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Essay Paper

A literature review of growth in a world of debt

Author : Loïc De Valkeneer

Thesis Director : Rigas Oikonmou

Thesis Reader : Vincent Bodart

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Université catholique de Louvain - Economics School of Louvain - Place Montesquieu 3, 1348 Louvain-La-Neuve

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1 Introduction

It is widely believed that high levels of debt affects long term growth rates, has adverse effects on agents utility and depresses long term output. This paper seeks to clarify how public debt interacts with long-run growth under various assumptions, planning horizons and fiscal policies.

This question is motivated by the observation that debt plays sharply different roles in neoclassical and endogenous growth models. In some settings, debt is merely a fiscal smoothing device. In others, it directly affects capital accumulation and long-run growth. Understanding these differences is essential for interpreting fiscal policy debates and for assessing the conditions under which debt can support or hinder economic development. The main result suggests that the role of debt on long run growth and utility hinges upon the structure of the economy.

Methodologically, this paper conducts a structured literature review of the main theoretical frameworks used to study debt and growth. It relies on well established models such as: Barro's Ricardian equivalence framework, Diamond's overlapping-generations model, Blanchard's finite-horizon model, Chamley-Judd's optimal taxation result, Greiner–Semmler's endogenous growth model with productive public capital and Saint-Paul's fiscal policy in an endogenous growth model. The analysis proceeds by reconstructing the logic of each model, identifying the mechanisms through which debt affects capital accumulation and comparing the resulting implications for growth and welfare. The review is analytical rather than empirical, and focuses on the internal consistency, assumptions and long-run predictions of each framework.

The remainder of this paper is organized as follows: the first chapter, *Debt in Neoclassical Growth Models*, highlights the importance of the planning horizon of agents. Accordingly, debt plays a distinct role as implied by the different planning horizons of households. Barro expounds how a planner resorts to debt to absorb shocks and smooth taxation rates over time when agents horizons are infinite. This follows from the so-called *Ricardian Equivalence*. However, agents rarely behave in such a way. Diamond advances an OLG model that exhibits a distinct purpose for debt, departing from Barro's model. Then issuing debt yields a Pareto improvement when an economy is characterized by overaccumulation of capital. Finally, Blanchard's model bridges both models by allowing for flexible planning horizons.

The second chapter, *Fiscal policy*, is motivated by the fact that fiscal policy triggers a corresponding fiscal response. The chapter is built upon the *Chamley-Judd result*. The latter states that capital should remain untaxed in the long-run. Since any capital income taxation causes a distortion on the intertemporal allocation of households. The ensuing papers challenge the Chamley-Judd result. From a welfare point of view, the redistribution of capital income provides insurance against idiosyncratic risk and shocks. Hence, taxing capital income is associated with a rise in aggregate utility. Even though a capital income tax introduces a distortion into the agents' planning horizon.

In addition, the chapter examines debt as a fiscal device. In the presence of state contingent debt, optimal policy prescribes the following: debt should be issued in such a way that is *time-consistent*. So that it binds successive governments to stick to the Ramsay solution. In the absence of state contingent debt, a government employs debt as a fiscal instrument to minimize distortions from a second best.

In the third and final chapter,*Endogenous growth models*, we describe how a planner cannot simultaneously reconcile the aims of optimizing growth and welfare through debt. However, that issue can be solved by resorting to an investment subsidy. Therefore, the focus is narrowed to the issue of growth. In addition, different budgetary regimes are formulated by which a government must stick. Increasing public capital through deficit financing can be achieved if the latter outweighs the negative feedback effects. These negative channels on growth encompass: internal and external crowding out effects. A growing level of debt requires a larger proportion of output to pay off interest payments. Hence a smaller proportion of resources can be devoted to the accumulation of private and public capital.

Finally, this paper suffers from several shortcomings. It is theoretically driven and produces very little empirical data. Nor does it provide any policy guidelines such as the optimal level of debt implied by a budgetary regime to optimize growth and welfare.

2 Debt in Neoclassical Growth Models

2.1 Public Debt Theory under Ricardian Equivalence

In *On the determination of public debt*, Barro presents a theory of public debt in which the *Ricardian equivalence* holds as a first order condition. It supposes that public debt is equivalent to the present value of future taxes. Hence, government bonds are not regarded as a source of net wealth to consumers. Since the present value of government bonds do not outweigh future tax liabilities. If so, what shapes a planner's decision making between taxes and debt ? This is the focal point of his paper.

Barro's answer relies on second-order considerations: the excess burden of taxation and the optimal smoothing of tax rates over time. I begin by outlining what the Ricardian equivalence encompasses and subsequently turn to the earlier argument.

Understanding whether the *Ricardian equivalence* holds, has major consequences on monetary and fiscal policies. A rise in public debt supposes a higher *perceived* disposable income. The latter induces agents to increase consumption. However, this all hinges on the assumption that agents perceive higher lifetime resources. Yet, rational agents might expect that a larger tax burden will be necessary to pay off the interest payments. Therefore, future tax liabilities offset the wealth effects from expansionary monetary policy. Even tough current disposable income has increased.

The Ricardian equivalence holds because agents act as though they were infinitely lived. That is, the utility of each member hinges upon the attainable level of consumption of their immediate descendant, which takes the form of a net bequest. When debt is included in the model, future interest payments must be financed. Barro assumes that future interest liabilities are shifted to the following generation through a lump-sum tax. Since the utility level of a member of the previous generation depends upon the attainable utility level of their immediate descendant. It follows that consumption falls as to maintain the desired net bequest after taxation. Of course, the latter depends upon the weight that is attributed to utility associated to current consumption and attainable utility of their immediate descendant. Thus future tax liabilities are fully capitalized in the current period.

This raises the question of how this principle fits into a theory of public debt. In this model, the excess burden of taxation is convex and a rising tax rate causes a disproportionate cost for a household. Hence, a government minimizes the excess burden by issuing debt. This allows a planner to smooth tax rates across time and is exactly captured by the *constancy argument*. The level of taxes is exogenously determined by the sequence of incomes Y. Thus, tax rates should be constant and deficits should absorb temporary shocks. These concepts are translated mathematically in the following paragraph.

Barro constructs a theory of public debt that determines the optimal level of debt issuance in each period. This theory is underpinned by the Ricardian equivalence as a first order condition. The model assumes a government that finances its expenditures through current taxation and public debt. The government's budget constraint is written as:

$$G_t + rb_{t-1} = \tau_t + (b_t - b_{t-1})$$

In addition, it rules out the no ponzi game condition. That is, it dismisses perpetual public expenditure through debt financing:

$$\sum_{1}^{\infty} \left[\frac{G_t}{(1+r)^t} \right] + b_0 = \sum_{1}^{\infty} \left[\frac{\tau_t}{(1+r)^t} \right]$$

Furthermore, the model assumes that taxes are associated with collection costs or excess burden, denoted by Z_t the real cost incurred. Moreover, both the first and second derivative with respect to tax revenue, τ_t are positive. Finally, by assuming that the homogeneity condition holds, then:

$$Z_t = F(\tau_t, Y_t) = \tau_t f\left(\frac{\tau_t}{Y_t}\right)$$

Or expressed in present value terms

$$Z = \sum_{t=1}^{\infty} \frac{\tau_t f\left(\frac{\tau_t}{Y_t}\right)}{(1+r)^t}$$

Observe that $f(\frac{\tau_t}{Y_t})$ denotes the average cost of taxation per unit, then by scaling it by τ_t we obtain the real cost incurred Z_t.

