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Firefighter Localization: A Lifesaving Geolocation System

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Abstract

During their interventions, firefighters are forced to evolve in dangerous and unpredictable conditions created by the environment of buildings in fire. Due to the threats of fire, thick smoke, and building collapses, there is a risk that they become disoriented, injured, or trapped inside the burning building. Being able to quickly and accurately localise the firefighters in trouble is essential to efficiently bring them assistance, which can make the difference between life and death in conditions of extreme heat and low oxygen. The objective of this master thesis is to design an indoor positioning system that is suited for the specific use case of firefighter interventions and simulate it to evaluate its performance. The propagation of electromagnetic waves in typical firefighter scenarios is modelled and simulated through ray-tracing to evaluate localisation impairments such as multipath that indoor environments are known to cause. To provide a realistic evaluation of the performance that can be achieved in those scenarios, a complete simulation of the system is implemented to consider the physical propagation and communication of wireless signals which are then used by positioning algorithms. A Multiple Signal Classification (MUSIC) algorithm is used to jointly estimate the time of arrival (ToA) and angle of arrival (AoA) of propagation paths, and the performance of traditional localisation algorithms in the firefighter scenario simulated are evaluated. A Random Sample Consensus (RANSAC) algorithm for a hybrid localisation scheme using TDoAs and AoAs is proposed to discard multipath components that do not correspond to a line-of-sight (LoS), and the use of reflected paths to improve localisation accuracy is explored. Their performances are evaluated by complete simulations in the modelled firefighter environment.

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List of Abbreviations

AoA	Angle-of-Arrival
AWGN	Additive White Gaussian Noise
BS	Base Station
CFR	Channel Frequency Response
CIR	Channel Impulse Response
CSI	Channel State Information
DFT	Discrete Fourier Transform
EM	Electro-Magnetic
FOR	First Order Reflection
IPS	Indoor Positioning System
LM	Levenberg-Marquardt
LoS	Line-of-Sight
MPC	MultiPath Component
MUSIC	MULTiple SIGNAL Classification
NLoS	Non Line-of-Sight
RF	Radio-Frequency
RANSAC	RANdom SAmple Consensus
RSS	Received Signal Strength
Rx	Receiver
TDoA	Time-Difference-of-Arrival
ToA	Time-of-Arrival
Tx	Transmitter
ZF	Zero-Forcing

Introduction

As emergency responders, firefighters have to intervene in hazardous situations, such as building fires or collapses, in order to secure locations, rescue people, and prevent additional damage. The dangerous environments in which they evolve often require them to put themselves at risk, and while they are well-trained to face such scenarios, it can happen in the midst of the chaos that a firefighter finds himself in a difficult position. For example, building fires create thick smoke, which drastically reduces the firefighter's field of vision, sometimes to less than a metre, causing the firefighter to be lost inside the building. Additionally, the collapse of a wall section, blocking or burying the firefighter, can also prevent him from moving and leaving the building, the most tragic scenario being that he runs out of his limited oxygen reserves before the help arrives. In those cases, it is crucial to be able to quickly help and ensure the security of the rescuer, as well as to keep the rest of the operation on track.

In that context, one of the key elements to ensure a proper response is access to an accurate localisation of the endangered firefighter. It allows his fellow rescuers to know where they must bring him the help he needs. To enable efficient and accurate positioning, the geolocation system must meet the needs of firefighters and overcome the challenges posed by the building fire environment. Some of the challenges to consider include smoke, system deployment, or indoor localisation. In case of fires, smoke has a high-density of black carbon aerosols, which attenuates electromagnetic waves, especially those at very-high frequency or optical waves. Because rescue operations are often a race against the clock, the system must be light enough for firefighters to deploy it quickly and respect other criteria like autonomy or cost. For example, if the system includes base stations, it means keeping the number of deployed antennas reasonable, as well as any calibration or synchronisation phases.

The principal challenge to ensure accurate localisation is the strong multipath effects caused by the numerous walls and other cluttering objects, as well as a high probability of having no line-of-sight (NLoS) between the transmitter and the receiver, which is typical of indoor scenarios. This is an important issue, as traditional localisation techniques often rely on measuring the time of arrival (ToA) or the angle of arrival (AoA) of the signal from the transmitter to the receiver. However, if the measured ToA or AoA does not correspond to the direct path but to a path with multiple reflections, it gives incorrect information about the distance separating the transmitter from the receiver or its direction. Consequently, algorithms that are robust to these impairments are needed.

With this in mind, the *Zone de Secours Brabant wallon* is interested in assessing the impacts of the difficult localisation conditions caused by indoor environments and firefighters specific requirements on a wireless positioning system. The goals of this work are to perform a first evaluation of the propagation of radio signals in indoor environments, formulate a system architecture suitable for the use of firefighters, evaluate the performance achieved by traditional localisation techniques such as multilateration, and propose some techniques that are robust to the principal challenges of localisation in firefighters scenario. The approach adopted in this work is to simulate the whole localisation process, which includes the propagation channel associated with typical building configurations, the wireless communication happening on that channel, and the localisation algorithm used to estimate the firefighter position.

Contributions. Using ray-tracing simulations on typical building configurations built with a 3D modelling software, realistic propagation channels are simulated to obtain a first insight into multipath presence and NLoS conditions. A system architecture that is adapted to the application of firefighters is formulated, and a simulation of the whole system functioning in the propagation environment obtained with ray-tracing is implemented. The performance of classical localisation algorithms is evaluated in the firefighter environment considered, and some methods that are robust to multipath conditions are developed.

Structure. First, Chapter 1 presents the propagation model that is used to represent the firefighter environment. It describes the theory of electromagnetic waves that is used to perform ray-tracing simulations. The assumptions made to model the firefighter environment considered in this work are described, and the simulated propagation channel corresponding to the modelled environment is analysed to evaluate its potential impact on localisation.

Then, the main criteria influencing the design of indoor positioning systems (IPS) for emergency responders are detailed in Chapter 2, and a review of existing techniques that can be applied for IPS is conducted. Based on this, it presents the specifications formulated with firefighters of *Zone de Secours Brabant wallon*, and a system architecture that respects those requirements is proposed. The implementation and main parameters of the localisation system are described and the methodology to realistically simulate its functioning in the firefighter scenario is proposed.

Next, Chapter 3 describes the different localisation algorithms that are used to obtain position estimates of firefighters. The MUSIC algorithm is used to jointly estimate the ToA and AoA of the different multipath components (MPC) that compose the propagation channel. Localisation algorithms that use TDoAs, AoAs, or a hybrid combination of both measurements are described, and their performance is evaluated for the firefighter scenario considered.

The severe multipath present in indoor scenarios can strongly degrade the performance of traditional localisation techniques. Chapter 4 proposes a method to eliminate NLoS paths using a RANSAC algorithm to apply a hybrid localisation using only the paths corresponding to a line-of-sight (LoS). Finally, the use of the other MPCs detected with the MUSIC algorithm is considered by introducing a model for paths having been subject to at most 1 reflection, and the final performance of the system using these algorithms in the firefighter scenario is evaluated.

Chapter 1

Propagation Model

Wireless localisation systems work by exploiting the propagation of electromagnetic (EM) waves to gain information on the location of a target of interest. For example, one of the most intuitive methods consists of transmitting a radio signal from a target to a base station (BS), measuring the time taken by the signal to propagate from the target to the base station, and computing the corresponding distance by multiplying it with the signal velocity (typically the speed of light). However, the environment in which the radio signal evolves affects its propagation, creating different types of interaction that modify its strength or trajectory. These interactions include signal attenuation, scattering, reflection, refraction, or diffraction, and can drastically influence the performance of the localisation system. In practical scenarios, which can be quite complex, it is difficult, if not impossible, to predict the exact interactions that will affect the radio signal. In that context, the environment in which a localisation system is deployed strongly influences its performance, and it is crucial to build a propagation model to estimate the different effects to which the radio signal is subject to design a robust and accurate localisation system.

This chapter describes the propagation model used to represent the typical environment in which firefighters intervene and the ray-tracing techniques that are used to simulate the wireless channel. First, Section 1.1 briefly explains the basic theory of EM waves and their propagation. Then, Section 1.2 defines the wireless channel model that represents the typical firefighter environment. Finally, Section 1.3 describes the modelling of the environment and analyses the corresponding wireless channel obtained through ray-tracing simulations with the Sionna RT library [2].

1.1 Electromagnetic waves

This section describes the theory of EM waves that is used for the model of the ray-tracing tool of Sionna. It is based on part of Orfanidis' book on EM waves and antennas [3], the recommendations of the ITU-R P.2040-4 [4], and follows a similar structure to the Sionna primer on electromagnetics [5].

1.1.1 Time-harmonic plane waves

Time-harmonic plane waves are one of the simplest types of EM waves that propagate in a fixed direction $\mathbf{x}$ in a homogeneous and potentially conductive medium. Even if true plane waves do not actually exist, they are incredibly useful to model practical phenomena like reflection and refraction, or approximate waves in the far-field region.

This type of EM wave has its coupled electric and magnetic fields $\mathbf{E}(\mathbf{x}, t)$, $\mathbf{H}(\mathbf{x}, t) \in \mathbb{C}^3$ that depend on time on a cosine or a sine. Starting with the electric field $\mathbf{E}(\mathbf{x}, t)$, this can be written in the phasor domain as

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x})e^{j\omega t} = \mathbf{E}_0 e^{-j\mathbf{k}^H \mathbf{x}} e^{j\omega t}, \quad (1.1)$$

where $\mathbf{E}_0 \in \mathbb{C}^3$ is the field phasor, $\omega = 2\pi f$ is the angular frequency, and $\mathbf{k} = k\hat{\mathbf{k}}$ is the wave vector, with $\hat{\mathbf{k}}$ a unit norm vector and $k = \omega\sqrt{\varepsilon\mu}$ is the wave number that depends on the permittivity ε and the permeability μ of the medium in which the wave propagates. They are defined as

$$\varepsilon = \eta\varepsilon_0 \quad (1.2)$$

and

$$\mu = \mu_r\mu_0 \quad (1.3)$$

with η and ε_0 the complex relative and vacuum permittivities, μ_r and μ_0 the relative and vacuum permeabilities. The complex relative permittivity depends on the real relative permittivity ε_r of a non-conductive dielectric medium, and also takes into account the conduction loss due to the conductivity σ of a lossy medium:

$$\eta = \varepsilon_r - j\frac{\sigma}{\varepsilon_0\omega}. \quad (1.4)$$

The speed of light of such a wave corresponds to the real part of $\frac{\omega}{k}$ given by

$$c = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu}} \left\{ \frac{1}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon_0\varepsilon_r} \right)^2} + 1 \right) \right\}^{-\frac{1}{2}}. \quad (1.5)$$

For non-conducting materials where $\sigma = 0$, it becomes

$$c = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu}}, \quad (1.6)$$

In the special case of vacuum, for which $\varepsilon_r = 1$, the speed of light is denoted as $c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \approx 3 \cdot 10^8$ m/s. The associated magnetic field $\mathbf{H}(\mathbf{x}, t)$

$$\mathbf{H}(\mathbf{x}, t) = \frac{\hat{\mathbf{k}} \times \mathbf{E}(\mathbf{x}, t)}{Z} = \mathbf{H}(\mathbf{x})e^{j\omega t} \quad (1.7)$$

where $Z = \sqrt{\frac{\mu}{\varepsilon}}$ is the wave impedance. In vacuum, the impedance $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 376.73\Omega$. In reality, the actual EM wave is composed of the real parts of $\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{H}(\mathbf{x}, t)$.

In order to quantify the amount of energy carried by an EM wave, the time-averaged Poynting vector, which describes the directional energy flux in W/m², is defined for a plane wave as

$$\mathbf{S}(\mathbf{x}) = \frac{1}{2} \text{Re} \{ \mathbf{E}(\mathbf{x}) \times \mathbf{H}(\mathbf{x}) \} = \frac{1}{2} \text{Re} \left\{ \frac{1}{Z} \right\} \|\mathbf{E}(\mathbf{x})\|^2 \hat{\mathbf{k}}. \quad (1.8)$$

1.1.2 Far field of transmitting antenna

It is assumed that the electric field of a transmitting antenna at an observation point $\mathbf{x} = (r, \theta, \phi)$ from its centre, with r being the distance, θ the zenith angle and ϕ the azimuth angle, can be described by a spherical wave:

$$\mathbf{E}(r, \theta, \varphi, t) = \mathbf{E}(r, \theta, \varphi)e^{j\omega t} = \mathbf{E}_0(\theta, \varphi) \frac{e^{-jk_0 r}}{r} e^{j\omega t}. \quad (1.9)$$

In contrast to a plane wave, the amplitude of the electric field decays as $\frac{1}{r}$, and $\mathbf{E}_0$ depends on the direction of observation, from which one can define the antenna field pattern $\mathbf{F}(\theta, \phi)$, which represents how the antenna spreads its power depending on the direction, as

$$\mathbf{F}(\theta, \varphi) = \frac{\mathbf{E}_0(\theta, \varphi)}{\max_{\theta, \varphi} \|\mathbf{E}_0(\theta, \varphi)\|}. \quad (1.10)$$

For a spherical wave, the time-averaged Poynting vector is defined as

$$\mathbf{S}(r, \theta, \varphi) = \frac{1}{2Z_0} \|\mathbf{E}(r, \theta, \varphi)\|^2 \hat{\mathbf{r}} \quad (1.11)$$

where $\hat{\mathbf{r}}$ is the radial unit vector. For an ideal isotropic antenna with input power P_T , it becomes

$$\mathbf{S}_{\text{iso}}(r, \theta, \varphi) = \frac{P_T}{4\pi r^2} \hat{\mathbf{r}} \quad (1.12)$$

From (1.9), (1.11) and (1.12), the directional antenna gain $G(\theta, \phi)$ of an antenna is defined as the ratio between the Poynting vector of the wave radiated by the antenna and the Poynting vector radiated by an ideal isotropic antenna, which can be written as

$$G(\theta, \phi) = \frac{\|\mathbf{S}(r, \theta, \varphi)\|}{\|\mathbf{S}_{\text{iso}}(r, \theta, \varphi)\|} = \frac{2\pi}{Z_0 P_T} \|\mathbf{E}_0(\theta, \varphi)\|^2 = G \|\mathbf{F}(\theta, \varphi)\|^2, \quad (1.13)$$

where the antenna gain G can be written as

$$G = \frac{2\pi}{Z_0 P_T} \max_{\theta, \varphi} \|\mathbf{E}_0(\theta, \varphi)\|^2 \quad (1.14)$$

For the transmitting antenna, one obtains the electric far field $\mathbf{E}_T$ as

$$\mathbf{E}_T(r, \theta_T, \varphi_T) = \sqrt{\frac{P_T G_T Z_0}{2\pi}} \frac{e^{-jk_0 r}}{r} \mathbf{F}_T(\theta_T, \varphi_T) \quad (1.15)$$

1.1.3 Receiving antenna

On the side of the receiving antenna, it is assumed that the received EM wave, originating from the transmitted spherical wave with an electric field $\mathbf{E}_T(r, \theta_T, \varphi_T)$, is a plane wave with an electric field $\mathbf{E}_R$, which does not depend on (r, θ, ϕ) . This assumption holds because even if the transmitting antenna radiates a spherical wave, the receiving antenna is supposed to be located in the far-field. The Poynting vector S_R of the received wave is then given by

$$\mathbf{S}_R = -\frac{1}{2Z_0} \|\mathbf{E}_R\|^2 \hat{\mathbf{r}}(\theta_R, \varphi_R) \quad (1.16)$$

where the minus sign indicates that the antenna receives power and $\hat{\mathbf{r}}(\theta_R, \varphi_R)$ is the radial unit vector in the spherical coordinate system of the receiver. The power received by the antenna P_R is given by the product of the energy flux density S_R and the direction-dependent effective area $A_R(\theta_R, \phi_R)$, also called aperture, of the antenna:

$$P_R = \|\mathbf{S}_R\| A_R(\theta_R, \phi_R). \quad (1.17)$$

With the aperture of an isotropic antenna A_{iso} being equal to $\frac{\lambda^2}{4\pi}$, the direction-dependant aperture of the receiving antenna $A_R(\theta_R, \phi_R)$ is given by

$$A_R(\theta_R, \phi_R) = A_{\text{iso}} G_R(\theta_R, \phi_R) = \frac{\lambda^2}{4\pi} G_R \|\mathbf{F}(\theta_R, \phi_R)\|^2 = A_R \|\mathbf{F}(\theta_R, \phi_R)\|^2 \quad (1.18)$$

with the aperture A_R defined as

$$A_R = A_{\text{iso}} G_R = \frac{\lambda^2}{4\pi} G_R. \quad (1.19)$$

The received power can then be expressed as

$$P_R = \frac{1}{2Z_0} \|\sqrt{A_R} \mathbf{F}_R(\theta_R, \phi_R)^R \mathbf{E}_R\|^2 = \frac{\lambda^2}{4\pi} G_R \frac{1}{2Z_0} \|\mathbf{F}_R(\theta_R, \phi_R)^H \mathbf{E}_R\|^2 \quad (1.20)$$

where the dot product $\mathbf{F}(\theta_R, \phi_R)^H \mathbf{E}_R$ takes into account the potential polarisation mismatch.

1.1.4 Propagation channel

In the case of free space propagation, the electric field received $\mathbf{E}_R$ is equal to the electric field transmitted $\mathbf{E}_T(r, \theta_T, \phi_T)$. In the general case, multiple scattering processes modify the radiated field $\mathbf{E}_T(r, \theta_T, \phi_T)$ before it reaches the receiving antenna. The succession of scattering processes can be described by a transformation matrix $\tilde{\mathbf{T}}$, as done in [5], which also includes the free space propagation term $\frac{e^{-jk_0 r}}{r}$. From (1.15), the expression of the received electric field becomes

$$\mathbf{E}_R = \sqrt{\frac{P_T G_T Z_0}{2\pi}} \tilde{\mathbf{T}} \mathbf{F}_T(\theta_T, \varphi_T). \quad (1.21)$$

The frequency response of the propagation channel $C(f)$ is defined as the ratio between the receive voltage V_R and the transmit voltage V_T :

$$C(f) = \frac{V_R}{V_T}. \quad (1.22)$$

For an antenna of impedance Z , the relation between the antenna power P and its voltage V is given by

$$P = \frac{|V|^2}{8 \operatorname{Re}\{Z\}}. \quad (1.23)$$

For a single propagation path, the received power P_R depends on the transmit power P_T , it can be expressed by injecting (1.15) and (1.21) in (1.20):

$$P_R = \left(\frac{\lambda}{4\pi}\right)^2 G_R G_T P_T \|\mathbf{F}_R(\theta_R, \phi_R)^R \tilde{\mathbf{T}} \mathbf{F}_T(\theta_T, \phi_T)^T\|^2. \quad (1.24)$$

Using (1.23), the voltage V_R can be expressed as a function of the voltage V_T

$$V_R = 8 \operatorname{Re}\{Z_R\} \sqrt{\left(\frac{\lambda}{4\pi}\right)^2 G_R G_T P_T} \mathbf{F}_R(\theta_R, \phi_R)^R \tilde{\mathbf{T}} \mathbf{F}_T(\theta_T, \phi_T)^T t, \quad (1.25)$$

and assuming that the transmit and receive antennas have the real parts of their impedances Z_T and Z_R equal, the channel $C(f)$ for a single propagation path is obtained as

$$C(f) = \frac{\lambda}{4\pi} \sqrt{G_R G_T} \mathbf{F}_R(\theta_R, \phi_R)^H \tilde{\mathbf{T}} \mathbf{F}_T(\theta_T, \phi_T). \quad (1.26)$$

In the presence of multipath, the channel becomes for K multipath components:

$$C(f) = \sum_{k=1}^K \frac{\lambda}{4\pi} \sqrt{G_R G_T} \mathbf{F}_R(\theta_{R,k}, \phi_{R,k})^H \mathbf{T}_k \mathbf{F}_T(\theta_{T,k}, \phi_{T,k}) e^{-j2\pi f \tau_k}. \quad (1.27)$$

where the matrix $\tilde{\mathbf{T}}_k$ was replaced with $\mathbf{T}_k = \tilde{\mathbf{T}}_k e^{jk r_k} = \tilde{\mathbf{T}} e^{j2\pi f \tau_k}$, with $\tau_k = r_k/c_k$ the propagation time equal to the distance travelled r_k divided by the average propagation speed c_k of the k^{th} multipath component. With the notation $\alpha_k = \frac{\lambda}{4\pi} \sqrt{G_R G_T} \mathbf{F}_R(\theta_{R,k}, \phi_{R,k})^H \mathbf{T}_k \mathbf{F}_T(\theta_{T,k}, \phi_{T,k})$, where α_k is named the path coefficient, the channel can be written in a more common way as

$$C(f) = \sum_{k=1}^K \alpha_k e^{-j2\pi f \tau_k}, \quad (1.28)$$

which in time domain gives the channel impulse response $c(t)$ expressed as

$$c(t) = \sum_{k=1}^K \alpha_k \delta(t - \tau_k). \quad (1.29)$$

1.1.5 Reflection and refraction

Reflection and refraction occur when an EM wave reaches an interface between two different media. For an incident plane wave with a direction of propagation $\hat{\mathbf{k}}_i$ that hits a plane interface with a normal $\hat{\mathbf{n}}$, part of it is reflected in a direction $\hat{\mathbf{k}}_r$, and part of it is transmitted, or refracted, to the second medium in a direction of propagation $\hat{\mathbf{k}}_t$.

In order to compute in which directions and in what proportion the incident wave is reflected and transmitter, the electric fields $\mathbf{E}_i$, $\mathbf{E}_r$ and $\mathbf{E}_t$ of the incident, reflected and transmitted waves are decomposed into two polarisation components as

$$\begin{aligned}\mathbf{E}_i &= E_{i,\perp} \hat{\mathbf{e}}_{i,\perp} + E_{i,\parallel} \hat{\mathbf{e}}_{i,\parallel}, \\ \mathbf{E}_r &= E_{r,\perp} \hat{\mathbf{e}}_{r,\perp} + E_{r,\parallel} \hat{\mathbf{e}}_{r,\parallel} \\ \mathbf{E}_t &= E_{t,\perp} \hat{\mathbf{e}}_{t,\perp} + E_{t,\parallel} \hat{\mathbf{e}}_{t,\parallel}\end{aligned}\tag{1.30}$$

where $\hat{\mathbf{e}}_{i,\perp}$, $\hat{\mathbf{e}}_{r,\perp}$ and $\hat{\mathbf{e}}_{t,\perp}$ are the vectors orthogonal to the plane of incidence, and $\hat{\mathbf{e}}_{i,\parallel}$, $\hat{\mathbf{e}}_{r,\parallel}$ and $\hat{\mathbf{e}}_{t,\parallel}$ are the vectors parallel to the plane of incidence. These vectors are defined as

$$\begin{aligned}\hat{\mathbf{e}}_{i,\perp} &= \frac{\hat{\mathbf{k}}_i \times \hat{\mathbf{n}}}{\|\hat{\mathbf{k}}_i \times \hat{\mathbf{n}}\|} = \hat{\mathbf{e}}_{r,\perp} = \hat{\mathbf{e}}_{t,\perp} \\ \hat{\mathbf{e}}_{i,\parallel} &= \hat{\mathbf{e}}_{i,\perp} \times \hat{\mathbf{k}}_i. \\ \hat{\mathbf{e}}_{r,\parallel} &= \frac{\hat{\mathbf{e}}_{r,\perp} \times \hat{\mathbf{k}}_r}{\|\hat{\mathbf{e}}_{r,\perp} \times \hat{\mathbf{k}}_r\|} \\ \hat{\mathbf{e}}_{t,\parallel} &= \frac{\hat{\mathbf{e}}_{t,\perp} \times \hat{\mathbf{k}}_t}{\|\hat{\mathbf{e}}_{t,\perp} \times \hat{\mathbf{k}}_t\|}.\end{aligned}\tag{1.31}$$

The directions of propagation $\hat{\mathbf{k}}_r$ and $\hat{\mathbf{k}}_t$ depend on the direction $\hat{\mathbf{k}}_i$ of the incident wave. For an angle θ_1 between $\hat{\mathbf{k}}_i$ and $\hat{\mathbf{n}}$, the angle formed by $\hat{\mathbf{k}}_r$ and $\hat{\mathbf{n}}$ is also equal to θ_1 , while the angle θ_2 formed by $\hat{\mathbf{k}}_t$ and $\hat{\mathbf{n}}$ is given by Snell's Law as

$$\sin(\theta_2) = \sqrt{\frac{\eta_1}{\eta_2}} \sin(\theta_1)\tag{1.32}$$

where η_1 and η_2 are the complex permittivities of the first and second media, respectively. The directions of propagation $\hat{\mathbf{k}}_r$ and $\hat{\mathbf{k}}_t$ are now expressed as

$$\begin{aligned}\hat{\mathbf{k}}_r &= \hat{\mathbf{k}}_i - 2 \left(\hat{\mathbf{k}}_i^\top \hat{\mathbf{n}} \right) \hat{\mathbf{n}} \\ \hat{\mathbf{k}}_t &= \sqrt{\frac{\eta_1}{\eta_2}} \hat{\mathbf{k}}_i + \left(\sqrt{\frac{\eta_1}{\eta_2}} \cos(\theta_1) - \cos(\theta_2) \right) \hat{\mathbf{n}}.\end{aligned}\tag{1.33}$$

This situation of reference for two dielectric media is illustrated in Figure 1.1.

Finally, for values of θ_1 smaller than the critical angle $\theta_c = \arcsin \sqrt{\frac{\eta_2}{\eta_1}}$ the Fresnel equations give the relations between the incident electric field and the reflected and refracted electric fields:

$$\begin{aligned}r_\perp &= \frac{E_{r,\perp}}{E_{i,\perp}} = \frac{\sqrt{\eta_1} \cos(\theta_1) - \sqrt{\eta_2} \cos(\theta_2)}{\sqrt{\eta_1} \cos(\theta_1) + \sqrt{\eta_2} \cos(\theta_2)} \\ r_\parallel &= \frac{E_{r,\parallel}}{E_{i,\parallel}} = \frac{\sqrt{\eta_2} \cos(\theta_1) - \sqrt{\eta_1} \cos(\theta_2)}{\sqrt{\eta_2} \cos(\theta_1) + \sqrt{\eta_1} \cos(\theta_2)} \\ t_\perp &= \frac{E_{t,\perp}}{E_{i,\perp}} = \frac{2\sqrt{\eta_1} \cos(\theta_1)}{\sqrt{\eta_1} \cos(\theta_1) + \sqrt{\eta_2} \cos(\theta_2)} \\ t_\parallel &= \frac{E_{t,\parallel}}{E_{i,\parallel}} = \frac{2\sqrt{\eta_1} \cos(\theta_1)}{\sqrt{\eta_2} \cos(\theta_1) + \sqrt{\eta_1} \cos(\theta_2)}.\end{aligned}\tag{1.34}$$

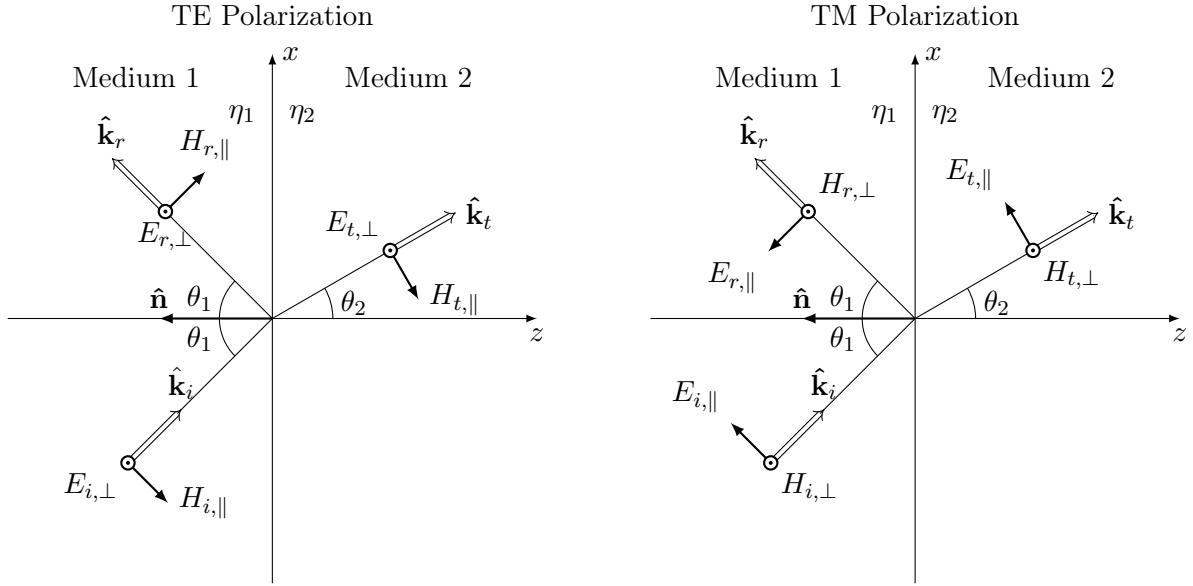


Figure 1.1: Situation of reference for the reflection and refraction of a plane wave by a plane interface.

1.1.6 Single-layer slab

A special scenario where reflection and refraction laws can be applied is when a plane wave propagating in the air encounters a single-layer slab. In that case, there are two interfaces where reflection and refraction occur, the first side of the slab where the wave comes from the air and goes through the slab, and the second side of the slab where the wave in the slab leaves it and goes back in the air. However, between the two interfaces, the wave trapped inside the slab can bounce back between the two sides, creating interferences. The reflection and transmission coefficients should be modified to take into account these interferences, [3] gives their expressions as

$$\begin{aligned} r &= \frac{r' (1 - e^{-j2q})}{1 - r'^2 e^{-j2q}} \\ t &= \frac{(1 - r'^2) e^{-jq}}{1 - r'^2 e^{-j2q}} \end{aligned} \quad (1.35)$$

with

$$q = \frac{2\pi d}{\lambda} \sqrt{\eta - \sin^2 \theta_0} \quad (1.36)$$

and r' being either $r_{\perp}$ or $r_{\parallel}$ depending on the polarisation of the electric field. With these coefficients, it is possible to model the slab as a single interface with a virtual thickness d and reflection and transmission coefficients $r(d)$ and $t(d)$ that depend on this thickness, which is the method used by the Sionna ray-tracing tool. Figure 1.2 illustrates the real propagation effects that occur in a single-layer slab the corresponding model. An approximation made by the model is to neglect the angular deflection of θ_2 and consider that the ray keeps the same trajectory. This is acceptable for thin objects like building walls.

