

**École polytechnique de Louvain**

# **Coupling a high-fidelity aerodynamics solver and a multi-agent formalism for the simulation of bird formation flight dynamics**

**Author: Arnaud CARBONI**

**Supervisors: Pr. Philippe CHATELAIN, Dr. Victor COLOGNESI, Pr. Renaud RONSSE**

**Reader: Pr. Grégoire WINCKELMANS**

**Academic year 2023–2024**

**Master [120] in Mechanical Engineering**





# Abstract

The formation flight of birds has long fascinated humanity. Engineers in the aeronautical sector have even adopted bird flight dynamics in aircraft design and formation flying practices. However, few studies explain the physical phenomena underlying bird formations in detail. Furthermore, there has been no numerical simulation of a bird flapping its wings in formation. This gap in research provides the motivation for this master's thesis. The aim of this dissertation is to couple a high-fidelity aerodynamics solver with a multi-agent formalism to simulate bird formation flight dynamics, using resources developed by the Université Catholique de Louvain. The objective is to create a model allowing birds to optimize energy savings under visual and distance constraints. Position and phase controllers are detailed and developed to direct the movement and stabilization of simulated birds. The simulations focus on the northern bald ibis, a migratory bird known for its formation flying. In our analyses, we are interested in the bird's attitude, the energy saved compared to flying alone, and the distribution of energy gains within the ranks of a formation. Results are compared and interpreted in relation to observed data for this species.

**Keywords :** Bird Flight, Flapping Flight, Flight Formation, Numerical Simulation, Optimization, Flight Control, Biomimicry.



# Acknowledgements

*My first thought goes to my parents and my sister, whom I thank for their unwavering support and for giving me the opportunity to study what I love, engineering. I am also grateful for their confidence in my academic journey, which began at the École Polytechnique de Louvain, continued at the University of Notre Dame, and will further extend next year at the University of Chicago.*

*I would like to express my gratitude to Pr Philippe Chatelain for proposing this master's thesis to me. His active involvement and proactive participation in meetings significantly improved the quality of my work. I am equally thankful to Dr Victor Colognesi for his dedication and availability throughout the year. I also extend my sincere thanks to Pr Renaud Ronsse for his constructive comments and insightful questions, and to Pr Julien Hendrickx for bringing mathematical rigor to the concepts described in this thesis. These four individuals have been instrumental in challenging and elevating my work to a higher level. It was a true pleasure to experience the research environment on this subject with such a dedicated team. Additionally, I would like to thank Pr Grégoire Winckelmans for agreeing to read this master's thesis.*

*My gratitude also extends to Dr Hirotaka Sakaue of the Aerospace-Mechanical Engineering Department at the University of Notre Dame for welcoming me into his laboratory. I am thankful to Dr Mitsugu Hasegawa for giving me the chance to assist him with his research, which allowed me to engage in-depth discussions and broaden my interest in fluid mechanics.*

*I acknowledge the computational resources provided by the supercomputing facilities of the Université Catholique de Louvain (CISM/UCL) and the Consortium des Équipements de Calcul Intensif en Fédération Wallonie Bruxelles (CÉCI), funded by the Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under convention 2.5020.11 and by the Walloon Region.*

*Finally, I would like to thank my long-standing friends and my partner, whose support made my years at Louvain-La-Neuve personally enriching.*

**Arnaud Carboni**

Louvain-La-Neuve, June 2024



# Contents

|                                                              |             |
|--------------------------------------------------------------|-------------|
| <b>Abstract</b>                                              | <b>i</b>    |
| <b>Acknowledgements</b>                                      | <b>iii</b>  |
| <b>Contents</b>                                              | <b>v</b>    |
| <b>List of Symbols</b>                                       | <b>vii</b>  |
| <b>List of Figures</b>                                       | <b>ix</b>   |
| <b>List of Tables</b>                                        | <b>xiii</b> |
| <b>1 Introduction</b>                                        | <b>1</b>    |
| 1.1 Biomimicry . . . . .                                     | 1           |
| 1.2 Formation flight . . . . .                               | 2           |
| 1.3 Objectives of the thesis . . . . .                       | 3           |
| 1.4 Outline . . . . .                                        | 5           |
| <b>2 Bird dynamics</b>                                       | <b>7</b>    |
| 2.1 Conventions and notations . . . . .                      | 7           |
| 2.2 Forces and bird stability . . . . .                      | 8           |
| 2.3 Numerical simulation of bird flapping flight . . . . .   | 13          |
| <b>3 Wake benefit topology</b>                               | <b>29</b>   |
| 3.1 Assumptions . . . . .                                    | 29          |
| 3.2 Wake benefit model . . . . .                             | 31          |
| 3.3 Constraints . . . . .                                    | 34          |
| 3.4 Gradient-based method . . . . .                          | 37          |
| 3.5 Application to the wake of a single bird . . . . .       | 38          |
| <b>4 Flight of two birds</b>                                 | <b>43</b>   |
| 4.1 Spatial control . . . . .                                | 43          |
| 4.2 Phase shift control . . . . .                            | 46          |
| 4.3 Two birds in formation . . . . .                         | 48          |
| <b>5 Formation flight analysis</b>                           | <b>53</b>   |
| 5.1 V-shaped flight . . . . .                                | 53          |
| 5.2 Discussions . . . . .                                    | 54          |
| 5.3 Echelon flight . . . . .                                 | 60          |
| 5.4 Discussions . . . . .                                    | 60          |
| 5.5 Impact of bird attitude on the formation shape . . . . . | 62          |
| <b>6 Conclusions and perspectives</b>                        | <b>65</b>   |
| 6.1 Contributions . . . . .                                  | 65          |
| 6.2 Discussions on future possibilities . . . . .            | 67          |
| <b>Bibliography</b>                                          | <b>71</b>   |
| <b>Appendices</b>                                            | <b>77</b>   |

|          |                                                                               |           |
|----------|-------------------------------------------------------------------------------|-----------|
| <b>A</b> | <b>Numerical integration on the wing surface</b>                              | <b>79</b> |
| <b>B</b> | <b>Resolution by gradient method</b>                                          | <b>81</b> |
| <b>C</b> | <b>Different views of the convergence path for position</b>                   | <b>83</b> |
| <b>D</b> | <b>Analytical calculation of step response for the phase shift controller</b> | <b>85</b> |
| <b>E</b> | <b>Induced velocity for semi-infinite and finite straight vortex filament</b> | <b>87</b> |

# List of Symbols

## Alphanumeric characters

|                                |                                                   |                   |
|--------------------------------|---------------------------------------------------|-------------------|
| $a$                            | Semi axis length of ellipsoid representing a bird | [m]               |
| $b$                            | Wing span                                         | [m]               |
| $\mathcal{B}_C, \mathcal{B}_V$ | Collision, vision constraint functions            |                   |
| $\bar{c}$                      | Mean chord length                                 | [m]               |
| $C_D, C_L$                     | Drag , lift coefficient                           | [/]               |
| $C_l, C_m, C_n$                | Rolling, pitching, yawing moment coefficients     | [/]               |
| $C_X, C_Z$                     | Streamwise and vertical force coefficients        | [/]               |
| $D, L, W, T$                   | Drag, lift, weight forces and thrust              | [N]               |
| $f$                            | Flapping frequency                                | [1/s]             |
| $F$                            | Flapping amplitude                                | [°]               |
| $\mathcal{G}$                  | Objective function                                |                   |
| $l$                            | Lift per unit span                                | [N/m]             |
| $\mathbf{M} = [L, M, N]^T$     | Rolling, pitching and yawing moments              | [Nm]              |
| $m$                            | Bird mass                                         | [kg]              |
| $p, q, r$                      | Rolling, pitching, yawing angular velocities      | [rad/s]           |
| $P_{ij}$                       | Relative position between the bird $i$ and $j$    | [m]               |
| $Re_c$                         | Chord based Reynolds number                       | [/]               |
| $S_w$                          | Wing surface area                                 | [m <sup>2</sup> ] |
| $t$                            | Time                                              | [s]               |
| $T$                            | Flapping period                                   | [s]               |
| $\mathbf{u} = [u, v, w]^T$     | Velocity field                                    | [m/s]             |
| $U$                            | Streamwise characteristic velocity                | [m/s]             |
| $\mathbf{V}$                   | Velocity vector                                   | [m/s]             |
| $\mathbf{x} = [x, y, z]^T$     | Position vector                                   | [m]               |
| $\mathbf{X} = [X, Y, Z]^T$     | Resultant force vector                            | [N]               |

## Greek letters

|                      |                                                             |                      |
|----------------------|-------------------------------------------------------------|----------------------|
| $\alpha$             | Angle of attack                                             | [°]                  |
| $\beta$              | Sideslip angle                                              | [°]                  |
| $\Gamma$             | Circulation                                                 | [m <sup>2</sup> /s]  |
| $\varGamma$          | Dihedral angle                                              | [°]                  |
| $\delta$             | Vision comfort angle                                        | [°]                  |
| $\delta_r, \delta_e$ | Rudder, elevator angles                                     | [°]                  |
| $\Delta\phi_f$       | Phase shift                                                 | [°]                  |
| $\zeta$              | Vorticity vector                                            | [1/s]                |
| $\eta$               | Angle between the axis of vision and the spanwise direction | [°]                  |
| $\kappa$             | Angle between upstream bird and axis of vision              | [°]                  |
| $\lambda$            | Spatial wave length                                         | [m]                  |
| $\Lambda$            | Sweep angle                                                 | [°]                  |
| $\nu$                | Kinematic viscosity                                         | [m <sup>2</sup> /s]  |
| $\rho$               | Density                                                     | [kg/m <sup>3</sup> ] |
| $\tau$               | Flapping angle                                              | [°]                  |
| $\phi, \theta, \psi$ | Roll, pitch, yaw angles                                     | [°]                  |

## List of Symbols

---

### Abbreviations

---

|      |                                     |       |
|------|-------------------------------------|-------|
| AR   | Aspect ratio                        |       |
| HR   | Heart rate                          | [bpm] |
| ILDL | Immersed lifting and dragging lines |       |
| PO   | Power output                        | [W]   |
| VPM  | Vortex particle mesh method         |       |
| WTS  | Wing tip spacing                    | [m]   |

### Subscripts

---

|           |                                           |
|-----------|-------------------------------------------|
| 0         | Initial value                             |
| $f$       | Follower                                  |
| $l$       | Leader                                    |
| $n$       | Normal                                    |
| $opt$     | Optimum                                   |
| $ref$     | Reference                                 |
| $ss$      | Steady state                              |
| $t$       | Target                                    |
| $x, y, z$ | Streamwise, spanwise, vertical directions |



# List of Figures

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |    |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 1.1  | Tree and column in the Sagrada Familia. (from [3]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 1  |
| 1.2  | L'Eole by Clément Ader. (from [9]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 1  |
| 1.3  | Crane echelon formation observed in Lorraine (France). (from [16]) . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 2  |
| 1.4  | Canada geese V-shaped formation. (from [17]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 2  |
| 1.5  | U-shaped formation of herons observed in Lyon (France). (from [18]) . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 2  |
| 1.6  | Block diagram of the methodology implemented for formation flight dynamics. The minimization problem (see section 3.2) takes as input the formation wake (see section 2.3.2), taking into account visual and collision constraints (see section 3.3). The output is the ideal position and phase shift that the controllers define as target (see section 4.1 & 4.2). Once the birds have been positioned, phase-shift and space controllers maintain the formation stable despite wake disturbances on the birds. The displacements implies a new wake. The new data is analyzed to add a new rank of one or two birds while satisfying their constraints. . . . . | 3  |
| 2.1  | Reference frame with reference axes $x$ , $y$ and $z$ . Resulting forces are respectively $X$ , $Y$ and $Z$ ; moments $L$ , $M$ and $N$ , rotational velocities $p$ , $q$ and $r$ ; velocity components of the bird relative to the atmosphere are $u$ , $v$ , $w$ . (adapted from [70]) . . . . .                                                                                                                                                                                                                                                                                                                                                                  | 7  |
| 2.2  | Reference frame for a V-shaped bird formation of 7 birds, including a leader and 6 followers in 3 ranks. . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 8  |
| 2.3  | Forces acting on a bird. Four forces are represented: lift $L$ , drag $D$ , weight $W$ and thrust $T$ . The bird's relative wind velocity vector is denoted $V$ . The angle of attack $\alpha$ is the angle between the velocity vector and the zero-lift line. (adapted from [68]) . . . . .                                                                                                                                                                                                                                                                                                                                                                       | 8  |
| 2.4  | Aircraft wing with relative wind $V$ , sideslip angle $\beta$ and sweep angle $\Lambda$ . (from [20,22]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 10 |
| 2.5  | Bird wing showing the deformation of the wing to produce a sweep angle. (from [21]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 10 |
| 2.6  | Plane showing the dihedral effect thanks to the $I'$ angle and undergoing a $\beta$ sideslip angle. (from [20,22]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 10 |
| 2.7  | Bird wing showing dihedral effect. (from [21]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 10 |
| 2.8  | Influence of wing-to-fuselage position during a $\beta$ sideslip on rolling stability. (from [20,22]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 11 |
| 2.9  | Wing position on the northern bald ibis. (from [23]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 11 |
| 2.10 | Aircraft tail showing elevator angle. (from [21]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 11 |
| 2.11 | Bird tail showing elevator angle. (from [21]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 11 |
| 2.12 | Aircraft tail showing rudder angle. (from [21]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 12 |
| 2.13 | Bird tail showing rudder angle. (from [21]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 12 |
| 2.14 | Illustration showing the anatomy of a bird's wing, showing the bones and nomenclature of the feathers attached to it. (from [27]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 13 |
| 2.15 | View in the $xy$ plane of the bird's multibody model. (adapted from [25,26]) .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 14 |
| 2.16 | Illustration of numerical bird kinematics for a flapping period from $t = 0$ to $t = T$ . The bird's body, tail and head are shown in gray. Bones and feathers are shown in blue. Leading and trailing edges are shown in green. The lifting line is red. (from [15]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                     | 15 |
| 2.17 | Cross-section of a bird's wing. (from [15]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 16 |

|      |                                                                                                                                                                                                                                                                                                                                                                                                                            |    |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 2.18 | Airfoil AS6092. (from [31]) . . . . .                                                                                                                                                                                                                                                                                                                                                                                      | 16 |
| 2.19 | Dimensionless coefficient of force resultants according to $-z$ and $x$ respectively $-C_Z$ and $C_X$ as well as that of pitching moment $C_m$ for a wing flapping period $T = 1/f = 0.25$ [s]. For the top figure, the solid line represents $-C_Z$ taking into account the bird's weight $W$ and the dotted line represents only the aerodynamic forces acting on the bird, <i>i.e.</i> without its weight $W$ . . . . . | 18 |
| 2.20 | Bird displacement adimensionalized along $-z$ and $x$ as well as pitch angle $\theta$ for a wingbeat period $T = 1/f = 0.25$ [s]. The solid line shows the numerical values as a function of time, while the dotted line shows the mean value over the period $T$ . . . . .                                                                                                                                                | 18 |
| 2.21 | Distribution of the $\Gamma$ circulation represented in blue along the lifting line aligned along $y$ on a grey elliptical wing. Each green line spaced by a distance $\Delta y$ between them delimits a surface of the vortex sheet called the trailing vortex. (from [36]) . . . . .                                                                                                                                     | 20 |
| 2.22 | Representation of the magnitude of vorticity $\zeta$ in the wake of an elliptical wing solved using the VPM method. (form [33]) . . . . .                                                                                                                                                                                                                                                                                  | 21 |
| 2.23 | Representation of the vertical velocity component $w$ of the wake cross section showing the upwash and downwash in the wake of an aircraft along a line in the spanwise direction. (adapted from [79]) . . . . .                                                                                                                                                                                                           | 21 |
| 2.24 | Oblique view of the volume rendering of vorticity magnitude in the wake for different times in the bird's flight sequence. The times $t/T = 0$ and $t/T = 1$ correspond to mid-downstroke, $t/T = 1/4$ to end-of-downstroke, $t/T = 1/2$ to mid-upstroke and $t/T = 3/4$ to end-of-upstroke. . . . .                                                                                                                       | 22 |
| 2.25 | Volume rendering of the vorticity magnitude for a wake of length $8\lambda$ allowing visualization in the $xz$ plane as well as the $xy$ plane. . . . .                                                                                                                                                                                                                                                                    | 23 |
| 2.26 | Representation of streawise vorticity and velocity in a cross-section of the $yz$ plane at different positions in the bird's wake $-x/\lambda = 1/4, 1/2, 3/4$ and 1 respectively from top to bottom image for mid-downstroke $t/T = 0$ . . . . .                                                                                                                                                                          | 25 |
| 2.27 | Representation of streawise vorticity and velocity in a cross-section of the $yz$ plane at different positions in the bird's wake $-x/\lambda = 1/4, 1/2, 3/4$ and 1 respectively from top to bottom image for the end of the downstroke $t/T = 1/4$ . . . . .                                                                                                                                                             | 26 |
| 2.28 | Representation of streawise vorticity and velocity in a cross-section of the $yz$ plane at different positions in the bird's wake $-x/\lambda = 1/4, 1/2, 3/4$ and 1 respectively from top to bottom image for mid-upstroke $t/T = 1/2$ . . . . .                                                                                                                                                                          | 27 |
| 2.29 | Representation of streawise vorticity and velocity in a cross-section of the $yz$ plane at different positions in the bird's wake $-x/\lambda = 1/4, 1/2, 3/4$ and 1 respectively from top to bottom image for the end of the upstroke $t/T = 3/4$ . . . . .                                                                                                                                                               | 28 |
| 3.1  | Illustration of the resultant forces acting on an airfoil for a relative velocity $\mathbf{V}$ with a vertical velocity component oriented along $-z$ . . . . .                                                                                                                                                                                                                                                            | 31 |
| 3.2  | Resultant of forces along $-z$ during a period of wing flapping represented as a solid line, approximated by a cosine with an offset as a dotted line. . . . .                                                                                                                                                                                                                                                             | 33 |
| 3.3  | Representation of the $P_{ij}$ position vector and ellipsoids in a top view. . . . .                                                                                                                                                                                                                                                                                                                                       | 34 |
| 3.4  | Top view of stress and collision zones in relation to the upstream bird. . . . .                                                                                                                                                                                                                                                                                                                                           | 34 |
| 3.5  | Illustration of the visual constraint and field of view of the bird's left eye when viewed from above. . . . .                                                                                                                                                                                                                                                                                                             | 36 |
| 3.6  | Illustration of the visual constraint and the bird's field of vision when viewed from the side. . . . .                                                                                                                                                                                                                                                                                                                    | 36 |

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                            |    |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 3.7  | Graph of convergence of the gradient method for the four minimization variables $\mathbf{x} = [x, y, z, \Delta\phi_f]^T$ . Red represents the initial guess $\mathbf{x}_0 = [-2.5, -1.4, -0.7, 150]^T$ . Green represents the initial guess $\mathbf{x}_0 = [-3.5, -1.0, 0.4, 50]^T$ . Orange represents the initial guess $\mathbf{x}_0 = [-3.0, -1.7, 1.0, 70]^T$ . Blue represents the initial guess $\mathbf{x}_0 = [-4.2, -1.1, 1.3, 20]^T$ . . . . . | 38 |
| 3.8  | Representation of a cross-section of the $\mathcal{G}$ function in the $xy$ plane for $(z, \Delta\phi_f) = (0.17 [m], 168 [^\circ])$ fixed. . . . .                                                                                                                                                                                                                                                                                                        | 39 |
| 3.9  | Representation of a cross-section of the $\mathcal{G}$ function in the $z\Delta\phi_f$ plane for $(x, y) = (-1.62 [m], 1.20 [m])$ fixed. . . . .                                                                                                                                                                                                                                                                                                           | 39 |
| 3.10 | Representation of a cross-section of the $\mathcal{G}$ function in the $yz$ plane for $(x, \Delta\phi_f) = (-1.62 [m], 168 [^\circ])$ fixed. . . . .                                                                                                                                                                                                                                                                                                       | 39 |
| 3.11 | Representation of a cross-section of the $\mathcal{G}$ function in the $y\Delta\phi_f$ plane for $(x, z) = (-1.62 [m], 0.17 [m])$ fixed. . . . .                                                                                                                                                                                                                                                                                                           | 39 |
| 3.12 | With $\tilde{L}_z = 1$ . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                           | 40 |
| 3.13 | With $\tilde{L}_z$ from eq. 3.8. . . . .                                                                                                                                                                                                                                                                                                                                                                                                                   | 40 |
| 3.14 | Representation of a cross-section of the $\mathcal{G}$ function in the $x\Delta\phi_f$ plane for $(y, z) = (1.20 [m], 0.17 [m])$ fixed. . . . .                                                                                                                                                                                                                                                                                                            | 40 |
| 3.15 | Cut with $(y = 0 [m], z = 0.17 [m])$ . . . . .                                                                                                                                                                                                                                                                                                                                                                                                             | 41 |
| 3.16 | Cut with $(y = 1.20 [m], z = 0 [m])$ . . . . .                                                                                                                                                                                                                                                                                                                                                                                                             | 41 |
| 3.17 | Representation of a section of the $\mathcal{G}$ function in the $x\Delta\phi_f$ plane for different $(y, z)$ . The dotted black line is the locus of the optimal points in the $x\Delta\phi_f$ plane of Fig. 3.14. . . . .                                                                                                                                                                                                                                | 41 |
| 4.1  | Representation of the control of a follower bird trying to reach a point in space at a relative distance $\Delta\mathbf{x}$ and with a target velocity $\mathbf{u}_t$ in the wake of a leader. . . . .                                                                                                                                                                                                                                                     | 43 |
| 4.2  | Schema block of the controller to reach position $\mathbf{x}_t$ and velocity $\mathbf{u}_t$ at the same location. (adapted from [67]) . . . . .                                                                                                                                                                                                                                                                                                            | 44 |
| 4.3  | Graph showing the evolution of the variables $(x, y, z)$ , averaged over a flapping period, as a function of time for a follower bird in the wake of a leader. . . . .                                                                                                                                                                                                                                                                                     | 45 |
| 4.4  | Graph showing the evolution of the variables $(\phi, \theta, \psi)$ , averaged over a flapping period, as a function of time of a follower bird present in the wake of a leader. . . . .                                                                                                                                                                                                                                                                   | 45 |
| 4.5  | Formation of two birds, the leader on the right and the follower on the left, showing the angle $\tau$ between the wing and the horizontal. . . . .                                                                                                                                                                                                                                                                                                        | 46 |
| 4.6  | Schema block representation of the control of a follower's phase shift and the bird's flapping frequency. . . . .                                                                                                                                                                                                                                                                                                                                          | 47 |
| 4.7  | Evolution of follower phase shift control to reach target phase shift as a function of time. . . . .                                                                                                                                                                                                                                                                                                                                                       | 47 |
| 4.8  | Evolution of a follower's flapping frequency following a change in its phase shift as a function of time. . . . .                                                                                                                                                                                                                                                                                                                                          | 47 |
| 4.9  | Power versus time for a flapping period $T$ . The solid line represents the power of the leader. The dotted line represents the first and only follower for this simulation. Horizontal straight lines represent the average power of the two birds. . . . .                                                                                                                                                                                               | 48 |
| 4.10 | Illustration of the Bio-Savart law applied to one of a bird's two trailing tip vortices. . . . .                                                                                                                                                                                                                                                                                                                                                           | 49 |
| 4.11 | Snapshots of the follower's flight sequence for four different times of a flapping period in an oblique view of the two-bird formation. (1) Mid-downstroke of the follower. (2) End-of-downstroke. (3) Mid-upstroke. (4) End-of-upstroke. . . . .                                                                                                                                                                                                          | 50 |

|      |                                                                                                                                                                                                                                                                                                              |    |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 4.12 | Representation of the vertical velocity component in the wake of two birds in formation in a section $z = 0.17 [m]$ . From top to bottom, we see four different times in the follower's flight sequence; (1) Mid-downstroke of the follower. (2) End-of-downstroke. (3) Mid-upstroke. (4) End-of-upstroke. . | 52 |
| 5.1  | Volume rendering of the vorticity magnitude for the wake of a V-shaped formation of seven selfish birds. Birds are stabilized, obeying the constraints of vision and collision. . . . .                                                                                                                      | 54 |
| 5.2  | Illustration of wing tip spacing (WTS) for two birds in the $xy$ plane. . . .                                                                                                                                                                                                                                | 55 |
| 5.3  | Nothern bald ibis formation in front view. (from [72]) . . . . .                                                                                                                                                                                                                                             | 55 |
| 5.4  | Top view of the V-shaped formation in the $xy$ plane for the case with and without constraints. . . . .                                                                                                                                                                                                      | 55 |
| 5.5  | Representation of the phase shift with respect to the leader $\Delta\phi_f$ as a function of $x$ in the case with and without constraints for a V-shaped flight. .                                                                                                                                           | 55 |
| 5.6  | Power as a function of the size of the V-shaped formation for different ranks. .                                                                                                                                                                                                                             | 56 |
| 5.7  | Power as a function of echelon formation size for different ranks. . . . .                                                                                                                                                                                                                                   | 56 |
| 5.8  | Efficiency $e_\mu$ as a function of position in the V-shaped formation. . . . .                                                                                                                                                                                                                              | 57 |
| 5.9  | Efficiency $e_\mu$ for the first rank in the V-shaped formation. . . . .                                                                                                                                                                                                                                     | 57 |
| 5.10 | Illustration of the Kutta-Joukowski theorem on the bird's lifting line. . . .                                                                                                                                                                                                                                | 57 |
| 5.11 | Illustration of the Biot-Savart law on the bird's lifting line. . . . .                                                                                                                                                                                                                                      | 57 |
| 5.12 | Oscillatory behavior of the $-z$ coordinate of rank 3 in a V-shaped formation of seven selfish birds. . . . .                                                                                                                                                                                                | 59 |
| 5.13 | Power versus time for a V-shaped formation of seven selfish birds. Followers are those of the starboard branch. Blue: Leader. Red: Follower 1. Orange: Follower 2. Purple: Follower 3. . . . .                                                                                                               | 59 |
| 5.14 | Volume rendering of the vorticity magnitude for the wake of an echelon formation of five selfish birds. . . . .                                                                                                                                                                                              | 60 |
| 5.15 | Top view of the echelon formation level in the $xy$ plane for the case with and without constraints. . . . .                                                                                                                                                                                                 | 61 |
| 5.16 | Representation of the $\Delta\phi_f$ relationship as a function of the streamwise position $x$ in the case with and without constraints for an echelon flight. .                                                                                                                                             | 61 |
| 5.17 | Efficiency $e_\mu$ as a function of position in the echelon formation. . . . .                                                                                                                                                                                                                               | 62 |
| 5.18 | Efficiency $e_\mu$ for the first rank in the echelon formation. . . . .                                                                                                                                                                                                                                      | 62 |
| 5.19 | Numerical simulation of a V-shaped (black circle) and U-shaped (white triangle) formation flight. (from [39]) . . . . .                                                                                                                                                                                      | 63 |
| 5.20 | Efficiency $e_\mu$ as a function of fixed wing position for V-shaped formation (white triangle) and U-shaped formation (black circle). (from [39]) . . . .                                                                                                                                                   | 63 |
| A.1  | Top view of an approximation of the wing surface $S_w$ during mid-downstroke at $t/T = 0$ . . . . .                                                                                                                                                                                                          | 79 |
| A.2  | Rear view of an approximation of the wing surface $S_w$ during end-upstroke at $t/T = 3/4$ . . . . .                                                                                                                                                                                                         | 79 |
| C.1  | Graphs of convergence of the gradient method for the four minimization variables $\mathbf{x} = [x, y, z, \Delta\phi_f]^T$ for four different initial guesses $\mathbf{x}_0$ . The top figure shows a view from above, the middle one from the side and the bottom one from the front. . . . .                | 83 |
| E.1  | Representation of the induced velocity of a semi-infinite straight vortex filament of strength $\Gamma$ . . . . .                                                                                                                                                                                            | 87 |
| E.2  | Representation of the induced velocity of a straight vortex filament of finite length of strength $\Gamma$ . . . . .                                                                                                                                                                                         | 87 |

# List of Tables

|     |                                                                                                                                                  |    |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 2.1 | Different yawing control configurations for the bird. $L_T$ stands for tail lift and $\delta_r$ for rotation about the $x$ axis. . . . .         | 12 |
| 2.2 | Numerical parameters representing physical characteristics used in northern bald ibis simulations. . . . .                                       | 13 |
| 5.1 | Coordinate $y$ and $-z$ for echelon and V-shaped formation, with or without constraints and according to position in a four-rank flight. . . . . | 54 |



# Chapter 1

## Introduction

### 1.1 Biomimicry

Human nature has always been intrigued by the world around us. We seek to understand it, but also to draw from it a source of inspiration capable of improving our living conditions. It is not uncommon, therefore, to see certain behaviors or characteristics that come directly from nature appear in all kinds of human creations, from the scientific to the artistic. This kind of phenomenon is called *biomimicry*.



Figure 1.1: Tree and column in the Sagrada Familia. (from [3])



Figure 1.2: L'Eole by Clément Ader. (from [9])

Biomimicry can be found in various fields of application, including art, medicine, architecture and engineering [1]. In the field of art and architecture, La Sagrada Familia in Barcelona (Spain) is a well-known example of biomimicry in that field. As we can see on Fig. [1.1](#), the building's supporting beams are a trunk at the base of the structure, and divide into leaves on the surface of the ceiling [2,3]. In medicine, biomimicry enables the creation of new tools, as in surgery. Indeed, researchers at the University of Illinois have developed suction cups inspired by octopus suction cups to move delicate tissue for transplants [4]. Still in the medical field, a study of the medical innovation market based on biomimicry by the British strategic consultancy *Precedent Research* estimated it for USD\$33 billion for 2022 and USD\$65 billion in 2032 [5]. This shows the importance of researching and studying nature to inspire creative design [6,7]. Finally, in the field of aeronautics and fluid mechanics, a shark-skin-like coating is being studied for the outer surface of aircraft wings. This would improve aircraft performance compared with conventional wings [8,9]. As a final example, let's recall that the first aircraft designs, and indeed still today, are an imitation of the physical characteristics of birds, as seen on each of the 16 machines designed by Otto Lienthal, or with Clément Ader's Eole, built in 1890 and resembling the wings of a bat on Fig. [1.2](#).

Many technological advances have been made possible by first studying nature and then applying the results to applications that are useful to mankind.

### 1.2 Formation flight

It is not uncommon to see migratory birds heading south in the sky between August and November [10]. Anyone who has seen this phenomenon may have wondered about the reason for the form of group flight, known as formation. Formation can take several forms, such as "echelon", "V" or "U"-shaped formation, all shown respectively in Fig. 1.3, Fig. 1.4 and Fig. 1.5. The reason for the different formation types seems trivial, but still hard to explain. Formation flying as such could be explained by the desire of the group or each individual to minimize their energy expenditure during long periods of flight. The explanation for the decrease in energy consumed during flight will be explained in a later chapter, as will the reasons for the different formations. Generally speaking, each bird present in the formation will exploit the wake produced by all the other birds to adapt its position and wing beats to minimize its energy expenditure. Given that this is a problem involving several individuals, they may be prone to selfishness, or rather to a cooperative attitude. Let's not forget that the constraints of collision and visibility make formation flying a dynamic equilibrium, whether in the wake itself or in the event of external disturbances. We should also bear in mind that subjects can be of different mass, sex, age and even species in certain cases [11]. Each bird therefore has different characteristics and physiology, and not all birds are equal when it comes to effort. Therefore, birds can take turns, but the decision-making process has yet to be explained, or the allocation of places in the formation based on the characteristics of each bird.



Figure 1.3: Crane echelon formation observed in Lorraine (France). (from [16])



Figure 1.4: Canada geese V-shaped formation. (from [17])



Figure 1.5: U-shaped formation of herons observed in Lyon (France). (from [18])



The study of bird formation flight, and more specifically the study of energy minimization taking advantage of the wake, could provide solutions in the aeronautical sector. For instance, to reduce problems concerning global energy consumption impacting  $CO_2$  emissions. Moreover, the International Energy Agency (IEA) has set out a scenario for reducing greenhouse gas production, the Net Zero Emissions by 2050 Scenario (NZE) [12]. The air transport sector, whether tourist or commercial, is a major contributor to the production of greenhouse gases. The aim of air transport today is therefore to reduce fuel consumption. There are two solutions. The first is to improve propulsion system efficiency. Safran and GE are currently studying a new design called the "open rotor" engine which could reduce fuel consumption by 20% [13]. The second one is formation flight. Airbus has successfully completed the first formation flight between Toulouse (France) and Montreal (Canada) in 2021, saving 6 *metric tonnes* of  $CO_2$  and more than 5% in fuel consumption [14].

Formation flying is therefore a real solution for reducing the aviation sector's carbon footprint, in line with the Net Zero Emissions Scenario. Consequently, it is a matter of studying it, and doing it directly from the source, *i.e.* by studying bird formation flight. This will enable us to understand the underlying physics and turn our understanding into applications for the aeronautic industry. It is clear that the flight dynamics of birds in formation are poorly understood, and there are still many grey areas to be clarified.

### 1.3 Objectives of the thesis

The models are based on the study and processing of data from a fluid disturbed by the passage of birds. These data are then used to determine the control parameters to be modified for subjects present in the fluid, in order to limit their energy consumption. This work presents a modular approach to the models developed. The aim of making the code modular means that it can be used in other environments or with other subjects. Fig. 1.6 explains the procedure for creating the structure of a formation flight for any kind of subjects.

The model we have developed would enable us to analyze the wake of a group of subjects, and then determine where in space to place another rank in order to minimize energy consumption for each new subjects. More generically, the model could also be applied to aircrafts and to all so-called rotary-wing vehicles, such as helicopters.

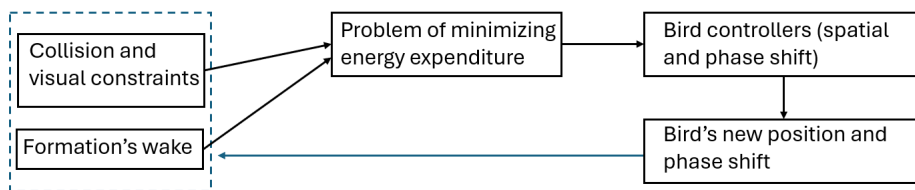


Figure 1.6: Block diagram of the methodology implemented for formation flight dynamics. The minimization problem (see section 3.2) takes as input the formation wake (see section 2.3.2), taking into account visual and collision constraints (see section 3.3). The output is the ideal position and phase shift that the controllers define as target (see section 4.1 & 4.2). Once the birds have been positioned, phase-shift and space controllers maintain the formation stable despite wake disturbances on the birds. The displacements implies a new wake. The new data is analyzed to add a new rank of one or two birds while satisfying their constraints.