A government faces the challenge of financing its expenditure each period. Moreover, it aims to minimize the present value of real costs incurred from collecting taxes subject to the present value of the government's budget constraint.

Hence, the problem amounts to choosing the tax revenue in each period. Note that the present value of taxation hinges on the timing of tax collection, since the marginal cost of tax collection varies across time. In that regard, Barro argues that minimizing the cost of tax collection requires the marginal cost $\frac{\partial Z_t}{\partial \tau_t}$ to be constant every period. The reason is intuitive: If the marginal cost of collecting taxes differs across time. Then a central planner is induced to shift taxes at times where the marginal cost is lower and vice versa. Moreover, this also implies that $\frac{\tau}{Y}$ is constant.

This follows from deriving the marginal cost of taxation:

$$\frac{\partial Z_t}{\partial \tau_t} = f\left(\frac{\tau_t}{Y_t}\right) + \frac{\tau_t}{Y_t} f'\left(\frac{\tau_t}{Y_t}\right)$$

Maintaining a constant marginal cost per unit of income throughout the entire horizon forces the ratio $\frac{\tau_t}{Y_t}$ to be constant across time. Since any deviation necessarily violates a constant marginal cost. Barro calls this the *constancy argument*.

As a consequence, a government issues debt in times of temporary fluctuations in government spending or income. Barro highlights two important dimensions: On the one hand the duration and on the other hand the magnitude of the departure. Given the constancy argument, the government budget constraint can be rewritten as:

$$G_t + rb_{t-1} = \tau_t + b_t - b_{t-1}$$

where

$$\tau = \frac{\tau_t}{Y_t} \iff \tau Y_t = \tau_t$$

then

$$G_t + rb_{t-1} - \tau Y_t = b_t - b_{t-1}$$

It captures the optimal debt-accumulation rule. Because the optimal taxation rule still applies, it follows that any departure in aggregate income entails a mechanical change in taxation τ_t.

Thus, a transitory deviation in government spending is fully absorbed by debt in order for tax rates to remain constant. This follows directly from the modified government budget constraint. In contrast, a permanent change in government spending requires a planner to revise the tax-income ratio.

The issue with Barro's analysis is that it relies on the controversial assumption that the Ricardian equivalence holds as a first-order condition. Under this assumption, agents act as though they were infinitely lived. Hence, they internalize the tax and debt structure and debt and taxes become perfect substitutes. Under this condition, debt plays a different role. A central planner issues debt to absorb shocks. This allows a government to smooth tax rates over time in order to satisfy the constancy condition, when facing temporary government fluctuations of aggregate income and expenditure. Permanent fluctuations in contrast require a reassessment of the tax rate. In reality, however, the Ricardian equivalence does not hold as a first-order condition. As a result, a planner behaves differently. The following subsection expounds how, the Ricardian equivalence as a first-order condition fails in an OLG model where agents are finitely lived.

2.2 OLG Model with Finitely Lived Agents

Diamond in his paper, *National debt in a neoclassical growth model*, examines long-run growth equilibrium by introducing a government that levies taxes and issues debt. He investigates the effects of government debt on this equilibrium.

Methodologically, Diamond develops a neoclassical growth model with overlapping generations.

It is key to recall that growth in such models occurs exogenously, driven solely by population expansion. This point is important to address because in the last chapter we inquire how debt behaves differently in an endogenous growth model. Having laid out these points, we can now turn to the model.

Diamond assumes an environment in which agents live for two periods. They work in the first and retire in the second. Each individual has an ordinal utility function $U(e^1, e^2)$. Technology is characterized by constant returns to scale and the labor force grows at a constant rate n given by:

$$L(t) = L_0(1+n)^t$$

A central planner attempts to preserve a *constant capital-to-labor ratio*. At the end of the production process, the central planner determines how the capital stock and production is allocated between consumption in C_t and the available capital in the next period K_{t+1}. In addition, total resources are further divided between the two generations :

$$Y_t + K_t = K_{t+1} + C_t = K_{t+1} + e_t^1 L_t + e_t^2 L_{t-1}$$

Preserving a constant capital-to-labor ratio requires output per worker to be constant over time. Finally, if capital-labor ratio is kept constant, then the economy is on what is called a *Golden Age Path*.

As noted earlier, economic growth occurs exogenously. As a result, the central planner's only concern is to maximize aggregate utility. This issue can be formalized as follows: What is the stock of capital that maximizes agents' level of consumption in steady state ? Formally the planner solves,

$$\Longrightarrow max\, C = F(K,L) - nK$$

Taking the derivative with respect to K

$$\Longrightarrow F_K(K,L) = n$$

The latter is referred to as the *Golden Rule Level*. Note that there can be as many *Golden Age Paths but, only one Golden Rule Level*. Intuitively the central planner faces the following allocation problem: maintaining a large stock of capital bolsters output. Consequently more resources can be devoted to consumption. However, supporting a large stock of capital requires a planner to commit a significant amount of resources to maintain it. Thereby, lowering levels of consumption.

Furthermore, every deviation from the *Golden Rule Level* would be inefficient. On the one hand, a higher marginal product than n implies a scarce stock of capital. Then steady-state utility can be increased by expanding the capital stock. On the other hand, a lower marginal product of capital requires excessive savings to maintain this capital-labor ratio. Consequently, fewer resources and existing capital are devoted to consumption and utility. Diamond calls this allocation as, *dynamically inefficient*. This might sound surprising, since it is widely believed that large savings increase the capital stock and benefit growth.

Naturally, the following question concerns how debt fits in Diamond's framework to examine the long-run effects of national debt. Until now, we have only a considered a central planner's allocation. First, however, it is key to understand whether the competitive equilibrium of the economy actually converges to the *Golden Rule Level*.

Because of what has been dicussed previously, the utility maximizing allocation of the central planner requires that:

$$\frac{\partial U}{\partial e^1} = (1+n)\frac{\partial U}{\partial e^2}$$

This Euler equation characterizes the utility maximizing allocation. The interest rate equals the population rate of growth. The allocation is the same as identified by Samuelson, which he referred to as the *biological optimum*. Since in this model every unit of foregone consumption of the young cohort must be compensated by one additional unit of functional capital in the next period, when that cohort turns old. Then the required return is therefore $1+n$, the growth rate of the cohort. This explains why it is referred to as the biological optimum.

Therefore, the utility gain from the increase in productive capital is spread across $1+n$ individuals. To study a competitive framework we shift from a centrally planned economy to a market, where agents decide how savings are allocated over the two periods. This is of particular interest since it allows to capture savings choices, capital accumulation and hence the long-run growth of the economy. Again, Diamond assumes that an agent in t earns a wage w_t equal to the marginal product:

$$w_t = F_L(K_t, L_t)$$

A fraction of that income is saved, denoted s_t for future consumption and lend to the capital market given an interest rate r_{t+1}, defined as the marginal product of capital. Hence, in the second period the agent consumes its savings plus the accrued interest:

$$e_t^2 = (1+r_{t+1})s_t$$

Finally, agents maximize their utility over time, computing the Euler equation we obtain :

$$\frac{\partial U}{\partial e^1} = (1+r)\frac{\partial U}{\partial e^2}$$

It implies that households are smoothing consumption across time such that the loss in utility in period 1 resulting from less consumption is compensated by additional utility in period 2 by consuming more. The supply of capital is defined as:

$$S_t = s_t L_t$$

and on the other hand the demand of capital as capital-to-labor ratio:

$$r_{t+1} = f'\left(\frac{K_{t+1}}{L_{t+1}}\right)$$

Combining both schedules:

$$r_{t+1} = f'\left(\frac{S_t}{L_{t+1}}\right) = f'\left(\frac{s(w_t, r_{t+1})}{(1+n)}\right)$$

We obtain the equilibrium condition. The key question is how the supply and demand schedules of capital interact. Understanding this issue is essential to evaluate whether the competitive equilibrium or the *Golden Age Path* satisfies the *Golden Rule Level*.

