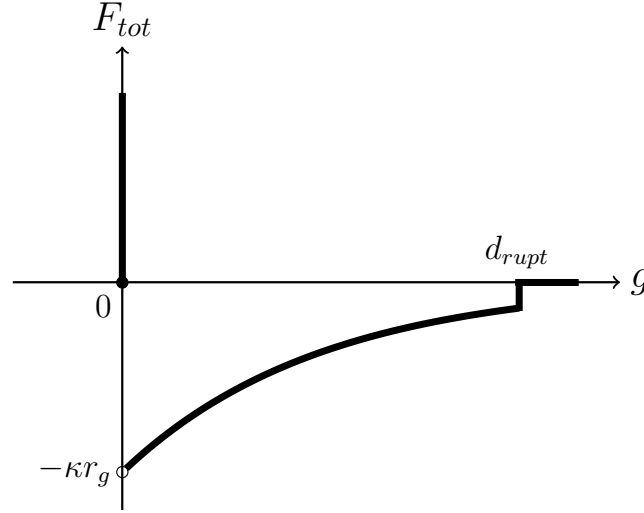


Figure 3.5 Signorini graph for the cohesive NSCD contact.

Furthermore, the relative velocity must be constrained by the maximum admissible velocity for given gap and time step, to ensure that particles do not penetrate each other:

$$\mathbf{U}_N^{i+1} + \mathbf{U}_c^{i+1} \leq \frac{g}{\Delta t}$$

The current local variation of the relative contact speed and the associated normal contact reaction can therefore be computed as:

$$\begin{aligned} \delta \mathbf{U}_N^{i+1} &= \mathbf{U}_N^{i+1} - \min \left(\mathbf{U}_N^{i+1} + \mathbf{U}_c^{i+1}, \frac{g}{\Delta t} \right) \\ \mathbf{R}_N^{i+1} &= \delta \mathbf{U}_N^{i+1} / (\mathbf{W}_{NN} \Delta t) \end{aligned}$$

3.3.3 Simulation results

In the pendular regime, saturation-induced cohesion affects how particles move around the intruder. As it can be seen in Fig. 3.6a, particles tend to stick more together and less to the surface of the intruder (since no wall-particle cohesion was implemented), resulting to a smaller amount of mixing between soil layers. It is also more localized, meaning that intrusion mobilizes grains within a smaller zone of influence around the foot, compared to the dry cases. This behavior can be attributed to cohesion, which keeps the grain attached together. This is illustrated by the fact that the particle walls of the second layer (yellow) did not collapse into the cavity dug by the foot. Another consequence of the liquid bridges is that the slope of the surface conical grain cavity is steeper than in the dry case. Indeed, cohesion increases the angle of repose of the granular medium, and this is particularly well illustrated in Fig. 3.6b.

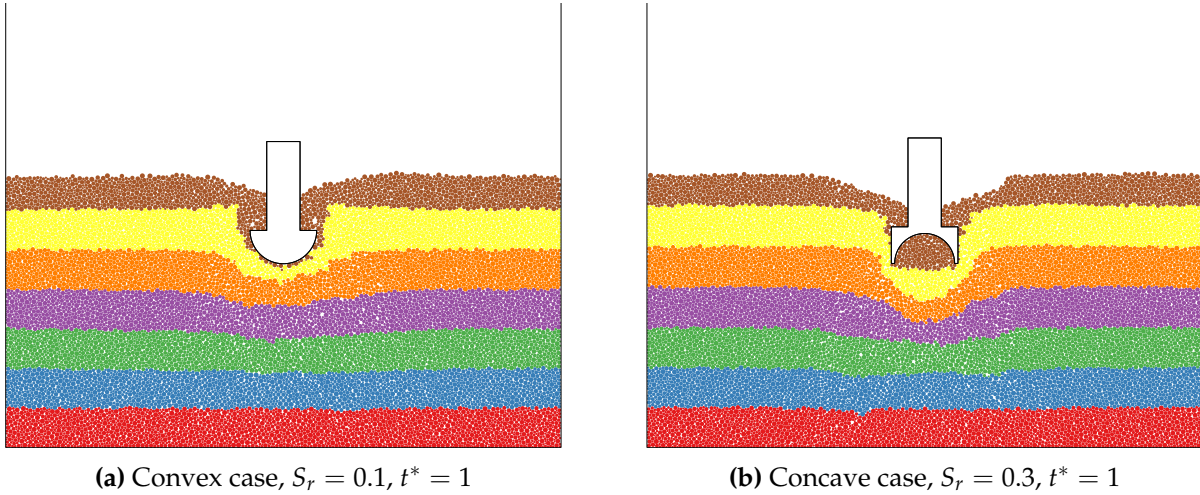


Figure 3.6 Low-saturated sand final configuration, for the convex and concave case.