In the case of bird formation flight, the birds are in air, and their movements induce a certain structure in the wake. To this end, we will take up the work carried out by Dr Victor Colognesi to simulate the wake produced by one or more birds [15]. The aim is to use this data to determine the position and phase shift of each bird. Thus, to dynamically move the subjects in such a way as to minimize their energy expenditure. We take into account the discomfort due to the risk of collision and their visibility. The implementation takes a reference bird configuration and then analyzes the wake to place the birds correctly rank by rank downstream of the first leader bird. The ideal locations are then input into the spatial and phase shift controllers of each bird to achieve this position and phase shift. Then, they remain there despite the disturbance caused by the wake. The structure of the approach is illustrated in Fig. 1.6. Afterwards, we will be able to see the reduction in energy expenditure in the ranks of the formation compared to flying alone, the wake structure within the formation and the exploitation of the wake as a function of space and phase shift by the subjects.

At the end of this master's thesis, we should be able to answer the following questions.

**Question 1:** How can we establish a model for bird formation flight?

- 1.i. : What effects do birds benefit from to reduce their energy expenditure?
- 1.ii. : Which bird control variables should be taken into account in the model?
- 1.iii. : What are the constraints to be taken into account when a bird flies in a group?

**Question 2:** What is the impact on the energy consumed during a formation flight for a first-rank or upper-rank bird?

- 2.i. : What is the difference in power output between a leader and a follower?
- 2.ii. : Do all followers have a similar reduction in energy expenditure? Is it better to be at the end of the formation or at the beginning?
- 2.iii. : Is bird formation more efficient than airplane formation in terms of energy savings achieved?

**Question 3:** Does energy minimization alone justify the shape and stability of formation flight numerically?

- 3.i. : Are the bird's variables at optimum always similar if the constraints are not respected? What about the stability of group flight in this case?
- 3.ii. : What are followers positions, phase shifts and energy savings in observations versus numerically?
- 3.iii. : How does the distribution of effort within the formation change according to the bird's attitude?

**Question 4:** What happens to the wake structure once the birds are in flight formation?

- 4.i. Where is the bird positioned in this wake and how does it adapt its phase shift to the upstream bird?
- 4.ii. What are the differences between a flapping wing wake and that of a fixed wing?

## 1.4 Outline

The rest of the dissertation is organised into five chapters as follows:

**Chapter 2: Bird dynamics** In this chapter following the introduction, we will look at the conventions, notations and vocabulary used in aeronautics. We will also review the forces acting on a bird and how it uses its various control surfaces to move and stabilize in space. We will also discuss the different characteristics of the bird used in our simulations, the northern bald ibis. We conclude this chapter by reviewing the notions of vorticity and upwash velocity, and then analyze the structure of the wake produced by a single bird. This will help us to introduce a model that can take advantage of the wake's characteristics to reduce the effort of a bird in a group.

**Chapter 3: Wake benefit topology** In this third chapter, we will explain the various components of our model. We begin by establishing the scope of application, setting out the various underlying assumptions. We will discuss the objective function known as the wake benefit model, based on a reflection of the power saved by a non-zero angle of attack. This model is made up of a force model and an average of the vertical velocity component on the bird's wings. The minimization problem is a constrained problem. It is constrained by a vision comfort zone centered around the axis of vision of each eye, and a collision zone which is linked to the risk of colliding with other birds. We then solve the problem using the gradient method and apply the model to the wake of a single bird. At the end, we analyze the results as a function of the minimization parameters.

**Chapter 4: Flight of two birds** In this chapter, we use the results of the previous chapter to position a second bird at the optimum location with the optimum phase shift. To do this, we explain the development of a spatial and phase-shift controller. The aim of which is to enable the transition from a reference configuration to the optimum configuration. Secondly, to ensure the stability of the formation by controlling each bird and ensuring that its flight remains stable despite disturbances caused by the presence of upstream birds.

**Chapter 5: Formation flight analysis** In this penultimate chapter, we move on to the analysis of formation flying as such. First, we will look at the case of V-shaped flight and then echelon flight. For both cases, we will analyze the differences and similarities in terms of the number of birds present, optimal positions, optimal phase shifts and energy saved within the formation. We will compare them with observations of real birds. Then, for each of the two cases, we will evaluate the impact of the constraints on the same measurements but also on the stability of the formation. Finally, the chapter concludes with a section on the impact of the attitude of each numerical bird in the formation on group structure and effort distribution.

**Chapter 6: Conclusions and perspectives** The dissertation concludes with a review of the results obtained, the model's limitations and ways for improvement. In the results obtained, we will discuss the differences and similarities with aircraft formation flight. We will then look at a number of special cases where the model could be applied, such as in the case of aircraft formation flight, in the wake analysis of rotary-wing vehicles in general, or in the control of flapping-wing drones.



## Chapter 2

# Bird dynamics

### 2.1 Conventions and notations

Before presenting and explaining the model developed and detailed in Chapter 3, it is essential to establish the reference frame in which we will be working. This is illustrated in Fig. 2.1 for a bird.

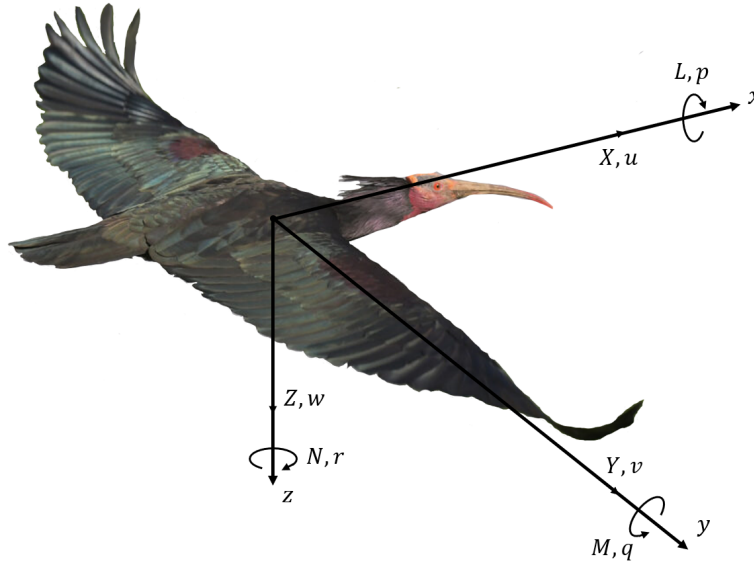


Figure 2.1: Reference frame with reference axes  $x$ ,  $y$  and  $z$ . Resulting forces are respectively  $X$ ,  $Y$  and  $Z$ ; moments  $L$ ,  $M$  and  $N$ , rotational velocities  $p$ ,  $q$  and  $r$ ; velocity components of the bird relative to the atmosphere are  $u$ ,  $v$ ,  $w$ . (adapted from [70])

Note that this is the convention where the  $x$  axis points forward and is aligned with the bird's longitudinal axis. The  $x$  position is called the streamwise position. For the  $y$  axis, the convention posits a positive  $y$  coordinate when we are on the starboard side of the bird. The position along  $y$  is called the spanwise position. Rotations around the axes of rotation  $x$ ,  $y$  and  $z$  are called roll, pitch and yaw respectively; the angles associated with these rotations are  $\phi$ ,  $\theta$  and  $\psi$ .

In formation flight, the reference frame is centered on the bird, where the origin coincides with its center of mass. In Fig. 2.2 the reference frame is attached to the first bird, called the leader. Birds downstream of the leader are called followers and are divided into  $n$  ranks. For each  $i$  rank, there may be one or two birds, depending on the structure of the formation, *i.e.* whether it is an echelon or V-shaped flight. Still on the same figure, we have the leader at  $(x, y, z) = (0, 0, 0)$  and 6 followers in  $n = 3$  ranks of 2 birds.

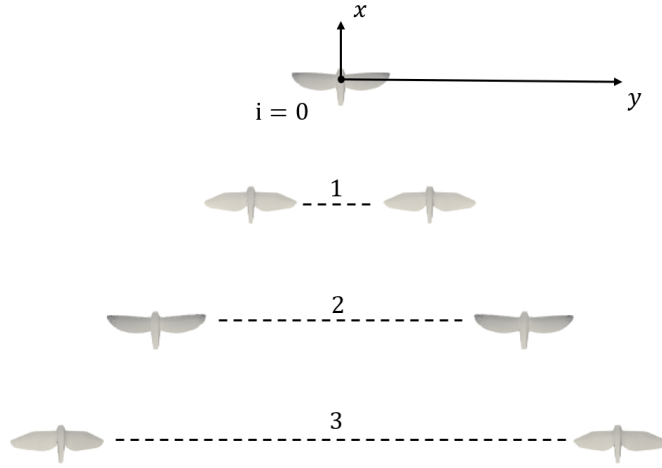


Figure 2.2: Reference frame for a V-shaped bird formation of 7 birds, including a leader and 6 followers in 3 ranks.

## 2.2 Forces and bird stability

### 2.2.1 Forces

The forces acting on a bird are similar to those acting on an aircraft. There are four of them: lift  $L$ , drag  $D$ , weight  $W$  and thrust  $T$ . All these forces are shown in Fig. 2.3

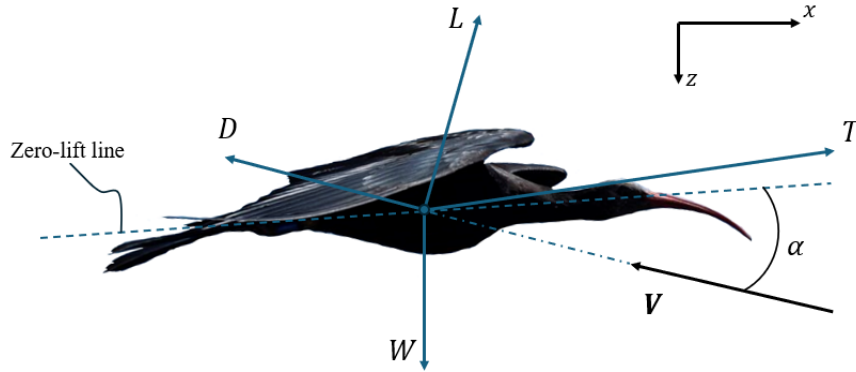


Figure 2.3: Forces acting on a bird. Four forces are represented: lift  $L$ , drag  $D$ , weight  $W$  and thrust  $T$ . The bird's relative wind velocity vector is denoted  $\mathbf{V}$ . The angle of attack  $\alpha$  is the angle between the velocity vector and the zero-lift line. (adapted from [68])

Lift  $L$  is defined as the resultant of forces perpendicular to the velocity vector  $\mathbf{V}$ . We define the lift coefficient  $C_L$  in eq. 2.1 with  $\rho$  the air density,  $S_w$  the wing surface area and  $U$  the characteristic velocity. Drag  $D$  is the resultant of forces opposing bird motion. We define the drag coefficient  $C_D$  in eq. 2.2

$$C_L = \frac{L}{\frac{1}{2}\rho S_w U^2} \quad (2.1)$$

$$C_D = \frac{D}{\frac{1}{2}\rho S_w U^2} \quad (2.2)$$

Thrust is the force created by the flapping of its wings to set the bird in motion. Because of the bird's flight sequence, *i.e.* wing kinematics over a flapping period, the perceived angle of attack along the wing changes over time. This has a direct impact on the variation of these forces over time. Lift, drag and thrust are not constant, unlike an aircraft in level flight, for instance. As we will discuss later, these variations in force over a flapping period induce displacements, forcing the bird to stabilize itself.

## 2.2.2 Bird stability

During a flapping period, the bird undergoes an imbalance created by the desire to propel itself by modifying the shape of its wings. This change in shape induces a variation in the forces applied to the bird as a function of time. Thus a displacement that can unbalance the bird's flight if disturbances act on it. To remain stable and combat external disturbances such as turbulence or crosswinds, it can also use what are known as control surfaces. These surfaces are simply what it sets in motion to stabilize itself or change direction. In this section, we will look at how birds use their feathers to combat disturbances, and how humans have copied the bird through biomimicry in aircraft design. To do this, we will perform a static stability for roll, pitch and yaw. Static stability is the study of the initial tendency for a vehicle to return to equilibrium after a disturbance.

### 2.2.2.1 Rolling control

To study rolling stability, we will first define the rolling moment coefficient with  $L$  the rolling moment.

$$C_l = \frac{L}{\frac{1}{2}\rho S_w b U^2} \quad (2.3)$$

with  $\rho$  the air density,  $S_w$  the wing surface area,  $b$  the wing span and  $U$  the characteristic velocity. In particular, we analyze stability in sideslip  $\beta$  perturbations, such as those exerted by a sidewind. We linearize  $C_l$  in eq. 2.4

$$C_l \approx \frac{\partial C_l}{\partial \beta} \beta = C_{l,\beta} \beta \quad (2.4)$$

For the  $\beta > 0$  perturbation to lead to a stabilization of  $C_l$ , we have as a criterion that  $C_{l,\beta} < 0$ .

#### Sweep effect

To contribute to  $C_{l,\beta}$ , aircraft can use the sweep effect as shown in Fig. 2.4. The sweep angle  $\Lambda$  is the angle between the horizontal axis  $y$  and a straight line located at a constant fraction (1/4) of the chord from the leading edge.  $V_n$  represents the velocity component  $V$  orthogonal to this wing reference line. As  $V_n$  is different on the two parts of the wing, the lift created at starboard is greater than that created at portboard. This results in a negative rolling moment. Since  $\beta > 0$  and  $C_l < 0$ , then  $C_{l,\beta} < 0$  is a stabilizing contribution [22]. Birds can also create a sweep angle as shown in Fig. 2.5 for the same stabilizing purpose as aircraft.

#### Dihedral effect

The dihedral effect, as illustrated in Fig. 2.6 is created by an angle  $\Gamma$  between the horizontal axis along span  $y$  and the wing reference line.

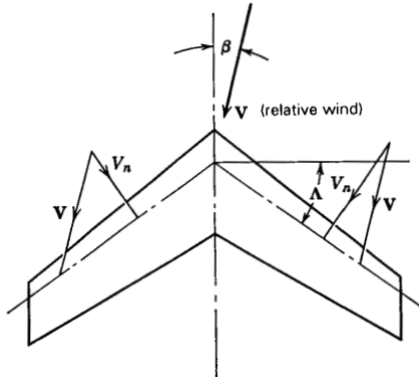


Figure 2.4: Aircraft wing with relative wind  $\mathbf{V}$ , sideslip angle  $\beta$  and sweep angle  $\Lambda$ . (from [20,22])

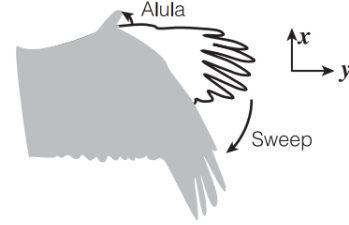


Figure 2.5: Bird wing showing the deformation of the wing to produce a sweep angle. (from [21])

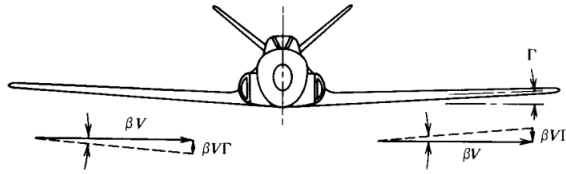


Figure 2.6: Plane showing the dihedral effect thanks to the  $\Gamma$  angle and undergoing a  $\beta$  sideslip angle. (from [20,22])

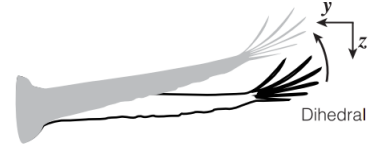


Figure 2.7: Bird wing showing dihedral effect. (from [21])

The velocity component perpendicular to the starboard wing,  $V_n$ , has as its component for  $\Gamma$  small as follows.

$$V_n = w \cos(\Gamma) + v \sin(\Gamma) \approx w + v\Gamma \quad (2.5)$$

For the portboard wing,  $V_n \approx w - v\Gamma$ . Since for  $\Gamma$  small,  $v = V \sin(\beta) \approx \beta V$  with  $V = |\mathbf{V}|$ . Therefore,  $V_n = w \pm \beta V \Gamma$ . The angle of attack  $\alpha$  is therefore not identical for the two wing panels. The variation  $\Delta\alpha = \pm\beta V \Gamma$  is shown in Fig. 2.6. The left wing, the starboard wing, will have lower lift than the portboard wing, inducing a negative rolling moment  $C_l < 0$ . Note that the rolling moment is proportional to  $\beta\Gamma$ ,  $\Delta C_l \propto \beta\Gamma$  [20,22]. For a fixed  $\beta > 0$  then we have  $C_{l,\beta} \propto \Gamma$ . If  $\Gamma > 0$ ,  $C_{l,\beta} < 0$  and we have a stable contribution. In Fig. 2.7, birds can change  $\Gamma$  more easily when gliding, since their muscles enable them to modify the phase of their wings and thus improve their rolling stability.

### Wing position

The position of the wings on an aircraft fuselage, as shown in Fig. 2.8, also influences rolling stability. The flow interacts with the fuselage to change the dihedral effect perceived by the wing. Let's imagine a cylindrical body centered with the  $x$  axis pointing towards the sheet, and focus solely on the cross-flow component  $v \approx V\beta$ . We can see that the presence of the cylindrical body induces a vertical velocity that would impact the local angle of attack along the wings. When we have high-wings, we find the dihedral effect, as the right wing will have more lift (vertical component of upward velocity) than the left wing (vertical component of downward velocity), inducing a negative rolling moment. Conversely, for the low wing, the flow will induce a positive rolling moment. Consequently, for the high wing,  $C_{l,\beta} < 0$  for  $\beta > 0$  stabilizing effect due to the secondary flow avoiding



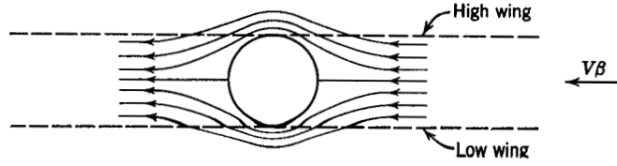


Figure 2.8: Influence of wing-to-fuselage position during a  $\beta$  sideslip on rolling stability. (from [20,22])



Figure 2.9: Wing position on the northern bald ibis. (from [23])

the fuselage as explained in [20]. In Fig. 2.9, the bird has its wings positioned high on its body, enabling it to take advantage of this rolling stabilization when gliding.

### 2.2.2.2 Pitching control

To study pitching control, we define the pitching moment coefficient  $C_m$  as follows.

$$C_m = \frac{M}{\frac{1}{2}\rho S_w \bar{c} U^2} \quad (2.6)$$

with  $\rho$  the air density,  $S_w$  the wing area,  $\bar{c}$  the mean aerodynamic chord length and  $U$  the characteristic velocity.

#### Elevator deflection

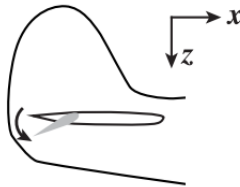


Figure 2.10: Aircraft tail showing elevator angle. (from [21])

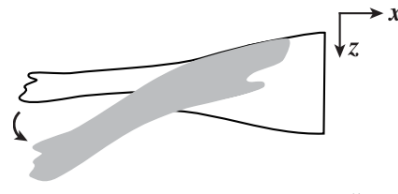


Figure 2.11: Bird tail showing elevator angle. (from [21])

The main control surface for varying the pitch angle  $\theta$  is on the tail of an aircraft, as shown in Fig. 2.10. The elevator angle  $\delta_e$  is positive, as shown on the same figure. When deployed  $\delta_e > 0$ , the wing camber increases, inducing positive lift (negative force according to  $z$ ). Since tail lift is applied at a certain distance from the aircraft's center of mass, this induces a variation in pitching moment  $\Delta C_m < 0$ , which consequently reduces  $\theta$ . Conversely, if  $\delta_e < 0$ ,  $\Delta C_m > 0$  increases  $\theta$ . Birds use exactly the same principle, moving their tails as illustrated in Fig. 2.11. When tilted downwards, it corresponds to the case  $\delta_e > 0$  and induces a  $\Delta C_m < 0$  to reduce  $\theta$ . When the tail is raised, the lift generated by the tail is negative (positive according to  $z$ ), which increases the bird's pitching moment.

### 2.2.2.3 Yawing control

To study yawing control, we define the yawing moment coefficient  $C_n$ :

$$C_n = \frac{N}{\frac{1}{2}\rho S_w b U^2} \quad (2.7)$$

with  $\rho$  the air density,  $S_w$  the wing surface area,  $b$  the wing span and  $U$  the characteristic velocity.

#### Rudder deflection

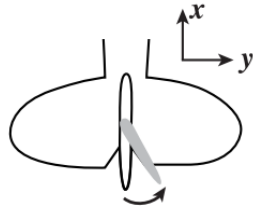


Figure 2.12: Aircraft tail showing rudder angle. (from [21])

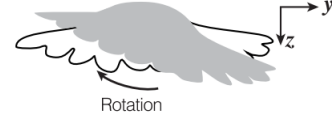


Figure 2.13: Bird tail showing rudder angle. (from [21])

Yawing control for aircraft is made possible by the rudder located on the tail of the aircraft, as shown in Fig. 2.12. In this figure, the rudder angle  $\delta_r$  is shown negatively. When  $\delta_r < 0$ , the vertical tail produces lift oriented along  $-y$ . As this force is applied at a certain distance from the aircraft's center of mass, it causes a variation in the yawing moment  $\Delta C_n > 0$ . Conversely, if  $\delta_r > 0$ , the yawing moment will decrease. Birds, on the other hand, do not have a vertical tail, but a horizontal tail that can be rotated around the  $x$  axis, as shown in Fig. 2.13. We need to separate the four cases shown in Table 2.1

|           | $\delta_r > 0$ | $\delta_r < 0$ |
|-----------|----------------|----------------|
| $L_T > 0$ | case 1         | case 2         |
| $L_T < 0$ | case 3         | case 4         |

Table 2.1: Different yawing control configurations for the bird.  $L_T$  stands for tail lift and  $\delta_r$  for rotation about the  $x$  axis.

For Case 1,  $L_T > 0$  means that the bird's tail is down ( $\delta_e > 0$ ), so lift production is positive (negative according to  $z$ ). As  $\delta_r > 0$ , we are in the configuration shown in Fig. 2.13 and lift has a positive  $y$  component. As the tail is far from the center of mass, the positive  $y$  component of the lift induces a negative yawing moment variation,  $\Delta C_n < 0$ . For case 2, the rotation along  $x$  is opposite, inducing a negative lift component along  $y$ . Consequently, the yawing moment variation is positive,  $\Delta C_n > 0$ . In Case 3, the bird's tail is raised, inducing a positive lift component along  $z$  and therefore pointing downwards. As the tail rotation is positive along  $x$ ,  $L_T$  has a negative component along  $y$ , resulting in a positive yawing moment variation,  $\Delta C_n > 0$ . Finally, in the last case, Case 4, the position along  $x$  is opposite,  $L_T$  has a positive component along  $y$  and therefore causes a negative yawing moment variation,  $\Delta C_n < 0$ .

## 2.3 Numerical simulation of bird flapping flight

### 2.3.1 Nothern bald ibis

The bird considered for the numerical simulations is the nothern bald ibis, also known by its Latin name *Geronticus eremita*. All its physical characteristics are listed in Table 2.2

| Parameter                    | Symbol    | Value                  |
|------------------------------|-----------|------------------------|
| Mass                         | $m$       | 1.20 [kg]              |
| Length                       | $l_b$     | 0.80 [m]               |
| Wing span                    | $b$       | 1.35 [m]               |
| Mean aerodynamic chord       | $\bar{c}$ | 0.18 [m]               |
| Wing surface                 | $S$       | 0.24 [m <sup>2</sup> ] |
| Aspect ratio                 | $AR$      | 7.5 [/]                |
| Flapping amplitude           | $F$       | 30 [°]                 |
| Flapping frequency           | $f$       | 4 [Hz]                 |
| Front binocular vision angle | $\eta$    | 15 [°]                 |
| Vision comfort angle         | $\delta$  | 35 [°]                 |

Table 2.2: Numerical parameters representing physical characteristics used in nothern bald ibis simulations.

Modular numerical simulations can be used to simulate the wake of any bird. However, to respect the Lifting line theory described in more detail in section 2.3.3, it is necessary to choose a bird with a large aspect ratio. This migratory bird, the nothern bald ibis, shown in Fig. 2.9 is chosen because it has an aspect ratio  $AR$  of 7.5. In addition, for the study and simulation of formation flight, it is necessary to choose a bird where there are observations of ibises flying in formation, and whose followers save energy by being present behind a leader [24].

### 2.3.2 Bio-mechanical model

The bio-mechanical model proposed by Dr Victor Colognesi in his articles [15,25,26] is based on the structure of a bird's bones and feathers, as shown in Fig. 2.14

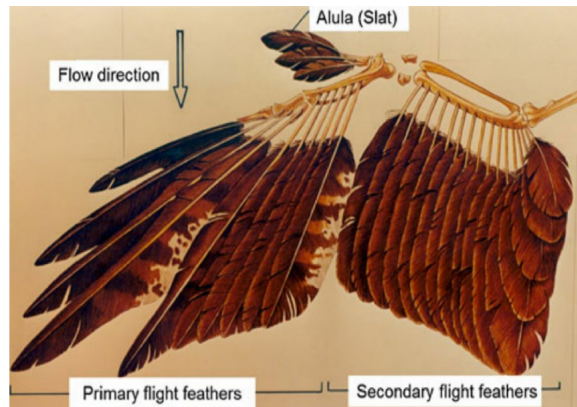


Figure 2.14: Illustration showing the anatomy of a bird's wing, showing the bones and nomenclature of the feathers attached to it. (from [27])

## 2. Bird dynamics

Not all feathers are equal in utility. In Fig. 2.14, the feathers attached to the bird's arm, particularly the ulna, the rightmost bone in the illustration between the elbow and the wrist, are called secondary flight feathers. Primary flight feathers are attached directly to the bird's hand and fingers. The feathers are attached to the bones by ligaments and muscles. These allow the feathers not to be fixed, but to rotate on themselves. In addition, due to aerodynamic loads, feathers can bend. This element is not taken into account in the modeling due to a lack of studies on feather bending. As a result, the camber of the wing varies according to its deformation due to the bird's flight sequence, but also because of the aerodynamic forces applied to the feathers.

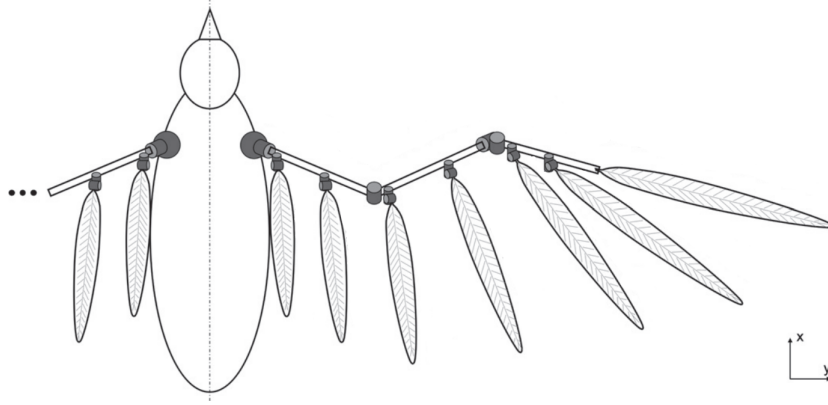


Figure 2.15: View in the  $xy$  plane of the bird's multibody model. (adapted from [25,26])

The bird model detailed in Fig. 2.15 shows the different joints linking bones and feathers according to the degrees of freedom observed in birds. The wings are made up of three rigid bodies representing the arm, forearm and hand bones. As for the feathers directly linked to the bones, the tendons at the roots of the feathers that hold them in place are represented by springs, due to their elastic nature. In the model, for simplicity's sake, these tendons are positioned at the tip. However, where the feathers are attached to the bones, the insertion is equivalent to a rotational spring, causing reaction forces between the feathers under aerodynamic load. To complete the wing model, a damping element is placed between the feathers to dissipate energy and prevent feather oscillation.

Dr Victor Colognesi assumed the kinematics of the wings to be imposed. Consequently, we do not solve the equations of state for the wing dynamics. For each displacement  $q_i(t)$  of a joint  $i$  as a function of time  $t$ , we have the eq. 2.8.

$$q_i(t) = q_{0,i} + A_i \sin(2\pi f t + \phi_{0,i}) \quad (2.8)$$

Each joint has offset  $q_{0,i}$ , amplitude  $A_i$  and phase  $\phi_{0,i}$ . To generate this framework, we use the *Robotran* tool [28] created by the Université Catholique de Louvain. The deformation of the wings and the movements of each joint over a flapping period  $T = 1/f$  can be visualized in Fig. 2.16. The wing flapping frequency is fixed at  $f = 4$  [Hz] based on observations made in [24,29]. However, it can be modified via a controller to change the phase shift of a bird in relation to another bird upstream, as detailed later in section 4.2.

### 2.3.3 Aerodynamic model

The model used to analyze the aerodynamic behavior of the bird and its wake is based on a formulation of the Navier-Stokes equations for incompressible flows known as the

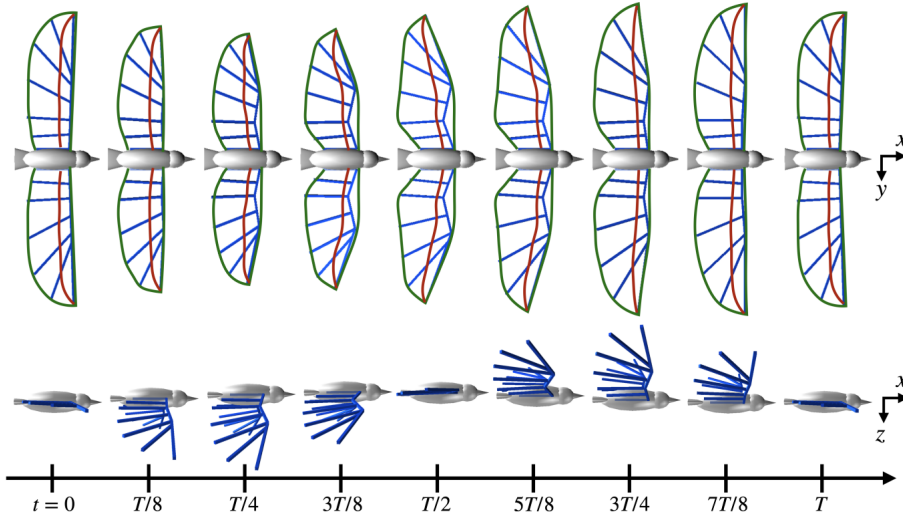


Figure 2.16: Illustration of numerical bird kinematics for a flapping period from  $t = 0$  to  $t = T$ . The bird's body, tail and head are shown in gray. Bones and feathers are shown in blue. Leading and trailing edges are shown in green. The lifting line is red. (from [15])

vorticity-velocity  $\boldsymbol{\zeta} - \mathbf{u}$  formulation with  $\mathbf{u} = [u, v, w]^T$ . Let's start by defining the vorticity  $\boldsymbol{\zeta}$  as the velocity field curl [30]:

$$\boldsymbol{\zeta} = \nabla \times \mathbf{u} \quad (2.9)$$

As far as the mass conservation equation is concerned, it can be simplified as the flow is incompressible and the air density is constant,  $\partial\rho/\partial t = 0$  and  $\nabla\rho = 0$  since no spatial or temporal variation in density is considered.

$$\frac{\partial\rho}{\partial t} + \mathbf{u} \cdot \nabla\rho + \rho\nabla \cdot \mathbf{u} = 0 \quad (2.10)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2.11)$$

Consequently, assuming incompressibility, the Navier-Stokes equations can be rewritten as follows.

$$\frac{\partial\boldsymbol{\zeta}}{\partial t} + \mathbf{u} \cdot \nabla\boldsymbol{\zeta} = (\nabla\mathbf{u}) \cdot \boldsymbol{\zeta} + \nu\nabla^2\boldsymbol{\zeta} + \nabla \cdot \mathbf{T}^M \quad (2.12)$$

with  $\nu$  the kinematic viscosity of air and  $\mathbf{T}^M$  the subgrid scales (SGS) model enabling Large Eddy Simulations (LES). We find the vortex stretching term  $(\nabla\mathbf{u}) \cdot \boldsymbol{\zeta}$  which is zero in a two-dimensional problem, since turbulence does not exist in 2D. The left-hand member represents  $\boldsymbol{\zeta}$  convection and diffusion is represented by the diffuse term  $\nu\nabla^2\boldsymbol{\zeta}$ . This implemented method is called the vortex particle-mesh method (VPM) with Immersed Lifting and Dragging Lines (ILD) [15,33]. Particles have a position  $\mathbf{x}_p$  and a strength  $\boldsymbol{\alpha}_p$  being the integral of the vorticity  $\boldsymbol{\zeta}$  over a material volume  $V_p$  assigned to the particle. Particles are influenced by convection, so their position and strength follow the various equations [2.13](#) and [2.14](#).

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}(\mathbf{x}_p) \quad (2.13)$$

$$\frac{d\boldsymbol{\alpha}_p}{dt} = \int_{V_p} \left( (\nabla\mathbf{u}) \cdot \boldsymbol{\zeta} + \nu\nabla^2\boldsymbol{\zeta} + \nabla \cdot \mathbf{T}^M \right) d\mathbf{x} \quad (2.14)$$

## 2. Bird dynamics

To calculate the  $\mathbf{u}$  velocity field, we need to solve the Poisson equation [2.15](#) below [15,30,33].

$$\nabla^2 \mathbf{u} = -\nabla \times \boldsymbol{\zeta} \quad (2.15)$$

For discretization, the two equations governing the temporal variation of  $\mathbf{x}_p$  and  $\boldsymbol{\alpha}_p$  are solved with third-order Runge-Kutta (RK3). Spatial integration over the volume  $V_p$  is solved with centered finite differences of order four. The Poisson equation is solved with a solver using Fourier space [32,33].