On the one hand, firms will demand savings up to the point where the marginal product of capital equals the interest rate. On the other hand, the slope of the supply curve is more ambiguous, because two forces are interacting: the substitution and income effect. It requires to work out which effect will dominate.

A higher wage in period t results in a larger quantity of savings, regardless of the interest rate. However, whether this leads to a higher equilibrium level of savings depends upon how the demand and supply schedules of savings interact. This relation between the interest rate in the next period r_{t+1} and the wage in the previous period is denoted, $r_{t+1} = \psi(w_t)$. This mechanism links factor prices across generations.

Finally, Diamond states that this *Golden Age Path* might not coincide with the *Golden Rule Level*. Since, households chose their savings levels to maximize utility over the planning horizon. Households maximize lifetime utility but do not consider how their savings affect future wages. Therefore the competitive equilibrium may have $r < n$ or $r > n$. However, they do not internalize that their savings choices affect factor prices across different generations. That is, the Euler equations in both settings:

$$\frac{\partial U}{\partial e^1} = (1+n)\frac{\partial U}{\partial e^2}$$

and

$$\frac{\partial U}{\partial e^1} = (1+r)\frac{\partial U}{\partial e^2}$$

Because these conditions are not generally equal, the competitive equilibrium may lead to either overaccumulation or underaccumulation of capital. Consequently, any deviation from the Golden Rule Level does not constitute a utility-maximizing allocation for society.

In examining the long-run effects of debt, two approaches are provided. On the one hand, *balanced-budget incidence* and on the other hand *differential incidence*. The former refers to a change in expenditure and financing while the latter corresponds to a comparison of alternative methods of financing for a given level of expenditure.
In addition, incidence alludes to the question who ultimately bears the burden of debt. However, only differential incidence is considered, since Diamond is interested in how two different forms of debt financing affect long-run equilibrium. That is, debt can either be external or internal. Both have a different set of consequences on the domestic economy with regard to the supply of capital and welfare.

In the case of *external debt*, taxes are levied to pay off the interest payment associated to the subsequent rise in debt. Consequently, households' disposable income are reduced. Taxes per worker in period t are expressed as:

$$T_t = (r_t - n)g$$

And disposable income is:

$$w_t - (r_t - n)g$$

Importantly, this induces a new stability condition. A lower disposable income affects savings and therefore capital accumulation, which in turn rises the interest rate up and lowers the supply of capital.
However, whether it causes utility to fall is ambiguous. It depends upon whether $r > n$ or $r < n$. In the former case, external debt pushes the interest rate away from the Golden Rule Level. Subsequently, it necessarily reduces agents' utility. The latter case is unclear. Since in reality two forces are at play. Although a fall in savings pushes r closer to n, therefore increasing utility. External debt reduces disposable income and therefore utility. Hence, we are unaware whether utility has risen or fallen.

In the case of *internal debt*, the same process occurs, however with an additional mechanism. Both external and internal debt reduce the supply of physical capital in the economy. Nevertheless this drop in physical capital under internal debt is amplified. The reason lies in the fact that bonds and physical capital must yield the same return so that agents hold both assets in their portfolio. Because bonds are a risk-free asset and pay the same return, households substitute government bonds for physical capital. This in turn reduces available capital for firms, capturing the *displacement effect*.

Again the change in utility is ambiguous. In order to correctly identify how utility is affected by internal debt, we consider several cases. On the one hand, if the solution is *dynamically inefficient* before the issuance of government bonds, then bonds absorb this excess capital, thereby, pushing the interest rate closer to the Golden Rule Level. This increases utility. On the other hand, when the solution was already efficient or $r > n$ before the issuance of government bonds. Then, internal debt drives in both situations the interest rate away from the Golden Rule Level. This reduces the utility levels of households.

In summary, debt in a neoclassical growth model matters in different ways depending on the underlying assumptions. That is, debt in a model where agents live infinitely, like Barro's model, perfectly internalize the government's intertemporal budget constraint. Consequently, debt and taxes become perfect substitutes. A central planner resorts to debt to absorb shocks and smooth taxation rates across time.

In contrast, in an OLG model, a central planner uses debt for different reasons. It uses debt as a fiscal instrument if it drives the interest rate closer to the growth rate of population. This suggests that debt is not neutral in Diamond's model as opposed to Barro's. Moreover, the welfare effects of debt in an OLG model have different implications. It hinges upon whether debt is held internally or externally and on whether the solution is efficient or inefficient.

The issue, however with Diamond's framework lies in the fact that he builds a model with representative agents, which implies no heterogeneity. Moreover, Diamond states that the misallocation comes from households not maximizing utility across generations as mentioned previously. Nonetheless, in reality agents face incomplete markets, income uncertainty and borrowing constraints. Both assumptions limit relevance in how savings are affected when key variables are perturbed.

Finally, it highlights that the importance of debt depends crucially on the horizon of agents.

2.3 Arbitrary Horizon

As mentioned earlier, the dynamics of government debt and interest rate depend critically on the horizon of agents. Blanchard provides additional insights in his paper, *Debt, Deficits and Finite Horizons*. Barro builds a model where the Ricardian equivalence holds as first-order condition in an infinite horizon problem. In contrast, Diamond develops an OLG model with finite horizons. However, Diamond's model abstracts from fiscal policy and from the household responses such policies would trigger. Hence, Blanchard's key contribution to extend Diamond's model to allow for fiscal policies by a planner and household reaction. Finally, Blanchard formulates a tractable model in which the horizon of agents is finite but age-independent.

The model that Blanchard develops faces the challenge aggregation. This stems from the fact that agents of different ages have in realistic life-cycle models distinct propensities to consume. For instance, Diamond solves the aggregation problem by assuming a two-period problem with two types of cohorts. Within each cohort, individuals display the same consumption behavior. Hence, the aggregation problem becomes trivial. In contrast, Blachard's approach hinges on two assumptions.

On the one hand, agents face an instantenous probability of death p. Nevertheless, as Blanchard himself emphasizes the model fails to capture a life-cycle model. This follows from the model design. Although two agents differ greatly in age, they face the same probability of death. This causes age-independent horizons and a collapse of heterogeneity. Age dependency is key to correctly capture a life-cycle model.

On the other hand, Blanchard assumes the presences of insurance companies. In the absence of bequest motives, agents have different saving and dissaving motives. For instance, old households, run down on their assets and young agents would accumulate assets. Importantly, it introduces age dependent behavior over the life-cycle. Age-dependent behavior does not support aggregation. Therefore, to tackle that issue, agents can seek insurance. The model assumes that insurance companies pay pw each period for a surviving agent and leave all their assets w upon death to the insurance company. This eliminates accidental bequests and ensures that wealth evolves according to the same differential equation for all agents. The effective return on wealth becomes $r + p$ identical across ages, collapsing heterogeneity. Thus the model behaves similarly to a permanent-income model because consumption is a linear function of total wealth and horizons are age-independent..