As for the internal stress in the granular medium, it increases due to the addition of cohesive forces to the normal contact forces, when particles are stuck together. As shown in Fig. 3.6 for the flat and concave case, the shape of the stress concentration zone remains approximately the same, but the intensity of the von Mises equivalent stress is significantly higher than in the dry case. Indeed, the maximum local von Mises equivalent stress reaches approximately $33000 [Pa]$ for $S_r = 10\%$ and $40000 [Pa]$ for $S_r = 30\%$. This increase is particularly pronounced in the core of the stress concentration zone, near the bottom of the feet. A noticeable increase also occurs in the outer regions of the stress cone. This phenomenon can be directly linked to the increase of vertical resistive force with saturation that can be observed at the macroscopic scale for the pendular regime.

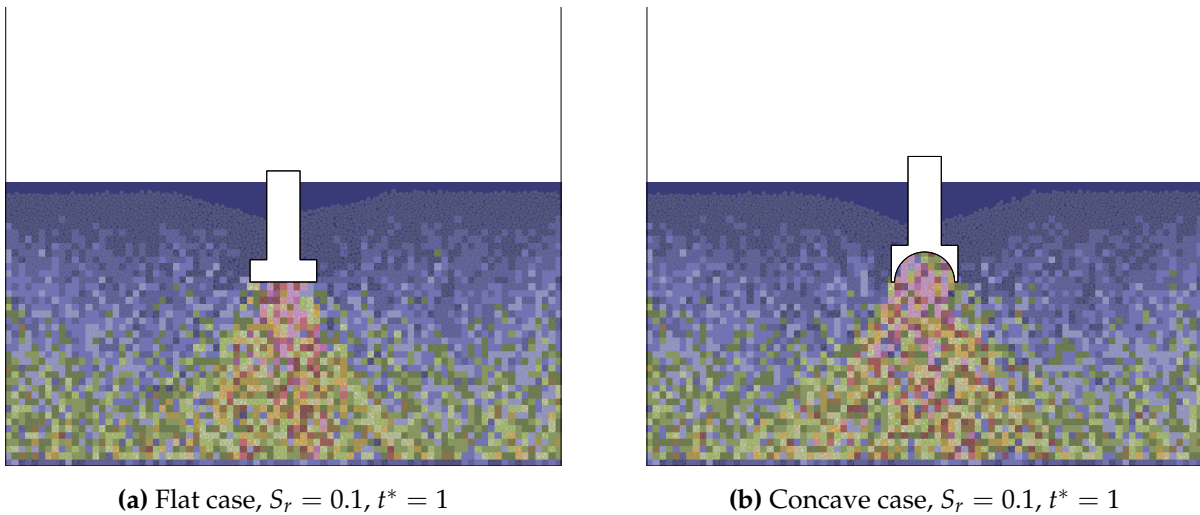


Figure 3.7 Discrete stress field in the low-saturated sand at the end of the intrusion of the flat and concave feet. Cell sizes and stress scale are the same as in Fig. 2.10. Local von Mises stress equivalents that exceeds $20000 [Pa]$ are displayed in light purple.

As for the force chain network, observations are similar to that of the discrete stress fields. As expected, an overall increase of inter-particle normal contact can be observed, leading to a more developed high force network below the feet, as observed in 3.8.