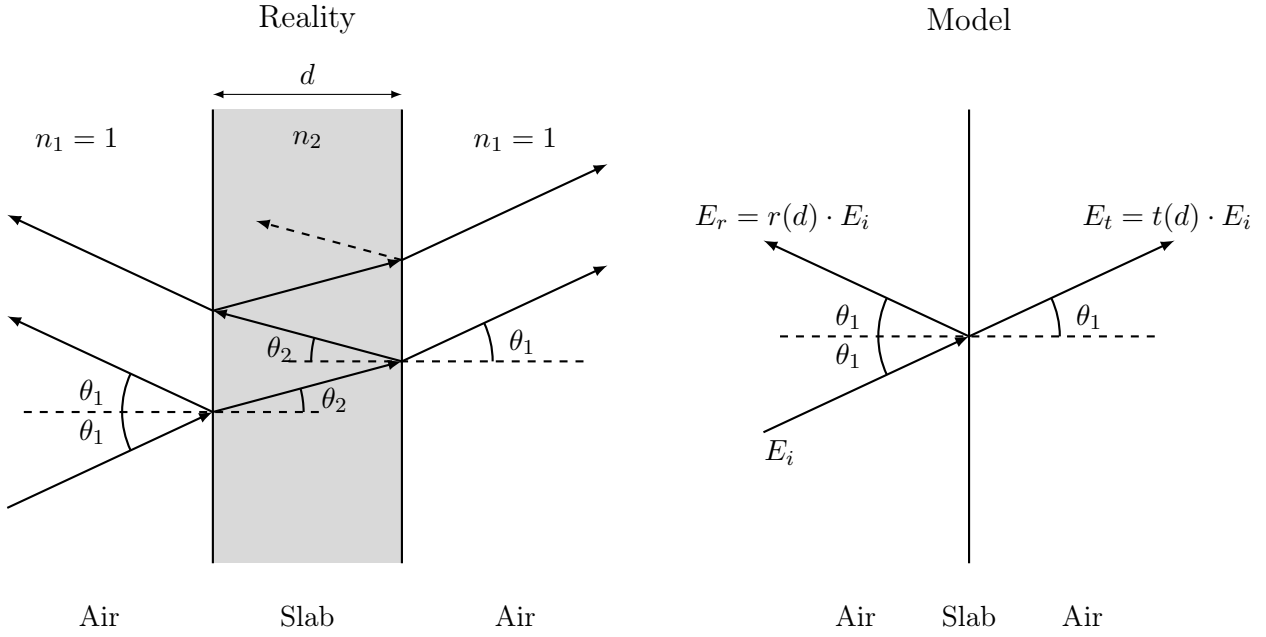


Figure 1.2: Reflection and refraction at a single-layer slab, reality vs ray-tracing model

1.2 Firefighter environment

In indoor scenarios, the propagation of EM waves is modified by walls or other cluttering objects that cause different scattering processes like reflection, refraction, scattering or diffraction. Compared to free space propagation where waves travel from a source to a destination in a straight line, those effects deflect emitting waves, causing them to take alternative paths.

The design of a localisation system is strongly influenced by the environment in which it is supposed to be deployed. The most common intervention scenario for firefighters, which is the one studied in this thesis, is the case of building fires. From the perspective of EM waves, there are two main elements specific to this environment that can influence their propagation, the smoke created by the burning objects and the walls of the buildings. This section discusses their impacts and explains how they are taken into account to simulate the channel with ray-tracing.

1.2.1 Building walls

Indoor environments are known for the presence of severe multipath and shadowing [6] due to the walls that create reflections and obstruct the radio signal. Depending on their material, thickness, or geometry, they affect the EM waves differently.

In this work, the building walls are considered to be made of concrete with windows made of glass. Typically, the thickness of concrete walls varies from 15 cm to 30 cm depending if it is load-bearer or not, the thickness of a window from 4 to 18 mm. According to the ITU-R recommendations [4], the relative permittivity ϵ_r and the conductivity σ of materials should be modelled with a frequency dependence as

$$\begin{aligned}\epsilon_r &= a f_{\text{GHz}}^b \\ \sigma &= c f_{\text{GHz}}^d.\end{aligned}\tag{1.37}$$

For frequencies below 100 GHz, the values recommended by the ITU-R for concrete and glass are reported in Table 1.1.

Material	a	b	c	d
Concrete	5.24	0	0.0462	0.7822
Glass	6.31	0	0.0036	1.3394

Table 1.1: Recommended values from the ITU-R to model electrical properties of materials

Due to the exponential, the loss when a wave goes through a concrete wall can quickly become important as the frequency increases. Figure 1.3 shows the transmission of a wave through a 20 cm concrete wall. One can see that, at 30 GHz, practically no signal manages to go through. This encourages the use of lower frequency waves that have a higher wall penetration for a localisation system. In anticipation of Chapter 2 which describes the architecture of the system, the carrier frequency used in this work is $f_c = 3.5$ GHz, similarly to typical 5G systems.

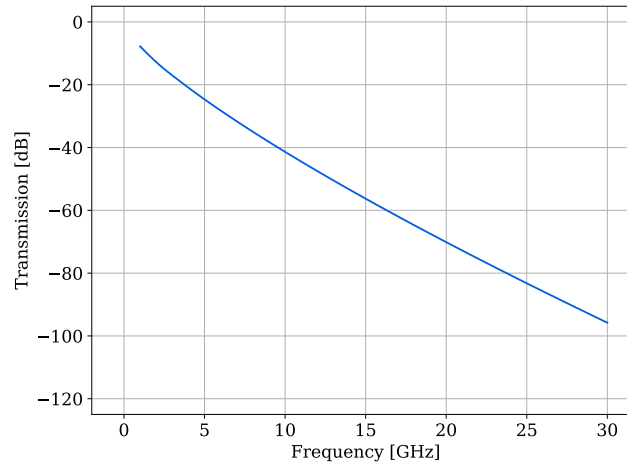


Figure 1.3: Transmission through a concrete wall of 20 cm thickness as a function of frequency

The material thickness also plays an important role in the attenuation that a wave is subject to when propagating through a lossy material. For a concrete wall, at 3.5 GHz, it ranges from 10 dB to 30 dB for a thickness varying between 15 and 30 cm for an angle of incidence perpendicular to the wall surface, as shown in Figure 1.4. For the modelling of the environment, the thickness of walls is considered to be of 20 cm, as it is a good average of reality. The corresponding attenuation is a bit under 20 dB.

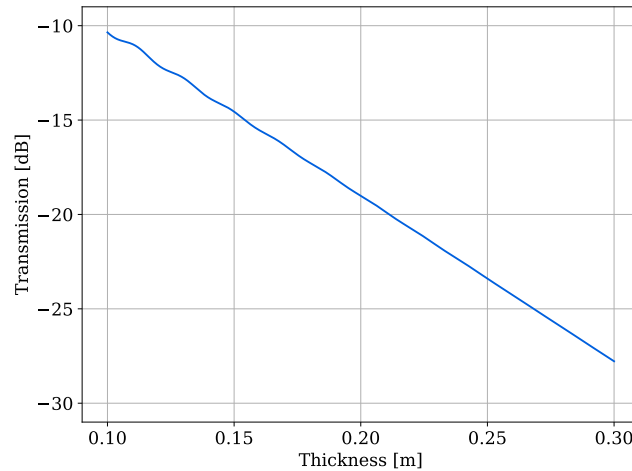


Figure 1.4: Transmission through a concrete wall at 3.5 GHz at normal incidence as a function of frequency

For a glass window at 3.5 GHz, the attenuation is significantly lower than that of concrete. This is due to its conductivity $\sigma_{\text{glass}} = 0.019$ being lower than that of concrete $\sigma_{\text{concrete}} = 0.123$. Figure 1.5 shows that the attenuation oscillates around 3 dB for typical thickness values. The oscillations are due to the multiple internal reflections that occur in the window, which interfere constructively or destructively, as explained in Section 1.1. Due to its low value compared to the attenuation of concrete, it is neglected in the environment modelling.

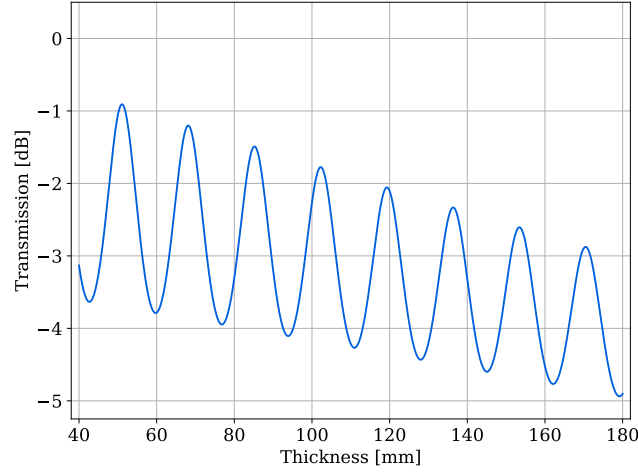


Figure 1.5: Transmission through a glass window at 3.5 GHz at normal incidence as a function of frequency

The last element that influences the interaction of the wave with the material is the angle of incidence with which it hits its surface. For a lossy material, part of the incident wave power is reflected, part of it is transmitted, and the rest is lost as heat. Depending on the angle, these proportions vary. For low angles of incidence, the transmitted power gradually decreases, but when the wave skims the surface, nearly all the power is reflected, as illustrated in Figure 1.6. In practical scenarios where a transmitter (Tx) tries to reach a receiver (Rx), it means that, depending on their position with respect to the walls, the amount of received power can quickly change, which makes it difficult to ensure a predictable scenario.

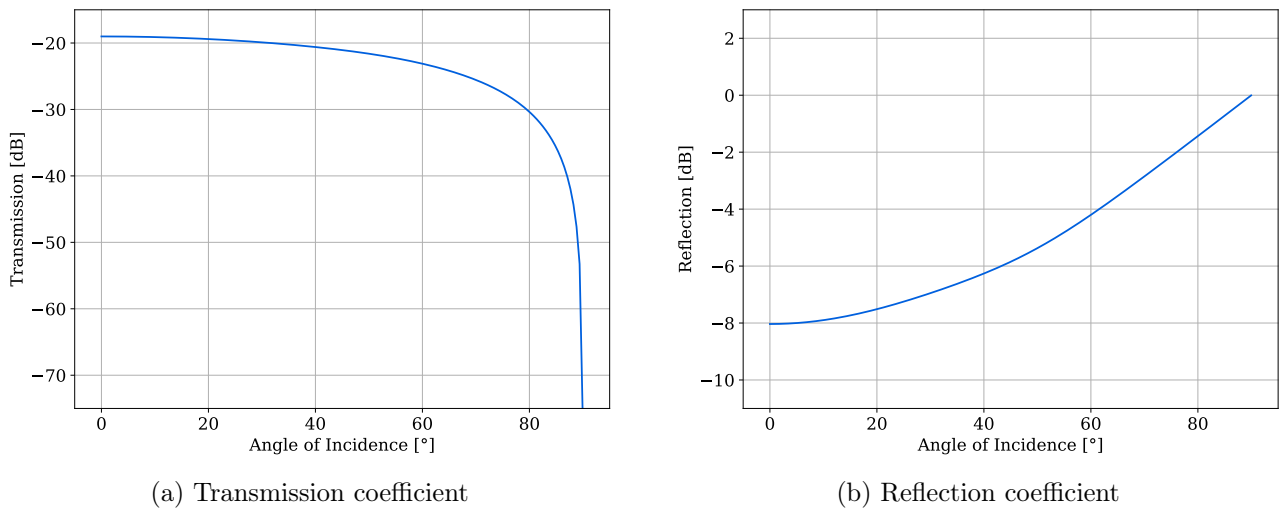


Figure 1.6: Transmission and reflection for a 20 cm concrete wall as a function of the angle of incidence

1.2.2 Smoke

The smoke released by building fires is commonly made of high-density black carbon aerosols that can reduce visibility to less than 0.1 m and have a significant impact on EM waves. The most obvious example is the propagation of optical waves that are completely attenuated, creating the poor visibility. In [7], the authors studied the impact of smoke in building fires on radar EM waves using simulations and experimental results and obtained values of attenuation coefficients for different frequencies. They reported an attenuation of approximately 0.03 dB/m at 2.95 GHz, 0.37 dB/m at 8.75 GHz, and 0.76 dB/m at 60 GHz.

In this work, the buildings considered have sizes of a few dozen metres. For a wave travelling a distance r , the attenuation of the electric field due to free space propagation is proportional to the ratio $\frac{1}{r}$ as shown in (1.15). For a distance r of 30 metres, it corresponds to an attenuation equal to $20 \log \frac{1}{r} \approx 30$ dB. In comparison, the attenuation due to smoke is only $0.03 \cdot r \approx 1$ dB at 2.95 GHz. Since the carrier frequency of the system designed in Chapter 2 is $f_c = 3.5$ GHz, the attenuation due to smoke is neglected for channel simulation.

1.3 Channel simulation

Ray-tracing is an efficient method to obtain realistic multipath affected channels for any arbitrary environment. It takes as input a virtual model of the scene of interest, where each object is represented as a mesh with assigned material properties. In this work, the virtual scene is modelled with the 3D modelling software Blender [8] to which the Sionna ray-tracing tool is applied [2].

Two different buildings are considered to evaluate how their sizes and configurations impact the radio signal propagation. The first building considered has dimensions of $15\text{m} \times 10\text{m}$ and the second one dimensions of $25\text{m} \times 15\text{m}$. For each of them, concrete walls are placed with windows and doors. To simulate the wireless channel, the transmitter, which is considered to be part of the firefighter's equipment, is placed inside the building, while the receivers are considered to be light base stations deployed around the building. Each device is represented as a half-wavelength vertical dipole, and they all lie in the same horizontal plane at $z = 1.5$ m. At a zero angle of elevation, the gain of a half-wavelength dipole is of 2.15 dBi for all azimuth angles. For the ray-tracing, the only scattering processes simulated are specular reflection and refraction, diffuse reflection and diffraction are not considered.

1.3.1 Small building scenario

The layout of the small building and the main propagation paths obtained by ray-tracing simulation for a specific transmitter position are shown in Figure 1.7. For readability reasons, the paths represented for each receiver are those that have path coefficients greater than -20 dB of the largest path coefficient. In this case, most receivers have a direct path to the transmitter with a few delayed paths due to reflections. The reflections that reach the receivers are the ones that go through doors or windows, since they do not suffer from the 20 dB attenuation of concrete walls. However, for Rx4, the direct path is not received as the radio signal would have to go through multiple walls. In contrast, the alternative paths due to reflections keep more of the signal energy.

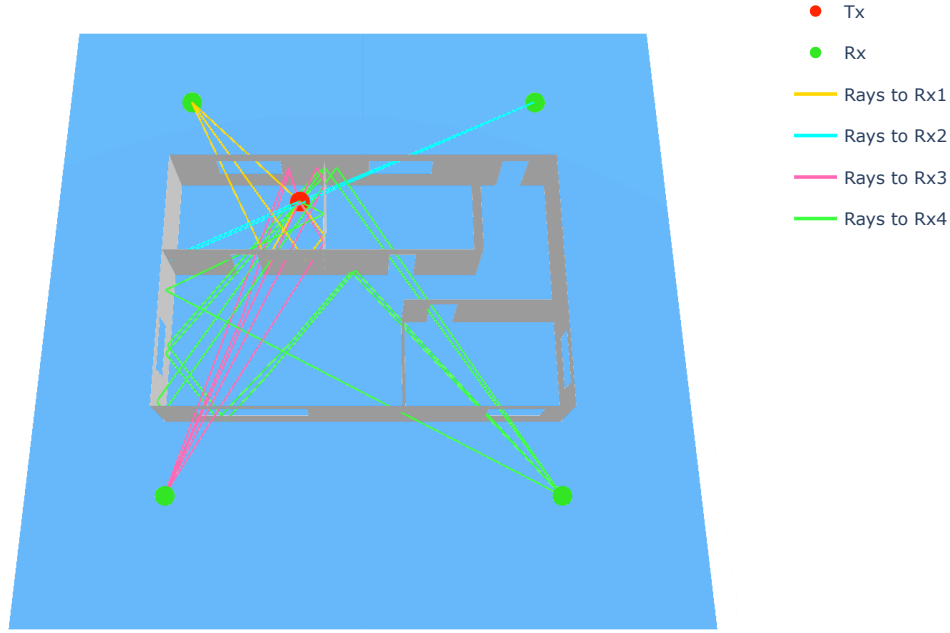
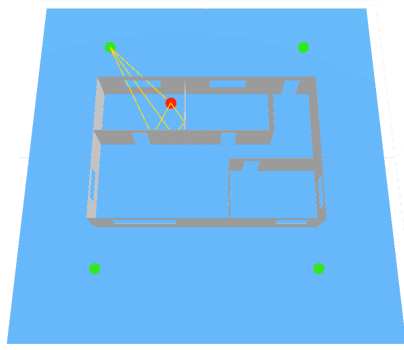


Figure 1.7: Model of firefighter environment and main propagation paths obtained with ray-tracing simulation

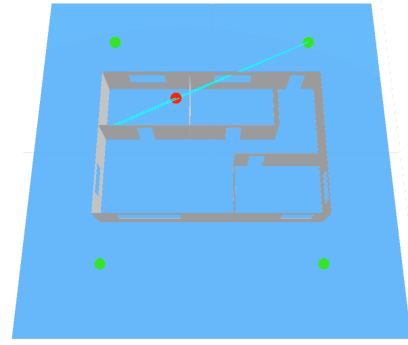
From Figure 1.8, one can see more clearly that the dominant multipaths are mainly paths with 1 or 2 reflections. In general, they come from different angles to the receiver, but those angles are relatively close to that of the dominant path. In the case of Rx2, the delayed path reaches it at the same angle as the main path, as it has bounced back in the opposite corner of the room. However, the delay of propagation is double that of the direct path. As a preliminary analysis for Chapter 3, one can anticipate that localisation techniques that use propagation delays are subject to more important outliers, i.e. measurement values that are far off the ones that should be given for the direct path, compared to techniques that use angles. This can significantly degrade positioning accuracy.

Another important element to consider is the value of the path coefficient. In practical scenarios, it needs to be large enough for the receiver to actually be able to detect the incoming signal from the ambient noise. In buildings and other scenarios, the distances are relatively small compared to other applications, losses due to free space propagation remain reasonable. However, the variability of losses is quite important. For a receiver that is separated from a transmitter by 1 metre or 10 metres, the difference of received power is 20 dB. In addition, the reflection and refraction of the signal by walls can also significantly reduce the signal power. This is illustrated by the Channel Impulse Response (CIR) of the different receivers in Figure 1.9, where one can see that Rx1 has a dominant path of above -60 dB, whereas the dominant path of Rx4 is of only -110 dB, since it is a lot further from the transmitter and the paths that reach it have been subject to reflection and refraction.

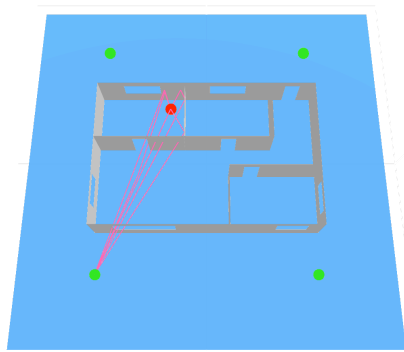
The CIRs also show that the propagation channel is quite dense, with a lot of multipath components that are sometimes very close to each other. Many of those MPCs have significantly lower coefficients, but some of them are comparable to the dominant one. The duration of the CIRs can be several times longer than the propagation delay of the first path, but it does not extend to more than 200 ns.



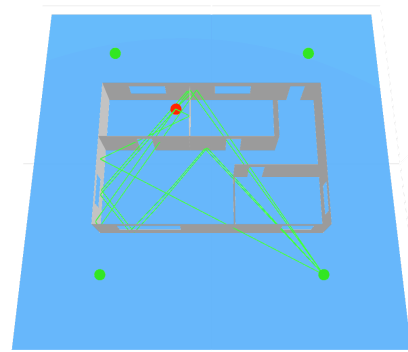
(a) Main paths for Rx1



(b) Main paths for Rx2



(c) Main paths for Rx3



(d) Main paths for Rx4

Figure 1.8: Main propagation paths for the different receivers

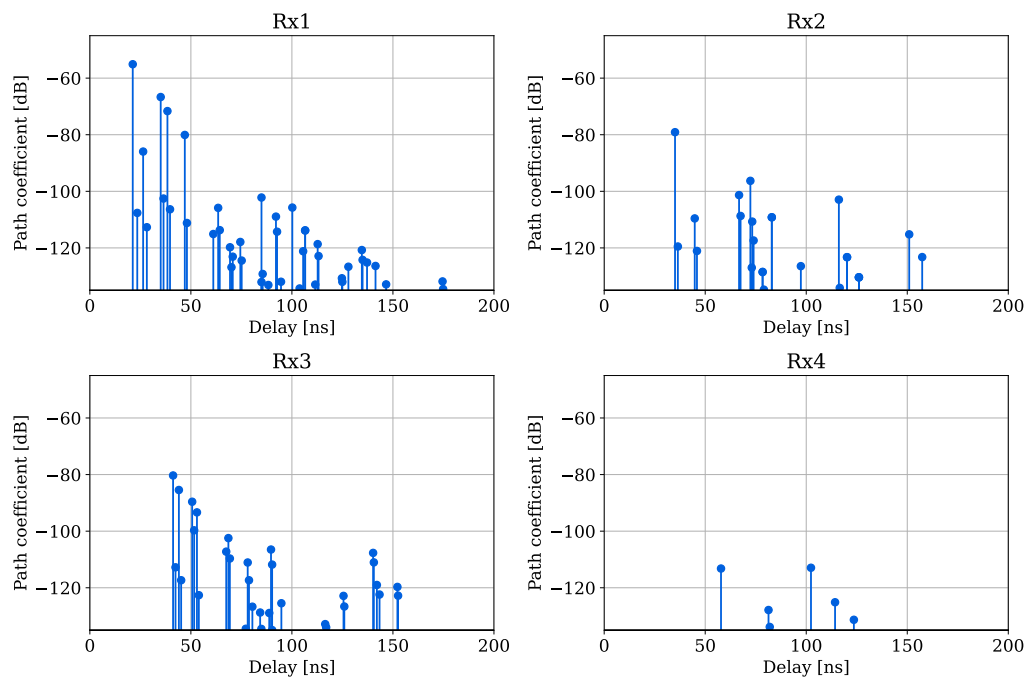


Figure 1.9: Channel impulse response of the different receivers

For a different target position, the propagation channels of the different receivers can drastically change, as shown in Figure 1.10 and Figure 1.11. Compared to the first case, Rx4 receives a direct paths, whereas Rx1 which is further receives paths that are subject to many reflections or refractions. This is mainly created by the corridor, which causes its channel to be a clutter of paths. for the other receivers, the behaviour is similar to the first case. For Rx3, the dominant paths come from reflections in the corridor, while for Rx2 and Rx4 the direct path is dominant as well as delayed paths.

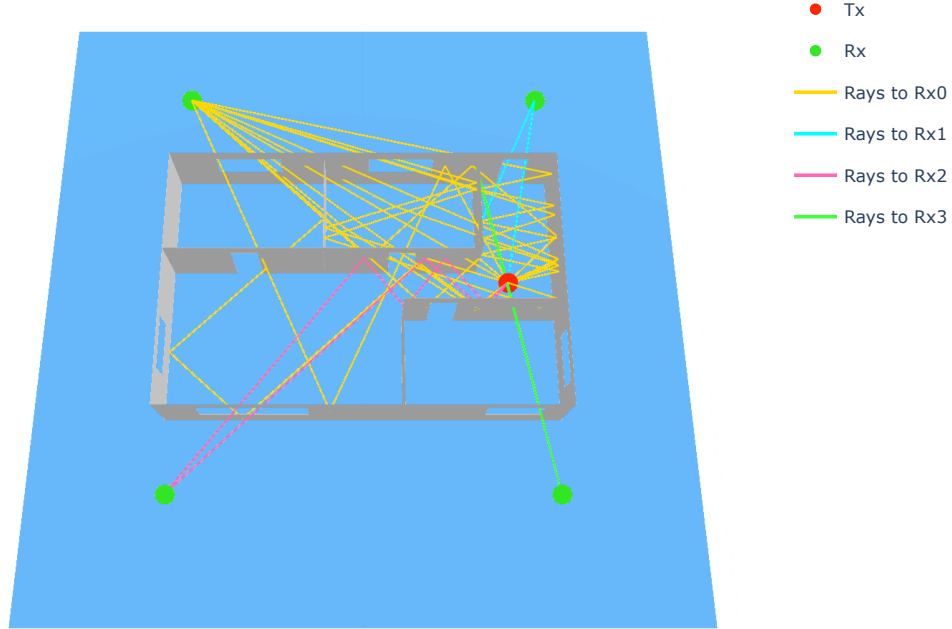
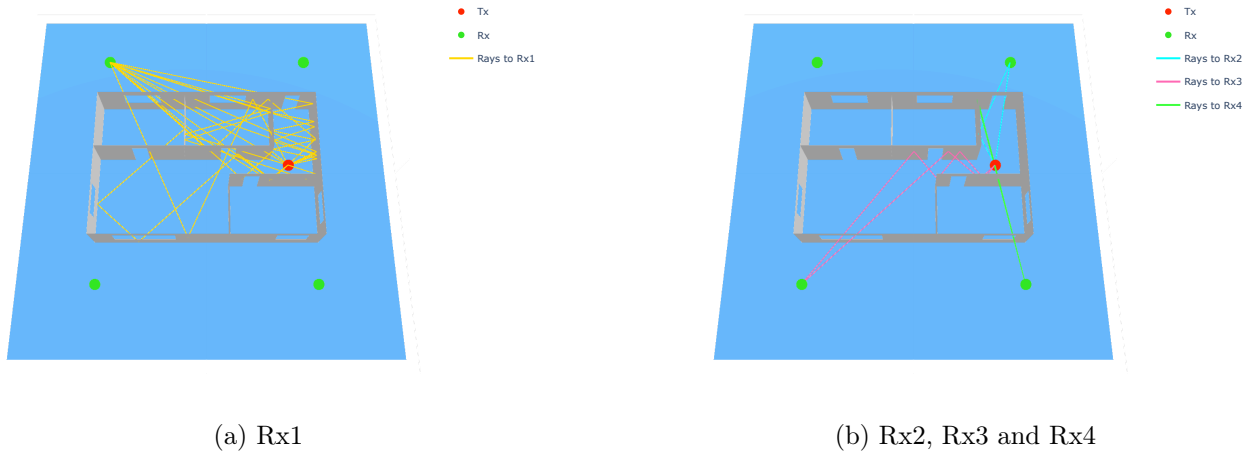


Figure 1.10: Main propagation paths for a different transmitter position



(a) Rx1

(b) Rx2, Rx3 and Rx4

Figure 1.11: Main propagation paths for the different receivers

Figure 1.12 shows that CIRs that are similar to the first case, but for different receivers. Rx1 now receives a clutter of paths with small path coefficients due to the multiple reflections created by the corridor and the larger distance that separates it from the transmitter. For Rx3, even though it is also located further from the Tx, the paths that reach it have relatively large coefficients compared to Rx1 due to the presence of less walls on their trajectory.

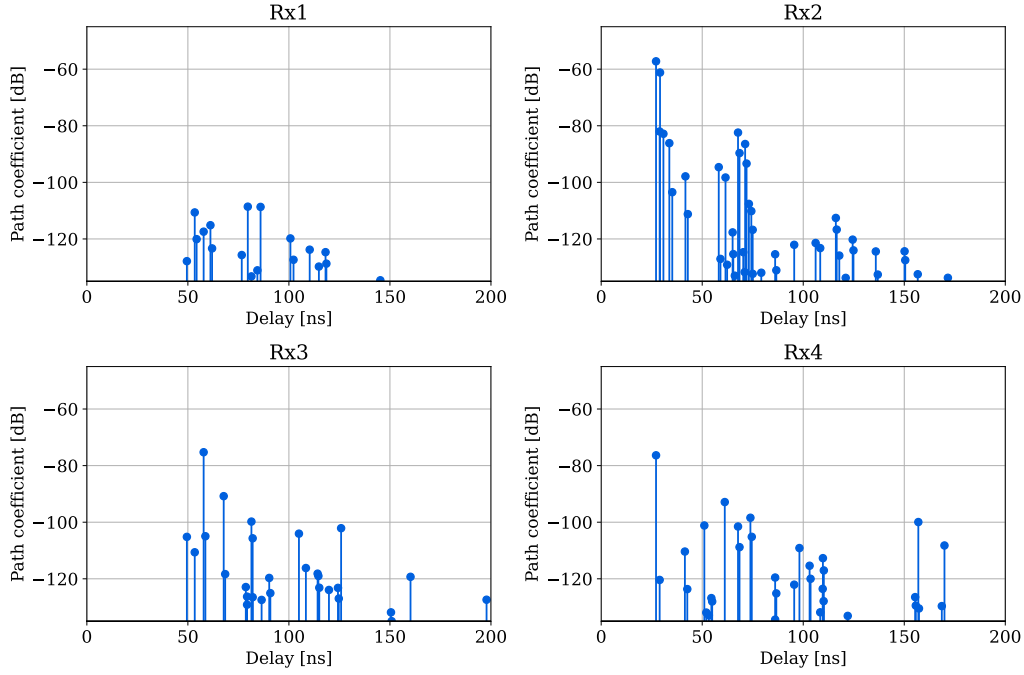


Figure 1.12: Channel impulse response of the different receivers

1.3.2 Large building scenario

The configuration of the large scene is represented in Figure 1.13 with the main propagation paths between the Tx and the different Rxs. The behaviour of the paths is globally the same as for the large scenarios, with some Rxs receiving direct paths and reflections, while others do not have the LoS as a dominant path. It can be noticed that the corridor has an effect somewhat similar to a waveguide, with the EM waves bouncing back between its two walls. Also, some of the paths come from reflections on the ground, which is for example the case of Rx1 and Rx4, their propagation time could be longer. However, Figure 1.13 shows that the additional distance travelled by a path reflected on the ground is actually quite small, as they essentially arrive at the same time. This is actually the case for many paths, as one may observe that they often come by pairs arriving at nearly the same instant. In practice, this means that the signals that reach the receivers by these two paths could interfere constructively, creating fading. Lastly, the path coefficients are globally lower than in the small building scenario due to the presence of more walls that can interfere with the waves propagation. The duration of the CIRs is also slightly increased because the scene is larger, but in general the maximum duration is not superior to 200 ns, and late paths are usually a few orders of magnitude below the dominant ones, so they could be neglected.