Now an agent maximizes the following utility function:

$$\int_t^{\infty} log\, c(s, v) e^{(\theta+p)(t-v)} dv$$

subject to the budget constraint

$$\int_t^{\infty} c(s, v) e^{-\int_t^v |r(\mu)+p| d\mu} dv = w(s, t) + h(s, t)$$

The solution is

$$c(s, t) = (p + \theta) [w(s, t) + h(s, t)]$$

where $(p + \theta)$ is the propensity to consume.
Blanchard introduces a government that finances expenditure either through lump sum taxes or debt. It has the following dynamic budget constraint:

$$\dot{D} = rD + G - T$$

Moreover, a government is required to satisfy the transversality condition:

$$lim_{t\to\infty} D_t e^{-\int_s^t r_v dv} = 0$$

Finally, combining the dynamic budget constraint of the government and transversality condition results in the equality of the present values of future levels of debt and surpluses:

$$D_t + \int_t^{\infty} G_s e^{-\int_s^t r_v dv} ds = \int_t^{\infty} T_s e^{-\int_s^t r_v dv} ds$$

A natural question concerns the effects of a reallocation of taxes. Here Blanchard considers a fall in taxation in t with an associated increase in $t+\tau$. Meanwhile, government expenditure and stock of debt remain unchanged. Sinc, it is necessary to satisfy the planner's intertemporal budget constraint. Then a fall in taxes in t must be compensated by an equivalent rise in present value of taxes in $t+\tau$. Thus,

$$\Longrightarrow d\left[\int_t^{\infty} (T_s - G_s) e^{-\int_s^t r_v dv} ds\right] = 0$$

$$\Longrightarrow dT_t e^{-\int_t^t r_v dv} + dT_{t+\tau} e^{-\int_t^{t+\tau} r_v dv} = 0$$

$$\Longrightarrow dT_t e^{-\int_t^t r_v dv} = -dT_{t+\tau} e^{-\int_t^{t+\tau} r_v dv}$$

$$\Longrightarrow -dT_t e^{\int_t^{t+\tau} r_v dv} = dT_{t+\tau}\ (*)$$

As a consequence of the assumptions made, the framework collapses to a permanent income model. That is, changes in the present value of wealth affect current consumption. Hence, this raises the question of how a reallocation of taxes alters the present value of wealth. Blanchard defines the latter as follows:

$$H_t = \int_t^{\infty} Y_s e^{-\int_t^s (r_v+p) dv} ds - \int_t^{\infty} T_s e^{-\int_t^s (r_v+p) dv} ds$$

Then,

$$\Longrightarrow \dot{H}_t = d\left[T_s e^{-\int_t^s (r_v-p) dv} ds\right]$$

$$\Longrightarrow -dT_t - dT_{t+\tau} e^{-\int_t^{t+\tau} (r_v+p} dv$$

By substituting $(*)$ in the previous result:

$$\Longrightarrow -dT_t + dT_t e^{\int_t^{t+\tau} r_v dv} e^{-\int_t^{t+\tau} (r_v+p) dv}$$

$$\Longrightarrow -dT_t(1 - e^{-p\tau})$$

The final result effectively summarizes the effect of a reallocation of taxes on current consumption. It is key to observe that it hinges on two parameters: The

probability of death p and the duration over which taxes are deferred τ. Both are essential in determining how the agent internalizes the tax structure and its response.
On the one hand, low levels of p imply a longer horizon over which decisions are evaluated. That is, the longer the intertemporal horizon the more likely they will bear the burden of future taxation. Moreover, they experience only a limited increase in the net value of their assets. Therefore, the strength of their response is diminished. Note that when p converges to zero, then the model collapses to Barro's model. As a result, debt and taxes become perfect substitutes. On the other hand, the argument for τ is built in a similar fashion.

Thus the Blanchard paper confirms what has been mentioned earlier by Barro and Diamond. The impact of debt issuance and deferred taxation on consumption responses is highly sensitive to the agents' intertemporal horizon. The model developed by Blanchard is helpful in its flexibility to adjust the planning horizon of households.

In summary, in neoclassical growth models, one could argue that in some cases debt can raise welfare. Since a balanced growth path may be dynamically inefficient. This argument follows from the fact that growth in such models is exogenous, solely driven by population expansion. Therefore, the planner's unique concern involves maximizing agents utility. In the final chapter, we contrast the role of debt in an endogenous growth model.

3 Fiscal Policy

In the previous section different models were presented in order to shed light on how growth evolves in the presence of debt according to different assumptions. This begs the question how fiscal policy fits into this context. A central planner uses labor and capital taxation to generate revenue. It finances government expenditure and manages intertemporal ressources. The issue, however, of labor and capital income taxation entails distortions in the optimal intertemporal allocation of households. Hence, debt is introduced as an alternative fiscal device to offset distortions. Debt influences the path of capital accumulation and the behavior of agents. This section reviews key contributions in the optimal fiscal policy literature. It provides guidance on how a planner should use taxes and debt to minimize distortions and support efficient long-run growth.

3.1 The Planner's Problem

Atkinson and Stiglitz explain in their paper *The design of tax structure* the difficulties that arise in designing an optimal tax policy: people differ in many ways. Then on the grounds of ethical and effciency principles agents should be taxed based on their different characteristics. Hence, if a government could costlessly and perfectly observe these characteristics, it could rely on a lump sum tax designed specifically for each agent. Thereby, achieving a first-best allocation without distorting households' labor supply or saving decisions.

In reality, however, those characteristics are not observed by a central planner. Hence, taxes are based on *surrogate characteristics* approximating the characteristics the tax system would ideally target. Nevertheless, they are imperfect, costly and often in control of the agent. This informational limitation creates the need for distortionary taxation rather than uniform lump-sum taxes. This motivates the search for an optimal tax structure that minimizes these distortions while raising the required revenue. In that context, there exists a rich literature on how to minimize these distortions caused by a second best taxation.

3.2 The Chamley-Judd Result

In their respective papers, although they differ in some respects, Judd (1985) and Chamley (1986) provide a remarkable guideline as policy: optimal capital income taxation should be equal to zero in the long-run. Although highly debated and controversial, it is worth expounding how this result was established in understanding how later papers questioned its theoretical implications. Both papers adopted distinct models. For simplicity, the result is constructed based on Chamley's paper, *Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives.*

That result relies on a *representative agent model*. The main assumptions of this paper are the following: On the one hand, current consumption levels have a negligible effect on preferences among consumption plans far in the future. On the other hand, the lifespan of agents is infinite. The utility function of this representative agent is a Bellman equation of the form:

$$J_t(X) = U(x_t, J_{t+1}(X)), t \geq 1$$

and the agent maximizes this utility function subject to the following budget constraint:

$$b_{t+1} = (1 + r_t)b_t + r_t k_t + w_t l_t - f(k_t, l_t) + g_t$$

. This yields the following first order conditions:

$$\bar{w}_t \frac{\partial U(c_t, l_t, g_t, J_{t+1})}{\partial c_t} + \frac{\partial U(c_t, l_t, g_t, J_{t+1})}{\partial l_t} = 0$$

$$\frac{\partial U(c_t, l_t, g_t, J_{t+1})}{\partial c_t} = (1 + \bar{r}_{t+1}) \frac{\partial U(c_t, l_t, g_t, J_{t+1})}{\partial J_{t+1}} \frac{\partial U(c_{t+1}, l_{t+1}, g_{t+1}, J_{t+2})}{\partial c_{t+1}}.$$

and

$$J_t - U(c_t, l_t, g_t, J_{t+1}) = 0$$

Chamley himself admits that the most restrictive property of this functional form may be the *limited noncomplementarity* between periods. This type of utility function is also called a *Koopmans utility function*. It is worthwhile to delve deeper in this utility function. Koopmans defines in his paper, *Stationary Ordinal Utility and Impatience*, the idea of a preference for advancing the timing of future satisfaction in economic theory.