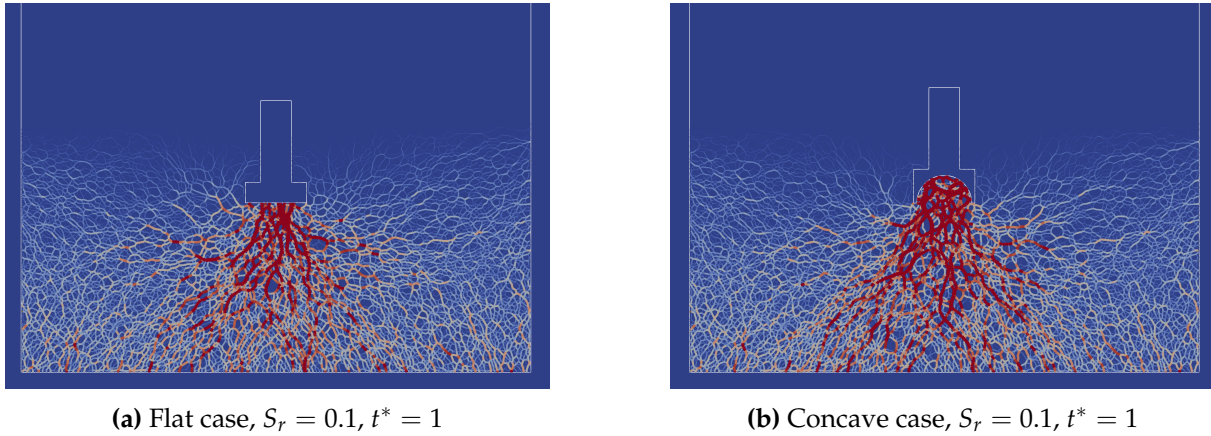


Figure 3.8 Force network in the low-saturated sand final, at the end of the intrusion of the flat and concave feet. The color scale representing normal force magnitude is the same as in Fig. 2.12.

Finally, since part of the momentum transmitted from one particle to another is used to overcome the cohesive force, velocity gradients in the pendular regime are lower than those observed in dry sand. However, the overall motion pattern remain consistent across for the different foot geometries.

3.4 Discussion

In this section, low saturation results from extended RFT and cohesive DEM simulations are compared to the experimental results. In Fig. 3.9, the force-depth curves for 10% and 30% saturation are shown. As expected from the experimental results, since both saturation levels fall within the pendular regime, the force-depth curves increase with both depth and saturation. For $S_r = 10\%$, the extended RFT model captures the linear trend of the experimental curves for the flat and convex feet with relatively small error, while it underestimates the force for the concave foot, particularly at higher saturation levels. The numerical simulations, on the other hand, overestimate the vertical resistive force for all foot shapes. However, cohesive DEM predicts the fact that the concave case exceeds the flat case for high intrusion depth. Similar observations can be made for the 30% saturation case. However, in this case, the experimental convex result is closer to the RFT model and the simulation results are closer to the experimental results, especially for the concave foot.

For all three approaches, there is a clear increase of the resistive force with the saturation level. It is illustrated in Fig. 3.10, which shows the difference in resistive force

