

École polytechnique de Louvain

Dynamic study of vehicles equipped with limited-slip differential

Comparison between open and Torsen differential

Author: **Corentin GUILIN**

Supervisors: **Paul FISETTE, Nicolas DOCQUIER**

Readers: **Gilles VANVECKHOVEN, Blaise MAYER, Alexandre Buset**

Academic year 2024–2025

Master [120] in Mechanical Engineering

Remerciements

Je tiens tout particulièrement à remercier certaines personnes sans qui ce travail n'aurait jamais vu le jour.

Tout d'abord, je souhaite exprimer ma profonde gratitude à l'ensemble de l'équipe encadrante du projet *Robotran*, et plus particulièrement à mon promoteur, Paul Fisette. Toujours disponible, quelle que soit l'heure, il a su répondre à mes questions avec rigueur, bienveillance et souvent une touche d'humour. Il a également trouvé les mots justes pour me confronter à mes difficultés de manière constructive, et je le remercie sincèrement d'avoir accepté ce sujet de mémoire, qui nous a permis de partager tout au long de l'année notre passion commune pour les Toyota. Je tiens également à remercier Marine Boxus pour ses conseils avisés et son accompagnement tout au long de la rédaction de ce travail. Je remercie également les participantes du *STEM Project Days*, avec qui j'ai pu partager pendant trois jours ma passion pour la *Formule 1*. Elles ont ainsi pu découvrir une facette du métier d'ingénieur lors d'une introduction au sujet de ce mémoire.

Je suis également extrêmement reconnaissant envers Blaise Mayer et Gilles Vanveckhoven, de chez JTEKT *Torsen*, pour leur soutien précieux dans la réalisation de ce travail. Leur accueil chaleureux sur le site de La Louvière, leur disponibilité constante et leur enthousiasme à l'égard de ce projet ont été d'une aide inestimable. Grâce à eux, j'ai pu approfondir ma compréhension du fonctionnement et de la modélisation des différentiels *Torsen*, adaptés aux spécificités de chaque client. Je les remercie pour cette collaboration passionnante, qui restera une étape marquante de mon parcours.

Enfin, il m'est cher de pouvoir remercier mes parents et ma soeur, ainsi que mes amis, notamment Steve Matout, Antoine Gourdange, Antoine, Maarten et Gilles, pour leur écoute, leur soutien indéfectible, et leur présence rassurante à chaque moment de doute. Je me considère extrêmement chanceux d'être aussi bien entouré, et de pouvoir compter sur chacun d'eux.

Malgré un parcours parfois semé d'embûches, je suis convaincu que la réussite d'un projet, d'une année ou d'un cursus universitaire ne se construit jamais seul. Elle résulte d'un engagement personnel profond envers un sujet qui m'a passionné tout au long de cette année, mais aussi du soutien continu de celles et ceux qui m'ont accompagnés pendant ces six années à l'Université. À toutes ces personnes, je dis merci. Sans vous, je ne serais certainement pas arrivé jusqu'ici.

Contents

1	Introduction	1
2	Theoretical Concepts	3
2.1	Wheel/Ground Contact	3
2.1.1	Preambles	3
2.1.2	<i>Bakker</i> and <i>Pacejka</i> Model	6
2.1.3	Combined Longitudinal and Lateral Forces	9
2.2	Friction Coefficient	10
2.3	Load Transfer	11
2.3.1	Longitudinal Load Transfer	11
2.3.2	Lateral Load Transfer	13
2.4	Steering Behaviour	15
2.4.1	Understeering and Oversteering	15
2.4.2	<i>Ackermann</i> Principle	16
2.4.3	Single Track and Double Track Model	17
3	The Differential	21
3.1	Origins and Functions	21
3.2	Mathematical Approach	22
3.2.1	Differential Torque Balance and Friction Power	23
3.2.2	Internal Efficiency and TBR	25
3.2.3	Tyres Interaction with The Ground and The Differential	25
3.2.4	Torque Distribution Determination	27
3.2.5	Differential Analogy	28
3.3	Open Differential	31
3.3.1	Bevel Gear Differential	31
3.3.2	Planetary Gear Differential	32
3.3.3	Locked Differential	33
3.4	Limited-slip Differential	35
3.4.1	Speed Sensing	35
3.4.2	Torque Sensing	37
3.5	<i>Torsen</i> Differential	40
3.5.1	Composition	40
3.5.2	Advanced <i>Torsen</i> differentials	42
3.5.3	Performance and Assets	44
3.6	Advantages, Limitations, and Conclusion on Differential Variants	46

4	Multibody Model	48
4.1	Upgrades to the Multibody Car Model	48
4.1.1	The Steering Wheel	49
4.1.2	The Engine	51
4.1.3	The Tyres and the Suspensions Representation	52
4.2	Differentials Implementation	53
4.2.1	The Open Differential	54
4.2.2	The Locked Differential Condition	56
4.2.3	Torque Balance Distribution	56
4.3	Controllers	61
4.3.1	Speed Control	61
4.3.2	Trajectory Control	62
5	Differentials Validation	65
5.1	Straight Line	65
5.1.1	Open Differentials	66
5.1.2	Locked and <i>Torsen</i> Differentials	67
5.2	Cornering	68
5.2.1	Constant Steering Angle	68
5.2.2	Driver Reaction on a Fixed Trajectory	73
5.3	Conclusion on the Influence of Differential Types on Vehicle Handling . . .	75
6	Cornering Performance	76
6.1	Preface of Cornering Behaviour	76
6.2	Influence of Engine Torque	77
6.2.1	Low Acceleration Rate	77
6.2.2	Transition Acceleration Rate	80
6.2.3	High Acceleration Cornering	83
6.2.4	Cornering Ability	87
6.3	Driver Cornering Influence	90
6.4	Throttle-off Reaction	93
6.5	Conclusion on Vehicle Cornering Performance	96
7	Split-μ Test: Open VS <i>Torsen</i>	97
7.1	On Wet Road	98
7.1.1	Low Acceleration Rate	98
7.1.2	Transition Acceleration Rate	103
7.1.3	High Acceleration Rate	105
7.1.4	Influence of The <i>TBR</i> and The Engine Torque	109
7.2	On Ice Road	114
7.2.1	Low Acceleration Rate	114
7.2.2	Transition Acceleration Rate	118
7.2.3	Influence of The <i>TBR</i> and The Engine Torque	121
7.3	Conclusion on the Performance of <i>Torsen</i> and Open Differentials on Low-Grip Surfaces	125
8	Conclusion	127

A The Differential	129
B Multi-Body Verification	134
C Additional Results for Validation of Differentials	138
D Additional Results for Cornering Behaviour Differences	141
E Additional Results for <i>Split</i> – μ Test	145

Table of Symbols

Symbol	Definition	Unit
a_x, a_y	Longitudinal and lateral acceleration	[m/s ²]
CBV	Carbody Velocity	[m/s]
E, t	Wheelbase and track of the vehicle	[m]
F_{long}, F_x	Longitudinal force	[N]
F_{lat}, F_y	Lateral force	[N]
F_n	Normal (vertical) force	[N]
FWD, RWD	Front/Rear wheels drive	[-]
FL, FR, RL, RR	Front Left, Front Right, Rear Left and Rear Right wheel	[-]
g	Gravity acceleration	9.81 [m/s ²]
G	Centre of gravity	[-]
G_{tot}	Total gain of <i>Torsen</i> differential compared to the open differential	[-]
$G_{Traction}$	Traction gain of <i>Torsen</i> differential compared to the open differential	$[\frac{Nm}{Nm}]$ or [-]
I^i	Inertial matrix of the body i	[kg.m ²]
k	Friction coefficient factor	[-]
LD	Locked Differential	[-]
m	Mass of the vehicle	[kg]
OD	Open Differential without friction	[-]
<i>Open</i>	Open Differential with friction	[-]
R	Radius of the cornering	[m]
R_{wheel}	Wheel's radius	[m]
T	Torque	[Nm]
T_m, T_{motor}	Engine torque applied by the driver	[Nm]
TBR	Torque Bias Ratio	[-]
u, v	Longitudinal and lateral linear velocity in the inertial frame	[m/s]
V_x, V_y	Longitudinal and lateral linear velocity	[m/s]
W_i, W_o	Power in and out	[W]
W_f	Friction power (lost)	[W]
α	Slip angle	[°]
β	Drift angle	[°]
δ	Steering wheel angle	[°]
ε_d	Locking effect	[%]
η	Efficiency	[-]
κ	Longitudinal slip ratio	[-] or [%]

Symbol	Definition	Unit
μ	Coefficient of friction	[-]
ρ	Corner curvature	[1/m]
ψ	Yaw	[°]
$\dot{\psi}$	Yaw rate	[°/s]
ω	Angular velocity	[rad/s]
$\dot{\omega}$	Angular acceleration	[rad/s ²]

Chapter 1

Introduction

During the 2024 Japanese Grand Prix qualifying session, Max Verstappen secured pole position with a mere 0.012 second lead over Oscar Piastri. In a discipline where every millisecond matters, the optimization of every mechanical component becomes essential [9].

From the icy stages of the Monte Carlo Rally to the dusty tracks of Mexico, Thierry Neuville adapts his driving to extreme road conditions and ultimately secures the 2024 WRC World Championship title. Such performance across variable terrains relies on efficient traction management [35].

In motorsport, every second counts. Every improvement determines the outcome of a race. As these two examples illustrate, racing teams and drivers in all disciplines optimise every mechanical system to their limits. Among the systems, the transmission of engine torque to the road under variable grip conditions is a decisive factor in performance and tyre preservation. Apart from sporting performance, everyday drivers also encounter dangerous situations such as aquaplaning or icy roads. Losing control of the vehicle in these situations can sometimes be fatal. Despite the presence of numerous electronic driving aids, such scenarios remain highly stressful and occasionally insufficient to ensure safety.

In this context, the development of mechanical solutions addressing traction and stability becomes increasingly relevant. This is particularly true in motorsport, where even the enhancement improvement can save time but also for consumer vehicles, where safety and reliability are essential. Unlike complex electronic systems, mechanical traction solutions offer robustness, consistency, and an improvement of the direct connection between the driver and his vehicle.

For these reasons and in the context of the Master's thesis, I proposed a comparative dynamic study of vehicles using an open differential versus a limited-slip differential. Conducted in collaboration with JTEKT *Torsen*, this work focuses especially on the comparison between a standard open differential and a *Torsen*-type differential manufactured in Belgium. The thesis begins by introducing the theoretical concepts used in this work. This will be followed by a presentation of differential operation and the various models, with particular emphasis on the *Torsen* differential. Finally, this Master's thesis will provide a detailed comparative analysis of the general behaviour of the various differentials. This will lead to a focused investigation into the performance differences between the open

differential and the *Torsen* differential when cornering and on variable road surfaces.

It is worth mentioning that artificial intelligence was used for repetitive tasks such as writing Python code for reading data. AI was also used for proofreading and reformulation.

Chapter 2

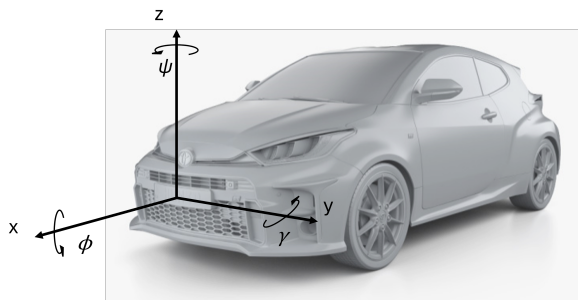
Theoretical Concepts

This chapter aims to review the theoretical concepts used in this Master's thesis. First, an overview of the tyre-ground contact will be presented, followed by an analysis of load transfer during cornering. The steering behaviour in cornering will conclude the chapter.

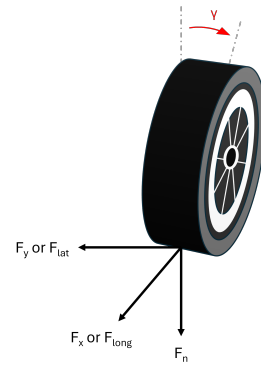
2.1 Wheel/Ground Contact

2.1.1 Preambles

The car's global reference frame has an upward z-axis, as shown in Figure 2.1a. Unlike the carbody, in the SAE axis system the z-axis of the wheel coordinate system points towards the ground (Figure 2.1b). This allows the tyre's ground penetration to be defined as positive when the tyre deforms.



(a) The *roll* angle ϕ describes the rotation along the x-axis. The *pitch* angle γ is the rotation along the y-axis and the *yaw* angle ψ is along the z-axis.



(b) The z-axis pointing towards the ground gives positive tyre penetration. The wheel camber angle is defined as γ .

Figure 2.1: Global car and local tyre/ground reference frames. Compared to the global system of axes, the z-axis for the tyre points toward the ground.

One of the first tyre lateral models was proposed by *Fromm* [13]. In this model, the tyre is considered as an assembly of springs arranged axially along the rim [10]. As Figure 2.2 shows, the lateral force developed by the tyre depends on the lateral elasticity and the lateral deformation distributed along the rubber tread. As regards the longitudinal force model, the

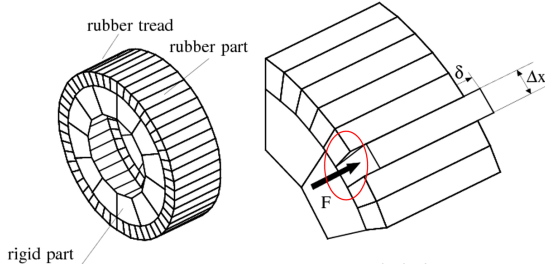


Figure 2.2: The *Fromm* model describes the tyre as an assembly of lateral springs [10].

ωR_{wheel} , the bristles stay straight (Figure 2.3a), resulting in rolling without slipping situation.

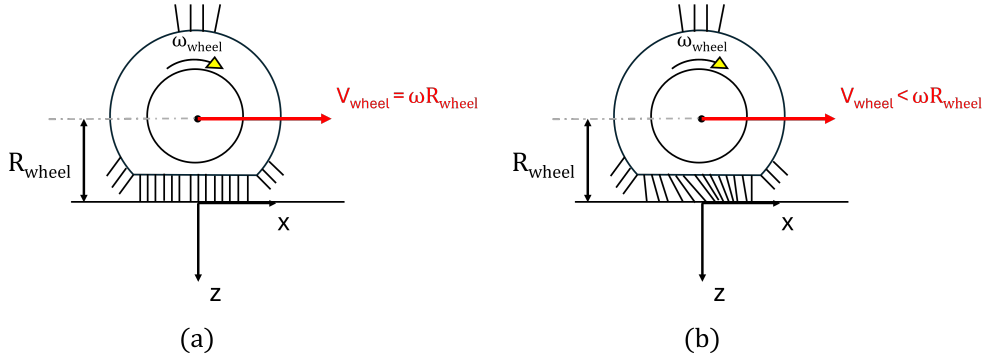


Figure 2.3: Inspired from [3].

The *RSG* (Rolling Without Slipping) principle helps to establish the **slip** of the tyre. This illustrates the bending of the bristles. First, we need to define the velocity of a material point on the wheel radius (Figure 2.4 top):

$$V_{m,wheel} = \omega_{wheel} R_{wheel} \quad (2.1)$$

As soon as the velocity of the wheel centre V_{wheel} exceeds the material point velocity, slip occurs (Figure 2.4 bottom). The bristles bending introduced in the *Brush* model starts as soon as slip appears. Because the contact between tyre and ground is modelled by an elliptic patch (details in [29]), the ellipse is split into 2 parts. Firstly, the front part of the contact patch that is in **adhesion**: rolling without slipping, the bristles bend while adhering to the road. This emphasizes the phenomenon of static friction between the tyre and the road. Secondly, the rear part of the contact patch, which experiences **slipping**: the bristles bend and slip on the road. The *Fromm* and the *Brush* models allow to define the bound between both areas

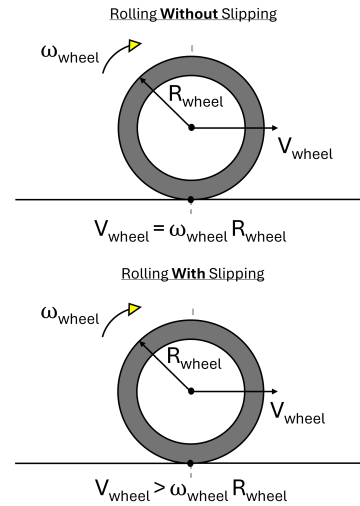


Figure 2.4: Slip of tyre is related to rolling without slipping principal.

in the lateral and the longitudinal direction, respectively. Indeed, this bound varies with the amount of slip. The force exerted by the tyre can be expressed as a function of the longitudinal slip κ for the longitudinal force F_{long} and of the slip angle α for the lateral part of the force F_{lat} , with a linear behaviour at small slip values.

Longitudinal Slip Contribution

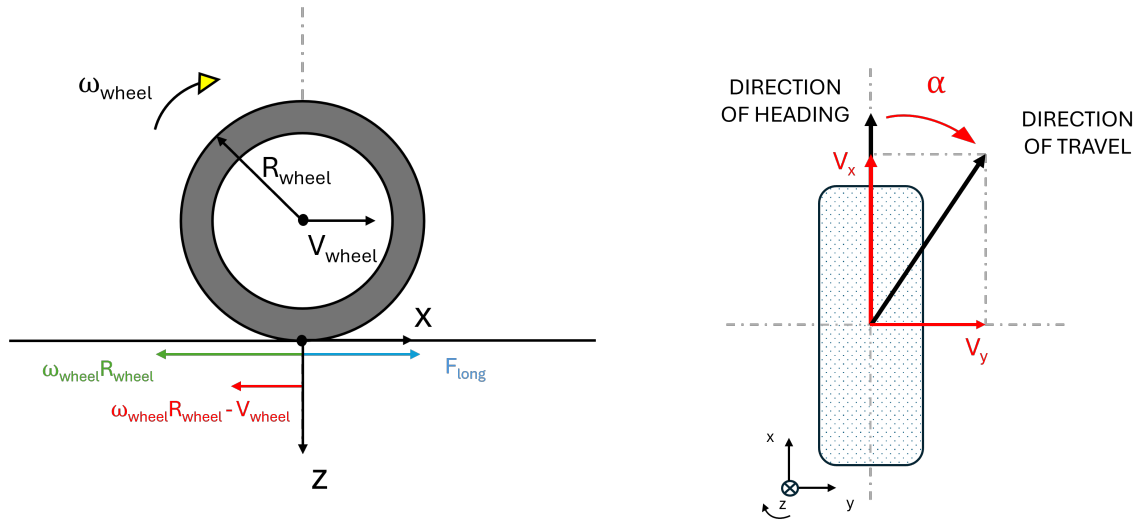
As mentioned above, the longitudinal slip ratio κ represents how much the tyre is slipping compared to an ideal rolling without slipping situation. It is defined as the ratio between the material contact point longitudinal velocity and the wheel's center velocity V_{wheel} (Figure 2.5a). It is expressed as ¹:

$$\begin{aligned}\kappa &\triangleq \frac{V_{wheel} - \omega_{wheel} R_{wheel}}{V_{wheel}} > 0 \quad [-] \quad \text{for breaking;} \\ \kappa &\triangleq \frac{V_{wheel} - \omega_{wheel} R_{wheel}}{\omega_{wheel} R_{wheel}} < 0 \quad [-] \quad \text{for accelerating;}\end{aligned}\tag{2.2}$$

Lateral Slip Contribution

The lateral slip angle α quantifies the angle in the horizontal plane between the wheel's direction of travel and the wheel's mid-plane (Figure 2.5b). Taking V_x the wheel center longitudinal velocity (also called the *forward* velocity) and V_y the wheel's lateral velocity, α is defined as follows [29]:

$$\alpha \triangleq -\arctan\left(\frac{V_y}{|V_x|}\right) \quad [^\circ]\tag{2.3}$$



(a) The numerator of the slip ratio κ represents the material contact point velocity (red arrow). (b) The slip angle compares the wheel's orientation with the wheel velocity.

Figure 2.5: The longitudinal slip κ represents a velocity ratio while the lateral slip α represents an angle.

¹This expression is simplified with respect to the multi-body point of view in order to highlight directly the longitudinal sliding via known variables.

In this kind of models, the force is limited by the friction coefficient between the tyre and the ground. The lateral force in the *Fromm* model will saturate beyond a slip angle value called α_{lim} . When the slip angle α reaches α_{lim} , the entire contact patch is sliding. Therefore, there is no longer *grip* between the tyre and the road because the tyre contact saturates.

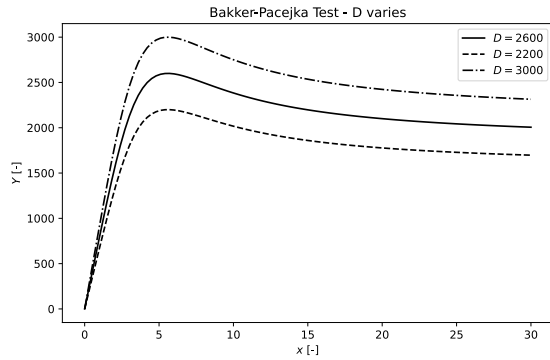
2.1.2 Bakker and Pacejka Model

Moving on to more advanced and more accurate models, the *magic* formula of *Bakker-Pacejka* enables establishing the longitudinal and the lateral forces as a function of the longitudinal slip ratio κ and the lateral slip angle α , respectively [30]. With this semi-empirical model, which is based on experimental results, most types of tyres can be studied by stretching the formula. The *magic* formula expresses the variable Y (for example, representing the longitudinal or the lateral force) as a function of a measured data x (typically, κ or α):

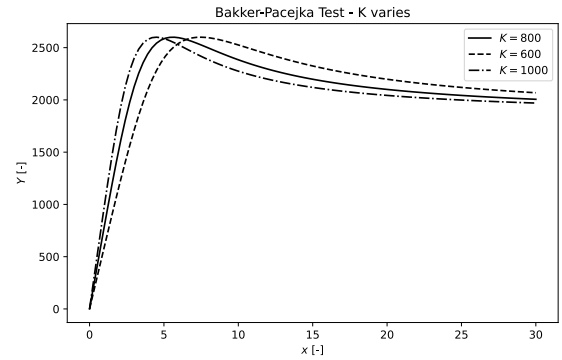
$$Y(x) = D \sin(C \arctan(Bx^* - E(Bx^* - \arctan(Bx^*)))) + S_v \quad (2.4)$$

with $x^* = x + S_h$

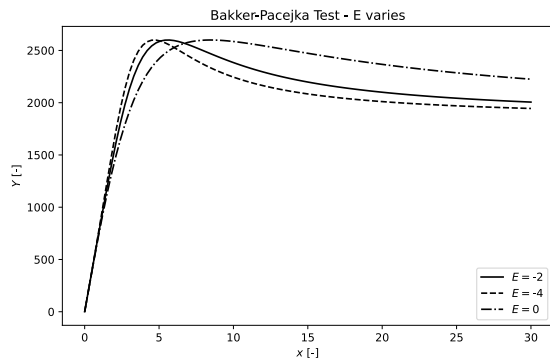
All the coefficients in the relation 2.4 must be determined to fit a specific tyre.



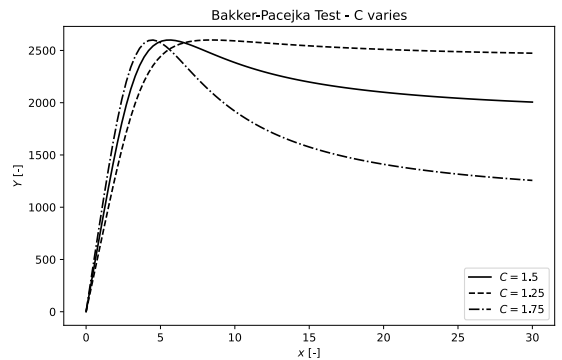
(a) D changes the peak value of Y . The peak depends on the friction coefficient between the tyre and the road.