In conclusion, the indoor environment presents severe multipath, with several reflections that reach receivers from very different angles and with a propagation time that can be several times that of the LoS path. The numerous walls that block the LoS can cause an Rx to not detect any signal, hence the receivers should ideally be placed in front of windows that let through much more of the signal power. Because of multipath, the algorithms used for localisation should be able to correctly estimate the main MPCs and be robust to against them as they do not provide the information about the target position that is needed.

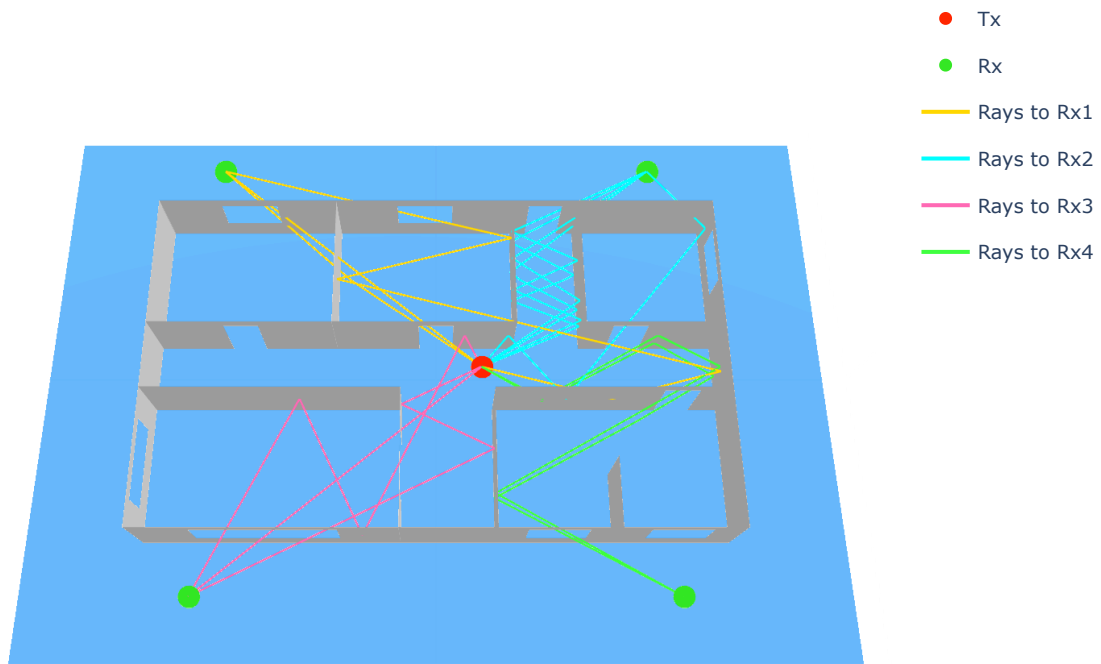


Figure 1.13: Model of firefighter environment and main propagation paths obtained with ray-tracing simulation

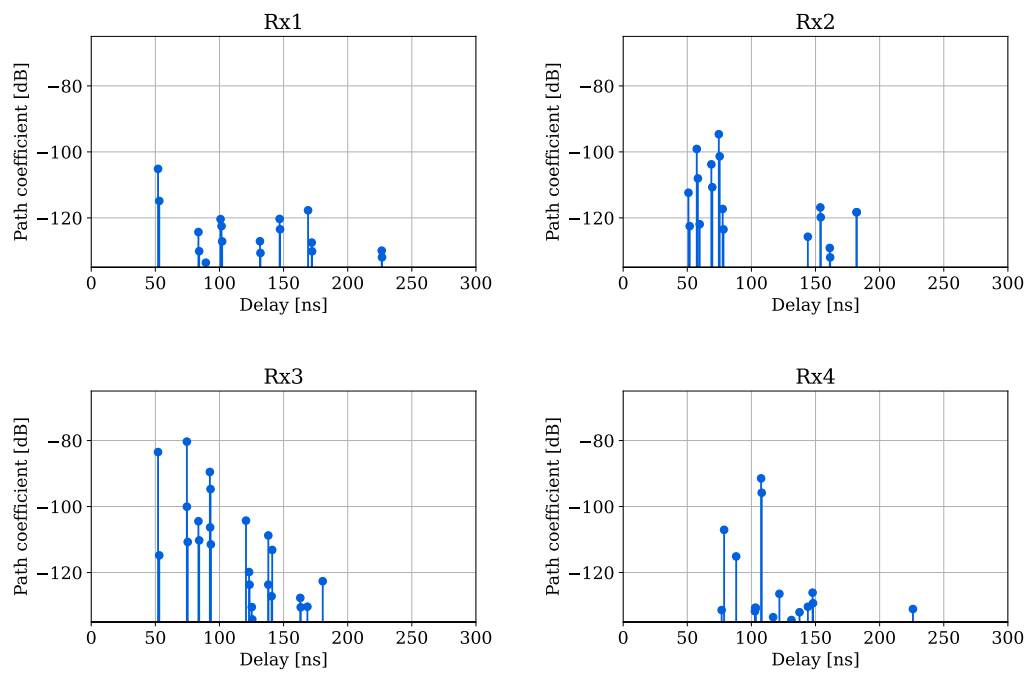


Figure 1.14: Channel impulse response of the different receivers

Chapter 2

System design

The ultimate goal of the system is to ensure an accurate and reliable localisation of firefighters in environments, which can be potentially large and complex buildings, during their rescue missions. Because classical localisation systems such as GPS are not suited to these scenarios due to multipath and NLoS conditions, there is a need to develop specific IPS that handle these effects. To achieve this, there exists a myriad of different techniques and technologies, each with their own strengths and weaknesses, advantages, and constraints. However, due to the specific conditions under which firefighters carry out their missions, every IPS does not necessarily respect the constraints and requirements inherent to rescue operations. Consequently, it is important to select those that are most in line with the needs and capabilities of the firefighter, and then design a system whose architecture is based on those specifications.

This chapter covers the main criteria that impact the quality and relevance of a localisation system for emergency responders and the design of the system that takes into account these considerations. First, Section 2.1 discusses the different characteristics of interest in the design of IPSs for emergency responders. Then, in Section 2.2, a brief review of existing indoor positioning systems is conducted and Section 2.3 presents the specifications that were formulated with the firefighters of Zone de Secours Brabant Wallon. Finally, Section 2.4 presents the proposed system architecture, which is used in the simulations, based on the discussions of the previous sections.

2.1 Localisation systems for emergency responders

To identify which techniques and technologies are relevant to firefighters, it is important to understand the different requirements that an IPS must meet to be applicable to their interventions. In [9], the authors have identified 6 main criteria that influence the design of IPS for emergency responders, which obviously include firefighters.

1. **Accuracy:** IPSs need sufficient accuracy on the position estimations to provide assistance to the responder efficiently. The precision level required can vary depending on the application, typically ranging from metre-level to room-level precision. A distinction can be made between horizontal accuracy and vertical accuracy, as it might, for example, be more important to know on which floor to send help. The reliability of the estimation is also crucial, as it is often better to have coarser but correct estimations than very precise ones with some outliers that are completely wrong.
2. **Information accessibility:** After the localisation process, the information about the responder location should be clearly communicated to the incident commander or the firefighter concerned, for example with a proper user interface. The latency of the position update should also be low enough to allow for a quick response. In addition, the transmission should also be secured against cyber threats.

3. **Adaptability:** The practical implementation of the localisation system is a major concern for emergency responders. Deployment should be efficient and manageable while also flexible for the different scenarios that firefighters can encounter, which can vary in terms of building infrastructure or coverage needed. It is also preferable that the system can adapt to dynamic changes of the environment, for example with an efficient re-deployment. Finally, potential requirements regarding pre-installed infrastructure or prior knowledge about the scene of intervention should be avoided to be suitable for all possible scenarios.
4. **Architecture:** The architecture of the system should be convenient for the usage of several rescuers simultaneously. For this, important aspects to consider are modularity, which impacts the integration, and scalability. IPSs should also be robust to the harsh conditions of incidents such as heat or flying debris. In addition, the size and weight of the equipment should be reasonable for transport and deployment.
5. **Autonomy:** Autonomy is a crucial consideration for IPSs, as they should be able to last the entire duration of the intervention, which can often stretch depending on the severity of the situation, and be easily powered. Hence, solutions that are battery-powered and energy-efficient are usually preferred, for autonomy reasons but also for practicability and cost considerations.
6. **Cost:** Affordability plays a central role in the adoption of an IPS. The cost of the system should be within the emergency response budget and allows the possibility of replacing it in the event of damage, which can be expected with the rescue environments. Similarly, maintenance costs of usable parts should be manageable.

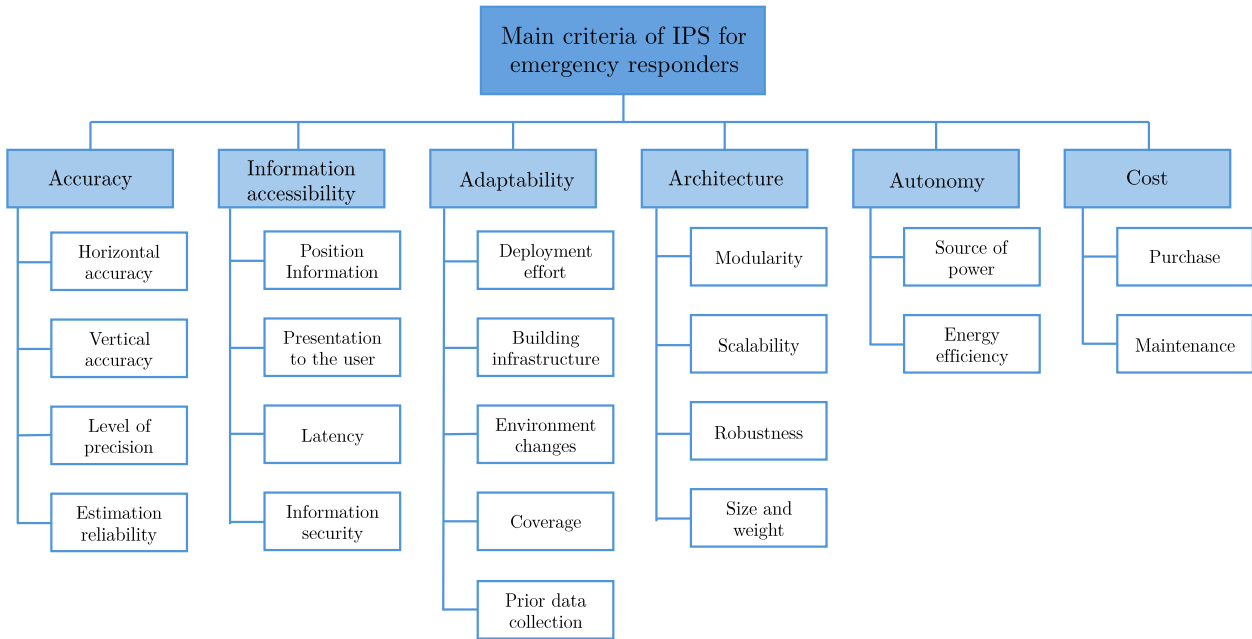


Figure 2.1: Summary of the main criteria that influence the design of IPS for emergency responders, adapted from [9].

2.2 Review of indoor positioning systems

In order to properly characterise indoor positioning systems, the analysis can be divided into three main parts, the distance measurement techniques that dictate the way information about the target location is acquired, the technology used to obtain those measurements, and the localisation methods

that are the actual algorithms that use those distance measurements to determine where the target is the most likely to be located. As some methods can be applied to different types of measurements, which can be obtained with different technologies, and one type of measurement can be given as input to several algorithms, a localisation system is a specific combination of a measurement technique and a localisation algorithm.

2.2.1 Distance measurements techniques

In order to localise the target, IPSs try to measure the distance separating it from the sensors. There exist several ways to acquire those distance estimations, some exploit the properties of radio signals, such as the propagation time or the angle of arrival of radio signals, while others work with measurements of physical quantities, for example with an Inertial Measurement Unit (IMU).

Propagation time

The first way to obtain information about the target position is by exploiting the propagation time of the radio signal from a transmitter to a receiver. From information on the delay of propagation τ , one can easily retrieve the corresponding distance d knowing the propagation speed of the wave, which is equal to the speed of light c_0 in vacuum, with $d = c_0\tau$. There exist different methods to exploit the propagation time of the waves.

The most direct method to use the waves propagation time is to measure the exact time of arrival (ToA) that the wave has taken to travel from a transmitter to a receiver. For one receiver, knowing the distance d that separates it from the transmitter gives the information that the locus of points where the transmitter could be located is a circle centred around the receiver of radius d , as illustrated in Figure 2.2. Combining the information of multiple receivers allows one to pinpoint the actual target position at the intersection of the loci. The main constraint of this method is that the transmitter and the i^{th} receiver must be synchronised, i.e., share the same time origin $t_0 = t_{0,i}$, so that when the wave reaches the receiver at instant t_i , it estimates the propagation time as $\tilde{\tau}_i = t_i - t_{0,i} = t_i - t_0$ which corresponds to the true propagation delay $\tau_i = t_i - t_0$. However, this is expensive to realise in practice, and a small synchronisation error can create non negligible errors in distance. For example, with $c \approx 3.10^8$ m/s, a synchronisation error of 1 nanosecond results in an error of 0.3 metres.

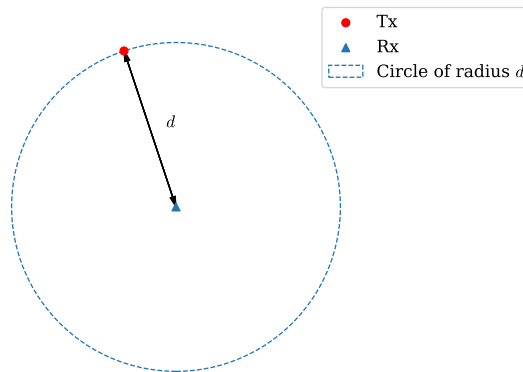


Figure 2.2: Circle corresponding to a ToA

Another method that addresses the synchronisation problem is to work with time differences of arrival (TDoA) between the different receivers. In contrast to using pure ToAs, which requires to have the time origin $t_{0,i}$ at the i^{th} receiver equal to t_0 , using TDoAs tolerates that they are not synchronised, i.e.

$t_{0,i} = t_0 + \Delta t_i$. TDoAs eliminate this variable by subtracting two measurements of different receivers to obtain the difference of distances $\delta_{i,j}$ that separate the target from the two receivers,

$$\delta_{i,j} = c(\tau_i - \tau_j) = c((t_i - t_0) - (t_j - t_0)) = c(t_i - t_j), \quad (2.1)$$

and estimate them as $\tilde{\delta}_{i,j}$ given by

$$\tilde{\delta}_{i,j} = c(\tilde{\tau}_i - \tilde{\tau}_j) = c((t_i - t_{0,i}) - (t_j - t_{0,j})) = c(t_i - t_j - \Delta t_i + \Delta t_j). \quad (2.2)$$

However, the different receivers still need to be synchronised together in order to have the same local time origin $t_{0,i}$ and $t_{0,j}$, or the same synchronisation error $\Delta t_i = \Delta t_j$ with the transmitter. The locus of point formed by this information is no longer a circle but a hyperbola, as shown in Figure 2.3. Similarly, one can also pinpoint the exact location of the target with the intersection of several loci. Because t_0 does not appear in the expression of $\delta_{i,j}$, the transmitter no longer needs to be synchronised with the receivers.

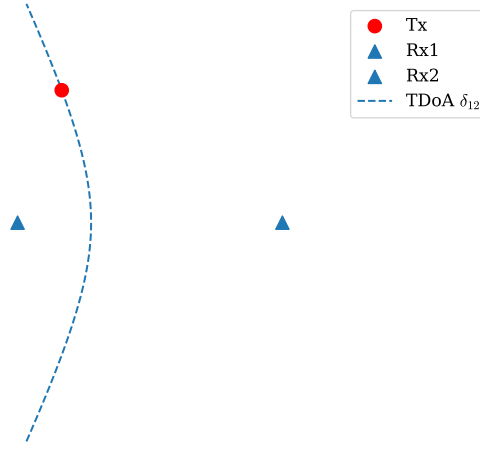


Figure 2.3: Hyperbola corresponding to a TDoA

Received signal phase or angle

A second technique consists of using the angle of arrival (AoA) or the angle of departure (AoD) of the radio signal to know along which direction the target is located. To do so, IPSs commonly use antenna arrays and measure the phase shifts at the different array elements to infer the angle at which the incoming signal hits the array [10]. In most scenarios, those systems make the assumption that the receive antenna is located in the far field of the transmitted signal. While this is generally the case for RF signal at low frequency, it can become an issue for higher frequencies or very large antenna arrays. Other methods include the use of directional antennas, which do not receive the same signal strength depending on the angle, to find the direction [10], or rotatable antennas to find a minimum received signal strength that corresponds to the AoA [11].

Wireless channel measurements

A third technique to exploit the radio signal is to use measurements of the physical layer of a communication network. They have the advantage of being directly available from the communication network and do not require any additional technique or hardware.

A first measurement is the Received Signal Strength (RSS), which represents the total power received from the channel. It gives information based on the physical principle that the amplitude of the received signal is inversely proportional to the distance it has travelled [12]. It has the advantage of

being simple to work with, as it is a single scalar value, but it is quite coarse and does not capture all the information on the channel. In addition, the received power depends not only on the distance but also on other elements, such as walls, that attenuate radio waves that pass through them. Such obstacles are typically present in indoor environments, which can cause RSS to not provide sufficient localisation accuracy.

A second measurement of the wireless channel is the Channel State Information (CSI), which is a more detailed channel measurement than RSS. It captures the evolution of the channel through time and allows one to discriminate its multipath components, which is particularly relevant for indoor environments that typically show severe multipath effects. CSI can take two different forms, the Channel Impulse Response (CIR) and the Channel Frequency Response (CFR), which are the time and frequency domain representations of multipath components. It can often replace RSS in an RSS-based system in the like of fingerprinting to offer better accuracy, at the expense of a more complex signal processing. CSI can also be used to retrieve other distance measurements, such as TDoAs, using algorithms like MUSIC (MUltiple SIgnal Classification) [13].

IMU

IMUs are electronic devices that are carried by a moving user or object that use a combination of gyroscopes, accelerometers, and magnetometers to obtain information on their motion. They are used to measure physical quantities such as linear acceleration, angular speed, or direction of the motion of a specific target, which is the person or object carrying the device [9]. These quantities do not provide information on the absolute position of the target. Instead, they give information relative to the initial position of the target, for example in which direction it has moved and at what speed. For that reason, IMUs are used in IPSs that have access to the initial target position, either by estimating it with a second type of measurements or by being given the information a priori, that then apply a tracking algorithm to follow the target with these relative measurements.

2.2.2 Wireless technologies

Measurements can be obtained with different technologies that use different types of hardware, configurations, costs, and other resources such as frequency bandwidth. The most commonly used technologies include Ultra-Wideband (UWB), Bluetooth Low Energy (BLE), Wi-Fi, Radio-Frequency Identification (RFID) or most recently 5th generation (5G) communication networks [6].

UWB

UWB uses ultra-short pulses, which occupy large bandwidths in frequency domain, typically at least 500 MHz, to measure ToAs or AoAs. It can achieve very high indoor accuracy, up to 20 cm. Because those pulses last a very short amount of time, UWB is quite robust to multipath because there is less overlap between successive pulses, hence it is easier to separate the different paths. Also, their sending rate is much lower than the actual pulse duration, thus the corresponding duty cycle is relatively low, typically 1:1000, which makes UWB tags quite energy efficient. However, it is relatively expensive, as the Tx and Rx hardwares must be complex enough to be able to accurately send and receive those ultra-short pulses.

BLE

BLE is widely used in short-distance scenarios, typically a few dozens of metres, for communication between devices, such as smartphones [14]. It is relatively cheap compared to other technologies like UWB, as it usually measures RSS to provide localisation functionality, which requires only simple hardware. However, this offers a lower precision in the range of a few metres [12]. More recent versions

of BLE also estimate AoAs [10], which requires slightly more expensive hardware due to the need for multiple antennas.

Wi-Fi

Indoor positioning can also be applied with Wi-Fi routers that can measure AoAs or ToAs from the CSI [14], which typically provide a metre-level accuracy. The main advantage of using Wi-Fi is the opportunity to reuse existing infrastructure, as most buildings are already equipped with Wi-Fi technology. One danger of relying on Wi-Fi in a building fire scenario is that routers can be damaged by heat or debris.

5G

5G networks are not only designed as data transmission technology, but also a positioning technology using TDoAs and AoAs using a relatively large bandwidth of about 100 MHz and antenna arrays. It typically operates between 3.5 and 6GHz, with the possibility of using millimetre waves [10], and is able to achieve sub-metre accuracy. However, 5G systems are quite expensive since they require specific infrastructure for and high hardware requirements.

2.2.3 Localisation methods

Localisation methods take as input distance measurements to provide the best estimate of the target position based on these measurements. Some algorithms are designed for specific types of measurements, while others are more generic and can be adapted to different kinds of measurement. Among the different methods, some are autonomous, meaning that they can directly produce a position estimate based on current measurements only, and some are training-dependent methods that require a learning phase before they can be applied.

Lateration

Lateration uses distance estimations that separate targets from base stations to estimate the target position. In order to obtain the distance measurements, ToAs or TDoAs can be used to recover the distance knowing the speed of the wave. Depending on the number of position coordinates to estimate, i.e. x and y for the 2D problem or x , y and z for the 3D problems, this technique requires at least 3 or 4 receivers, respectively, to estimate the position of the target. In the case of TDoAs, it can happen that there is an ambiguity between two positions, but it can often be solved by specifying the region of interest, or simply by adding an additional receiver. In practice, IPSs often use more than the minimum number of receivers to obtain a more precise estimation. As mentioned previously, the drawback of using those kinds of technique is the need for synchronisation between receivers, and sometimes with the transmitter. In addition, it is quite sensitive to multipath which creates delayed copies of the radio signal, corrupting the estimation.

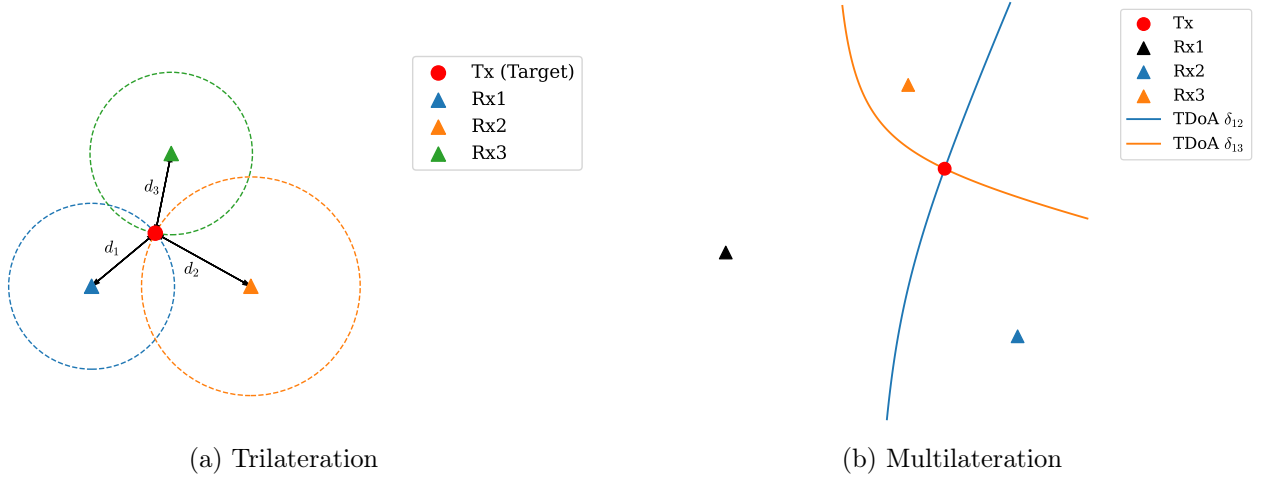


Figure 2.4: Illustration of lateration techniques

Triangulation

Triangulation uses the geometric properties of triangles to find the position of a point. It does so by measuring the angles between that point and other reference points for which their position is known, as illustrated in Figure 2.5. For localisation in a 2D plane, triangulation requires only two receivers to find a position estimate, and it has the advantage of not requiring any synchronisation. Although it does not require any synchronisation, it requires a more complex hardware due to the multiple antennas needed to measure the AoAs. It is also sensitive to multipath, since the reflections typically come from other directions than the LoS, which gives incorrect information about the position of the target [10].

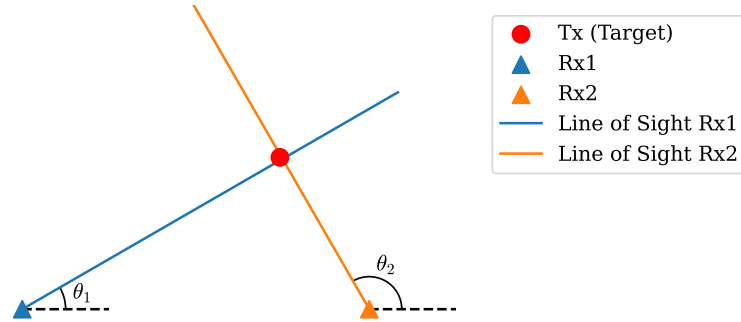


Figure 2.5: Illustration of triangulation technique

Proximity

Proximity localisation method works with a network of antennas deployed on the site in advance. The position of the target is given by the antenna that receives the strongest signal [12]. The accuracy of this method is directly related to the density of the network. The main constraint of such a system is that it needs to be deployed in advance. In addition, to improve the precision, the number of antennas deployed should be increased, which can rapidly become expensive. Another downside is that in case of building fires or collapses, the network nodes can be damaged, reducing the system performance.

Pedestrian Dead Reckoning

Pedestrian dead reckoning uses IMUs measurements to obtain information relative to the previous position of the target. Thus, it demands to know the initial position of the target, from which the IMUs measurements can derive its current position, typically using tracking algorithms to follow the target motion [15]. The main weakness of PDR is that it is subject to a position drift error caused by the accumulated errors of the IMUs that are propagated by the tracking algorithm. To mitigate this, hybrid systems fuse the IMU measurements of PDR with traditional wireless localisation such as UWB [16], [17] or WLAN [18] that have the same uncertainty at each instant. In turn, PDR tracking is useful to filter outliers produced by wireless technologies in severe multipath conditions. In [16], a PDR-based system designed for firefighters coupled with a wireless sensor network is implemented. The WSN is deployed using a breadcrumb method that drops nodes dynamically, but this might require firefighter intervention and distract him, or he can run out of nodes that can also get destroyed.

Fingerprinting

Fingerprinting is a technique that learns to match a set of measurements, typically the RSS or CSI of several sensor nodes, to a position. More precisely, for a specific site, it uses a database of positions and their corresponding measurements obtained by surveying the site, for example by a technician who stands at each possible position, transmits a radio signal, and records the measurements obtained at the sensor nodes, and identifies the position that is closest to the set of measurements using a distance metric [19] or a machine learning algorithm [20]. Hence, fingerprinting works in two phases, an offline phase to survey the site, build the database, and potentially apply a machine learning algorithm, and an online phase in which the measurements obtained at the different sensor nodes are matched to a position of the database. This method performs quite well in complex NLoS environments [21], but requires a pre-deployment of the system and a time-consuming surveying phase for each different site.

Machine learning

With the rise of AI, researchers are also starting to apply Machine learning techniques to localisation problems. Indoor environments are an interesting scenario for applying those techniques, as complex relations such as those created by multiple reflections and the absence of LoS are known to be surprisingly well captured by those algorithms. For example, in [22], a deep learning model that exploits AoAs is implemented for multipath environments. Although those techniques can offer high accuracy in indoor environments, they often behave as black boxes, making it difficult to generalise them to other scenarios. In addition, they require expensive training and access to sufficient data, which is an obstacle to their adoption.

2.3 Firefighter specifications

Different systems can be designed to provide positioning to firefighters, and some works have already been performed to propose an IPS that is suitable for their interventions. In [16], an algorithm using UWB signals and IMUs with an extended Kalman filter. In the optic of limited costs an RFID-based system was designed in [23], and [24] implemented and tested a system based on PDR and breadcrumbs, which are sensor nodes, that are deployed by the firefighter along its path to correct the drift error of IMU sensors. While being promising, these systems also have downsides, such as cost, accuracy, or practicability.

The system choices made in this thesis are motivated and influenced by different criteria that were highlighted during discussions with the Zone de Secours Brabant Wallon. They concern various parts of the system, including hardware requirements, firefighter convenience issues, or precision. Here are the main specifications formulated by the firefighters:

- **No pre-training phase:** firefighters have to intervene in a large variety of locations. Techniques such as machine learning or fingerprinting that require a specific training or building a position database for each location become unrealistic, making them unsuitable for the firefighter's use.
- **No a priori information:** knowledge about building plans is not guaranteed for all locations, and acquiring those plans for all possible places is fastidious. Due to these issues, firefighters prefer a system that does not require this type of information.
- **Radio signal:** technologies with large bandwidths like UWB provide better time resolution than others with smaller bandwidths like BLE or WiFi, in return of being more expensive. To have a trade-off between accuracy and cost, the considered bandwidth and carrier frequency are fixed similarly to existing 5G systems. In addition, designing the system based on an existing communication protocol could enable the use of existing equipment for a potential implementation.
- **Sensing infrastructure:** in order to ensure sufficient localisation precision, it is decided that each firefighter will be equipped with a transmitter, as wearable systems offer higher accuracy than contactless ones. The transmitter must be light enough to not bother the firefighter and energy-efficient to have the required autonomy. Hence, its transmit power should be kept relatively low, and tracking algorithms that require it to be constantly emitting should be avoided. For the receiving side of the chain, the number of receivers should be reasonable to provide enough redundancy while being readily deployable by firefighters. Finally, synchronisation between receivers is available, but not with the transmitter.
- **System accuracy:** the precision of the localisation should be high enough to be able to find the lost or hampered firefighter, but it does not need to be centimetre-level. Typically, firefighters are equipped with a sound alarm that activates when they are in trouble, and although it cannot alone enable to find the firefighter, combining that with a metre-level precision is generally sufficient to bring the help needed, even in conditions where smoke drastically reduces the visibility to less than a metre. On the other hand, system robustness is critical, as sending help to a wrong location loses the few minutes that can make the difference.