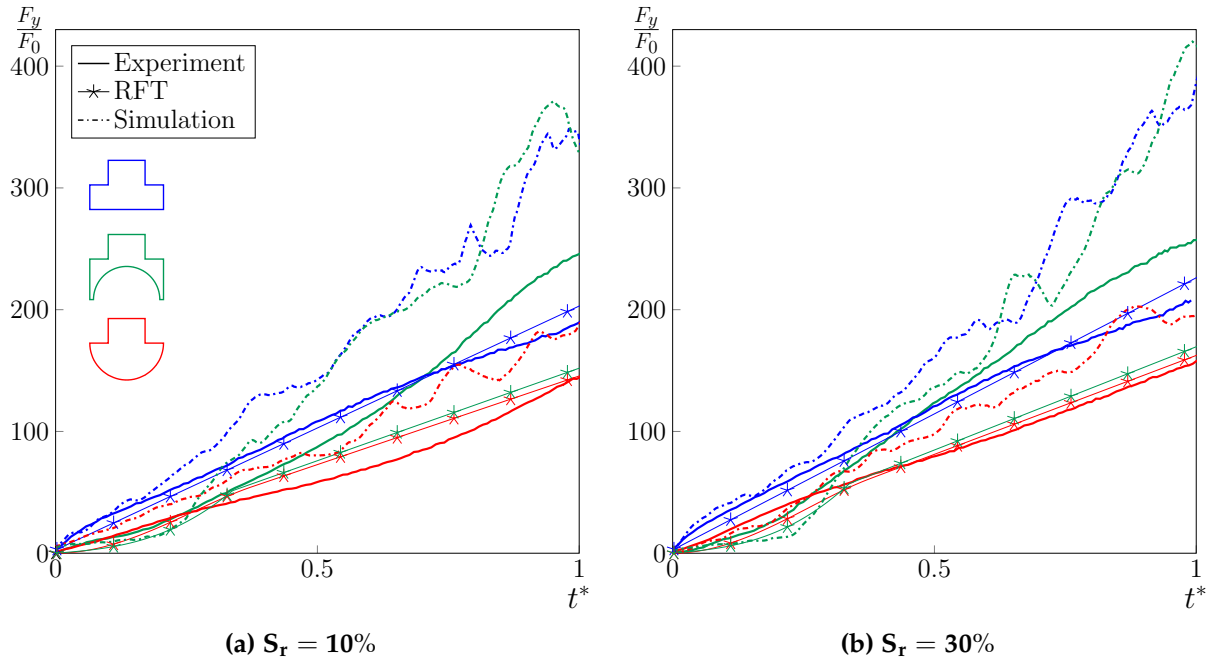


Figure 3.9 Dimensionless vertical resistive force in function of the dimensionless time are presented here for the different foot shapes and two saturation levels that fall within the pendular regime.

compared to the dry case for the three foot shapes. This increase appears relatively linear in the experimental results, motivating the choice of a linear relationship between resistive force and saturation in the extended RFT mode. This linearity is particularly evident for the flat foot. However, for the concave and convex feet, the experimental results show that increases in saturation can sometimes lead to slightly lower resistive forces than in the dry case. This could be due to the difficulty in maintaining a homogeneous saturation level throughout the intrusion depth. As for the numerical simulations, the modeling of cohesion in the medium induces a significant increase of resistive force, but the increase of resistive force due to the saturation is not evident. Indeed, force-depth curves often intersects for the two studied saturation levels. A clear distinction between the two curves is reached for $t^* \simeq 0.25$ in the convex case and for $t^* \simeq 0.45$ in the flat case. For the concave case, there is no specific intrusion depth at which the $S_r = 30\%$ curve consistently exceeds the $S_r = 10\%$ curve. Indeed, the increment of resistive force is very noisy in this case.

Therefore, it can be concluded that the extended RFT model predicts the experimental results at low saturation levels with appreciable accuracy, particularly for the flat and convex feet. The numerical simulations capture the overall trend but overestimates the resistive force in the pendular regime, especially at high depths. It is likely due to a bad tuning of the cohesive law parameters, highlighting the difficulty of modeling the pendular regime with DEM simulations. Nevertheless, this approach still remains a useful tool for studying particle dynamics in low saturation conditions.