(b) The stiffness factor K impacts the slope curve. The curve is more sharp when K increases.



(c) The coefficient E affects the curvature. It determines the position of the x peak. A more negative value of E produces a sharper peak.



(d) C is the shape factor. With a higher value of C , the saturation part of the curve will vanish more steeply.

Figure 2.6: Inspired from *Steve Matout* and *François Blase* Master's thesis [11].

The coefficient K is called the *stiffness factor*. It is introduced instead of the coefficient B and is defined as:

$$K \triangleq BCD \quad (2.5)$$

As previously mentioned, each coefficient affects the shape of the curve. The resulting shape alterations are shown in Figure 2.6:

- K : impacts the slope curve (Figure 2.6b). For the longitudinal force, K_{long} is the longitudinal tyre stiffness and for the lateral one K_α is the cornering tyre stiffness. If K increases, the slope of the curve will be more sharp and the saturation peak will be closer to the origin.
- C : represents a *shape factor*. It affects the saturation value and the location of the curve peak. The saturation value appears after the peak value of Y . Regarding Figure 2.6d, the saturation part of the tyre will vanish more steeply with $C > 1.5$.
- E : is the *curvature coefficient*. It affects the x peak position without influencing the peak saturation value. As presented in Figure 2.6c, a more negative value of E results in a sharper peak. The tyre stiffness is maintained up to larger values of x .
- D : is the *peak amplitude*. This coefficient, as described in Figure 2.6a changes the peak value. Therefore, D directly links the normal tyre load F_n with the maximum coefficient of friction μ_{max} between the tyre and the ground in the longitudinal and the lateral direction:

$$\mu_{max} = \frac{D}{F_n} \quad (2.6)$$

For instance, the value of μ will widely vary between wet asphalt and ice-covered asphalt.

- S_h and S_v : are offset coefficients. They allow non-zero forces at the origin, even at κ or α null. These coefficients are employed to take account of wheel camber γ (Figure 2.1b) for lateral force and rolling resistance for longitudinal force.

In sum, the *Bakker* model can be used to characterize both lateral and longitudinal forces (Y) as a function of the slip angle ($x = \alpha$) and the slip ratio ($x = \kappa$), respectively. An example with a normal load of $F_n = 4$ kN is illustrated in Figures 2.7 and 2.8.

The lateral force distribution is divided into several parts. At low slip angles α , the lateral force is a linear function, as in the *Fromm* model. When α grows in the transition area and reaches α_{lim} , the lateral force becomes saturated. In Figure 2.7, this saturation is quite moderate, meaning the force does not decrease too much after the peak. After the saturation limit, the majority of the contact patch is in sliding [30, 32].

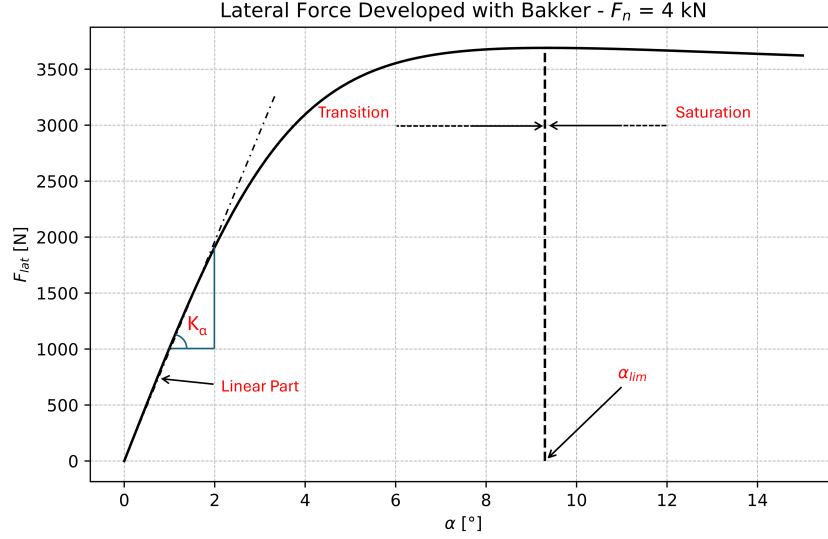


Figure 2.7: Typical example of a lateral force curve with $F_n = 4 \text{ kN}$. Once the saturation limit α_{lim} is exceeded, the car is sliding and losses grip.

The longitudinal force distribution can also be established by the *Bakker* model. As the lateral force, the longitudinal component of the tyre's forces is drawn in the example of Figure 2.8. The saturation for the longitudinal force is more pronounced than for the lateral force. Therefore, modern systems such as the Anti-Lock Braking System (*ABS*) help to remain close to the top of the rising part. Once the saturation is exceeded, the vehicle's longitudinal force decreases, and the car begins to slip. This last phenomenon highlights the transition between static and kinetic friction (i.e. adhesion to slipping region).

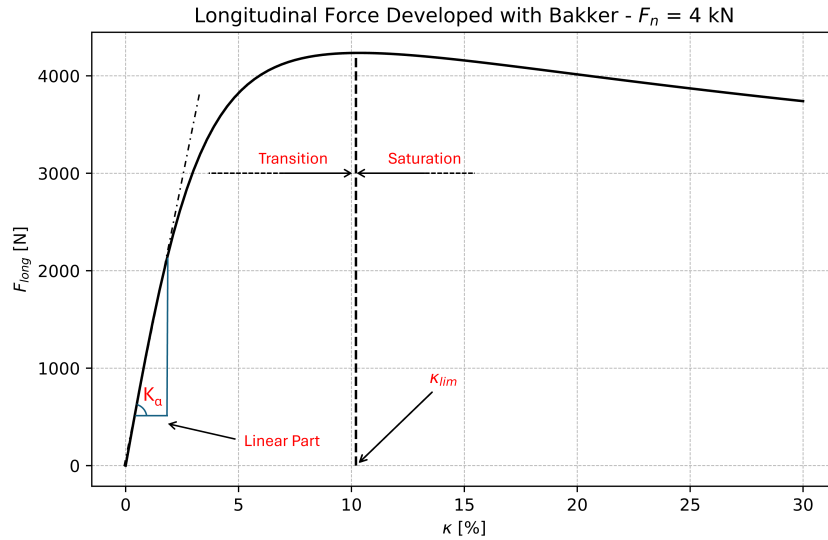


Figure 2.8: The longitudinal force distribution, according to the *Bakker* model, has a more pronounced saturation than for the lateral force distribution. In particular, the *ABS* aims to keep the longitudinal braking force before the saturation limit.

2.1.3 Combined Longitudinal and Lateral Forces

To optimise the tyre's behaviour on the road, it would be ideal to maximise both the longitudinal and the lateral forces. Nevertheless, it is impossible to dissociate the longitudinal force component from the lateral component. The concept of Friction Limit Surface represents the maximal frictional force developed by the contact surface in each direction. The maximum frictional force depends on the normal load F_n and the maximal friction coefficient μ_{max} between the tyre and the road surface [4, 21]:

$$F_R = \sqrt{F_{long}^2 + F_{lat}^2} < \mu_{max} F_n \quad (2.7)$$

As presented in Figure 2.10, the tyre's friction surface is closer to an ellipse. Therefore, a tyre can generate more forces in either the longitudinal or lateral directions. The combination of lateral and longitudinal forces can be represented into a resultant force F_R , also called the *grip force vector*. As Figure 2.9 shows with the blue arrow, it is unphysical to maximize separately lateral and longitudinal forces. As soon as the tyre is in a mixed slip condition (when $\alpha \neq 0$ and $\kappa \neq 0$), the longitudinal force and the lateral force must be coupled. Therefore, the coupling coefficients $s_{xs}(\kappa, \alpha)$ and $s_{ys}(\kappa, \alpha)$ multiply the longitudinal and the lateral forces [19]:

$$\begin{aligned} F_{long,final} &= s_{xs}(\kappa, \alpha) F_{long}(F_n, \kappa) = \frac{|\kappa|}{\sqrt{(\kappa^2 + \tan^2(\alpha))}} F_{long}(F_n, \kappa) \\ F_{lat,final} &= s_{ys}(\kappa, \alpha) F_{lat}(F_n, \alpha, \gamma) = \frac{|\alpha|}{\sqrt{(\kappa^2 + \tan^2(\alpha))}} F_{lat}(F_n, \alpha, \gamma) \end{aligned} \quad (2.8)$$

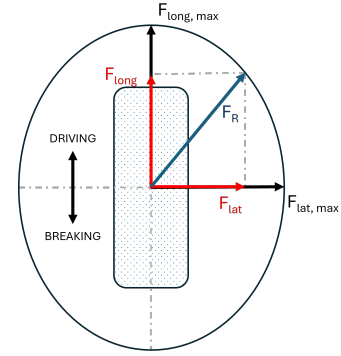


Figure 2.9: The resultant force F_R can not maximize both the longitudinal force and the lateral force independently.

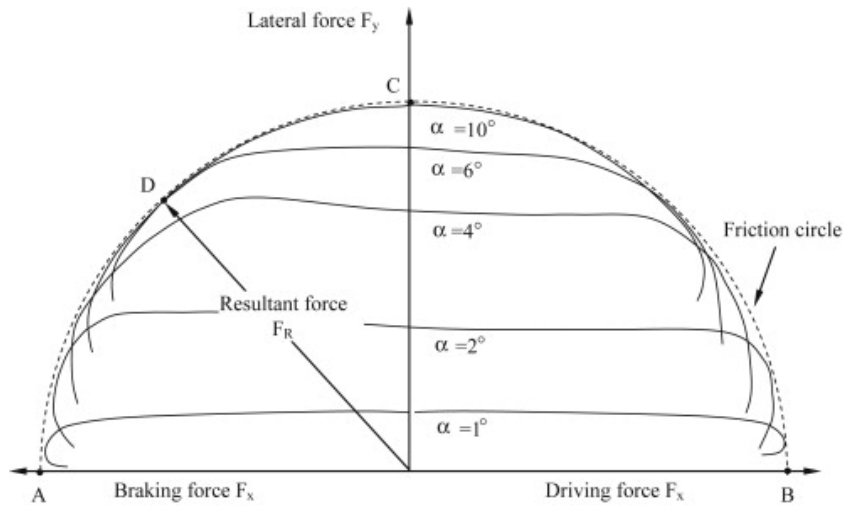


Figure 2.10: Friction circle detailed in [4]. The resultant F_r of the lateral force (F_y) and the longitudinal force (F_x) are limited by the maximal friction and the normal load.

2.2 Friction Coefficient

The friction coefficient limits the forces applied by the tyre on the road. This coefficient depends on the road surface, the weather and the tyre performances. As we can see for the *dry asphalt* in Table 2.1, the static friction coefficient can exceed 1 under high-performance conditions (e.g. sports tyres, ...) [12]:

Static friction coefficients	
	μ_s
Dry Asphalt	0.7 - 1.1
Wet Asphalt	0.4 - 0.6
Snow	0.2 - 0.3
Ice	0.1 - 0.2

Table 2.1

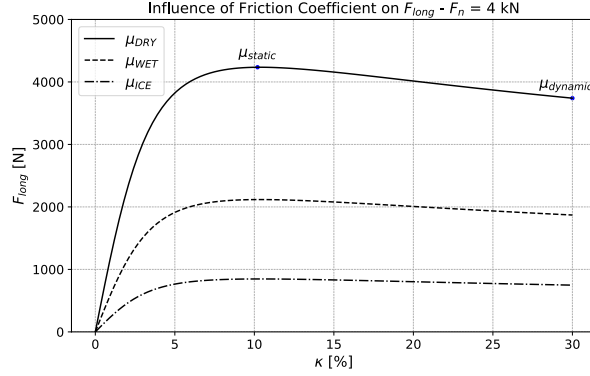


Figure 2.11: A lower coefficient of friction decreases the peak value of F_{long} for the same value of $F_n = 4$ kN.

As introduced before, the maximal friction coefficient is responsible for the peak value in the *Bakker* model. Moreover, the value of the friction coefficient depends on the wheel's slip. When the tyre is sliding, the value of the coefficient of friction then changes from *static* to *dynamic* and the value of μ decreases. For example, Figure 2.11 represents the influence of the friction coefficient on the longitudinal force: a lower friction coefficient results in a lower maximum value of the longitudinal force. Therefore, vehicle behaviour will be affected.

An useful method of testing the impact of the coefficient of friction on the car's handling is the *Split- μ* test (Figure 2.12). On a specially designed track, a car is launched either at a constant speed or while accelerating, in a straight line or while turning, with a section of the track being wet, snowy, or configured to simulate these climatic conditions. The purpose is to change the road surface on one wheel and to check the torque distribution on the other wheel: with a specific choice of differential, it could change the torque balance between wheels or axles (front or rear). It will be demonstrated through the chapter 7.

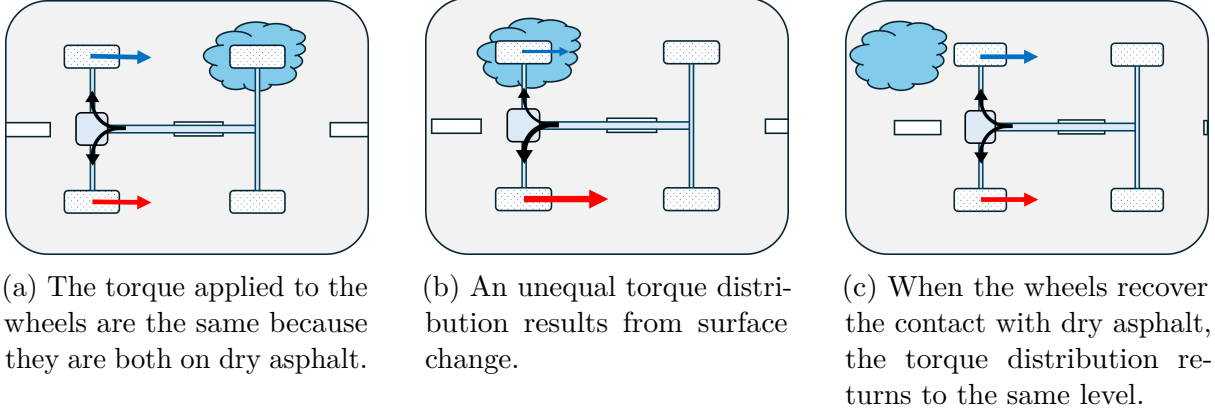


Figure 2.12: Representation of the *Split-μ* test. The blue cloud represents a variation of surface friction coefficient as a pool of water or a patch of ice. Torque distribution changes with the type of vehicle's transmission. Inspired from JTEKT *Torsen* slides.

2.3 Load Transfer

The load transfer refers to the distribution of the normal load F_n between the four tyres. It is the second variation parameter of F_R in the relation 2.7. The distribution along the car of the normal load affects the tyre's reaction. The load transfer varies depending on the sign and value of lateral and longitudinal acceleration.

2.3.1 Longitudinal Load Transfer

With the driver's torque input injected via the acceleration pedal, the car takes longitudinal positive acceleration a_x . It becomes negative when the driver pushes the braking pedal. This acceleration induces *pitch* γ (rotation of the carbody along the y-axis, see Figure 2.1a) which affects the load distribution along the wheelbase E of the vehicle. The longitudinal acceleration of the gravity centre G is defined as follows [20]:

$$a_x = \dot{u} - v\dot{\psi} \quad \left[\frac{m}{s^2} \right] \quad (2.9)$$

with u the longitudinal velocity of the car, $\dot{u}$ its longitudinal acceleration, $\dot{\psi}$ is the yaw rate (the rotational velocity along the z-axis) and v the lateral velocity, as detailed in Figure 2.13. The last part of the equation 2.9 can be transformed with the *drift angle* β and the curvature of the cornering ρ :

$$a_x = \dot{u} - u^2 \rho \beta \quad (2.10)$$

with $\beta = \arctan\left(\frac{v}{u}\right) \quad [^\circ] \quad \& \quad \rho = \frac{1}{R} = \frac{\dot{\psi}}{u} \quad \left[\frac{1}{m} \right]$

The drift angle β illustrates how the velocity of the car's gravity centre G tends to be lateral compared to the longitudinal midplane of the vehicle. As observable in Figure 2.14, it measures the car's deviation in cornering and illustrates the overall *understeering* or *oversteering* behaviour of the vehicle (more details in section 2.4.1).

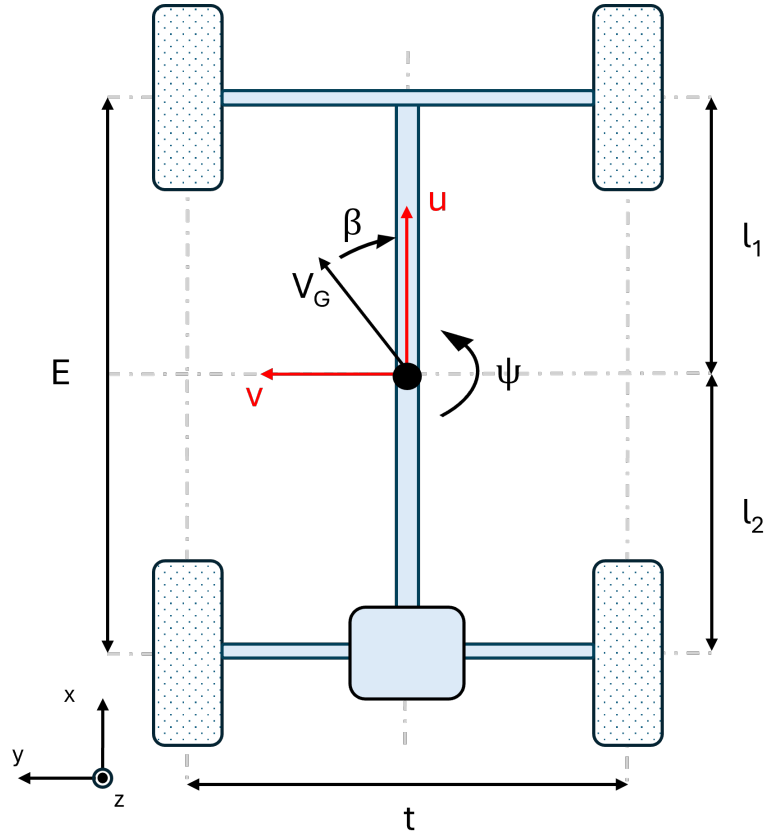


Figure 2.13: Global car schema, the black point represents the gravity centre G . Based on [19].

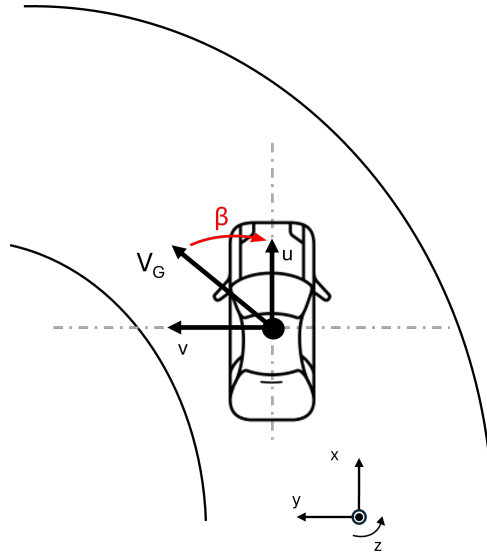


Figure 2.14: The drift angle β measures the car's deviation in cornering.

The pitch induced by the longitudinal load transfer is presented in Figure 2.15. If $a_x = 0$, the normal load is distributed between the front and rear, according to the distribution of mass in the vehicle. This is denoted by $R_{front,static}$ for the front and $R_{rear,static}$ for the

rear. When the longitudinal acceleration a_x is negative, the dynamic load generated is transferred to the front of the vehicle ($R_{front} = R_{front,static} + T_{load}$). Conversely, when the acceleration is positive, the load is transferred to the rear of the vehicle, unloading the front ($R_{rear} = R_{rear,static} + T_{load}$).

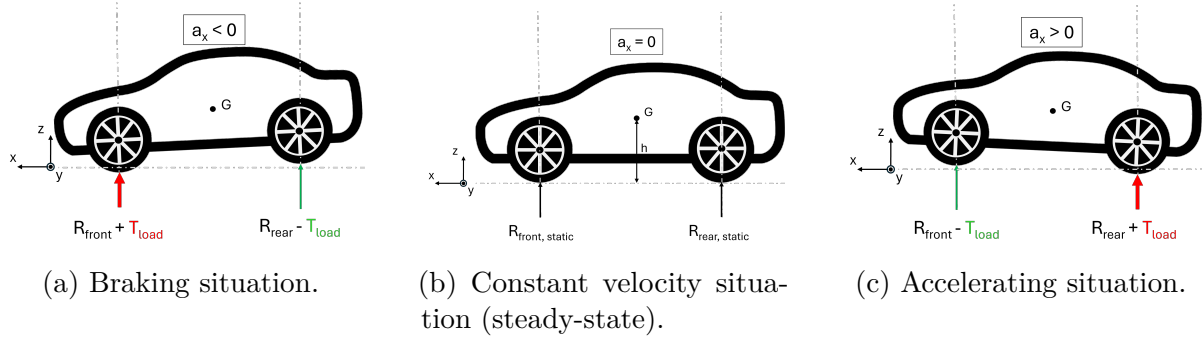


Figure 2.15: The longitudinal acceleration a_x plays on the normal load distribution along the wheelbase of the vehicle.

For a rigid carbody with infinitely rigid suspensions, the expressions of the longitudinal load transfer are [10, 20, 36]:

$$\text{Static loads:} \quad R_{front,static} = \frac{mgl_2}{E} \quad [N] \quad \& \quad R_{rear,static} = \frac{mgl_1}{E} \quad [N] \quad (2.11)$$

$$\text{Load Transfer:} \quad T_{load} = \frac{mha_x}{E} \quad [N]$$

The longitudinal load transfer T_{load} is determined by the mass m of the vehicle, the position of the gravity centre (l_1 , l_2 and h), the wheelbase E and the longitudinal acceleration a_x .

2.3.2 Lateral Load Transfer

The lateral load transfer appears as soon as a lateral acceleration a_y is produced by the vehicle, when the driver turns the steering wheel. Therefore, this acceleration induces *rolling motion* ϕ of the carbody (rotation along the x-axis, see Figure 2.1a). The lateral acceleration of G can be defined as [19]:

$$a_y = \dot{v} + u\dot{\psi} \quad \left[\frac{m}{s^2}\right] \quad (2.12)$$

with $\dot{v}$ the lateral acceleration of the car. Using the relation 2.10 with the definition of the drift angle β , one can write:

$$a_y = u\dot{\beta} + \dot{u}\beta + u\dot{\psi} \quad (2.13)$$

The lateral load transfer applies to the left or right of the vehicle, depending on the sign of the lateral acceleration a_y . When the car is in a straight line ($a_y = 0$), the static load on the left R_{left} and on the right R_{right} varies according to the mass distribution of the car. As shown in Figure 2.16, if a_y is positive, then the dynamic load T_{load} (also called ΔT between wheels) caused by lateral acceleration is directed to the right of the vehicle. On the other hand, if a_y is negative, then the left-hand side of the vehicle is the most heavily loaded.