2.4 System architecture

Between the different localisation techniques and methods presented above, the technique selected has to respect the needs and constraints of firefighters. Although there is no unique solution, the system architecture studied in the rest of this thesis is chosen to be tag-based by equipping firefighters with an active transmitter, and the receiver side uses deployable base stations around the scene of the intervention that are synchronised together, as illustrated in Figure 2.6. The number of receivers deployed can strongly influence the IPS performance, as adding redundancy increases the accuracy and robustness. Firefighters do not have a fixed constraint, but should be kept reasonable for them to deploy the system quickly enough. Hence, the rest of this work will consider between 4 and 8 antennas. The transmitter should be energy efficient, so its transmitted power P_T is set to 1 mW, and the base stations should be light enough to be easily deployable by firefighters.

To provide sufficient accuracy, a hybrid localisation algorithm using TDoAs and AoAs is considered. Using TDoAs avoids expensive synchronisation with the transmitter, but still requires synchronisation with the receiver. To measure AoA, each receiver is considered to have an antenna array of 4 elements to keep its size and cost reasonable.

The system uses CSI measurements of the physical layer to jointly estimate the TDoAs and AoAs of the different multipath components using the MUSIC algorithm [25], [26]. One advantage of using measurements provided by the physical layer is that the localisation system could potentially be integrated into an existing wireless communication system. The MUSIC algorithm provides a time resolution

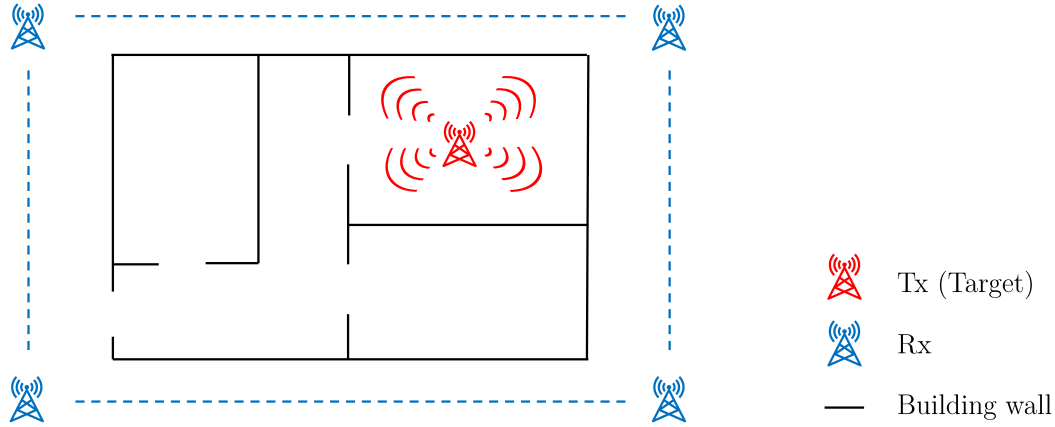


Figure 2.6: Global system architecture

that is not strictly limited by the bandwidth, but increasing it still improves the performance. For these reasons, the system is considered to work with typical 5G parameters, with a carrier frequency $f_c = 3.5$ GHz and a bandwidth $B = 100$ MHz.

The choice of modulation is motivated by the presence of strong multipath, while also being convenient to compute the CSI that is given as input to the MUSIC algorithm [13]. This algorithm takes as input the CFR, and the SC-CP modulation allows an easy Zero-Forcing (ZF) equalisation in frequency domain. Using a single carrier, in contrast to OFDM, allows to keep the complexity at the transmitter limited, while also limiting the PAPR. The summary of the different system parameters is presented in Table 2.1.

System parameter	Symbol	Value
Number of receivers	N	4-8
Number of array elements	M	4
Carrier frequency	f_c	3.5 GHz
Antenna spacing	d	$\lambda/2$
Bandwidth	B	100 MHz
Transmit power	P_T	1 mW
Modulation	/	SC-CP

Table 2.1: Summary of system parameters

The rest of this section describes the implementation and considerations for the infrastructure part of the IPS designed, i.e. the wireless communication and the receiver configuration, the localisation techniques are described in Chapter 3. Finally, it describes the methodology used to simulate the system.

2.4.1 Wireless communication

The designed system relies on the CSI measurements from the wireless communication. The wireless transmission chain considered in this thesis is described in Figure 2.7. Because the system does not involve data transmission and is interested in the CSI, the chain only sends pilots to estimate the channel. Then, the cyclic prefix is inserted and the pulse shaping filter is applied to the symbols. The signal propagates through the multipath channel and is corrupted by additive white gaussian noise to arrive to the receiver which performs matched filtering. Finally, the chain discards the cyclic prefix and computes the CSI, assuming that it has been able to correctly detect the arrival of packet and synchronise with the beginning of the pilot sequence.

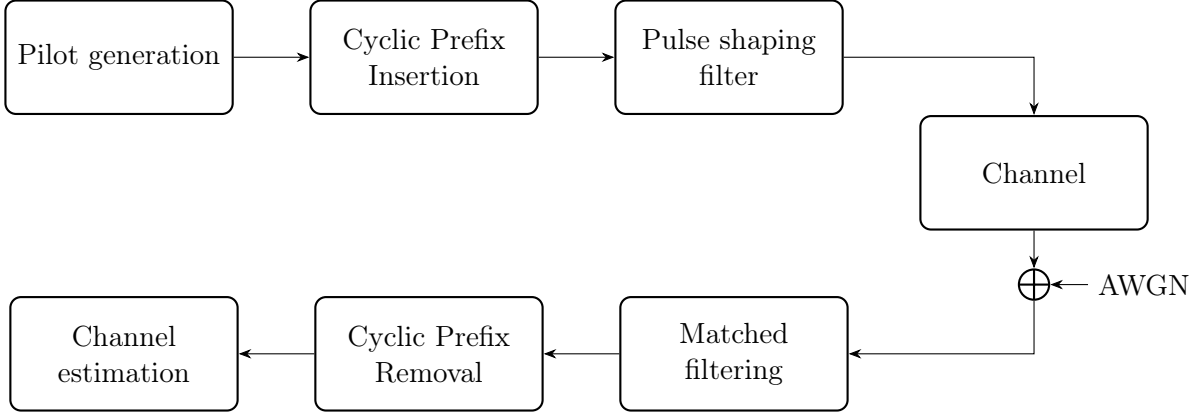


Figure 2.7: Simulated wireless transmission chain

Modulation

As explained previously, the desire to keep the complexity limited at the transmitter while being able to easily have access to the CFR led to use a single-carrier modulation with cyclic prefix (SC-CP). Indeed, the cyclic prefix allows a simple equalisation step in the frequency domain with a ZF equaliser, keeps the PAPR limited as well as the computational complexity, in contrast to OFDM that requires a heavy IFFT operation before the transmission.

The pulse shaping filter $g(t)$ and the matched filter $g^*(t)$ are root-raised-cosine (RRC) filters to keep the signal band-limited and respect the Nyquist criterion, they are described in Appendix 1. Those filters depend on a parameter α called the roll-off factor, which dictates the width of the transition band of the filter frequency response $G(f)$, thus the effective bandwidth B_{eff} taken by the signal. For a symbol period T , which would give an ideal bandwidth $B = \frac{1}{T}$, the effective bandwidth B_{eff} is given by

$$B_{eff} = \frac{1 + \alpha}{T} . \quad (2.3)$$

The frequency response of $G(f)$ for different values of α is illustrated in Figure 2.8. One can already notice that the CSI will provide an estimate of the composite channel $H(f)$, i.e. the concatenation of the pulse shaping filter, the physical channel and the matched filter $H(f) = |G(f)|^2 C(f)$. Because localisation is only interested in the pure physical channel $C(f)$, it is preferable to keep the larger possible part of the channel unaffected by the pulse and the matched filter. This translates into picking smaller values of α . For that reason, the roll-off factor is set to $\alpha = 0.1$. The downside is that the oscillations take more time to attenuate. This is detailed more precisely in the rest of this section.

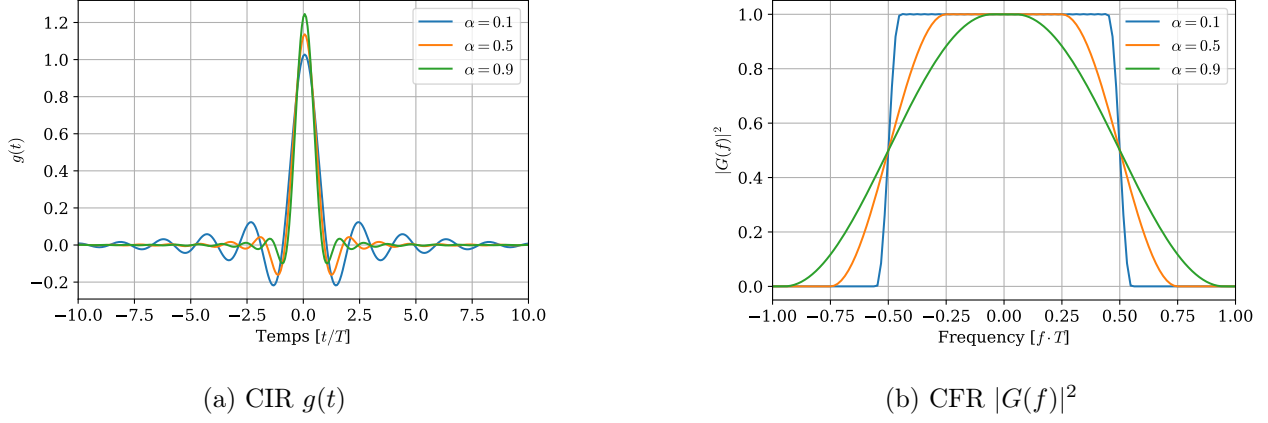


Figure 2.8: CIR of an RRC filter and squared magnitude of its CFR

Wireless channel

As presented in the Chapter 1, the indoor environment in which firefighters perform their rescue mission creates severe multipath due to the numerous scattering processes created by building walls. This motivates to model the wireless channel as a frequency-selective channel with a channel impulse response $c(t)$ and a channel frequency response $C(f)$ defined as

$$c(t) = \sum_{k=0}^{K-1} \alpha_k \delta(t - \tau_k), \quad (2.4)$$

$$C(f) = \sum_{k=0}^{K-1} \alpha_k e^{-j2\pi f \tau_k}. \quad (2.5)$$

where K is the number of multipath components that comprise the channel. In order to deal with the frequency-selectivity of the channel, the SC-CP modulation uses a cyclic prefix that serves as a buffer for the delayed previous packets that corrupt the beginning of the current packet send, while also circularising the channel, enabling a simple frequency domain equalisation.

For a transmitter that needs to send a sequence of symbols $I[m]$, the signal $s(t)$ to be emitted is obtained by shaping the symbols with the pulse $g(t)$ as

$$s(t) = \sum_m I[m] g(t - mT). \quad (2.6)$$

It then propagates through the physical channel $c(t)$, reaches the receiver and is corrupted by a thermal noise of power spectral density $N_0 = k_B T$, where $k_B = 1.38 \times 10^{-23}$ is the Boltzmann constant and $T = 290K$ is the ambient temperature. Furthermore, it is assumed that the noise figure N_F at the receiver is given by $N_F = 10$ [dB]. For a transmitted signal $s(t)$, the received signal $x(t)$ is then given by

$$x(t) = s(t) * h(t) + w(t) = \sum_m I[m] g(t - mT) * c(t) + w(t) \quad (2.7)$$

where the noise $w(t)$ is considered as an AWGN with bilateral power spectral density $\gamma_w(f) = 2N_0 N_F$ in baseband. The signal $x(t)$ goes through the matched filter $g^*(-t)$ which gives as output the signal

$y(t)$ expressed as

$$\begin{aligned} y(t) &= x(t) * g^*(-t) \\ &= \sum_m I[m] \left(g(t - mT) * c(t) * g^*(-t) \right) + \int_{\tau} w(\tau) g^*(\tau - t) d\tau \\ &= \sum_m I[m] h(t - mT) + v(t), \end{aligned} \quad (2.8)$$

where the composite channel $h(t)$ is defined as

$$\begin{aligned} h(t) &= g(t) * c(t) * g^*(-t) \\ &= \sum_{k=1}^K \alpha_k C_g(t - \tau_k), \end{aligned} \quad (2.9)$$

with the autocorrelation $C_g(t)$ of the filter $g(t)$ defined as

$$C_g(t) = \int_{-\infty}^{\infty} g(\tau) g(\tau - t) d\tau, \quad (2.10)$$

and where the filtered noise $v(t)$ has a power σ_v^2 given by $\sigma_v^2 = 2N_0N_F B$. Finally, the signal $y(t)$ is sampled every period T . Denoting $y[n] = y(nT)$, $h[n] = h(nT)$, $C_g[n] = C_g(nT)$ and $v[n] = v(nT)$, the sampled signal $y[n]$ is obtained as

$$y[n] = \sum_m I[m] \sum_{k=1}^K \alpha_k C_g[n - m - \frac{\tau_k}{T}] = \sum_m I[m] h[n - m] + v[n]. \quad (2.11)$$

Considering that the discrete channel $h[l]$ has a finite response of L symbols, i.e. $h[l] = 0 \quad \forall l < 0$ and $h[l] = 0 \quad \forall l \geq L$, (2.11) can be rewritten as

$$y[n] = \sum_{l=0}^{L-1} h[l] I[n - l] + v[n]. \quad (2.12)$$

The length L of the composite channel $h(t)$ depends on the duration $T_c = \max(\tau_k)$ of the physical channel $c(t)$, as well as the tails of the filters $g(t)$ and $g^*(-t)$. The length L_{CP} of the cyclic prefix must be long enough compared to L to combat intersymbol interference, i.e. $L_{CP} \geq L - 1$. As a remark, the tails become longer as the roll-off factor α decreases, increasing L , which creates a trade-off between the length of the cyclic prefix and the width of the filter frequency response.

Pilot sequence and channel estimation

To estimate the channel, it is necessary to send a sequence of pilots. In this chain, the pilot sequence used is a Zadoff-Chu (ZC) sequence, which is commonly used in wireless systems for key functions such as synchronisation and channel estimation [27]. The ZC sequence $z_u[n]$ of length N parameterised by the root index u is expressed as

$$z_u[n] = e^{-\frac{j\pi un^2}{N}} \quad (2.13)$$

if N_{ZC} is even or

$$z_u[n] = e^{-\frac{j\pi un(n+1)}{N}} \quad (2.14)$$

if N_{ZC} is odd, for $n = 0, 1, \dots, N - 1$ [28]. This type of sequence has the interesting CAZAC property, which consists of having a constant amplitude, which has the benefit of limiting the PAPR, and a zero cyclic auto-correlation, which is useful for synchronisation [29]. In addition, the DFT $Z_u(f)$ of a ZC

sequence is also a ZC sequence, which is particularly useful for frequency domain equalisation, as it does not amplify the noise. Finally, ZC sequences are cyclic, which can be written as

$$z_u[n + N] = z_u[n]. \quad (2.15)$$

The sequence of symbol I_m to be transmitted is now the ZC sequence with its cyclic prefix inserted, $\bar{z}_u[m] = z_u[m \bmod N]$ for $m = -L_{CP}, \dots, N - 1$. From (2.6), the signal sent $s(t)$ becomes

$$s(t) = \sum_{m=-L_{CP}}^{N-1} \bar{z}_u[m] g(t - mT). \quad (2.16)$$

and the received signal $y[n]$, using (2.11), can now be expressed as

$$y[n] = \sum_{l=0}^{L-1} h[l] \bar{z}_u[n - l] + v[n] \quad (2.17)$$

Removing the cyclic prefix, which simply means keeping the values of $y[n]$ for $n = 0, \dots, N - 1$, and assuming that $L_{CP} \geq L - 1$, the channel becomes circularised:

$$y[n] = \sum_{l=0}^{L-1} h[l] z_u[(n - l) \bmod N] + v[n]. \quad (2.18)$$

In the frequency domain, it is equivalent to the element-wise multiplication of the DFTs, the N-point DFT of $y[n]$ is then given as

$$Y[q] = H[q] Z_u[q] + V[q], \quad (2.19)$$

where $H[q]$, $Z_u[q]$ and $V[q]$ are the DFTs of $h[n]$, $z_u[n]$ and $v[n]$, respectively. With this pilot sequence, the estimated channel $\hat{H}[q]$ is obtained with Zero-Forcing (ZF) equalisation:

$$\hat{H}[q] = \frac{Y[q]}{Z_u[q]} = H[q] + \frac{V[q]}{Z_u[q]} \quad (2.20)$$

where $Y[q]$ is obtained with the FFT of the first N symbols after the cyclic prefix removal. $\hat{H}[q]$ can then be given as input to the superresolution algorithm. However, due to the filters and to sampling, $\hat{H}[q]$ does not represent exactly the true physical channel $C(f)$. In the continuous domain, from (2.9), $H(f)$ depends on $C(f)$ as

$$H(f) = |G(f)|^2 C(f) \quad (2.21)$$

After sampling, the DFT $H[q]$ of the composite channel is given by

$$\begin{aligned} H[q] &= \frac{1}{T} \sum_{m=-\infty}^{\infty} H\left(\frac{q}{NT} - \frac{m}{T}\right) \\ &= \frac{1}{T} \sum_{m=-\infty}^{\infty} \left| G\left(\frac{q}{NT} - \frac{m}{T}\right) \right|^2 C\left(\frac{q}{NT} - \frac{m}{T}\right), \end{aligned} \quad (2.22)$$

where aliasing occurred due to the transition bands of the filter $G(f)$. Fortunately, for the central frequencies, i.e. $-\frac{1-\alpha/2}{T} \leq f \leq \frac{1-\alpha/2}{T}$, the copied versions of $G(f)$ do not overlap, leaving the spectrum intact as

$$H[q] = |G[q]|^2 C[q] = C[q] \quad (2.23)$$

where $|G[q]|^2$ is the DFT of $C_g[n]$ and $C[q] = C(\frac{q}{NT})$ is the sampled Fourier transform of (2.5), and because $|G(f)|^2 = 1$ at these frequencies. Consequently, a truncated version of the channel is sent to the MUSIC algorithm so that the true channel is actually represented.

2.4.2 Receiver structure

To obtain information on the angle of arrival, it is necessary to sample in the spatial domain, i.e. to use antenna arrays. The chosen array configuration is the standard uniform linear array, with 4 array elements spaced by a distance $d = \frac{\lambda}{2}$ where $\lambda = \frac{c}{f_c}$ is the carrier wavelength. For this antenna array, the reference for an angle of arrival θ defined with respect to the normal of the array is represented in Figure 2.9.

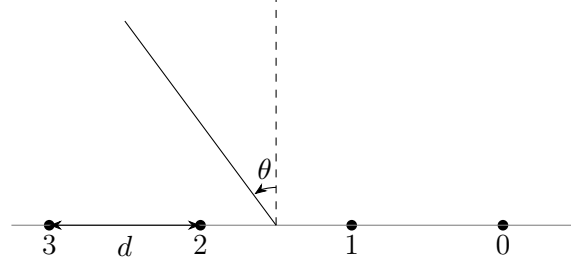


Figure 2.9: Uniform Linear array

Uniform linear arrays work by measuring the phase shifts at the different antennas. It uses the fact that for a wave that has a phase $\phi = \phi_0$ at a certain point, travelling an additional distance l from that point changes its phase to $\phi = \phi_1$, the phase shift $\Delta\phi$ between ϕ_0 and ϕ_1 is given as

$$\Delta\phi = \phi_1 - \phi_0 = -\frac{2\pi}{\lambda}l. \quad (2.24)$$

Assuming that the incoming signal is a plane wave that arrives at an angle θ , one can find that, using Figure 2.10 as a reference and (2.24), the phase shift from antenna i to antenna $i + 1$ is given by

$$\Delta\phi_m = \phi_{m+1} - \phi_m = \frac{2\pi}{\lambda}d \sin \theta. \quad (2.25)$$

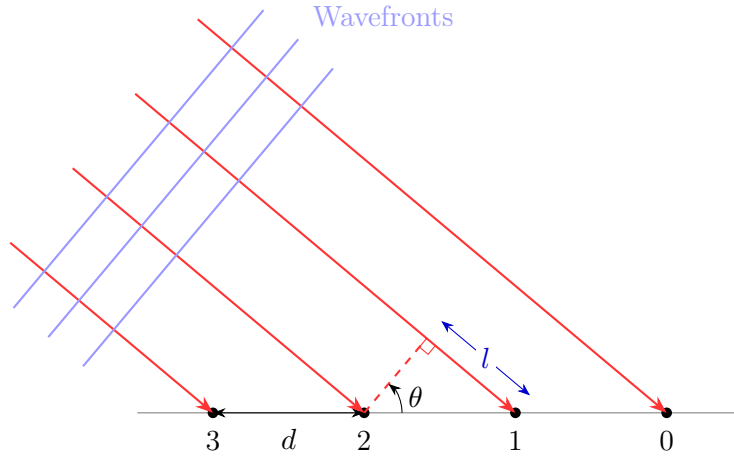


Figure 2.10: Uniform Linear array

Consequently, if the phase shift $\Delta\phi$ is known between two antennas, it is possible to find the angle of arrival by reversing (2.25):

$$\theta = \arcsin \left(\frac{\lambda \Delta\phi}{2\pi d} \right). \quad (2.26)$$

However, an important point to take into account is that, for a given value of $\Delta\phi$, there is an ambiguity regarding the corresponding θ . Indeed, the arcsine function returns an angle comprised between $-\pi/2$

and $\pi/2$, the second possible value of θ' is given by

$$\theta' = \pi - \arcsin\left(\frac{\lambda\Delta\phi}{2\pi d}\right). \quad (2.27)$$

The angle θ' actually corresponds to the mirror of θ with respect to the array. Hence, it is necessary to have prior information on the side of the array from which the signal is supposed to arrive. Typically, it is done by knowing which side of the array faces the region where the target is supposed to be, and by supposing that no signal can come from the back of the array. In addition, the localisation algorithm requires the system to know the orientations $\theta_{0,i}$ of the different receivers, i.e. the orientations of their antenna arrays, with respect to its own reference to compute the actual direction from which the signal reaches the i^{th} receiver as $\theta_{0,i} + \theta_i$, with θ_i being the angle θ measured at the i^{th} receiver.

Finally, depending on the angle of the incoming wave, small variations of θ can affect more or less the variation of ϕ . Indeed, the first derivative of ϕ with respect to θ is given by

$$\frac{\partial\phi}{\partial\theta} = \frac{2\pi}{\lambda}d\cos\theta, \quad (2.28)$$

which is maximal when $\theta = 0$. This means that it is easier to measure variations of θ around $\theta = 0$ because they create larger variations of ϕ . Thus, it is preferable that the antenna arrays face the scene of intervention. With this in mind, the system requires to know the orientations of the receivers that are placed such that their antenna arrays face the scene. This allows discarding ghost angles that would come from the back of the array and improving the angle precision. The downside is that firefighters must pay attention to the setup of receivers during deployment.

2.4.3 System simulation

One purpose of this work is to simulate the entire localisation system, from the radio signal propagation to the localisation using those signals. The approach followed is illustrated in Figure 2.11.

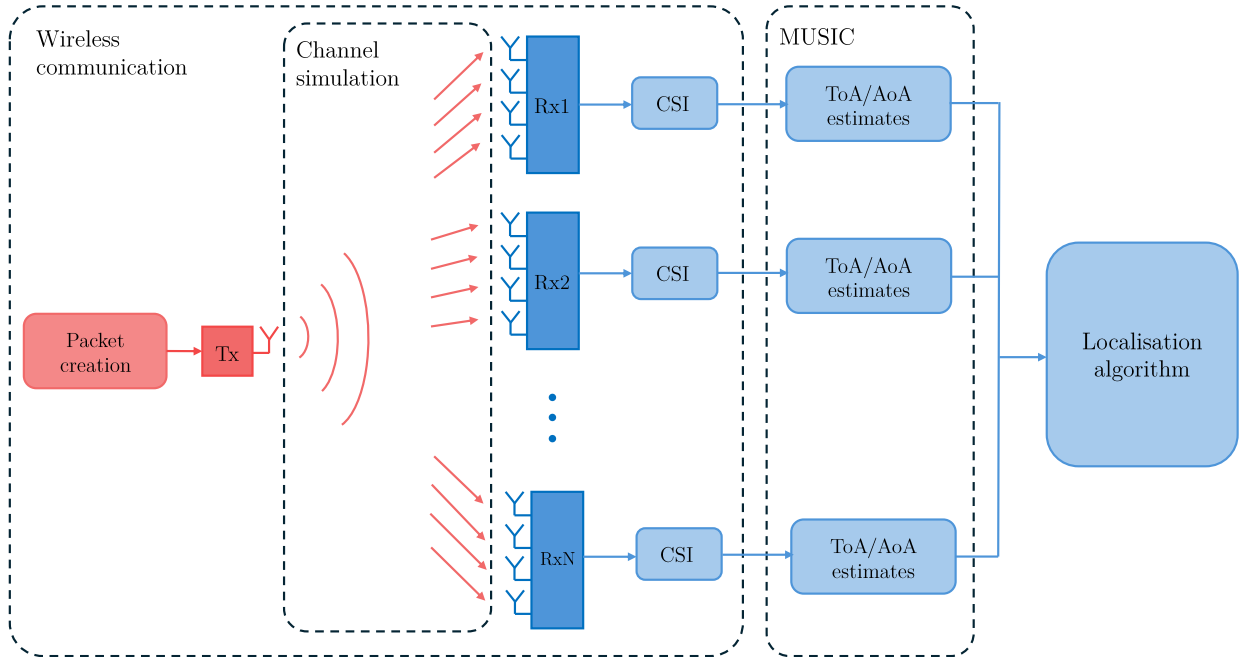


Figure 2.11: Organisation of the system simulation

First, the physical channels between each base station and the firefighter are obtained with ray-tracing simulations in the modelled firefighter environment, as presented in Chapter 1, for a particular scenario, i.e., for specific positions of the receivers and of the target.

Then, the transmitter sends its pilot sequence to each receiver. The wireless communication through that channel is simulated for each antenna of each base station, and each receivers perform a ZF equalisation to obtain the CSI. The MUSIC algorithm is applied at each receiver to extract the delays and angles of the paths that compose its channel. Finally, these estimations are centralised to apply the localisation algorithm that outputs a position estimate.

Chapter 3

Localization techniques

Indoor environments are known to have a strong multipath effect, which can severely degrade the accuracy of the positioning. Building fires in which firefighters intervene are an example of such indoor scenarios. Thus, it is interesting to study what performance classical localisation techniques can achieve in this type of environment.

This chapter covers the different localisation techniques that are used by the IPS designed in Chapter 2. First, Section 3.1 describes the MUSIC algorithm that is used to extract TDoAs and AoAs from the CSI estimated at the different receivers. In a second time, algorithms that use TDoAs, AoAs or a combination of both measurements, are detailed, and the performances they achieve in the firefighter scenario considered are evaluated.

3.1 MUSIC Algorithm

The MUSIC algorithm is an algorithm that uses subspace decomposition of the received signal to provide what is called superresolution, which means that the resolution achievable is not strictly limited by the bandwidth or the size of the antenna array. However, higher bandwidths and larger antenna arrays still improve the performance of MUSIC [30].

MUSIC can be used to enhance time resolution [13], [30], as well as angular resolution [31], [32]. It is also possible to apply MUSIC for joint estimation of multipath components delays and angles of arrival [25], [26]. First, the measurement models for the separated estimation of AoAs and ToAs are described. As a point of clarification, this section uses the term of ToA and not TDoA, because the MUSIC algorithm essentially estimate the time instants at which a signal was received. However, since those ToAs are not synchronised with the transmitter, they are then used to compute TDoAs that can be given as input to the positioning algorithm. Then the subspace decomposition, estimation of the number of incoming signals, and frequency smoothing that compute the AoAs and ToAs are detailed. Finally, the joint ToA/AoA estimation is derived from the models of ToAs and AoAs.

3.1.1 AoA measurement model

The AoA estimation problem originally presented in [31] consists in determining the angles of arrival θ_k of K different signals that arrive simultaneously to the M antennas that comprise the antenna array of a receiver. The measurement model assumed by the MUSIC algorithm is given by

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} \mathbf{a}(\theta_1) & \mathbf{a}(\theta_2) & \cdots & \mathbf{a}(\theta_K) \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_K \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \quad (3.1)$$

or

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{w}, \quad (3.2)$$

where $\mathbf{x} \in \mathbb{C}^M$ is a vector that contains the signals received at the M different array antennas, $\mathbf{s} \in \mathbb{C}^K$ is the vector containing the K different signals reaching the receiver, the matrix $\mathbf{A} \in \mathbb{C}^{M \times K}$ contains the steering vectors $\mathbf{a}(\theta_k)$ of size M that correspond to how the k^{th} signal is received for a specific angle of arrival θ_k , and $\mathbf{w}$ is an AWGN vector of variance σ_n^2 .