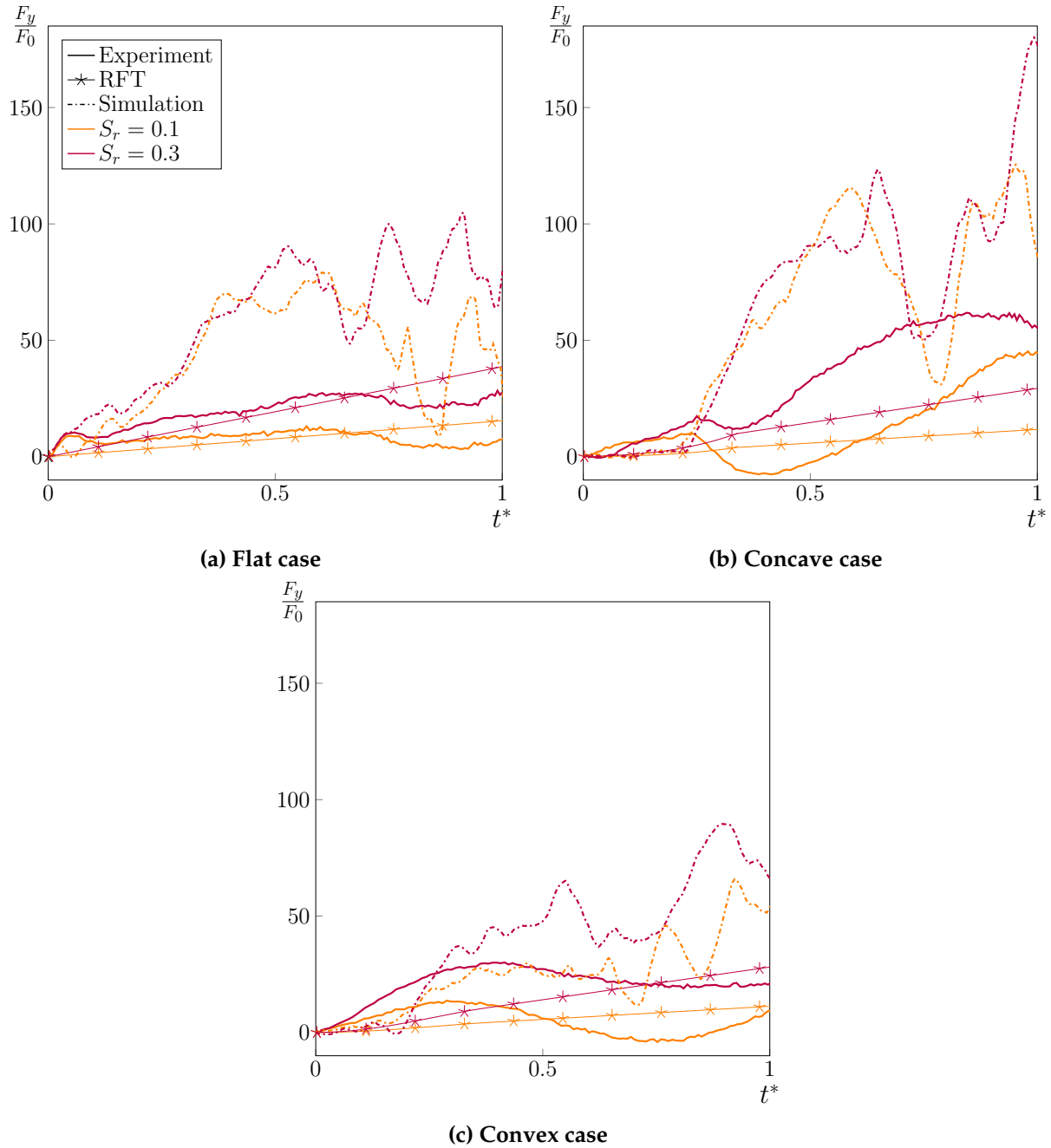


Figure 3.10 Influence of the saturation on the resistive force, presented here by taking the difference between the saturated cases and the dry case, which is used as a reference. This difference is relative and does not mix the different approaches (experimental, analytical, and numerical). It is presented here for the low saturation cases ($S_r = 10\%$ and $S_r = 30\%$), for the three different foot geometries.

4

Conclusion

There is a growing need for robots capable of operating on granular media presenting various saturation levels such as coastal zones, riverbanks, semi-flooded disaster areas, and construction sites for applications in agriculture, biology, the military and the construction industry. The success of their various mission, ranging from search and rescue missions to data-gathering, depends directly on the performance of their feet on challenging terrain that constitute granular media. Therefore, this thesis aims to provide a better understanding of the influence of feet design on their performance in sandy soil with varying saturation levels. This performance evaluation was conducted using various approaches, including experiments, analytical modeling and numerical simulations.

In the first chapter, the state of the art in the field of terramechanics was presented, with a specific focus on legged locomotion and wet granular soils since it was observed that legged systems present advantages on other means of locomotion, such as wheel. The existing analytical and numerical approaches to model these conditions were reviewed, indicating a lack of study on concave foot geometries, and low saturation conditions. The complexity of this kind of granular media motivated a pluralistic approach. Therefore, three feet were designed, including a concave foot, and only the vertical resistive force was studied within the scope of this master thesis, for both experimental and analytical approaches. In order to gain insights on the underlying grain dynamics, Discrete Element Method (DEM) simulations were used and compared to other approaches to critically validate the results.

In the second chapter, the dry sand case is addressed. Experimental intrusion tests for each foot in dry, coarse, loose sand showed a linear increase in vertical resistive force with the intrusion depth, which is also predicted by the Resistive Force Theory (RFT). Both experimental and simulation results showed the convergence of flat and concave case curves at higher intrusion depths, confirming that RFT model fails to capture the behavior of concave geometries in granular media. It was also demonstrated that wall

friction has a negligible contribution in the vertical resistive force, compared to the bottom surface of the foot. Investigating the discrete stress field and the force network in the granular soil showed that a stress concentration zone appear below the feet bottom part. This zone is smaller for foot geometry that induces less resistive force such as the convex foot, while it adopts a larger conical shape for flat and concave geometries. Studying the particle velocity field shows the apparition of small vortices that allows better momentum transmission from the intruder surface to the soil surface for convex geometries. The velocity gradients are higher and induce more complex patterns in the concave case, such as vortex in the cavity during its filling and shear bands. Once fully submerged, grains are trapped in the cavity, leading to the formation of a stagnation zone, similarly to the flat case.