In particular, he introduces the concept of noncomplementarity, which is characterized mathematically as follows:

$$\frac{\partial^2 U}{\partial x_t \partial J_{t+1}} = 0$$

Formally, it states that marginal utility today does not depend on future utility. By adding a series of assumptions Koopmans proves that the aggregator becomes affine in the continuation value and hence a separable utility function. More precisely, we can separate the current utility from future utility levels.

Essentially, a central planner endeavors to minimize the distortion from a second best taxation. The key insight is that capital taxation distorts the intertemporal margin because it enters the Euler equation and alters the tradeoff between current and future consumption. Hence, modifying the optimal savings rule alters: optimal savings, output and consumption. This distortion compounds over time, making capital taxation increasingly costly in the long-run. Therefore, capital income taxation is set to zero in the long run.

In that regard two key questions arise: Why does only capital income taxation cause distortions and not labor income taxation ? And why should capital income taxation be equal to zero only in the long run. The former lies in the fact that labor income taxation yields a distortion in the intratemporal substitution and does not affect intertemporal substitution. Hence it exclusively adjusts the inputs within the Euler equation but does not reshape the equation itself.

Finally, the latter stems from the assumption Chamley makes about the capital stock. That is, the capital stock is initially predetermined or a state variable in the short run. As a result capital does enter in the euler equation. However, because capital is inelastic, capital income taxation does not occur. Consequently, a central planner abandons capital income taxation as fiscal device once capital has become a control variable. This marks the long run.

These ideas are formalized in the following two theorems:

THEOREM 1: When the representative individual's utility function has the form

$$J_t(X) = U(x_t, J_{t+1}(X)), t \geq 1$$

, and the second-best dynamic path converges to a steady state, the tax rate on capital income is equal to zero in the long run

THEOREM 2: Assume that the utility function of the private agent is additively separable and iso-elastic in consumption. If the fiscal policy is efficient, there is a time r such that for t ¡ r, the constraint rFt 0 is binding, and for t¿ r, capital income is untaxed

3.3 Response to the Chamley-Judd Result

The Chamley-Judd result sparked a considerable literature on optimal taxation. Here I discuss several papers that overturn the Chamley-Judd result by questioning its theoretical foundations.

In *Taxing capital ? Not a bad idea after all !* the authors invalidate the the zero capital income in the long run. They use a life-cycle model in which households face borrowing constraints and idiosyncratic risk with incomplete markets. Since the authors claim that capital income taxation is optimal, a social welfare criterion is necessary. In this model agents benefit from: the redistributive power of assets of the central planner and its concern regarding insurance against idiosyncratic shocks and risk.

Hence, the central planner faces a tradeoff between its concern for providing insurance against shocks, redistributing assets and distortions caused from taxation. In their model, households face uninsurable productivity shocks and permanent ability differences. This framing is essential to grasp why taxing capital income is optimal. Thus the central planner optimizes labor income and capital income taxation that maximizes the social welfare function. In the last chapter, we consider how taxation plays in addition to providing welfare a role in generating growth.

The authors claim that endogenous labor-leisure choice breaks the Chamley-Judd result in a life-cycle model. It is reinforced by incomplete markets and idiosyncratic shocks. When labor is exogenously supplied which does not vary with life-cycle dynamics, the central planner does not engage in capital income taxation. Since it can levy uniform lump-sum taxes on labor income. Therefore the central planner is not required to impose a capital income tax that is distortionary.

By contrast, when labor supply is coupled to life-cycle dynamics, the central planner would ideally impose an age-dependent labor tax. Because labor elasticities and productivity differ substantially across ages: young workers have high elasticities and low wages, middle-aged workers have high productivity and older workers have low elasticities and high asset holdings. Nonetheless, as established in Atkinson and Stiglitz the central planner faces informational limitations such that it can not tax conditonal on age. As a consequence, it can not exclusively tax labor income.

This deserves special care. The result stems from the life-cycle model itself. Young households own very little capital, as they get older their capital stock grows larger. Hence imposing a capital income tax shifts the burden to older households and becomes an implicit labor income taxation that is age dependent. This constitutes the core reason for a capital income taxation. Exclusively operating a labor income tax would disproportionately affect the young households in a life-cycle model.

In addition, the presence of borrowing constraints and incomplete markets strengthen this result. As already mentioned before, the social welfare criterion encompasses: the redistribution of assets and the concern of a central planner to provide insurance against idiosyncratic risk. Young households own few assets and face tight borrowing constraint. Then taxing their labor income heavily exacerbates their ability in smoothing their consumption in the face of idiosyncratic shocks. Hence shifting the tax burden to older households eases these constraints on the young households. Conesa, Kitao and Kreuger show in their model that capital income taxation aimed at redistribution and providing insurance against idiosyncratic shocks rises aggregate utility. Importantly it outweighs the distortion from capital income taxation.

A second paper that overturns the Chamley-Judd result is *Positive Long Run Capital Taxation: Chamley-Judd Revisited* of Straub and Werning. Their approach involves challenging Chamley's second theorem which was already mentioned before using the same model. In particular, they state that under certain conditions the constraint on the return of capital income can bind in the long run. That is, $\tau_t = 1$ such that

$$\bar{r_t} = r_t(1 - \tau_t)$$

equals $\bar{r_t} = 0$ in the long run. Hence contradicting the Chamley-Judd result.

They derive that result using a continuous utility function and restrict their attention to additive seperable utility function:

$$\int_0^{\infty} e^{-\rho t} U(c_t, n_t) dt$$

and the resource constraint

$$c_t + \dot{k_t} + g_t = f(k_t, n_t) - \delta k_t$$

, where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. Moreover, they adopt an iso-elastic utility function over consumption or commonly known as a CRRA. Finally, to enforce the bounds on capital taxation Straub and Werning follow Chamley and impose the following conditions:

$$\dot{\theta_t} = \theta_t(\rho - \bar{r_t}),$$

$$\bar{r_t} \geq 0,$$

where $\theta_t = u'(c_t)$ which denotes the marginal utility of consumption and $\bar{r_t}$ denotes the return of capital after tax. Chamley states that the constraint $\bar{r_t} \geq 0$ can not be binding forever. Chamley explains that if it did, marginal utility of consumption would diverge to infinity. From a mathematical point of view, it would violate the transversality condition and the Hamiltonian would be unbounded. This captures the logic why Chamley establishes the zero capital income taxation in the long run.

Their key insight is that Chamley implicitly assumes the government is never forced to raise the maximum possible revenue from capital taxation. Nevertheless, Straub and Werning question this result under certain conditions. More precisely, they consider cases for different debt levels and intertemporal elasticities of substition. The former plays a key role. Straub and Werning define the maximum burden of taxes that agents can finance given a threshold for initial government debt, $\bar{b}$. Specifically, $\bar{b}$ is the highest level of debt that a central planner can incur given the constraint for capital tax income is binding, $\tau = 1$.

Straub and Werning disprove the Chamley-Judd result on two grounds. On the one hand, the constraint might be binding in the long run. It hinges on the position of initial debt level, b_0, realtive to $\bar{b}$ and the IES. For initial levels of debt, $b_0 \geq \bar{b}$, the central planner is forced to set the capital income taxation to one. This is necessary to satisfy the central planner's intertemporal budget constraint. Consequently, the feasibility condition overrides the transversality condition. The latter stems from the fact that implementing the optimality condition would necessarily violate the solvency constraint of the central planner. Therefore, $\bar{r} \geq 0$ might be binding forever, disproving the Chamley-Judd result.