With regard to the aerodynamics of the wing, the numerical simulation of the bird's wake uses an AS6092 airfoil [31], as shown in [Fig. 2.18](#). This airfoil resembles the cross-section of a bird wing shown in [Fig. 2.17](#). Since the ulna and radius bones are much smaller in diameter than the mean aerodynamic chord  $\bar{c}$ . The thickness of the wing near the leading edge is thin, exactly as in the AS6092 airfoil. Moreover, since secondary feathers occupy most of the surface area of the bird's wing, and the thickness of these feathers is of the order of a few millimetres, the result is an equally thin airfoil for the simulations. This assumption of a very thin airfoil makes it possible to approximate the flow around the airfoil as essentially two-dimensional as explained in [15]. Consequently, the numerical bird present in the simulations is considered as a line called the lifting line, shown in red in [Fig. 2.16](#). It is on this lifting line that the main aerodynamic variables are calculated as a function of Reynolds number  $Re$  and local angle of attack  $\alpha$ . These main variables are the lift, and drag coefficients with symbols  $C_L$  and  $C_D$  respectively.

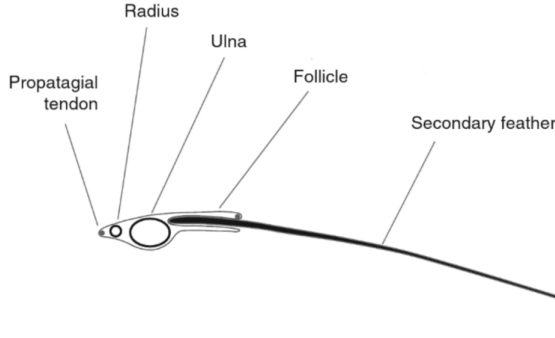


Figure 2.17: Cross-section of a bird's wing. (from [15])



Figure 2.18: Airfoil AS6092. (from [31])

As the bird is represented by a line, it interacts with the fluid in the same way as the ILDL mentioned above. For the VPM method, when we want to know the lift produced by each particle, we need to know the  $\Gamma$  circulation distribution along the wing.  $\Gamma$  is called the velocity circulation around the airfoil. Given a  $\mathcal{C}$  contour surrounding the airfoil and an infinitesimal  $d\mathbf{l}$  vector tangent to the  $\mathcal{C}$  contour, we define the  $\Gamma$  circulation as follows. [34]

$$\Gamma = \oint_{\mathcal{C}} \mathbf{u} \cdot d\mathbf{l} \quad (2.16)$$

By Stoke's Theorem, it is possible to link the  $\Gamma$  circulation with the  $\boldsymbol{\zeta}$  vorticity by transforming the contour integral by a surface integral where  $\mathcal{S}$  is the surface bounded by the  $\mathcal{C}$  contour.

$$\Gamma = \oint_{\mathcal{C}} \mathbf{u} \cdot d\mathbf{l} = \int_{\mathcal{S}} (\nabla \times \mathbf{u}) \cdot (\hat{\mathbf{e}}_n dS) = \int_{\mathcal{S}} \boldsymbol{\zeta} \cdot (\hat{\mathbf{e}}_n dS) \quad (2.17)$$

Now, according to the Kutta-Joukowski theorem, the lift per unit span  $l$  depends directly on the relative velocity to the airfoil, the circulation distribution, the air density and on the assumption that the flow around the airfoil is steady and unseparated:

$$l = \rho \mathbf{V} \times \Gamma \quad (2.18)$$

The relative airfoil velocity  $\mathbf{V}$  is calculated directly from the velocity field  $\mathbf{u}$  determined by the VPM method and the lifting line following the wing beat as shown in red in Fig. 2.16. We note that the ILDL varies as a function of time, so its discretization also varies as a function of time. For drag, a similar reasoning is used as for lift, considering the momentum deficit in the wake of a wing as explained in [15] and in detail in [33].

### 2.3.4 Forces and moments

We now proceed to the numerical analysis of the forces and moments acting on a numerical bird. The bird is positioned at  $(x, y, z) = (0, 0, 0)$  [m] in a spatial domain of dimension  $[-2; 8] \times [-3; 3] \times [-2; 2]$  and for a flight time  $t = 80$  [s]. The bird moves forward at a constant speed of  $U = 15$  [m/s]. Since the bird is fixed in the frame of reference, this means that it perceives a velocity of  $-15$  [m/s] in the streamwise direction. The characteristics of the bird, *i.e.* the northern bald ibis, are given in Table 2.2. The properties of air are density  $\rho = 1.225$  [kg/m<sup>3</sup>] and kinematic viscosity  $\nu = 1.48 \cdot 10^{-5}$  [m<sup>2</sup>/s]. Consequently, we can calculate the Reynolds number based on the mean chord  $\bar{c}$ .

$$Re_c = \frac{U\bar{c}}{\nu} = 1.82 \cdot 10^5$$

As far as forces are concerned, we will consider the resultant of forces along  $x$  denoted  $X$  and the resultant of forces along  $z$  denoted  $Z$  according to the reference frame shown in Fig. 2.1. We define the force coefficients  $C_X$  and  $C_Z$  as follows. For the calculation of the  $C_m$  coefficient, which consists in the adimensionalization of the pitching moment  $M$ , this was defined in eq. 2.6.

$$C_X = \frac{X}{\frac{1}{2}\rho S_w U^2} \quad C_Z = \frac{Z}{\frac{1}{2}\rho S_w U^2} \quad (2.19)$$

Let's start by analyzing the resultant of the forces that apply along  $-z$  shown in the top figure on Fig. 2.19. As a reminder, as defined in Fig. 2.1 the  $z$  direction is positive when the force is pointed downwards at the bird. Consequently, the resultant force  $-Z$  corresponds to the force that allows the bird to be pushed upwards. As shown in Fig. 2.3 the resultant force  $-Z$  comprises  $L_z$  the lift component along  $-z$ , the drag component  $D_z$ , the thrust component  $T_z$  and the bird's weight  $W$ . By balancing the forces acting on the bird in the  $-z$  direction, we obtain:

$$-Z = -W + T_z(t) + L_z(t) + D_z(t) \quad (2.20)$$

Interpreting Fig. 2.19, we see that  $-C_Z$  is maximum around  $t = 0$  [s]. Comparing this with the bird's flight sequence shown in Fig. 2.16 this corresponds to the mid-downstroke of the wings, *i.e.* when the wings are in their descent phase between  $t/T = 3/4$  and  $t/T = 1/4$ . At  $t/T \approx 0.2$ ,  $-C_Z < 0$  this means that the bird's weight is no longer compensated by the aerodynamic forces that allow the bird to rise, which changes the concavity of the  $-z/c$  curve shown in Fig. 2.20. This is explained by Newton's second law:

$$-Z = m \frac{d^2}{dt^2}(-z) \quad (2.21)$$

$$-C_Z = \frac{m}{\frac{1}{2}\rho U^2 b} \frac{d^2}{dt^2} \left( \frac{-z}{\bar{c}} \right) \quad (2.22)$$

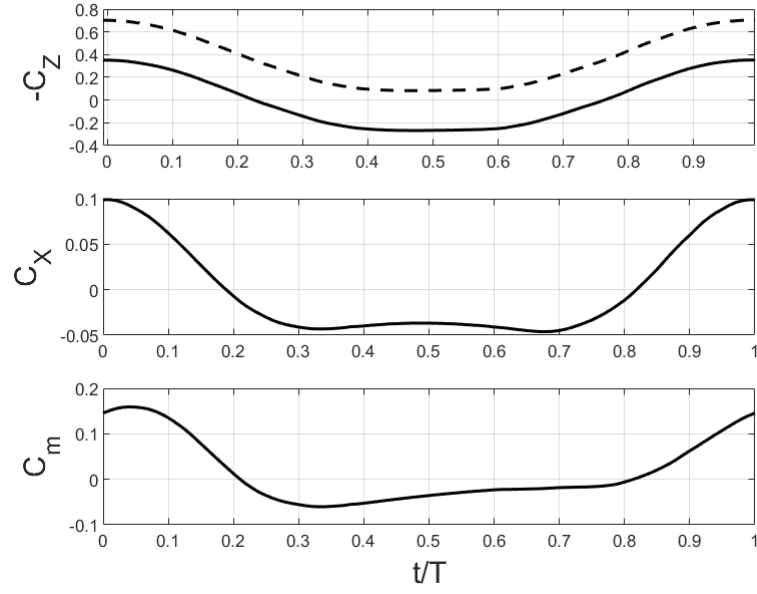


Figure 2.19: Dimensionless coefficient of force resultants according to  $-z$  and  $x$  respectively  $-C_Z$  and  $C_X$  as well as that of pitching moment  $C_m$  for a wing flapping period  $T = 1/f = 0.25$  [s]. For the top figure, the solid line represents  $-C_Z$  taking into account the bird's weight  $W$  and the dotted line represents only the aerodynamic forces acting on the bird, *i.e.* without its weight  $W$ .

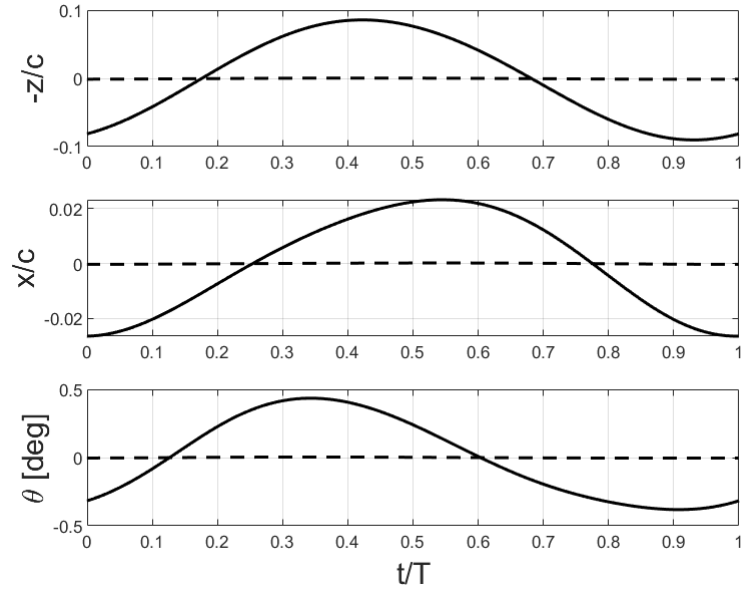


Figure 2.20: Bird displacement adimensionalized along  $-z$  and  $x$  as well as pitch angle  $\theta$  for a wingbeat period  $T = 1/f = 0.25$  [s]. The solid line shows the numerical values as a function of time, while the dotted line shows the mean value over the period  $T$ .



The bird therefore oscillates along  $-z$  during a flapping period, as can be seen in Fig. 2.16. In the same figure, we can see that the angle of attack perceived locally by the lifting line varies as the wing deforms during a period. This angle peaks at the onset of mid-downstroke in the vicinity of  $t/T = 0$ . The mid-downstroke is the major contribution to lift during the bird's flight sequence. It is for this very reason that the resultant of the forces along  $-z$  is maximal at this time, since lift is linearly cross-correlated with angle of attack for small values [20,22]. When we remove the contribution of weight  $W$  at  $-C_Z$  as shown in dotted line on Fig. 2.19, we obtain an approximation of the lift component along  $-z$ ,  $L_Z$ . Note that it is never zero. However, when we look at Fig. 2.28 showing the streamwise vorticity created during mid-upstroke, the trailing tip vortices are in the opposite direction to those of the mid-downstroke in Fig. 2.26. This can be explained by the opposite direction of circulation  $\Gamma$ , resulting in negative lift during mid-upstroke according to eq. 2.18. Interestingly, despite the negative lift created around  $t/T = 1/2$ , the vertical component of the force resultant is still positive along  $-z$ . The lowest value of the angle of attack is also at  $t/T = 1/2$  during the mid-upstroke, *i.e.* when the bird raises its wings. Finally, despite the variations in lift, we can see from Fig. 2.20 that  $-z/c$  is on average zero, proving the bird's stability in  $z$  during a level flight.

Let's move on to the analysis of the resultant forces according to  $x$ ,  $C_X$ . Like  $-C_Z$ , we note that the maximum of  $C_X$  is reached around  $t/T = 0$ , *i.e.* in mid-downstroke. This has the effect of pushing the bird forward, as we can see from the middle graph in Fig. 2.20 illustrating the variation of  $x/c$  as a function of time  $t/T$ . This means that most of the thrust produced by the bird is during downstroke. As  $C_X$  decreases and becomes negative, the bird is pushed backwards. The bird's streamwise  $x/c$  position thus oscillates slightly over a flapping period, but the average over time is zero, as shown in dotted line on the same figure. Finally, the pitching moment  $C_m$  peaks during downstroke shortly after  $t/T = 0$  and oscillates as a function of time. This is why the pitch angle  $\theta$  shown in Fig. 2.20 also varies as a function of time, but at a zero mean. In the end, although the bird is positioned at  $(x, y, z) = (0, 0, 0)$ , its behavior is oscillatory due to the forces and moments that are not constant. On average, however, the bird is located at this point in space.

Finally, we can calculate the power consumed by the bird over time. This can be calculated as the sum of the product of the rotational velocities  $\dot{q}_i(t)$  of each joint  $i$  with their respective torque actuation  $Q_{c,i}(t)$  calculated using *Robotran*. For our single-bird simulation, we can calculate the average power over a flapping period with  $N$  the number of joints shown in Fig. 2.15

$$\bar{P}(t) = \frac{1}{T} \int_{t-T}^t \left( \sum_{i=1}^N Q_{c,i}(t') \dot{q}_i(t') \right) dt' \quad (2.23)$$

We obtain a power  $\bar{P}(t) = 25.25$  [W] and therefore an energy consumed over a flapping period of  $\bar{E}(t) = \bar{P}(t)T = 6.31$  [J] numerically. Knowing that the bird has a mass of  $m = 1.20$  [kg], we have 21.04 [W/kg]. This value is similar to observations for birds [80].

### 2.3.5 Wake structure of a single bird

#### Wake of a gliding bird

To understand the wake produced by a wing, it is necessary to understand the various underlying concepts. We will take a look at the Lifting line theory, a mathematical model that uses a fictitious line to evaluate the  $\Gamma$  circulation distribution for a given wing geometry. It is this  $\Gamma$  circulation distribution that induces a wake downstream of the wing. When a bird glides, the wing shape does not evolve over time. We can therefore apply this theory to the case of a gliding bird and by neglecting its body.

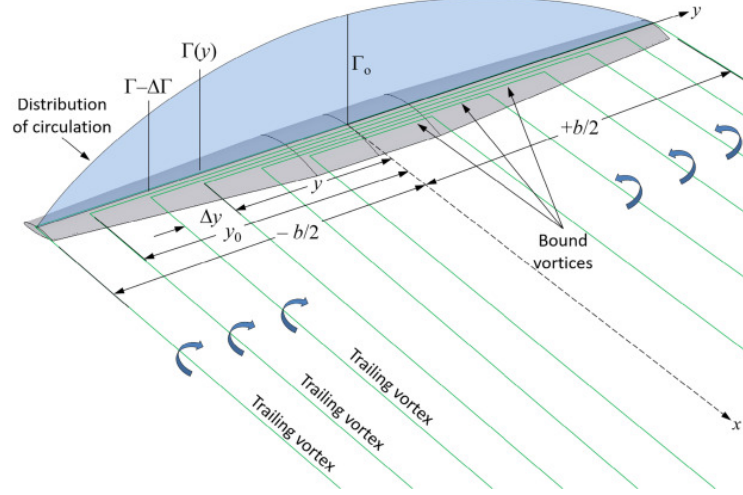


Figure 2.21: Distribution of the  $\Gamma$  circulation represented in blue along the lifting line aligned along  $y$  on a grey elliptical wing. Each green line spaced by a distance  $\Delta y$  between them delimits a surface of the vortex sheet called the trailing vortex. (from [36])

The  $\Gamma$  circulation along a wing's lifting line is not uniform, as shown in Fig. 2.21. Consequently, this also means that the lift is not uniformly distributed on the wing by application of the Kutta-Joukowski theorem (eq. 2.18). This distribution can be explained by the continuity of vortex tubes. According to Helmholtz's first theorem, "the circulation  $\Gamma$  of a vortex tube is the same at all locations along the tube" [35]. In other words, the vorticity  $\zeta$  must be divergent free:

$$\nabla \cdot \zeta = \nabla \cdot (\nabla \times \mathbf{u}) = 0 \quad (2.24)$$

The vortex tube is the set of contours  $\mathcal{C}$  of each airfoil making up the wing. The airfoil positioned at  $y$  is the cross-section of the wing at this point, and has a  $\Gamma(y)$  circulation as shown in Fig. 2.21. The lifting line is simply the sum of points, each of which is a replacement for a vortex point of the same circulation. Given the  $\Gamma$  distribution along the lifting line, we know that  $\Gamma(y) \neq \Gamma(y + dy)$ . In all generality,

$$\Gamma(y + dy) = \Gamma(y) + \frac{d\Gamma}{dy}(y)dy \quad (2.25)$$

Therefore, since vorticity is divergent free,  $(d\Gamma(y)/dy)dy$  is the circulation of elementary vortex tube shed shown in green on the figure. By the principle of vortex tube continuity, a vortex sheet must be shed. As kinetic energy is transmitted to the wake, by conservation of energy, a resistance to the gliding bird's movement is also created and is called induced drag due to lift. [35]

In the case of a gliding bird, assuming elliptical wings and neglecting profile drag, its wake, *i.e.* the magnitude of its vorticity, can be seen in Figure 2.22. Since the wing is of finite length, the vortex sheet rolls up at the wing tips to form two tip vortices of greater magnitude, *i.e.* shown in red on the figure, spaced by a distance  $b_0 < b$  apart and of opposite sign. As their signs are opposite, they induce a downwash velocity that pushes them mutually downwards (positive velocity according to  $z$ ) according to the Biot-Savart law.



Figure 2.22: Representation of the magnitude of vorticity  $\zeta$  in the wake of an elliptical wing solved using the VPM method. (form [33])

Formation flight, whether bird or aircraft, takes advantage of a particular feature in the wake. This particularity is called upwash velocity, which is the upwardly oriented vertical  $-w$  velocity component. In Fig. 2.23, when we are looking between the two trailing tip vortices, there is downwash. This would change the relative velocity perceived by an aircraft positioned there, increasing its drag and making formation flight inefficient. With regard to a position outside the trailing tip vortices, these produce upwash. This follows from Biot-Savart law that vorticity produces induced velocity, as shown in the figure. This upwash modifies the angle of attack perceived by an aircraft located at this position, which would benefit from “free” thrust. Since, indeed, a component of the lift would be positively oriented along  $x$ , as detailed in a later section of Chapter 3. As a result, the follower’s thrust input would be reduced for the same flight performance. Thus its energy expenditure on formation flight will be reduced.

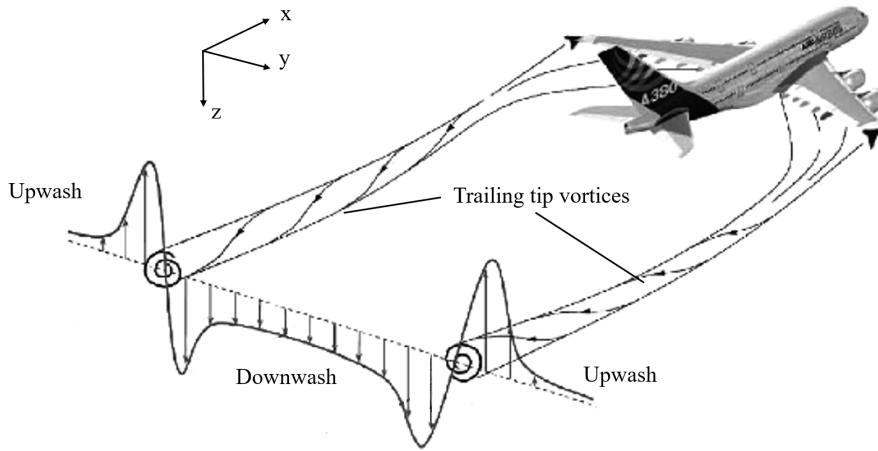


Figure 2.23: Representation of the vertical velocity component  $w$  of the wake cross section showing the upwash and downwash in the wake of an aircraft along a line in the spanwise direction. (adapted from [79])

### Wake of a flapping bird

To look at the wake of a flapping bird, we run exactly the same simulation as in section 2.3.4. There is a numerical bird at  $(x, y, z) = (0, 0, 0)$  and this bird has the characteristics of a northern bald ibis as shown in Table 2.2. As a reminder, the model does not capture ambient turbulence or unsteady aerodynamics. The bird only sees a “clean” wind with a

## 2. Bird dynamics

relative streamwise velocity of  $-15$  [m/s]. As the bird flaps its wings periodically, the result is a wake that is also periodic, unlike the wake made by a gliding bird as in Fig. 2.22. Indeed, as we can see in Fig. 2.24, the wake at  $t/T = 0$  and  $t/T = 1$  is identical and corresponds to the bird's mid-downstroke. Since the evolution of the wake is as much in time as in space, we define  $\lambda$  as the spatial wavelength of the wake between the air particles coming from two consecutive mid-downstrokes. The bird velocity is  $U = 15$  [m/s] and knowing the flapping period  $T = 0.25$  [s], the spatial wavelength  $\lambda = U/T = 3.75$  [m]. We therefore find in  $-x/\lambda = 0$ , the vorticity generated during the mid-downstroke, in  $-x/\lambda = 1/4$ , the vorticity generated during the end of the downstroke and similarly for the upstroke with  $-x/\lambda = 1/2$  and  $-x/\lambda = 3/4$ .

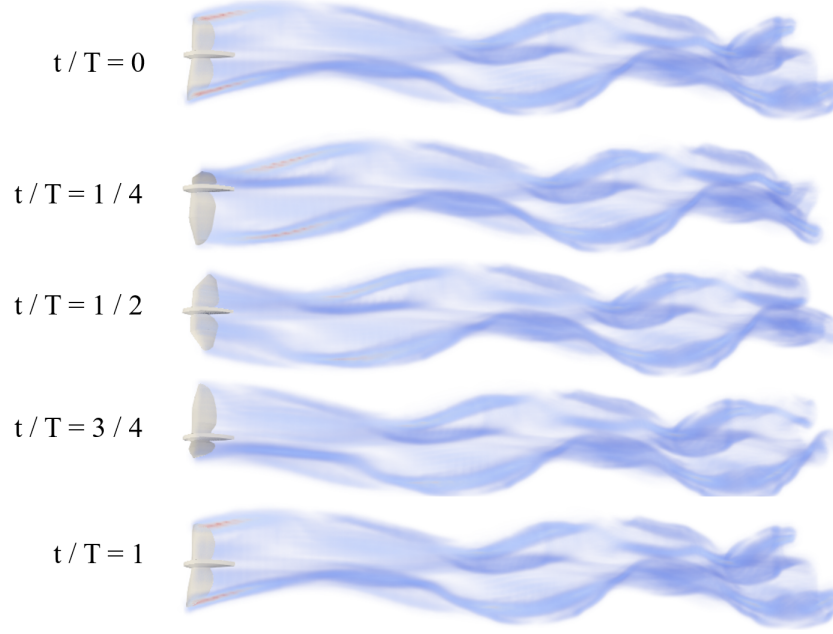


Figure 2.24: Oblique view of the volume rendering of vorticity magnitude in the wake for different times in the bird's flight sequence. The times  $t/T = 0$  and  $t/T = 1$  correspond to mid-downstroke,  $t/T = 1/4$  to end-of-downstroke,  $t/T = 1/2$  to mid-upstroke and  $t/T = 3/4$  to end-of-upstroke.

Still on Fig. 2.24, we can see a magnitude of maximum vorticity at the tip during mid-downstroke in  $t/T = 0$ . Indeed, as the circulation distribution decreases sharply close to the tip, this increases the shedding of a vortex sheet which rolls up very quickly behind the bird to form tubes, *i.e.* called trailing tip vortices. This effect is even more pronounced during mid-downstroke. As the circulation is greater, we can see a red tinge in the wake just behind the wing, synonymous with high vorticity intensity. We note that the sign of the vortex tubes changes during the bird's wingbeat. Indeed, during mid-downstroke at  $t/T = 0$ , the vortex direction is such that upwash is present towards the outside of the wake, as can be seen in Fig. 2.26. On the other hand, during mid-upstroke at  $t/T = 1/2$ , the circulation around the wing changes, producing negative lift and inducing the opposite rotation of the trailing tip vortices. In other words, downwash is generated towards the outside of the wake, as shown in Fig. 2.28. Furthermore, twin vortex tubes originating from the same time in the flight sequence will always be opposite in sign if they are not produced by the same wing, as we can see from the difference in sign of the vorticity in figures 2.26, 2.27, 2.28 and 2.29.

We can see from Fig. 2.25 that the bird's body also causes vorticity in the wake due to the change in circulation distribution caused. Indeed, Dr Victor Colognesi [15] has taken into account the impact of the bird's body as a kind of penalty applied to the lifting line. This penalty degrades the circulation distribution on this same lifting line, inducing vorticity due to the bird's body in the wake. The penalty has been set arbitrarily, so the analysis of wakes induced by this penalty must be carried out with care. For larger values of  $-x/\lambda$ , we can see that strongly interacting tube vortices of the same sign tend to merge together to form a single vortex, due to the effect of air viscosity. Viscous effects also diffuse vorticity over time. Indeed, when we compare the vorticity present in vortex tubes close to the wing, it has a large magnitude in a reduced tube. For large values of  $-x/\lambda$ , the tube increased its radius and the vorticity diffused through. This is exactly what we can see on the trailing tip vortices in Fig. 2.26

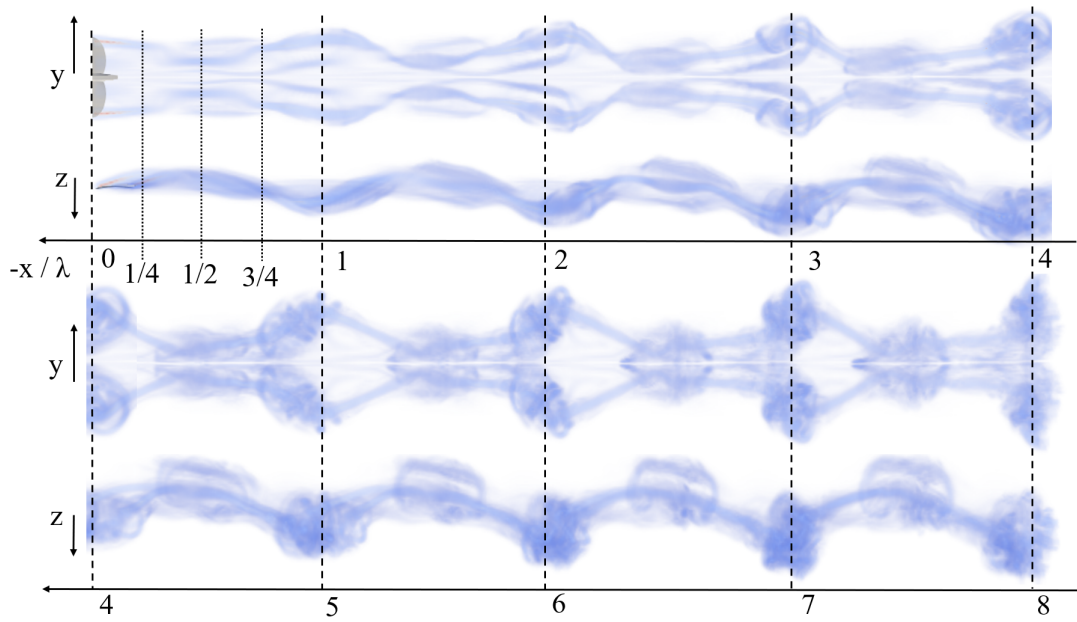


Figure 2.25: Volume rendering of the vorticity magnitude for a wake of length  $8\lambda$  allowing visualization in the  $xz$  plane as well as the  $xy$  plane.

When we consider the wake as a whole by looking at Fig. 2.25, the almost sinusoidal motion of the bird's wing is not returned to the wake. After  $-x/\lambda \approx 1$ , the interaction between the vortices and the diffusion of vorticity involves a change in shape and an increase in the space occupied by the vortex tubes. Firstly, the trailing tip vortices push each other downwards as they generate downwash between them as can be seen in the Fig. 2.26 as  $-x/\lambda$  grows. Then, for the trailing tip vortices produced during mid-upstroke in Fig. 2.28 as the circulation is opposite, these push each others upwards. Finally, the vortices generated by the reduced circulation due to the presence of the body have less strength but apply exactly the behavior as the trailing tip vortices. Bear in mind that uncaptured elements such as environmental turbulence or the impact of the body on the wake would increase the decay of the wake as mentioned also in [15].

As stated in Chapter 1, the aim of this master's thesis is to reproduce and analyze bird formation flight dynamics numerically. Observations made on migratory birds all agree that one of the motivations for formation is to reduce the energy expenditure of

## 2. Bird dynamics

---

individuals [24,29,43,50,52-55]. Moreover, this is achieved by drawing on the kinetic energy present in the wake. This kinetic energy can be used by considering upwash. Therefore, we are looking to create a model that could take advantage of a bird's wake as represented in Fig. 2.25 with the aim of placing other birds to reduce their energy expenditure. One way is to take advantage of the upwash generated by trailing tip vortices. To do this, we will comment on figures 2.26 to 2.29 representing a cross-section in the  $yz$  plane of the velocity and vorticity generated by different parts of the bird's flight sequence at different  $-x/\lambda$  locations. We can see that maximum upwash is generated during mid-downstroke ( $t/T = 0$ ) in Fig. 2.26. Conversely, during mid-upstroke ( $t/T = 1/2$ ), at the same point, downwash is generated, as shown in Fig. 2.28. Placing a follower is therefore not an easy task, and a model for doing so based on upwash will have to take into account the spatio-temporal behavior of the wake. The development of the model will be the central topic of the next chapter, so that we can then create our own flights in numerical bird formation to analyze their performance and see whether or not it corroborates with observations of northern bald ibis flights.



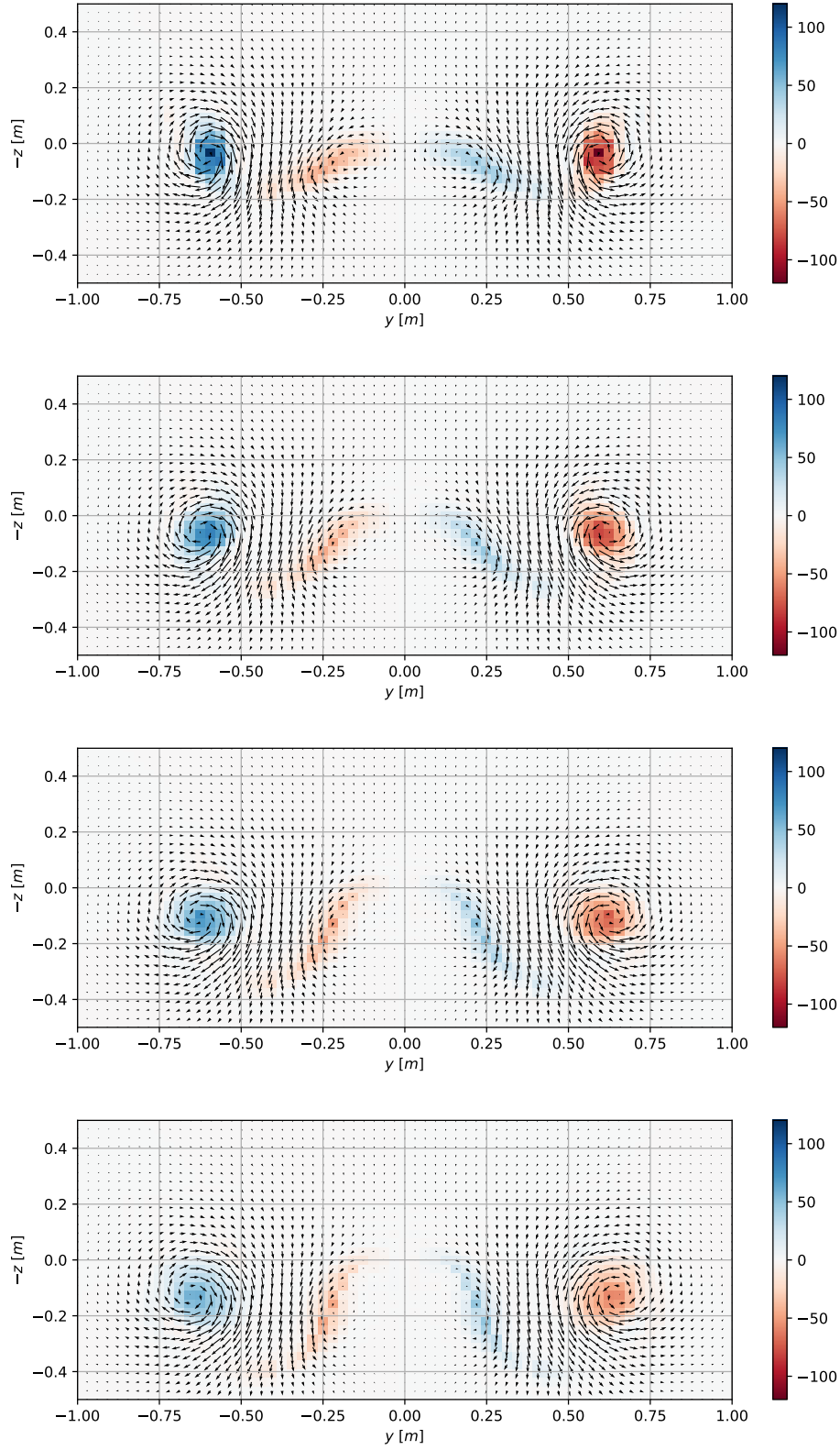


Figure 2.26: Representation of streamwise vorticity and velocity in a cross-section of the  $yz$  plane at different positions in the bird's wake  $-x/\lambda = 1/4, 1/2, 3/4$  and  $1$  respectively from top to bottom image for mid-downstroke  $t/T = 0$ .