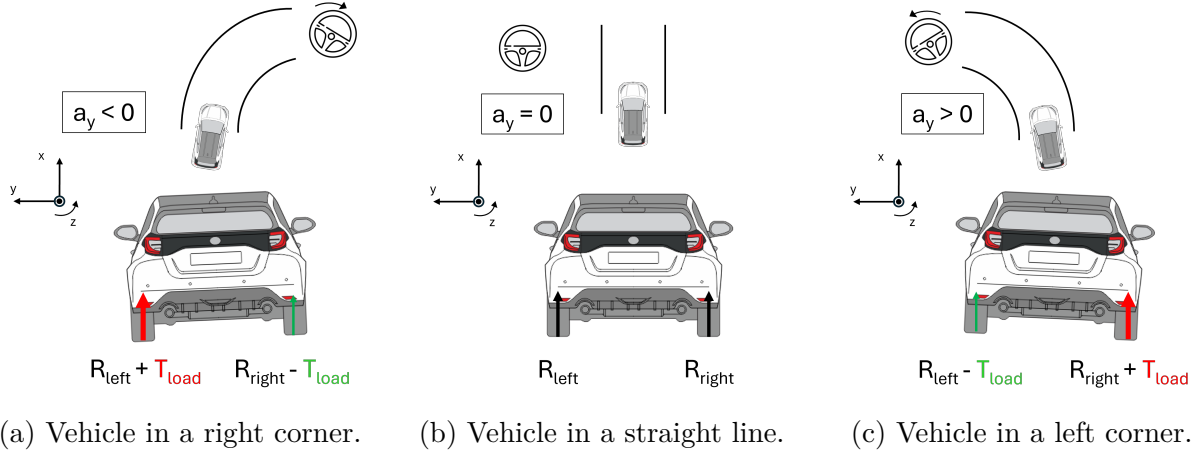


Figure 2.16: The lateral acceleration a_y plays on the normal load distribution along the track t of the vehicle. Considering a perfectly mass repartition in the car, the wheels are therefore equally loaded if $a_y = 0$. The car drawing comes from [18].

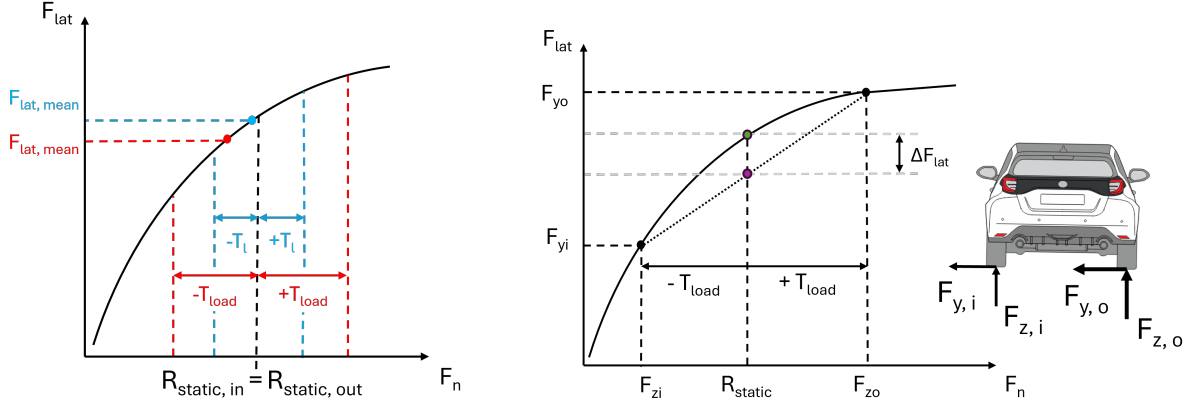
In steady-state cornering (with $\dot{u} = 0$ and $\dot{v} = 0$), the expressions of the lateral load transfer terms for a rigid car are [10, 36]:

$$\text{Static loads:} \quad R_{in,static} = \frac{mg}{2} \quad [N] \quad \& \quad R_{out,static} = \frac{mg}{2} \quad [N] \quad (2.14)$$

$$\text{Load Transfer:} \quad T_{load} = \frac{mha_y}{t} \quad [N]$$

Considering a perfect mass repartition along the track t of the vehicle. $R_{in,static}$ means the static load on the inner wheels, while $R_{out,static}$ represents the static load on the outer wheels. In turns, the lateral dynamic load T_{load} is transferred to the outer wheel of the vehicle (Figure 2.16).

An important parameter that influences the lateral loading is the *anti-roll bar* of the car. This rod prevents the car from rolling due to lateral load transfer thanks to its own torsion. As "stiffness attracts force", a stiffer anti-roll bar tends to increase the lateral load transfer, while a less stiff anti-roll bar will reduce it. The anti-roll bar can be installed on both front and rear axles. Figure 2.17a illustrates the impact of the anti-roll bar stiffness on the lateral force F_{lat} (in steady-state cornering). Due to the nonlinearity between F_n and F_{lat} , the dynamic load transfer T_{load} induces a smaller mean value of F_{lat} for the red points with a higher anti-roll bar stiffness than for the blue points with a lower stiffness [10, 36]. This results in the vehicle being more understeered (see 2.4.1).



(a) Higher stiffness in the anti-roll bar (red points) results in a lower $F_{lat, mean}$ compared to a lower stiffness (blue points). Based on [10].

(b) The mean lateral force in cornering (purple point) is lower than the mean lateral force in straight line (green point). Inspired from [36].

Figure 2.17: The lateral force F_{lat} is a non-linear function of the normal force F_n . Therefore, the mean lateral force $F_{lat, mean}$ decreases during cornering compared to a car travelling in a straight line. This mean lateral force $F_{lat, mean}$ also decreases if the anti-roll bar stiffness is higher [36]. The subscript i refers to the inside wheels during a turn, while the subscript o refers to the outside wheels.

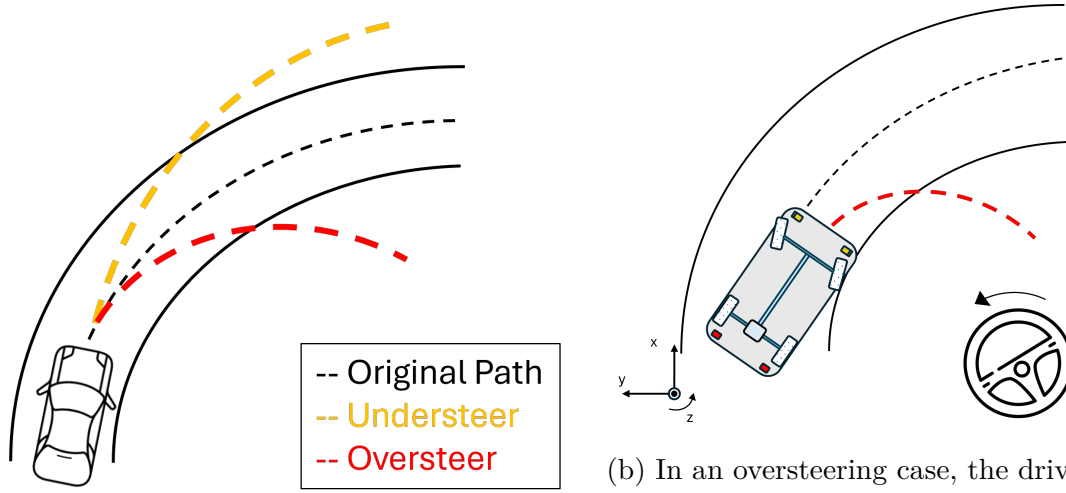
In Figure 2.17b, F_{zi} and F_{zo} respectively represent the vertical load on the inner and the outer wheel, while F_{yi} and F_{yo} respectively denote the lateral force on the inner and the outer wheel. When the vehicle enters a corner, the normal load F_n increases on the outer wheels and decreases on the inner wheels ($F_{zo} > F_{zi}$). Therefore, the mean lateral force in straight line (green dot) decreases by a ΔF_{lat} to a lower mean value (purple dot) when cornering. This reduces the tyre's ability to maintain grip during cornering: the slip angle required for the tyre increases.

2.4 Steering Behaviour

To modify the trajectory of a four wheels vehicle, the driver turns the steering wheel to orient the front wheels in the desired direction. However, the car's actual direction can be affected by the speed of the car, which influences the load transfer. The driver controls the car's longitudinal acceleration using the engine torque pedal. Therefore, it is recommended to take into account the steering wheel angle and the level of engine torque applied by the driver when analysing the vehicle's cornering behaviour.

2.4.1 Understeering and Oversteering

The understeering and oversteering of a car in cornering characterize the handling of the vehicle reacting to the driver's inputs (the steering angle δ and the engine torque T_m) [40]. Depending on the car's transmission (to the front wheels or the rear wheels), the vehicle's handling can vary.



(a) Depending on the driver's inputs, the car can follow a different trajectory than what the driver really wants.

(b) In an oversteering case, the driver must turn the steering wheel in the opposite direction of the turn to follow as close as possible the desired path of the curve.

Figure 2.18: The two distinct driving situations induce different driver's reaction. These situations are triggered when the vehicle starts to slip around a corner. In the case of understeering, the main solution to recover the trajectory is to slow down in the turn. This allows the driver to recover grip. In the case of oversteering, the driver can maintain the trajectory by counter steering without braking. Almost all standard vehicles are built understeered due to the braking natural reaction when a non experimented driver loses grip with his car.

As it is observable in Figure 2.18a, *understeering* arises when the actual trajectory of the car is larger than the desired trajectory (*original path*) of the driver. It occurs when the front tyre slip exceeds the saturation limit. For example, this situation arises when a front-wheel-drive vehicle enters a corner too fast. Therefore, the driver has to slow down or increase the steering wheel angle to recover the control of his vehicle. On the other hand, *oversteering* occurs when the vehicle's trajectory is shorter than intended by the driver. The rear tyres lose grip, and the vehicle tends to spin up. For example, this can be caused by too much input torque at the turning entry with a rear-wheel-drive vehicle. The car starts to spin. The solution for the driver is therefore to counter-steer in order to recover the initial trajectory (Figure 2.18b).

2.4.2 Ackermann Principle

The Ackermann principle is used to define the steering angles of the front wheels. For a curvature of radius R , the steering angles of the inner wheel δ_i and of the outer wheel δ_o can be defined as follows (Figure 2.19):

$$\begin{aligned}\delta_o &= \arctan\left(\frac{E}{R + \frac{t}{2}}\right) \quad [^\circ] \quad \text{for the outer wheel;} \\ \delta_i &= \arctan\left(\frac{E}{R - \frac{t}{2}}\right) \quad [^\circ] \quad \text{for the inner wheel;}\end{aligned}\tag{2.15}$$

Therefore, the average front steer angle is:

$$\delta_a = \arctan\left(\frac{E}{R}\right) \quad [^\circ] \quad (2.16)$$

This angle is called the *Ackermann* angle. This idealized model can only be applied in the case of low speeds because it does not take into account the slip angles.

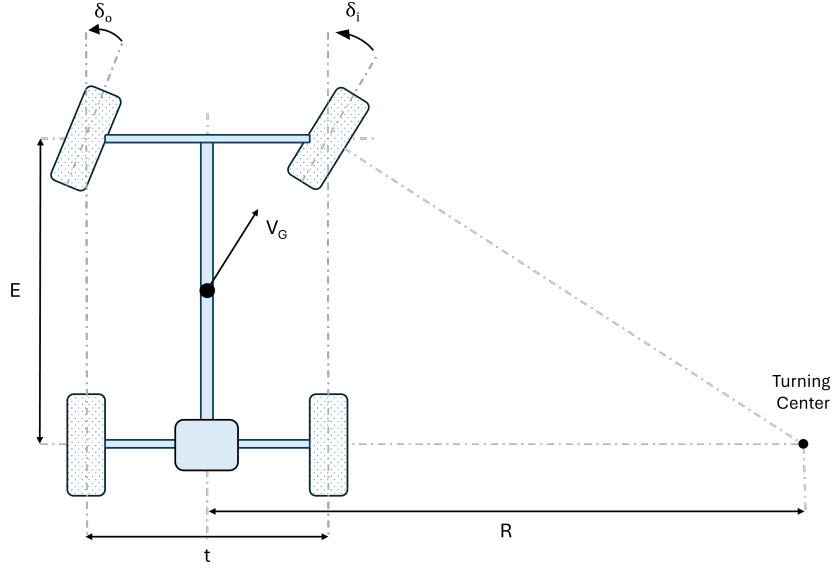
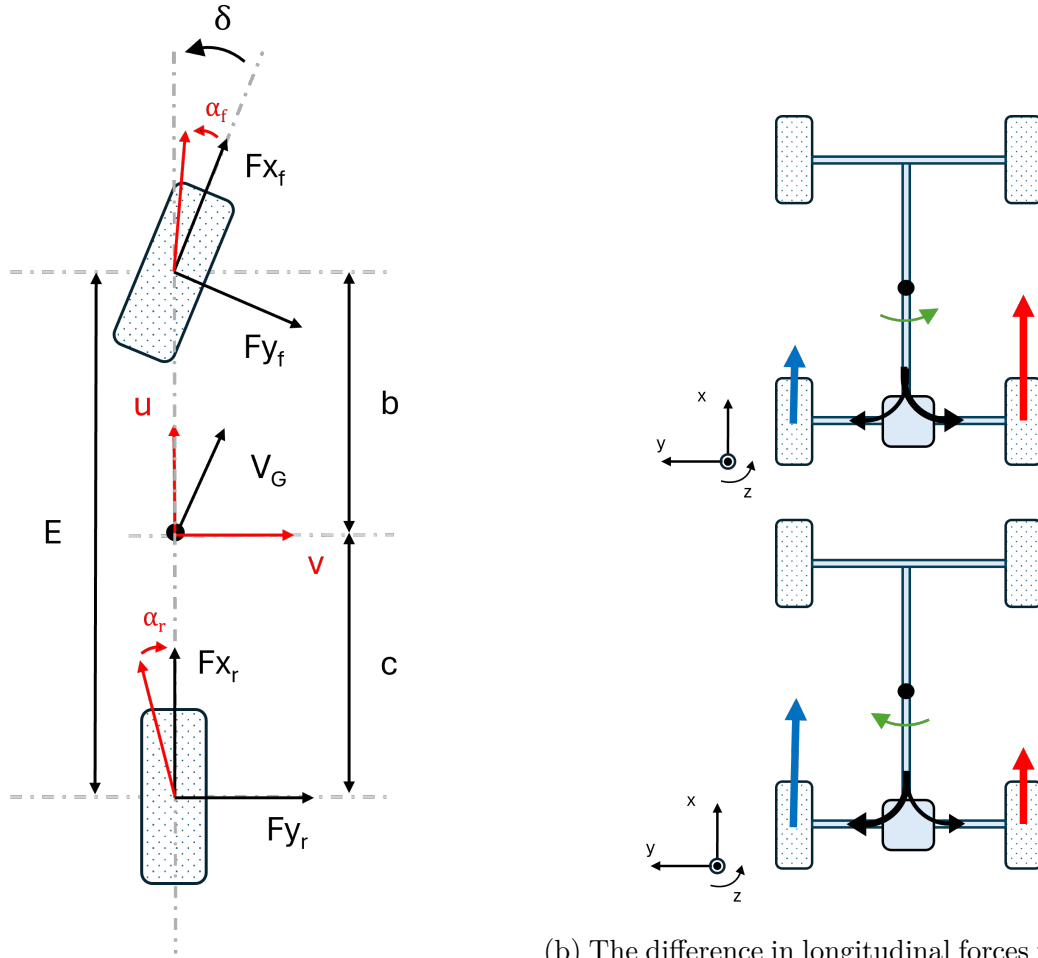


Figure 2.19: The *Ackermann* principle relates the steering angles to the radius of curvature R . The steering angle of the inner wheel δ_i is greater than that of the outer wheel.

2.4.3 Single Track and Double Track Model

For higher speed turns, the *Single Track Model* (also called the *Bicycle Model*) demonstrates the vehicle's cornering handling, as reference [36] employs. The hypothesis applied to this model can not be fully respected in this work. Because the *Single Track Model* assumes the same ground forces on the left and the right wheel, it simplifies a four-wheeled vehicle into a two-wheeled car (like a bicycle). This only holds true if the vehicle is equipped with a perfectly open differential with no friction. As we will prepare our simulation car with a diverse torque distribution between the wheels, the *Single Track Model* can not be adopted to demonstrate the cornering behaviour of the simulation vehicle model. Nevertheless, it remains relevant to develop the vehicle handling parameters highlighted in the *Bicycle Model*.

Single Track



(a) The *Single Track Model* establishes an average of the forces exerted by the left and right wheels on one side.

(b) The difference in longitudinal forces induced by the torque distribution creates a yaw moment, which is positive or negative depending on the distribution.

Figure 2.20: The *Single Track Model* can not highlight the yaw moment produced by variations in the torque distribution of the wheels.

Firstly, Figure 2.20a shows that a part of the lateral force F_{y_f} brakes the car due to the steering angle δ . During steady-state cornering, the longitudinal force F_{x_f} must be adjusted in order to maintain a constant velocity. Furthermore, the *Single Track Model* highlights the *Understeer Gradient* coefficient K . The vehicle equilibrium in steady-state cornering is (Figure 2.20a) [19, 29]:

$$\text{Newton:} \quad F_{y_f} + F_{y_r} = m \frac{V_G^2}{R} \quad (2.17)$$

$$\text{Euler:} \quad F_{y_f} b - F_{y_r} c = 0 \quad \longrightarrow \quad F_{y_f} = \frac{F_{y_r} c}{b} \quad (2.18)$$

with m the equivalent mass of the vehicle. Inserting the relation 2.18 into 2.17 with $F_y = K_\alpha \alpha$, we can highlight the front and the rear slips:

$$\alpha_f = \frac{W_f}{K_{\alpha f}} \frac{V_G^2}{gR} \quad \& \quad \alpha_r = \frac{W_r}{K_{\alpha r}} \frac{V_G^2}{gR} \quad (2.19)$$

The mass distribution between the front and rear axles is represented by $\frac{W_f}{g}$ and $\frac{W_r}{g}$. As the speed increases, wheel slip affects the steering angle (Figure 2.20a). Therefore, the steering angle ² is defined as follows:

$$\delta \simeq \arctan\left(\frac{E}{R}\right) + \alpha_f - \alpha_r = \arctan\left(\frac{E}{R}\right) + \hat{\alpha} \quad (2.20)$$

The last term of the previous relation indicates the vehicle's cornering behaviour. If $\hat{\alpha} > 0$, the car is understeered: the driver has to steer more as he accelerates through the turn. The oversteering begins when $\hat{\alpha} < 0$. This situation requires the driver to steer less when he accelerates in a turn. Regarding the relation 2.20, the *Understeer Gradient* K in steady-state cornering can be figured out using the lateral acceleration a_y :

$$\delta = \left(\frac{E}{R}\right) + \underbrace{\left(\frac{W_f}{K_{\alpha f}} - \frac{W_r}{K_{\alpha r}}\right)}_K \underbrace{\left(\frac{V_G^2}{gR}\right)}_{\frac{a_y}{g}} \quad (2.21)$$

The coefficient K is positive for an understeered vehicle. With the relation 2.21, the *Single Track Model* highlights the two main factors that impact vehicle handling:

- The longitudinal load balance between the front and rear axles. As the motor accounts for the largest proportion of the vehicle's weight, its position on the car body emerges as a characteristic that influences understeering or oversteering. For example in a sport car, the motor is typically rear-mounted, inducing oversteering behaviour.
- The difference between the front and rear tyres reactions on the ground. Depending on the sign of $\hat{\alpha}$, the car may be more understeered or oversteered.

Double Track

The *Double Track Model* considers the four wheels independently, as demonstrated in [19]. As shown in Figure 2.20b, a *yaw moment* is caused by the difference in longitudinal force between the two wheels. This difference is produced by an uneven distribution of torque in the differential. As previously explained, the *Single Track Model* can not take this significant factor into account in the vehicle handling equations.

²The $\simeq$ appears due to the simplification of the *Single Track Model* compared to the *Double*. The *Single Track Model* does not take into account the *Ackermann* correction caused by the five-arm steering [21].

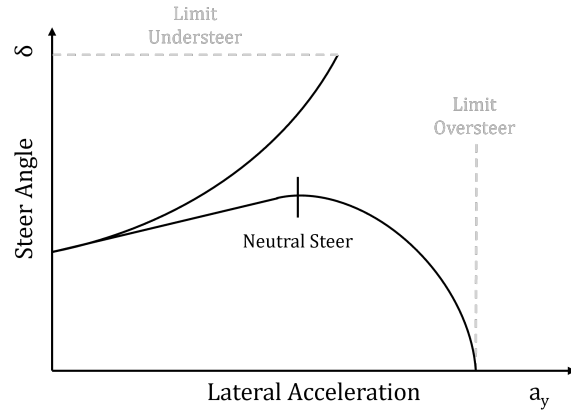


Figure 2.21: To obtain the curves, the behaviour of an accelerating car taking a constant radius corner is examined. With a different torque distribution, we can expect to encounter different limits [36, 10]. This graph is qualitative and does not represent specific values.

Figure 2.21 reveals the steering angle limits of understeering and oversteering (driver's limits). The upper limit is reached when the steering angle is twice that of *Ackermann's*. The oversteer limit is reached when the driver has to countersteer (Figure 2.18b). These curves assume a perfect torque distribution. With an unequal torque distribution between wheels, Section 6 will show that the understeer and oversteer limits will vary depending on the differential used.

Chapter 3

The Differential

The purpose of this chapter is to present the origins and fundamental principles of the differential from broad mathematical and functional perspectives. It begins with a description of the differential's basic functions, providing essential context. This is followed by a general mathematical analysis of how it operates. Finally, the chapter provides an overview of the various types of differential used in modern vehicles, highlighting the key benefits and limitations of different limited-slip differential designs.

3.1 Origins and Functions

The word *differential* comes from the abbreviation of *differentiation*. This word was chosen to describe the principal feature of a differential. This mechanical device enables the wheels to turn at different speeds. This is very useful when cornering, as the outer wheel needs to rotate faster than the inner one (the origin of the difference in rotational speed is presented in Appendix A.1).

The differential was invented in 1827. Its designer, *Onésiphore Pecqueur*, was a French mechanic who used the first model of the differential on steam engines. The widespread implementation of the differential in the automotive industry originated with the Ford Model T. Before using differentials, some examples of vehicles that do not use the differential as a differentiating device can be found. Notable examples include cyclecars from the 1920s, which sometimes had three wheels, and go-karts, which favour increased chassis flexibility. Another notable system is the variomatic system found in *DAF* trucks. The differential system has been gradually improved over the years by several renowned engineers and automotive companies, such as Ferdinand Porsche, ultimately leading to the modern designs that will be presented throughout this chapter [5, 39].

As previously introduced, the differential is a mechanical device used in automotive for its two particular features [28, 15]:

- The differential allows the wheels to rotate at two different speeds. Without it, it would be very difficult for the driver to achieve the correct turning radius for any type of turn. This will be illustrated in the section 3.3.3.
- The differential aims to split the torque coming from the engine between the two wheels (Figure 3.1). This mechanical feature can vary depending on the differential

type studied. The most common type of differential is called the **open differential**.

The torque produced by the motor goes through the gearbox in the case of a **thermal** vehicle, or in a reduction system in the case of an **electrical** vehicle. Then, the multiplied torque enters the differential via the rotation of the differential's housing (Figure 3.1). The various mechanical components of the differential depend on its type and will be presented later.

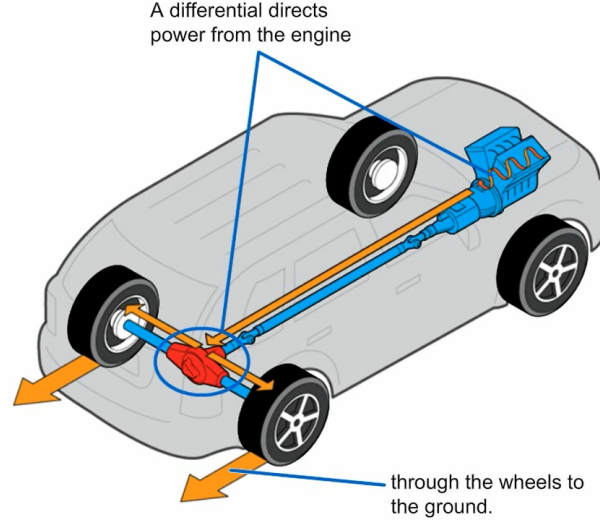


Figure 3.1: Path of torque transmission. The differential splits the torque coming from the engine between the two wheels [7].