For a ULA with an element spacing d and signals with a carrier wavelength λ , the steering vectors $\mathbf{a}(\theta_k)$, from (2.25), are given by

$$\mathbf{a}(\theta_k) = \begin{bmatrix} 1 & e^{j\frac{2\pi d}{\lambda} \sin \theta_k} & e^{j2\frac{2\pi d}{\lambda} \sin \theta_k} & \cdots & e^{j(M-1)\frac{2\pi d}{\lambda} \sin \theta_k} \end{bmatrix}^T \quad (3.3)$$

3.1.2 ToA measurement model

Considering the variant of estimating the ToA, the problem becomes to identify the L different signals that reach a single receiver with different delays τ_k , for example due to the presence of multipath. The signal $h(t)$ arriving at the receiver is the sum of the different signals shifted by their delays τ_k ,

$$h(t) = \sum_{k=1}^K \alpha_k \delta(t - \tau_k), \quad (3.4)$$

with the Fourier transform $H(f)$ of $h(t)$ given by

$$H(f) = \sum_{k=1}^K \alpha_k e^{-j2\pi f \tau_k}. \quad (3.5)$$

At the receiver, a similar measurement model to (3.1) can be built with the sampled frequency-domain received signal $x[n]$ given by

$$X[l] = H(f_l) + w[l] = \sum_{k=1}^K \alpha_k e^{-j2\pi f_l \tau_k} + w[l] \quad (3.6)$$

where $l = 0, \dots, L-1$, $f_l = l\Delta f$ and $w(l)$ a sampled AWGN of variance σ_n^2 . The measurement model is then given by

$$\begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{L-1} \end{bmatrix} = \begin{bmatrix} \mathbf{e}(\tau_1) & \mathbf{e}(\tau_2) & \cdots & \mathbf{e}(\tau_K) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_K \end{bmatrix} + \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{L-1} \end{bmatrix} \quad (3.7)$$

or

$$\mathbf{X} = \mathbf{E}\boldsymbol{\alpha} + \mathbf{w}, \quad (3.8)$$

where $\mathbf{X} \in \mathbb{C}^L$ is a vector containing L consecutive time samples of the received signal $X[l]$, $\boldsymbol{\alpha} \in \mathbb{C}^K$ is the vector containing the K different path coefficients of the multipath components, the matrix $\mathbf{E} \in \mathbb{C}^{L \times K}$ contains the steering vectors $\mathbf{e}(\tau_k)$ of size L that correspond to the exponentials of the Fourier transform and $\mathbf{w} \in \mathbb{C}^L$ is the AWGN vector. This formulation has the same structure as (3.1), but with different steering vectors

$$\mathbf{e}(\tau_k) = \begin{bmatrix} 1 & e^{-j2\pi \Delta f \tau_k} & e^{-j2\pi \cdot 2 \Delta f \tau_k} & \cdots & e^{-j2\pi \cdot (L-1) \Delta f \tau_k} \end{bmatrix}^T. \quad (3.9)$$

3.1.3 Subspace decomposition

Taking the case of AoA estimation, the goal of the MUSIC algorithm is to find the K different signals s_k that correspond the most to the M received signals $\mathbf{x}$. To do so, MUSIC works by analysing the eigendecomposition of the measurement covariance matrix $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{R}_x$. In practice, this covariance matrix is estimated with the sample-covariance matrix as

$$\mathbf{R}_x \approx \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i \mathbf{x}_i^H \quad (3.10)$$

where N is the number of observations, i.e. the number of times the values of $\mathbf{s}$ at the M antennas are measured. Because the received signal $\mathbf{x}$ is the sum of the K transmitted signals s_k and the noise w , its covariance matrix $\mathbf{R}_x$ is equal to

$$\mathbf{R}_x = \mathbf{A} \mathbf{R}_s \mathbf{A}^H + \sigma_n^2 \mathbf{I}. \quad (3.11)$$

with $\mathbf{R}_s = \mathbb{E}\{\mathbf{s}\mathbf{s}^H\}$. If the signals s_K are independent, the matrix R_s is diagonal and of rank K . When the number of array elements M is larger than the number of incoming signals K , the matrix $\mathbf{A} \mathbf{R}_s \mathbf{A}^H$ is singular. Among its eigenvalues λ_i , the first K of them are non zero, with their corresponding eigenvectors $[\mathbf{v}_1, \dots, \mathbf{v}_K]$ spanning the signal subspace, and the last $M - K$ eigenvalues are equal to zero, with their eigenvectors $[\mathbf{u}_{K+1}, \dots, \mathbf{u}_M]$ spanning the null subspace. For those vectors,

$$\mathbf{A} \mathbf{R}_s \mathbf{A}^H \mathbf{u}_i = \lambda_i \mathbf{u}_i = 0 \quad (3.12)$$

which also implies that

$$\mathbf{A}^H \mathbf{u}_i = 0, \quad (3.13)$$

which implies that the eigenvectors $\mathbf{u}_i$ are orthogonal to the steering vectors $\mathbf{a}(\theta_k)$ corresponding to the received signals.

After the addition of noise, one can look at what become the eigenvectors of $\mathbf{R}_x$. Multiplying $\mathbf{R}_x$ by an eigenvector $\mathbf{v}_i$ of the signal subspace, one has

$$\mathbf{R}_x \mathbf{v}_i = \mathbf{A} \mathbf{R}_s \mathbf{A}^H \mathbf{v}_i + \sigma_n^2 \mathbf{I} \mathbf{v}_i = (\lambda_i + \sigma_n^2) \mathbf{v}_i, \quad (3.14)$$

which means that the K eigenvectors $\mathbf{v}_i$ of the signal subspace are also eigenvectors of the measurement covariance matrix $\mathbf{R}_x$, with their eigenvalues increased by σ_n^2 . Similarly, by multiplying R_x by an eigenvector $\mathbf{u}_i$ of the null subspace, one can show that

$$\mathbf{R}_x \mathbf{u}_i = \mathbf{A} \mathbf{R}_s \mathbf{A}^H \mathbf{u}_i + \sigma_n^2 \mathbf{I} \mathbf{u}_i = \sigma_n^2 \mathbf{u}_i, \quad (3.15)$$

and that the eigenvectors $\mathbf{u}_i$ are also eigenvectors of R_x spanning the null subspace, which becomes the noise subspace. Finally, the MUSIC pseudospectrum $P_{MU}(\theta)$ is defined as

$$P_{MU}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{U}_N \mathbf{U}_N^H \mathbf{a}(\theta)}, \quad (3.16)$$

where $\mathbf{U}_N = [\mathbf{u}_{K+1} \ \dots \ \mathbf{u}_M]$ is the matrix that spans the noise subspace. The denominator corresponds to the squared norm of the projection of the steering vector for a specific angle θ . When the pseudospectrum P_{MU} is evaluated at angles $\theta = \theta_k$, the denominator should be equal to zero, as (3.13) shows that the eigenvectors of the noise subspace are orthogonal to the steering vectors $\mathbf{a}(\theta_k)$. Hence, the values of θ_k can be identified as the local maxima of P_{MU} that correspond to a zero denominator. To do so, a grid search can be performed to evaluate P_{MU} at different values of θ and identify the maxima. The level of precision depends on the density of the grid, and a higher precision demands a higher time-complexity. One solution to avoid this additional computational cost is a variant of

the MUSIC algorithm called root-MUSIC, which avoids the computation of the pseudospectrum and instead computes the roots of a polynomial derived from the signal correlation matrix [25], but this is not explored in this work.

Finally, once the values of the angles θ_k are found, the signal amplitudes s_k can be retrieved using a standard least squares estimator for (3.1), where the matrix $\mathbf{A}$ is known as the θ_k are known. The same method can be applied to find the different values of τ_k and their corresponding α_k .

3.1.4 Estimation of the number of signals

In the algorithm presented previously, it is assumed that the number of signals K that reach the receiver was known. For example, in the case of AoA estimation, it allows to build the matrix $\mathbf{U}_N$ with the $M - K$ eigenvectors that correspond to the smallest eigenvalues. In practice, the number of components created by multipath is not known in advance, as it is highly variable depending on the transmitter and receiver positions, as well as the environment. Hence, it is necessary to estimate the number of components that compose the received signal.

One way to estimate the number of incoming signals is to use information theoretic criteria such as the Akaike Information Criteria (AIC) or the Minimum Description Length, as done in [33]. The authors developed the two criteria for the case of signal detection and showed that, asymptotically, MDL leads to consistent estimates, while AIC tends to overestimate the number of signals. Hence, the MDL is chosen as the number of signal estimator for the system.

The MDL criterion $\text{MDL}(k)$ gives a confidence measure about the probability that k different signals actually compose the received signal. From [33], the MDL criterion is computed from the eigenvalues λ_i of the sample-covariance matrix and is expressed as

$$\text{MDL}(k) = -\log \left(\frac{\prod_{i=k+1}^p \lambda_i^{1/(p-k)}}{\frac{1}{p-k} \sum_{i=k+1}^p \lambda_i} \right)^{(p-k)N} + \frac{1}{2}k(2p-k) \log N, \quad (3.17)$$

where p is the dimension of the data, i.e. M for the AoAs, L for the ToAs or ML for the joint ToAs/AoAs, and N is the number of observations used to build the covariance matrix. The optimal value of k is the one that minimises $\text{MDL}(k)$. This value can then be given as input for the subspace decomposition, where $\mathbf{U}_N$ can be built with the $M - k$ eigenvectors that correspond to the smallest eigenvalues.

3.1.5 Frequency smoothing

The last consideration to be taken when computing the MUSIC algorithm is that the K different signals might be correlated, destroying the rank of the covariance matrix $\mathbf{R}_s$. This is typically the case in multipath environments, as the different components originate from the same source. Smoothing is a preprocessing scheme that restores the rank of $\mathbf{R}_s$ to K . There are two different types of smoothing, spatial smoothing (along the antennas) and frequency smoothing (along the CFR samples), both of which can be applied (even simultaneously) and are based on the same principle. Because the system works with a limited number of receive antennas, 4 by receiver, only frequency smoothing is applied, as it is easier to gather a larger number of samples by just extending the sampling duration, which is better for smoothing.

To restore the rank of the covariance matrix, smoothing as presented in [34] divides the data vector $\mathbf{X}$ of size L into overlapping subvectors of size P , with the first subvector formed with the data samples

$\{1, \dots, P\}$, the second with the samples $\{2, \dots, P+1\}$, etc. In matrix form, the q^{th} subvector $\mathbf{X}_q$ can be written as

$$\mathbf{X}_q = \mathbf{E}\mathbf{D}^{(q-1)}\boldsymbol{\alpha} + \mathbf{w}_q, \quad (3.18)$$

where $\mathbf{D}^q$ is the q^{th} power of the $q \times q$ diagonal matrix representing the displacement of the subvector:

$$\mathbf{D} = \text{diag}(e^{-j2\pi\Delta f\tau_1}, \dots, e^{-j2\pi\Delta f\tau_q}). \quad (3.19)$$

The covariance matrix of the q^{th} subvector becomes

$$\mathbf{R}_{\mathbf{x},q} = \mathbf{A}\mathbf{D}^{(q-1)}\mathbf{R}_s\mathbf{D}^{H(q-1)}\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (3.20)$$

and the frequency-smoothed covariance matrix is defined as the sample mean of the subvector covariances:

$$\bar{\mathbf{R}}_{\mathbf{x}} = \frac{1}{Q} \sum_{q=1}^Q \mathbf{R}_{\mathbf{x},q} = \mathbf{A}\bar{\mathbf{R}}_s\mathbf{A}^H + \sigma^2\mathbf{I} \quad (3.21)$$

where $Q = L - P + 1$ and $\bar{\mathbf{R}}_s$ is the modified covariance matrix of the signals given by

$$\bar{\mathbf{R}}_s = \frac{1}{Q} \sum_{q=1}^Q \mathbf{D}^{(q-1)}\mathbf{R}_s\mathbf{D}^{H(q-1)}. \quad (3.22)$$

The authors of [34] then showed that this matrix can be rewritten as $\bar{\mathbf{R}}_s = \frac{1}{Q}\mathbf{B}\mathbf{B}^H$ where $\mathbf{B}$ is a Vandermonde matrix. If the number of columns Q of $\mathbf{B}$ is greater than the number of rows K , this type of matrix has full row rank, thus $\bar{\mathbf{R}}_s$ also has a rank of K . The same algorithm as before can then be applied to the smoothed covariance matrix $\bar{\mathbf{R}}_{\mathbf{x}}$. The cost of this is that the size of the data vectors $\mathbf{X}$ are reduced, which decreases the ability to separate close multipath components.

Combining the different elements presented, an example of a MUSIC pseudospectrum for a multipath channel obtained through a ray-tracing simulation of the modelled firefighter environment is represented in Figure 3.1, and the path coefficients α_k computed for the delays τ_k identified with a peak finding algorithm are reported in Figure 3.2. One can see that the algorithm is able to identify the paths that have an energy above the noise floor, but that close paths are merged into a single one. In addition, a path above the noise floor but close to a path with higher energy is not identified.

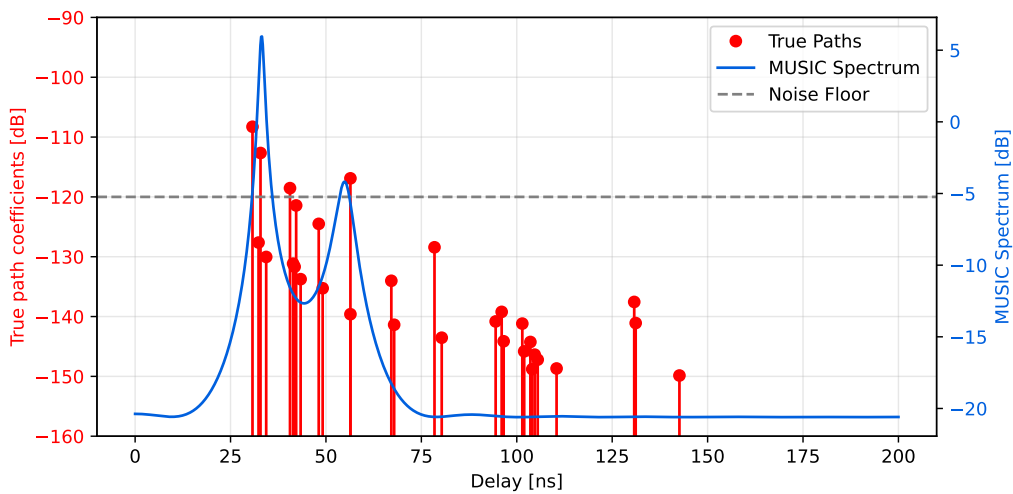


Figure 3.1: Example of MUSIC spectrum for channel paths estimation

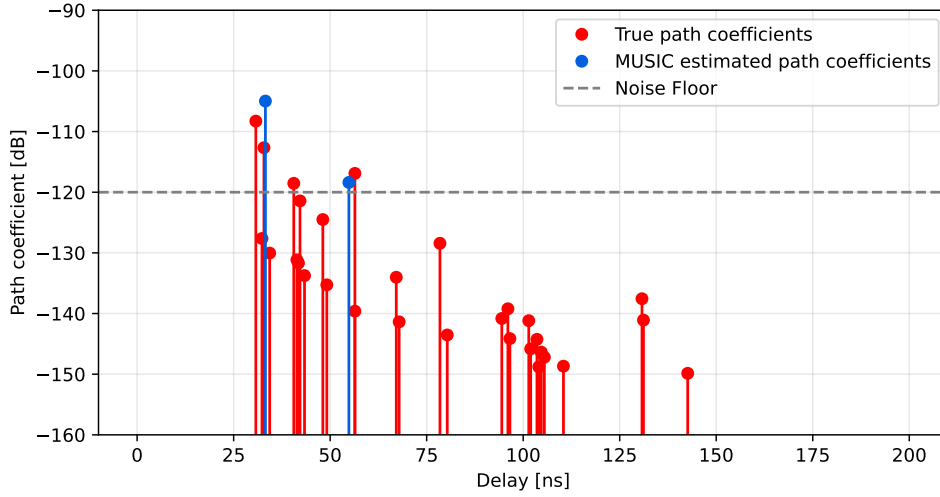


Figure 3.2: Example of channel paths identified with the MUSIC algorithm

The ability of the MUSIC algorithm to resolve channel paths depends on the noise power, as it can hide paths with smaller energy. Figure 3.3 shows the MUSIC pseudospectrum obtained for the same scenario but with a reduced noise power. The spectrum now shows more peaks, since some paths are not hidden anymore. For the rest, the same behaviour as before can be observed, closed paths are merged and smaller ones are hidden by larger ones. This means that, in practical scenarios, the direct path can be hidden by reflections with a higher energy arriving just after it, which reduces the performance of the localisation algorithm.

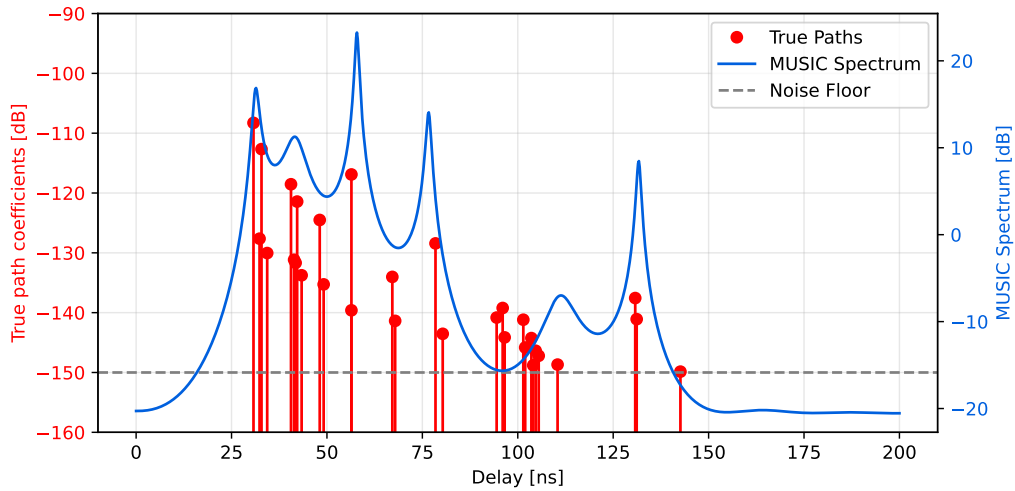


Figure 3.3: Example of MUSIC spectrum for channel paths estimation for a lower noise power

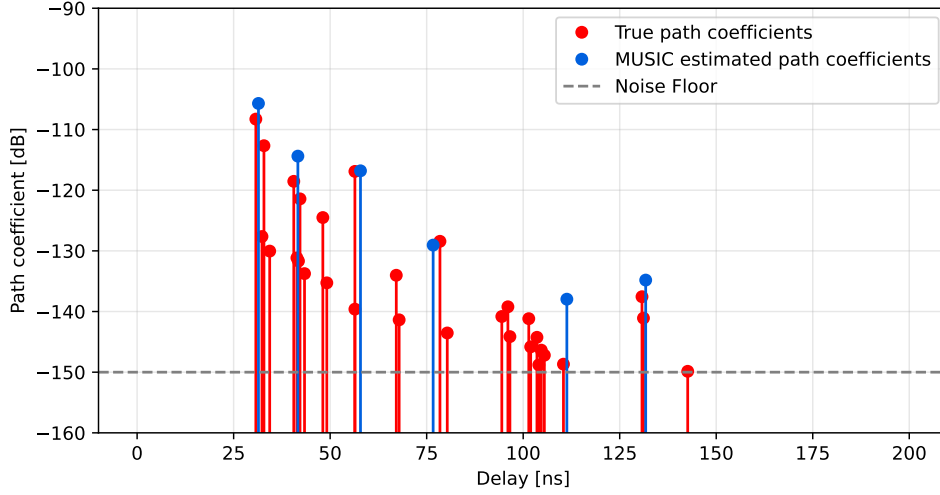


Figure 3.4: Example of channel paths identified with the MUSIC algorithm for a lower noise power

3.1.6 Hybrid AoA/ToA measurement model

The problem now consists of jointly estimating the delays τ_k and the angles of arrival θ_k of K different multipath components that reach the receiver at different times and from different directions. This can be done by combining the two measurement models presented in the previous sections. The new model consists of collecting N time samples of the signals received for each of the M antennas. Each antenna measures the a different channel $H_m(f)$ due to the different phase shifts of the antennas. Taking the channel sampled at the first antenna $H_1(f)$ as the reference channel $H(f)$:

$$H_1(f) = H(f) = \sum_{k=1}^K \alpha_k e^{-j2\pi f \tau_k}, \quad (3.23)$$

the sampled channel $H_m(f)$ at the m^{th} antennas is given by

$$H_m(f) = \sum_{k=1}^K a_m(\theta_k) \cdot \alpha_k e^{-j2\pi f \tau_k}, \quad (3.24)$$

where $a_m(\theta)$ is the m^{th} element of the steering vector $\mathbf{a}(\theta)$ in (3.3). For each antenna, the measurement model becomes, from (3.7),

$$\mathbf{X}_m = \begin{bmatrix} X_{m,0} \\ X_{m,1} \\ \vdots \\ X_{m,N-1} \end{bmatrix} = \begin{bmatrix} a_m(\theta_1)\mathbf{e}(\tau_1) & a_m(\theta_2)\mathbf{e}(\tau_2) & \cdots & a_m(\theta_K)\mathbf{e}(\tau_K) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_K \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \quad (3.25)$$

or

$$\mathbf{X}_i = \mathbf{E}_{\mathbf{a}_i} \boldsymbol{\alpha} + \mathbf{w}_i \quad (3.26)$$

The global measurement model can then be obtained by stacking the measurement vectors $\mathbf{X}_m$ of the M antennas as

$$\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_M \end{bmatrix} = \begin{bmatrix} a_1(\theta_1)\mathbf{e}(\tau_1) & a_1(\theta_2)\mathbf{e}(\tau_2) & \cdots & a_1(\theta_K)\mathbf{e}(\tau_K) \\ a_2(\theta_1)\mathbf{e}(\tau_1) & a_2(\theta_2)\mathbf{e}(\tau_2) & \cdots & a_2(\theta_K)\mathbf{e}(\tau_K) \\ \vdots & \vdots & \ddots & \vdots \\ a_M(\theta_1)\mathbf{e}(\tau_1) & a_M(\theta_2)\mathbf{e}(\tau_2) & \cdots & a_M(\theta_K)\mathbf{e}(\tau_K) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_K \end{bmatrix} + \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_N \end{bmatrix}, \quad (3.27)$$

or equivalently as

$$\mathbf{X} = \mathbf{E}_\mathbf{A} \boldsymbol{\alpha} + \mathbf{w}. \quad (3.28)$$

The MUSIC pseudospectrum that is computed based on the covariance matrix of $\mathbf{E}_\mathbf{A}$ now depends on two variables, θ and τ , since the goal is to find the pairs (θ_k, τ_k) of K multipath components. It is defined as

$$P_{MU}(\theta, \tau) = \frac{1}{\mathbf{e}_\mathbf{a}^H(\theta, \tau) \mathbf{U}_N \mathbf{U}_N^H \mathbf{e}_\mathbf{a}(\theta, \tau)}, \quad (3.29)$$

In contrast to the estimations of only AoAs or only ToAs that require a 1D grid search, the problem becomes a 2D grid search, which increases the computational burden. Similarly to the 1D problem, root-MUSIC can be applied to reduce the time complexity [25].

3.1.7 Application to system measurements

The 2D MUSIC algorithm just described can be applied to the measurements obtained for the firefighter scenario and system architecture presented in Chapter 2. For one receiver that has 4 antennas, it takes as input the 4 CSI estimations $\mathbf{X}_i$, $i = 1, 2, 3, 4$, then builds the matrix $\mathbf{E}_\mathbf{A}$ knowing the antenna spacing $d = \lambda/2$ and the frequency sampling step $\Delta f = 1/T$, with T the symbol period. The 2D MUSIC pseudospectrum is then built using (3.29) after having estimated the number K of paths that compose the channel using the MDL criterion. An example of a MUSIC spectrum is shown in Figure 3.5, which corresponds to a case where the true channel paths those represented in Figure 3.6. Amplitude peaks of the spectrum indicate that a path arrived from the corresponding pair of delay-angle, one can see that they are located at the same places as the most dominant paths.

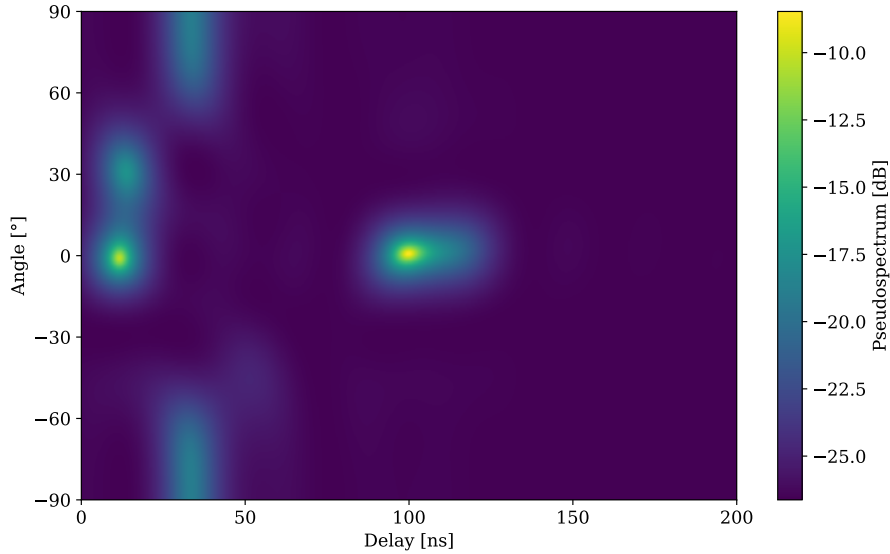


Figure 3.5: 2D MUSIC pseudospectrum

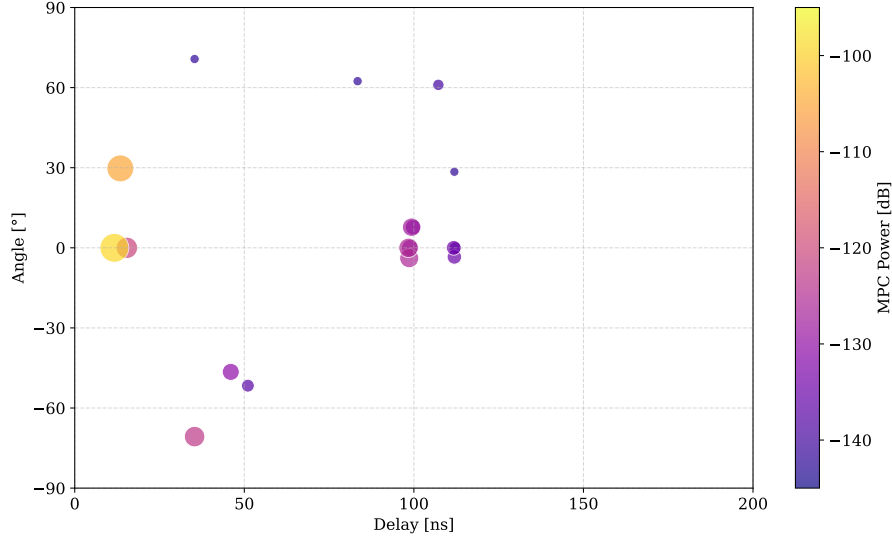


Figure 3.6: True paths of the channel

One remark must be made about the angles of arrival that are evaluated between -90° and 90° and not -180° and 180° . Indeed, as mentioned in Section 2.4, there is an ambiguity of 180° on the angle as the same phase shifts are obtained for a mirrored angle coming from the back of the antenna, which creates the need to have a priori information on which side of the antenna array the signal is supposed to reach. This is illustrated in Figure 3.7, where it can be observed that the MUSIC spectrum is mirrored around -90° and 90° . In addition, the phase shift for an angle of $\theta = 90$ given from (2.24) as $\Delta\phi = \pi$ for a half-wavelength antenna spacing, and $\theta = -90$ gives a phase shift $\Delta\phi = -\pi$, which gives the same $e^{-j\phi} = -1$. Consequently, it is important to estimate the pseudospectrum between -90° and 90° , while keeping in mind that an angle θ close to those values will "leak" at the opposite side of the spectrum, which can be done by searching the local maxima of the spectrum on a circularised version of the spectrum.

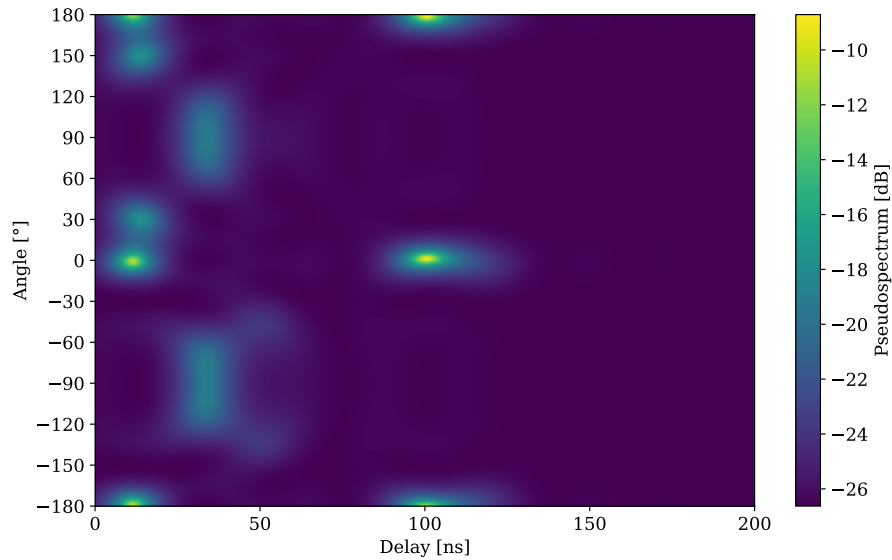


Figure 3.7: 2D MUSIC pseudospectrum

Once the spectrum is obtained, the angles and delays of the paths can be identified by finding its K largest local maxima, and their amplitudes can then be estimated using a standard LLS. For the example considered, Figure 3.8 shows the paths identified using the MUSIC algorithm. The most

dominant paths have been correctly detected, and one can see that the clutter of paths around 100 ns has been considered as a single larger path. Similarly to the 1D case, the algorithm does not see smaller paths that are close to larger ones. However, this is now the case if the paths are close in time and in angle. Indeed, the two dominant paths that arrive early are both identified because they arrive from different directions. Finally, the smaller paths that are too small compared to the noise power are not detected.