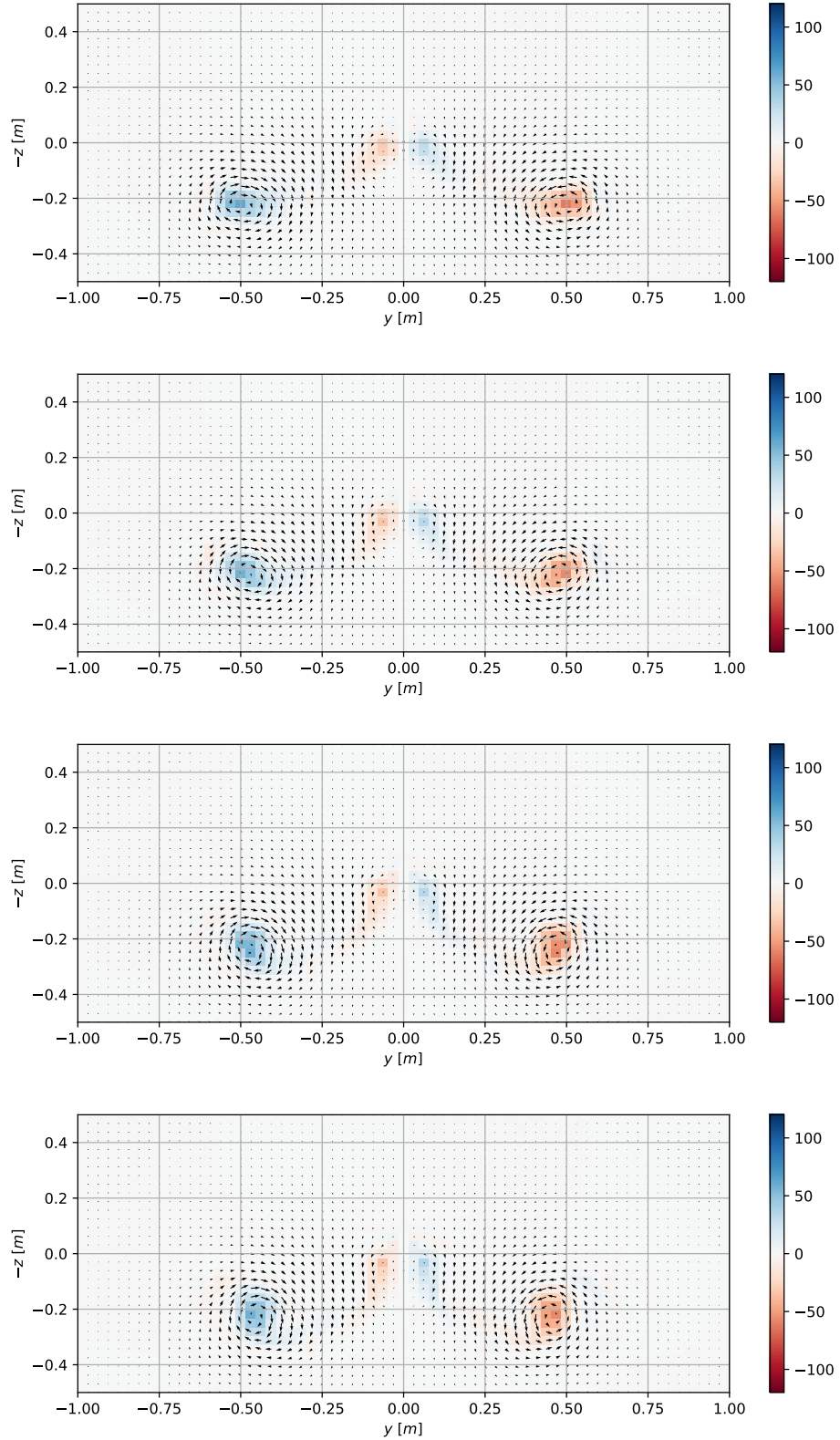


Figure 2.27: Representation of streamwise vorticity and velocity in a cross-section of the  $yz$  plane at different positions in the bird's wake  $-x/\lambda = 1/4, 1/2, 3/4$  and  $1$  respectively from top to bottom image for the end of the downstroke  $t/T = 1/4$ .



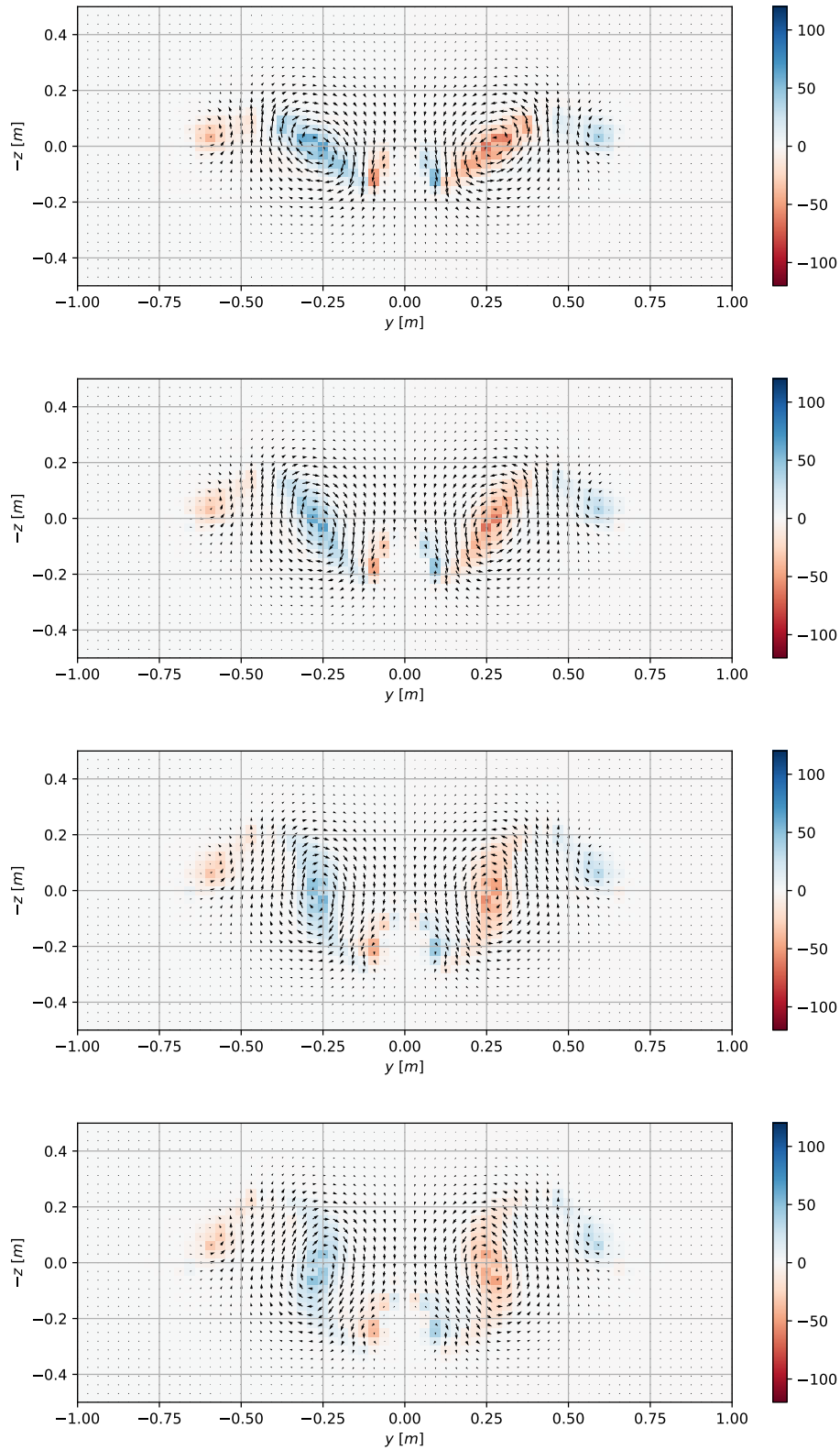


Figure 2.28: Representation of streamwise vorticity and velocity in a cross-section of the  $yz$  plane at different positions in the bird's wake  $-x/\lambda = 1/4, 1/2, 3/4$  and  $1$  respectively from top to bottom image for mid-upstroke  $t/T = 1/2$ .

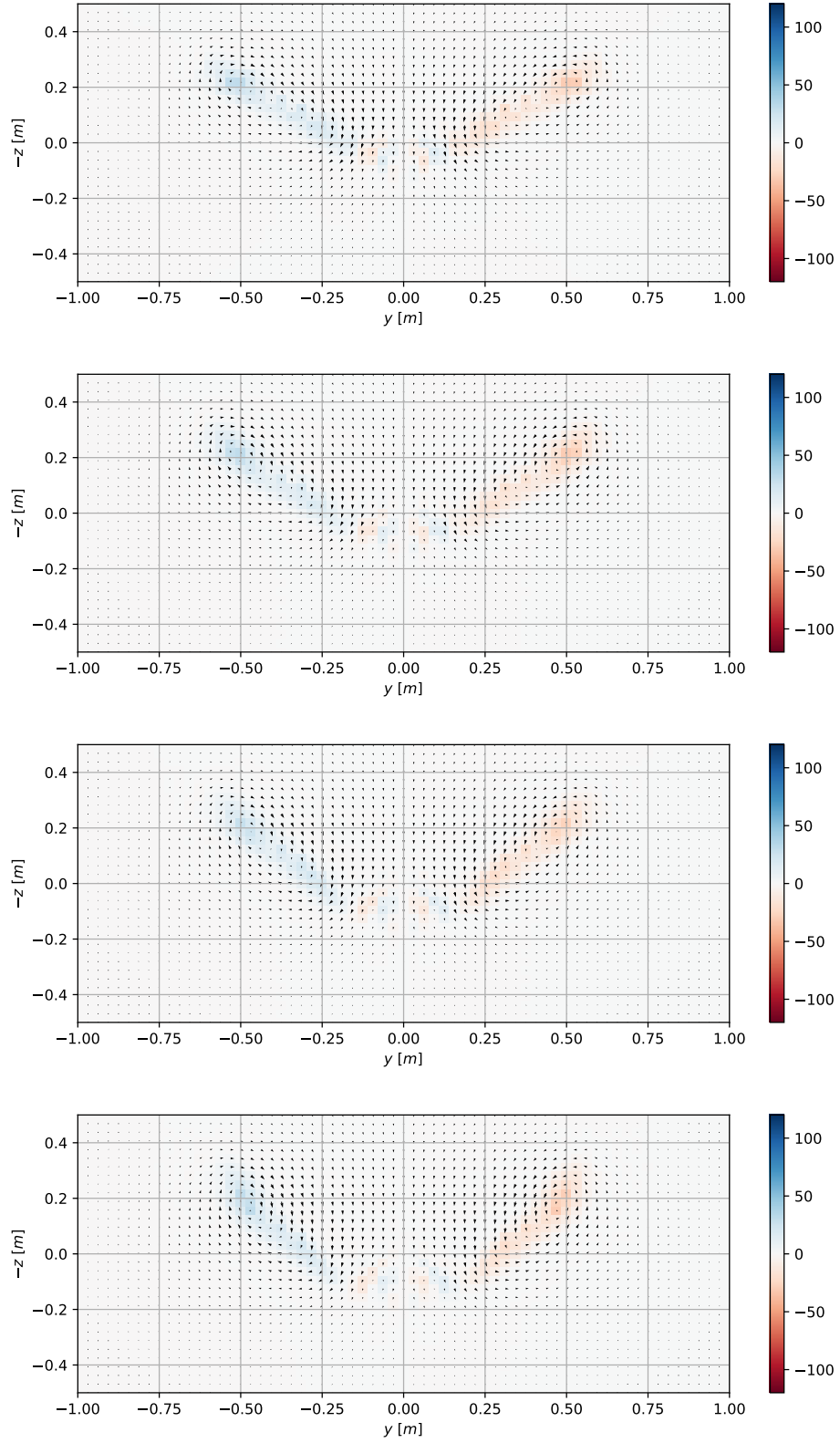


Figure 2.29: Representation of streamwise vorticity and velocity in a cross-section of the  $yz$  plane at different positions in the bird's wake  $-x/\lambda = 1/4, 1/2, 3/4$  and  $1$  respectively from top to bottom image for the end of the upstroke  $t/T = 3/4$ .

## Chapter 3

# Wake benefit topology

Numerous models have already been established to explain the dynamics of flight in aircraft formation [37-40] or bird formation [41-45]. However, all these models are based on fixed wings and therefore neglect the impact of wing flapping on the wake and on the energy saving of followers. In this master's thesis, the aim of which is to study the dynamics of birds formation flight. We will carry out one of the first studies of a numerical model taking wing flapping into account. This will lead us to simulate the wake produced by a formation and study follower variables, as detailed in Chapter 5.

### 3.1 Assumptions

In the first section of this chapter, we devote it to the development of assumptions for a model designed to take advantage of the upwash generated by birds in the wake. Therefore, we list the various assumptions associated with the model and the simulations. First, we will look at the general assumptions. We will then move on to the physical characteristics of the birds present in the numerical formation, and finally to their possible attitudes.

#### 3.1.1 General assumptions

For the simulations, we consider a steady flow upstream of velocity  $U = -15 [m/s]$  along the  $x$  axis. Like the previous simulations we made before, we will neglect the destabilizing effects on the equilibrium of bird formation flight and its impacts. Consequently, there will be no noise, no crosswinds, no atmospheric turbulence, and no updrafts or downdrafts.

Birds in observed formation flights have a phase shift between the different ranks. This means birds are slightly behind or ahead in their flight sequence, *i.e.* wing kinematics during a flapping period, with the upstream bird. These observations can be seen in [24,29]. Given that the phase shift varies to optimize the flight of birds in the formation, the flapping frequency  $f$  cannot be considered as necessarily constant during a bird's search for an optimal location. On the other hand, the flight sequence illustrated in Fig. 2.16 is considered constant. This means that the bird maintains this flight sequence and does not try to modify its wing deformation during a flapping period. We also assume that visual comfort and distance from the upstream bird are causes of formation flight shape, as discussed in [47-49] and in more detail in section 3.3.

#### 3.1.2 Numerical bird characteristics

The birds used in the simulations are all northern bald ibises, with the same physical characteristics as in Table 2.2. We will simulate for small-scale flocks of less than ten birds present in the formation.

We assume that numerical birds are all identical in age, sex, species, mass, size, visual capacity and comfort in relation to distance and vision. All these variables could disrupt the homogeneity of formation flight as assumed in [50]. Indeed, age, mass and height are directly related. It was observed in [50,51] that young birds (therefore of low mass and small size) were regularly behind adults and not directly behind a rank of other juveniles. In this way, they can benefit from the kinetic energy transmitted in the wake of an adult, which is greater than that of a smaller juvenile. As a result, formations are

sometimes asymmetrical due to the difference in span  $b$  between a juvenile and an adult [42,43]. There are also formation flights where a majority species accepts another minority species in the formation, as observed in [52]. Due to the different physical characteristics of the two species, this also implies an asymmetrical formation shape [42,43]. To avoid these distributions of factors influencing the shape of formation flight, we choose to be completely general by using identical numerical birds.

#### 3.1.3 Attitude of birds

We assume that the attitude of the birds has an impact on the flight formation shape [41,43,48,50,53-55]. We consider the attitude of the birds present in the formation to be either selfish, cooperative or empathetic. These three attitudes imply a change of mentality in the search for an optimum place to position oneself in order to reduce energy expenditure.

If all the  $2n + 1$  birds present, distributed over  $n$  ranks in the V-shaped formation, have a so-called selfish attitude [41,48,53-55]. Then, each individual positioned on rank  $i$  will seek to minimize a certain beneficial function  $G$  by modifying their parameters  $P_i$  knowing only their personal  $G$  function. We can translate this minimization problem as follows, with  $P_{i,opt}$  the optimal parameters of the individual on rank  $i$ . For each individual in the formation, we therefore have the following resolution with  $i = 1, \dots, n$ . For V-flight, recall that the leader is positioned on rank  $i = 0$  and his position there is fixed as shown in Fig. 2.2

$$P_{i,opt} = \underset{P_i}{\operatorname{argmin}} G(P_i) = \{P_i | G(P_i) = \min_{P'_i} G(P'_i)\} \quad (3.1)$$

For the cooperative attitude [41,43,48,50], this is an optimization considering that each bird is aware of the energy expenditure of all birds present in the formation. In other words, for a formation flight of  $n$  ranks. Each individual on rank  $i$  seeks to adapt its parameters  $P_i$  to minimize the function  $G$  of all the individuals present in the formation. Consequently, each individual in rank  $i$  knows not only his beneficial function  $G(P_i)$ , but also all those of the other individuals  $G(P_j)$  with  $j \neq i$ . The problem can be written as follows.

$$\{P_{i,opt}\}_{i=1}^n = \underset{\{P_i\}_{i=1}^n}{\operatorname{argmin}} G(\{P_i\}_{i=1}^n) \quad (3.2)$$

The hypothesis that each bird in the formation can assess the energy expenditure of all the other individuals also present in the formation is difficult to prove. Indeed, it means that this sharing of information would have to take place through some form of communication such as a call, a visual signal, ... However, no scientific paper has been able to prove such in-flight communication. It seems more likely that information sharing can take place in the vicinity of each bird. Let's assume that each individual of rank  $i$  can know the energy expenditure information of an individual located one rank  $i - l$  upstream and that of an individual of rank  $i + l$  downstream. The optimal parameters  $P_{i,opt}$  minimizing the objective minimization function of the bird in rank  $i$  will therefore be solved as a cooperative but local problem with the energy expenditure information known from ranks  $i - l$  to  $i + l$ .

$$P_{i,opt} = \underset{\{P_j\}_{j=i-l}^{i+l}}{\operatorname{argmin}} G(\{P_j\}_{j=i-l}^{i+l}) \quad (3.3)$$

The behavior of animals in groups, taking into account the movements of other animals in a given vicinity, has been analyzed and studied [56-58]. Radakov's theory, which is

equally applicable to the flight formation of birds or schools of fish, states that each animal takes into account the movements of its neighbors in order to steer and avoid collisions.

Given the lack of information on communication between the ranks of a formation flight, we will assume a flight composed of selfish individuals as presented in the first case of this subsection. At the end of Chapter 5, we will look at the differences in the shape of formation for selfish and cooperative birds. In the cooperative case we will look at numerical results from articles considering communications between ranks in bird formations.

### 3.2 Wake benefit model

In the last section of Chapter 2, we saw in figures Fig. 2.26 2.27 2.28 and 2.29 that the velocity field vectors induced by the airflow on the lifting line have a vertical component along  $-z$  called upwash. The follower bird could therefore use this kinetic energy present in the wake to reduce its energy expenditure.

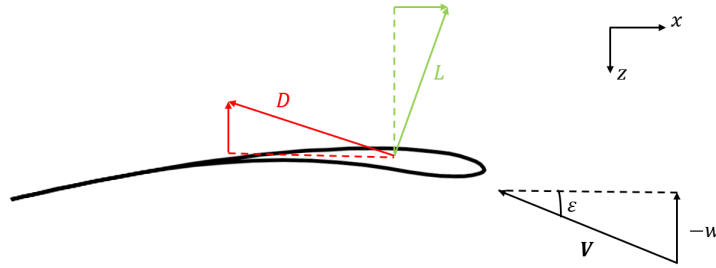


Figure 3.1: Illustration of the resultant forces acting on an airfoil for a relative velocity  $\mathbf{V}$  with a vertical velocity component oriented along  $-z$ .

As illustrated in Fig. 3.1, when the relative airfoil velocity  $\mathbf{V}$  has an upwash component, this orients the lift vector  $L$  forward and therefore has a horizontal component in the  $x$  direction. Upwash therefore changes the angle of attack. As a result, lift has a positive component along  $x$ . This streamwise component of lift is a thrust created solely by the upwash in the wake. As the bird acquires "free" thrust, it can reduce its effort and energy expenditure to move forward. The bird will produce less effort for similar flight performance. The power saved at a given time  $t$  can be approximated by the airfoil shown in Fig. 3.1. Let's denote  $P_e(t)$ , the power saved at time  $t$ . Let  $L_x$  be the horizontal component of the lift  $L$ .  $P_e(t)$  is therefore the product of the bird's speed  $U = 15 [m/s]$  and the "free" thrust due to the upwash,  $L_x$ .

$$P_e(t) = UL_x \quad (3.4)$$

We can calculate  $L_x$  using trigonometric relations and the angle  $\epsilon$ , which is the angle between the relative velocity  $\mathbf{V} = [u, v, w]^T$  and the horizontal axis  $x$ . We assume that  $\epsilon$  is small.

$$L_x = L_z \tan(\epsilon) \approx L_z \frac{-w}{u} \quad (3.5)$$

Consequently, we calculate  $P_e(t)$  assuming  $U \approx u$ . Moreover, we know that upwash  $-w$  is a function of time and so is  $L_z$ , since lift varies with time during a flapping period.

$$P_e(t) \approx -L_z(t)w(t) \quad (3.6)$$

### 3. Wake benefit topology

Our model therefore consists in maximizing the power saved by the upwash  $P_e(t)$ , which amounts to minimizing  $-P_e(t) = L_z(t)w(t)$ . As the wing beat is periodic during a trimmed bird flight, we average over the period  $T$ . In addition, we average the upwash over the surface of the bird's wings and denote this  $\bar{w}$ . The minimization problem can be rewritten as follows, with  $P_e$  the average power saved over a flapping period.

$$-P_e(t) \approx \frac{1}{T} \left( \int_t^{t+T} L_z(t') \bar{w}(t') dt' \right) \quad (3.7)$$

Section 3.2.2 will focus on the  $L_z(t)$  function, known as the force model, and section 3.2.3 on the  $\bar{w}(t)$  function, known as the upwash model. Therefore, over the time of the formation flight, thanks to this average power saved per period, the follower will expend less energy than when flying alone for the same flight performance.

#### 3.2.1 Variables of the minimization problem

There are several variables that can influence a follower's ability to take advantage of the upwash present in the wake created by other birds. For instance, we have its relative position with respect to the upstream bird  $(x, y, z)$ , its rotation with respect to the reference axes, *i.e.* roll, pitch and yaw angles, or its wing flapping frequency  $f$ .

Numerous models and observations on birds have demonstrated the impact of stream-wise position  $x$  and spanwise position  $y$  in formation flight [24,41,48,53-55]. As discussed in Chapter 2 when analyzing the wake, it oscillates according to  $z$  due to the bird's flight sequence and wing beat, with different vorticity production over the period. Consequently, the vertical position  $z$  must be considered to find the ideal position for a follower. We assume that the roll, pitch and yaw angles of the follower do not influence the average power saved during a flapping period. Therefore, we do not consider rotations as variables in the minimization problem. Finally, a change in flapping frequency  $f$  will induce a phase shift noted  $\Delta\phi_f = \phi_f - \phi_l$  with  $\phi_f$  the follower phase and  $\phi_l$  that of the upstream bird. The phase shift is synonymous with the delay or advance between the flight sequence of the upstream bird and that of the follower. For example, on a flight with a follower and a leader, if  $\Delta\phi_f = 0$ , then in  $t/T = 0$ , they will both be in mid-downstroke and then have the same flight sequence. If  $\Delta\phi_f = \pi$ , in  $t/T = 0$ , the leader will be in mid-downstroke and the follower in mid-upstroke. Observations of ibis formation flight show that phase shift is a variable in the energy expenditure minimization problem [24,29]. This phase shift is constant over time when the bird is at a given position. Consequently, its flapping frequency is constant. The flapping frequency is therefore not a variable in the minimization problem.

The four variables considered are the follower's position in space relative to an upstream bird  $(x, y, z)$  and its phase shift relative to this same bird  $\Delta\phi_f$ .

#### 3.2.2 Force model

The function  $L_z(t)$  is therefore the evolution of the vertical component of lift, see Fig. 3.1 as a function of time. This aerodynamic force is represented in Fig. 3.2 during a period of wing flapping.

Our model takes into account the wake of a bird or group of birds. Then it determines where to place an additional bird to take maximum advantage of the upwash to reduce its power consumption during a flapping period. In eq. 3.7 we will approximate the vertical component of lift  $L_z(t)$  by a  $\tilde{L}_z(t)$  function of unit mean. We therefore consider a weighting function  $\tilde{L}_z(t)$  used to weight the upwash averaged  $\bar{w}(t)$  over the bird's wings

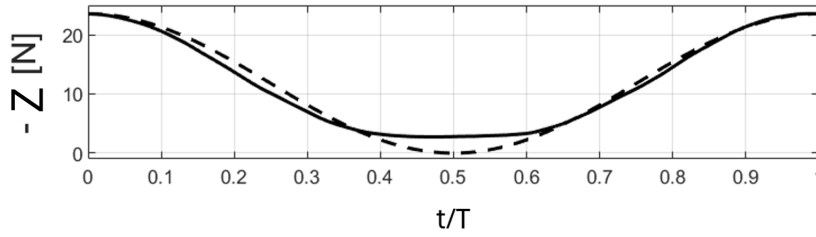


Figure 3.2: Resultant of forces along  $-z$  during a period of wing flapping represented as a solid line, approximated by a cosine with an offset as a dotted line.

at time  $t$ . We can see in Fig. 3.2 the resultant of the forces along  $-z$ , which consists mainly of the vertical component of lift  $L_z$ . Assuming,  $-Z \approx L_z$ , we can approximate  $L_z$  by a consinusoid function with an offset as represented on the same dotted figure. Consequently,  $\tilde{L}_z(t)$  will be a cosine with the same frequency of  $f = 4$  [Hz] and an offset of 1. The flight sequence can be out of phase with the upstream bird, and so lift production can be out of phase by  $\Delta\phi_f$ .

$$\tilde{L}_z(t, \Delta\phi_f) = 1 + \cos(2\pi ft + \Delta\phi_f) \quad (3.8)$$

The offset of 1 ensures compliance with the unit average imposed by the definition of a weighting function.

$$\frac{1}{T} \left( \int_t^{t+T} \tilde{L}_z(t', \Delta\phi_f) dt' \right) = \frac{1}{T} \left( \int_t^{t+T} 1 + \cos(2\pi ft' + \Delta\phi_f) dt' \right) = 1 \quad (3.9)$$

### 3.2.3 Upwash model

In this sub-section, we will focus on the  $\bar{w}$  term of the eq. 3.7. This term refers to the upwash averaged over the bird's wing surface area, noted  $S_w$ . This wing surface area  $S_w$  is a function of time  $t$ , since the wings are flapping and this area is therefore moving in space. This surface also depends on the phase shift  $\Delta\phi_f$  with the flight sequence of the bird present on the upstream rank, so  $S_w = S_w(t, \Delta\phi_f)$ .

For the average upwash on the wing surface,  $\bar{w}$ , this average depends on the upwash present in the region close to the bird's  $(x, y, z)$  position. This varies as a function of time  $t$  too. The wing surface area also depends on the phase shift  $\Delta\phi_f$  so  $\bar{w} = \bar{w}(x, y, z, \Delta\phi_f, t)$ . We define  $\bar{w}$  as follows with  $dA(t, \Delta\phi_f)$  an infinitesimal surface element of the surface  $S_w$ .

$$\bar{w}(x, y, z, \Delta\phi_f, t) = \frac{1}{S_w(t, \Delta\phi_f)} \int \int_{S_w(t, \Delta\phi_f)} w(x', y', z', t') dA(t', \Delta\phi_f') \quad (3.10)$$

For this spatial numerical integration, we use Simpson's method on a two-dimensional surface [59,60]. For the numerical resolution of eq. 3.10, this is detailed in *Appendix A*.

We define the objective function  $\mathcal{G}$  as the function of the approximate minimization problem  $-P_e$  described in eq. 3.7. Function  $\mathcal{G}$  is therefore an approximation of  $-P_e$  by approximating  $L_z$  by  $\tilde{L}_z$  defined in eq. 3.8 and  $\bar{w}$  defined in eq. 3.10. Finally,  $\mathcal{G} = \mathcal{G}(x, y, z, \Delta\phi_f)$  as explained in section 3.2.1 showing the minimization variables.

$$\mathcal{G}(x, y, z, \Delta\phi_f) = \frac{1}{T} \left( \int_t^{t+T} \tilde{L}_z(t', \Delta\phi_f) \bar{w}(x, y, z, \Delta\phi_f, t') dt' \right) \approx -P_e \quad (3.11)$$



### 3.3 Constraints

Thanks to studies carried out on the dynamics of formation flight based on previous observations [43,47,61], the authors assert that the shape of formation flight results not only from good visual resolution. But also from minimizing the risk of possible collision. This theory even goes by the name of *communication hypothesis* in some papers, as discussed in [43]. Other articles even postulate that the birds' visual comfort on the upstream bird is the major constraint imposing the shape of a formation flight. Indeed, the visual angle is determined by the position of the bird's retinas and the location of its eyes [49,62, 63]. They also assert that the optimal angle is close to the bird's axis of vision. This means opposite to the left/right of the bird, but not directly in front of it [62,64].

Some existing models assume that stress can accelerate a bird's metabolism [48,61] and thus increase energy expenditure. They identify two stresses. The first is the stress associated with colliding with another bird and being too close to it. The second is stress due to visual discomfort. We do not really know the impact of bird stress on energy expenditure, due to a lack of studies on the subject. Consequently, we prefer to call it a distancing comfort zone. This will be discussed in more detail in section 3.3.1. As well as a visual comfort zone, which we explain in section 3.3.2. These are constraints, because if a zone rich in upwash is located in a situation of discomfort for the bird, then we assume that this could counterbalance the beneficial effect of taking advantage of the kinetic energy present in the wake.

#### 3.3.1 Collision comfort constraint

For the risk of collision constraint, we assume two birds  $i$  and  $j$ . Bird  $j$  is in a rank upstream of bird  $i$ , *i.e.*  $|x_j| < |x_i|$ . We note,  $P_{ij}$  the relative position vector between the center of mass of bird  $i$  and that of bird  $j$  as shown in Fig. 3.3

$$P_{ij} = [x_i - x_j, y_i - y_j, z_i - z_j]^T \quad (3.12)$$

The numerical birds are ellipsoids centered on the bird's center of mass, symbolizing the volume occupied by the bird over time. These are shown in black in Fig. 3.3 and 3.4 with semi-major axes  $a_x = l_b/2$ ,  $a_y = b/2$  and  $a_z = 0.15$  [m] set arbitrarily. We find the values of  $l_b$  and  $b$  in Table 2.2

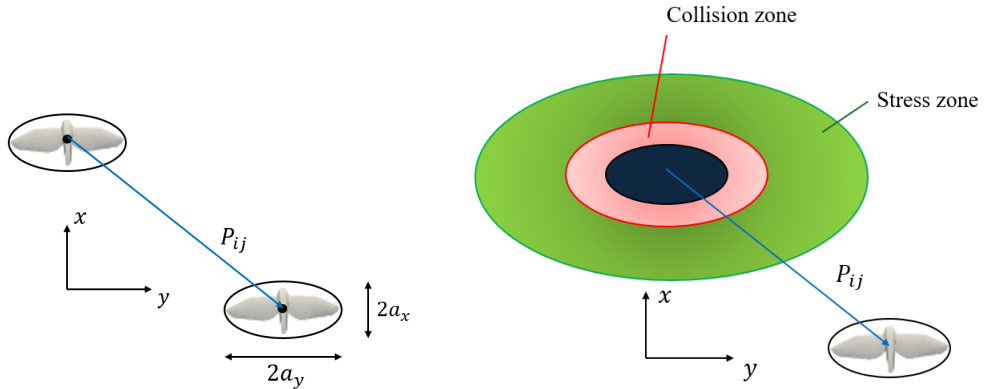


Figure 3.3: Representation of the  $P_{ij}$  position vector and ellipsoids in a top view.

Figure 3.4: Top view of stress and collision zones in relation to the upstream bird.



We assume that two birds collide as soon as their respective ellipsoids are tangent. This is the same condition as if the follower's center of mass were to enter an ellipsoid with half major axes double those of the ellipsoid representing the leader. We call this ellipsoid the collision zone shown in red in Fig. 3.4. The semi-major axes of this ellipsoid are therefore twice those of the black ellipsoid, *i.e.*  $2a_x$ ,  $2a_y$  and  $2a_z$ .

We define a variable  $\mathcal{D}$  linked to the collision criterion. There is no collision if  $\mathcal{D}$  is strictly greater than 1, as shown below.

$$\mathcal{D} := \left( \frac{x_i - x_j}{2a_x} \right)^2 + \left( \frac{y_i - y_j}{2a_y} \right)^2 + \left( \frac{z_i - z_j}{2a_z} \right)^2 > 1 \quad (3.13)$$

The green zone in Fig. 3.4 is the stress zone. This is an ellipsoid in which each half major axis of the black ellipsoid has been multiplied by a constant  $H > 2$ . When the follower's center of mass is outside this ellipsoid, the bird has no discomfort. When the follower's center of mass is inside this ellipsoid, but outside the collision zone, it has discomfort. This increases as it approaches the collision zone boundary shown in red. We use the components of the relative position vector  $P_{ij}$  to determine whether we are inside the collision zone.

$$\left( \frac{x_i - x_j}{Ha_x} \right)^2 + \left( \frac{y_i - y_j}{Ha_y} \right)^2 + \left( \frac{z_i - z_j}{Ha_z} \right)^2 = \left( \frac{2\mathcal{D}}{H} \right)^2 > 1 \quad (3.14)$$

As a result, the variable  $\mathcal{D}$  contains all the information on the area where the follower bird is located. Indeed, if  $\mathcal{D} \leq 1$ , the two birds collide. If  $1 < \mathcal{D} \leq (H/2)^2$  then the bird is in the stress zone. Finally, if  $\mathcal{D} > (H/2)^2$ , the follower suffers no collision discomfort due to the presence of the upstream bird.