The differential can be mounted either at the front or at the rear of the vehicle. We call *traction vehicles* when the torque applied by the engine goes to the front wheels, and *propulsion vehicles* when it goes to the rear wheels. Rear-wheel drive (**RWD**) is found in trucks, sports cars, and performance luxury cars (Figure A.2). Front-wheel drive (**FWD**) is used in passenger cars and urban SUVs. A differential can be placed on both axles to convert the car into a four-wheel-drive vehicle (**4WD**). In this case, the vehicle needs a **transfer box** [34] between the two axles. This splits the torque between the front and rear axles. This type of transmission is commonly employed for pickups and off-road vehicles.

3.2 Mathematical Approach

In addition to the above qualitative description of the differential, it is both useful and necessary to establish a general mathematical approach to its operation. The objective of this section is to provide a quantitative justification for the prevalence of different types of differential [21, 34, 14, 33].

A differential essentially comprises a housing combined with two output shafts. Each wheel is connected to a shaft. The alignment of the shafts enables *epicycloidal* movement, meaning that the housing can rotate. Assuming w_l , w_r and w_d for the angular velocity of

the left wheel, right wheel and the housing ¹, respectively (Figure 3.2a). According to the differential principle, the left and right wheels can rotate at different speeds. Nevertheless, due to the mechanical relation between the housing and the output shafts, the speed difference remains constant. If one wheel rotates faster with a positive $\Delta\omega$, the other wheel will rotate more slowly by $\Delta\omega$ (Figure 3.2b). It is crucial for the vehicle to behave in the same manner when turning left or right.

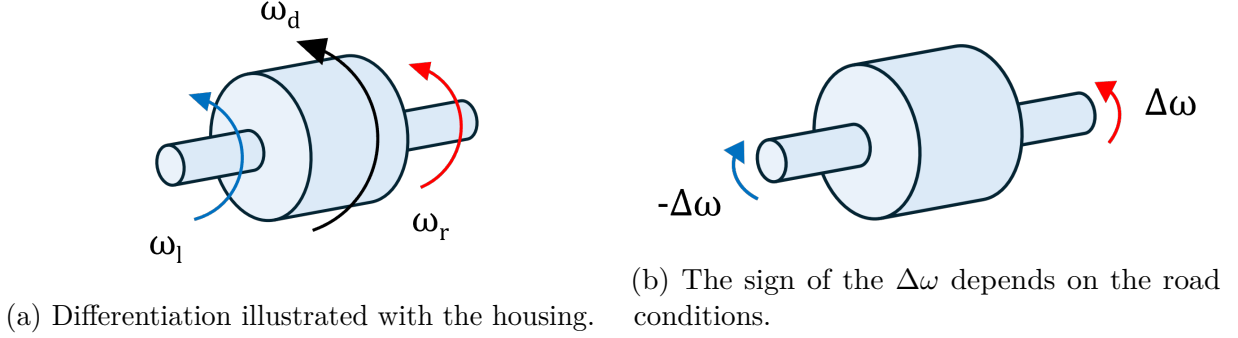


Figure 3.2: Differential angular velocities.

Therefore, using the *Willis's* formula, we can write the following relations:

$$\omega_d = \frac{\omega_r + \omega_l}{2} \quad \& \quad \frac{\omega_l - \omega_d}{\omega_d - \omega_r} = 1 \quad (3.1)$$

The difference of velocity is:

$$\Delta\omega = |\omega_l - \omega_d| = |\omega_r - \omega_d| = \frac{|\omega_l - \omega_r|}{2} \quad (3.2)$$

Because differential has 2 degrees of freedom, the *Willis's* formula can not evaluate the amplitude of $\Delta\omega$.

3.2.1 Differential Torque Balance and Friction Power

The torque balance in the differential is essential to describe the interaction between the requested driver's commands and the road. In this work, we treat the differential as a *black box*. The differential is represented as a device that transmits engine torque to the wheels.

Engine torque T_m is transferred to the differential via a gear ratio. The torque in the differential is denoted T_d . This is then split by the internal mechanical relations, producing a torque on the left wheel T_l and a torque on the right wheel T_r . We will neglect the internal dynamics of the differential because our focus is on its effect on vehicle handling. For each wheel, the torque is related to the longitudinal force F_x as follows:

$$T_i = F_{x,i} R_{wheel} \quad [Nm]$$

¹d means differential, the velocity applied on the housing can be viewed as the differential overview velocity.

The sign of T_d depends on the driver's command. If the vehicle is accelerating, the differential torque T_d is defined as positive. When decelerating (throttle-off), the value of T_d will be negative. The torque difference between the output shafts is:

$$\Delta T = |T_l - T_r| \quad (3.3)$$

This ΔT , which is created by the friction in the differential, can not be evaluated using only this equation. The result will depend on the type of differential.

Power Loss Induced by Internal Friction

Friction is responsible for the torque difference between the right and left wheels. Nevertheless, it also causes a power loss W_f in the differential (Figure 3.3) [14, 21]. A power balance between the left and right wheels occurs as soon as $\omega_l \neq \omega_r$. This balance results in a power loss W_f due to internal gear contact. This will be important when discussing limited-slip differentials.

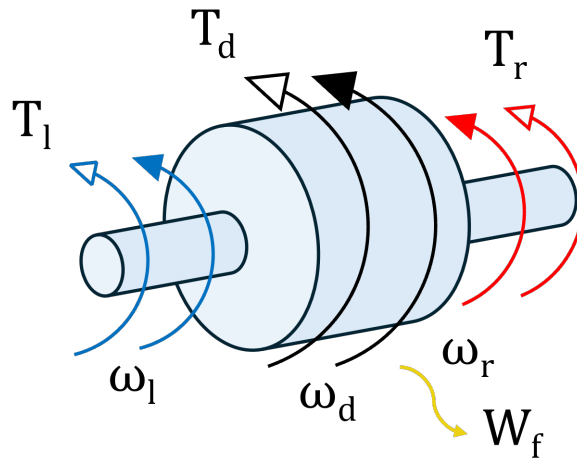


Figure 3.3: W_f is the power loss in the differential due to the internal friction.

The friction power is defined as follows [19]:

$$\begin{aligned} W_f &= (T_l - T_r) \left(\frac{\omega_r - \omega_l}{2} \right) \quad [W] \\ &= \Delta T \frac{\Delta \omega}{2} \end{aligned} \quad (3.4)$$

The sign of ΔT follows the opposite sign of $\Delta \omega$. However, this can be reversed depending on the road conditions and the car's handling. Therefore, we will consider the *fastest wheel* ω_f and the *slowest wheel* ω_s :

$$\Delta T = T_s - T_f > 0 \quad \& \quad \frac{\Delta \omega}{2} = \frac{\omega_f - \omega_s}{2} > 0 \quad (3.5)$$

We can determine the **torque balance** between the slowest and fastest wheel:

$$\begin{aligned} T_s &= \frac{T_d \pm \Delta T}{2} = \frac{T_d}{2} \pm \frac{W_f}{\omega_f - \omega_s} \quad [Nm] \\ T_f &= \frac{T_d \mp \Delta T}{2} = \frac{T_d}{2} \mp \frac{W_f}{\omega_f - \omega_s} \quad [Nm] \end{aligned} \quad (3.6)$$

The sign in the relation 3.6 will be determined depending on the tyres' interaction with the ground (see Section 3.2.3). Firstly, the internal efficiency must be considered in order to determine the amount of torque transfer ΔT .

3.2.2 Internal Efficiency and TBR

The friction power W_f dissipating inside the housing represents the difference between the *input power* W_i (provided by the slowest wheel), and the *output power* W_o (received by the fastest wheel).

$$W_i - W_o = W_f \quad W_i > W_o > 0 \quad (3.7)$$

The **internal efficiency** η_d of the differential is therefore:

$$\eta_d(T_d) = \frac{W_o}{W_i} = \frac{W_i - W_f}{W_i} \quad [-] \quad (3.8)$$

In general, η_d depends on the differential input torque T_d . The efficiency of the differential is often known because it is associated with a fixed quantity of the differential called the **Torque Bias Ratio (TBR)**. This is another manner of expressing the differential efficiency. This ratio is commonly used in industry to illustrate how much torque can be transferred from one wheel to another via the differential:

$$TBR = \frac{1}{\eta_d} \quad [-] \quad > 1 \quad (3.9)$$

This ratio determines how the wheels interact together with the available friction in the differential. Therefore, we can view the friction as a *locking effect* between the wheels. This locking effect depends on the TBR and can be expressed as a *locking coefficient*:

$$\varepsilon_d = \frac{1 - \eta_d}{1 + \eta_d} = \frac{TBR - 1}{TBR + 1} \quad [-] \quad (3.10)$$

$$\eta_d = \frac{1 - \varepsilon_d}{1 + \varepsilon_d} \quad (3.11)$$

The locking effect ε_d saturates as the TBR grows (Figure A.3 in Appendix A.4). A clear example allowing to understand the principle of friction in the differential will be presented in the section 3.2.5.

3.2.3 Tyres Interaction with The Ground and The Differential

The interaction between the tyres and the differential directly affects the amount of torque available on each shaft. Only the interaction between the wheels and the road can determine the values of T_s , T_f , ω_s and ω_f (i.e. T_l , T_r , ω_l and ω_r depending on the situation).

As mentioned previously, it is often incorrect to assume that the inner wheel necessarily has a lower forward velocity than the outer wheel during cornering situation [34, 19]. Indeed, we have to take into account the potential longitudinal slip of both tyres (introduced in Section 2.1). The angular velocities of the wheels follow these relations:

$$\omega_l = \frac{V_{x,l}}{(1 - \kappa_l)R_{wheel}} \quad \& \quad \omega_r = \frac{V_{x,r}}{(1 - \kappa_r)R_{wheel}} \quad (3.12)$$

Where R_{wheel} is the real wheel radius (considering the tyre penetration in the ground), $V_{x,i}$ the longitudinal velocity and κ the longitudinal slip ratio (considered positive in an accelerating situation). In the relation 3.12, $V_l < V_r$ does not necessarily mean that $\omega_l < \omega_r$. For example, consider a vehicle travelling at a high speed in a turn. Due to the lateral load transfer, the inner wheel may barely touch the ground, causing it to spin. In this case, the inner wheel would have the highest rotational speed, despite the outer wheel having a greater longitudinal velocity, as it does not lose road contact (facilitated by the supplementary lateral load). From this example, it is possible to derive a **general rule** valid for all situations. The wheel with the lowest torque is always the fastest:

$$(T_s - T_f)(\omega_f - \omega_s) > 0 \quad (3.13)$$

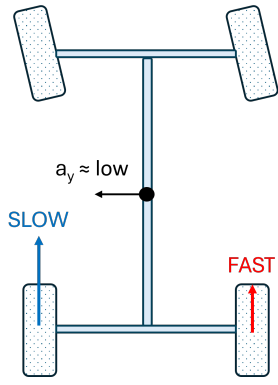
Therefore, at low lateral acceleration a_y , the inner wheel rotates more slowly and receives the highest torque. The longitudinal force is thus greater on the inner wheel (Figure 3.4a):

$$F_{x,inner} = TBR.F_{x,outer} \quad (3.14)$$

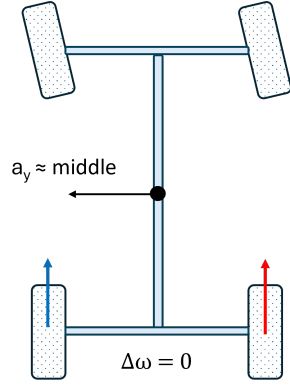
By contrast, at higher lateral acceleration a_y , the inner wheel is the fastest and receives the lowest torque (Figure 3.4c). The outer wheel experiences the greatest longitudinal force:

$$F_{x,inner} = \frac{F_{x,outer}}{TBR} \quad (3.15)$$

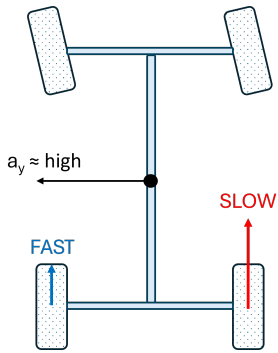
There is a range of lateral acceleration between which the difference in rotational velocity $\Delta\omega$ is 0 (Figure 3.4b). As mentioned in the section 2.4.3, the difference in torque between the wheels produces an additional yaw moment that contributes to the vehicle's understeered or oversteered characteristics. For the power-off condition (Figure 3.4d), the situation always remains the same. The angular velocities of the wheels persist as positive. The inner wheel always rotates more slowly than the outer wheel. Since both wheels receive a negative torque, the longitudinal forces are negative ($F_{x,inner} > F_{x,outer}$) and the general rule remains true.



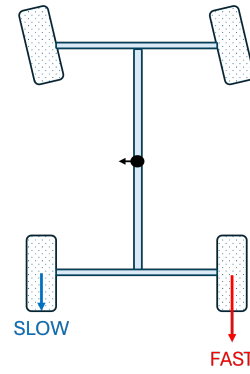
(a) Power-on with low lateral acceleration in left corner. The torque distribution induces an understeered moment.



(b) Power-on with middle lateral acceleration in left corner.



(c) Power-on with high lateral acceleration in left corner. This situation induces an oversteered moment.



(d) Power-off situation in a left corner. The supplementary negative torque is applied to the outside wheel and creates an understeered moment.

Figure 3.4: Cornering differential behaviour for power-on and power-off.

3.2.4 Torque Distribution Determination

As mentioned previously, there are two operating modes according to the driver's input. Both situations are illustrated in Figure 3.5:

1. *Power-on*: when the driver pushes the accelerator, he requests positive driving torque T_m from the engine. Thus, we have $W_i > W_o > 0$:

$$W_i = T_s \frac{\Delta\omega}{2} \quad \& \quad W_o = T_f \frac{\Delta\omega}{2} \quad (3.16)$$

2. *Power-off*: when the driver does not press the accelerator, a negative engine braking torque T_m is applied to the differential. Thus, we have $T_f > T_s > 0$, and so $W_i > W_o > 0$. These can be defined as follows:

$$W_i = T_f \frac{\Delta\omega}{2} \quad \& \quad W_o = T_s \frac{\Delta\omega}{2} \quad (3.17)$$

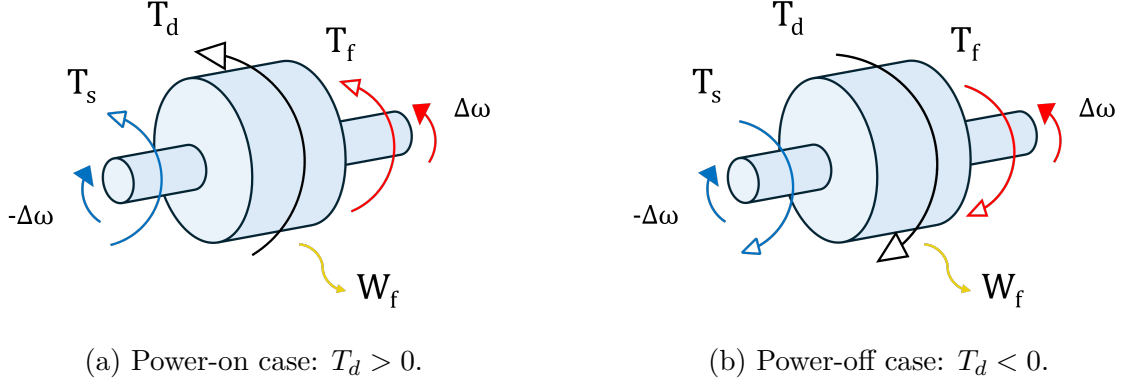


Figure 3.5: The differential torque distribution is induced by the sign of T_d . The direction of the quantities involved in the differential varies according to T_d .

Taking into account the efficiency η_d (assume known), we can write:

$$\begin{aligned}
 \text{power-on:} \quad & \frac{T_f \Delta\omega}{T_s \Delta\omega} = \frac{T_f}{T_s} = \frac{1}{TBR} = \eta_d(T_d) \\
 \text{power-off:} \quad & \frac{T_s \Delta\omega}{T_f \Delta\omega} = \frac{T_s}{T_f} = \frac{1}{TBR} = \eta_d(T_d)
 \end{aligned} \tag{3.18}$$

The final expression of the torque balance ΔT^2 :

$$\begin{aligned}
 \eta_d &= \left(\frac{T_d - \Delta T}{T_d + \Delta T} \right)^{\text{sign}(T_d)} \\
 TBR &= \left(\frac{T_d + \Delta T}{T_d - \Delta T} \right)^{\text{sign}(T_d)}
 \end{aligned} \tag{3.19}$$

$$\Delta T = \left(\frac{1 - \eta_d}{1 + \eta_d} \right) |T_d| = \left(\frac{TBR - 1}{TBR + 1} \right) |T_d| = \varepsilon_d |T_d| \tag{3.20}$$

For example, if $\eta_d = \frac{1}{3} \Leftrightarrow TBR = 3$, we obtain:

$$\varepsilon_d = \frac{3 - 1}{3 + 1} = \frac{1}{2}$$

Therefore, the torque distribution between the slowest and fastest wheel is:

$$\Delta T = \frac{T_d}{2} \quad \& \quad T_s = 3T_f$$

3.2.5 Differential Analogy

Let us consider a simple example to illustrate the differential operation: imagine a pulley system used to lift two weights, pulled by an external force [33]. In Figure 3.6, the weights represent the grip force available at the wheels (i.e. depending on the friction available between the tyre and the ground), the pulley represents the differential, and the

²In the exponent of the expression, the sign of the expression depends on the driver's input.

external force represents the driver's input.

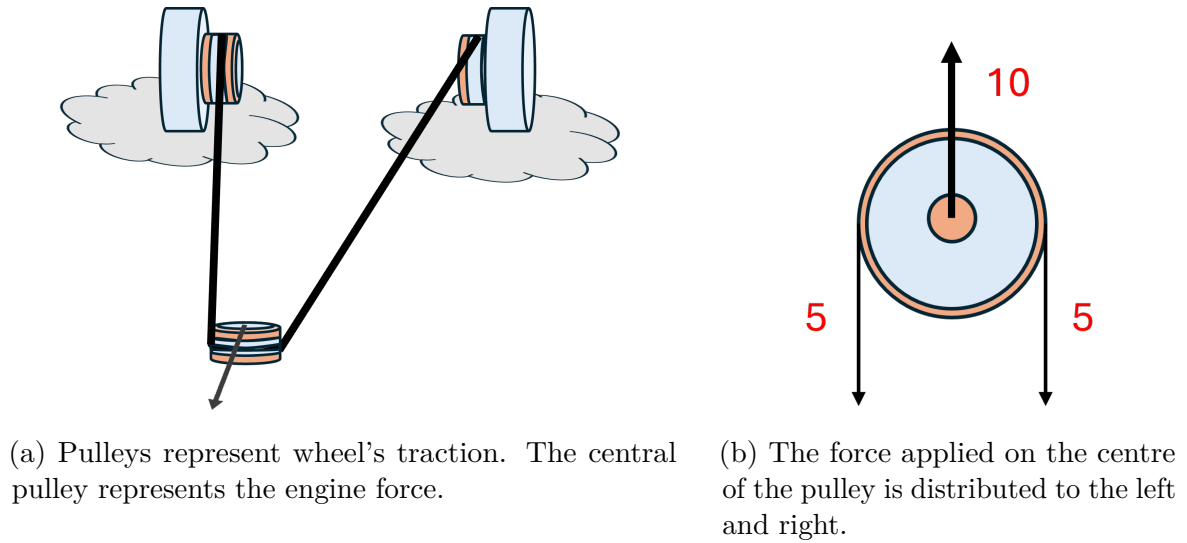


Figure 3.6: Pulley's analogy.

We will consider three different cases:

- The first case represents the use of an open differential. This case is further divided into three sub-cases (Figure 3.7).
- The second case involves a limited-slip differential with a sufficiently high TBR (Figure 3.8b).
- The third case shows a limited-slip differential with an insufficient TBR (Figure 3.8a).

First case

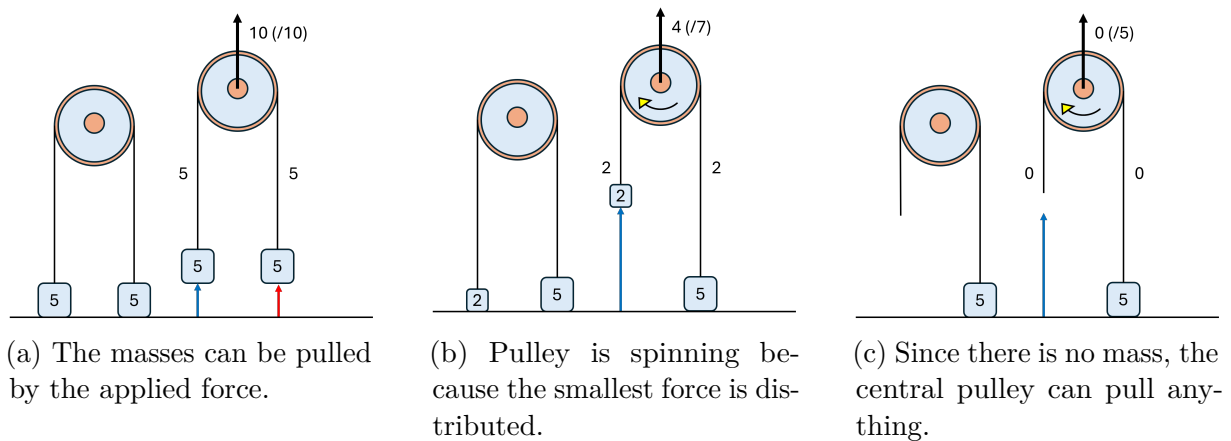


Figure 3.7: Pulley's analogy with an open differential.

In the case of a perfect open differential (OD) presented in Figure 3.7, there is almost no torque difference between the left and right wheels. The open differential distributes the

torque equally to both the left and right wheels. If one of the pulleys (wheels) loses grip, the other wheel is directly affected. The central pulley can only transfer a force equivalent to the smallest available force on either side. Since the force applied to the central pulley (by the engine with the driver's request) is greater than the total available force (weight), the pulley spins on itself, creating a friction torque (yellow arrow). Compared to previous developments, an open differential can not create a torque bias ΔT between wheels because of the no internal friction in this type of differential.

In the most extreme cases, such as when there is no mass on one side of the pulley (e.g. a wheel on ice or not touching the ground), no force is transmitted to the weights. Since the force transmitted to the other side must equal the smallest available force, the pulley rotates on itself without moving anything.

Second case

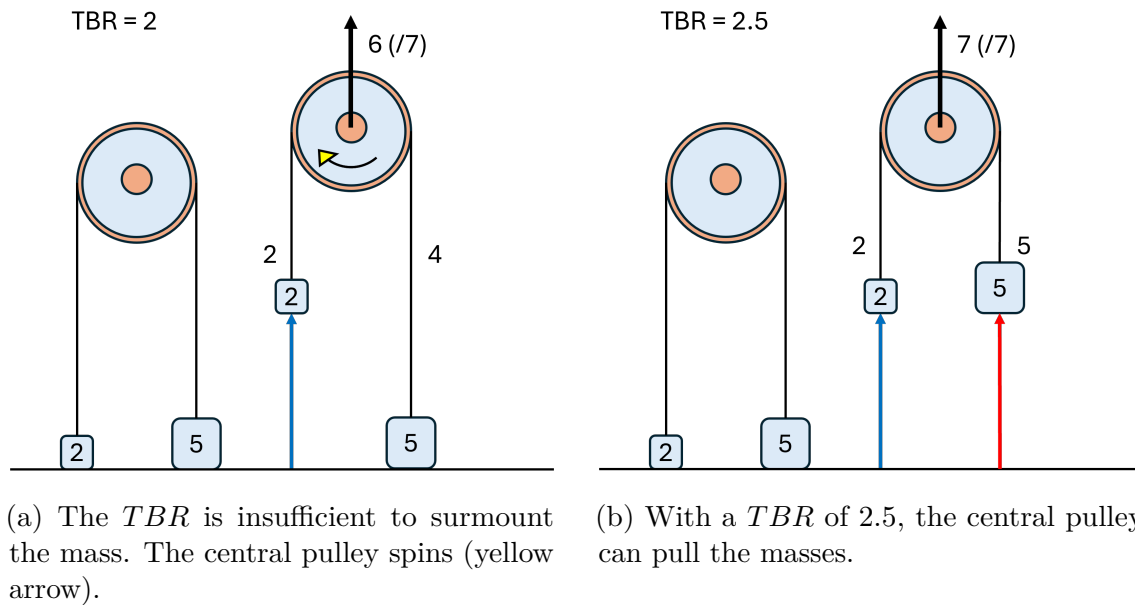


Figure 3.8: Pulley's analogy with a limited-slip differential.