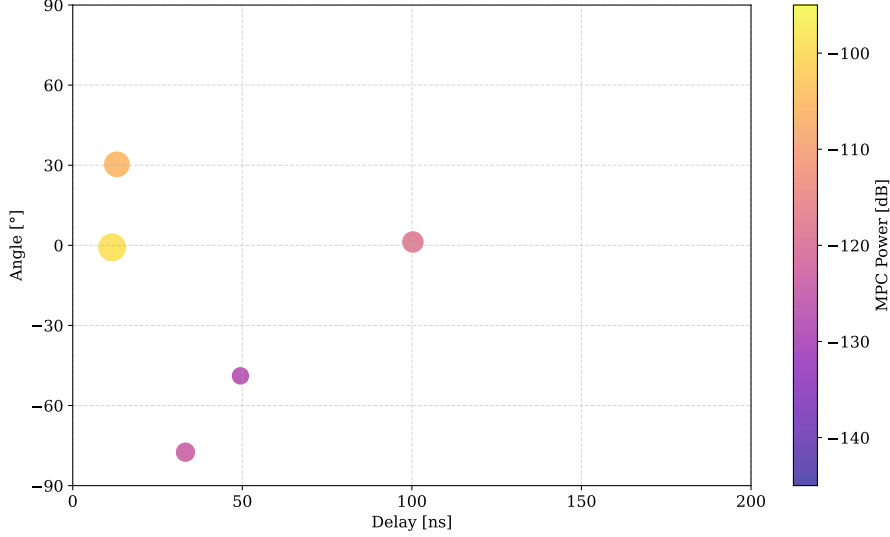


Figure 3.8: Estimated paths

3.2 Multilateration

Multilateration techniques use the difference in times of arrival of the radio signal sent by a transmitter to different receivers to obtain information about the position of a target, in contrast to trilateration techniques that used the absolute times of arrival. An advantage of using TDoAs instead of pure ToAs is the synchronisation requirements. In the case of ToAs, the delay τ_i taken by the signal to propagate from the transmitter to the i^{th} receiver is calculated by

$$\tau_i = t_i - t_0, \quad (3.30)$$

where t_0 is the time at which the transmitter sent the signal and t_i is the time at which it reaches the i^{th} receiver. These times need to be defined in relation to the same time origin t_0 . Consequently, the receiver needs to be synchronised with the transmitter so that they share the same t_0 . For TDoAs, the uncertainty about the time origin is eliminated by taking the difference $\delta_{i,j}$ between two measurements, provided that the receivers are synchronised together, as

$$\delta_{i,j} = t_i - t_j. \quad (3.31)$$

3.2.1 Linear Least Squares Estimator

It is possible to obtain a close form expression for multilateration [35]. For a scenario with N receivers, the target located at an unknown position $\mathbf{x}$ is separated from the i^{th} receiver located at position $\mathbf{x}_i$ by a distance d_i . In the 2D problem, the target position to estimate is $\mathbf{x} = [x \ y]^T$ and the position of the i^{th} receiver is $\mathbf{x}_i = [x_i \ y_i]^T$. The goal is to find the position $\mathbf{x}$ that respects the following relations:

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 = d_1^2 \\ (x - x_2)^2 + (y - y_2)^2 = d_2^2 \\ \vdots \\ (x - x_N)^2 + (y - y_N)^2 = d_N^2 \end{cases} \quad (3.32)$$

where the distances d_i are unknown. Because the receivers are not synchronised with the target, it is not possible to directly estimate d_i with Equation 3.30. Instead, one of them is taken as a reference, here the receiver 1. New target and receiver positions can be defined based on the reference receiver:

$$\begin{aligned} x' &= x - x_1 \\ y' &= y - y_1 \end{aligned} \quad (3.33)$$

and

$$\begin{aligned} x'_i &= x_i - x_1 \\ y'_i &= y_i - y_1. \end{aligned} \quad (3.34)$$

The TDoA measurements can also be defined based on receiver 1 as

$$\delta_{i,1} = c(t_i - t_1) = c((t_0 + \tau_i) - (t_0 + \tau_1)) = d_i - d_1, \quad \forall i \in \{2, \dots, N\}. \quad (3.35)$$

The system computes those TDOAs, the d_i terms in (3.32) can then be replaced with (3.35). By also replacing the target and receiver positions with (3.33) and (3.34), the system with x , y and d_1 as unknowns becomes

$$\begin{cases} x'^2 + y'^2 = d_1^2 \\ (x' - x'_2)^2 + (y' - y'_2)^2 = (d_1 + \delta_{2,1})^2 \\ \vdots \\ (x' - x'_N)^2 + (y' - y'_N)^2 = (d_1 + \delta_{N,1})^2. \end{cases} \quad (3.36)$$

After some development and reordering of the terms, the system becomes

$$\begin{cases} x'^2 + y'^2 = d_1^2 \\ 2x'_2x' + 2y'_2y' + 2d_1\delta_{2,1} = x'^2 + x'^2_2 + y'^2 + y'^2_2 - \delta_{2,1}^2 - d_1^2 \\ \vdots \\ 2x'_Nx' + 2y'_Ny' + 2d_1\delta_{N,1} = x'^2 + x'^2_N + y'^2 + y'^2_N - \delta_{N,1}^2 - d_1^2. \end{cases} \quad (3.37)$$

By injecting the first equation into the last $N - 1$ linear equations to simplify the right terms, the system can be written in matrix form in terms of x' , y' and d_1 as

$$\begin{bmatrix} x'_2 & y'_2 & \delta_{2,1} \\ x'_3 & y'_3 & \delta_{3,1} \\ \vdots & \vdots & \vdots \\ x'_N & y'_N & \delta_{N,1} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ d_1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x'^2_2 + y'^2_2 - \delta_{2,1}^2 \\ x'^2_3 + y'^2_3 - \delta_{3,1}^2 \\ \vdots \\ x'^2_N + y'^2_N - \delta_{N,1}^2 \end{bmatrix} \quad (3.38)$$

The unknown d_1 can be defined as parameter from which the other unknowns depend. The system becomes

$$\begin{bmatrix} x'_2 & y'_2 \\ x'_3 & y'_3 \\ \vdots & \vdots \\ x'_N & y'_N \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x'^2_2 + y'^2_2 - \delta_{2,1}^2 \\ x'^2_3 + y'^2_3 - \delta_{3,1}^2 \\ \vdots \\ x'^2_N + y'^2_N - \delta_{N,1}^2 \end{bmatrix} + r_1 \begin{bmatrix} -\delta_{2,1} \\ -\delta_{3,1} \\ \vdots \\ -\delta_{N,1} \end{bmatrix}, \quad (3.39)$$

which can be described as

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b} + d_1\mathbf{c} \quad (3.40)$$

From (3.40), the least squares solution is given by

$$\boldsymbol{\theta} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T (\mathbf{b} + d_1 \mathbf{c}), \quad (3.41)$$

which can be rewritten as

$$\boldsymbol{\theta} = (\mathbf{u} + d_1 \mathbf{v}). \quad (3.42)$$

Finally, knowing that

$$\|\boldsymbol{\theta}\|^2 = x'^2 + y'^2 = d_1^2 = \|\mathbf{u} + d_1 \mathbf{v}\|^2, \quad (3.43)$$

which gives

$$d_1^2 = \|\mathbf{u} + d_1 \mathbf{v}\|^2 = \mathbf{u}^T \mathbf{u} + 2d_1 \mathbf{u}^T \mathbf{v} + d_1^2 \mathbf{v}^T \mathbf{v}, \quad (3.44)$$

we obtain the quadratic equation for d_1

$$(1 - \mathbf{v}^T \mathbf{v})d_1^2 - (2\mathbf{u}^T \mathbf{v})d_1 - (\mathbf{u}^T \mathbf{u}) = 0. \quad (3.45)$$

Once the solution of this equation is found, the position of the target can be found with (3.42), remembering that

$$\boldsymbol{\theta} = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad (3.46)$$

3.2.2 Non-linear least squares estimator

One downside of the linear least squares estimator is that it has to take a receiver as a reference. If the measurement of this receiver is corrupted, its error will be injected into all the TDoAs, in contrast to the other receivers that inject their error only into their own TDoA. This is particularly an issue in the presence of multipath where a receiver can measure a reflection and not a direct path, leading to a large error. To avoid this problem, another approach consists of defining the unknown time origin t_0 as a variable and working with variable ToAs $\tau_i(t_0)$ that depend on t_0 as

$$\tau_i(t_0) = t_i - t_0. \quad (3.47)$$

The least-squares curve fitting problem, which in this case consists in finding the best set of values $\boldsymbol{\beta} = (x, y, t_0)$ with N receivers, can be solved by minimising the sum of the squares of the residuals $S(\boldsymbol{\beta})$ written as

$$S(\boldsymbol{\beta}) = \sum_{i=1}^N r_i(\boldsymbol{\beta})^2, \quad (3.48)$$

where the residual r_i of an estimate $\boldsymbol{\beta}$ with respect to the measurement of the i^{th} receiver located at position $\mathbf{x}_i$ is the difference between the estimate $\mathbf{x}$ and the data model from the measurements. In the case of TDoAs, the residual is given by

$$r_i(\boldsymbol{\beta}) = \sqrt{(x - x_i)^2 + (y - y_i)^2} - c(t_i - t_0). \quad (3.49)$$

Indeed, as illustrated in Figure 3.9, the closest position $\mathbf{x}^*$ that lies on the circle of radius $d = c\tau_i$ is separated from a distance $e_i = |(\|\mathbf{x} - \mathbf{x}_i\| - d)| = |\sqrt{(x - x_i)^2 + (y - y_i)^2} - c(t_i - t_0)| = |r_i|$. The sign of the residual r_i can be negative, it then disappears when taking the square of the residual. The sum of squared residuals $S(\boldsymbol{\beta})$ is then written as

$$S(\boldsymbol{\beta}) = \sum_{i=1}^N \left(\sqrt{(x - x_i)^2 + (y - y_i)^2} - c(t_i - t_0) \right)^2 \quad (3.50)$$

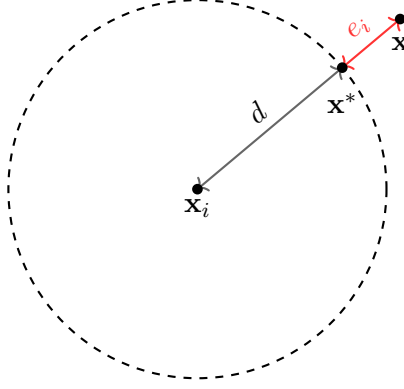


Figure 3.9: Error for a ToA

This function is non linear because x and y are inside a square root, but not t_0 , which makes it difficult to derive a closed-form description. To solve it, iterative solvers should be used to minimise the cost function $S(\boldsymbol{\beta})$ using techniques such as gradient descent. In this work, $S(\boldsymbol{\beta})$ is minimised with the Levenberg-Marquardt (LM) algorithm, which is commonly used for non-linear least squares. This algorithm requires to know the Jacobian matrix $\mathbf{J}$ of $\mathbf{r}(\boldsymbol{\beta})$, where $\mathbf{r}(\boldsymbol{\beta}) = (r_1(\boldsymbol{\beta}), r_2(\boldsymbol{\beta}), \dots, r_N(\boldsymbol{\beta}))$ is the residual vector. The row $\mathbf{J}_i$ is the gradient of the residual r_i defined as

$$\mathbf{J}_i = \nabla r_i(\boldsymbol{\beta}) = \left(\frac{\partial r_i(\boldsymbol{\beta})}{\partial x} \quad \frac{\partial r_i(\boldsymbol{\beta})}{\partial y} \quad \frac{\partial r_i(\boldsymbol{\beta})}{\partial t_0} \right). \quad (3.51)$$

The first derivatives of $r_i(\boldsymbol{\beta})$ with respect to x , y and t_0 are given by

$$\begin{aligned} \frac{\partial r_i(\boldsymbol{\beta})}{\partial x} &= \frac{x - x_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}, \\ \frac{\partial r_i(\boldsymbol{\beta})}{\partial y} &= \frac{y - y_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}, \\ \frac{\partial r_i(\boldsymbol{\beta})}{\partial t_0} &= c. \end{aligned} \quad (3.52)$$

The LM algorithm works iteratively by updating its current estimate $\boldsymbol{\beta}$ to a new estimate $\boldsymbol{\beta} + \boldsymbol{\delta}$, where the increment $\boldsymbol{\delta}$ is chosen to decrease $S(\boldsymbol{\beta} + \boldsymbol{\delta})$ as

$$\boldsymbol{\delta} = (\mathbf{J}^T \mathbf{J} + \lambda \mathbf{I})^{-1} \mathbf{J}^T \mathbf{r} \quad (3.53)$$

with λ the damping factor that is a modifiable parameter to adjust the direction of the increment $\boldsymbol{\delta}$ and $\mathbf{I}$ the identity matrix. This method combines the Gauss-Newton (GN) algorithm with the term $\mathbf{J}^T \mathbf{J}$, which helps improve convergence speed, and the gradient descent with the term $\lambda \mathbf{I}$, which is less sensitive to the starting location of the algorithm [36].

3.3 Triangulation

Triangulation techniques use the direction, or angle, between a transmitter and a receiver to obtain information about the position of a target. Depending on the system architecture, they can use angles of arrival or angles of departure. For both methods, the basic principle is the same, a transmitter sends a signal to a receiver, and the receiver measures the phase shifts created by an antenna array to calculate the corresponding direction of transmission. The difference lies in the roles of the transmitter and the receiver. When using AoAs, the target sends a signal to the different base stations, each equipped with an antenna array, that measure the angle from which the signal arrives. Using AoDs reverse the roles, the target now receives different signals and measures the angles from which they have departed

from the base stations. Systems that use AoAs are typically designed for scenarios where the position needs to be known on the base station side, in contrast to AoD-based systems that are designed to give information on the target side.

In the context of this work, the localisation system should give the position of endangered firefighters to the commanding officer so that it can send support to the correct location. For this reason, and as explained in Chapter 2, the designed system uses AoAs, along with TDoAs, where the different base stations measure the angle from which they receive the signal transmitted by the target, here the firefighter.

In this system, the i^{th} receiver located a position $\mathbf{x}_i$ measures the angle θ_i with respect to the unknown position of the target $\mathbf{x}$. Considering the 2D localisation problem where $\mathbf{x}_i = [x_i \ y_i]^T$ and $\mathbf{x} = [x \ y]^T$, the angle θ_i is given by

$$\theta_i = \arctan\left(\frac{y - y_i}{x - x_i}\right). \quad (3.54)$$

The expression (3.54) is not linear in terms of x and y . It can be rewritten as

$$x \sin \theta_i - y \cos \theta_i = x_i \sin \theta_i - y_i \cos \theta_i. \quad (3.55)$$

Combining the measurements of the N receivers gives a set of N linear equations that can be written in the matrix form $\mathbf{A}\mathbf{x} = \mathbf{b}$, where $\mathbf{A}$ is an $N \times 2$ matrix with each row $\mathbf{A}_i$ given by

$$\mathbf{A}_i = [\sin \theta_i \quad -\cos \theta_i] \quad (3.56)$$

and $\mathbf{b}$ is an $N \times 1$ vector where each element b_i is given by

$$b_i = x_i \sin \theta_i - y_i \cos \theta_i \quad (3.57)$$

The solution $\mathbf{x}$ can then be computed with a standard Least Squares $\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$. In order for $\mathbf{A}$ to be invertible, it must have its rank equal to 2, which is the minimum number of receivers required, assuming that its rows are not linearly dependent, for example if the receivers are aligned with the target.

Similarly to what is done in Section 3.2 for the multilateration problem, another possible formulation is to minimise the mean square error with a sum of squared residuals $S(\boldsymbol{\beta})$ with the estimate $\boldsymbol{\beta} = (x, y)$. In the case of triangulation, the residuals are linear with respect to the variables, which makes this formulation equivalent to the standard least squares. However, this formulation is useful for Section 3.4. The cost function $S(\boldsymbol{\beta})$ can be expressed as

$$S(\boldsymbol{\beta}) = \sum_{i=1}^N r_i(\boldsymbol{\beta})^2 = \sum_{i=1}^N \left((x - x_i) \sin \theta_i - (y - y_i) \cos \theta_i \right)^2, \quad (3.58)$$

with

$$r_i(\boldsymbol{\beta}) = (x - x_i) \sin \theta_i - (y - y_i) \cos \theta_i. \quad (3.59)$$

Indeed, as illustrated in Figure 3.10, the error e committed for an angle measurement $\hat{\theta}_i$ by the estimated point X is the distance from the closest position $X^* = [x^* \ y^*]^T$ along the direction θ_i . One can obtain the following relations:

$$\begin{aligned} x - x_i &= (x^* - x_i) + e \sin \theta_i = d \cos \theta_i + e \sin \theta_i \\ y - y_i &= (y^* - y_i) - e \cos \theta_i = d \sin \theta_i - e \cos \theta_i \end{aligned} \quad (3.60)$$

where d is the distance between X_i and X^* . From there, by isolating d and equalling the two relations:

$$\begin{aligned} d \cos \theta_i &= (x - x_i) - e \sin \theta_i \\ d \sin \theta_i &= (y - y_i) + e \cos \theta_i \end{aligned} \quad (3.61)$$

$$\sin \theta_i [(x - x_i) - e \sin \theta_i] = \sin \theta_i [(y - y_i) + e \cos \theta_i] \quad (3.62)$$

The expression of e is obtained as

$$e = (x - x_i) \sin \theta_i - (y - y_i) \cos \theta_i, \quad (3.63)$$

which is equivalent to (3.59).

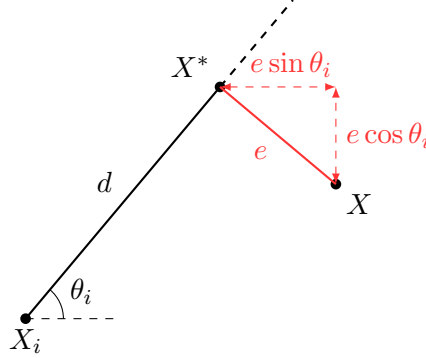


Figure 3.10: Error for an AoA

The same was as for the multilateration problem, the sum of squares $S(\beta)$ can be minimised with the Levenberg-Marquardt algorithm. The first derivatives of $r_i(\beta)$ with respect to x and y are simply given by

$$\begin{aligned} \frac{\partial r_i}{\partial x} &= \sin \theta_i, \\ \frac{\partial r_i}{\partial y} &= \cos \theta_i. \end{aligned} \quad (3.64)$$

3.4 Hybrid multilateration and triangulation

In order to improve the accuracy of the position estimation, a logical and intuitive idea is to fuse the two types of measurement into a single algorithm. It combines the information of the TDoAs, which inform about the distance between the target and the receivers, and the AoAs that indicate in which direction the target is located. In addition, a hybrid system only requires two receivers, as shown in [37].

The sum of squares functions defined in Section 3.2 and Section 3.3 make it quite straightforward to combine the two types of measurements. The residuals $r_{t,i}$ of the ToAs and $r_{\theta,i}$ of the AoAs are all expressed in metres, making them directly comparable. Hence, it is possible to create a sum of squares function $S(\beta)$ that is simply the sum of (3.50) and (3.58), which does not favour one type of measurement over the other. The hybrid sum of squares is then given as

$$\begin{aligned} S(\beta) &= \sum_{i=1}^N r_{t,i}(\beta)^2 + r_{\theta,i}(\beta)^2 \\ &= \sum_{i=1}^N \left(\sqrt{(x - x_i)^2 + (y - y_i)^2} - c(t_i - t_0) \right)^2 + \left((x - x_i) \sin \theta_i - (y - y_i) \cos \theta_i \right)^2 \end{aligned} \quad (3.65)$$

To apply the LM algorithm to this new sum of squares, one must still have access to the Jacobian $\mathbf{J}$ of the vector of residuals $\mathbf{r}(\beta)$. For N receivers, it can now be expressed as

$$\mathbf{r}(\beta) = (r_{t,1}(\beta) \ r_{\theta,1}(\beta) \ r_{t,2}(\beta) \ r_{\theta,2}(\beta) \ \dots \ r_{t,N}(\beta) \ r_{\theta,N}(\beta)) . \quad (3.66)$$

The derivatives of these residuals are the same as the one given by (3.52) and (3.64), the Jacobian can then be constructed with each row being the gradient of a residual and the iterative solving of the LM algorithm is applied as before.

3.5 Application to firefighter environment

Each of the techniques described previously are tested with the system architecture described in Chapter 2 and in the firefighter environment described in Chapter 1. The exact same parameters are kept, which include the bandwidth or the transmit power, to simulate as realistically as possible the firefighter scenario. For the two buildings considered, a grid of positions is tested, as shown in Figure 3.11, and the localisation algorithm is applied 10 times for each position to average random processes like AWGN.

As said in Chapter 2, the number of receivers deployed is not a fixed parameter, so each technique is also evaluated for different numbers of base stations, 4, 6 and 8, to estimate an appropriate number for the system. For scenarios with 4 Rxs, the Rxs are placed at the 4 corners of the building, for scenarios with 6 Rxs, 2 additional Rxs are placed at the left and right of the building, and for 8 Rxs, the last 2 Rxs are placed at the top and bottom of the building.

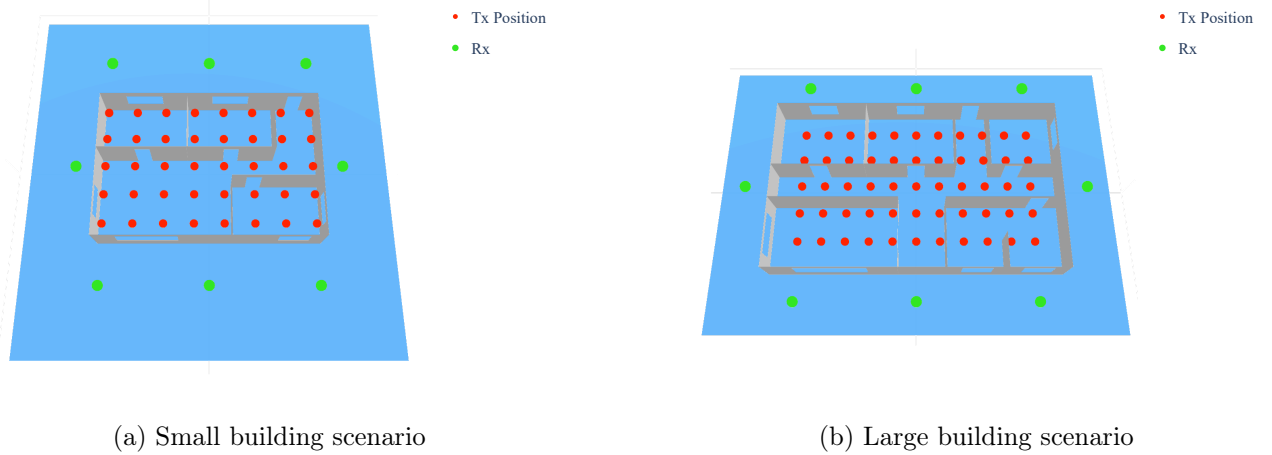


Figure 3.11: Grids of positions tested for localisation techniques evaluation

3.5.1 Multilateration

Multilateration has been applied successfully to many different applications such as GPS. However, its efficiency is only guaranteed as long as there is a line-of-sight between a transmitter and a receiver so that the radio signal travels along the direct path, as this technique assumes that the propagation time measured corresponds to physical distance between the transmitter and the receiver. In the case where the radio signal that reaches the receiver does not follow a direct path but an alternative one with reflections, the distance it travels, which is now longer, does not correspond to the actual physical distance between the transmitter and the receiver, breaking the measurement model. This typically happens in indoor environments where the multiple walls or objects create multipath.

The two estimators presented above, LLS and NLLS, are applied with the ToA measurements provided by the MUSIC algorithm. For each receiver, the MUSIC algorithm can detect several paths, but only the first ToA, i.e. the path that arrived the earliest, is given as input to the localisation algorithm, because it is assumed that it corresponds to the LoS path. The paths that arrive later are necessarily NLoS paths, but it could also be the case that the first ToA detected also corresponds to an NLoS path and that the direct path did not reach the receiver, which would then give incorrect information

to the localisation algorithm. With the ToAs given by MUSIC, TDoAs are computed since the Rx's are not synchronised with the Tx.

Because multipath makes the localisation algorithm produce many implausible solutions, the metric used here to characterise its performance is the cumulative density function (CDF) of the errors committed, instead of a metric like RMSE that is very sensitive to outliers. The CDFs of the errors made by the two estimators for different numbers of Rx in the small building scenario are reported in Figure 3.12.

The general trend observed is that the CDF increases steadily as the errors committed become larger, and at one point (around errors of 8 metres), it reaches a plateau. It means that, in the majority of the cases, the multilateration algorithm manages to find an estimation, but this estimation may not be very precise due to NLoS paths given as input to the algorithm. In the rest of the cases, multipath and shadowing created by building walls actually cause some Rx's to either detect paths that create TDoAs that do not respect the triangle inequality, i.e. the absolute value of the TDoA is larger than the physical distance between the receivers, or either to not detect any signal at all.

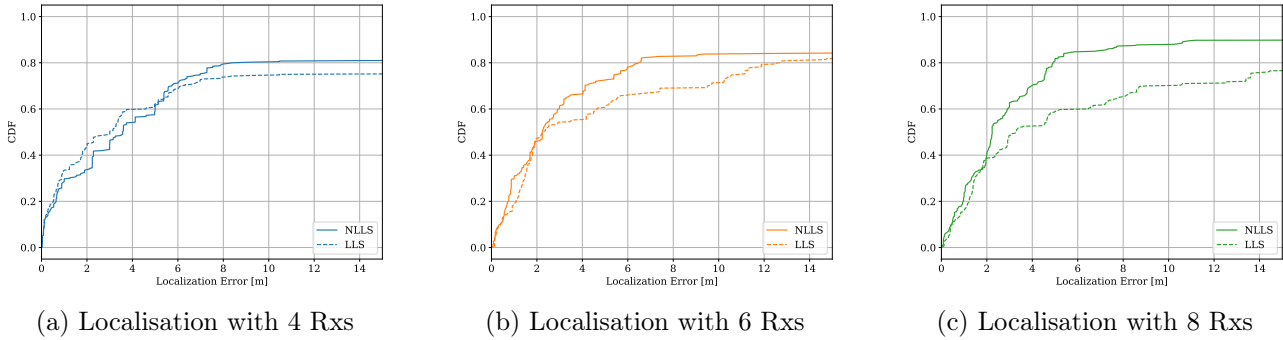


Figure 3.12: Comparison of the NLLS and the LLS estimators of multilateration for different number of Rx's in the small building scenario

One can see that the NLLS estimator globally outperforms the LLS estimator, especially when the error starts to increase. Indeed, those cases typically occur because NLoS paths are used by the algorithm, and because the LLS estimator takes 1 Rx as reference, its error can corrupt all the TDoAs computed, which creates large inaccuracies if the path of that reference Rx was an NLoS path. On the other hand, the NLLS considers all variable ToAs (with t_0 as an unknown) equally, which avoids giving too much importance to an NLoS path.

Increasing the number of Rx's slightly improves the performance, as it reduces the number of cases where not enough measurements are available. However, NLoS paths remain a problem and still strongly corrupt the estimation. Although in most of the cases the error is under 5 metres, it is not a decent enough precision, especially compared to the dimensions of the buildings that are $15\text{m} \times 10\text{m}$.

The redundancy provided by additional receivers gets more important for the large scenario environment, as the increased distances and number of walls cause more Rx's to not detect the direct path or any path at all. As shown in Figure 3.13, using only 4 Rx's only allows to get an estimate 20 % of the time, and this estimate is often inaccurate. The performance improvement when increasing the number of Rx's is more significant than in the small building scenario, and using 8 Rx's gives a nearly similar performance in both scenarios. The other observations remain true in the large scenario, using only multilateration is not satisfying for the firefighter application.

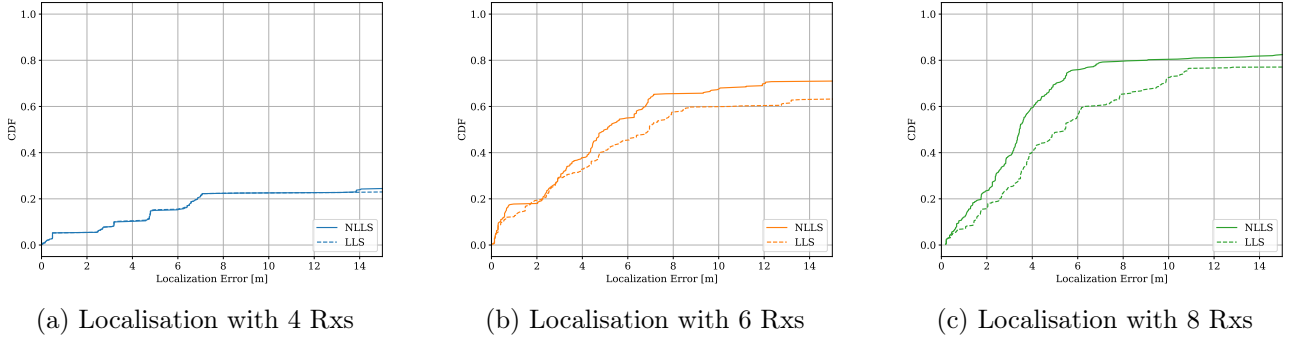


Figure 3.13: Comparison of the NLLS and the LLS estimators of multilateration for different number of Rxs in the large building scenario

3.5.2 Triangulation

The LLS estimator is applied to both scenarios with the AoA estimations of the MUSIC algorithm that corresponds to the paths with the smallest associated ToA, as it should ideally be a direct path. The CDFs of the errors committed for different number of receivers are shown in Figure 3.14. Unlike multilateration, there are no plateau observed due to the invalidity of some measurements such as TDoAs that are larger than the distance between receivers, as AoAs always correspond to a physical location, even if it is incorrect. Hence, it does not suffer from convergence issues. However, the range of the errors remain the same, and the precision is not sufficient for the use of firefighters. Increasing the number of Rxs also slightly improves performances, but does not solve the problem of multipath and the absence of LoS. Similarly to multilateration, triangulation performs worse in the large building scenario due to increased distances and number of walls.