On the other hand, governement debt introduces uncertainty relative to future consumption paths. In particular, households characterized with a low IES or $\sigma > 1$ strongly dislike intertemporal fluctuations in their consumption patterns. Large levels of debt might induce a fiscal policy of sudden high taxation to pay off interest payements. Therefore taxing capital income rather than accumulating debt would avert such situations. Hence Straub and Werning show that there exists a unique optimum even when $b_0 < \bar{b}$ for IES less than one which involves a binding constraint in the long run. In contrast to the previous subsection where taxation of capital income was necessary on the grounds of feasibility, here it is carried out on grounds of optimality. In doing so the Chamley-Judd result is invalidated.

3.4 Debt as a Fiscal Instrument

Until now the emphasis has been on understanding optimal fiscal policy regarding capital and labor income. However, since tax receipts are generally not equal to government consumption in each period. This issue forces us to consider in addition to the theory of optimal taxation, the theory of optimal public debt as well. In that context, two papers consider optimal public debt policy under different set of assumptions. On the one hand, *Optimal Fiscal and Monetary Policy in an Economy without Capital* of Lucas and Stokey. And on the other hand, *Optimal Taxation without State-Contingent Debt* of Rao Aiyagari, Albert Marcet, Thomas J. Sargent and Juha Seppälä.

In *Optimal Fiscal and Monetary Policy in an Economy without Capital* the authors greatly simplify the analysis by abstracting from capital in a simple barter economy. This allows to circumvent the issues raised by capital levies. Nevertheless, it provides an insightful framework on how public debt policy should be conducted. Lucas and Stokey assume a representative agent Ramsey model. The representative agent has the following preferences and budget constraint:

$$E\left\{\sum_{t=0}^{\infty}\beta^t U(c_t, x_t)\right\} = \sum_{t=0}^{\infty}\beta^t \int U(c_t(g_t), x_t(g_t))dF(g),$$

$$c_t + x_t + g_t \leq 1,\; t = 0, 1, 2, ...\, all\, g^t$$

Government consumption follows a stochastic process, the realizations $g = (g_0, g_1, g_2, ...)$ of which have the joint distribution F. Lucas and Stokey decribe the structure of the market as one where the government issues state-contingent claims. That is, the central planner issues bonds whose payoffs depend on the realized state of the world at maturity. Moreover, the authors assume the presence of complete market securities. Hence the representative household purchases Arrow-Debrue securities for every possible realized state in the future that exactly matches its consumption pattern in the next period. Consequently, the planner's problem reduces to choosing allocations that satisfy feasibility and the implementability constraint derived from household optimality. The latter is constructed by substituting the first order conditions of the household in the central planner's feasability constraint. Therefore every allocation the central planner picks that satisfies the implementability constraint ensures that the agent voluntarily adopts it. The resulting allocation constitutes a competitive equilibrium.

The central insight of Lucas and Stokey is the following: optimal fiscal policy smooths labor taxes across states, and state-contingent debt is the instrument that makes this possible. In times of high government spending, the central planner incurs debt rather than raising distortionary labor income taxes. In contrast, low government spending states are devoted on cutting down debt. Thus debt serves as a shock absorber.

Lucas and Stokey state that optimal fiscal policies are time consistent. That is, a government defines a fiscal policy that will be carried over the entire horizon. Nonetheless, in democratic societies future governements are not bound by fiscal polcies set by previous governments. Time consistency requires that all following governments carry on the fiscal policy set in $t = 0$. This claim follows from Kydland's and Prescott's paper, *Rules rather than discretion: The inconsistency of optimal plans*. In that paper Kydland and Prescott prove that in a Ramsey model optimal fiscal policy over the entire horizon rather than optimal state contingent fiscal policy generates more utility. As a result, optimal fiscal policy consists of issuing debt levels that bind successive governments in adopting the optimal fiscal policy as in a representative agent Ramsey model. Doing so minimizes distortions from labor income taxation.

Nevertheless, we observe that this model suffers from several shortcomings. Complete markets rarely do exist. Therefore, agents devote resources to assets to smooth consumption against shocks. Subsequently, *Optimal taxation without state-contingent debt* relaxes these constraints, culminating in a different optimal fiscal policy.

In contrast to the previous paper, *Optimal Taxation without State-Contingent Debt*, the authors adopt a model with incomplete markets. Yet it largely retains the same features as that of Lucas and Stokey. In the previous model the central planner maximizes utility by conducting a fiscal policy subject to the implementability constraint. Notwithstanding, incomplete markets require maximizing an objective function subject to a different implementability constraint:

$$E_t\left(\sum_{j=0}^{\infty}\beta^j\frac{U_{c,t+j}}{U_{c,t}}s_{t+j}\right), \, measurable\, with\, respect\, to\, g^{t-1}$$

This constitutes the core issue. In contrast to complete markets, a central planner can only issue debt based on available information up to t, not on the shock in $t+1$. A planner cannot condition debt on future states. Furthermore, to make the contrast even starker. In complete markets, the central planner provides securities for each possible state. Hence, it simply issues debt up to the point where each possible state is covered.

A planner solves then the following Lagrangian:

$$L = E_0\sum_{t=0}^{\infty}\beta^j\left[u(c_t, 1-c_t-g_t-\psi_t u_{c,t}s_t + u_{c,t}(\nu_{1t}\overline{M}-\nu_{2t}\underline{M}+\gamma_t b^g_{t-1}\right]$$

and

$$\psi_t = \psi_{t-1}+\nu_{1t}-\nu_{2t}+\gamma_t$$

The latter is the law of motion of the multiplier of the implementability constraint. Observe that its functional form is a martingale. If the upper debt limit binds, ν_{1t}, then ψ_t grows. This implies that the implementability constraint becomes more restrictive. It characterizes a situation where the central planner has hit the debt limit. In contrast, hitting the lower debt limit relaxes the implementability constraint. It portrays a situation where a governement has accumulated an excessive stock of savings. Finally, γ is the multiplier of the measurability constraint. That is it provides a measure of the degree to which a central planner can rely on state-contingent debt.

In complete markets, the problem reduces to $\nu_{1t}=\nu_{2t}=\gamma_{t+1}=0\, for\, all\, t\geq 0$. Hence the optimal tax rate amounts to a time-invariant function of g_t only. More precisely, in a setting of complete markets the multiplier of the implementability constraint becomes $\psi_t=\psi_{t-1}$. This implies that the functional form of taxation is constant across all states. Note that in complete markets the borrowing constraints are by definition slack. Because a government can emit Arrow-Debreu securities, the planner perfectly tailors debt repayments to future shocks.

Incomplete markets significantly increase the complexity of the problem. On the one hand, in that type of market setting γ_t is nonzero for all time periods by definition of incomplete markets. On the other hand, a debt constraint might bind in each period. The latter occurs, since the planner can exclusively emit risk-free bonds that pays the same return in each possible state. Consequently, a government might borrow too little or excessively. This raises the probability of hitting a borrowing constraint. Observe that the multiplier ψ_t becomes a *risk-adjusted martingale*. That is, the multiplier evolves according to its previous value on which we superimpose variables that vary randomly.