With a limited-slip differential, there can be a difference in torque between the wheels (pulleys) due to friction inside. As shown in Figure 3.8a, the TBR helps to multiply the smallest force of one pulley so that it can be transferred to another. However, if the value of the TBR (or locking effect) is not high enough, the central pulley can not pull all the available weights. If the force applied to the central pulley is greater than the forces acting on the wheels (weights), the central pulley spins.

Third case

If the Torque Bias Ratio is greater under the same conditions, the transferred force is larger. In Figure 3.8b, the force on the right is equal to $2 \cdot TBR = 5$ because the $TBR = 2.5$. The central pulley can pull the entire system without spinning.

These three situations demonstrate the difference between the open differential and the limited-slip differentials. In order to understand why a limited-slip differential, particularly a *Torsen* differential, is more interesting than an open differential, it is important to examine the characteristics of all variants.

3.3 Open Differential

The open differential is the most widespread type of differential in the world because it is the simplest. As explained in the introduction to this chapter, the differential allows the two wheels to rotate at different velocities (see Appendix A.1).

In the open differential, the engine torque applied to the differential T_d is distributed equally between the driving wheels in all handling conditions. This attribute enables the following torque relation [14, 20]:

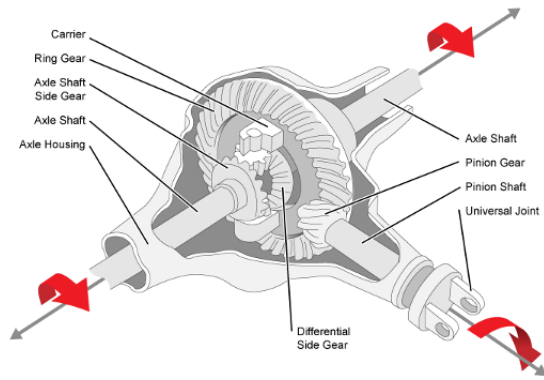
$$T_d = T_r + T_l \quad \& \quad T_r = T_l = \frac{T_d}{2} \quad (3.21)$$

As proven in Section 3.2.5, the value of the transmitted torque is determined by the wheel with the poorest grip on the ground. Depending on the road surface, the wheels may be in contact with different types of ground (with a different friction coefficient between the left and right wheels). Therefore, the maximum torque distributed through the wheels in the differential depends on the contact forces between the wheels and the ground.

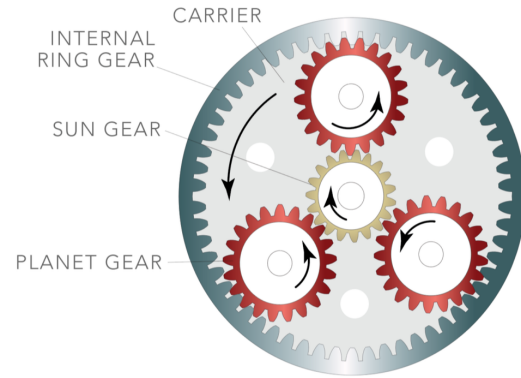
This expression can be found in many scientific works dealing with vehicle dynamics. In practice, this would mean that the efficiency η_d of an open differential is close to 1 and the dissipated friction W_d is close to 0. However, regarding the composition of an open differential, these assumptions are somewhat too strong. Due to the friction between the various gears, an open differential has a Torque Bias Ratio of around 1.1–1.2. Although this remains low compared to a limited-slip differential, the natural locking effect is not negligible. The gear configuration in an open differential can be one of two types: with bevel gears or with planetary gears.

3.3.1 Bevel Gear Differential

The most widely adopted differentials are the *bevel gear* type because bevel gear sets are cheaper to manufacture. Engine torque is transferred to the differential housing via the *pinion* gear to the *ring* gear (also called the *crownwheel*), which is attached to the differential housing. As detailed in the Figure 3.9a, two *bevel side* gears mesh with the *bevel planetary pinions* which are fixed to the housing by the *carrier*. Typically, the two *side* gears have the same characteristics in order to transmit the same level of torque, as they are connected to the wheel shafts. The *bevel planetary pinions* can rotate freely. Three situations can be studied to understand the overall approach to the bevel gear rotation in relation to the driving situations [15, 28, 2]:



(a) The torque is achieving via the *ring gear* on the housing [27].



(b) The torque is achieving via the *sun gear* [22].

Figure 3.9: They are the two configurations of an open differential. The *bevel gears* (on the left) is the most widespread open differential because of its low manufacturing cost.

- In straight line: the *side gears* are driven by the differential housing thanks to the movement of the *planetary pinions* around them. In this case, the *planetary pinions* do not spin on their own axes. The planetary pinions can be seen as a key: they do not rotate relatively to the *side gears*.
- In cornering: the *planetary pinions* compensate for the difference in speed between the left and right wheels by rotating on their own axes.
- In case of losing wheel/ground contact: the 'crazy' wheel (the one lifted off the ground) rotates at twice the speed of the differential housing (in respect of the *Willy's formula*). The *planetary pinions* rotate around the *side gear* attached to the wheel that is always touching the ground. According to the open differential principle, the torque on the 'crazy' wheel is null as is the torque on the other wheel.

3.3.2 Planetary Gear Differential

In the industry, planetary gear differentials are sometimes chosen due to their interesting torque distribution characteristic. There is a central *sun gear* and a *ring gear* with internal teeth, concentric to the *sun gear*. *Planetary gears* are fixed to the *planet gear carrier*. This configuration allows the *sun* and the *ring gear* to be connected (Figure 3.9b). This kind of *epicycloidal* configuration can be useful in the case of a central differential. The size of the gears can be adjusted to create an unequal torque distribution.

Central differential is employed in the case of a 4WD transmission. The engine sends torque to both the front and the rear axles, and the central differential helps to distribute this torque. The front axle differential is connected to the *internal ring gear*, while the rear axle is connected to the *sun gear*. The input torque comes from the *planetary gear carrier*. As the meshing of gears can have different ratios, the torque distribution can be tuned between the front and rear axle. Most of the equipped cars with this kind of differential are sports cars or luxury 4x4, such as the *Lamborghini Gallardo*, *Porsche Cayenne* (first version before 2007) or *BMW X5 (E53)*.

One of the main issues with open differentials is the torque distribution between wheels. The mathematical approach and the previous presentation of the open differential mechanism demonstrate the limits of this variant of differential in some situations. For example, when the coefficient of friction between the wheel and the ground is low (on water, ice, or mud) or during high-speed cornering (when the inner wheel lifts off the ground), the torque applied to the road is significantly lower than the vehicle's grip capability. To solve these issues, one possible solution is to lock the differential.

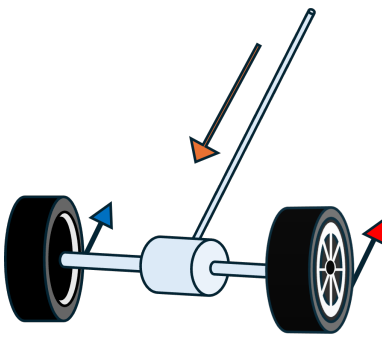
3.3.3 Locked Differential

When the differential is locked, the output shafts can be seen as one single axle (*spool* axle). Thus, employing the notations in the mathematical approach, we impose $\Delta\omega = 0$.

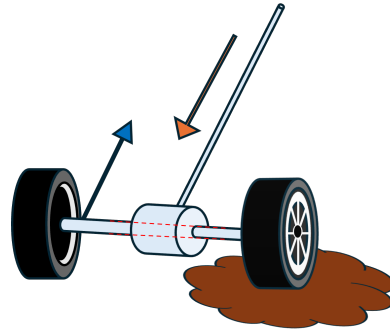
$$\Delta\omega = 0 \quad \Rightarrow \quad \omega_l = \omega_r = \omega_d \quad (3.22)$$

$$T_d = T_l + T_r \quad (3.23)$$

The effect of the locked differential does not directly involve the transfer torque ΔT . Considering a simple example, when a vehicle has a wheel in the mud. As the friction coefficient between the tyre and the road is low, the wheel starts to spin when the driver pushes the torque pedal. With an open differential, the maximum transmittable torque depends only on the available torque on the fastest wheel (i.e. the 'muddy' wheel). If this torque is zero, the torque on the second wheel will also be zero. However, as soon as the wheels are forced to rotate at the differential housing velocity ω_d , the entire engine torque is sent to the slowest wheel. This allows the vehicle to get out of this tricky situation, as described in Figure 3.10.



(a) The open differential equally splits the torque regardless the road conditions.



(b) When the differential is locked, all the torque goes to the slower wheel.

Figure 3.10: Differential locking effect. The locking of the wheels allows to get out of tricky situations.

The wheels are locked together even when the vehicle is turning. Therefore, wheel's *longitudinal slip* κ appears. The development of the longitudinal slip ratio difference is outlined in Appendix A.3. As the longitudinal slip between the two wheels varies, a yaw moment is produced by the difference in longitudinal forces F_{long} acting on the wheels:

$$M_z = (F_{x,r} - F_{x,l})t \quad (3.24)$$

In the context of the right-turning (Figure A.1), the relation shown in 3.24 indicates that a *spool* axle results in an understeered vehicle behaviour. This effect is directly proportional to the cornering radius. Therefore, at small radii (such as parking manoeuvres), the *spool* tends to restrict the driver's steering action. This creates more friction between the wheels and the road, resulting in excessive tyre wear. Furthermore, when the driver negotiates a "normal" turn on the road, the lateral acceleration increases. Consequently, the difference in the vertical forces applied to the right and left wheels of the vehicle becomes significant. The longitudinal force on the outer wheel (on the left) increases while that on the inner wheel decreases. The yaw moment remains understeered, but the value is reduced. In fact, in high-velocity cornering, this phenomenon causes a change in the sign of the yaw moment since the outer wheel becomes more loaded than the inner wheel. The vehicle increases its oversteered tendency.

Consequently, it is clear that the effectiveness of the spool is only relevant within a certain speed range and under specific conditions. The handling behaviour of a vehicle equipped with a locked differential is mainly influenced by the lateral acceleration a_y and the injected engine torque T_m . Locking the differential can be useful in some specific situations, such as off-road conditions. For this reason, mechanical devices that lock or unlock the differential have largely been developed for military 4x4 off-road vehicles such as the *Willys MB* (WWII's *Jeep*) and the *Mitsubishi Pajero*.

Manual differential locking

One of the main locking systems operates like the selection of gears in a vehicle gearbox, as presented in Figure 3.11: a *command fork* drives a *sweeper* (dog clutch actuator) [2]. This links one shaft to the differential housing. The sweeper is equipped with inner teeth that lock the torsion between the shaft and the housing. This prevents the *planetary* gears of a conventional differential from rotating on the side gears. They then act as keys, locking the relative movement between the left and right wheels.

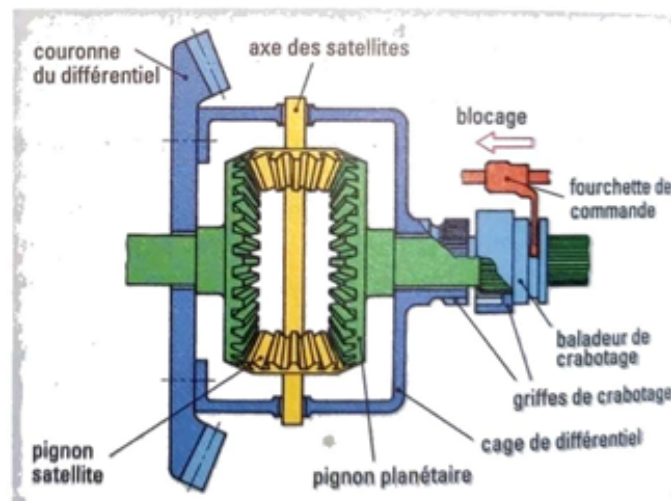


Figure 3.11: The locking effect is produced by the dog clutch actuator. It locks the relative rotation between the two side gears (called *pignons planétaires* in French) [2].

In addition to causing significant mechanical stresses, locking the differential under standard

driving conditions does not address the initial issue of the difference in angular velocity of the wheels while the vehicle is turning. The solution of combining the advantages of open and locked differentials lies in the mechanism between the two: the limited-slip differential.

3.4 Limited-slip Differential

The limited-slip differential, also known as a *self-locking differential*, can behave like either an open differential or a locked differential, taking advantage of the aforementioned benefits. Due to the internal friction, it can create and transfer a certain amount of torque ΔT to the slowest wheel (the one with the best traction). This ΔT , as explained in the mathematical approach, mainly depends on the torque applied to the differential housing and its type. There are two categories [17, 34]:

- Speed sensing: the locking of the differential is triggered once it detects a **speed difference** between left and right wheel. Hence, the torque balance is a function of $\Delta\omega$:

$$\Delta T = f(\Delta\omega) \quad (3.25)$$

- Torque sensing: the locking effect is proportional to the **torque difference** between both wheels. The torque balance depends on the input torque and the type of friction in the differential:

$$\Delta T = f(T_d) \quad (3.26)$$

3.4.1 Speed Sensing

Visco-coupling

The most common system of speed sensing differential employs a viscous coupling system. Inside the differential casing, a viscous fluid begins to transfer torque due to a temperature increase within the box. This temperature change is caused by the speed difference between the left and the right wheels. As presented in Figure 3.12, the box comprises perforated plates mounted on the wheel shaft (internal discs) and on the differential carter (external discs). Silicone fluids are often preferred because of their mechanical properties.

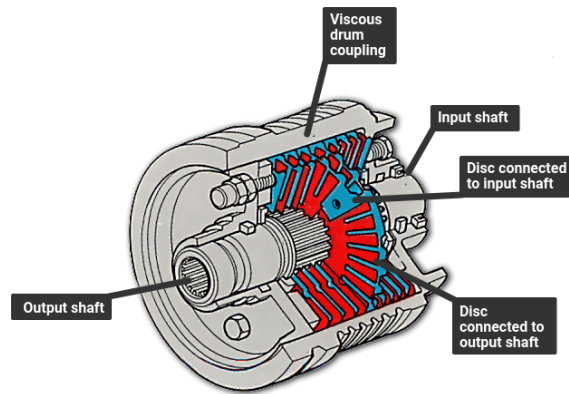


Figure 3.12: The input and output shaft could be differential gear and one wheel, respectively. [42]

In the context of a *RWD/FWD* car, the visco-coupling box completes an open differential on one side (left or right wheel). Speed differences induce shearing between the internal and external discs. This increases the temperature and pressure in the box. The fluid viscosity decreases. This pressure change generates a force on the disks that pushes the internal and external discs to be closer. Once the pressure becomes sufficient to create contact between the discs, Coulomb's friction law between two plates applies (similar to a clutch). The torque difference is amplified by the thermal expansion of the fluid. This principle is called the **hump phenomenon** [2, 28].

Even though the main advantages are the compact size and the easy fabrication, some of the previously mentioned aspects could affect vehicle handling and driving convenience:

- **Ambient temperature:** the torque distribution depends on the shearing of the viscous fluid. Since fluid shear is induced by temperature change, the properties of the torque balance depend on the ambient temperature. Therefore, the velocity differentiation could vary during vehicle use.
- **Slow response:** the main problem with a speed-sensitive differential is the time response of the torque transfer when slip occurs. Undesirable wheel slip occurs well before the detection of a speed difference between the wheels. Therefore, the ΔT could be applied too late to prevent the car losing traction [17].
- **Amount of torque transfer:** the amount of torque transfer is shown in Figure 3.13. At the start of speed differentiation (when $\Delta\omega > 0$), a large amount of ΔT is already transferred to the slowest wheel ω_s . On low-friction surfaces, this characteristic can be beneficial as it requires minimal driver intervention. However, under 'normal' road conditions at low speeds (e.g., parking), manoeuvrability is restricted, which makes the driver's actions more difficult. Moreover, since the curve tends to stabilise as $\Delta\omega$ increases, the value of viscous-coupling torque distribution becomes limited at high $\Delta\omega$. It means that the vehicle's handling is not significantly enhanced [14].

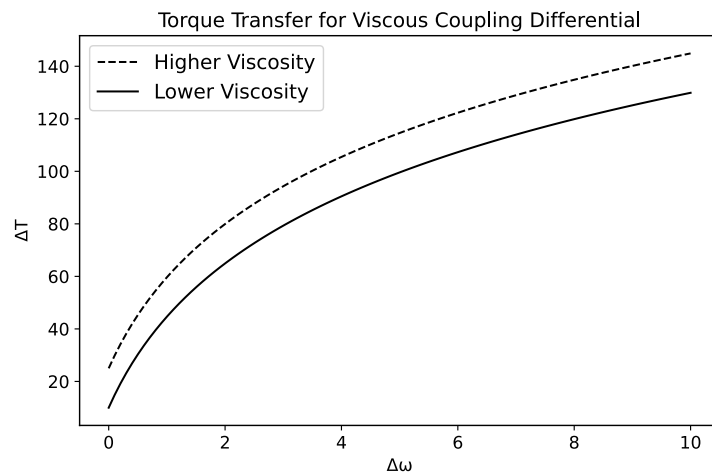


Figure 3.13: The torque transfer tends to remain constant as $\Delta\omega$ increases. The values are arbitrary. Inspired from [24].

Piston - Haldex example

A piston can be used to push a clutch system comprising multiple discs between the *side* gear and the differential housing. The clutch is composed of friction plates with internal teeth (linked to the *side* gear shaft) and external teeth (that mesh with the differential housing). Only axial movement between the housing and the clutch discs is permitted, and this displacement is achieved via an actuator mechanism.

The *Haldex* clutch differential is used for central differential applications (in *4WD* cars). The piston squeezes the friction discs in order to distribute torque between the front and rear axles (Figure 3.14). When the wheels are rotating at the same speed, the piston pump produces no flow. As soon as slip begins, an annular piston pump reacts to the velocity difference and injects flow proportional to this difference. The piston pushes the friction discs together to balance the torque, coupling the front and the rear axles. The speed difference is reduced and the torque is distributed according to the available grip on each shaft. This type of torque distributor can be found in the *Audi-TT 1998* and the *Volvo XC-90 2002* [28, 2, 14].

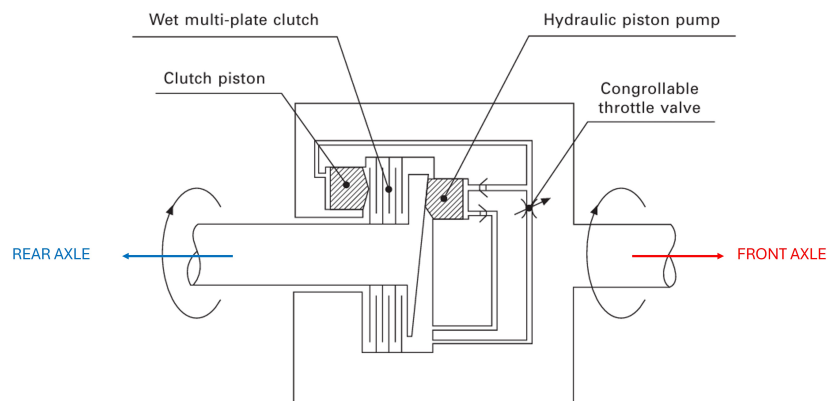


Figure 3.14: The *Haldex* system prevents speed differentiation between the axles by locking them together using a piston system [28].

This example of torque distribution mechanism can be installed in an open differential for a *FWD* or *RWD* car. This is known as an **automatic locking differential** and the action of the piston is triggered by electronic devices. When the sensors detect a speed difference between the wheels, the ECU (*Electronic Control Unit*) of the differential unit turns on the hydraulic piston. This action occurs when there is a difference of 2 km/h between the driven wheels or between the front and rear axles.

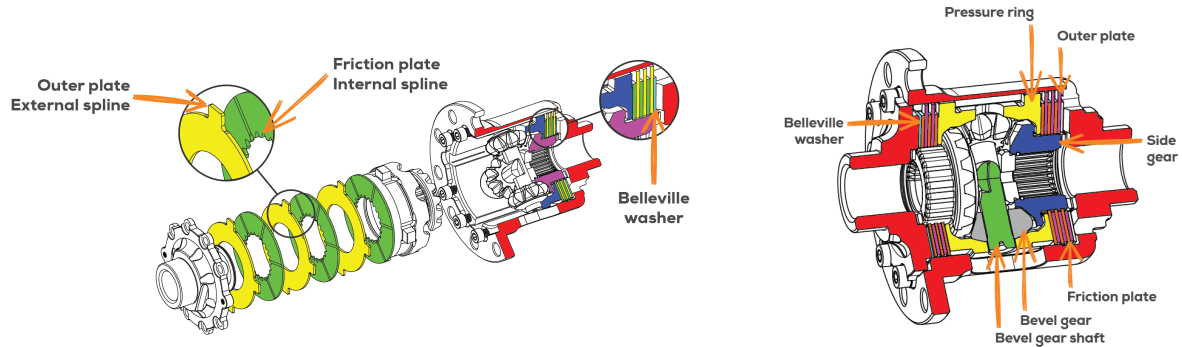
3.4.2 Torque Sensing

The second trigger for torque amplification is the *torque difference sensing*. In this case, the locking effect begins as soon as there is a difference in torque between both wheels.

Clamping ring

As in the previous example, the clamping ring differential employs a clutch system (Figure 3.15a). The clutches are positioned on either side of the open differential elements.

The friction discs with internal teeth are connected to the output shaft, while the friction discs with external teeth are linked to the differential housing. As detailed in Figure 3.15b, the *planetary* gears (also known as *bevel* gears) are positioned between each clamping ring (pressure ring). Each ring meshes with the housing. This allows axial displacement between the clamping rings and the differential housing [37, 5, 14, 2].



(a) Friction discs are composed of plates with internal and external teeth and Belleville springs [37].

(b) Bevel gear pushes on the clamping ring (pressure ring) [37].

Figure 3.15: Multi-clutch differential with clamping ring. Belleville springs can be tuned for the pre-loading.

Both clamping rings are designed with triangular notches that lock the *planetary* gears in place. Following the movement of the gears in an open differential (see 3.3.1), when the *planetary* gear begins to rotate, the *bevel* gears also move inside the notches. As they are V-shaped (Figure 3.16), the up/down displacement of the gears creates an axial force that pushes the clamping ring towards the clutch discs (i.e. the friction discs). For example, imagine that the right wheel is slipping. In this case, as there is a difference in speed and torque between the two wheels, the *planetary* gears turn. Their axes push the clamping ring on the friction discs. Depending on the input torque and the *load*, the axial force creates a friction torque in the right clutch, as the internal friction discs rotate faster than the external discs. This torque is transferred to the left wheel via the differential housing, the left *side* gear, and the left friction discs. This torque balance is added to the engine torque that has already been transferred to the left wheel.

One of the main parameters is the *loading* in the differential. This prevents movement between the *side* gears and the housing. Depending on the friction capabilities of the clutch, more torque can be transferred to the slowest wheel (number of plates, friction coefficient of the discs, ...). Nevertheless, since the locking effect is proportional to the input torque, the load is minimal when the driver does not press the pedal. This is why the clamping-ring differential is often equipped with Belleville springs, which provide a preload. This creates a certain amount of friction torque, even when the input torque is very low.

The final level of loading (after the Belleville springs and friction discs) is the pressure angle of the *bevel* gear shaft. The contact force grows as the contact angle decreases (Figure 3.16) [37]. This force enables the clamping ring to compress the friction discs. The

value of the contact angle often changes between accelerating and braking.