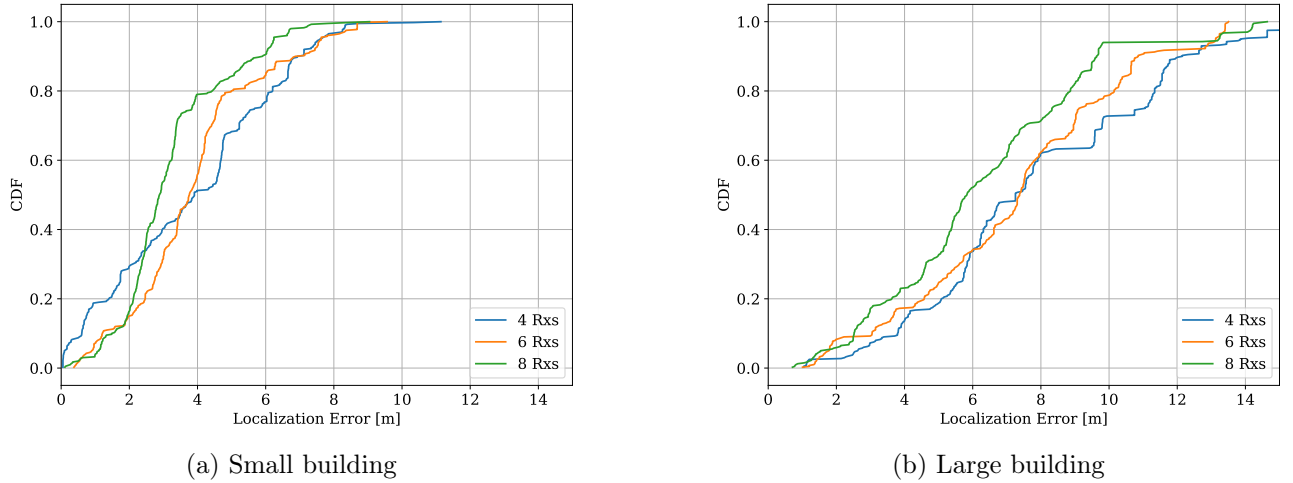


Figure 3.14: Performance of triangulation for different number of Rxs in the firefighter environment

3.5.3 Hybrid localisation

The system was designed to estimate jointly the ToAs and the AoAs of MPCs detected by the receivers. The hybrid localisation takes as input the estimates for the paths that arrived first at each receiver and minimises the sum of squares (3.65) with the LM algorithm. The CDFs obtained for different numbers of receivers are shown in Figure 3.15. Compared to the simple multilateration or triangulation algorithms, the hybrid algorithm performs noticeably better. The combination of the two types of measurement makes the algorithm more robust to cases where few receivers actually detect correct paths. In the small building scenario, the error when 8 Rxs are deployed does not exceed 4 metres, but this has to be compared with the actual dimensions of the building that are also quite short. In the same way

as the other localisation techniques, the errors committed in the large building scenario are logically larger, and increasing the number of Rxs improves the performance, but not drastically.

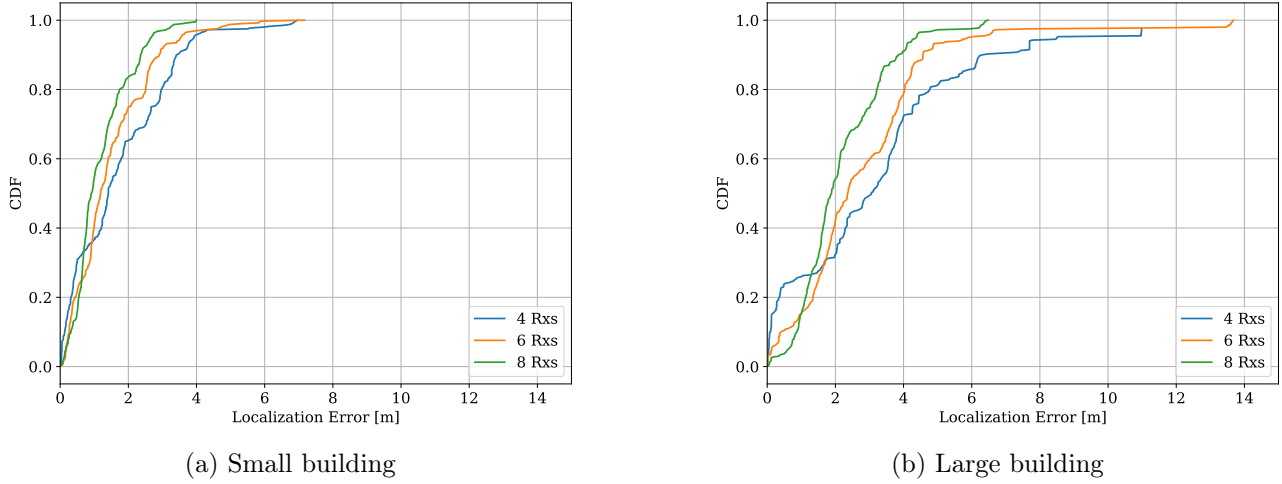


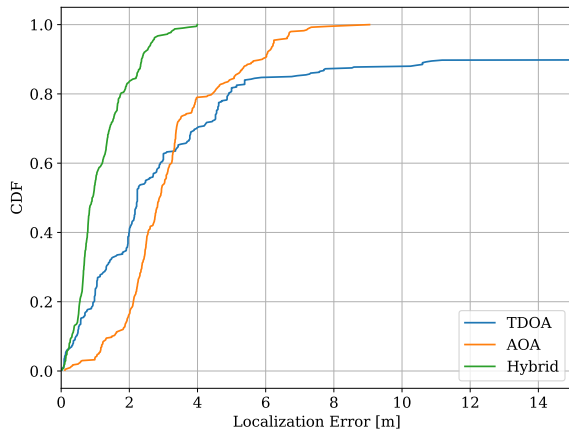
Figure 3.15: Performance of hybrid localisation for different number of Rxs in the firefighter environment

3.5.4 Summary of results

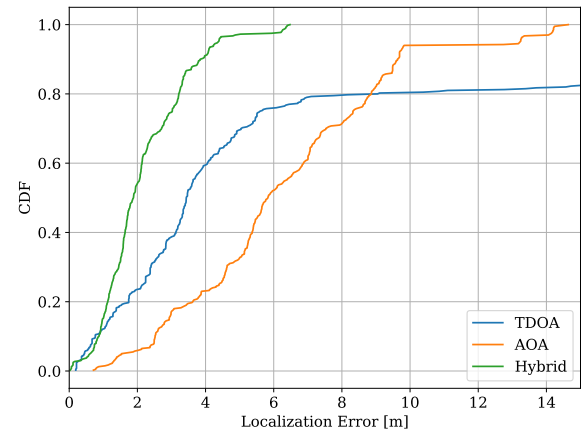
This chapter developed the different techniques that can be used by the IPS designed in Chapter 2. The MUSIC algorithm enables to jointly estimate the ToA and AoA of MPCs that have enough power compared to the noise, so NLoS conditions and shadowing due to walls are an issue against reliable localisation. Then, multilateration, triangulation, and hybrid localisation were applied to the firefighter scenario to evaluate what performance could be achieved by classical localisation techniques.

It was observed that using only TDoAs suffered from convergence errors due to MPCs that give incorrect information to the algorithm. Using AoAs does not suffer from that problem, but its performance remains similar to that of TDoAs. Combining the two types of measurement increases accuracy, but this remains unsatisfactory for the precision required by firefighters. In addition, larger scenarios lead to a decrease in performance due to increased distances, multipath components, and NLoS conditions.

Finally, a larger number of receivers improves performance by adding redundancy but does not solve the core problems of indoor environments. Figure 3.16 compares the performance achieved by the different techniques for the largest number of receivers considered, i.e. 8 Rxs. The final performance achieved by these traditional localisation techniques remains insufficient to provide a reliable position estimate to firefighters. Consequently, there is a need to develop alternative localisation algorithms that are robust to the difficult indoor conditions. This problem is tackled in Chapter 4.



(a) Small building



(b) Large building

Figure 3.16: Performance comparison of the different localisation techniques for 8 RxS in the firefighter environment

Chapter 4

Multipath awareness

In Chapter 3, it was highlighted that the multipath present in indoor environments, such as firefighter intervention locations, could significantly reduce the performance of classical localisation techniques such as multilateration or triangulation. This was due to detected signals that do not correspond to a direct path between the transmitter and the receiver and to the shadowing created by building walls that simply prevents any signal to be detected.

4.1 NLoS mitigation

NLoS paths that are misinterpreted as direct paths strongly degrade the performance of classical localisation algorithms that rely on this assumption, even if a good proportion of the paths given as input to the algorithm are correct. Hence, it can be beneficial to try to identify which measurements, ToA or AoA, correspond to direct paths, and which of them are actually outliers corresponding to NLoS paths.

There are different ways to try mitigating NLoS paths. Some methods use a priori information on the NLoS statistics and develop a maximum likelihood estimator to detect NLoS paths [38]. However, this information is rarely directly accessible, it needs to be approximated with empirical values, which are not always available. To solve this, [38] introduces a two-step algorithm for TDoAs and AoAs that first computes a position estimation with all measurements, then compares the residuals of each measurement to a threshold, and discards measurements with too large residuals. A similar approach is used in [39]. The downsides of this approach are that it needs a sufficiently large number of measurements, that the value of the threshold impacts the detection rate, and that a too large number of outliers can completely corrupt the first estimate, invalidating the second step.

The method presented in [40] builds a vector of outliers that is supposed to be sparse and iteratively minimises a reweighted least squares algorithm with LASSO regularisation to give smaller importance to outliers. A weighted least squares in combination with a heuristic firefly algorithm is also implemented in [41]. Other systems, as the one presented in [42], detect the type of environment, either LoS or NLoS, based on a threshold and apply either a MUSIC algorithm to use ToAs and AoAs or a fingerprint algorithm with extreme learning machine. Tracking algorithms, typically based on Kalman filters, allow to filter measurement outliers that occur sporadically [43].

The approach chosen for this system is to use a RANdom SAmple Consensus (RANSAC) algorithm, originally developed for image analysis applications [44], for the detection of outliers. This algorithm is known to be quite robust with respect to the number of outliers and versatile as it can be applied to many different applications.

4.1.1 RANSAC algorithm

The basic principle of this algorithm is to randomly select a subset of samples, or measurements, fit the model of interest with these samples, and count the number of inliers, i.e. samples that match with the model up to a defined threshold. Repeating this for several iterations, the best model selected is the one that gave the highest number of inliers.

Figure 4.1 illustrates a scenario in which the data is supposed to follow a straight line, the corresponding model is thus a linear function. The RANSAC algorithm randomly selects 2 data points, as it is the smallest number of samples required to fit a linear model, and fits the model. It then looks at all the data points and labels those that lie within a certain distance from the fitted line as inliers. The corresponding set of inliers is called the consensus set. When the 2 selected data points are inliers, the fitted model corresponds quite well to the real one, and many data points lie close to the fitted line, the algorithm labels a large number of them as inliers. In the case where an outlier is selected, the fitted model does not correspond to reality and only a few points are labelled as inliers. The RANSAC algorithm looks at the cardinality of the consensus set of a model and, if it is above a certain threshold, considers it as valid, otherwise it tries again with another random pair. Finally, the model can be fitted with all the inliers identified to improve its quality.

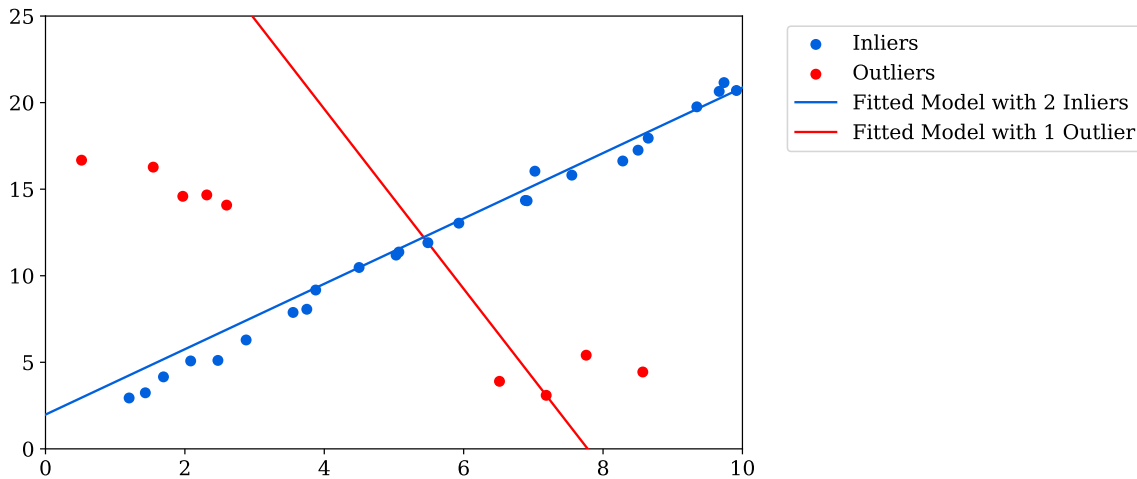


Figure 4.1: Example of RANSAC algorithm for a 2D linear model

One downside of this classical approach is that the threshold for considering a number of inliers as sufficient is not always easy to determine. In the case of a localisation system with only a few base stations, just slightly increasing or decreasing this threshold can have a strong impact. For this reason, the RANSAC algorithm used here is a variant of the original without the threshold on the cardinality of the consensus set. Instead, the algorithm iterates a fixed number of k times over random subsets of samples, computes their consensus sets, and keeps as the best estimate the model with the largest consensus subset. At the end, it refits the model with the largest consensus set. This algorithm, for a set of samples D , a model M that takes a minimum of n samples to fit, k iterations, and a threshold t to determine inliers, is described as follows.

RANSAC algorithm

1. Initialize the set of inliers I to $I = \emptyset$.
2. Repeat the following steps k times:
 - 2.1 Randomly sample n points from D .
 - 2.2 Fit a model to these n points.

- 2.3 Create an empty consensus set C . For every other point in D , calculate its distance to the fitted model. If the distance is smaller than a threshold t , add the point to C .
- 2.4 If the size of C is larger than the current size of I , update $I = C$
3. Train the final model M with the set of inliers I .

There exist more advanced versions of the RANSAC algorithm that use more complex indicators than the number of inliers to evaluate the goodness of a model fit **ransac**, but they are not considered in this work.

For the application of the RANSAC algorithm to the localisation problem, the model to fit is the measurement model for the hybrid algorithm with parameters $\beta = (x, y, t_0)$ that needs to be chosen optimally. The data points are the different receiver positions and their measurements, which form the dataset $D = \{D_1, D_2, \dots, D_N\}$ for N receivers, where each $D_i = (x_i, y_i, t_i, \theta_i)$. With (3.65), this model can be fitted by minimising a sum of squares.

For each iteration k , a subset D^k is built with 2 receivers, i.e. 2 D_i , as it provides 4 residuals for 3 unknowns, which is enough so that the problem is not underdetermined. The sum of squares $S_k(\beta)$ to minimise at the k^{th} iteration is simply given from (3.65) as

$$S_k(\beta) = \sum_{i \in D^k}^N r_{t,i}(\beta)^2 + r_{\theta,i}(\beta)^2 \quad (4.1)$$

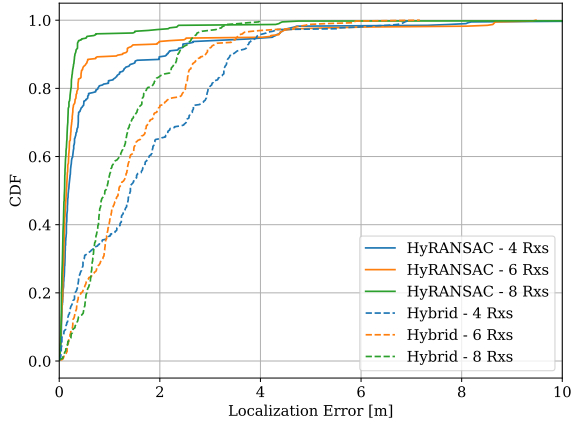
After the minimisation with the LM algorithm, the RANSAC algorithm checks which samples D_i are inliers and add them to a consensus set C_k . To determine if a receiver sample D_i with respect to the parameters found β , it compares the distance between the estimated position (x, y) and the position predicted by D_i for the estimated time origin t_0 to a certain threshold t . The comparison can be written as

$$\sqrt{\left(x - (x + c(t_i - t_0) \cos \theta_i)\right)^2 + \left(y - (y + c(t_i - t_0) \sin \theta_i)\right)^2} < t \quad (4.2)$$

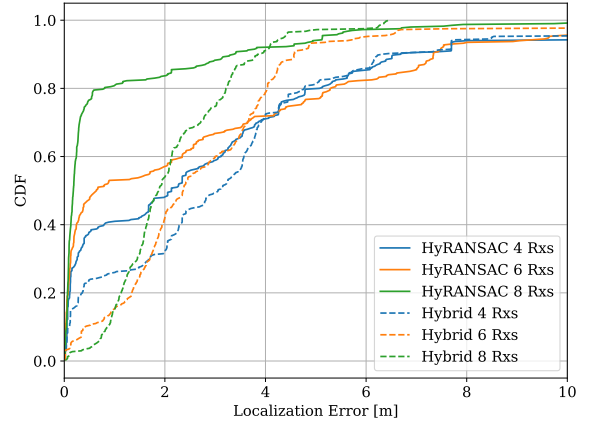
If the number of inliers is larger than the current best set of inliers I , $|C_k| > |I|$, then I is updated as $I = C_k$. After the k iterations, the sums of squares is minimised one last time for the set of inliers I , and the position estimation (x, y) is obtained. If after the iterations, I is still empty, for example, because the majority of measurements were outliers and no good model was found, the final estimate is computed with the classic hybrid algorithm of Section 3.4.

4.1.2 Application to firefighter scenario

The RANSAC algorithm using hybrid measurements, which is called here HyRANSAC, can be compared to the basic hybrid localisation scheme. For both scenarios, Figure 4.2 shows the CDFs of errors committed by both localisation schemes with the same procedure as the one described in Chapter 3. Compared to the basic hybrid algorithm, HyRANSAC achieves better accuracy for cases where the errors committed by the hybrid scheme are small. However, at one point, it is caught up by the hybrid scheme. It can be explained by the fact that the small error cases correspond to the cases where enough measurements of sufficient quality were acquired by the receivers, HyRANSAC can then choose those that correspond to LoS paths and discard the others. When the error gets larger, this corresponds to the case where not enough LoS paths were detected by the receivers, HyRANSAC no longer has the opportunity to select enough correct paths, thus it becomes actually equivalent to the basic hybrid scheme.



(a) Small building

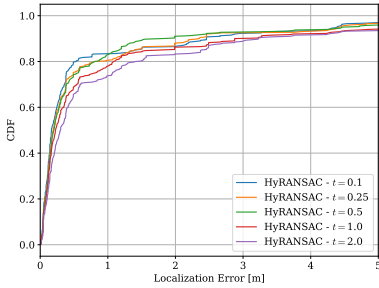


(b) Large building

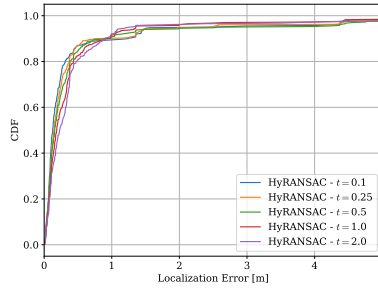
Figure 4.2: Performance of triangulation for different number of Rxs in the firefighter environment

One element that can potentially influence the HyRANSAC performance is the value at which the threshold t is set. If chosen too tight, paths estimated with some inaccuracies will be considered as outliers, and if chosen too large, some NLoS paths will get treated as direct paths. Different values of t are tested in both scenarios for 4, 6, and 8 Rxs. The CDFs obtained are reported in Figure 4.3 and Figure 4.4. As a first observation, the performance does not change drastically depending on the threshold value, which gives some margin in the choice of the threshold value.

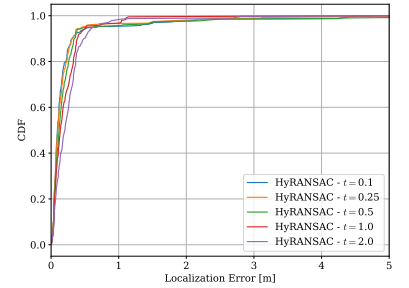
In the different cases, one can see that for small errors, low values of t gives slightly more precise positioning. This is logical, as for those cases, most of the estimated paths are LoS paths, and the NLoS paths have similar ToAs and AoAs as the direct path. This also means that the estimations provided by the MUSIC algorithm are quite precise, as inliers are detected even for a threshold as low as $t = 0.1$. When the errors become larger, higher values of t give better results, as they are more tolerant to small inaccuracies. To summarize, although the threshold value needs to be fixed carefully, the HyRANSAC algorithm offers a bit of tolerance regarding that value without drastic change of performance.



(a) Localisation with 4 Rxs



(b) Localisation with 6 Rxs



(c) Localisation with 8 Rxs

Figure 4.3: Comparison of the HyRANSAC algorithm for different threshold values in the small building scenario

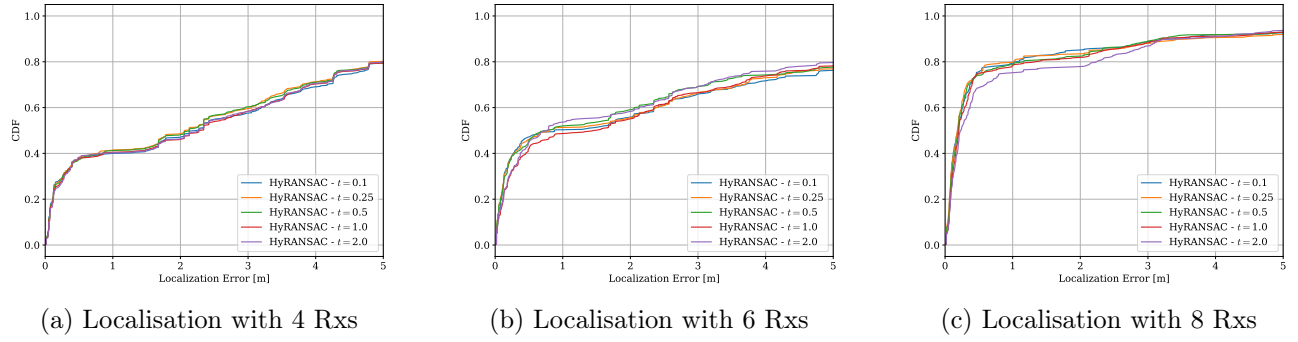


Figure 4.4: Comparison of the HyRANSAC algorithm for different threshold values in the large building scenario

4.2 First-order reflections

In the previous section, the aim of the RANSAC algorithm is to detect outliers so that they are not given as input to the localisation algorithm, which would otherwise corrupt the estimation. However, even though NLoS paths are not suited for the classical measurement models that expect information about direct paths, they still contain some information about the scene and the position of the target. For example, if an MPC is known to have a reflection at a certain place, it can be used to adjust the measurement model rather than incorrectly considering it as a direct path. In [45], a model to exploit the reflections is developed assuming that the position of reflectors are known, which is not the case for the firefighter scenario considered, and [46] builds an NLoS model based on a stochastic distribution of MPC delays and amplitudes.

This section considers the use of First Order Reflections (FOR) to improve the performance of the HyRANSAC algorithm presented in the previous section. FORs are paths that have only been subject to 1 reflection, which implies that the propagating wave changes only once of direction. At the different Rx, the FORs arrive from a certain direction and at a certain time that generally does not correspond to the actual direction between the Rx and the Tx or to the time the signal should have taken to propagate directly between them. For a certain ToA and AoA, the actual position of the Tx can vary significantly depending on the location where the reflection happened and the angle of incidence with which the propagating wave hit the reflecting surface. As illustrated in Figure 4.5, for the same propagation delay and angle of arrival, the actual Tx position cannot be known if the orientation and the location of the reflecting wall is not known. The goal of this section is to develop a measurement model for FORs that can be used to improve the position estimation.

4.2.1 Measurement model

One issue against the use of FORs is that all the MPCs detected by the MUSIC algorithm are not necessarily FORs, they could be paths with several reflections. Thus, it is necessary to identify which ones are actually FORs from other paths. The HyRansac algorithm developed in the previous section is actually well suited for this task. In the same way it tried to identify which paths were actually direct paths and which ones were outliers to the measurement model of direct paths, it can do the same with the other paths for the measurement model of FORs that is developed in this section.

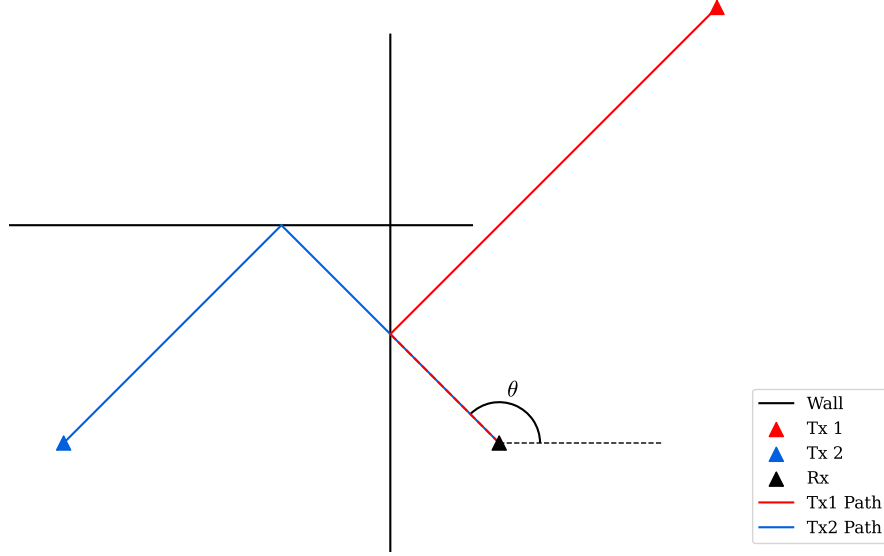


Figure 4.5: Example of potential Tx positions for the same ToA and AoA

In order to use the first-order reflections, some a priori information about the reflecting surface is needed. In the firefighter environment modelled in Chapter 1, the reflections are created by the walls of the building. In practice, firefighters usually do not have access to the plans of the building, and it is difficult to visually estimate the exact position of walls, particularly those of the interior, as it requires judging absolute distances. On the other hand, estimating their orientation is somewhat easier, because orientation is relative to the observer's perspective rather than an absolute measurement. In addition, most buildings are built such that their walls are parallel or perpendicular to each other, which makes only 2 possible orientations.

The approach considered is that, during the system deployment, firefighters should also indicate to the system the orientations of the walls, which can be observed from the exterior walls. Although this is an additional constraint, it is not that significant since, for the system designed in Chapter 2, firefighters already have to specify the orientations of the different receivers.

In order to use an FOR of Rx_i with position (x_i, y_i) , ToA t_i and AoA θ_i , it is assumed here that the propagation delay τ_i is known, i.e., the time origin t_0 of the receiver is known to compute $\tau_i = t_i - t_0$ and the distance $d_i = c\tau_i$ that separates it from the Tx, as well as the orientation of the wall, horizontal or vertical. In that case, it can be shown that the locus of points where the Tx could be located is a line segment perpendicular to the wall. Figure 4.6 illustrate the case where the wall is vertical. If the wave were not reflected, the Tx would be located at $(x, y) = (x_i + c\tau_i \cdot \cos \theta_i, y_i + c\tau_i \cdot \sin \theta_i)$. However, since it is reflected by a vertical wall, its direction along $\hat{x}$ is changed, but not along $\hat{y}$. Because the position of the wall is not known, it is not possible to know when the wave changed direction, so how much it has travelled along $\hat{x}$ and along $-\hat{x}$, and x cannot be determined, but y remains equal to $y_i + c\tau_i \cdot \sin \theta_i$. Thus, the Tx can be located on a horizontal straight line of equation $y = y_i + c\tau_i \cdot \sin \theta_i$. This line is of course finite due to the travelled distance d_i that is also finite, the bounds are $x_{\min} = x_i - c\tau_i \cdot |\cos \theta_i|$ and $x_{\max} = x_i + c\tau_i \cdot |\cos \theta_i|$. The same reasoning can be done for a horizontal wall, which gives the locus of points that is a vertical line $x = x_i + c\tau_i \cdot \cos \theta_i$ and the bounds $y_{\min} = y_i - c\tau_i \cdot |\sin \theta_i|$ and $y_{\max} = y_i + c\tau_i \cdot |\sin \theta_i|$. In addition, this can be extended to paths that bounce on several walls that each have the same orientation, so that the direction changes occur only along $\hat{x}$ or $\hat{y}$.