We define the constraint function  $\mathcal{B}_C$  related to collision discomfort due to the presence of another bird as below. The variables  $(x, y, z) = (x_i, y_i, z_i)$  are the position of the follower we wish to place in the upstream bird's wake.

$$\mathcal{B}_C(\mathcal{D}(x, y, z)) = \begin{cases} 0 & \text{if } \mathcal{D} > (H/2)^2 \\ g(\mathcal{D}) & \text{if } 1 < \mathcal{D} \leq (H/2)^2 \end{cases} \quad (3.15)$$

The function  $g(\mathcal{D})$  is an arbitrary function defined as follows with  $k_C$  of the order of unity.

$$g(\mathcal{D}) = k_C \left( \frac{\mathcal{D} - (H/2)^2}{\mathcal{D} - 1} \right)^2 \quad (3.16)$$

This is a monotonic function that increases as  $\mathcal{D}$  decreases. As this function is considered as an discomfort due to the risk of collision with an upstream bird, this discomfort tends towards infinity when  $\mathcal{D}$  tends towards 1. Since if  $\mathcal{D} = 1$ , there is a collision. The function is zero when  $\mathcal{D} = (H/2)^2$ , so the two birds are far enough apart and the follower has no discomfort caused by the presence of the upstream bird. Finally, the discomfort is zero when the follower is far enough away from the other bird, *i.e.*  $\mathcal{D} > (H/2)^2$ . In other words, the feeling of discomfort due to the risk of collision increases drastically and monotonically as the follower gets closer to another bird.

### 3.3.2 Vision comfort constraint

As mentioned in [48,49], Canada geese, migratory birds similar to ibises, tend to maintain visual contact with the bird in the rank above them. Therefore, it is close to their axis of vision. Furthermore, [49] hypothesizes that this bird has fixed eyes or moves them only slightly. Thus, having the upstream bird close to its axis of vision would enable it to improve visual resolution on the latter and thus keep it in a visual comfort zone.

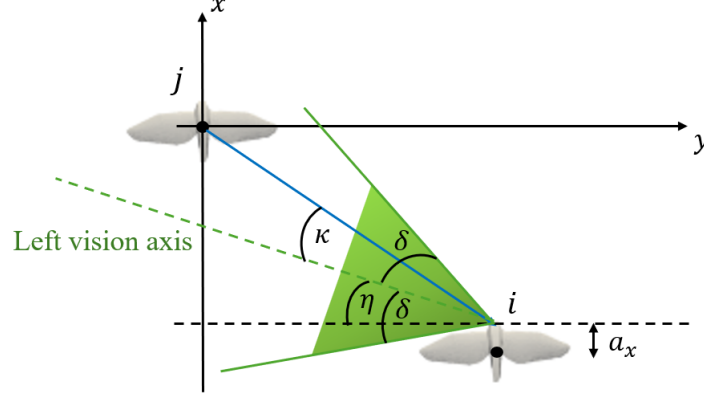


Figure 3.5: Illustration of the visual constraint and field of view of the bird's left eye when viewed from above.

We assume the bird's eye to be fixed [48,49] with a vision axis at an angle  $\eta$  to the  $y$  axis as shown in Fig. 3.5. The bird has a comfort zone of angle  $\pm\delta$  around its axis of vision [48,49]. This represents a cone whose vertex coincides with the bird's eye and its axis with the axis of vision. We hypothesize that the bird's head is perfectly aligned with the  $x$  axis, so the axis of vision has no  $z$  component, as shown in Fig. 3.6. We also note that the bird's eyes are at a distance  $a_x$  along  $x$  from the bird's center of mass  $i$  with the bird  $j$  in its vision. The visual comfort constraint is therefore respected if  $|\kappa| < \delta$ . Therefore bird  $j$  will be in bird  $i$ 's visual comfort zone.

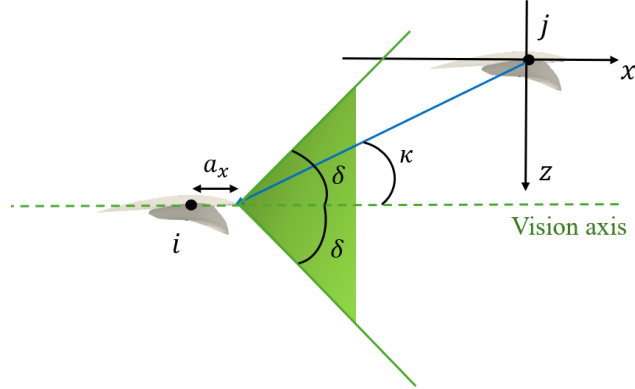


Figure 3.6: Illustration of the visual constraint and the bird's field of vision when viewed from the side.

To find the value of  $\kappa$ , we will use the properties of the scalar product between the axis of vision  $\mathbf{A}$  and the relative position  $\mathbf{P}$  represented in blue between the bird's center of mass  $j$  and the bird's eyes  $i$ . Based on Figs. 3.5 and 3.6, it is possible to give a direct expression for these two vectors.

$$\mathbf{A} = [\sin(\eta), \pm\cos(\eta), 0]^T \quad (3.17)$$

$$\mathbf{P} = [x_i - x_j + a_x, y_i - y_j, z_i - z_j]^T \quad (3.18)$$

We know the position of the upstream bird  $(x_j, y_j, z_j)$  as well as the bird's visual parameters  $\eta$  and  $\delta$  from Table 2.2. We look for the position where to place a follower with a general coordinate  $(x, y, z)$ . We then find the relationship  $\kappa = \kappa(x, y, z)$  that the follower must satisfy as follows.

$$|\kappa(x, y, z)| = |\arccos\left(\frac{\mathbf{P} \cdot \mathbf{A}}{\|\mathbf{P}\| \cdot \|\mathbf{A}\|}\right)| \leq \delta \quad (3.19)$$

We reason in a similar way to section 3.3.1, imposing an arbitrary function  $\mathcal{B}_V$  which imposes a discomfort, when  $|\kappa| > \delta$ . Consequently when the leader leaves the follower's visual comfort zone. To write this visual constraint function, we use a variable  $s(\kappa) = \tan(\kappa)$ . The condition thus becomes  $s^2(\kappa) \leq \tan^2(\delta)$ .

$$\mathcal{B}_V(s(x, y, z)) = \begin{cases} 0 & \text{if } s^2 \leq \tan^2(\delta) \\ k_V (s^2(x, y, z) - \tan^2(\delta)) & \text{if } s^2 > \tan^2(\delta) \end{cases} \quad (3.20)$$

We have  $k_V$  of the order of unity and a monotonically increasing function as  $s^2$  increases. This means that as the leader moves further away from the  $2\delta$  visual comfort zone, the discomfort on the follower increases. That makes it more difficult to position the upstream bird further from the follower's eye axis.

### 3.4 Gradient-based method

The aim of our model is to simulate formation flight and compare numerical results with empirical observations, in order to unravel the underlying dynamics of bird formation flight. We set up a minimization model based on scientific papers demonstrating the need for birds to reduce their energy expenditure by drawing on the kinetic energy present in the wake created by the formation. The objective function  $\mathcal{G} = \mathcal{G}(x, y, z, \Delta\phi_f)$  is an approximation of the power saved by a follower bird in the wake of another bird. This objective function is composed of an average of the upwash on the bird's wings, weighted by a force model of the same form as the lift function over a period of time. Due to the  $z$  convention, upwash is negative and oriented along  $-z$ . Consequently, for an upwash-rich position in the optimization space  $(x, y, z, \Delta\phi_f)$ ,  $\mathcal{G}$  will be negative. We therefore aim to minimize  $\mathcal{G}$  so that it is as negative as possible. The constraint functions are the one related to possible collisions  $\mathcal{B}_C = \mathcal{B}_C(x, y, z)$  and the one related to vision  $\mathcal{B}_V = \mathcal{B}_V(x, y, z)$ . These two functions are strictly positive in the bird's discomfort range, and zero when the bird has no discomfort. The minimization problem is therefore written as follows.

$$\min_{x, y, z, \Delta\phi_f} \mathcal{G}(x, y, z, \Delta\phi_f) \quad (3.21)$$

$$\text{subject to} \quad \mathcal{B}_C(x, y, z) \quad (3.22)$$

$$\mathcal{B}_V(x, y, z) \quad (3.23)$$

To solve this minimization problem, we use the gradient method [66]. We define  $\mathbf{x} = [x, y, z, \Delta\phi_f]^T$  as the variable vector and  $k < 0$  as the iteration step to converge.

$$\mathbf{x}_{i+1} = \mathbf{x}_i + k (\nabla\mathcal{G}(\mathbf{x}_i) + \nabla\mathcal{B}_C(\mathbf{x}_i) + \nabla\mathcal{B}_V(\mathbf{x}_i)) \quad (3.24)$$

The first iteration at  $i = 0$  is given by an arbitrary guess  $\mathbf{x}_0$ . We stop the iteration of eq. 3.24 when  $|\mathbf{x}_{i+1} - \mathbf{x}_i| < \epsilon$  with  $\epsilon > 0$  a certain tolerance giving the convergence criterion. The gradients of the various functions are calculated using a centered finite difference of order two. Full details of the gradient method are given in *Appendix B*.

### 3.5 Application to the wake of a single bird

We apply this minimization problem to the wake of an ibis positioned at  $(x, y, z) = (0, 0, 0)$  and with a velocity of  $U = 15$  [m/s]. The wake of this bird was detailed and analyzed in the last section of the previous chapter. In this section, we use this wake as an input to the problem, to place a second bird that takes full advantage of the upwash present in the wake.

#### 3.5.1 Results and discussions

We apply the gradient method to solve the problem of minimizing the function  $\mathcal{G}$  constrained by  $\mathcal{B}_V$  and  $\mathcal{B}_C$ . We find as ideal variables minimizing the constrained function the optimal parameter vector  $\mathbf{x}_{opt}$  given below.

$$\mathbf{x}_{opt} = [x, y, z, \Delta\phi_f]^T_{opt} = [-1.62 \text{ [m]}, 1.20 \text{ [m]}, 0.17 \text{ [m]}, 168 \text{ [}^\circ\text{]}]^T \quad (3.25)$$

We can see the iterations of the gradient method in Fig. 3.7 and for different planes of space in Appendix C. We took four  $\mathbf{x}_0$  candidates to run the iterations and all converged to  $\mathbf{x}_{opt}$  after a few thousand iterations. The further  $\mathbf{x}_0$  is from  $\mathbf{x}_{opt}$ , the more iterations it will take to converge. Indeed, since  $k$  is a constant, if the local gradients are small, the method will take longer to converge. An improvement to the model could use variable-step gradient descent to optimize compilation time.

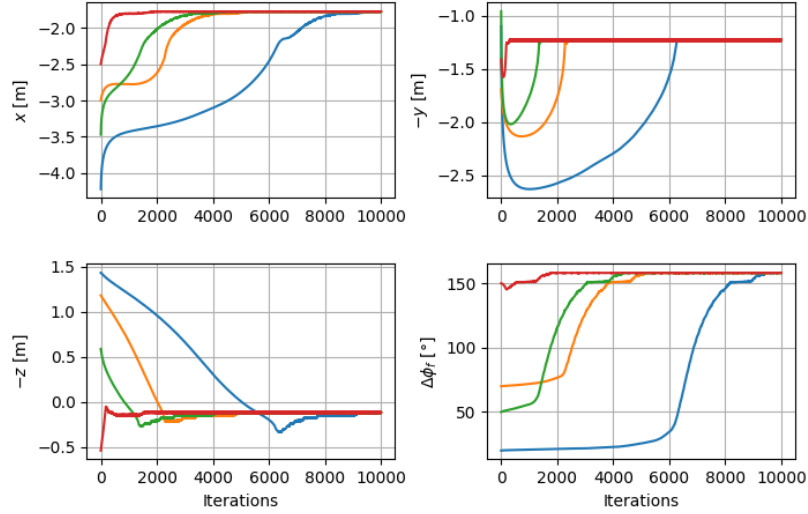


Figure 3.7: Graph of convergence of the gradient method for the four minimization variables  $\mathbf{x} = [x, y, z, \Delta\phi_f]^T$ . Red represents the initial guess  $\mathbf{x}_0 = [-2.5, -1.4, -0.7, 150]^T$ . Green represents the initial guess  $\mathbf{x}_0 = [-3.5, -1.0, 0.4, 50]^T$ . Orange represents the initial guess  $\mathbf{x}_0 = [-3.0, -1.7, 1.0, 70]^T$ . Blue represents the initial guess  $\mathbf{x}_0 = [-4.2, -1.1, 1.3, 20]^T$ .

We will analyze Figs. 3.8 to 3.11. As a reminder, since we are looking to minimize  $\mathcal{G}$ , we are looking for the strongly negative zones, *i.e.* in red on these same figures. We will only compare numerical results with ibis observations. The physical interpretations and explanations behind these observations will be dealt with in Chapters 4 and 5.

For Fig. 3.8, we see two optimal zones at  $x \approx -2$  and  $x \approx -6$  spaced by a distance  $\Delta x = \lambda$  in the cross-section for  $(z, \Delta\phi_f) = (0.17 [m], 168 [^\circ])$  fixed. This means that for the same variables  $(z, \Delta\phi_f)$ , there is an equally energetically favorable zone in which to place a follower at a spatial wavelength  $\lambda$  in the wake. Conversely, for the same  $x$ , the position just behind the leader at  $y = 0$  is the most unfavorable. Steven Portugal's observations in [24,29] show that the angle between two birds in a rank following each other and the horizontal axis is  $\arctan(1/1.35) \approx 36.52 [^\circ]$ . For our simulation, we have  $\arctan(y_{opt}/|x_{opt}|) \approx 36.53 [^\circ]$ . Moreover, observations suggest an overlap between the leader and follower wings for the spanwise position  $y$ . We also have this feature since  $y_{opt} < b$  and  $y_l = 0$ . For Fig. 3.9, we note that the follower seeks to adapt its  $z$  coordinate. The phase shift induces the bird's wingtip position to follow the trajectory of the nearest trailing tip vortex.

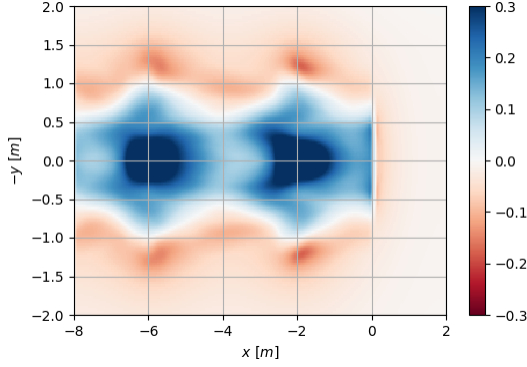


Figure 3.8: Representation of a cross-section of the  $\mathcal{G}$  function in the  $xy$  plane for  $(z, \Delta\phi_f) = (0.17 [m], 168 [^\circ])$  fixed.

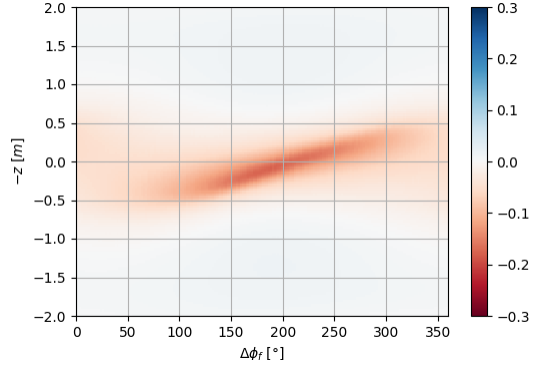


Figure 3.9: Representation of a cross-section of the  $\mathcal{G}$  function in the  $z\Delta\phi_f$  plane for  $(x, y) = (-1.62 [m], 1.20 [m])$  fixed.

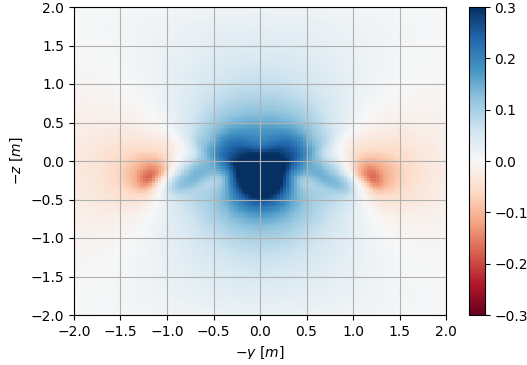


Figure 3.10: Representation of a cross-section of the  $\mathcal{G}$  function in the  $yz$  plane for  $(x, \Delta\phi_f) = (-1.62 [m], 168 [^\circ])$  fixed.

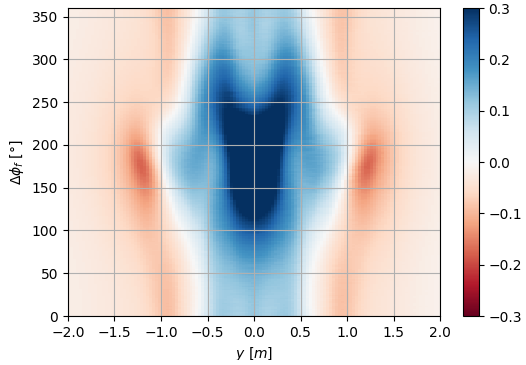


Figure 3.11: Representation of a cross-section of the  $\mathcal{G}$  function in the  $y\Delta\phi_f$  plane for  $(x, z) = (-1.62 [m], 0.17 [m])$  fixed.

We can see from Fig. 3.10 that the ideal zone is slightly below the leader since  $z_{opt} > 0$  as in the observations. Indeed, the trailing tip vortices push each other downwards due to the downwash they generate. In Fig. 3.11, the ideal phase shift  $\Delta\phi_{f,opt}$  for  $y_{opt} = 1.20 [m]$  is the worst case when  $y = 0$  as also observed in [24,29] for the V-shaped formation flight. Moreover, at  $y_{opt} = \pm 1.20 [m]$ , the phase shift approaches phase opposition, as observed.

### 3. Wake benefit topology

In fact, it is this search for an optimal phase shift that motivates the development of a phase shift controller, as detailed in Chapter 4. The intuition behind these observations, and confirmed in Chapter 4, is that the follower seeks to adapt its wing position to coincide as closely as possible with the trailing tip vortices produced by the bird upstream. In particular, the bird seeks to adapt its phase to take advantage of the vortex created by the upstream bird's downstroke. The follower seeks during its downstroke to be close to the vortex produced by the upstream bird's downstroke. Thus, a linear relationship emerges between the phase shift relative to the leader  $\Delta\phi_f$  and the spanwise direction  $x$  of the follower, as illustrated in Fig. 3.13. Indeed, if  $x = -\lambda$ , we have  $\Delta\phi_{f,opt} = 0$  [°] meaning we are in phase. At this position in space and being in phase, the follower intercepts during its downstroke the vortex produced by the upstream bird's downstroke. For the same reason, for  $x$  being an integer multiple of  $-\lambda$ , the phase shift will be zero as observed in [24,29] and in Fig. 3.13.

We now turn to the  $\tilde{L}_Z$  force model defined in eq. 3.8. This model is a function intended to resemble the vertical component of lift during a flapping period. As this weighting function gives extra weight to a large lift production, *i.e.* produced during the leader's downstroke, it encourages the search for this downstroke. This explains why, when we compare Fig. 3.15 and 3.16, we see a stronger gradient in the zone close to the optimum when  $\tilde{L}_Z$  is defined as eq. 3.8. This improves the speed of convergence of the gradient method. In addition,  $\tilde{L}_Z$  disfavors upstroke. As a result, the force model allows us to better target the leader's downstroke and move away from his upstroke in the search for the optimum.

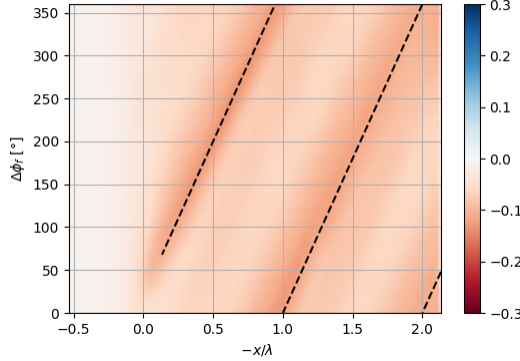


Figure 3.12: With  $\tilde{L}_z = 1$ .

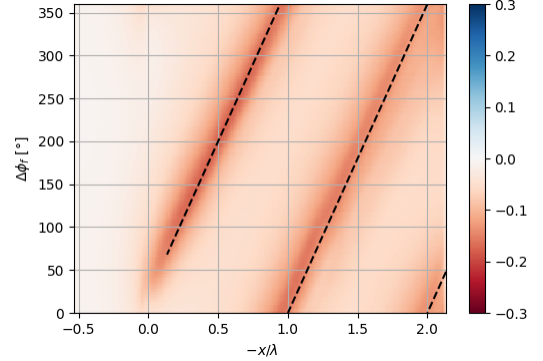


Figure 3.13: With  $\tilde{L}_z$  from eq. 3.8

Figure 3.14: Representation of a cross-section of the  $\mathcal{G}$  function in the  $x\Delta\phi_f$  plane for  $(y, z) = (1.20 [m], 0.17 [m])$  fixed.

In his observations, S. Portugal shows that there is a linear relationship between  $x$  and  $\Delta\phi_f$  as illustrated in Fig. 3.14

$$\frac{\partial}{\partial x} (\Delta\phi_f) \approx \frac{-2\pi}{\lambda} \quad (3.26)$$

However, his measurements proving this model in [24,29] seem to have a high statistical variability. This may be due to atmospheric disturbance or instrument measurement errors. We therefore perform different cuts to see if the relationship eq. 3.26 is still valid when  $y$  or  $z$  vary. We starts with the optimal configuration and cutting with  $y = 0$ , Fig. 3.15, then varying  $z = 0$ , Fig. 3.16. By comparing Fig. 3.17 with Fig. 3.13, we can

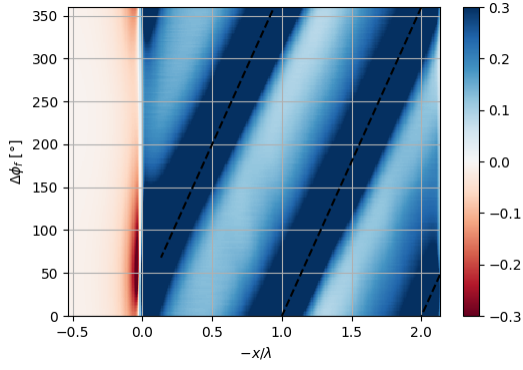


Figure 3.15: Cut with  $(y = 0 [m], z = 0.17 [m])$ .

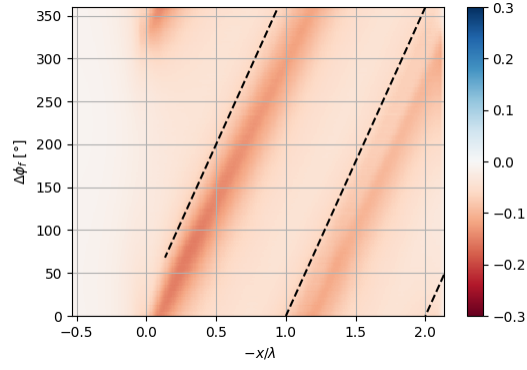


Figure 3.16: Cut with  $(y = 1.20 [m], z = 0 [m])$ .

Figure 3.17: Representation of a section of the  $\mathcal{G}$  function in the  $x\Delta\phi_f$  plane for different  $(y, z)$ . The dotted black line is the locus of the optimal points in the  $x\Delta\phi_f$  plane of Fig. 3.14.

hypothesize that there is an offset of the ideal phase shift as a function of  $x$  that is caused by a variation of  $y$  and  $z$ . Consequently, the model is still valid. Moreover, we can add that we should take into account the dependence in  $y$  and  $z$ . We therefore introduce a function  $h$  that reflects this dependence.

$$\Delta\phi_f \approx 2\pi \frac{-x}{\lambda} + h(y, z) \quad (3.27)$$

In [24,29], there is no measure according to  $z$  but there is for  $y$ . Observations indicate that when the follower is in  $-x/\lambda \approx 0.5$  just behind the leader ( $y = 0$ ), the follower tends to fly in phase concordance. Conversely, when flying in V-shaped formation, they tend to be close to phase opposition. In our case, for the V-shaped, we have  $\Delta\phi_f = 168 [^\circ]$  in  $-x/\lambda = 0.43$  and therefore close to phase opposition. In Fig. 3.15 showing the beneficial function of a follower just behind a leader ( $y = 0$ ), in  $-x/\lambda = 0.43$ , we have the optimal phase shift  $\Delta\phi_f \approx 0$  and therefore well in phase concordance. Our results are therefore in line with observations. In addition, for the same  $x$ , we have several  $\Delta\phi_f$  for different  $y$  and  $z$ , proving the  $y$  and  $z$  dependence of eq. 3.27.

### 3.5.2 Discussions on assumptions

In section 3.1.1, we have considered that the flight sequence does not vary when a follower is in the wake of a leader. This means that the kinematics of the wings over a period of wing beats is constant. However, this is not necessarily the case. Indeed, in a scientific article [46] analyzing the flight sequence of pigeons during a close flight of two pigeons, it is mentioned that a bird might change its flight sequence when it is a follower for reasons of visual comfort. In fact, the birds studied in this article, when in follower flight and therefore at a relatively close distance from the leader, the latter reduced the time of their upstroke to decrease their vertical oscillation along  $z$  by 22%. As discussed in the previous chapter, the component of forces along  $z$  shown in Fig. 3.2 is negative during the upstroke and tends the bird to head downwards. Limiting this oscillation would increase the visual stability and resolution of the image it perceives of the leader in front of it. We assume that the slight oscillations in  $z$  of the order of less than  $0.1\bar{c}$ , see Fig. 2.20, can neglect the visual discomfort associated with vertical displacement. Thus, we assume a flight sequence identical to flight alone.

### 3. Wake benefit topology

---

Finally, in our assumptions, we have included our simulations in a case where there is no atmospheric turbulence. As an avenue for further development of the model, a study of the impact of atmospheric turbulence on the dynamics of formation flight could tell us whether or not equilibrium would still be possible under excessive turbulence. Also, to what extent this would affect the followers' energy savings.



## Chapter 4

# Flight of two birds

In the previous chapter, we established a model for determining where to position a bird in a wake with a given phase shift to minimize its energy expenditure. This chapter presents the implementation of numerical bird controllers for simulations. The purpose of the controllers is twofold. The first is to enable movement from an arbitrary position to a desired position for the bird, as illustrated in Fig. 4.1. This will move the bird from its initial position to the position determined by the minimization problem. The second goal is that, once the bird has been positioned in space, we want it to stabilize despite the presence of forces that could destabilize it. These forces are induced by the velocity field present in the wake of other birds.

### 4.1 Spatial control

In this section, we will look at the controllers used to manage the numerical bird's six degrees of freedom: its position  $(x, y, z)$  and the associated angles of rotation  $(\phi, \theta, \psi)$ . As illustrated in Fig. 4.1, the controllers are used to position a follower at a target location  $\mathbf{x}_t = (x, y, z)$  determined by the minimization problem. Since we want the bird to remain at the same relative position to the upstream bird, at the desired location  $\mathbf{x}_t$  it must have the same velocity as the upstream bird. In this figure, the upstream bird has a velocity of  $\mathbf{u}_t$ , so at  $\mathbf{x}_t$ , the follower must have the same velocity.

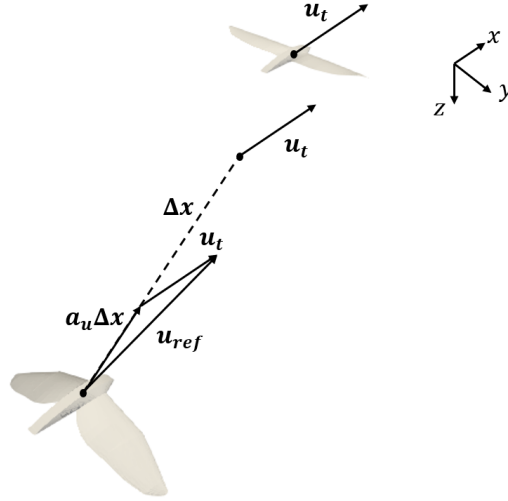


Figure 4.1: Representation of the control of a follower bird trying to reach a point in space at a relative distance  $\Delta \mathbf{x}$  and with a target velocity  $\mathbf{u}_t$  in the wake of a leader.

To achieve this, the controller is implemented in Fig. 4.2. This is a stabilization controller presented in [15,67]. The controller takes a target position  $\mathbf{x}_t$  and compares it with the bird's actual position  $\mathbf{x}$ . The result is a position error  $\Delta \mathbf{x}$ . The block  $C_x$  induces a velocity based on the error  $\Delta \mathbf{x}$  and the target velocity  $\mathbf{u}_t$ . The result is a reference velocity  $\mathbf{u}_{ref}$ . In Fig. 4.1, we can see that  $\mathbf{u}_{ref}$  is the vector sum of the target velocity  $\mathbf{u}_t$

#### 4. Flight of two birds

with a velocity allowing displacement to reduce the  $\Delta \mathbf{x}$  error. We therefore wish to apply a  $\mathbf{u}_{ref}$  velocity to the bird positioned at  $\mathbf{x}$ . The  $T_u$  block takes this velocity as input to calculate the forces the bird must produce. This block is composed of a proportional integrator (PI) controller whose integral component reduces the impact of parasitic forces. Since a variation in  $\mathbf{u}_{ref}$  must be caused by a variation in the bird's thrust. This block also includes a function for wing kinematics. The thrust variation constrains the variable  $x$ . As output, the bird actually produces a velocity  $\mathbf{u}$  which will induce a new location  $\mathbf{x}$  where the bird is. As we also need to control the direction of the bird, we control it with forces according to  $y$ . These forces are produced by changing the roll angle  $\phi$  using a proportional controller.

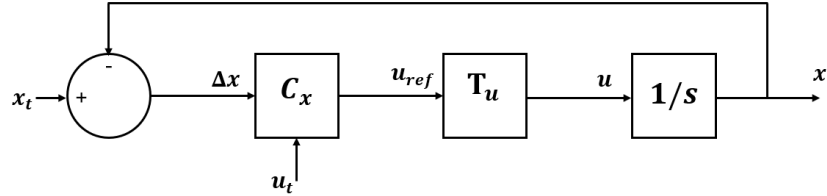


Figure 4.2: Schema block of the controller to reach position  $\mathbf{x}_t$  and velocity  $\mathbf{u}_t$  at the same location. (adapted from [67])

For the other three degrees of freedom,  $(z, \theta, \psi)$ , these are controlled by three separate controllers. For  $z$ , based on a  $\Delta z$  error, we determine a reference force for the bird. The bird then produces a force based on this reference force, which induces a displacement. For the angles  $\theta$  and  $\psi$ , we calculate the error between the target and actual angles to induce respective moments. This variation in moment consequently induces a variation in angle. Note that for the yaw angle  $\psi$  controller, as explained in [67], a variation in direction caused by the roll angle control will induce a sideslip angle  $\beta \neq 0$ . The yaw angle controller therefore takes this into account to cancel the sideslip angle.

The calculation of the aerodynamic forces that the bird must produce is based on a reference velocity, force or moment. This is explained in [15] and [67] as well as for the calculation of controller gains.

In the previous chapter, we determined that the ideal position to place the first follower with a leader positioned at  $(0, 0, 0)$  is  $(x, y, z) = (-1.62, 1.20, 0.17)$  [m]. In Fig. 4.3, we see that all three variables converge to the desired position. The order of time for the convergence of  $x$  and  $y$  is  $\Delta t = 30$  [s], while for  $z$  it is 60 [s]. The longer convergence time for  $z$  may be caused by the wake velocity field, which involves aerodynamic forces that disrupt the bird's vertical motion. Indeed, as the leader flaps its wings, its wake oscillates according to  $z$  and therefore induces vorticity which also changes according to  $z$  as explained in Chapter 2.

For bird orientation, in Fig. 4.4, we also see that convergence is of the order of 30 – 40 [s]. The pitch angle  $\theta$  and yaw angle  $\psi$  tend well towards 0 on average over a flapping period. In contrast, the roll angle  $\phi$  does not tend towards 0. Since the  $\phi$  controller is only proportional, it is subject to a static error. This results in an error between the target value and the actual value at equilibrium. Physically, this can be explained by the difference in lift exerted on the bird's two wings, which induces a positive rolling moment on average over a flapping period. This difference in lift on the wings is induced by a perceived relative velocity, which is different for each because of the wake.

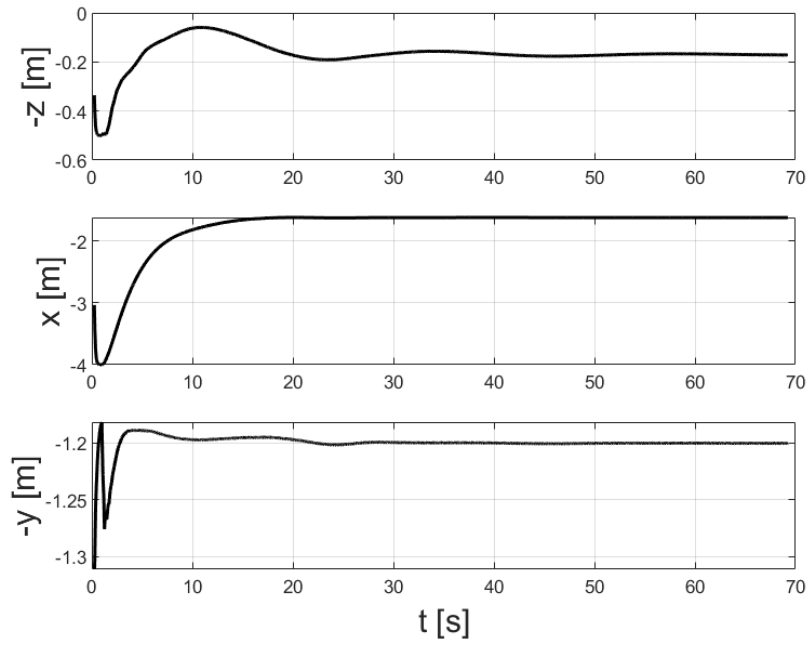


Figure 4.3: Graph showing the evolution of the variables  $(x, y, z)$ , averaged over a flapping period, as a function of time for a follower bird in the wake of a leader.