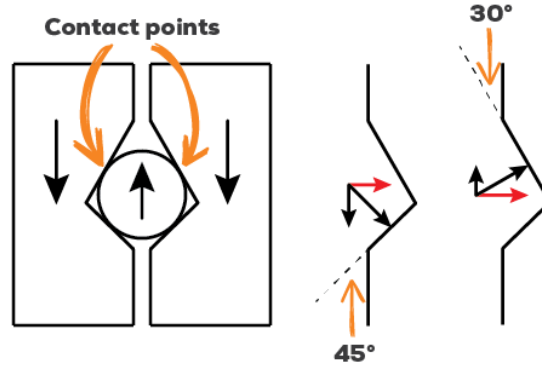
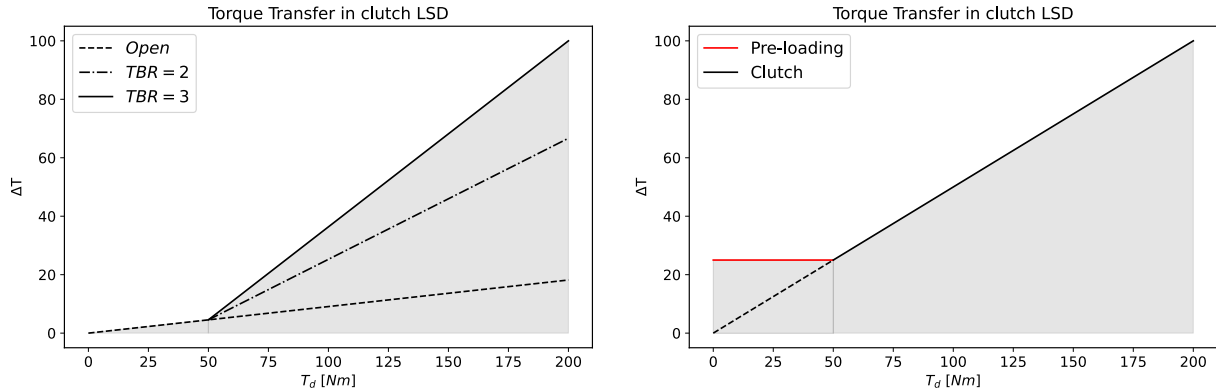


Figure 3.16: The axial force acting on the clamping ring varies according to the contact angle of the V-shaped notches.

Without pre-loading, a huge amount of input torque is required to trigger the torque balance on the slowest wheel (Figure 3.17a). Nevertheless, pre-loading can also lead to some issues, such as with the *viscous-coupling* application. As we can observe in Figure 3.17b, the slowest wheel will always receive more torque from the differential. In the even of positive input torque, the clutch-differential encounters the handling difficulty problem described for the *viscous-coupling* system at low speeds. Furthermore, in coasting and braking modes, when a small amount of torque is injected into the differential, the difficulty of speed differentiation between the wheels remains the same. This can create a torque 'wind-up' between the drive axles and interfere with the **ABS** (Anti-Block System) [17, 20].

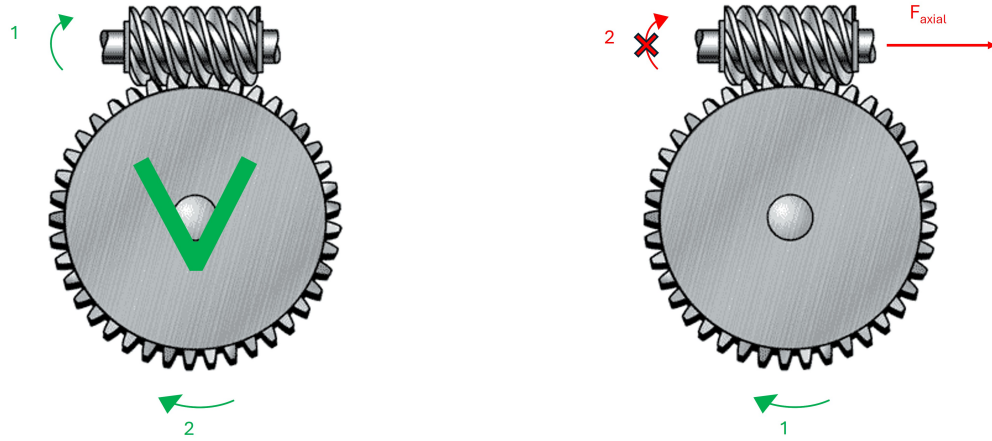


(a) In an open differential (without preload), (b) With a pre-loading in a clutch differential, there is a small value of ΔT created by the friction of gears. there is a ΔT even without speed differentiation.

Figure 3.17: The clutch differential with pre-loading induces a higher ΔT at the beginning of the wheel's speed difference. The ΔT in the clutch differential without preloading at low T_d is very small.

3.5 Torsen Differential

The *Torsen* differential, for *Torque-Sensing*, is a geared LSD. It was created in 1956 by *Vernon Gleasman*. It was then developed and produced by *Gleason Works* in Belgium. The slip limitation of a *Torsen* differential relies on the concept of *invex gearing* [33, 8].



- (a) The worm gear (1) can drive the worm wheel (2). (b) The worm wheel (1) cannot drive the worm gear (2). It creates axial force and movement like a rack and pignon assembly.

Figure 3.18: The basic principle of *Torsen* differential. It uses the *invex* gearing principle. The gear drawings come from [1].

The *Torsen* differential bases its torque transfer on the relation between the worm gear and the worm wheel. If the worm gear (1) is driven first, it forces the worm wheel (2) to rotate (Figure 3.18a). In the second configuration (Figure 3.18b), if the worm wheel tries to drive the worm gear, the latter can not rotate due to the helical meshing between the two gears. In a *Torsen* differential, it is this configuration of gears that forms the core of the mechanical components, rather than the bevel pinions found in an open differential [17].

3.5.1 Composition

Torsen differential can be either central or axle-mounted. Input torque is injected through the differential *ring* gear (*crown wheel*), which is welded to the housing. Each side of the differential contains a worm gear (called *side gear*) that meshes with pairs of worm wheels (called *element* gears) and *spur* gears. The number of gear pairs around the *side* gear depends on the overall torque capacity and the available space. The *side* and *element* gears are designed with helical teeth to provide instantaneous elliptical contact. This reduces mechanical stresses and enhances contact time between the teeth [17, 8].

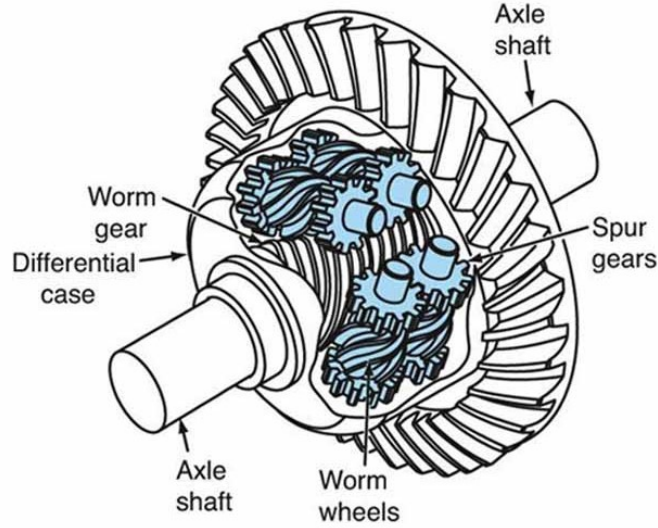


Figure 3.19: The *Torsen* differential is composed by worm wheels and worm gears, as described before. Moreover, there are thrust washers at each *side* of worm gears to enhance friction between differential housing and worm gears [23].

The Figure 3.19 shows the oldest and most basic version of the *Torsen* differential (type A). The housing is machined to hold the *element* gears and their shafts within cavities. Three *element* gears (worm wheels) and six *spur* gears are arranged around the *side* gears (worm gears) at 120° intervals. Each *spur* gear, mounted on either *side* of an *element* gear, meshes with the *spur* gear of the adjacent *element* gear (mounted around the second *side* gear). This allows for an equal but opposite direction of the relative rotation of the left and right shafts. The *side* gear shafts connect the differential to the left and the right wheels. In the middle of the differential, the *side* gears are not connected but are held in place by thrust washers mounted on each shaft (like friction disk clutch). Finally, washers are found between each *side* gear and the differential housing [17, 41].

The friction inside the differential is produced by two main forces. These induce four contacts with different friction coefficients. The first component of friction is caused by the axial and radial forces generated by the helical meshing of the *side* and *element* gears (Figure 3.18b). The axial force pushes the *element* gear shaft against the housing, and the radial force component presses the *element* gear onto the housing cavities/holes in which they are inserted. As there is a certain amount of friction between the two elements, friction torque is produced. This relative sliding of the *element* gears contributes to the locking effect of the *Torsen* differential. The second important force is due to the worm wheel/worm gear relation between the *side* gear and its respective *element* gears. When there is a relative speed difference between the two axles, the *element* gears try to drive the *side* gear. As the *side* gear is considered the worm gear (Figure 3.18b), an axial force acts on the thrust washers between the *side* gear and the housing, or between the *side* gears themselves. Depending on the situation (drive or coast), there are four operating modes of the *Torsen* differential, which allow the thrust washers to be tuned for each mode. This can alter the locking effect ε_d for each operating mode. These operating modes are detailed and illustrated in [8].

3.5.2 Advanced *Torsen* differentials

Torsen C-type

The *Torsen* type C is a central differential, especially exploited by the *Audi's Quattro System*. Compared to the type A, the type C is a compact epicyclic gear differential, similar to an open planetary gear differential (c.f.r 3.3.2). It distributes the engine torque asymmetrically between the front and rear axles by virtue of the different number of teeth on the *ring* gear and the *sun* gear [41, 16].

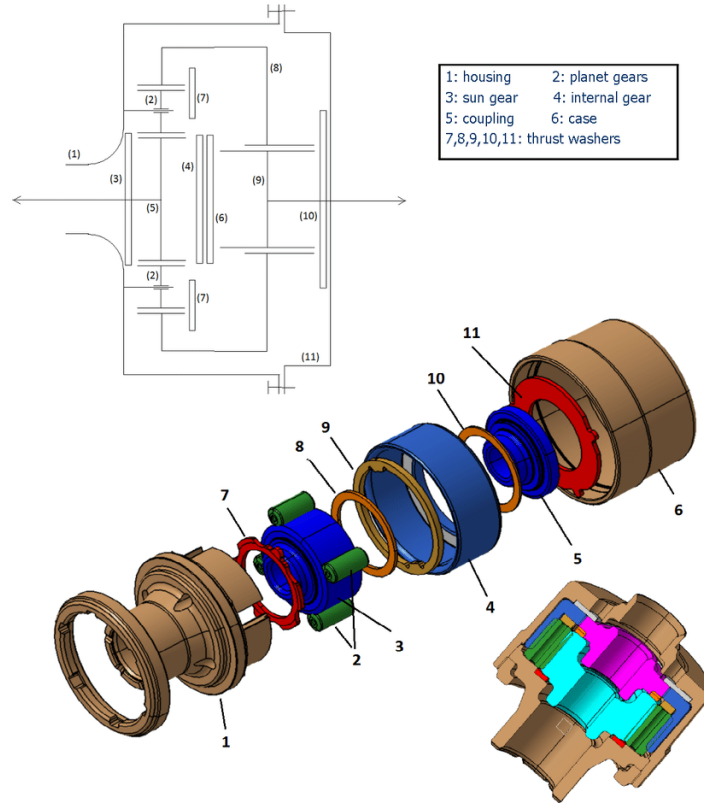


Figure 3.20: Exploded view of a type C *Torsen* differential, with a kinematic diagram [41]. It is designed by JTEKT *Torsen*.

The differential shown in Figure 3.20 tends to provide more torque to the rear axle (58 %) than to the front. As with the type A mechanism, the locking effect is the result of the axial forces generated by the helical mesh and the friction between the *planet* gears (i.e. the element *gears* in the type A) and the housing holes. The axial displacement of the *sun* gear and the *planet* gears on their respective thrust washers produces friction (which varies according to the friction coefficient of the washer). This limits the relative velocity between the axles. This friction helps to slow down the axle that speeds up. However, the *planet* gears are not mounted on a specific shaft. They can move freely radially between the *internal* gear (with internal teeth) and the *sun* gear (with external teeth). Contact between these elements generates the friction that creates the locking effect. The four transmission modes are presented in Figure 3.21 [33, 41, 16].

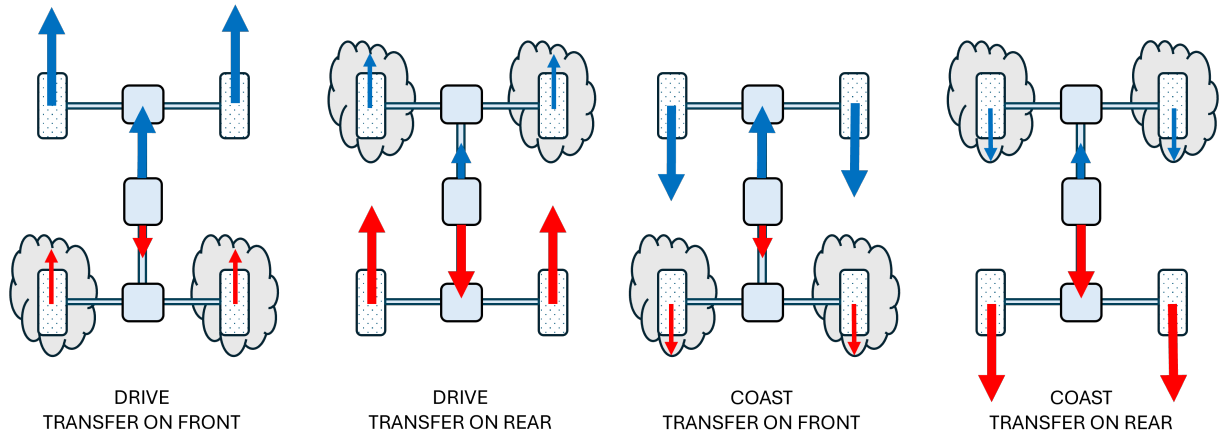


Figure 3.21: Torque transfer for the 4 modes with a *Torsen* type C. The clouds represent a low surface's friction coefficient.

Torsen B-type

The *Torsen* type B is an evolution of the type A. This differential is only used as an axle differential unlike type A [28]. The mechanism of this *Torsen* type is symmetric because it transfers the torque between left and right wheels (in normal road conditions, there are no reasons to prefer one side to the other). In contrast to the *element* gear in type A, the *element* gears of type B are parallel to the *side* gear (Figure 3.22). The *element* gears are formed of two parts: a slick reduced section and a portion with teeth meshing with one of the two *side* gears. The *element* gears interlock with each other at their two edges because they are mounted by pairs. The *side* gears are linked to the wheels.

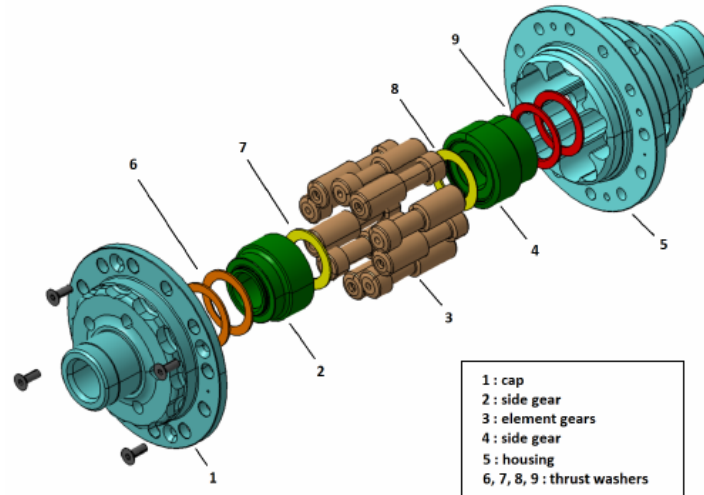


Figure 3.22: Unlike the *Torsen* differential of type A, the planet gears of the type B are parallel to the side gear [41].

As with the previous version of the *Torsen*, the locking effect is the combination of the friction between the *element* gears and the housing cavities, and the contact between the *side* gears and thrust washers [33, 41]. This internal friction reduces the relative speed

difference between each wheel. Appendix A.5 presents the various internal computations of the *Torsen* gears for each driving situation. As discussed above, the *Torsen* differential's torque management allows for a more interesting distribution of torque to the wheels. Depending on the grip conditions, more torque can be distributed to the wheel with the best grip (Figure A.5). Thanks to load transfer, the *Torsen* differential can also induce an understeered moment followed by an oversteered moment, which can be advantageous when turning at high speed (Figure A.4). This will be highlighted in the section 6.

Today, *Torsen* type B differentials are often installed in common sport cars because of their compact size and reduced weight compared to type A. Examples can be found in the *Toyota GR Yaris 2020*, the *Peugeot 308 GTI* and the *Ford Mustang 5.0*. However, the *TBR* is limited by the size of the housing and is thus smaller than that of the *Torsen* type A. This type of differential is found in trucks or military vehicles that require more torque bias [41, 31, 33].

3.5.3 Performance and Assets

The torque balance ΔT is the result of the mechanical gearing relations in the *Torsen* differential. In Figure 3.23, the ΔT arrives before any wheel spin because the *Torsen* is a geared *LSD* and has a faster timing response than viscous or pumping limited-slip differentials. The *Torsen* differential has an instantaneous response.

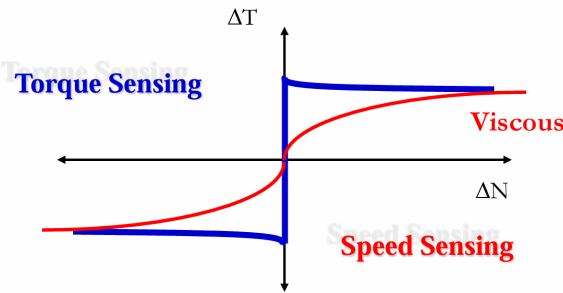


Figure 3.23: Torque bias law of a *Torsen* differential. Torque transfer occurs **before** there is a difference in wheel speed. Here, ΔN means $\Delta\omega$ [33].

As the mathematical approach demonstrates, the *TBR* strongly influences the energy loss (i.e. the efficiency) due to the internal friction. In a straight line, the *Torsen* is said *closed* or *locked*³. There is no relative speed ($\Delta\omega = 0$) between the wheels, resulting in less energy loss than with an open differential. Conversely, when the *Torsen* is opened (or *unlocked*, because $\Delta\omega \neq 0$) like in turning or grip losses, the locking effect generation induces a reduction in internal efficiency. The global efficiency therefore depends on the vehicle situation [19, 33, 16].

The two previous situations determine the *Torsen* operation. Before the *opening* of the differential, the actual *TBR* (known as $TBR_{measured}$) does not exceed the fixed *TBR* of the *Torsen* (named TBR_{fixed}). For example, if the torque coming from the engine is $T_d =$

³The reader must pay attention to the meaning of locked/unlocked as defined for the *Torsen* application, and differentiate it from the locked differential presented in section 3.3.3.

200 Nm, the available torque on the fastest wheel is $T_f = 100$ Nm and the TBR_{fixed} is 3.0 ($\varepsilon_d = 0.5$). The torque on the slowest wheel is:

$$T_d = 200Nm; \quad T_f = 100Nm \quad \rightarrow \quad T_s = 200 - 100 = 100Nm$$

$$\text{By 3.20 :} \quad TBR_{measured} = \frac{T_s}{T_f} = \frac{100}{100} = 1 < 3$$

$$T_d = T_s + T_f$$

The $TBR_{measured}$ is not exceeding the available torque bias capacity of the differential. Therefore, the differential remains *closed*. This is the first part of the curve in Figure 3.25b. Now consider $T_d = 450.0$ Nm. In this case, the torque transferred to the slowest wheel is constrained by the TBR_{fixed} , as it is not possible to generate more friction between the axles than that available in the *Torsen* differential. Consequently, the differential opens (becoming *unlocked*) and the excess torque accelerates the fastest wheel. The *Torsen*'s locking capacity is exceeded and the wheels start to rotate at different speeds. This is the second part of the curve in Figure 3.25b:

$$T_d = 450Nm; \quad T_f = 100Nm \quad \rightarrow \quad T_s = TBR_{fixed} \cdot T_f = 300Nm$$

In this case, the *Torsen* differential will apply the following torque to the wheels:

$$T_f = \frac{450 - 225}{2} = 112.5Nm \quad \& \quad T_s = \frac{450 + 225}{2} = 337.5Nm$$

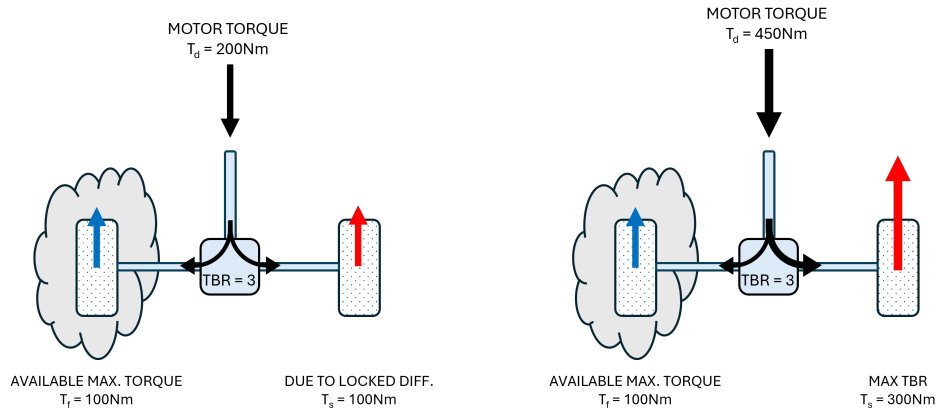
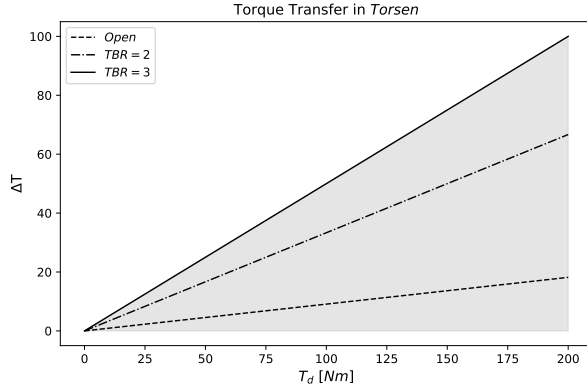
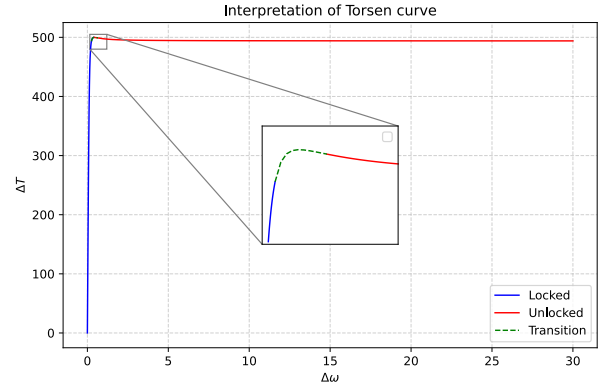


Figure 3.24: The torque balance is limited by the Torque Bias Ratio. The cloud represents a smaller road adherence. If the engine torque is too high, the locking capacity of the *Torsen* is exceeded and the wheels start to rotate at different speeds. The wheel with the lowest grip will start to spin.

Finally, unlike the viscous coupling or clamping ring application, the *Torsen* differential does not apply a torque balance when the input torque T_d is zero (Figure 3.25a). Therefore, the *Torsen* differential does not interfere with the **ABS** system's operation. Nowadays, the friction power produced by the *Torsen* can be converted into useful power in coasting mode. In electric vehicle applications, the friction power W_f (relation 6.1) can be useful for energy recuperation, as in the *Alpha Romeo Junior* or *Abarth 600e* (*JTEKT Torsen* source).