In the third chapter, the wet sand case is investigated. The addition of water due to low-saturation regime induces the apparition of cohesion in the granular media. Therefore, two regimes were observed for the experimental vertical resistive force of all feet. Increasing the saturation level caused the resistive force to increase in low saturation conditions (0 – 60%), while in high saturation conditions (70 – 100%), it led to a decrease in resistive force. Within the pendular regime (0 – 30%), a linear increase was observed, and an extended RFT was proposed for low-saturated sand. This model captures the performance of flat and convex geometries with appreciable accuracy. In these conditions, both experiments and simulations showed that concave foot performs similarly to flat foot at high intrusion depth, and eventually outperforms it. An increase in the size and intensity of the stress concentration zone was observed, compared to the dry sand case. Regarding the velocity gradients, their values decrease compared to the dry case, indicating a less fluidized behavior of the granular medium.

This master thesis conducted an evaluation of the performance of various feet geometries in dry and low-saturated sand, providing insights and guidelines for engineers designing legged robots that operate in such challenging terrains. Results show that a pluralistic approach is necessary to obtain both qualitative and quantitative predictions that are acceptable. An extension of this work using 3D DEM simulations and modeling of wall-particle cohesion could yield more precise and detailed results. Horizontal and diagonal intruder motion, as well as more complex gait patterns, should also be explored to provide a more global evaluation of robot locomotion performance. Finally, further research is needed to explore the other intermediate saturation regimes of sand, in order to comprehensively evaluate foot performance across the full range of saturation levels.

Appendices

A

Soil Evaluation Tool

Since hand-mixing was used to prepare the granular soil samples, it was necessary to ensure that the mixing and saturation were homogeneous. To achieve this, two tools were employed: a Torque Driver FTD2-S (see Fig. A.1a) and a Soil Moisture Meter DM-18R (see Fig. A.1b).



(a) Torque Driver FTD5CN-S used for soil characteristics homogeneity check.



(b) Soil Moisture Meter DM-18R used to assess the homogeneity of saturation level.

Figure A.1

The Torque Driver FTD5CN-S is used for shear vane test. It measures the maximum torque required to cause continuous rotation of a cylindrical sample of soil, which is indicative of the soil's shear strength. The torque value measurements range from 0.5 to 5.0 [$cN \cdot m$], with a resolution of 0.1 [$cN \cdot m$] and a precision of 3%. Four blades of 0.5×1 [cm^2] are mounted on the tip of the instrument and the mixing was validated when the torque values were consistent across all the given testing locations (at four points evenly spaced around the foot intrusion site, each positioned at a distance of four times the radius R and with an intrusion depth of 20 mm). The measurements provided torque value of about 0.6 [$cN \cdot m$]. An estimation of the peak shear stress for the loose sand condition below the surface can be computed using:

$$\tau = \frac{T_{tool}}{2\pi R_{blade}^2 \cdot H_{blade}}$$

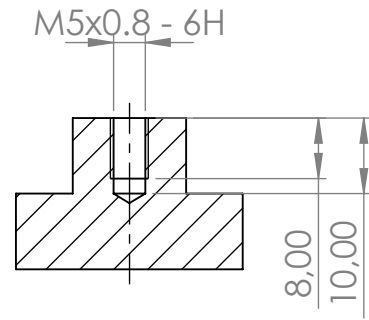
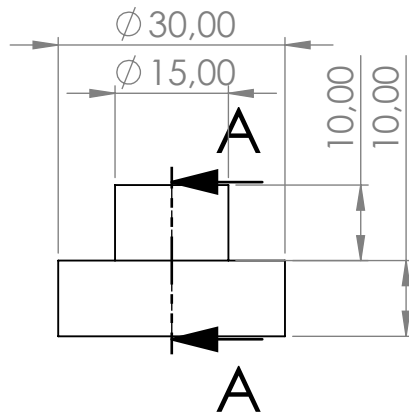
where $T_{tool} = 0.6 [cN \cdot m]$, is the measured torque, $R_{blade} = 5 [cm]$ is the radius of the the cylindrical effective area under shear (equal to the the width of the blade), and $H_{blade} = 2.0 [cm]$, is the height of the blade. This give an approximate peak shear stress of $1909 [Pa]$.