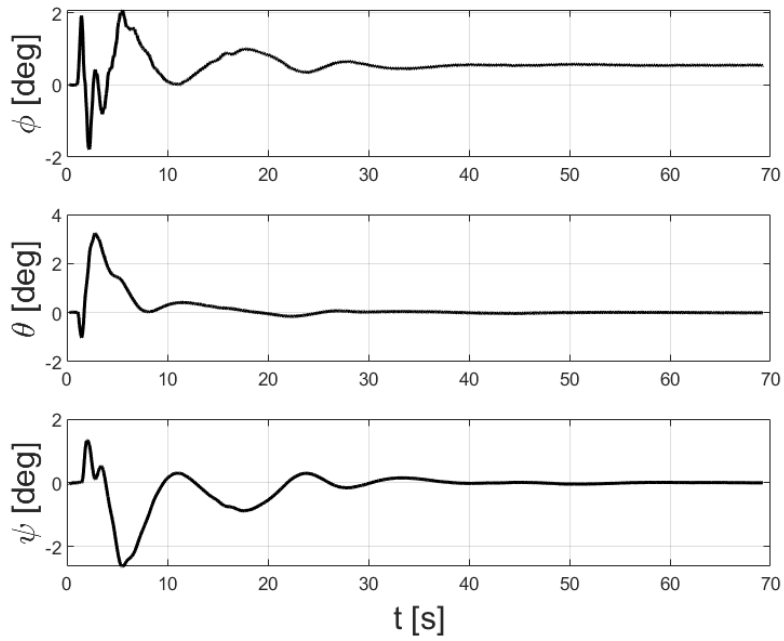


Figure 4.4: Graph showing the evolution of the variables  $(\phi, \theta, \psi)$ , averaged over a flapping period, as a function of time of a follower bird present in the wake of a leader.

## 4.2 Phase shift control

The fourth minimization variable is the phase shift of the bird relative to the upstream bird. The aim of phase control is to change this phase shift to reach a target phase shift at equilibrium. We define  $\Delta\phi_f$  as the difference between the phase of a bird  $\phi_f$  and that of the upstream bird  $\phi_l$ .

$$\Delta\phi_f(t) = \phi_f(t) - \phi_l(t) \quad (4.1)$$

As illustrated in Fig. 4.5, the phase shift is the offset between the bird's sequence and the one upstream. The flight sequence varies the  $\tau$  angle. Therefore it depends on time and phase shift for the first follower, and only on time for the leader, as it has no bird upstream.

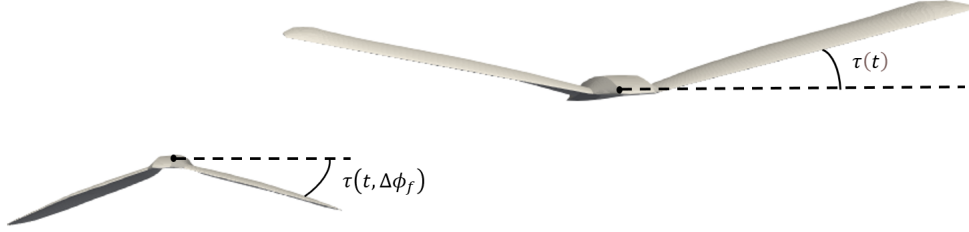


Figure 4.5: Formation of two birds, the leader on the right and the follower on the left, showing the angle  $\tau$  between the wing and the horizontal.

The structure of the closed-loop phase-shift controller is shown in Fig. 4.6. The controller takes as input the targeted phase shift  $\Delta\phi_{ref} = \Delta\phi_{f,opt}$  determined by the minimization problem. We calculate the error between the current phase shift  $\Delta\phi$  and the desired phase shift  $\Delta\phi_{ref}$ . To reduce this error, we use a proportional controller  $C_\phi$  to induce an angular frequency variation  $\Delta w = 2\pi\Delta f$ .

$$\Delta w = k_p(\Delta\phi_{ref} - \Delta\phi) \quad (4.2)$$

The value of  $k_p$  is determined on the basis of observations of northern bald ibis. The observations made in [68] show that when the bird is in a stable position in a formation for steady level flight, its flapping frequency is  $f_0 = 4 [Hz]$  as indicated in Table 2.2. The measurements in [68] also show that the observed frequency variation is  $\pm 15 [\%]$  around  $f_0$ , so  $\pm 0.6 [Hz]$ . The maximum phase shift dictated to the bird is  $\pm\pi [rad]$ . To match ibis observations as closely as possible, we calculate  $k_p$  as follows.

$$k_p = 2\pi \frac{0.6}{\pi} = 1.2 [s^{-1}] \quad (4.3)$$

We have a differentiation block to determine the angular frequency  $w$ , of the follower using eq. 4.4

$$w = \Delta w + \frac{d\phi_f}{dt} \quad (4.4)$$

Once we have obtained this new angular frequency  $w$ , all we have to do is integrate it to determine the new follower phase  $\phi_f$ .

For the same simulation as the previous section of the two identical numerical birds, we impose a phase shift  $\Delta\phi_{ref}$  with an initial phase shift  $\Delta\phi = 0 [^\circ]$ . In Fig. 4.7, the follower reaches its target phase shift in 7 [s]. We compare this with the evolution of frequency  $f = w/2\pi$  as a function of time. We see that a large variation in phase shift induces a large variation in frequency. Moreover, when the targeted phase shift is reached

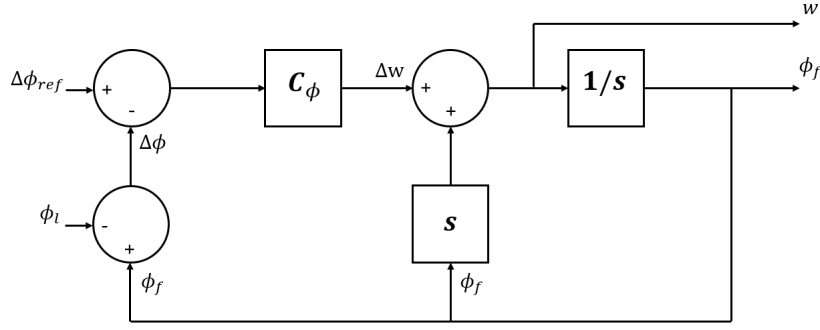


Figure 4.6: Schema block representation of the control of a follower's phase shift and the bird's flapping frequency.

$\Delta\phi = \Delta\phi_{ref}$ , frequency  $f$  returns to its steady-level-flight value of  $f_0 = 4$  [Hz] since  $d\phi_f/dt = 0$  and  $\Delta w = 0$ .

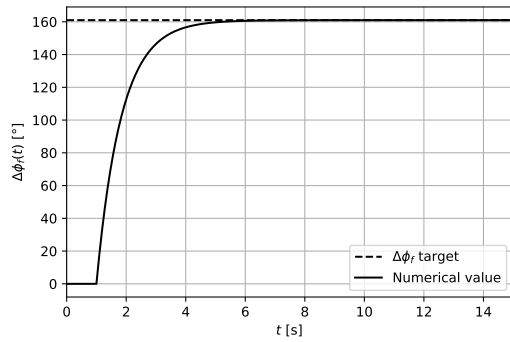


Figure 4.7: Evolution of follower phase shift control to reach target phase shift as a function of time.

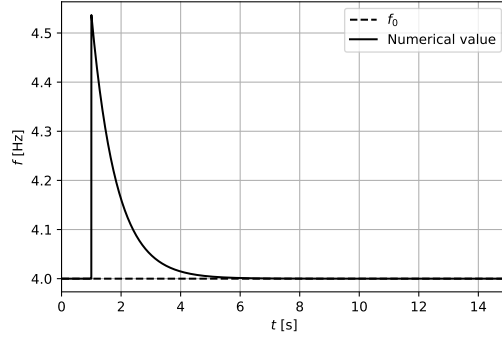


Figure 4.8: Evolution of a follower's flapping frequency following a change in its phase shift as a function of time.

In the special case of two birds in formation with a leader not changing his flapping frequency, it is possible to find the analytical equation of  $f$  and  $\Delta\phi_f$  as detailed in *Appendix D*.

$$\Delta\phi_f(t) = (\Delta\phi_{ref} - \Delta\phi_{f,0}) (1 - e^{-k_p t}) \quad (4.5)$$

$$f(t) = f_0 + (\Delta\phi_{ref} - \Delta\phi_{f,0}) \frac{k_p}{2\pi} e^{-k_p t} \quad (4.6)$$

An increase in the value of  $k_p$  would induce a greater increase in the frequency of the bird's wings beating. Or a decrease if the phase shift has to be reduced, *i.e.* if  $\Delta\phi_{ref} - \Delta\phi_f < 0$ . Since the change in frequency  $f$  is greater, equilibrium is reached more quickly. Indeed, the negative exponential would tend towards zero more rapidly.

### 4.3 Two birds in formation

#### 4.3.1 Results and discussions

After stabilizing the follower behind the leader with the variables  $(x, y, z, \Delta\phi_f)$  determined by the energy expenditure minimization problem, we can analyze the results.

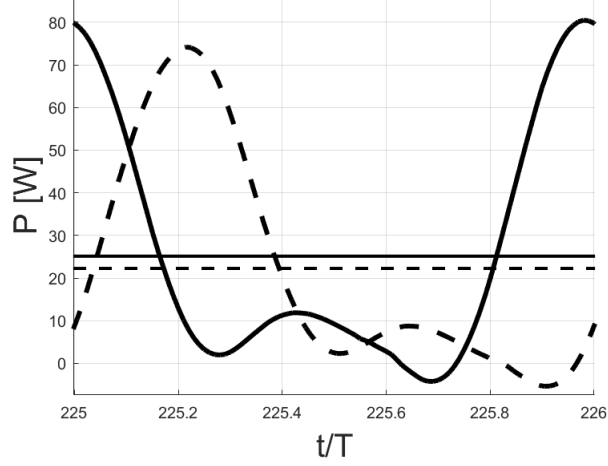


Figure 4.9: Power versus time for a flapping period  $T$ . The solid line represents the power of the leader. The dotted line represents the first and only follower for this simulation. Horizontal straight lines represent the average power of the two birds.

As shown in Fig. 4.9, the follower's average power over a period of wing-flapping has decreased compared with flying alone. When flying alone, the bird's level-flight power is 25.25 [W]. The follower, on the other hand, develops a power of 22.28 [W], which represents a saving of 11.76 [%]. There are no observations on the measurement of power exerted by a bird in a formation that could confirm the results obtained in the simulation. However, there are observations of bird heart rate [69] for pelicans and [70] for northern bald ibis. We now seek to relate these heart rate measurements to the power calculated during our simulations. To do this, we will use the models developed by Pr Julien Hendrickx in his article [71]. This enables us to model the heart rate of a human undergoing physical effort. We will then compare the metabolism of a human with that of the northern bald ibis to see if the model is applicable.

The model developed by Pr Hendrickx predicts heart rate as a function of power output  $PO$ . The case of interest is the steady state. Although power varies over a period as shown in Fig. 4.9, we assume the variation in power is not perceived by the metabolism. Since the variation is repeated every  $T = 0.25$  [s]. Bird's metabolism only perceives average power. In other words, and as observed in [70], after stabilization of the bird at a constant beat frequency, the bird's heart rate reaches a steady state. The steady state reached by the bird's heart is a function of the average power expended by the bird over a flapping period. Of course, this is limited by the maximum heart rate  $HR_{max}$ . The same applies to a sportsman. His steady-state heart rate  $HR_{ss}$  is a function of constant power output  $PO$ , but cannot exceed his maximum heart rate  $HR_{max}$ . Pr Hendrickx describes the steady-state model as eq. 4.7 with  $m$  [bpm/W] the slope coefficient.

$$HR_{ss}(PO) = \begin{cases} HR_{rest} + m PO & \text{if } HR_{rest} + m PO < HR_{max} \\ HR_{max} & \text{otherwise} \end{cases} \quad (4.7)$$

For the northern bald ibis, its maximum heart rate  $HR_{max}$  can reach 600 [bpm] whereas its resting rate  $HR_{rest}$  is around 100 [bpm] [70]. For a single ibis flapping its wings and reaching cardiac equilibrium, this is achieved for  $HR_{ss}(PO) \approx 450$  [bpm] according to the observations of [70]. If we estimate this power output to be of the order of 30 [W], the slope coefficient  $m \gg 1$  [bpm/W]. This assumption is realistic for birds, whose mass power can vary between 20 – 25 [W/kg] [80]. As a reminder, our bird has a mass of 1.20 [kg]. Consequently, the ratio between two power outputs  $PO_1$  and  $PO_2$  can be simplified as follows for  $HR_{ss}(PO_1) < HR_{max}$  and  $HR_{ss}(PO_2) < HR_{max}$  as is the case in formation flight.

$$\frac{HR_{ss}(PO_1)}{HR_{ss}(PO_2)} = \frac{HR_{rest} + m PO_1}{HR_{rest} + m PO_2} \approx \frac{PO_1}{PO_2} \quad (4.8)$$

This simplification is possible for two similar birds with identical metabolisms since  $mPO \gg HR_{rest}$ . Therefore, thanks to this result, a 11.76 [%] reduction in power would correspond to a 12 [%] reduction in heart rate as observed in formation flight [69]. These same observations also indicate that the reduction in energy expenditure would be between 11 and 14 [%].

### 4.3.2 Discussions on the physical interpretation of results

In this section, we will interpret the results by explaining the physical principles underlying the phenomenon of flight in bird formation.

As a reminder, the lift per unit span  $l$  is given by the Kutta-Joukowski theorem in eq. 2.18. For a bird, the majority of lift is created during the bird's downstroke, as discussed by looking at the resultant of forces  $Z$  during a flapping period in Fig. 3.2. Therefore, by application of the theorem, circulation creation  $\Gamma$  is great during downstroke. As the vorticity is divergent free, this circulation is transmitted in the bird's wake. This can be seen by looking at the strength of the trailing tip vortices in Fig. 2.26 compared with the streamwise vorticity at other times in the bird's flight sequence in Fig. 2.27 - 2.29 as discussed in Chapter 2.

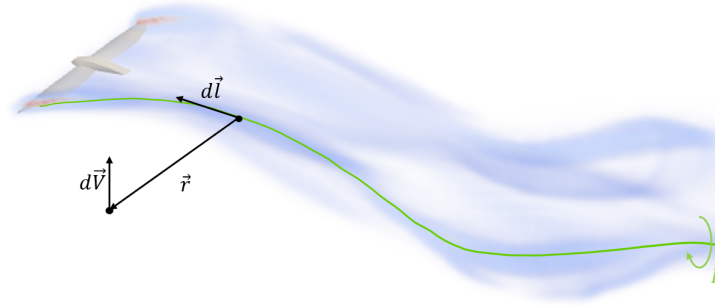


Figure 4.10: Illustration of the Biot-Savart law applied to one of a bird's two trailing tip vortices.

Now we dig into the impact of this vorticity in a bird's wake. We will apply the Biot-Savart law directly. This law applies to inviscid and incompressible flows. We are indeed in an incompressible flow, since  $U = 15$  [m/s] is much smaller than the speed of sound. We also apply this law even though we are not inviscid. However, as the Reynolds number  $Re$  is large, viscous effects are negligible compared with convection. Moreover, the follower is at a distance of 1.62 [m] and therefore perceives the wake produced by the leader in  $\Delta t = 0.1$  [s], leaving visible viscous effects to intervene.

#### 4. Flight of two birds

We define  $d\mathbf{l}$  as a segment of infinitesimal length along the center of the trailing tip vortices. The vector  $\mathbf{r}$  connects a point in space to this segment  $d\mathbf{l}$ . At this point in space, this infinitesimal segment induces a velocity  $d\mathbf{V}$  deduced by the Biot-Savart law as illustrated in Fig. 4.10

$$d\mathbf{V} = \frac{\Gamma}{4\pi} \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3} \quad (4.9)$$

We note that the induced velocity  $d\mathbf{V}$  depends on the sign of the trailing tip vortices, which may therefore be a downwash or upwash as discussed in Chapter 2. Now, we have positioned a follower just behind a leader bird so that it can take advantage of this upwash. This allows it to have an induced thrust component, thus reducing its power consumption.

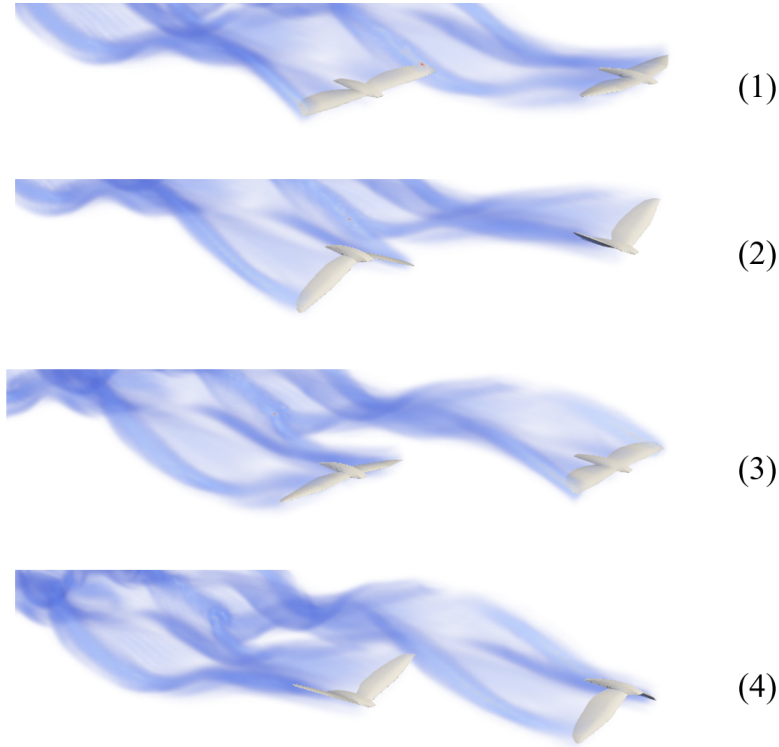


Figure 4.11: Snapshots of the follower's flight sequence for four different times of a flapping period in an oblique view of the two-bird formation. (1) Mid-downstroke of the follower. (2) End-of-downstroke. (3) Mid-upstroke. (4) End-of-upstroke.

In Fig. 4.11, we see that the follower follows the trailing tip vortex produced by the leader's right wing. Conversely, it moves away from the trailing tip vortex produced during leader's upstroke. In fact, during mid-upstroke (Fig. 2.28), the lift is negative, which changes the sign of the trailing tip vortices (application of Kutta Joukowski's theorem) and induces downwash on the follower. By moving its wings away from the vortex created by the upstroke. The follower thus avoids as much as possible the downwash generated by the leader.



We continue our observations by looking at the upwash in the wake in Fig. 4.12. In a cross-section in the  $xy$  plane for  $z = z_{opt}$ , the position of the follower, we can observe two phenomena. During the follower's mid-downstroke, (1) in Fig. 4.11 and 4.12 it picks up the upwash (shown in red in Fig. 4.12) generated by the vortex created by the leader's mid-downstroke. This allows the bird to take advantage of the maximum velocity induced during the peak of its lift production. This reduces the amount of lift required during the downstroke, and thus its power output, as illustrated in Fig. 4.9. In fact, the two power curves are farthest apart during the mid-downstroke. This means that the follower saves more energy during mid-downstroke. Then, in order to make the most of the upwash, the follower places the tip of his wing as close as possible to the center of the trailing tip vortex, without going any further. As Bio-Savart law can be approximated by a relationship in  $1/h$ , see *Appendix E*, with  $h$  the distance orthogonal to the vortex filament. If  $h$  tends towards 0 so the induced velocity increases drastically the closer we get to the center of the vortex. We also note that the wing does not protrude beyond the vortex. Otherwise, the wing would perceive the downwash (shown in blue). This would also be close to the center and therefore of high intensity, putting the bird at a disadvantage.

Most of the follower's energy savings come from its downstroke. When flying alone, this is the part of the flapping period that requires the most energy. In a formation, it requires less lift and thrust for the same flight performance. Consequently, it requires less power as illustrated in Fig. 4.9. This power is drawn from the wake where the follower positions its wingtip during its downstroke as close as possible to the center of the trailing tip vortex generated during the leader's downstroke. In addition, the follower avoids the trailing tip vortex produced during the leader's upstroke to limit the presence of downwash on his wings.

#### 4. Flight of two birds

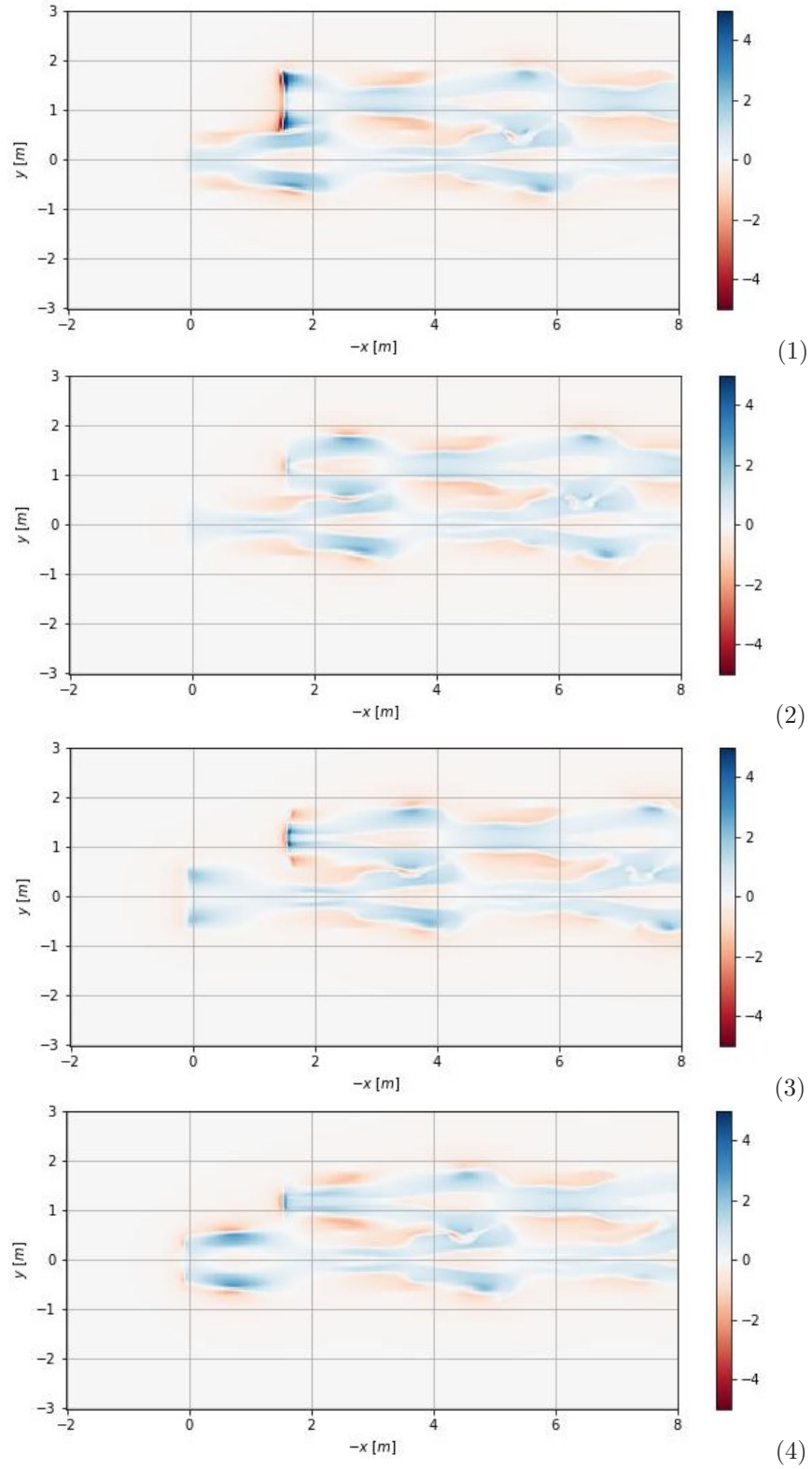


Figure 4.12: Representation of the vertical velocity component in the wake of two birds in formation in a section  $z = 0.17 [m]$ . From top to bottom, we see four different times in the follower's flight sequence; (1) Mid-downstroke of the follower. (2) End-of-downstroke. (3) Mid-upstroke. (4) End-of-upstroke.

## Chapter 5

# Formation flight analysis

Numerous models have been developed to simulate the formation flight of birds [39,41-45,48]. However, they are all based on a simplified model of the bird's wake. In the articles [39,41-43,45,48], the authors have represented the wings of their numerical birds by fixed wings represented by an horseshoe vortex. Paper [44] uses a rigid wing, but this time employs a  $k$ - $\epsilon$  model solution of the Navier-Stokes equations to obtain the wake. These models are all theoretical, as they do not take into account the flapping of the bird's wing.

With regard to the control of birds in formation, articles [42,43] arbitrarily fix the position of their numerical birds to form a V-shaped or U-shaped formation. They also study the impact on the formation if the birds have different wingspans, as in the case of birds of different ages or species flying in formation. In [45], the author models a controller for bird control in a V-shaped formation based on the aerodynamic advantages present in the wake and the bird's desire to track its predecessor. In [41], the authors use diffusion adaptation to start from a reference configuration and then converge on a formation. This model is based on the birds' interest in perceiving upwash and avoiding collision by imposing positional constraints. This paper also takes into account the attitude of numerical birds, *i.e.* whether they are selfish or cooperative, to study the impact on formation shape. In [39], the author represents the formation in a two-dimensional plane where the streamwise position and spanwise position are determined by maximizing the power induced in the wake, starting from a reference configuration. He also investigates the impact of relays and leader changes on the distribution of energy expenditure within the formation. Finally, in [48], the author pays attention to the attitude of birds in their need to maximize the upwash they perceive in the wake, while taking into account collision and vision constraints for numerical birds.

All these theoretical models do not take into account the flapping of birds' wings. As a result, they are to some extent unrepresentative of the dynamics of formation flight in birds. Thanks to the work of Dr Victor Colognesi in [15] and explained in Chapter 2, we can simulate a bird's wake with high fidelity. Our model is based on an approximation of the power saved for each numerical bird thanks to the upwash present in the wake. This model is also based on assumptions derived from observations of the birds' visual comfort, distancing and attitude, enabling us to numerically simulate a bird formation.

### 5.1 V-shaped flight

For the numerical simulations presented in this section, we assume all birds to be identical with the characteristics shown in Table 2.2. Once stabilized in the formation, the birds will have a displacement speed relative to the environment of 15 [m/s]. The birds' attitude is selfish, meaning that they have no information other than their personal energy expenditure. They do not seek to reduce that of others through their position.

Formation starts with the introduction of a leader bird at  $(x, y, z) = (0, 0, 0)$  [m], which is stabilized by the controllers. The wake of this bird is then calculated numerically using the VPM method and the bio-mechanical model explained in Chapter 2 and in [15]. Based on this wake, we analyze it and determine where to position the first follower rank using the minimization problem detailed in Chapter 3. The variables  $(x, y, z, \Delta\phi_f)$  are

then determined and input to the controllers to bring them in and stabilize. Once they have stabilized in the leader's wake, we repeat the process. This time, we analyze the wake produced by the three birds to position the new second rank of followers. Fig. 5.1 shows the result for a V-shaped formation of three ranks and a leader. Each bird shown is stabilized by its position and phase shift controllers to stay in that position.

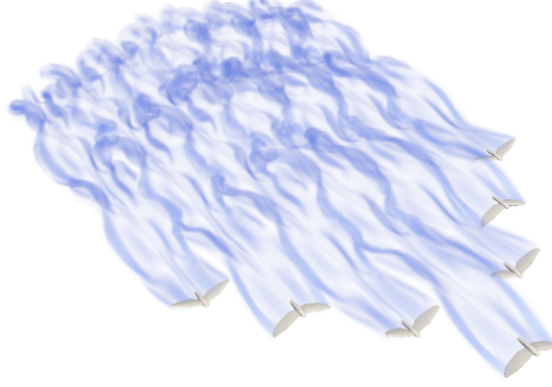


Figure 5.1: Volume rendering of the vorticity magnitude for the wake of a V-shaped formation of seven selfish birds. Birds are stabilized, obeying the constraints of vision and collision.

## 5.2 Discussions

### 5.2.1 Comparisons with observations

In this section, we will analyze the results of simulations taking into account collision and vision constraints. As a first result, when we look at Fig. 5.4 we see that the constrained results are very close to the observations made by Steven Portugal in [24]. The angle between the branches of the V-shaped flight and the  $x$  axis is indeed  $36.53^\circ$  against  $36.5^\circ$  in the observations [24,62].

| Position | Echelon |         |       |         | V-shaped |         |       |         |
|----------|---------|---------|-------|---------|----------|---------|-------|---------|
|          | z [m]   |         | y [m] |         | z [m]    |         | y [m] |         |
|          | with    | without | with  | without | with     | without | with  | without |
| 0        | 0       | 0       | 0     | 0       | 0        | 0       | 0     | 0       |
| 1        | 0.17    | 0.12    | 1.20  | 1.20    | 0.17     | 0.12    | 1.20  | 1.20    |
| 2        | 0.35    | 0.40    | 2.40  | 2.40    | 0.34     | 0.39    | 2.41  | 2.41    |
| 3        | 0.48    | 0.60    | 3.60  | 3.30    | 0.47     | 0.58    | 3.62  | 3.32    |
| 4        | 0.61    | n.a.    | 4.80  | n.a.    | 0.60     | n.a.    | 4.83  | n.a.    |

Table 5.1: Coordinate  $y$  and  $-z$  for echelon and V-shaped formation, with or without constraints and according to position in a four-rank flight.

We can also see on the same figure that the relative positions in the  $xy$  plane are similar but not identical between each bird, with a variation of a few centimetres. Then, looking at Table 5.1 again in the constrained case, the relative variation in  $y$  is  $1.21$  [m] between each individual. Given that the wing span  $b = 1.35$  [m], this means that there is an overlap between the wing of the upstream bird and the opposite wing of the downstream follower. This results in a negative wing tip spacing (WTS). WTS is the distance in  $y$  between the upstream bird's tip and the follower's tip as illustrated in Fig. 5.2

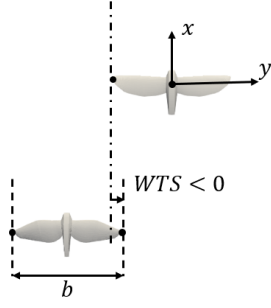


Figure 5.2: Illustration of wing tip spacing (WTS) for two birds in the  $xy$  plane.



Figure 5.3: Nothern bald ibis formation in front view. (from [72])

As observed in [24,45,53,73] analyzing the formation flight of ibis and Canada geese, the wing tip spacing between individuals is negative, reflecting an overlap between the wings. In our simulations,  $WTS/b = -0.104$  [/]. In observations from Portugal [24], it is  $-0.096$  [/]. For fixed-wing simulations [42-44], wing tip spacing is also negative. Several articles [42,45,53] discuss the optimal WTS for the fixed-wing case represented by an horseshoe vortex. This can be calculated analytically if the wing is elliptical. Indeed, the distance between its trailing tip vortices will be  $b_0 = \pi/4b < b$ . Since the wing's optimum is to position its tip closest to the center of the vortex,  $WTS_{opt}/b = 0.5(\pi/4 - 1) = -0.107$ . This value is therefore close to the simulations but not identical. Although the numerical bird's wing is neither fixed nor perfectly elliptical. The two values are relatively close, since in both cases the birds seek to position their downstroke wing's tip as close as possible to the trailing tip vortex center produced during the downstroke of the upstream bird. On average, the center of the trailing vortex is almost at the same place according to  $y$  in the case of fixed and flapping wings. This could explain the similarity of the WTS, even though the wings in our simulations are not elliptical.

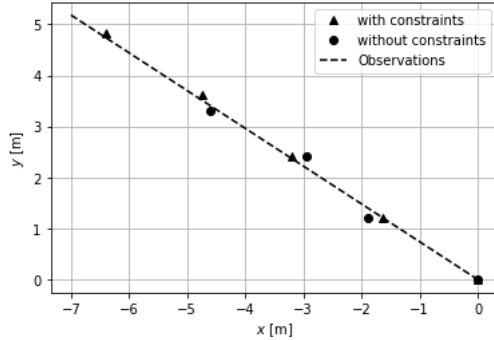


Figure 5.4: Top view of the V-shaped formation in the  $xy$  plane for the case with and without constraints.

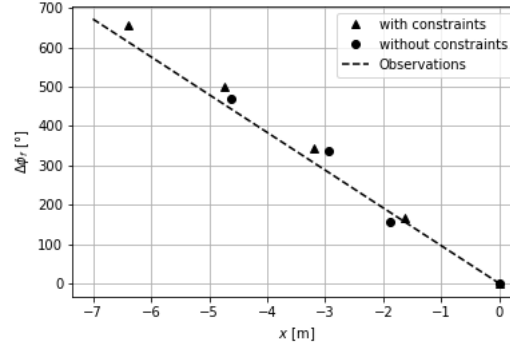


Figure 5.5: Representation of the phase shift with respect to the leader  $\Delta\phi_f$  as a function of  $x$  in the case with and without constraints for a V-shaped flight.

As for the vertical distance according to  $z$  between constrained individuals, this is not constant between each bird as shown in Table 5.1. It is also always positive, meaning that the numerical birds are one below the other, exactly as in real ibis formation flights (Fig. 5.3). As the trailing tip vortices produced by the upstream birds push each other downwards due to their downwash, this explains why the birds are one below the other. Since they try to follow these trailing tip vortices.

For the relationship between the cumulative phase shift in relation to the leader and the streamwise position of each bird in the formation shown in Fig. 5.5 we also observe a linear relationship, as discussed in Chapter 3. This is justified by the fact that each bird attempts, during its downstroke, to intercept the upwash generated by the upstream bird's downstroke. In our observations, we noted a slope of  $2\pi/\lambda$  with a possible offset as also discussed in Chapter 3. On the same figure, we can see that the first rank is on the dotted line, but the other ranks are on a straight line with a slight offset with same slope. This linear relationship is also observed for the ibis in observations by Steven Portugal [24,29] and [74]. That suggests that the birds' wing tips follow the path of those of the upstream birds, thus confirming the linear relationship between phase shift and streamwise position.

### 5.2.2 Average follower power

For the power consumed by followers, we will analyze the numerical results for different V-shaped formation sizes and for the different ranks present. Fig. 5.6 shows the power produced by a rank as a function of the number of ranks in the formation. We can see that the leader decreases his energy expenditure when a rank is added behind him. This decrease continues until two ranks are added downstream. This trend also applies to the other ranks when a bird is added after them. This observation is also observed in the numerical simulations of [39,41-43] for the leader.