(a) The torque transfer in a *Torsen* differential is larger when TBR grows [19].



(b) The transition zone corresponds to the transition between static friction and dynamic friction, by Coulomb's law.

Figure 3.25: The torque balance of the *Torsen* changes with the input engine torque, the available torque on the wheels, and the *TBR*. A transition zone between static and dynamic friction arises as soon as a speed difference occurs.

3.6 Advantages, Limitations, and Conclusion on Differential Variants

The presentation of the main LSD types on the market highlights the advantages and drawbacks of the *Torsen* differential compared to the other differential types. Table 3.1 summarises the characteristics of each LSD introduced in this chapter. It explains why comparing a *Torsen* differential with a standard (open) differential is more interesting than comparing it with other limited-slip differentials.

Comparison Table			
LSD Point	Viscous-coupling	Clamping Ring	<i>Torsen</i> A & B
Ambient Temperature	The torque transfer could change with the ambient temperature, because of the chemical and mechanical properties of the viscous fluid.	No influence on the clamping-ring performance.	No influence on the gears in the <i>Torsen</i> differential.
Response	Slow because of the speed-sensing behaviour. The system reacts after wheel slip.	Rapid but huge amount of torque apply at the beginning of transfer due to the pre-load.	Fast response because of gearing. The response is linearly proportional to the input torque T_d and smooth at the beginning of torque transfer (Figure 3.25a). The torque transfer is triggered before wheel spin.
Torque Transfer	There is torque transfer before any $\Delta\omega$. On low friction coefficient surface, it could be interesting but not on normal road conditions. It could influence the ABS action. Moreover, the value of viscous-coupling torque distribution becomes limited at high $\Delta\omega$.	Without pre-loading, it is necessary to have a certain amount of T_d to start the torque balance. With pre-loading, there is a constant torque transfer that can be inefficient for manoeuvrability at low speed. In addition, this basic torque balance creates a torque 'wind-up' troublesome for the ABS.	The amount of torque balance depends only on T_d and TBR . When the input torque is zero, no torque is distributed to the wheels. Therefore, <i>Torsen</i> does not induce torque if the ABS has to engage.
Weight, Complexity and Wear	Smaller and simpler than the others, the maintenance is easier because there is no friction between parts. Its fluid maintenance management can, however, present challenges.	Because the friction disks continually resist to differentiation, their wear can alter the torque transfer.	Due to the gears, the <i>Torsen</i> weight has to be optimised during the design process. Internal maintenance requires the housing disassembly. It is built to last the entire lifespan of the vehicle.

Table 3.1

Chapter 4

Multibody Model

The purpose of this section is to introduce the car model used in the analysis part of this thesis. The objective of this work is to analyse the differences in vehicle handling between an open differential and a *Torsen* differential. This car model is based on an existing vehicle model previously developed in earlier works. This vehicle model has been implemented using the multibody simulation software *Robotran* [6]. This chapter presents the important changes brought to the vehicle model to precisely simulate a car with open and *Torsen* differentials. Firstly, this section begins with a general introduction to the various modifications made to the base vehicle model. Next, the implementation of the different differential variants will be justified in order to determine the most suitable mathematical model for the simulations. The chapter concludes with an explanation of how the speed and position controllers have been configured.

4.1 Upgrades to the Multibody Car Model

As already mentioned, the vehicle model established for the simulations was adapted from an existing car project on *Robotran*. *Robotran* is a multi-body software developed by *EPL* teachers and *Quandyga*. It enables the computation of the motion equations of multi-body systems for the study of their dynamics. The 3D model of the car comprises all the standard components found in a car. The four wheels are linked to the carbody by an independent suspension system. The designed vehicle features a double wishbone suspension of the *MacPherson* type, equipped with anti-roll bars at both the front and rear (Figure 4.5). The axles and wheels are independent. The steering system comprises a five-bar mechanism and a rack-and-pinion system. Each body in the model has its own mass and inertia, enabling the simulation of the full dynamic behaviour of the model under all possible driving scenarios. In this case, the following general assumptions typical of a road car can be made:

- **Rear-wheel drive vehicle:** the differential is always installed on the rear axle. The torque engine T_m is only transmitted to the rear wheels.
- **No electronic systems:** the car system has no ABS system or Electronic Stability Program (ESP). The focus is on the mechanical behaviour of the vehicle without electronic aids.
- **Non aerodynamic forces:** the aerodynamic forces are not taken into account.

Most parameters, such as the car body mass or wheel radius, are taken from the previous model. These global parameters are presented in Table 4.1.

Model Parameters		
$\backslash$	Value	Unit
Chassis Mass	1480.0	kg
Total Mass	1761.0	kg
Wheelbase	3.88	m
Track	1.5	m
Front to G distance	1.36	m
Rear to G distance	2.52	m
G Height	0.35	m
Wheel Mass	15.0	kg
Real Front Wheel Radius	0.2779	m
Real Rear Wheel Radius	0.2816	m
Open Differential Mass	5.6325	kg
<i>Torsen</i> Differential Mass	7.0827	kg

Table 4.1

4.1.1 The Steering Wheel

Firstly, the steering wheel was not modelled. The steering system was controlled by the rack's displacement value. The steering wheel enables the driver's reaction to be analysed for all the situations that will be tested. Therefore, it needs to be implemented in the 3D car model. The link between the rack-and-pinion system and the steering wheel is:

$$x_{rack} = \tau_{steering} \delta$$

The rack displacement x_{rack} pushes the right and left steering arms (Figure 4.1). The driver controls the steering wheel, which modifies the steering wheel angle δ . The ratio between these two movements is $\tau_{steering}$.

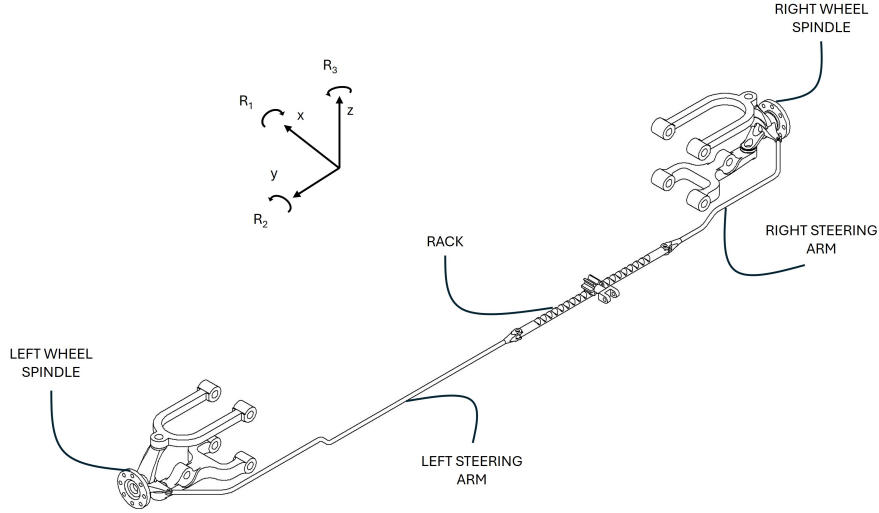


Figure 4.1: The steering wheel turns the pinion on the rack. The rack displacement determines the orientation of the front wheels.

The ratio between the steering angle and the rack must be measured experimentally. Using a *Yaris GR 2020*, we take the data on ground tyre displacement and the wheel radius to compute the steering ratio $\tau_{steering}$. As observable in Figure 4.2, the tyre displacement is measured by the angle θ . The steering ratio is obtained by turning the steering wheel to 180° as follows:

$$\tau_{steering} = \frac{\theta}{\text{Steering Angle}} = \frac{\arctan\left(\frac{p}{R_{wheel}}\right)}{\pi}$$

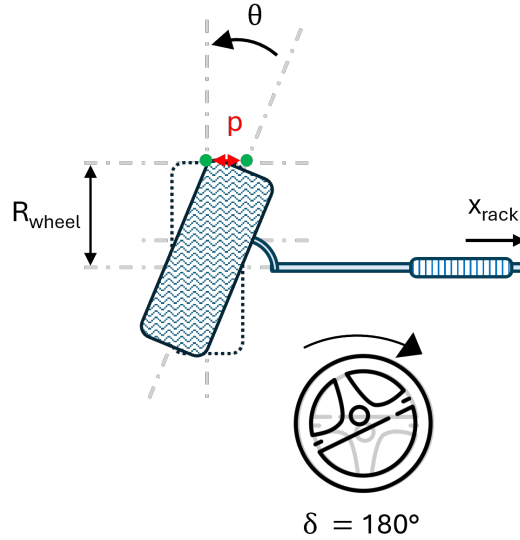
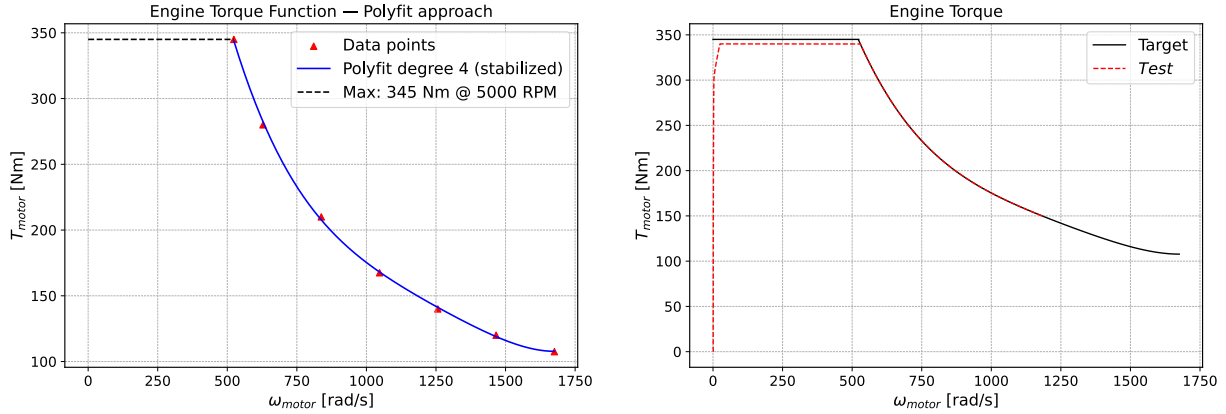


Figure 4.2: The distance p between the two points on the tyre when the wheel is turning helps to compute the ratio between the rack and the steering wheel angle δ .

The ratio calculated is $\tau_{steering} = 1.611 \cdot 10^{-4}$. It is the value used in the car model. The steering system is now controlled by the steering wheel.

4.1.2 The Engine

In the context of the collaboration with JTEKT *Torsen*, the car is equipped with an electric motor with a specific torque characteristic curve (the torque curve is in Appendix B.1). As Figure 4.3a shows, the maximal torque achieves 345 Nm, decreasing when the motor speed reaches $w_{motor} = 524$ rad/s (5000 RPM). We do not know all the torque values for each engine speed. Therefore, a polynomial fit of the torque curve is required. The torque applied by the driver must not exceed the engine's maximum performance. Therefore, the driver's input must be verified and corrected if necessary.



- (a) The polynomial fitting found a polynome of degree four to be more precise with the data points of JTEKT *Torsen*. The maximum engine torque can reach 345 Nm.
- (b) The driver accelerated with an injected torque of 340 Nm in straight line during all the test. The red line is the final torque returned by the motor. It follows the polynomial fitting.

Figure 4.3: The torque curve fitted is represented. The engine speed is verified at all times and the engine torque T_m requested by the driver is compared to the value returned by the polynomial expression. If the requested torque exceeds the limit calculated by the polynomial function, the returned torque is the value calculated by the function.

A quadrinomial stabilised fitting was applied to the data points supplied by the company in order to interpolate the torque curve (Figure 4.3a). The function is as follows:

$$T_{motor} = 3.10350.e^{-10}w_{motor}^4 - 1.5755.e^{-6}w_{motor}^3 + 3.0392.e^{-3}w_{motor}^2 - 2.7544w_{motor} + 1155.7328$$

Throughout the entire process, the torque requested by the driver is consistently verified in relation to the torque curve. Figure 4.3b shows how the torque is corrected for a vehicle travelling in a straight line and an engine torque of 340 Nm. Initially, the constant engine speed results of the difference between the torque requested by the driver and the engine torque required to respect the initial velocity of the wheels at the start of the test.

As described in the introduction of the section 3, the transmission involves a reduction box between the engine and the differential, which amplifies the motor torque on the wheels. The reduction ratio between the gears is:

$$\frac{\omega_d}{\omega_{motor}} = \frac{T_{motor}}{T_d} = \tau_{reduction} = 0.10232 \quad \text{JTEKT } Torsen \text{ data;} \quad (4.1)$$

4.1.3 The Tyres and the Suspensions Representation

The Tyres Model

As explained in the theoretical point 2.1, the *Bakker – Pacejka* tyre model is used for the four wheels of the car model. The tyre coefficients will vary in order to simulate a change in road surface for the *Split – μ* test. In this case, the coefficient D is divided by a specific factor k to adjust the friction coefficient according to the road surface:

$$D_i = \frac{D_{init}}{k} \quad \& \quad \mu_i = \frac{D_i}{F_{n,i}} \quad (4.2)$$

The characteristics of the longitudinal and lateral forces vary according to the tyre coefficients (Figure 4.4). The longitudinal and lateral saturation peaks for the case of the ice are well below those for dry asphalt. Therefore, wheel saturation will occur earlier. The tyre coefficients are the same as in the basic vehicle model (see Appendix B.2). Figure 4.4 shows the longitudinal and the lateral forces developed by the rear left wheel of the vehicle during a straight-line test with dry and icy asphalt. The driver applies a constant engine torque of $T_m = 100$ Nm.

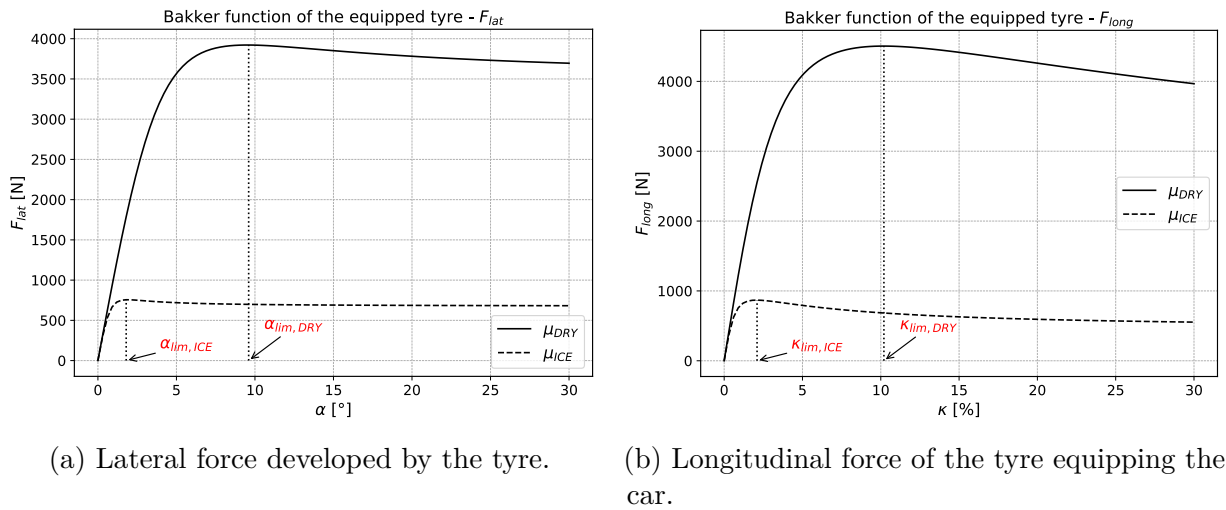


Figure 4.4: The ice curve reaches its saturation peak well before the dry asphalt does.

The suspensions

The car's suspensions are modelled as a spring and damper systems (Figure 4.5a). The characteristics are the same for the four wheels. In addition to the existing suspension system and anti-roll bars, a limitation on the maximum displacement of the front and rear suspensions was required. Indeed, due to the high torque loads exerted on the chassis by the electric engine, the vehicle pitch can become significant. To prevent excessive pitch and maintain all four tyres in contact with the ground, mechanical suspension bump stops were implemented. This prevents the front end of the vehicle from lifting. We designed the *blocks* as a spring-damper system: they apply a force in the opposite direction to the car lifting up when the angle of the upper wishbone α_{susp} exceeds the limit α_{lim} . The

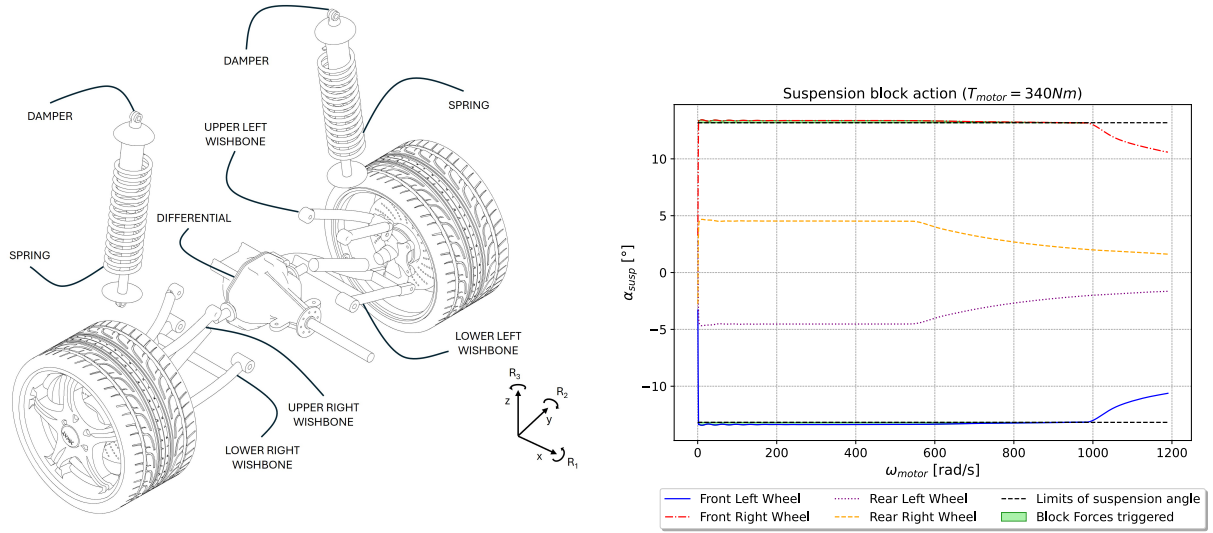
forces of the suspension and the blocks are calculated as follows:

$$\begin{aligned} F_{susp} &= -K(x_{susp,i} - L_0) - D(\dot{x}_{susp,i}) \\ F_{block} &= -K(\alpha_{susp,i} - \alpha_{lim}) - D(\omega_{susp,i}) \\ F_{antiroll\ bar, i} &= -C x_{antiroll\ bar, i} \end{aligned} \quad (4.3)$$

With x_i as displacement of the suspension and $\dot{x}_i$ its velocity. The angle α_{susp} represents the movement of the upper wishbone and ω_{susp} its velocity. As observable in Figure 4.5b, the green surfaces show when the block forces F_{block} start to act on the upper wishbones. The technical features of the suspensions are presented in Table 4.2:

Spring/Damper coefficients					
\	$K \left[\frac{N}{m} \right]$ or $\left[\frac{N}{^\circ} \right]$	$D \left[\frac{N}{ms} \right]$ or $\left[\frac{N}{^\circ s} \right]$	$L_0 [m]$	$C \left[\frac{N}{m} \right]$	$\alpha_{lim} [^\circ]$
Suspensions	33000.0	9160.0	0.66	-	-
Blocks	33000.0	9160.0	-	-	13.178
Anti-roll Bar	-	-	-	4000	-

Table 4.2



- (a) Each wheel is suspended by a set of double wishbones (upper and lower), combined with a spring and damper.
- (b) α_{lim} is fixed to 13.178° (0.23 rad). The green surface shows when the block starts to push on the wishbones.

Figure 4.5: The angle α_{susp} represents the angle of the upper left and right wishbones. We set the limit according to the maximum engine torque. The test is performed with a constant engine torque of 340 Nm in straight line.

4.2 Differentials Implementation

Before integrating a differential into the model, the user applied torque directly to the wheels to exert longitudinal acceleration a_x on the vehicle. As Figure 4.6 presents, the

engine now drives the differential crown wheel via the reductor. Because this study focuses on the vehicle handling with a differential rather than gear interactions, the differential is represented as a single entity with a mass and an inertia. The engine is located at the carbody's centre of gravity. The differential is installed on the rear axle.

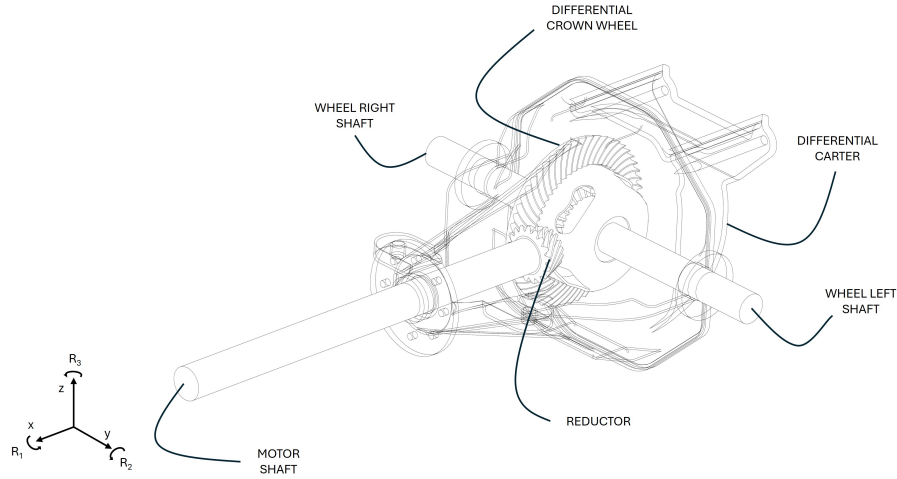


Figure 4.6: The engine drives the crown wheel of the differential. The differential is modelled as an entire body with mass and inertia.

4.2.1 The Open Differential

By mechanical connection, the velocity relation 3.1 remains true for all types of differential. Therefore, as we will analyse in the *Split- μ* test, when a wheel encounters an ice patch, the rotation of both wheels may differ, but this is limited by the differential velocity. Figure 4.8 illustrates the velocity constraint and the torque distribution of a **perfect** open differential (with a *TBR* of 1.0) in a right turn test. In this test, the driver steers at an angle of $\delta = 45^\circ$ and applies an engine torque of $T_m = 100$ Nm (Figures 4.7 and 4.9). The test confirms the respect of the torque distribution detailed in the expression 3.21 (Figure 4.8b) and the difference in rotational wheel speed when the vehicle starts to corner (Figure 4.8a).