Naturally, this raises the question on how to conduct optimal fiscal policy in an environment of incomplete markets. By taking the derivative of the Lagrangian with respect to b_t^g we obtain the following optimality condition:

$$E\left[u_{c,t+1}\gamma_{t+1}\right] = 0$$

This optimality condition takes the form of a risk-adjusted martingale. It is key to note that the multiplier of the measurability constraint can take negative and positive values. The intuition behind that result emanates from the market setting itself. Since the central planner can not issue state-contingent debt, emitted debt might be insufficient or excessive. More precisely, in a bad state a planner might have an insufficient amount of debt to absorb the shock, translating in a positive multiplier. In contrast to a good state, it might have capitalized by diminishing the stock of debt. It translates in a negative multiplier. Thus, optimal fiscal policy should be conducted the following way: distortions caused by the absence of state-contingent debt should be zero on average.

To sum up, optimal taxation is determined jointly by g_t, b_{t-1}^g and the multiplier of the implementability constraint ψ_t. An economy experiences exogenous expenditure shocks g_t that follow a Markov process, with transition probabilities $P(g'|g)$ and initial distribution π. A central planner issuing risk-free debt in incomplete markets induces distortions. It stems from the fact that a mismatch occurs between the level of debt and exogenous expenditure shock. Either providing excessive or insufficient insurance. The strength of this distortion essentially depends upon the flexibility of the central planner to emit debt as a function of the total stock of debt. Substantial indebtedness curtails the planner's flexibility to issue debt to smooth shocks. In addition, it raises the probability of hitting the borrowing constraint, captured by ν_{1t} and ν_{2t}.

Moreover, the persistence of government expenditure captured by the Markov process, affects ψ_t through γ_t. When exogenous expenditure shocks decay slowly to the mean, accordingly the central planner is required to finance a sequence of high expenditures. However, in the absence of state-contingent debt, the distortion grows larger. This is captured by the multiplier of the measurability constraint γ_t, where the latter enters in ψ_t.

In addition, recalling the optimality condition

$$E\left[u_{c,t+1}\gamma_{t+1}\right] = 0$$

debt issuance will greatly depend upon the curvature of households utility function. That is, bad states translate in high marginal utility and good states in low marginal utility. As a result, the curvature affects the weight of states in the optimality condition.

Finally, depending on the strength of these constraints, the Ramsey model with incomplete markets is close to the Ramsey model with complete markets.

4 Endogenous Growth Models

The role of debt in neoclassical growth models is critically shaped by the planning horizon of agents and whether the economy is dynamically inefficient. A planner issues debt to absorb shocks and smooth taxation rates over time, when the planning's horizon of agents is indefinite . By contrast, a finite planning's horizon entails a different purpose for debt. We argued that an increase in public debt enhances agents welfare if it drives the economy closer to the *Golden Rule Level.* That is an economy could be improved by consuming a part of its capital stock.

Here two papers spell out how public debt plays a different role in an endogenous growth model.

4.1 Welfare vs Growth

In his paper *Fiscal policy in an endogenous growth model* Saint-Paul describes how a planner simultaneously aiming at enhancing households welfare and maximizing growth creates a conflict. As a result, a central planner faces a tradeoff. Nevertheless, under certain conditions an investment subsidy can reconcile both policies.

The Saint-Paul model builds on the Blanchard model (1985) to allow for endogenous growth. It is key to recall that growth in a neoclassical model is driven exogenously and population growth expands aggregate output. Hence, the private and social return coincide. Agents' choices are not internalized, because neoclassical growth models abstract from externalities. This is especially critical in an environment characterized by learning dynamics. Therefore, in endogenous growth models private returns do no coincide with social returns as a result of spillovers.

In a neoclassical growth model, a central planner issues public debt such that the private return gets closer to the *Golden Rule Level.* Consequently, enhancing aggregate welfare of the economy. However, as Saint-Paul explains, rising public debt would indeed increase households welfare, yet cut growth rates.

To see this, when the private rate of return is below the growth rate of capital, then the economy is dynamically inefficient. From a household's perspective, the economy maintains an excessive stock of capital that requires too much resources that could have otherwise been devoted to consumption. Note,

$$h(t,s) = hs(t,s) - ht(t,s)$$

where $hs(t,s)$ is the present discounted value of future gross wages while $ht(t,s)$ is the present discounted value of future taxes. Moreover, as long as the interest rate is unaffected by a policy measure, the welfare of a given generation depends only on its permanent income. Hence, by issuing public debt, the planner effectively shifts the tax burden to future periods. Under log utility, consumption is proportional to permanent income, so a rise in permanent income causes an immediate jump in consumption.

$$c(t,s) = (p+\theta)(h(t,s) + w(t,s))$$

and

$$s(t,s) = h(t,s) - c(t,s)$$

Thus utility of current generations rises by maintaining a smaller stock of capital. This however, triggers a decline in savings, lower capital accumulation and therefore less growth. It pushes the social return further away from the growth rate of capital. As this sequence unfolds, later generations possess a lower stock of capital. Output becomes insufficient to commit further resources for consumption. Even though current generations benefit from an increase in public deficit it certainly harms future ones.

By a similar reasoning, Saint-Paul describes how reducing the stock of debt bolsters growth. Yet does not constitute a Pareto improvement. Because two contradicting forces are at play. Enhancing households current welfare requires increasing permanent income of current generation. If a governement is planning on reducing the level of debt, it forces the latter to levy taxes on current generations to finance that gap. Inevitably, taxation places a burden on older cohorts, who are unlikely to experience the subsequent rise in the capital stock. Thereby, the higher consumption possibilities it generates. This translates in a permanent drop in lifetime utility. Thus the planner fails in reconciling both goals.

The reason public debt fails lies in the fact that it does not tackle the wedge between private and social return. Saint-Paul claims that an investment subsidy could reconcile both aims. It suggests that a central planner pays additional return on interest income such that it narrows the gap between the private and the social return.

$$\hat{r} = \alpha A + q = r + q$$

where q is the amount of the interest cost for each unit of capital. A larger private return incentivizes households to save more. It speeds up the accumulation of productive capital and therefore growth. This enables a government to not resort to taxation. Because it would necessarily harms at least a current generation to reduce the level of debt.

Saint-Paul assumes that the investement subsidy is financed by a constant tax rate τ on labor income. Then, $(\hat{r} - r)K_t = \tau\omega_t$, where $\omega_t = (1-\alpha)Y_t$ is the marginal product of labor. By rearranging we obtain:

$$\tau = \frac{(\hat{r} - r)}{A(1 - \alpha)}$$

The investment subsidy triggers a rise in the stock of capital and therefore growth. The central planner successfully reconciles both goals if and only if the investement subsidy raises aggregate utility of households of current and future generations. It benefits future generations, since an expansion of the capital stock translates in an increase in consumption. An enlargement of the stock of capital is solely feasible by inducing current generations to consume less. To remedy this issue, the investment subsidy must be designed in such a way that induces agents to initially consume less. However, in a subsequent stage their initial decline in consumption is compensated by a sufficiently large increase in interest income. This is captured by the following condition:

$$\frac{dg}{d\hat{r}} \leq A\frac{(1 - \alpha)(1 - \tau)}{\hat{r} + p + \beta - g}$$

4.2 Budgetary Regimes: Abstracting from Welfare

In the previous subsection an endogenous growth model was examined. The central planner proved unable to reconcile growth and welfare through debt. The following paper of Greiner and Semmler, *Endogenous Growth, Government Debt and Budgetary Regimes* abstracts from welfare considerations and analyzes how fiscal policy affects the balanced growth rate of an economy, subject to the constraints imposed by a given budgetary regime.