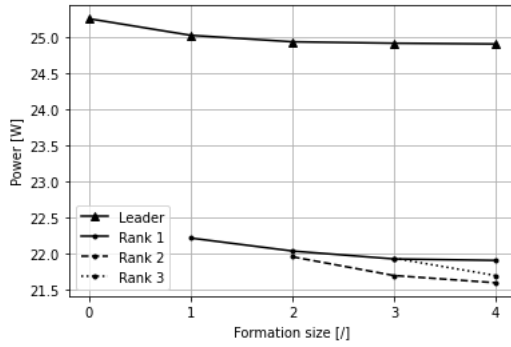


Figure 5.6: Power as a function of the size of the V-shaped formation for different ranks.

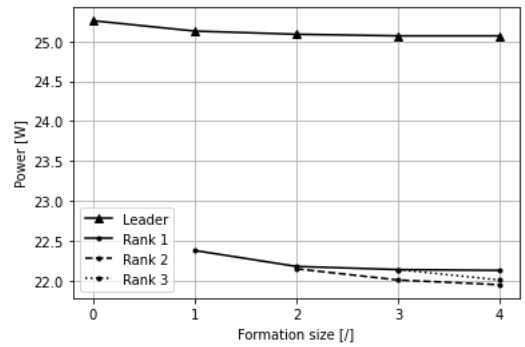


Figure 5.7: Power as a function of echelon formation size for different ranks.

As for the distribution of energy gains in the formation, these are shown in Fig. 5.8 with  $e_\mu$  defined as the benefit saved compared to flying alone. In the case of a V-shaped formation of nine birds divided into  $n = 4$  ranks and a leader, we find that the ideal position is in the middle of the formation on the second rank. The energy-saving benefits in the ranks range from 11 [%] to 14 [%], as also observed in [24,53,69,74].

In [74], the author discusses observations where bird heart rate measurements in the middle of a formation were lower, signifying energy savings. We also note that the  $e_\mu$  curve of our simulations based on a high-fidelity solver and an optimization model on an approximation of the power saved by a bird in formation gives a curve similar to [39] (cfr. Fig. 5.20) and [41-43]. Although these simplified models are based on a rigid elliptical wing to represent the birds in formation. Additionally, the bird in last position is at a disadvantage compared with birds in the middle ranks for formation bigger than 2 ranks. When we look only at the values of  $e_\mu$  for the leader and the first rank in Fig. 5.9 we see

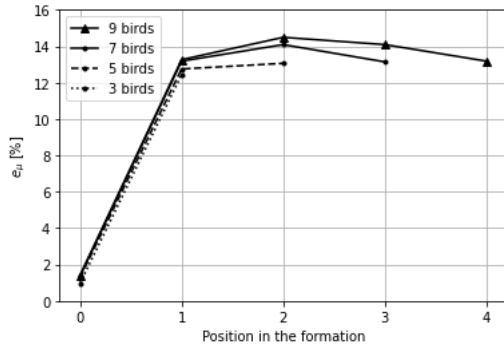


Figure 5.8: Efficiency  $e_\mu$  as a function of position in the V-shaped formation.

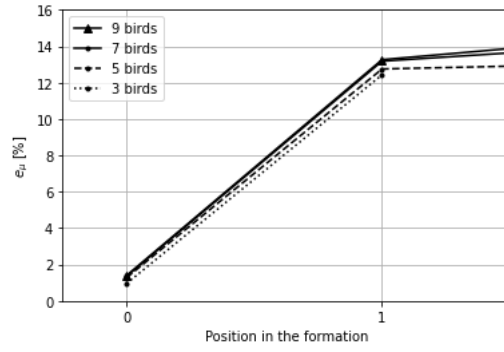


Figure 5.9: Efficiency  $e_\mu$  for the first rank in the V-shaped formation.

that the more birds there are in the formation, the greater the benefit. However, this is with asymptotic behavior as shown in Fig. 5.6. The leader can save between 1.5 [%] compared to flying alone. The same theoretical simulations estimate leader savings of up to 5 [%].

We attempt to explain the non-uniqueness of the  $e_\mu$  gains in a bird formation, as well as the differences from theoretical simulations of elliptical fixed wing. Thus, we will once again use the Kutta-Joukowski theorem and the Biot-Savart law. By applying the theorem to the lifting line of the bird shown in Fig. 5.10 we can see the sign of the  $\Gamma$  circulation on the lifting line. Then, by applying the law to this lifting line with circulation, we notice that upwash is generated upstream of each bird present. This is confirmed in Fig. 4.12, where upwash is shown in red in front of the follower. As a result, upwash is generated not only by the trailing tip vortices in a bird's wake, but also by the bird's lifting line.

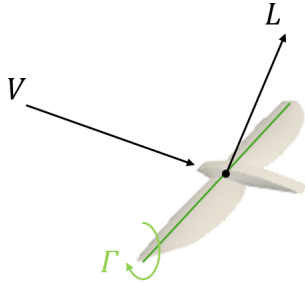


Figure 5.10: Illustration of the Kutta-Joukowski theorem on the bird's lifting line.

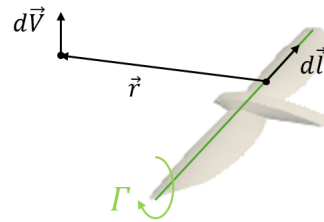


Figure 5.11: Illustration of the Biot-Savart law on the bird's lifting line.

Let's take the case of the bird in last position in a formation of nine birds. He benefits from the upwash generated by the trailing tip vortices of the birds ahead of him. The bird in penultimate position benefits from the upwash generated by the last bird, and therefore has a higher benefit. As illustrated in Fig. 5.6 the benefit of adding a downstream bird is noticeable up to two ranks downstream. As a result, the bird in the second rank benefits from the upwash of the last and second-to-last, making its position energetically more favorable.



To explain the difference in  $e_\mu$  values between the rigid elliptical wing simulations and our own, we use the same reasoning for the upwash generated by the lifting line. In fact, the leader does not benefit from any trailing tip vortices from upstream birds, since it is the first in the formation. Consequently, the benefit he perceives seems to come from the presence of upwash-producing downstream followers. For numerical simulations using an horseshoe vortex representation, lift is assumed to be constant as a function of time. Consequently, the upwash generated by the lifting line is also constant as a function of time. Furthermore, for straight vortex filaments, upwash evolves in  $1/h$  with  $h$  the distance orthogonal to the trailing tip vortices or lifting line, as demonstrated in *Appendix E*. In these simulations, the upwash generated by the lifting line is just as important as that produced by the trailing tip vortices. In contrast, in our simulations, lift changes as a function of time, so the upwash produced by lifting is not constant and is even downwash, since the bird's upstroke at some point produces negative lift. Since the influence of downstream bird upwash is overestimated by the fixed-wing hypothesis, this explains the 5 [%] difference in leader profit in their simulation versus 1.5 [%] in ours.

### 5.2.3 Comparison with simulated aircraft formations

Engineers understood the energetic benefits of formation flying and began to apply them to aircraft flight. Initially, formation flying was only used in military applications. Today, the principle is being studied for the formation flight of commercial aircraft, to reduce fuel consumption and  $CO_2$  emissions.

The first formation flight on a commercial aircraft was carried out by Airbus in 2021 on a flight from Toulouse to Montreal [14]. Fuel consumption was reduced by 5 [%] despite landing and take-off, and without being at an optimum position in the wake either. We define the thermal efficiency of a turbofan engine as the ratio between the change in kinetic energy required to move the aircraft forward and the energy input from fuel combustion [76]. If we assume that thermal efficiency is constant for the aircraft in flight alone and for the aircraft in formation, a 5 [%] decrease in the energy input to combustion induces a 5 [%] decrease in the forces required to move the aircraft forward. There are other simulations of optimal routing for formation flight of commercial aircraft [37,38,40,75,77]. In [37], the author established a fixed-wing model with a lift distribution for airliners such as the Boeing 727-200. He also took into account different flight missions, including take-off, level flight and landing. The study then simulated different routes linking certain cities in the USA, without taking into account the impact of wind. For a two-rank formation flight, the first rank at  $y/b = 0.9$  saved 13 [%] of fuel, compared with 13.5 [%] for the second rank at  $y/b = 1.8$ . In [75], the study refers to the experience of a one-rank formation on a transatlantic flight which saved up to 10 [%] in fuel. The aim of this study was to investigate the uncertainty associated with take-off time and fuel savings. In [38], the study established a method for taking wind into account, and found a saving of 8.6 [%] for the follower of a single-rank flight.

In Fig. 5.8 for our simulations in the case of five birds, the last was the best placed energetically with 13 [%] in  $y/b = 1.78$  and the first rank in  $y/b = 0.89$  saved 12.75 [%]. These values are close to those of simulated aircraft formation flight. However, these simulations take into account take-off and landing, whereas our simulations are based on level flight. In [77], Dr Ignace Ransquin estimates the saving in fuel burned compared with flying alone in a level flight at 22 [%]. Aircraft in formation, once stabilized at a given point, benefit from constant upwash on their wing. In the case of birds, given the nature of their wake, which follows the flapping of their wings, followers do not perceive a constant upwash. In addition, the oscillations of the wake induce a decay more important. As a result, aircraft formation flying appears to be more efficient than that of birds.



### 5.2.4 Influence of constraints

We then study the influence of the constraints on the position and phase shift determined by the energy minimization problem of the numerical birds present in formation. When we remove the collision constraint, the results are perfectly identical. This can be understood by looking at the birds in Fig. 5.1, which are far enough apart at equilibrium.

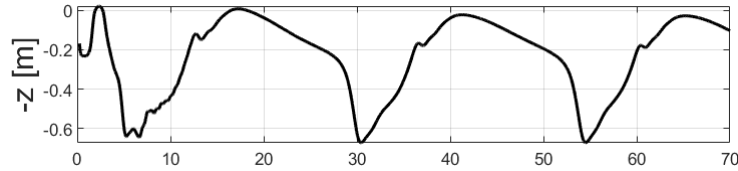


Figure 5.12: Oscillatory behavior of the  $-z$  coordinate of rank 3 in a V-shaped formation of seven selfish birds.

On the other hand, when we remove the vision constraint linked to the bird's comfort of having the upstream bird close to its axis of vision, the formation becomes unstable. For formations of less than three ranks, stabilization is possible. As we can see from Fig. 5.4, birds in the  $xy$  position are close to the observed values. Similarly in Fig. 5.5 the phase shift relative to the leader for each bird is well linear with their streamwise  $x$  position. However, for the birds in the third rank, their spanwise distance from the second rank is  $0.92 < 1.20$ , which positions the bird further inside the formation. Since it is further in, its distance from the trailing tip vortex of the upstream bird is smaller, inducing aerodynamic forces that destabilize it. The controller is unable to stabilize these third-rank birds, as shown in Fig. 5.12. As the bird tries to stabilize itself, it has to exert more effort and its power increases, as shown in Fig. 5.13. Since the bird is not stable, it is not possible to add another rank downstream of its position.

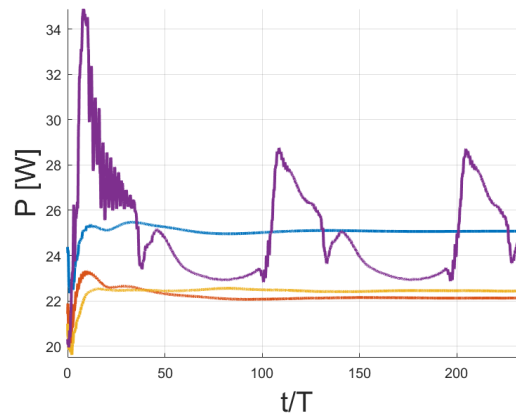


Figure 5.13: Power versus time for a V-shaped formation of seven selfish birds. Followers are those of the starboard branch. Blue: Leader. Red: Follower 1. Orange: Follower 2. Purple: Follower 3.

We note a limitation of the model. In this case, the energy minimization model determined that the position of the third-rank follower was energetically favorable. However, in the simulations, this bird has a higher power as it seeks to stabilize itself.

## 5. Formation flight analysis

The limitation is that the model does not take into account the imbalances that the bird might experience at the location determined by the model. In addition, the numerical birds in the simulations, unlike the ibis, have no tail. However, we saw in Chapter 2 that birds can use their tails to stabilize themselves in the face of disturbances. To avoid this unstable behavior, the model could be improved by taking the bird's tail into account, both in terms of control and wake impact.

### 5.3 Echelon flight

We now turn to the simulation results for echelon formation flight. To do this, we proceed analogically with V-shaped formation flight. We position a leader at  $(x, y, z) = (0, 0, 0)$  [m]. Once stabilized, we add a rank of a single bird at  $y > 0$  whose coordinates and phase shifts have been determined by the energy minimization problem developed. We repeat the process to place the ranks one after the other. The result for an echelon flight with five individuals, including a leader, and four ranks of one bird is shown in Fig. 5.14

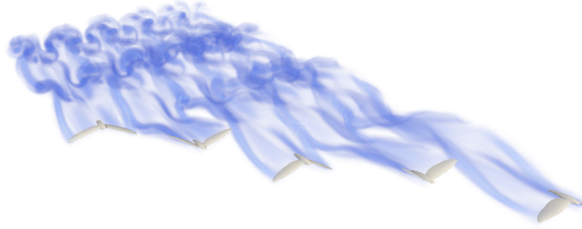


Figure 5.14: Volume rendering of the vorticity magnitude for the wake of an echelon formation of five selfish birds.

## 5.4 Discussions

### 5.4.1 Comparisons with observations

We begin by comparing the results with the observations of Steven Portugal [24], looking at Fig. 5.15. In this figure, the constrained results of the echelon simulation coincide with the observations in the  $xy$  plane. When compared with the spanwise direction  $y$  shown in Table 5.1, the V-shaped formation has a slightly greater variation  $y$  between ranks of the order of 1 [cm] than the echelon flight. Also in this table, V-shaped flight birds are slightly higher in  $z$  than those in echelon flight. This is because, for the first few ranks of the V-shaped formation, the two branches are close together, and their trailing tip vortices interact with each other. For the port branch, for example, the velocity induced by the port wing of the upstream rank will compensate for the velocity induced by the starboard wing of the downstream rank. As a result, the downwash pushing trailing tip vortices down will be created by those furthest away. As they are further away, their downwash is weaker, which tends the vortices downwards less than in the case of the echelon. As a result, the vertical positions of echelon flight are lower than those of V-shaped flight. However, if the rank is far enough into the V-shaped formation, it will be referred to as an echelon formation. Since interaction between the two branches tends towards zero.

We note a limitation in our model. Indeed, still on the Table 5.1 we can see that the position of the first follower is identical for the V-shaped and the echelon formation. Since it is on the basis of the leader's wake that we place one or two birds to form the first rank. However, for the first rank, the two branches of the V-shaped flight are close

together, and the bird on the other branch could induce a variation for the upwash. Then, for the other ranks, this limitation is less and less present as the two branches move further and further apart. Note that this limitation is also not present in the case of echelon flight, since there is only one branch.

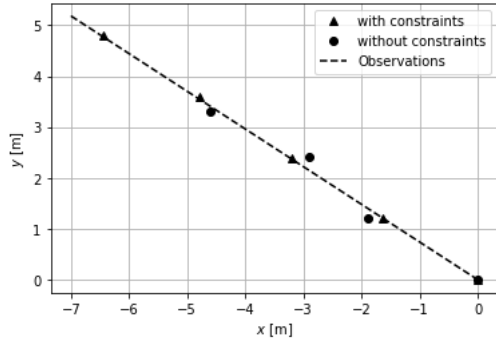


Figure 5.15: Top view of the echelon formation level in the  $xy$  plane for the case with and without constraints.

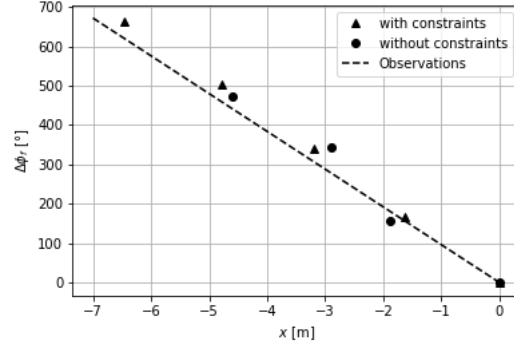


Figure 5.16: Representation of the  $\Delta\phi_f$  relationship as a function of the streamwise position  $x$  in the case with and without constraints for an echelon flight.

The relationship between the phase shift relative to the leader and the streamwise position shown in Fig. 5.16 is also linear, like the V-shaped formation and as observed in [24,29].

#### 5.4.2 Average follower power

To compare power averaged over an equilibrium flapping period between the V-shaped formation and the echelon, we will compare Figs. 5.6 and 5.7. When we compare the power evolution of the leader and the first follower rank, we see that for V-shaped flight, the power decrease stabilizes after the addition of two ranks downstream. It is only one rank for echelon flight. In V-shaped flight, the first ranks benefit from the upwash generated by the lifting line of the downstream bird, as well as from birds on the other branch at a relatively close distance. Conversely, in an echelon flight, there is no second branch. Therefore the first ranks only benefit from the upwash generated by the lifting line of the downstream birds present on this same branch. This reduces the amount of upwash received. Thus this reduces also the influence on energy expenditure within the formation when adding ranks. As illustrated, for example, by the leader's  $e_\mu$ , which was above 1 [%] in Fig. 5.9 and is below in Fig. 5.18. Since the leader has two birds behind it in the case of V-shaped formation flight and only one for echelon flight.

For a four-rank formation, such as a V-shaped flight, the ideal position is in the middle of the second rank with  $e_\mu \approx 13.7$  [%]. However, this value is lower than that for the same rank and configuration as for a V-shaped flight, which is over 14 [%]. We also note that for an echelon flight of three birds, the last rank is the most advantaged, but not by much compared with the first rank.

#### 5.4.3 Influence of constraints

The influence of discomfort constraints is identical to that of V-shaped formation flight. Indeed, removing the collision constraint does not change the results. However, when the visual comfort constraint is removed, the results are slightly different but still close to the observations in Figs. 5.15 and 5.16 for formations of less than three ranks. As discussed

## 5. Formation flight analysis

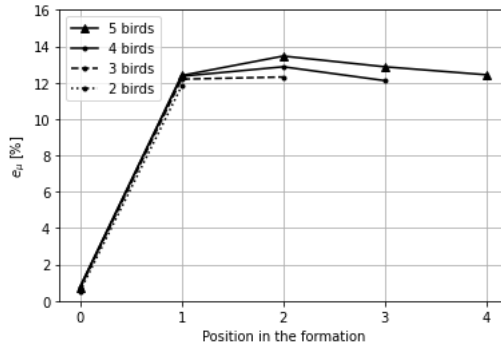


Figure 5.17: Efficiency  $e_\mu$  as a function of position in the echelon formation.

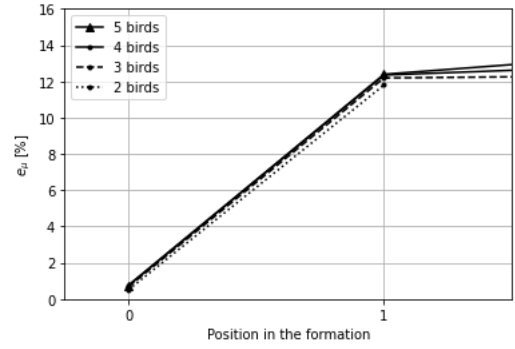


Figure 5.18: Efficiency  $e_\mu$  for the first rank in the echelon formation.

earlier, adding a fourth rank is made impossible because the bird in the third rank is unable to stabilize. The limitation of the model is also present here. Since for a position that is supposed to give a position and phase shift minimizing its average power during a flapping period. Once the numerical bird is in the wake, it fails to stabilize and therefore consumes more energy.

It should be remembered that visual constraint has clearly been observed as a factor in the stable, formation flight of migratory birds [49,62,63]. One reason is birds' visual resolution is best around their axis of vision [65]. Therefore, in order to keep the bird upstream in its axis of vision, its flight should be constrained.

### 5.5 Impact of bird attitude on the formation shape

For our simulations, we treated the case where the birds present in the formation were all selfish. In Chapter 3, we discussed the possibility of birds becoming cooperative. In other words, a bird that seeks to adapt its position and phase shift so as to minimize the energy expenditure of the whole group. To achieve this, we need to assume that the birds are able to communicate, or at least become aware of each other's efforts in the formation.

In [39], the author establishes a model based on a fixed-wing representation for numerical birds, with the aim of minimizing the energy expenditure of the entire group. To achieve this, the simulation begins with an arbitrary V-shaped formation as shown in Fig. 5.19. At this initial moment, the distribution of the reduction in energy expenditure  $e_\mu$  is given in Fig. 5.20. Note that this curve shape is similar to the formation flight of our simulations shown in Fig. 5.8. Next, the model converges on an optimal formation reminiscent of the U-shaped formation also observed in nature. As we can see in Fig. 5.20, this U-shaped formation is characterized by an identical  $e_\mu$  for all the birds in the formation. This looks like a cooperative behavior in which each bird makes the same effort.

We compare the results of a V-shaped formation and U-shaped formation flight for the relative  $e_\mu$  saving compared to flight alone. We notice that apart from the first rank of birds and the leader, all other birds have a reduced energy saving for U-shaped formation. This result of monicity of energy gain in formation is also presented in other bird simulations represented by fixed wings [41,43] with  $e_\mu$  greater than 10 [%].

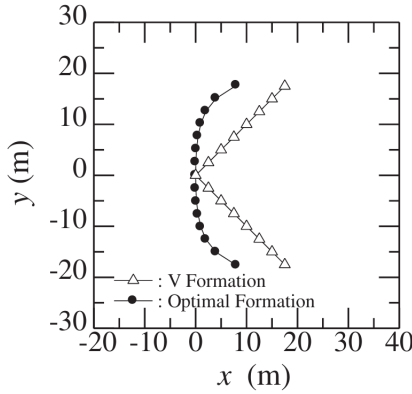


Figure 5.19: Numerical simulation of a V-shaped (black circle) and U-shaped (white triangle) formation flight. (from [39])

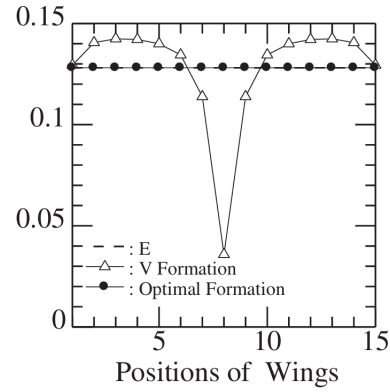


Figure 5.20: Efficiency  $e_\mu$  as a function of fixed wing position for V-shaped formation (white triangle) and U-shaped formation (black circle). (from [39])

A physical interpretation behind this phenomenon can be explained by the very close distance in the streamwise direction between the birds in this formation. This would allow the followers to always benefit from the upwash created by the trailing tip vortices of the upstream birds. But it would also increase the importance of the upwash produced by the bird's lifting line in the energy minimization problem. Indeed, since the birds are closer according to  $x$  than in the case of a V-shaped formation flight, the leader, for example, receives more upwash from the rank behind him. So much so that his energy gain is identical to that of the ranks behind it, which benefit from the upwash of the trailing tip vortices.

A question might arise as to why V-shaped formation flying exists if U-shaped seems to benefit formation the most. Firstly, we do not know whether in-flight communication of the effort they produce is possible or not. Secondly, we do not really know whether a migratory bird is more selfish, cooperative or empathetic. Finally, since U-shaped flight is optimal for each individual, no bird would want to change position to gain an advantage. However, given that one wing is working harder than the other as it benefits from the upwash, the bird may experience muscular discomfort or fatigue. That could prompt it to change position to spread the effort over both wings. This brings us to the notion of changing position.

In [39], the authors discuss the results of U-shaped flight and turn their attention to V-shaped flight with leader relay. Since in V-shaped flight, the birds in our simulations are selfish, the leader would also like to take advantage of the wake of the other birds. This could disrupt the stability of the formation, but also instigate a relay to change leader. As far as echelon flight is concerned, this would be considered as a transition formation, as explained in [47], allowing a relay for V-shaped flight. Furthermore, in [39], the relays carried out numerically in the simulations pass through the echelon formation to change leader. At the end of the simulation, when each bird has been leader for the same period of time, the average  $e_\mu$  benefit is identical for each individual. This result is exactly as U-shaped formation. However, this benefit is lower than that shown in Fig. 5.20 for U-shaped. The mechanism of relays and relay passages remains poorly studied and understood.

One of the possible perspectives to add to the model described in this master's thesis would be a behavior of collaboration between birds or empathy as detailed in Chapter 3. This will aim to confirm or refute the results of [39] for simulations of cooperative

## 5. Formation flight analysis

---

numerical birds with no fixed wing representation. In this way, we will be able to see whether a cooperative bird allows U-shaped formation. We will also be able to compare the value of the energy gains of each bird present in the formation with [39].

## Chapter 6

# Conclusions and perspectives

### 6.1 Contributions

The aim of this work is to understand the underlying mechanism of bird formation flight dynamics, by proposing a model that tells the numerical birds in a simulation where to position themselves. As we said, no scientific article exists on the simulation of birds flapping their wings in formation. Those that do simulate formation flight are based on the assumption that the bird does not flap its wings, *i.e.* has a fixed wing, and assume that their theoretical results are representative of reality. These theoretical models are based also on the recovery of upwash present in the wake of a formation. However, we have seen that the wake of a flapping wing is far more complex than that of a fixed wing. The purpose of this work is therefore to couple a high-fidelity aerodynamics solver and a multi-agent formalism for the simulation of bird formation flight dynamics, with subsequent comparison with observations of migratory birds in formation.

The first chapter underlined the importance of biomimicry in the world of science and engineering in particular. This chapter also set the ecological and economic context for the formation flight of commercial aircraft. These commercial formation flights are directly inspired by the formation flights of birds. Migratory birds formation observations underline a reduction in the energetic effort of the individuals present within it. It also highlighted the gaps in our understanding of bird formation flight. With regard to Chapter 2 it discusses the tools available to simulate a numerical bird, using the VPM method to simulate its wake and Dr Victor Colognesi's bio-mechanical model [15] applied to the northern bald ibis.

We can now answer the questions posed in Chapter 1.

**Question 1:** How can we establish a model for bird formation flight?

- 1.i. : What effects do birds benefit from to reduce their energy expenditure?
- 1.ii. : Which bird control variables should be taken into account in the model?
- 1.iii. : What are the constraints to be taken into account when a bird flies in a group?

The model explained in Chapter 3 is a model developed on the basis of numerous observations of migratory birds known to fly in formation. These observations explain the impact of the desire to save energy by creating a formation, avoiding collisions and maintaining visibility of the bird ahead. The model is therefore constrained by a comfort level linked to the distance between bird and vision. To reduce the energy expenditure of the birds, we base our model on an approximation of the power saved by the upwash present in a wake. Indeed, upwash changes the angle of attack perceived by the bird's wings, inducing a positive streamwise component for lift. This results in free thrust. So, based on this approximation, the bird will provide less energy when in formation for similar flight performance to flying alone. Still based on observation, we identified the parameters of the minimization problem as being the streamwise position  $x$ , the spanwise position  $y$ , the position according to  $z$  and the phase shift  $\Delta\phi_f$  with the upstream bird. This model, based on an approximation of the power saved for each bird, enables us to

determine where to position a new rank behind existing ranks. Once this position has been determined, we need to move the birds to this optimum position with the optimum phase shift. We also need to stabilize the birds at a given position and phase shift, despite the destabilizing effects present in the wake. Chapter 4 explains the phase shift and position controllers implemented for each bird.

**Question 2:** What is the impact on the energy consumed during a formation flight for a first-rank or upper-rank bird?

- 2.i. : What is the difference in power output between a leader and a follower?
- 2.ii. : Do all followers have a similar reduction in energy expenditure? Is it better to be at the end of the formation or at the beginning?
- 2.iii. : Is bird formation more efficient than airplane formation in terms of energy savings achieved?

We saw in Chapter 5 that for a selfish bird formation flight, this results in a V-shaped formation. From our simulations, we have a saving of 11 to 14 [%] in energy compared with flight alone. However, the energy gains are not evenly distributed between the formation ranks. For small formations of less than two ranks, the ideal position is last. For larger formations, the ideal position is in the middle rank. As explained in Chapter 5, birds benefit from the upwash induced by the trailing tip vortices of the upstream bird, but also from the upwash induced by the lifting line. This means that the bird positioned in the middle benefits from the upwash provided by the upstream bird's trailing tip vortices, but also from the upwash provided by the downstream bird lifting line. Ultimately, dissipation effects are greater in the oscillating wake of a flapping bird. Furthermore, unlike aircraft flight, the upwash produced by the wings is not constant. As a result, flying in an aircraft formation appears to be more efficient, and can achieve savings of up to 22 [%] compared with flying alone.

**Question 3:** Does energy minimization alone justify the shape and stability of formation flight numerically?

- 3.i. : Are the bird's variables at optimum always similar if the constraints are not respected? What about the stability of group flight in this case?
- 3.ii. : What are followers positions, phase shifts and energy savings in observations versus numerically?
- 3.iii. : How does the distribution of effort within the formation change according to the bird's attitude?

In Chapter 5, we hypothesize that the bird's attitude can change the shape of the formation. Indeed, whether the bird is selfish or cooperative, this could result in either a V-shaped formation or a U-shaped formation at equilibrium. This also has an impact on the distribution of gains, which are evenly distributed in a cooperative bird flight but not in a selfish one. The results are similar to the observations. We also find the linearity relationships between streamwise position and phase shift, as well as the linear relationship between  $x$  and  $y$ . When we study the impact of constraints on formation flight structure, we find an unstable formation when the visual comfort constraint is removed. However, this is not a realistic case, since visual comfort has been demonstrated in numerous observations.



**Question 4:** What happens to the wake structure once the birds are in flight formation?

- 4.i. Where is the bird positioned in this wake and how does it adapt its phase shift to the upstream bird?
- 4.ii. What are the differences between a flapping wing wake and that of a fixed wing?

As mentioned in Chapter 3, the wake of a flapping bird is more complex than that of a fixed wing. The bird does not produce a constant lift over a flapping period. Moreover, it even produces negative lift during upstroke. This has the effect of changing the circulation of its wings, and thus impacting the velocity field in the wake. The wake of a fixed-wing aircraft is called “frozen wake”, since it does not vary with time because the lift creation of the wing is constant. In contrast, a bird’s wake is periodic. Finally, our numerical observations show that the bird positions itself so that when it downstrokes, it can position the tip of its wing as close as possible to the trailing tip vortex produced by the downstroke of the upstream bird.

## 6.2 Discussions on future possibilities

### 6.2.1 Limitations

The model developed in this master’s thesis has a few limitations which we are taking into consideration for future improvement possibilities.

**Position of first rank** As mentioned in Chapter 5, the position of the first rank behind the leader is identical for V-shaped and echelon flight. Indeed, the model positions the first rank on the basis of the leader’s wake to position one bird in the case of echelon or two birds in the case of V-shaped. Consequently, the model does not take into account the interaction between the two birds in the same rank for the V-shaped. As the two branches are close together, the ideal position could be different. In fact, when the second rank is positioned for the V-shaped, the model positions it based on the wake of the two upstream branches. The position of the second rank is different from that of the echelon.

**Positions determined rank by rank** A second limitation linked to the first is that the positions once determined by the model for each rank are fixed. Then, each bird’s controller allows it to stabilize at these positions. An improvement to the model could be to make it more dynamic. By more dynamic, we mean using the model at each time step to analyze the wake over a beat period, and using the model to determine the optimal location for positioning the bird. If the position and phase shift are different from the current values, these are modified as new controller targets. The bird will move, and for each subsequent time step, it will deduce from the model whether it needs to change its targets or not. In this way, the model will be more dynamic. This will also avoid the first limitation, as it will take into account the impact of the presence of the two branches when searching for the ideal position of the first rank.

**Potential imbalance of the numerical bird at optimum** We noted in Chapter 5 that when the vision constraint is removed, the position determined by the model is not stable in the wake. This means that the model intended to determine the position that minimizes the power consumed during a flapping period does not take into account the aerodynamic forces that could destabilize the bird once in this position. As a reminder, the model is based on an approximation of the power

saved by the bird to maintain a constant speed by recovering the kinetic energy present in the wake of other birds. This is only an approximation, and does not take into account the destabilizing forces acting on the bird. One solution would be to implement a penalty function for locations in space with a large variation in velocity over time, or a large velocity gradient. Thus it represents areas where the destabilizing forces are of greater intensity. Another solution could be to directly modify the position controller to make it more robust to disturbances. We saw in Chapter 2 how a bird and an aircraft can react to disturbances by biomimicking. The bio-mechanical model in [15] has no tail. We have seen that the tail plays an important role in stabilizing birds. It would therefore be interesting to add a tail to the bio-mechanical model, so that it can be taken into account in the position controller to make it more robust.

### 6.2.2 Future possibilities to investigate bird formation flights

There are also opportunities for further study by adding enhancements to the models.

**Atmospheric turbulence** In Chapter 3, we assumed that there were no crosswinds, turbulence, updrafts or downdrafts. As an improvement to the model, it would be interesting to study the impact of these disturbances on the dynamics of flight in bird formation. In particular, we could analyze the stability of the formation and the difference in energy gains compared to the case with clean wind upstream.

**Birds' attitudes** : Future upgrade possibilities are also possible, not least with regard to the bird's attitude. We saw in the last section of Chapter 5 that flight with cooperative birds results in U-shaped flight. Results for fixed wing simulations showed that this formation shape would give a uniform distribution of energy gains. One possibility would therefore be to implement the cooperative attitude discussed in Chapter 3 to confirm or refute these numerical observations.

**Gliding effect** Other observations indicate that migratory birds, such as the northern bald ibis, may alternate periods of gliding and wing-flapping to save even more energy. We could study the impact of an arbitrary or non-arbitrary gliding time initiated just after the mid-downstroke. The wake would be different, and followers would have to adapt their phase shift and gliding time to try and stay as close as possible to the trailing tip vortices. Such assumptions should be confirmed or disproved by taking the gliding effect into account in the model.

**Influence of numerical bird characteristics** The model developed is modular in the sense that the birds can be any birds. The only constraint is that the aspect ratio must be large enough to guarantee consistent results from the aerodynamic solver. Observations indicate that formations can be composed of birds of different mass, size and species. We could therefore study the impact of these characteristics on the energy gains of birds in the formation. For example, in formations with juveniles, it was observed that young birds were always behind adults. This could be explained by the fact that an adult generates more upwash in its wake than a juvenile. Consequently, a juvenile behind an adult will see a greater reduction in energy. All this remains to be confirmed or invalidated by numerical simulations.