As for the Soil Moisture Meter DM-18R, it is used to measure the moisture content of the soil samples. The device provides a range of 12.1% to 58% moisture content, and it was used to check the homogeneity of the saturation level at four testing points similarly to the torque measurements procedure. It uses an electrical resistance method to measure the moisture content, which is based on the principle that the electrical resistance of the soil decreases as the moisture content increases.

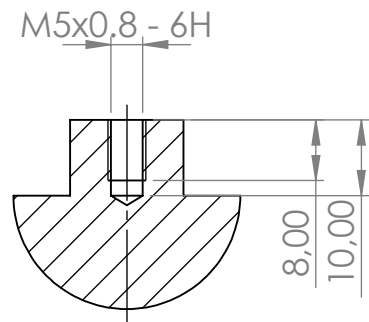
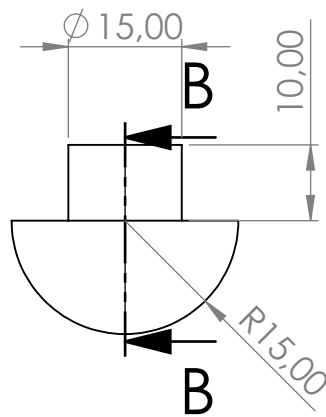


Feet technical drawings

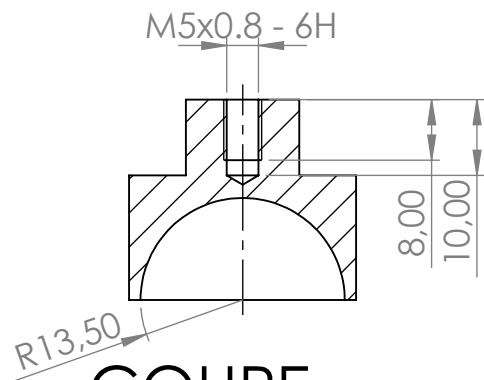
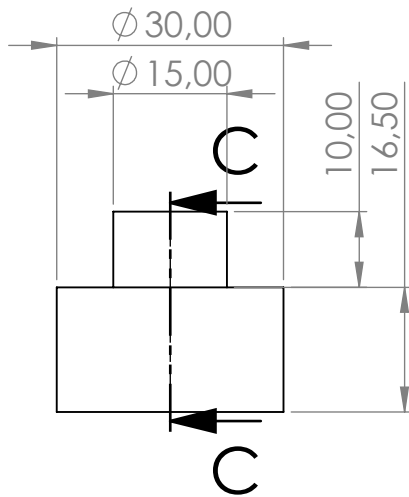
In this appendix, the technical drawings used to manufacture the three experimental feet are presented. The feet are made of aluminum, with standard tolerances and a surface roughness of Ra 3.2. They all present same frontal area A when intruded vertically in the granular media. The feet are mounted to the actuator through a threaded shaft and tapped hole assembly, such that the cylindrical upper part of the foot and the lower cylindrical part of the actuator form a continuous cylinder with a diameter of 15 mm and height 40 mm. This configuration ensures that the final intrusion depth z_f is reached for each foot without the actuator F/T sensor coming into contact with the granular surface.



COUPE A-A

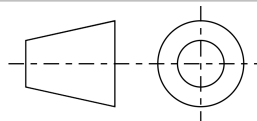


COUPE B-B



COUPE C-C

SAUF INDICATION CONTRAIRE:
LES COTES SONT EN MILLIMETRES
ETAT DE SURFACE: Rg 3.2
TOLERANCES:
LINEAIRES: 0.1
ANGULAIRES: 0.5



CASSER LES
ANGLES VIFS

NE PAS CHANGER L'ECHELLE

REVISION

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AUTEUR	V. GUILMAIN		20/12/24	
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Pieds - Plan général
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FEUILLE 1 SUR 1