The authors define three distinct budgetary regimes by which a government must stick. The first regime requires to finance all government expenditure through taxation. This encompasses transfers, public consumption and interest payments.

$$rB + C_p + T_p = \phi_0 T, \, with \, \phi_0 < 1$$

This regime also called the *Golden Rule of Public Finance*. It entails that it can not run any deficits that would finance non-productive capital. The second regime, called B, is obtained by relaxing the constraint on interest payments. More precisely, interest payments are partly financed through taxes.

$$rB + C_p + T_p = \phi_0 T \, with \, \phi_4 \in (0, 1)$$

The third and final regime, called C, is derived by assuming that interest payments are financed entirely by issuing new debt.

$$C_p + T_p = \phi_0 T \, , \phi_0 < 1$$

Thus, the authors assume that a government sticks to a particular budgetary regime and discuss the growth implications.

The equilibrium outcome of this economy is a balanced growth path. It implies that all variables share the same growth rate. Note that growth in this economy depends on the following ratio,

$$x = \frac{G}{K}$$

the ratio of public capital relative to private capital and the growth rate

$$g = (1 - \alpha)x^{\alpha}$$

In addition all variables' growth rates depends on the ratio $x = \frac{G}{K}$. Since the authors show how the growth rates of debt and consumption in steady states are all functions of x.

The preceding argument naturally leads to the question how fiscal policy affects the balanced growth path of the economy. More precisely, how does a deficit financed public investment and a government's choice of budgetary regime alter this ratio.

It is key to observe that public investment through debt induces feedback effects that diminish the actual growth rate of public capital. On the one hand, raising the level of debt reduces the proportion of resources available that can be devoted for the accumulation of public capital, because the volume of due interest liabilities expands. It follows from the fact that a larger share of taxes is required to finance non-productive expenditures. Therefore the ratio $x = \frac{G}{K}$ shrinks. This development is defined as *internal crowding out*. Note that this effect does not occur in regime C. Since a government finances interest payments solely with additional debt.

On the other hand, a larger stock of debt entails fewer resources available for private investment. The same reasoning applies for regime B and the golden rule for public finance. However it departs from regime C. It araises from the fact that the latter doesn't finance interest payments through taxation. The logic goes as follows: a larger stock of debt relative to private capital implies a larger share of output is devoted to paying off interest payments. The preceding argument stems from the government's budget constraint,

$$\dot{B} + T = rB + C_p + T_p + \dot{G}$$

Observe that a growing stock of debt B leaves a diminishing share of resources for other purposes. Consequently, a shrinking share of output is available for the accumulation of private capital. This second feedback channel is called *external crowding out*. It occurs in all three regimes.

Hence, when does a government resort to a deficit financed increase in public investment? The authors provide the following proposition:

PROPOSITION 1. A deficit financed increase of investment in public capital raises (reduces) the balanced growth rate in regime (A or Golden Rule of Public Finance) if

$$Z \equiv \tau(1-\phi_0)(1+(1-\alpha)b)[(\phi_3+((\phi_3-1)/(b(1-\alpha))] > (<)1$$

.

This identity is constructed by differentiating the following two steady-state growth rates:

$$0 = q(.) \equiv \frac{\dot{b}}{b}$$

and

$$0 = q_1(.) \equiv \frac{\dot{x}}{x}$$

Therefore the balanced growth rate is defined as follows:

$$\Longrightarrow -\left(\frac{\partial q(.)}{\partial b}\right)\left(\frac{\partial q_1(.)}{\partial \phi_3}\right)+\left(\frac{\partial q_1(.)}{\partial b}\right)\left(\frac{\partial q(.)}{\partial \phi_3}\right) \ (*)$$

Previously we mentioned the negative feedback effects from a deficit financed increase in public debt. Using the preceding arguments, public capital effectively grows if it outweighs the internal and external crowding out effects captured by $-\left(\frac{\partial q(.)}{\partial b}\right)\left(\frac{\partial q_1(.)}{\partial \phi_3}\right)$. If balanced growth rate is larger than 0 deficit-financed public capital expands. Conversely, if smaller than zero, deficit-financed public capital shrinks.

Economic growth responds differently as a function of the various budgetary regimes. Moreover, already highly leveraged economies financing public capital with debt are more likely to produce negative growth effects. The preceding argument comes from the fact that the negative feedback channels only grow stronger.

Another key consideration concerns how fiscal policy fits in an endogenous growth model. *Should income be taxed* ? Since a share of taxes is devoted to the accumulation of public capital, it appears unlikely that income should remain untaxed. It is captured by the following expression:

$$\frac{\partial g}{\partial \tau} = \frac{1-\alpha}{\sigma}x^{\alpha}\left[-1+\alpha\frac{1-\tau}{\tau}\frac{\partial x}{\partial \tau}\frac{\tau}{x}\right]$$

Then optimal taxation depends on the elasticity $\frac{\partial x}{\partial \tau}$. A large elasticity of the ratio public to private capital relative to taxation entails that a slight increase in taxation expands the stock of public capital. Hence, this suggests that taxation is suitable for bolstering the stock of public capital. The negative feedback channels are dominated by the positive growth effects. The opposite reasoning applies for a low elasticity.

Thus, economic growth behaves differently across the various regimes. It is necessary to emphasize that the authors do not claim to have identified the most appropriate regime for sustained growth. It is not a normative paper and does not provide any policy guidelines.

Nevertheless it opens the way for further inquiry regarding the most conducive regime for sustained economic growth. One might argue that less restrictive budgetary regimes are more convenient, since a smaller share of taxes are devoted to non-productive expenditures. However, it abstracts from a larger debt to private capital ratio. Although regime B and C are subject to a less stringent internal crowding out effect or even absent as in regime C. It necessarily translates in a more significant external crowding out effect.

A government adopting a budgetary regime must consider the following factors: first, a deep understanding of the cumulative feedback channels on growth. And second, how strongly public capital accumulation responds to taxation. Once the magnitude of those forces have been correctly assessed. Then only, can a government adopt the most suitable budgetary regime for sustained economic growth.

5 Conclusion

The goal of this paper was to shed light on how debt interacts with long-run growth under various assumptions, planning's horizons and fiscal policies. The main result suggests that the effects and the role of debt on long run growth and wealth hinges upon the structure of the economy.

In this paper two distinct perspectives were advanced to understand the interaction of debt with long run growth and wealth alongside the prominent role of fiscal policy. Neoclassical growth models emphasized the importance of the planning's horizon of agents. Whether it is indefinite or not. Accordingly, in the former situation a planner resorts to debt as a shock absorber. Whereas in the latter, to correct dynamic inefficient allocations.

Alongside growth models it appeared natural to investigate the role of fiscal policy. Since, any government issuing debt as a fiscal device triggers a corresponding fiscal response. The Chamley-Judd result was invalidated. Capital taxation provides insurance against idiosyncratic risk and shocks through the redistribution of assets. Therefore, taxation plays a role in generating aggregate welfare. Lucas and Stokey claim that debt should be issued in such a way that binds successive governments to stick to the Ramsay solution. In the absence of state contingent debt, a government employs debt as a fiscal instrument to minimize distortions from a second best.

In an endogenous growth model Saint-Paul spelled out how a planner couldn't simultaneously reconcile the aims of optimizing growth and welfare through debt. However, that issue could be solved by resorting to an investment subsidy. It is key to note that increasing public capital deficit financed can be achieved, if the latter outweighs negative feedback effects. In light of the fact that growth will respond differently according to each budgetary regime.

Finally, further insights could be gained by extending the endogenous growth model. How debt as a fiscal device can optimize public capital growth in the different regimes. This could involve defining a threshold that characterizes the optimal level of debt for an economy as implied by a particular framework.

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