### 6.2.3 Applications in other fields

The concepts developed are not necessarily applicable only to flight simulation in bird formation. There are other fields in aeronautics where they could be applied, as listed as follows, and whose results could be studied.

**Aircraft formation flights** Engineers have designed modern aircraft using bird biomimicry. It is therefore possible to do the same by applying the model developed for aircraft formation flight. Indeed, the aircraft can be considered as a bird with fixed wings, whose phase shift with the aircraft in the rank ahead is zero. Therefore, by numerically simulating the wake of an aircraft, the model may be able to position a second rank downstream. We can iterate the process to position several ranks. One of the main challenges of commercial formation flying remains safety and the risk of collision due to the proximity between aircraft. As far as collision constraints are concerned, the risk of collision zone can be determined on the basis of minimum distance standards between aircraft. Or, it could be on the basis of studies evoking the risks associated with a distance close to another aircraft. Vision constraints may or may not be adapted. The vision axis would therefore be that of the pilots, parallel to the streamwise axis. These models could therefore be applied to flight simulation in commercial aircraft formation. These aircraft formation studies could be made to meet the challenge of reducing  $CO_2$  emissions in the aeronautical transport sector.

**Formation flight investigation of rotary wing vehicles** The model has been developed in particular for wings moving periodically in space. It could therefore also be adapted for rotating-wing vehicles such as helicopters. Studies show that helicopters produce upwash which could be used for helicopter formation flying. To do this, the model will have to be adapted to respect the geometry of the problem, and the constraints will be linked to safety to avoid collision. Finally, the controllers will have to be adapted to be robust to helicopter flight.

**Flapping-wing drone** Flapping-wing drones are new inventions whose primary purpose is observation, thanks to the cameras they incorporate. In the field of public security, the French Ministry of the Armed Forces has published articles on this type of drone in its Defense Innovation Agency [78]. They state that the strong point of flapping-wing drones is their ability to perform flapping, gliding and quasi-stationary flights, which fixed wing unmanned aerial vehicles (UAVs) cannot do. In the field of research, these drones are used to observe animals. In fact, their stealth capabilities enable them to get as close as possible to the animal species under study. Our model is perfectly suited to the formation flight of these drones. This would not only save energy, but also make them appear even more real, thus enhancing their stealth capabilities. This could also encourage migratory birds to join the formation, increasing the potential for observing the dynamics of bird formation flight.

To conclude this master's thesis, we emphasize that many questions remain unanswered. The study of the bird formation flight dynamics is far from being completely covered. As mentioned, many gray areas persist, particularly regarding the attitude of the birds and their sensory ability to perceive the aerodynamic effects present in the wake. However, each additional study adds another piece to the puzzle. The work presented in this document thus opens up a variety of possibilities for model improvement, as well as useful applications for humans.



# Bibliography

- [1] Jangsun Hwang, Yoon Jeong, Jeong Min Park, Kwan Hong Lee, Jong Wook Hong and Jonghoon Choi (2015) Biomimetics: forecasting the future of science, engineering, and medicine. *International Journal of Nanomedicine*, 5701-5713, 10.2147/IJN.S83642.
- [2] Aanzeraey (2018) Le Biomimétisme. *Le Blog d'Histoire de l'Art des AI2*.
- [3] Snaves (2018) Biomimetic Architecture: Sagrada Familia. *Steemit*.
- [4] Catherine Picart (2023) Quand la médecine régénératrice imite la nature. *Polytechnique Insights*.
- [5] Unknown (2023) Medical Biomimetics Market Size To Hit USD 65.73 Bn By 2032. *Precedence Research*.
- [6] Valentin Hisler, Marie Palmieri, Vincent Le Houerou, Christian Gauthier, Michel Nardin, and al. (2013) Contacts adhésifs sur surfaces texturées d'élastomères: effet d'échelle. CFM 2013- 21ème Congrès Français de Mécanique, Bordeaux, France. hal-03439961.
- [7] Renaud Ronsse (2018) LMECA2755—Industrial automation Introduction to (Industrial) Robotics. *Ecole Polytechnique de Louvain*.
- [8] Domel AG, Saadat M, Weaver JC, Haj-Hariri H, Bertoldi K, Lauder GV. (2018) Shark skin-inspired designs that improve aerodynamic performance. *J. R. Soc. Interface* 15: 20170828.
- [9] Saint-Paul and Pradels (2017) Le biomimétisme dans l'aviation. *Lycée Louise Michel Narbonne*.
- [10] Unknown (2024) Animaux accueillis et relâchés. *Hegaldia, Association départementale pour la sauvegarde de la faune sauvage*.
- [11] V. V. Gavrilov (2013) Advantages and Disadvantages of Interspecies Associations of Migratory Northern Sandpipers (Charadrii, Aves). *Zhurnal Obshchei Biologii*, 2014, Vol. 75, No. 3, pp. 204–213, 10.1134/S207908641501003X
- [12] International Energy Agency (2023) Net Zero Emissions by 2050 Scenario (NZE). *Global Energy and Climate Model*.
- [13] Dominic Gates (2021) GE and Safran tout new 'open rotor' engine future for sustainable aviation. *The Seattle Times*.
- [14] Gil Bousquet (2021) Airbus réussit le premier vol en formation entre Toulouse et Montréal et économise six tonnes de CO2. *LaDepeche.fr*.
- [15] Colognesi, Victor. (2022) Modeling and simulation of bird flapping flight : control, wakes and formation. Prom. : Chatelain, Philippe ; Ronsse, Renaud <http://hdl.handle.net/2078.1/266563>
- [16] Bastien Munch (2022) Mais quels sont ces oiseaux migrants aperçus par centaines dans le ciel en Lorraine ?. *France Bleu*.

- [17] Chad Merda (2019) Nature curiosity: Why do some birds fly in a V formation?. *The Buzz*.
- [18] Nicolas Balais (2021) Hérons garde-bœuf : le vol blanc des « travailleurs pendulaires » dans le Sud-Ouest lyonnais. *Le Progres*.
- [19] Burak Ergocmen (2019) Fault tolerant flight control applications for a fixed wing UAV using linear and non linear approaches. *ResearchGate*, 10.13140/RG.2.2.25180.03205
- [20] Bernard Etkin and Lloyd Duff Reid (1996) Dynamic of flight - Stability and control. *JOHN WILEY & SONS, INC*, 3rd edition.
- [21] Christina Harveya, Lawren L. Gambleb, Christian R. Bolanderc, Douglas F. Hunsakerc, James J. Jood and Daniel J. Inmana (2022) A review of avian-inspired morphing for UAV flight control. *Progress in Aerospace Sciences*, 10.1016/j.paerosci.2022.100825.
- [22] Philippe Chatelain and Pierre Schrooyen (2022) LMECA2830-Aerospace Dynamics. *Ecole Polytechnique de Louvain*.
- [23] Reiner Gogolin (2022) Migrating northern bald ibis fly in 'V' formation behind microlight. *Why do birds, Birds in flight, Birds flying*.
- [24] Steven J. Portugal, Tatjana Y. Hubel, Johannes Fritz, Stefanie Heese, Daniela Trobe, Bernhard Voelkl, Stephen Hailes<sup>1</sup>, Alan M. Wilson<sup>1</sup> and James R. Usherwood (2014) Upwash exploitation and downwash avoidance by flap phasing in ibis formation flight. *Nature*, 10.1038/nature12939.
- [25] Gianmarco Ducci, Victor Colognesi, Gennaro Vitucci, Philippe Chatelain and Renaud Ronsse (2021) Stability and Sensitivity Analysis of Bird Flapping Flight. *Journal of Nonlinear Science* , 10.1007/s00332-021-09698-1.
- [26] Victor Colognesi, Renaud Ronsse and Philippe Chatelain (2020) Model coupling biomechanics and fluid dynamics for the simulation of controlled flapping flight. *Bioinspirations and Biomimetics*, 10.1088/1748-3190/abdd9c.
- [27] Dan Liu, Bifeng Song, Wenqing Yang, Xiaojun Yang, Dong Xue and Xinyu Lang<sup>1</sup> (2021) A Brief Review on Aerodynamic Performance of Wingtip Slots and Research Prospect, *Journal of Bionic Engineering*, 10.1007/s42235-021-00116-6.
- [28] Nicolas Docquier, Antoine Poncelet, and Paul Fiset. Robotran: a powerful symbolic generator of multibody models.
- [29] Steven J. Portugal, Tatjana Y. Hubel, Johannes Fritz, Stefanie Heese, Daniela Trobe, Bernhard Voelkl, Stephen Hailes<sup>1</sup>, Alan M. Wilson<sup>1</sup> and James R. Usherwood (2014) Supplementary information. *Nature*, 10.1038/nature12939.
- [30] Meng Wang (2023) AME40532-Computational Fluid Dynamics. *University of Notre Dame*.
- [31] Gavin K. Ananda and Michael S. Selig (2018) Design of Bird-Like Airfoils. *American Institute of Aeronautics and Astronautics*, 10.2514/6.2018-0310.
- [32] Philippe Chatelain and Petros Koumoutsakos (2009) A Fourier-based elliptic solver for vortical flows with periodic and unbounded directions. *Journal of Computational Physics*, 10.1016/j.jcp.2009.12.035.
- [33] Denis-Gabriel Caprace, Gregoire Winckelmans and Philippe Chatelain (2019) An Immersed Lifting and Dragging Line Model for the Vortex Particle-Mesh Method. *Theoretical and Computational Fluid Dynamics*, 10.1007/s00162-019-00510-1.

- 
- [34] Grégoire Winckelmans (2015) LMECA2322- Fluid Mechanics II. *Ecole Polytechnique de Louvain*.
  - [35] Philippe Chatelain and Grégoire Winckelmans (2023) LMECA2323-Aerodynamics of external flows. *Ecole Polytechnique de Louvain*.
  - [36] Snorri Gudmundsson (2012) To present the lifting line theory of Prandtl and Lanchester for a finite-span wing. *Fluid Mechanics*, 5th edition.
  - [37] Geoffrey C. Bower, Tristan C. Flanzer and Ilan M. Kroo (2009) Formation Geometries and Route Optimization for Commercial Formation Flight. *American Institute of Aeronautics and Astronautics*, AIAA 2009-3615.
  - [38] Thomas E. Kent and Dr. Arthur G. Richards (2013) On Optimal Routing for Commercial Formation Flight. *American Institute of Aeronautics and Astronautics*, 10.2514/6.2013-4889.
  - [39] Hiroyasu Kawabe (2007) A Study on Optimal Pattern and Leader Shift of Formation Flight. *The Japan Society for Aeronautical and Space Sciences*, Vol. 50, No. 168, pp. 134–140.
  - [40] Ning, A., Flanzer, T., and Kroo, I. (2011) Aerodynamic Performance of Extended Formation Flight. *Journal of Aircraft*, Vol. 48, No. 3, pp. 855–865, 10.2514/1.C031046.
  - [41] Federico S. Cattivelli and Ali H. Sayed (2011) Modeling Bird Flight Formations Using Diffusion Adaptation. *IEEE TRANSACTIONS ON SIGNAL PROCESSING*, VOL.59,NO. 5.
  - [42] W.J. McCroskey (1996) The Characterisation & Modification of Wakes from Lifting Vehicles in Fluids. *Advisory group for aerospace research & development*, AGARD-CP-584.
  - [43] D. Hummel (1982) Aerodynamic Aspects of Formation Flight in Birds. *Academic Press Inc. (London) Ltd*, 00225193/83/190321.
  - [44] H.P.Thien, M.A.Moelyadi, and H. Muhammad (2007) Effects of Leader’s Position and Shape on Aerodynamic Performances of V Flight Formation. *ICIUS*, ICIUS2007-A008.
  - [45] Pete Seiler, Aniruddha Pant, and Karl Hedrick (2002) Analysis of bird formations. *Conference on Decision and Control*, TuA04-4.
  - [46] Taylor LA, Taylor GK, Lambert B, Walker JA, Biro D, Portugal SJ (2019) Birds invest wingbeats to keep a steady head and reap the ultimate benefits of flying together. *PLoS Biol* 17 (6): e3000299, 10.1371/journal.pbio.3000299.
  - [47] Alma Streefkerk (2009) On clusters and formations: birds flying in groups. *Animal Ecology - University of Groningen*.
  - [48] Mingming Shi, Ignace Ransquin, Philippe Chatelain and Julien Hendrickx (2023) Whose energy cost would birds like to save? a revisit of the migratory formation flight, 10.1101/2023.03.17.533072.
  - [49] Frank Heppner, Jeffrey Convissar, Dennis Moonan and John Anderso (1984) Visual Angle and Formation Flight in Canada Geese (*Branta canadensis*), p.195-198.
  - [50] Malte Andersson and Johan Wallander (2004) Kin selection and reciprocity in flight formation. *Behavioral Ecology*, Vol. 15 No. 1: 158–162, 10.1093/bheco/arg109.
  - [51] Anders Hedenström (1995) Optimal Flight Speed of Birds. *Philosophical Transactions*, 10.1098/rstb.1995.0082.



- [52] D. Hummel (1995) Formation Flight as an energy-saving mechanism. *Israel Journal of Ecology and Evolution*, Vo 41. p261-269.
- [53] C. J. Cutts and J. R. Speakman (1994) Energy savings in formation flight of pink-footed geese. *The Company of Biologists*, J. exp. Biol. 189, p.251–261.
- [54] John P. Badgerow (1988) An Analysis of Function in the Formation Flight of Canada Geese. *Oxford University Press*, The Auk , Oct., 1988, Vol. 105, No. 4 (Oct., 1988), pp. 749-755.
- [55] Steven Portugal (2016) Lissaman, Shollenberger and formation flight in birds. *Journal of Experimental Biology*, 10.1242/jeb.148114.
- [56] David Sumpter, Camille Buhl, Dora Biro and Iain Couzin (2007) Information transfer in moving animal groups. *Theory Biosci*, 10.1007/s12064-008-0040-1.
- [57] Iain D. Couzin and Jens Krause (2003) Self-Organization and Collective Behavior in Vertebrates. *ADVANCES IN THE STUDY OF BEHAVIOR*, VOL. 32.
- [58] Peter Friederici (2009) How a Flock of Birds Can Fly and Move Together. *AUDUBON*.
- [59] Ian Cooper, Computation of two-dimensional integrals: double of surface integrals. *School of Physics, University of Sydney*.
- [60] Vincent Legat (2020) LEPL1104-Méthodes numériques. *Ecole Polytechnique de Louvain*.
- [61] Iztok Lebar Bajec and Frank H. Heppner (2009) Organized flight in birds. *Animal Behaviour*, 78(4), 777-789, 10.1016/j.anbehav.2009.07.007.
- [62] Lisa Lofland Gould and Frank Heppner (1974) The Vee Formation of Canada Geese. *The Auk*, Vol. 91, No. 3, pp. 494-506.
- [63] John P. Badgerow (1988) An Analysis of Function in the Formation Flight of Canada Geese, *The Auk*, Vol. 105, No. 4, pp. 749-755.
- [64] Hans G. Wallraff (1978) Social Interrelations Involved in Migratory Orientation of Birds: Possible Contribution of Field Studies. *Oikos*, Vol. 30, No. 2, Current Bird Migration Research. Proceedings of a Symposium at Falsterbo, Sweden, pp. 401-404.
- [65] Esteban Fernandez Juricic (2016) The role of animal sensory perception in behavior-based management. *Conservation Behavior: Applying Behavioral Ecology to Wildlife Conservation and Management*, p.150-175.
- [66] François Glineur (2021) LINMA1702-Modèles et méthodes d'optimisation I, *Ecole Polytechnique de Louvain*.
- [67] Victor Colognesi (2023) Flight controller for stabilization onto a moving target.
- [68] Helena Wehner, Johannes Fritz and Bernhard Voelkl (2022) Soaring and intermittent flap-gliding during migratory flights of Northern Bald Ibis. *Journal of Ornithology*, p.671-681, 10.1007/s10336-022-01978-1.
- [69] Henri Weimerskirch, Julien Martin, Yannick Clerquin, Peggy Alexandre and Sarka Jiraskova (2001) Energy saving in flight formation. *Nature*, Vol 413.
- [70] Ortal Mizrahy-Rewald, Elisa Perinot, Johannes Fritz, Alexei L. Vyssotsk, Leonida Fusani, Bernhard Voelkl and Thomas Ruf (2022) Empirical Evidence for Energy Efficiency Using Intermittent Gliding Flight in Northern Bald Ibises. *Original Research*, 10.3389/fevo.2022.891079.



- [71] Dimitri de Smet, Marc Francaux, Julien Hendrickx, and Michel Verleysen (2016) Heart rate modelling as a potential physical fitness assessment for runners and cyclists. *MLSA @ PKDD/ECML*.
- [72] C. Esterer (2023) Return of the Nothern Bald Ibis.
- [73] F. Reed Hainsworth (1986) Precision and dynamics of positioning by Canada geese flying in formation. *The Company of Biologists*, J. exp. Biol. 128, 445-46.
- [74] Bernhard Voelkl, Steven J. Portugald, Markus Unsölde, James R. Usherwoodd, Alan M. Wilsond, and Johannes Fritz (2014) Matching times of leading and following suggest cooperation through direct reciprocity during V-formation flight in ibis. *PNAS*, vol. 112, no. 7, p.2115–2120.
- [75] Maria Cerezo-Magana, Alberto OLivares and Ernesto Staffetti (2022) A Stochastic Switched Optimal Control Approach to Formation Mission Design for Commercial Aircraft. *IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS*, VOL.58,NO.5, p. 4342-4360.
- [76] Saeed Farokh (2014) Aircraft Propulsion, *Wiley*, 2nd edition.
- [77] Ignace Ransquin, Denis-Gabriel Caprace, Matthieu Duponcheel and Philippe Chatelain (2021) Wake Vortex Detection and Tracking for Aircraft Formation Flight. *American Institute of Aeronautics and Astronautics*, 10.2514/1.G006028.
- [78] Ministère des Armées françaises (2022) BIOFLY : un drone à ailes battantes à découvrir à La Fabrique Défense, <https://archives.defense.gouv.fr/aid/actualites/biofly-le-drone-a-ailes-battantes.html>
- [79] Li Jianbing, Gao Hang, Wang Tao, and al.. (2017) A survey of the scattering characteristics and detection of aircraft wake vortices[J]. *Journal of Radars*, 6(6): 653–672, 10.12000/JR17068.
- [80] Ryan M. Shelton, Brandon E. Jackson and Tyson L. Hedrick (2014) The mechanics and behavior of cliff swallows during tandem flights. *Journal of experimental biology*, 10.1242/jeb.101329.



# **Appendices**



## Appendix A

# Numerical integration on the wing surface

For the numerical integration for  $\bar{w}$  in eq. 3.10, we assume rectangular wings of width  $\bar{c}$  and length  $b$  with the values given in Table 2.2. We can see from Fig. A.1 that this surface is flat at mid-downstroke in  $t/T = 0$  during the flight sequence. Consequently,  $S_w(t = 0, \Delta\phi_f = 0) = \bar{c}b$ . Nevertheless, this is an approximation. This approximation is realistic when we compare the flight sequence shown in Fig. 2.16 with  $\tau(t, \Delta\phi_f = 0) = 0$  in  $t/T = 0, 1/2, 1$  and maximum in absolute value in  $t/T = 1/4, 3/4$ .

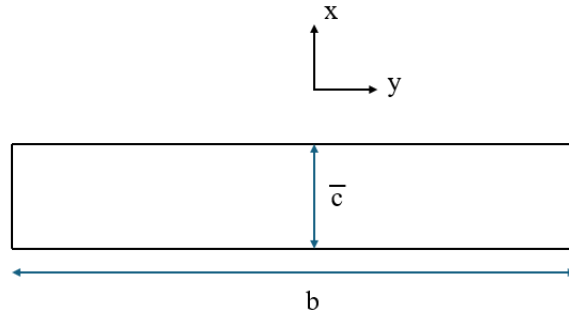


Figure A.1: Top view of an approximation of the wing surface  $S_w$  during mid-downstroke at  $t/T = 0$ .

We also assume that the angle between the horizontal and the bird's wing  $\tau$ , shown in Fig. A.2, is a variable of time but also of the phase shift with the upstream rank such that  $\tau = \tau(t, \Delta\phi_f)$  which we approximate as below with the flapping amplitude  $F$  taken from Table 2.2

$$\tau(t, \Delta\phi_f) = F \sin(2\pi f t + \Delta\phi_f)$$

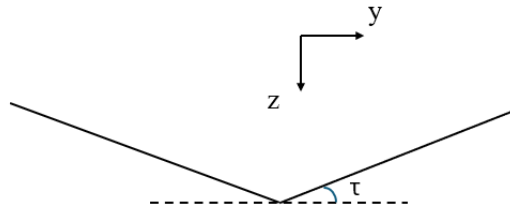


Figure A.2: Rear view of an approximation of the wing surface  $S_w$  during end-upstroke at  $t/T = 3/4$ .

For the spatial discretization along  $x$ ,  $dx$  is constant with a number of elements  $N$  determined on the basis of the numerical discretization of the aerodynamic solver presented in section 2.3.3. For  $dy$  and  $dz$ , based on Fig. A.2 and trigonometric relations, we have what follows.

## A. Numerical integration on the wing surface

---

$$dx = \frac{\bar{c}}{N-1} \quad ; \quad dy = \frac{b \cos(\tau(t, \Delta\phi_f))}{N-1} \quad ; \quad dz = dy \tan(\tau(t, \Delta\phi_f))$$

We now use Simpson's method [59,60] as a weighted sum to approximate the integral, with  $S_{mn}$  the two-dimensional Simpson's coefficient matrix for  $N$  and  $w_{mn}$  the upwash value at the evaluated location on the wing mesh:

$$\bar{w}(x, y, z, t) \approx \sum_m \sum_n \frac{dx \sqrt{dy^2 + dz^2}}{9} S_{mn} w_{mn} = \sum_m \sum_n \frac{dx dy \sqrt{1 + \tan^2(\tau(t, \Delta\phi_f))}}{9} S_{mn} w_{mn}$$

## Appendix B

# Resolution by gradient method

To solve the problem of minimizing the function  $\mathcal{G}$  constrained by  $\mathcal{B}_C$  and  $\mathcal{B}_V$ , we use the gradient method with the vector of variables  $\mathbf{x} = [x, y, z, \Delta\phi_f]^T$ . The iterative resolution is described as follows.

$$\mathbf{x}_{i+1} = \mathbf{x}_i + k (\nabla\mathcal{G}(\mathbf{x}_i) + \nabla\mathcal{B}_C(\mathbf{x}_i) + \nabla\mathcal{B}_V(\mathbf{x}_i))$$

To start the iterations, we fix an arbitrary  $\mathbf{x}_0$ . The convergence criterion is set at  $\epsilon = 10^{-4}$  so that the gap  $|\mathbf{x}_{i+1} - \mathbf{x}_i|$  is at the limit of the discretization of the solution of the aerodynamic wake problem. Finally, for  $k < 0$ , we assume a constant iteration step. A large absolute  $k$  would improve convergence speed for the first iterations, but the  $\epsilon$  criterion might not be met. On the other hand, a  $k$  too small in absolute value would drastically increase the number of iterations before convergence is reached. We therefore chose to compromise by taking  $k = -5 \cdot 10^{-2}$ . The problem can be rewritten in matrix form as shown below.

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ \Delta\phi_{f,i} \end{bmatrix} = \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \\ \Delta\phi_{f,i+1} \end{bmatrix} + k \left( \begin{bmatrix} \frac{\partial}{\partial x}\mathcal{G}(\mathbf{x}_i) \\ \frac{\partial}{\partial y}\mathcal{G}(\mathbf{x}_i) \\ \frac{\partial}{\partial z}\mathcal{G}(\mathbf{x}_i) \\ \frac{\partial}{\partial(\Delta\phi_f)}\mathcal{G}(\mathbf{x}_i) \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial x}\mathcal{B}_C(\mathbf{x}_i) \\ \frac{\partial}{\partial y}\mathcal{B}_C(\mathbf{x}_i) \\ \frac{\partial}{\partial z}\mathcal{B}_C(\mathbf{x}_i) \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial x}\mathcal{B}_V(\mathbf{x}_i) \\ \frac{\partial}{\partial y}\mathcal{B}_V(\mathbf{x}_i) \\ \frac{\partial}{\partial z}\mathcal{B}_V(\mathbf{x}_i) \\ 0 \end{bmatrix} \right)$$

For the numerical computation of the operator  $\nabla = [\partial/\partial x, \partial/\partial y, \partial/\partial z, \partial/\partial(\Delta\phi_f)]^T$ , we compute the gradient in the optimization space with centered finite differences of order two. For example, for an arbitrary function  $f = \{\mathcal{G}, \mathcal{B}_C, \mathcal{B}_V\}$  and for an arbitrary discretization  $d\mathbf{x} = [dx, dy, dz, d(\Delta\phi_f)]^T$ , we use centered finite difference of order two as below.

$$\nabla f(\mathbf{x}) = \frac{\partial}{\partial \mathbf{x}} f(\mathbf{x}) \approx \frac{f(\mathbf{x} + d\mathbf{x}) - f(\mathbf{x} - d\mathbf{x})}{2 d\mathbf{x}}$$





## Appendix C

### Different views of the convergence path for position

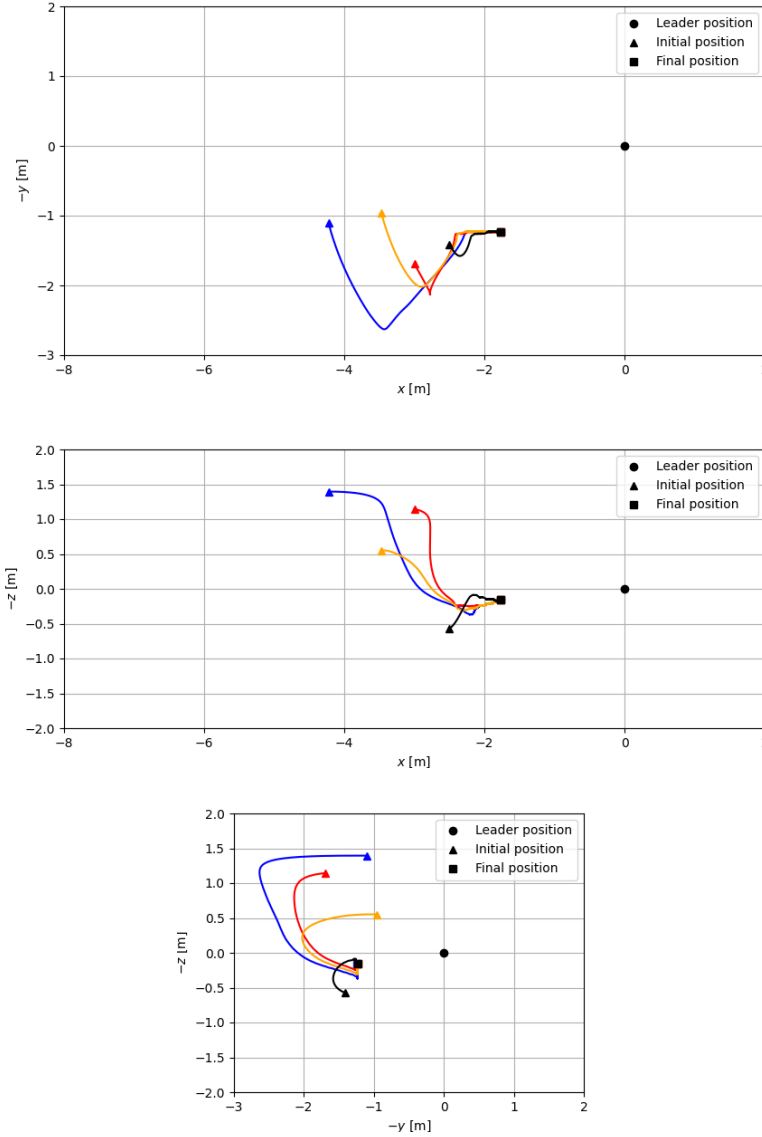


Figure C.1: Graphs of convergence of the gradient method for the four minimization variables  $\mathbf{x} = [x, y, z, \Delta\phi_f]^T$  for four different initial guesses  $\mathbf{x}_0$ . The top figure shows a view from above, the middle one from the side and the bottom one from the front.



## Appendix D

# Analytical calculation of step response for the phase shift controller

In the particular case of a formation flight with two birds, one of which is the leader with a constant wingbeat frequency  $f_0$ , we can analytically find the function  $\Delta\phi_f(t)$  and  $f(t)$  for the follower. The  $\phi_l$  phase of the leader is then as below, with  $\phi_{0,l}$  the phase of the leader at  $t = 0$ .

$$\phi_l(t) = \phi_{0,l} + 2\pi f_0 t$$

Since the phase shift of the follower is defined as the difference between its phase and that of the leader, *i.e.*  $\Delta\phi_f = \phi_f - \phi_l$ , we can determine the differential equation to be solved as follows.

$$\frac{d}{dt}(\Delta\phi_f(t)) = \frac{d}{dt}\phi_f(t) - \frac{d}{dt}\phi_l(t) = \frac{d}{dt}\phi_f(t) - 2\pi f_0$$

The angular frequency of the follower will vary by  $\Delta w$  relative to  $2\pi f_0$ . Moreover, the control to achieve a phase shift  $\Delta\phi_{ref}$  is made with a proportional controller so  $\Delta w = k_p(\Delta\phi_{ref} - \Delta\phi_f)$ . We can find the equation of  $d(\Delta\phi_f)/dt$  since it is equal to  $\Delta w$ .

$$\frac{d}{dt}(\Delta\phi_f(t)) = k_p(\Delta\phi_{ref} - \Delta\phi_f)$$

Simply integrate once according to  $t$  to find the phase shift.

$$\Delta\phi_f(t) = (\Delta\phi_{ref} - \Delta\phi_{f,0})(1 - e^{-k_p t})$$

As  $\Delta\phi_f = \phi_f - \phi_l$ , we can find the phase of the follower.

$$\phi_f(t) = \phi_{0,l} + 2\pi f_0 t + (\Delta\phi_{ref} - \Delta\phi_{f,0})(1 - e^{-k_p t})$$

Now all we have to do is derive its angular frequency as a function of time during control. For the function of frequency versus time, we divide  $w$  by  $2\pi$ .

$$w(t) = 2\pi f(t) = \frac{d}{dt}\phi_f(t) = 2\pi f_0 + k_p(\Delta\phi_{ref} - \Delta\phi_{f,0})e^{-k_p t}$$



## Appendix E

# Induced velocity for semi-infinite and finite straight vortex filament

In this appendix, we describe the analytical resolution for determining the velocity induced by a filament vortex for a point in space.

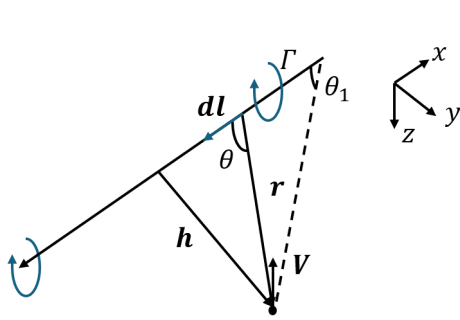


Figure E.1: Representation of the induced velocity of a semi-infinite straight vortex filament of strength  $\Gamma$ .

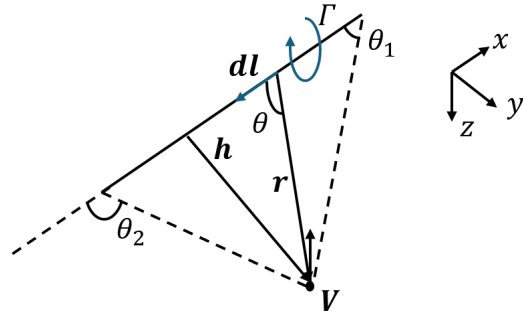


Figure E.2: Representation of the induced velocity of a straight vortex filament of finite length of strength  $\Gamma$ .

We define  $d\mathbf{l}$  as a segment of infinitesimal length of the filament shown in Figs. [E.1](#) and [E.2](#). The vector  $\mathbf{r}$  connects a point in space to this segment  $d\mathbf{l}$ . At this point in space, this infinitesimal segment induces a velocity  $d\mathbf{V}$  deduced by the Biot-Savart law.

$$d\mathbf{V} = \frac{\Gamma}{4\pi} \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}$$

We begin by calculating the induced velocity  $\mathbf{V}$  by applying this law to the case shown in Fig. [E.1](#).

$$\mathbf{V} = \int_{-\infty}^0 \frac{\Gamma}{4\pi} \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}$$

By definition of the vector product, we can calculate the velocity intensity  $V = |\mathbf{V}|$  for any point in space.

$$V = \int_{-\infty}^0 \frac{\Gamma}{4\pi} \frac{dl \sin(\theta)}{r^2}$$

We introduce the distance  $h = |\mathbf{h}|$  as the orthogonal distance between a point in space and this straight vortex filament. Thus,  $h = r \sin(\theta)$  and  $l = h / \tan(\theta)$ . Consequently, we find  $dl$  as follows.

$$dl = -\frac{h}{\sin^2(\theta)} d\theta \quad ; \quad r = \frac{h}{\sin(\theta)}$$

We can now change the variable in the integral.

$$V = \int_{-\infty}^0 \frac{\Gamma}{4\pi} \frac{dl \sin(\theta)}{r^2} = -\frac{\Gamma}{4\pi h} \int_{\pi}^{\theta_1} \sin(\theta) d\theta$$

$$V = \frac{\Gamma}{4\pi h} (1 + \cos(\theta_1))$$

For the case of the velocity magnitude induced by the finite-length filament vortex shown in Fig. [E.2](#), we change the bounds of the integral.

$$V = -\frac{\Gamma}{4\pi h} \int_{\theta_2}^{\theta_1} \sin(\theta) d\theta$$

$$V = \frac{\Gamma}{4\pi h} (\cos(\theta_1) - \cos(\theta_2))$$

The two cases studied therefore have an induced velocity magnitude inversely proportional to  $h$ , *i.e.* the orthogonal distance from a point in space to this straight vortex filament.

UNIVERSITÉ CATHOLIQUE DE LOUVAIN  
École polytechnique de Louvain

Rue Archimède, 1 bte L6.11.01, 1348 Louvain-la-Neuve, Belgique | [www.uclouvain.be/epl](http://www.uclouvain.be/epl)