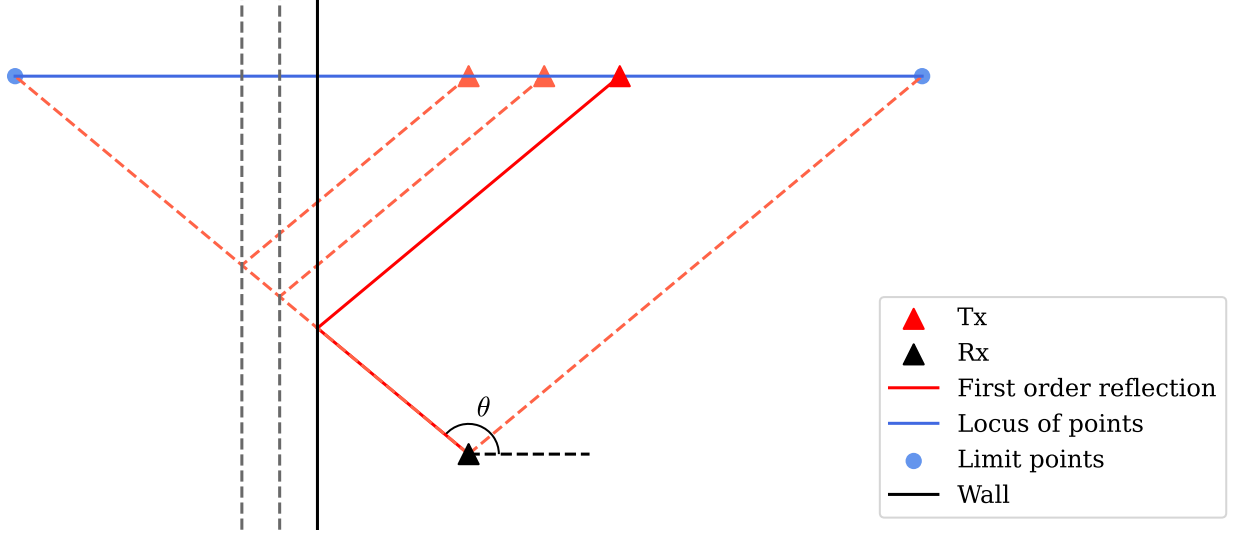


Figure 4.6: Example of First Order Reflection

To integrate this type of measurement to the sum of squares formulation of (3.65), the residual $r_{i,j}(\beta)$ of the path that reached Rx_i at the j^{th} position, considering the ordering $t_{i,j} < t_{i,j+1}$, can be expressed as

$$r_{v(i,j)}(\beta) = \begin{cases} \sqrt{\left(y - (y_i + c(t_{i,j} - t_0) \cdot \sin \theta_{i,j})\right)^2 + \epsilon^2} & \text{if } x_{\min} < x < x_{\max} \\ \sqrt{\left(y - (y_i + c(t_{i,j} - t_0) \cdot \sin \theta_{i,j})\right)^2 + (x - x_{\min})^2 + \epsilon^2} & \text{if } x < x_{\min} \\ \sqrt{\left(y - (y_i + c(t_{i,j} - t_0) \cdot \sin \theta_{i,j})\right)^2 + (x - x_{\max})^2 + \epsilon^2} & \text{if } x > x_{\max} \end{cases} \quad (4.3)$$

if it is an FOR that was reflected on a vertical wall, where ϵ is a small constant to ensure that $r_{h(i,j)}(\beta)$ is continuously differentiable, or as

$$r_{h(i,j)}(\beta) = \begin{cases} \sqrt{\left(x - (x_i + c(t_{i,j} - t_0) \cdot \cos \theta_{i,j})\right)^2 + \epsilon^2} & \text{if } y_{\min} < y < y_{\max} \\ \sqrt{\left(x - (x_i + c(t_{i,j} - t_0) \cdot \cos \theta_{i,j})\right)^2 + (y - y_{\min})^2 + \epsilon^2} & \text{if } y < y_{\min} \\ \sqrt{\left(x - (x_i + c(t_{i,j} - t_0) \cdot \cos \theta_{i,j})\right)^2 + (y - y_{\max})^2 + \epsilon^2} & \text{if } y > y_{\max} \end{cases} \quad (4.4)$$

if it was reflected on a horizontal wall. The first derivatives of $r_{v(i,j)}(\beta)$, with $y_{\text{ref}} = y_i + c(t_{i,j} - t_0) \cdot \sin \theta_{i,j}$, $\frac{\partial y_{\text{ref}}}{\partial t_0} = -c \sin \theta_{i,j}$, $\frac{\partial x_{\min}}{\partial t_0} = -c |\cos \theta_{i,j}|$, and $\frac{\partial x_{\max}}{\partial t_0} = c |\cos \theta_{i,j}|$, are given by

$$\frac{\partial r_v(\beta)}{\partial x} = \begin{cases} 0 & \text{if } x_{\min} \leq x \leq x_{\max} \\ \frac{x-x_{\min}}{\sqrt{(x-x_{\min})^2+(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x < x_{\min} \\ \frac{x-x_{\max}}{\sqrt{(x-x_{\max})^2+(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x > x_{\max} \end{cases} \quad (4.5)$$

$$\frac{\partial r_v(\beta)}{\partial y} = \begin{cases} \frac{y-y_{\text{ref}}}{\sqrt{(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x_{\min} \leq x \leq x_{\max} \\ \frac{y-y_{\text{ref}}}{\sqrt{(x-x_{\min})^2+(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x < x_{\min} \\ \frac{y-y_{\text{ref}}}{\sqrt{(x-x_{\max})^2+(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x > x_{\max} \end{cases} \quad (4.6)$$

$$\frac{\partial r_v(\beta)}{\partial t_0} = \begin{cases} \frac{-(y-y_{\text{ref}})\frac{\partial y_{\text{ref}}}{\partial t_0}}{\sqrt{(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x_{\min} \leq x \leq x_{\max} \\ \frac{-(x-x_{\min})\frac{\partial x_{\min}}{\partial t_0} - (y-y_{\text{ref}})\frac{\partial y_{\text{ref}}}{\partial t_0}}{\sqrt{(x-x_{\min})^2+(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x < x_{\min} \\ \frac{-(x-x_{\max})\frac{\partial x_{\max}}{\partial t_0} - (y-y_{\text{ref}})\frac{\partial y_{\text{ref}}}{\partial t_0}}{\sqrt{(x-x_{\max})^2+(y-y_{\text{ref}})^2+\epsilon^2}} & \text{if } x > x_{\max} \end{cases} \quad (4.7)$$

Similar expressions can be obtained for $r_{h_{(i,j)}}(\beta)$.

4.2.2 Wall orientation

In practice, the orientation of the wall on which the path was reflected is not known, one has to estimate it. In this work, the method developed to identify the orientation of a wall for an FOR is based on comparing its AoA with the AoA of the direct path between the Rx and the Tx. This requires a priori information on the direct path. If the path considered came after another one, then the the first path provided by the MUSIC algorithm may be considered as the direct path, and its AoA can be used for the comparison. However, as highlighted in Chapter 1, it is often the case that the first path that reaches the receiver does not correspond to the LoS. For that reason, the approach followed in this work is to do a first estimation without considering FORs using the HyRANSAC algorithm. Then, the algorithm is applied a second time using the FORs and the first position estimate to improve the final estimation.

The knowledge of the AoA of the LoS path and of the AoA of the FOR is enough to identify if the wall orientation is horizontal or vertical. First, Figure 4.7 defines the different quadrants that are used in the remainder of the section. In function of the quadrants of the AoAs of the direct path and the FOR, it is possible to know the wall orientation of this FOR.

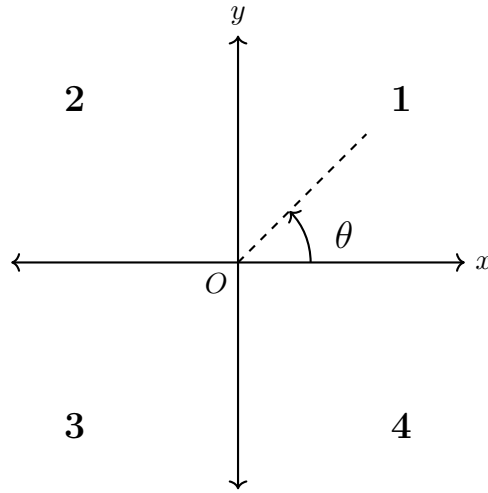


Figure 4.7: Reference of quadrants

Figure 4.7 illustrates a scenario where the direct path arrives from the Tx to the Rx with an AoA in the first quadrant. If the Rx receives an FOR, there are only 4 reflections possible, 2 of which that reach the Rx with an AoA in a different quadrant, named here as Type 1 FOR, and 2 that reach the Rx from the same quadrant named Type 2 FOR. If the FOR is of Type 1, then the wall orientation can be found directly by checking the combination of quadrants of the direct path and of the FOR. In the case of a direct path arriving in Quadrant 1, a Type 1 FOR that arrives from Quadrant 2 must have been reflected by a vertical wall, an FOR that arrives from Quadrant 4 must have been reflected by a horizontal wall, and it is not possible to have an FOR that arrives from Quadrant 3. For a Type 2 FOR, one needs to look at the actual values of the angles. If the angle of the FOR is larger than that of the direct path, the wall must be horizontal, and if it is smaller the wall must be vertical.

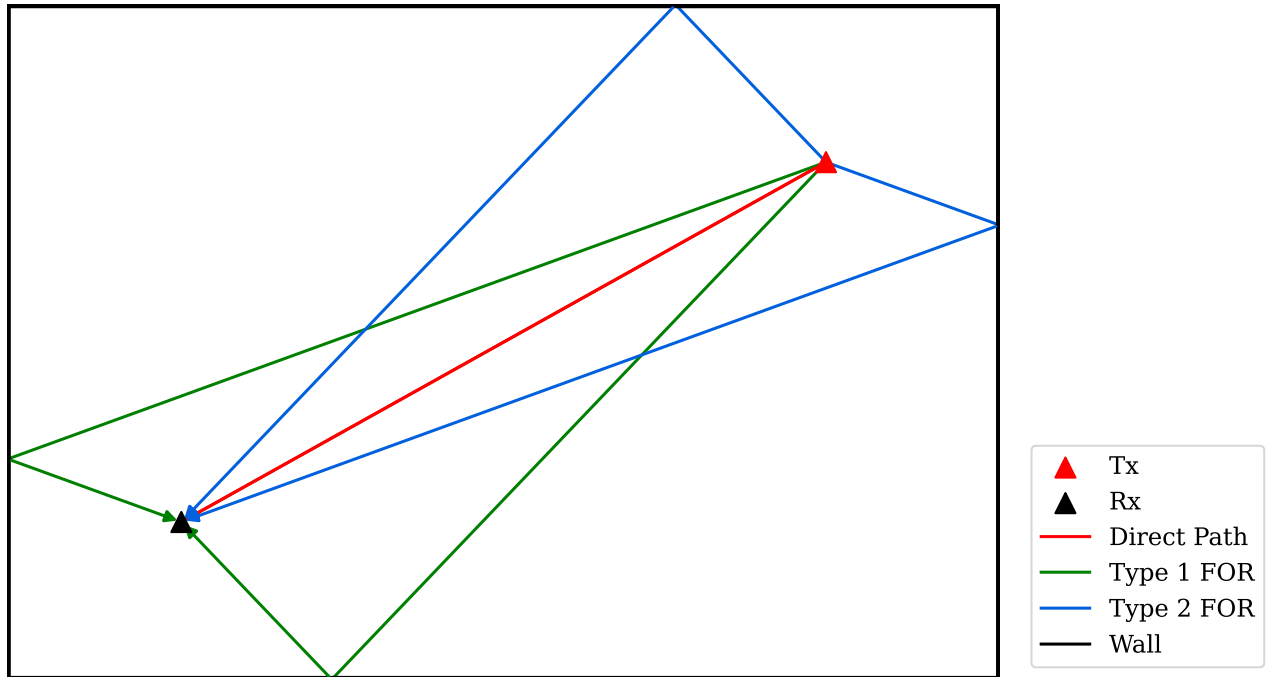


Figure 4.8: FORs between a receiver and a transmitter for a direct path AoA in Quadrant 1

The same reasoning can be applied for direct path AoAs in different quadrants. Table 4.1 reports the

different combinations of quadrants and the corresponding wall orientation. When the FOR is of Type 2, a comparison between θ_{direct} and θ_{FOR} is needed. Table 4.2 shows the wall orientation in function of the comparison result.

Direct path / FOR	Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
Quadrant 1	Angle comparison	Vertical	/	Horizontal
Quadrant 2	Vertical	Angle comparison	Horizontal	/
Quadrant 3	/	Horizontal	Angle comparison	Vertical
Quadrant 4	Horizontal	/	Vertical	Angle comparison

Table 4.1: Wall orientation in function of AoA quadrants of direct path and FOR

Type 2 Quadrant	Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
$\theta_{\text{direct}} < \theta_{\text{FOR}}$	Horizontal	Vertical	Horizontal	Vertical
$\theta_{\text{direct}} > \theta_{\text{FOR}}$	Vertical	Horizontal	Vertical	Horizontal

Table 4.2: Wall orientation in function of AoA comparison for Type 2 FOR

4.2.3 Integration into RANSAC algorithm

After the identification of the orientation of the reflecting walls for the potential FORs, a second RANSAC algorithm is applied to discard the MPCs that do not correspond to the FOR measurement model, and keep the ones that do. For this RANSAC, the dataset of measurements is now the union between the original set D that contains the potential direct paths, and the set F that contains the potential FORs, which can be divided into two sets F_v and F_h that contain, respectively, the FORs that were identified as having been reflected by a vertical wall and the FORs having been reflected by a horizontal wall. Because the first paths are not necessarily direct paths, the set F also contains their measurements, but that are treated as FORs. At each iteration k , 3 paths in total are randomly selected from the different sets to form the sets D^k , F_v^k and F_h^k , which can be empty. The sum of squares to minimise, with $F_{v(i,j)}$ and $F_{h(i,j)}$ being the path reaching receiver Rx i at the j^{th} position reflected by a horizontal or a vertical wall, is given by

$$S(\beta) = \sum_{i \in D_k} r_{t,i}(\beta)^2 + r_{\theta,i}(\beta)^2 + \sum_{(i,j) \in F_v^k} r_{v(i,j)}(\beta)^2 + \sum_{(i,j) \in F_h^k} r_{h(i,j)}(\beta)^2. \quad (4.8)$$

In the same way as for the HyRANSAC algorithm, the inliers are identified by comparing the distance of the estimate from what is predicted by the measurement. For paths $F_{v(i,j)}$ and $F_{h(i,j)}$, the residuals of (4.3) and (4.3) are compared to a threshold t .

4.2.4 Application to firefighter scenario

Using the same procedure as for the other algorithms, Figure 4.9 shows the CDFs obtained in the small firefighter scenario for the algorithm presented above, which is named here HyRANSAC-FOR. It can be observed that using FORs only has a few impact on the performance. For 6 and 8 Rxs, the knee of the CDF is slightly higher, which indicates the additional measurement provided by an FOR give additional precision and avoid using an NLoS path as a direct path. However, for 4 Rxs, HyRansac-FOR performs slightly worse than HyRANSAC. This can be explained by two elements, the

level of redundancy and the prior information. First, not all MPCs are FORs, and in the case where only a few measurements are available, they are less likely to be detected, thus, the system cannot use any additional information, or might even try to classify some MPCs as FORs even if they are not. Secondly, prior information is needed to classify potential FORs as vertical or horizontal, and the accuracy of the first estimate is generally worse for lower numbers of deployed BSs. This can lead to a worse identification of FORs orientations, and the algorithm cannot use those incorrect measurements.

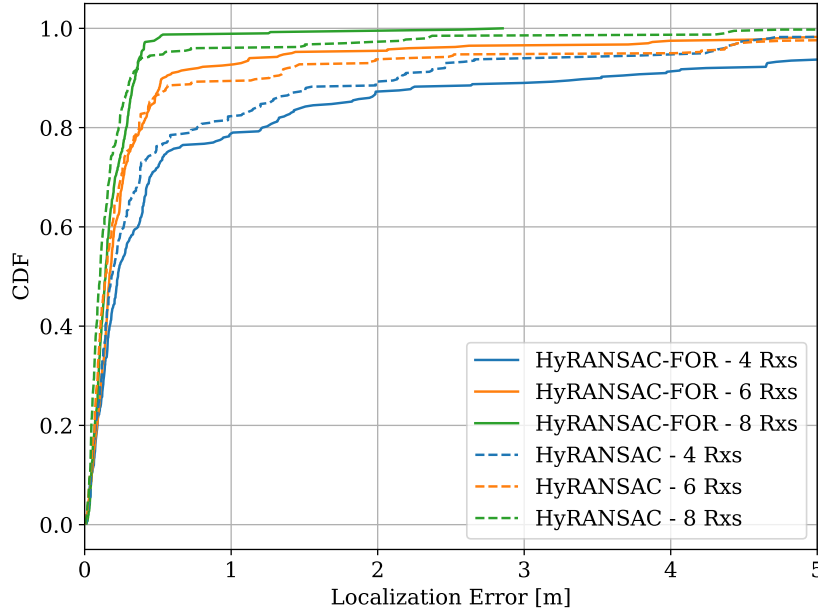


Figure 4.9: Performance of HyRANSAC-FOR in the small building scenario

In the large building scenario, the improvement due to the use of FORs is a bit more noticeable. As shown in Figure 4.10, the CDFs have their knees slightly above those of the HyRANSAC algorithm. It means that the additional information provided by FORs allows to refine the position estimation when measurements are good enough, but not when the prior information obtained with the first RANSAC algorithm is too far from the true firefighter position.

However, the performance increase obtained by using FORs is not large enough compared to that of the HyRANSAC to conclude that there is a real benefit in using them. Indeed, for the cases with 4 Rxs, HyRANSAC-FOR actually performs worse than HyRANSAC, which means that some MPCs are considered as FORs by the algorithm because their residual is smaller than the threshold t , but that they actually do not correspond to that model. One limitation to using this model is that only a small proportion of MPCs are actually FORs, which makes it difficult to effectively exploit them.

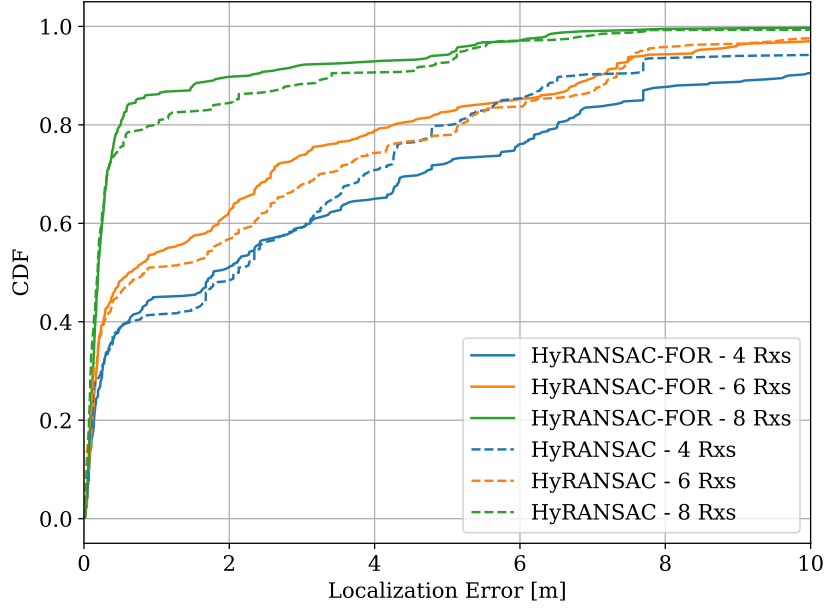


Figure 4.10: Performance of HyRANSAC-FOR in the large building scenario

4.3 Final results

Compared to the classic hybrid algorithm, the HyRANSAC method developed allows to effectively mitigate the NLoS paths that corrupt the measurement model. In most of the cases, offers an accuracy under 1 metre, which satisfies the precision requirements of firefighters, but this result has to be put into perspective with the relatively small dimensions of the intervention scenes. The system performance is quite dependent on the number of BS deployed, with 4 BSs being not always sufficient to provide reliable measurements due to the obstructions and reflections created by the walls of the buildings, especially in larger scenarios that are more challenging. For that reason, the final choice for the number of receivers that must be deployed by firefighters is to use 8 BSs, as it adds the redundancy needed to acquire enough good quality measurements.

The performance of the different algorithms studied in both scenarios is summarized in Figure 4.11 and Figure 4.12 when using 8 BSs. In the small building scenario, the error committed by the HyRANSAC and HyRANSAC-FOR algorithms in more than 95% of the cases is below 1 metre, which is a significant improvement compared to the basic hybrid localisation algorithm where it is only the case for approximately 50% of the cases. The largest error is under 5 metres, which is still quite large compared to the $10\text{m} \times 15\text{m}$ dimensions of the building.

In the large building scenarios, a precision of 1 metre is achieved in 85% of the cases. Similarly to the small scenario, the HyRANSAC algorithms achieve significantly better performance than the basic hybrid scheme, but when the measurements acquired by the base stations become too sparse or incorrect, their performances become similar. The use of FORs to provide additional measurements was tested, but it did not show a considerable improvement, some work must still be done to effectively exploit the information contained in MPCs. Nevertheless, the performance achieved by the system in simulation is promising for real case application opens the door for the design and field testing of a prototype in future works.

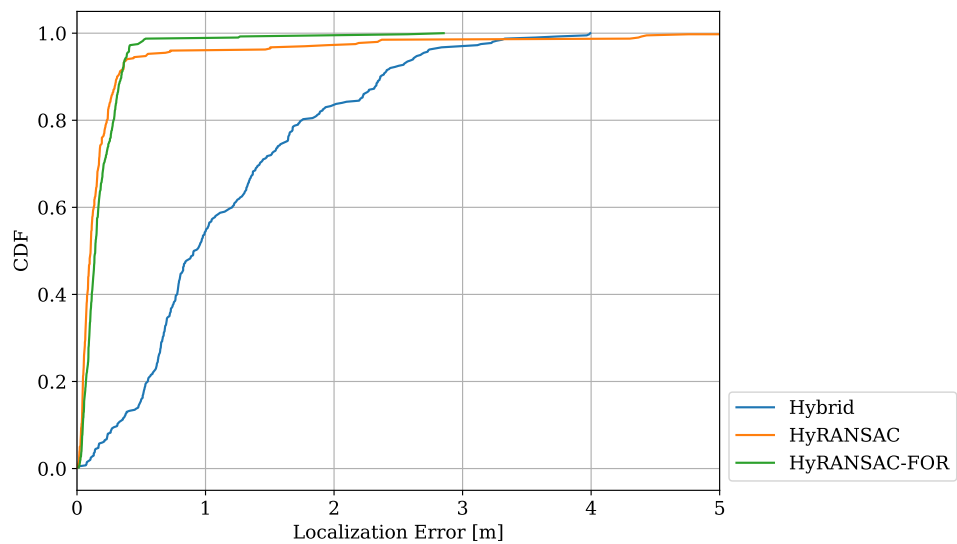


Figure 4.11: Performance achieved by the indoor positioning system with 8 Rxs in the small building scenario

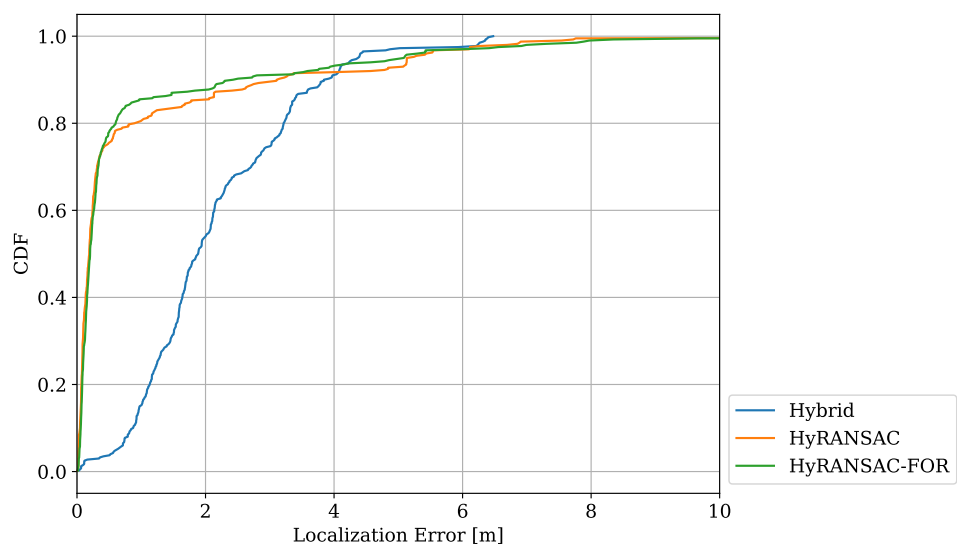


Figure 4.12: Performance achieved by the indoor positioning system with 8 Rxs in the large building scenario

Conclusion

In this work, the design of an indoor positioning system for the localisation of firefighters trapped in building fires has been studied by taking into account the requirements and impairments that are specific to the intervention scenarios of firefighters.

First, a propagation model was formulated to represent the way typical indoor environments affect the propagation of radio signals. The impact of building walls that attenuate and reflect EM waves was discussed, and the attenuation of high-concentration smoke aerosols was identified as negligible in comparison. Then, this model was simulated using the ray-tracing library Sionna [2] in 2 types of building to obtain realistic propagation channels associated to typical intervention scenarios of firefighters. It was highlighted that walls created severe multipath due to the numerous reflections and refractions they create. For the case of a transmitter located inside the building and a receiver placed outside the building, the LoS path is often obstructed, and the reflections that reach the receiver can be misinterpreted as direct paths. Furthermore, in some cases, a receiver may not receive any signal at all. It was concluded that these issues could strongly impact the performance and reliability of traditional localisation techniques.

Then, a system architecture that is adequate to the firefighter application was formulated by taking into account the main criteria that influence the design of IPS and the specifications. This system was then simulated entirely, from the physical propagation channel using ray-tracing to the wireless communication and the CSI estimation. These measurements are then given as input to localisation algorithms. The first part consisted in extracting the ToAs and AoAs of MPCs using a 2D MUSIC algorithm with frequency smoothing and an MDL criterion to estimate the number of incoming signals. This allowed to accurately find the paths that carry sufficient energy compared to the thermal noise.

Finally, localisation techniques that use TDoAs, AoAs or a combination of both were developed and applied to the designed system in the firefighter scenario. It confirmed that multipath and shadowing had a strong impact on the achieved performance of those techniques, which made them insufficient to provide a reliable position estimate of the firefighters. To solve this, a RANSAC algorithm was proposed to filter out NLoS paths, which improved the estimation when the base stations gave enough correct measurements, but did not solve the issue that, in some cases, the system simply does not detect enough direct paths. Consequently, a measurement model for first order reflections was developed to obtain additional measurements that can be exploited by the RANSAC hybrid localisation algorithm. Although a slight performance increase was observed, the use of FORs still needs further investigation to evaluate the performance gain it can actually offer. Nevertheless, the performance achieved by the system with 8 deployed base stations offered sufficient performance in both scenarios for a potential usage by the firefighters, with a sub-metre level accuracy in nearly in more than 95% in the small building scenario and 85% in the large building scenario.

Future works

The system designed in this work was completely simulated to obtain realistic predictions of what could be achieved in practical scenarios. The system provided sufficient accuracy for the firefighter application in simulation, which serves as a good proof of concept for a practical implementation. The logical continuity of this work is to develop a prototype of this system and to test it in real-world conditions to validate its effectiveness and operational feasibility.

Another area for improvement is to consider the non-idealities were not taken into account in the system characterised in this work. For example, some research such as in [47] can be conducted to evaluate the errors caused by inaccuracies on the position of receivers. This is especially relevant to firefighters that need to rapidly deploy base stations around the scene of intervention, which makes them prone to commit errors when encoding the positions and orientations of the receivers.

Finally, multipath and NLoS conditions remain important issues against a robust localisation, and even though the HyRANSAC algorithm provides an accurate estimate in most of the cases, it is not unlikely to have scenarios where the signal transmitted by the firefighter does not reach some receivers. The use of MPCs to improve the estimation was explored, but some work must still be done to properly exploit the information they carry.

Appendix A

Wireless communication simulation

The expression of the impulse response of a root-raised-cosine filter of symbol period T is given as

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}} \left[1 + \alpha \left(\frac{4}{\pi} - 1 \right) \right] & \text{if } t = 0, \\ \frac{\alpha}{\sqrt{2T}} \left[\left(1 + \frac{2}{\pi} \right) \sin \left(\frac{\pi}{4\alpha} \right) + \left(1 - \frac{2}{\pi} \right) \cos \left(\frac{\pi}{4\alpha} \right) \right] & \text{if } t = \pm \frac{T}{4\alpha}, \\ \frac{1}{\sqrt{T}} \frac{\sin \left(\frac{\pi(1-\alpha)t}{T} \right) + 4\alpha \frac{t}{T} \cos \left(\frac{\pi(1+\alpha)t}{T} \right)}{\pi \frac{t}{T} \left[1 - (4\alpha \frac{t}{T})^2 \right]} & \text{else.} \end{cases} \quad (\text{A.1})$$

The frequency content of a RRC filter is larger than $\frac{1}{T}$, the chain needs to be simulated by oversampling the signal. In this work, the oversampling rate $osr = 8$ is used to simulate the physical transmission. The signal is then decimated to the rate $\frac{1}{T}$ after the matched filtering to perform the channel equalisation.

In simulation, generating gaussian noise samples of variance σ_n^2 at an oversampling rate osr happens in discrete time. Thus, its power is spread over the Nyquist bandwidth $B \cdot osr$, which gives the noise power spectral density $\gamma_n(f)$ as

$$\gamma_n(f) = \frac{\sigma_n^2}{B \cdot osr}. \quad (\text{A.2})$$

For a bilateral power spectral density equal to $\frac{N_0}{2}$ in RF, the corresponding power spectral density in baseband is given by

$$\gamma_n(f) = 2N_0. \quad (\text{A.3})$$

Combining (A.2) and (A.3), one obtains that the variance used to generate the noise for a desired N_0 should be equal to

$$\sigma_n^2 = 2N_0 B \cdot osr. \quad (\text{A.4})$$

At the matched filtering, the convolution must be normalised by osr to keep the energy constant. Indeed, the filtering, which consists in continuous time of performing the following convolution:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot g(t - \tau) d\tau, \quad (\text{A.5})$$

has the integral over dt replaced by a summation in discrete time, with dt being replaced by the sampling interval $\Delta t = T$. Discrete convolutions are often normalised by $\frac{1}{T}$, which allows one to omit the Δt factor when working at the sample rate $\frac{1}{T}$. However, when working with oversampled signals where $\Delta t = \frac{T}{osr}$, there should remain a factor $\frac{1}{osr}$ to keep the energy constant.

It is important to test the soundness of the simulated wireless chain. To test that the noise is correctly scaled, the wireless chain is tested by transmitting QPSK symbols on an AWGN channel by using $c(t) = \delta(t)$. The probability of error of a QPSK modulation over an AWGN channel is given by

$$p_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{\operatorname{SNR}}{2}}\right), \quad (\text{A.6})$$

with the SNR given as $\operatorname{SNR} = \frac{\sigma_I^2}{\sigma_n^2}$. The result of the simulation is shown in Figure A.1, where it can be observed that the simulation matches the theory.

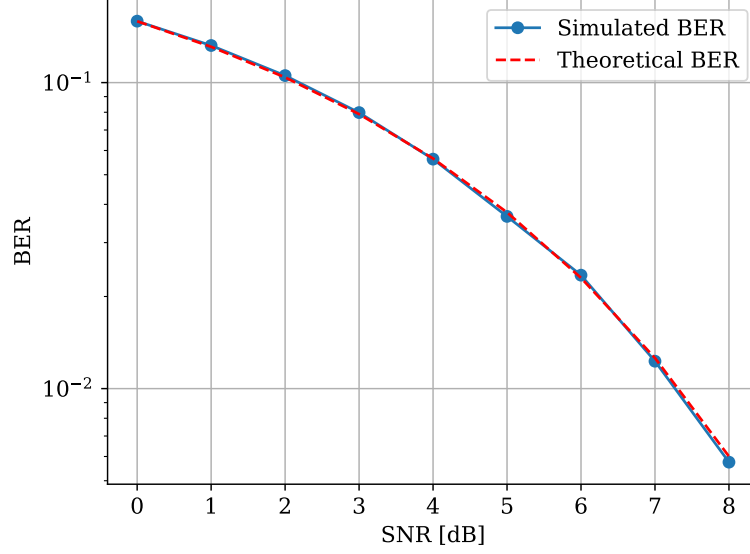


Figure A.1: Simulated and theoretical BERs for a QPSK modulation in an AWGN channel

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