Convex and Concave feet RFT

3.1 Convex case

To find the resistive force acting on the convex foot for an intrusion depth z , the integral of a function $f(\theta)$ has to be solved on the surface of a spherical cap of a radius R . we can separate the problem in 2 case: the case where the spherical part is not fully immersed ($z \leq R$) and the one where it is the case ($z > R$). For the first case, using the spherical coordinate, the integral can be expressed in the moving frame $\mathcal{O}'$ as:

$$I = \int_0^{2\pi} \int_{\theta_0}^{\frac{\pi}{2}} f(\theta) R^2 \sin \theta d\theta d\varphi$$

Knowing that in the convex case, the angle θ_0 between the origin of the moving frame and the surface is equal to:

$$\theta_0 = \arcsin \left(\frac{R - z}{R} \right)$$

The variable change $z_{\mathcal{O}'} = R \sin \theta$ can be applied:

$$I = 2\pi R \int_{R-z}^R f(\theta) \frac{z_{\mathcal{O}'}}{\sqrt{R^2 - z_{\mathcal{O}'}^2}} dz_{\mathcal{O}'}$$

Finally, the vertical resistive force can be computed as:

$$F_z = 2\pi Rz \int_{R-z}^R \alpha_z(\beta) \frac{z_{\mathcal{O}'}}{\sqrt{R^2 - z_{\mathcal{O}'}^2}} dz_{\mathcal{O}'}$$

with $\beta = \pi/2 - \theta$.

The second case is solved very simply: when the foot is fully immersed, the contact area doesn't vary with the intrusion depth anymore, then the integral becomes constant, and the resistive force F_z varies linearly with z . It only requires to modify the varying lower limit of the integral to take into account this case. Finally, the vertical resistive force for the convex case can be expressed as follows:

$$F_z = 2\pi Rz \int_{g(z)}^R \alpha_z(\beta) \frac{z_{\mathcal{O}'}}{\sqrt{R^2 - z_{\mathcal{O}'}^2}} dz_{\mathcal{O}'} \quad (\text{C.1})$$

with:

$$g(z) = \begin{cases} R - z & \text{when } 0 < z \leq R \\ 0 & \text{when } z > R \end{cases}$$

3.2 Concave case

The concave case is solved quite similarly to the convex case, except that the integral has a varying upper limit instead of a lower one, and the radius R_e is involved instead of R . Moreover, the resistive force also has a contribution from the flat annulus around the hemispherical concavity, which has an area $A_e = \pi(R^2 - R_e^2) = 1279.41 \text{ [mm}^2\text{]}$. This gives the following integral to solve:

$$I = \int_0^{2\pi} \int_0^{\theta_f} f(\theta) R_e^2 \sin \theta d\theta d\varphi$$

Knowing that in the convex case, the angle θ_f between the origin of the moving frame $\mathcal{O}'$ and the surface is equal to:

$$\theta_f = \arcsin\left(\frac{z}{R_e}\right)$$

The variable change $z_{\mathcal{O}'} = R_e \sin \theta$ can be applied:

$$I = 2\pi R \int_0^z f(\theta) \frac{z_{\mathcal{O}'}}{\sqrt{R^2 - z_{\mathcal{O}'}}^2} dz_{\mathcal{O}'}$$

Finally, the vertical resistive force can be computed as:

$$F_z = \alpha_{z,max} A_e z + 2\pi R z \int_0^z \alpha_z(\beta) \frac{z_{\mathcal{O}'}}{\sqrt{R^2 - z_{\mathcal{O}'}}^2} dz_{\mathcal{O}'}$$

with $\beta = \pi/2 - \theta$.

Similarly to the convex case, we can model the varying upper limit of the integral by a function $h(z)$ to take into account the case when the cavity is entirely submerged.

$$F_z = \alpha_{z,max} A_e z + 2\pi R z \int_0^{h(z)} \alpha_z(\beta) \frac{z_{\mathcal{O}'}}{\sqrt{R^2 - z_{\mathcal{O}'}}^2} dz_{\mathcal{O}'} \quad (\text{C.2})$$

with:

$$h(z) = \begin{cases} z & \text{when } 0 < z \leq R_e \\ R_e & \text{when } z > R_e \end{cases}$$

Note

AI tools such as OpenAI's ChatGPT and GitHub Copilot were used in the writing of this thesis, for helping to improve the clarity of several sections of the paper and for assistance with code development and debugging.

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École polytechnique de Louvain

Rue Archimède, 1 bte L6.11.01, 1348 Louvain-la-Neuve, Belgique | www.uclouvain.be/epl