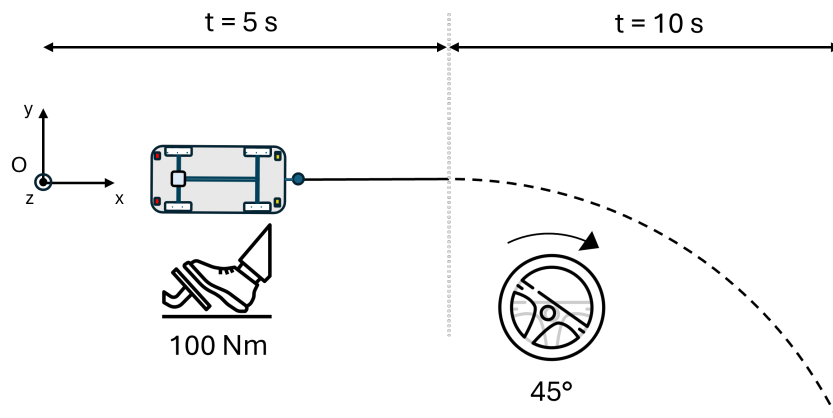
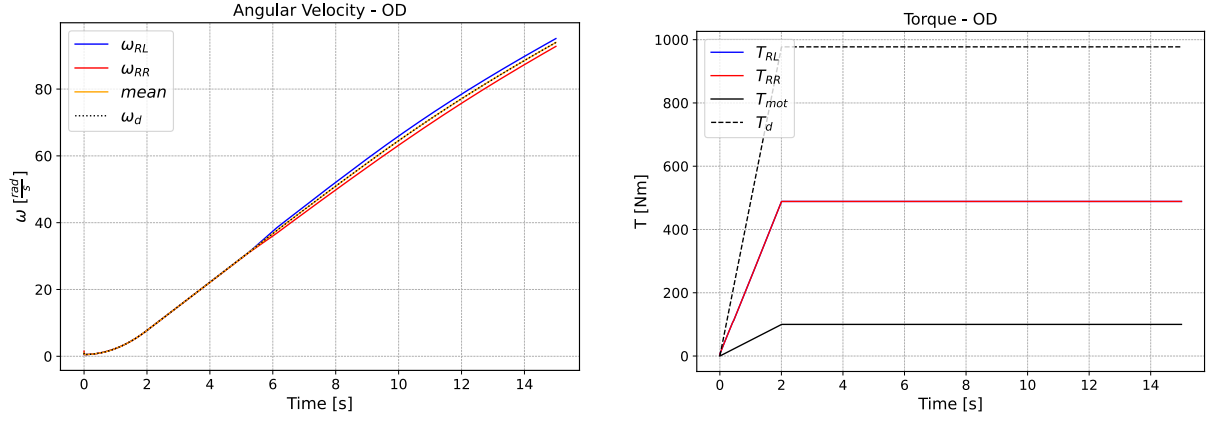


Figure 4.7: In this test, the driver applies an engine torque of $T_m = 100$ Nm. After 5 seconds, the driver steers at an angle of $\delta = 45^\circ$.



(a) When the vehicle starts to turn on the right, the left wheel (outer one) begins to rotate faster than the right wheel (inner one). However, the mean of the two speeds follows the differential rotational speed.

(b) The torque on the left and the right wheels are the same throughout the test, even when the driver turns the steering wheel. The sum of the wheel torques is equal to the differential torque T_d , as shown in the relation 3.21.

Figure 4.8: The driver chooses a steering angle of 45° at the 5th second of the test. The engine torque reaches the target value of 100 Nm after two seconds.

Figure 4.8a confirms that the velocity constraint is respected. The *mean* curve (the orange one) represents the equation:

$$\omega_d = \frac{\omega_{RR} + \omega_{RL}}{2}$$

Thanks to the velocity constraint, the torque distribution of the open differential is also validated. The torque on both rear wheels is equal during cornering (Figure 4.8b). Additionally, the reader will observe that equation 3.21 also applies for the first 15 seconds of the test.

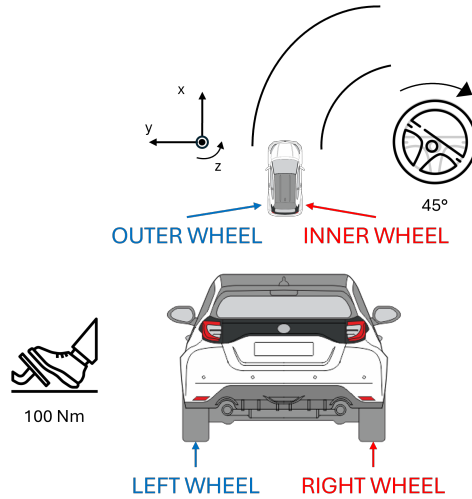


Figure 4.9: The right wheel is on the inside of the corner (inner wheel), while the left wheel is on the outside (outer wheel).

4.2.2 The Locked Differential Condition

The velocity relation no longer holds for the locking of the differential because there is no differential. In this case, the wheel velocities are the same:

$$\omega_{RR} = \omega_{RL} \quad (4.4)$$

The result of the differential locking will be demonstrated in the next section of this Master's thesis.

4.2.3 Torque Balance Distribution

As detailed in previous chapters, the friction inside the differential produces a torque balance between the left and the right wheels. Since there is also friction in the open differential, the torque balance law operates for both the open and the *Torsen* differential, with a variation in the *TBR* (locking effect).

To sculpt the shape of the distribution (Figure 3.25b), multiple mathematical shapes can be tested. Nevertheless, because *Robotran* uses numerical integration to compute the motion equations, attention must be paid to the stiffness of the initial part of the torque law (when the *Torsen* differential opens). If the slope of the selected function is too extreme, the numerical integration will be too challenging, resulting in calculation errors.

Linear Decoupling

The first possibility to model the torque balance is to decompose the *Torsen* law into two linear parts (Figure 4.10).

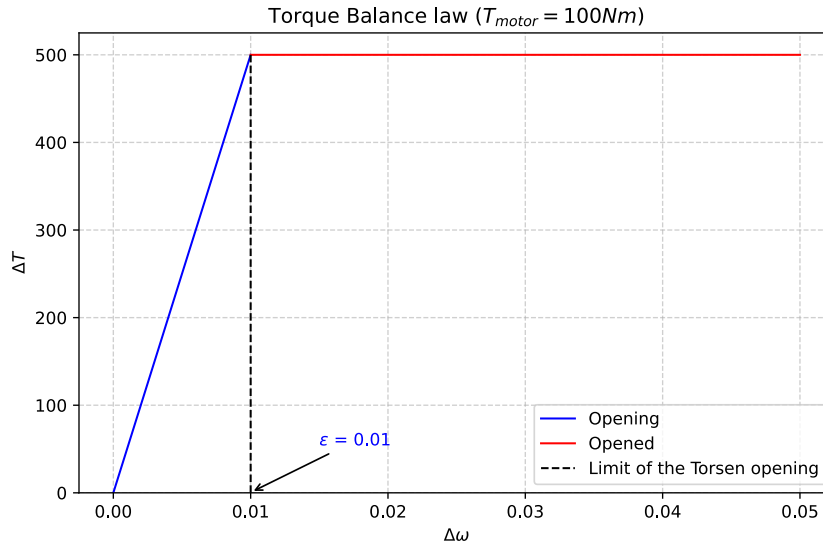


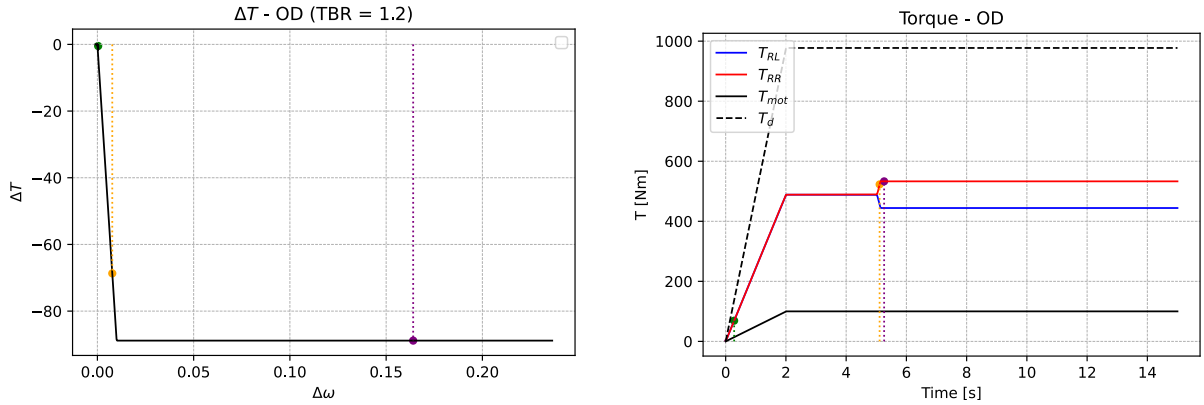
Figure 4.10: This torque distribution is valid for an engine torque T_m of 100 Nm and a locking effect of 50%. The opening threshold is fixed at $\epsilon = 0.01$: the speed differentiation starts when there is a $\Delta\omega$ of 0.01 rad/s (0.0955 rpm) between the rear wheels. The negative part of the graph is symmetrical.

In this case, the slope of the blue line is determined by the opening limit ϵ . This slope controls the stiffness of the numerical integration. If the slope (i.e. the opening limit) is too steep (i.e. too small), the integration of the problem will become stiff, which can result in calculation errors. The mathematical decomposition is as follows:

$$\Delta T(\Delta\omega) = \begin{cases} T_d \frac{\epsilon_d}{\epsilon} \Delta\omega & \text{if } \{\Delta\omega\} \leq \epsilon \\ T_d(\epsilon_d) & \text{else} \end{cases} \quad \text{with} \quad \epsilon_d = \frac{TBR - 1}{TBR + 1} \quad (4.5)$$

As it is explained in the section 3, the friction of the bevel gears in an open differential (OD) induces a TBR of 1.2. Therefore, the following test sets a TBR of 1.2 with the vehicle in the same configuration as in the previous tested ($\delta = 45^\circ$ and $T_m = 100$ Nm recalled in Figure 4.9). To verify the torque distribution of the linear decoupling (Figure 4.11), the torque difference is defined as follows:

$$\Delta T_{measured} = T_{rl} - T_{rr} \quad \& \quad \Delta\omega = \omega_{rl} - \omega_{rr}$$



(a) The torque difference between both wheels respect the linear decoupling. The negative value arises from the definition of $\Delta T_{measured}$. Only the initial section of the turn is represented in the plot.

(b) When the vehicle starts to corner, the torque balance is positively applied to the inner wheel (the right-hand wheel because of the right turn). The coloured points follow the opening of the differential.

Figure 4.11: The coloured points help to track the differential's mechanical behaviour during cornering. The green point appears on the straight line when the differential is closed. The yellow point indicates the opening process, while the purple point highlights the moment when the differential is fully open.

With an engine torque of 100 Nm and the fixed TBR , the torque balance corresponds to:

$$T_m = 100Nm \quad \rightarrow \quad T_d = 977,33Nm$$

$$\text{With } TBR = 1.2: \quad \Delta T = 977.33 \left(\frac{1.2 - 1}{1.2 + 1} \right) = 88.85Nm$$

Figure 4.11a shows that the shape of the torque distribution correctly reflects the decoupling in Figure 4.10 . The maximum value of ΔT is the computed result.

Hyperbolic Tangent Function

To perform a hyperbolic tangent function, the numerical method advises verifying the *rank* of the function:

$$rank = \frac{2}{v^*} \quad \text{For reaching 96\% of the maximal value of } \tanh(x) \text{ at } x = v^* \quad (4.6)$$

with v^* being the limit of differential opening (as ϵ in the linear decoupling model).

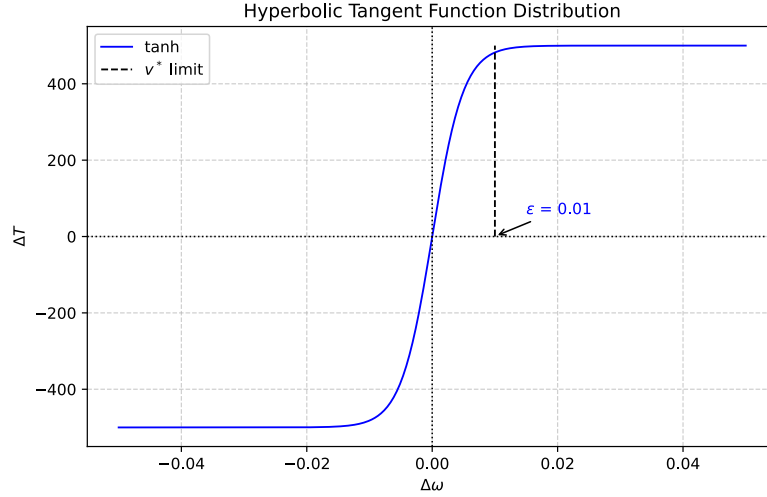
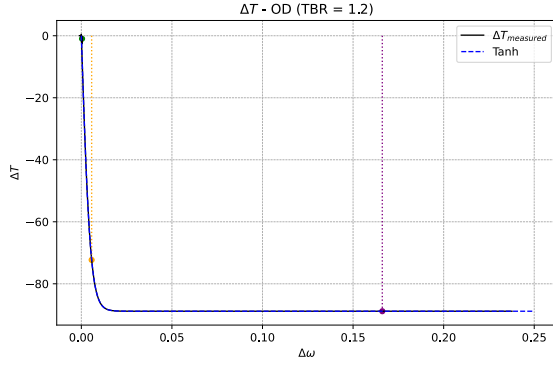
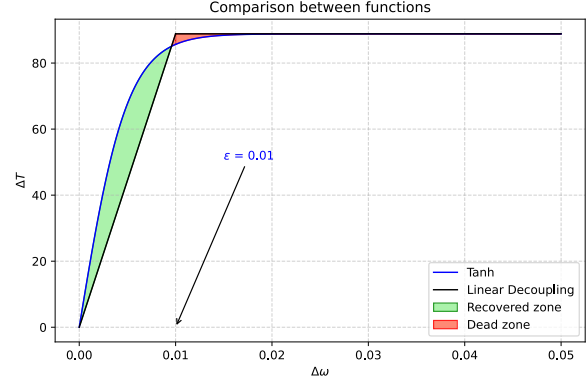


Figure 4.12: The torque distribution is valid for an engine torque of 100 Nm and a locking effect of 50%. The opening threshold is fixed at $\epsilon = v^* = 0.01$.

The hyperbolic tangent function provides a suitable approximation of the transition characteristic of the *Torsen* torque distribution law (Figure 4.12). One advantage lies in its ability to describe the full distribution curve continuously, without requiring a piecewise definition. The torque distribution performs successfully, as Figure 4.13a shows (the result for the hyperbolic tangent function is available in the annex B.3). Compared to the linear decoupling, the hyperbolic tangent function results in a more abrupt differential response. Ideally, the torque bias would occur instantaneously (even before any significant wheel speed difference arises). However, the hyperbolic tangent introduces a smooth transition region that delays the torque bias from reaching its maximum value immediately. This region is called the *dead zone* in Figure 4.13b.



(a) The $\Delta T_{measured}$ accurately replicates the implemented curve. The rear wheel torque distribution behaves similarly to that of the linear decoupling model (see Appendix B.3).



(b) The recovered zone represented by the green area, is more beneficial than the lost area, represented by 'dead zone'. The value of ϵ remains the same for both functions.

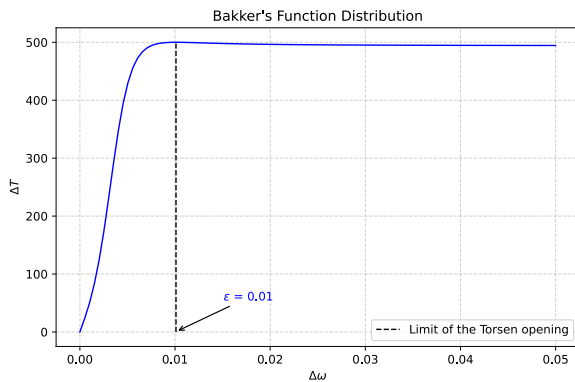
Figure 4.13: The hyperbolic tangent seems to offer a better definition of the *Torsen* torque distribution law compared to the linear decoupling.

Bakker's Function

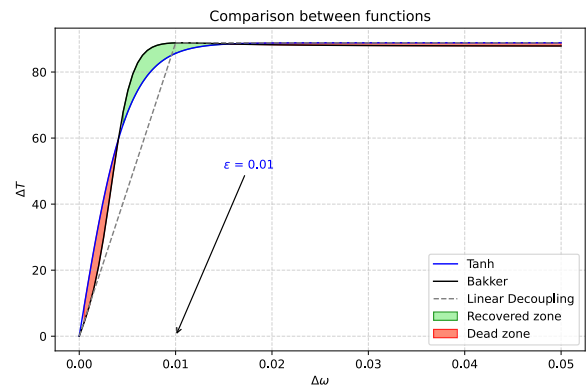
The interest behind the *Bakker's* function lies in the interpretation of the friction transition zone (the green curve in Figure 3.25b). The function satisfies the equation 2.4 with the following coefficients:

Bakker's coefficients for torque bias				
$\backslash$	C_{ref}	D_{ref}	K_{ref}	E_{ref}
First values	1.10	500	45000	-45.0

Table 4.3



(a) The torque distribution holds for an engine torque of 100 Nm and a locking effect of 50%. The opening threshold ϵ highlights where the peak function appears.



(b) The *Bakker's* function recovers the friction transition and the dead zone identified when applying the *tanh* function, but shifts away from the ideal vertical alignment.

Figure 4.14: The *Bakker's* function does not solve all the issues of the *Torsen* distribution.

The Bakker’s function is presented in Figure 4.14a. Compared to the hyperbolic tangent distribution (Figure 4.14b), *Bakker’s* distribution introduces more *dead* zones. These zones shift the torque balance distribution away from the actual distribution in the *Torsen* differential. Ideally, even if the friction transition zone is represented, the function should be close to a vertical line at $\Delta\omega = 0$.

Numerical Stiff Problems and Final Choice

Although *Bakker’s* function appears to be the good choice to respect the *Torsen* distribution law, the values employed earlier for Figure 4.14 induce a numerical stiffness problem (even with a variable time-step integrator). The results of stiff integration problem on the torque distribution can be found in Appendix B.3. As previously explained, the slope of the curve before ϵ acts as a key parameter in numerical integration issues. Consequently, the selected function must remain effective under the most extreme conditions of the torque bias law, i.e. when subjected to the highest *TBR* and maximum engine torque ($T_m = 340$ Nm and *TBR* = 4.0). The maximum values of the linear decoupling function and the hyperbolic tangent distribution provide the optimal threshold between stable solutions.

Their closed reciprocity with Figure 3.21 is presented in Figure 4.15. The selected ϵ and v^* result in a parametric test that verifies the potential stiff problem and the stability of the wheel torque distribution (the objective was to avoid torque distribution similar to that in B.3). The linear decoupling strategy confers the best choice for torque distribution recovery compared to the hyperbolic tangent function. The lost zone (the distance between the theoretical torque bias and the linear distribution) seems problematic when compared directly to the theoretical plot (Figure 3.21). However, the theoretical model does not account for the elasticity of the gears, which induces a small transition window between the onset of wheel speed difference and the application of torque bias. This behaviour was discussed with the *Torsen* team, and we concluded that the linear curve with the selected parameters offers the best compromise.

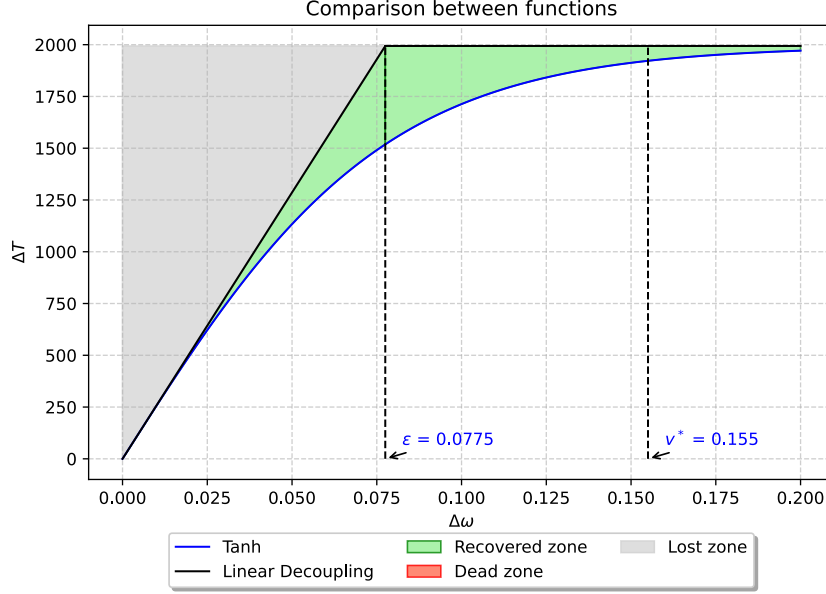


Figure 4.15: The torque distribution is valid for an engine torque of 340 Nm and a locking effect of 60%. Achieving undisturbed simulation results requires setting $\epsilon = 0.0775$ for the linear decoupling strategy and $v^* = 0.155$ when applying the hyperbolic tangent function.

4.3 Controllers

PID controllers were independently implemented for the further manoeuvres to control both the speed and the trajectory of the car.

4.3.1 Speed Control

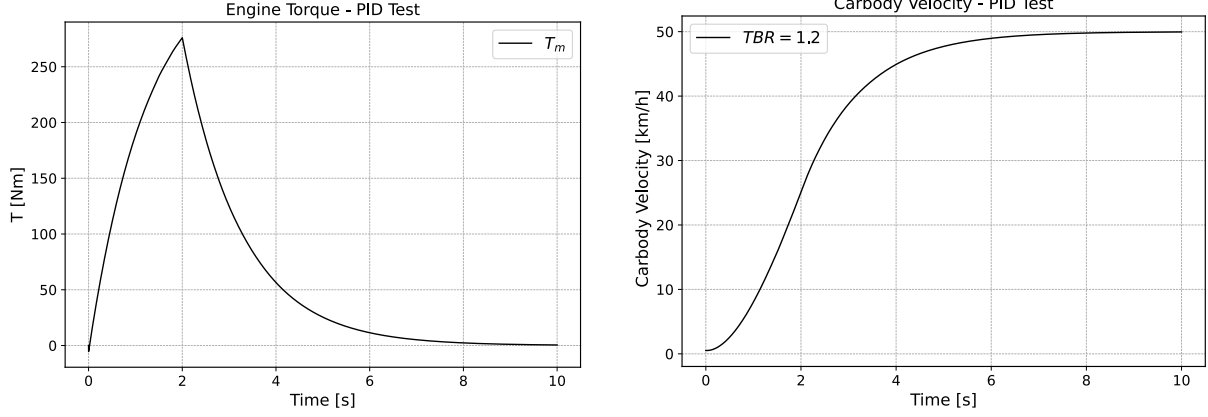
The vehicle speed is controlled through the implementation of a PID controller, with periodic reset of the accumulated error compared to the target velocity v_{target} . In straight-line motion, only the power proportional K_P is active, whereas in cornering situations, the velocity is further adjusted using derivative and integral gains K_D and K_I based on the error. The PID controller injects the corrected engine torque T_m in proportion to the error as follows:

$$T_m = K_P \cdot e + K_D \cdot \dot{e} + K_I \cdot \int_0^t e dt \quad (4.7)$$

And the errors are:

$$e = v_{target} - \sqrt{u^2 + v^2} \quad \dot{e} = \frac{e - e_{previous} + t_c \cdot \dot{e}_{previous}}{t + t_c} \quad \int_0^t e dt = \sum_i e_i \cdot t \quad (4.8)$$

Here, t is defined as the time-step numerical integration and t_c is the time-step derivative. The engine torque limiter (see point 4.1.2) continuously monitors the corrective torque generated by the PID controller. Figure 4.16 demonstrates the PID action in the simplest situation: achieving a target velocity of 50 km/h in a straight line. It takes the car 8 seconds to reach the target velocity (Figure 4.16b).



(a) The engine torque is limited to 345 Nm until it reaches the speed limit of 5000 RPM. (b) The vehicle reaches the target velocity after 8 seconds of simulation in straight line.

Figure 4.16: Evolution of the engine torque and the carbody velocity for a straight line during 10 seconds. Because there is no drag force in straight line, the engine torque arrives to 0 when the carbody velocity is near 50 km/h .

4.3.2 Trajectory Control

As the cornering behaviour of the vehicle will be studied in the next section, the driver needs to follow a specific trajectory in order to make a comparison with the vehicle in the same situation with a different differential. A PD controller adjusts the steering wheel angle to control the y-position and yaw angle of the car.

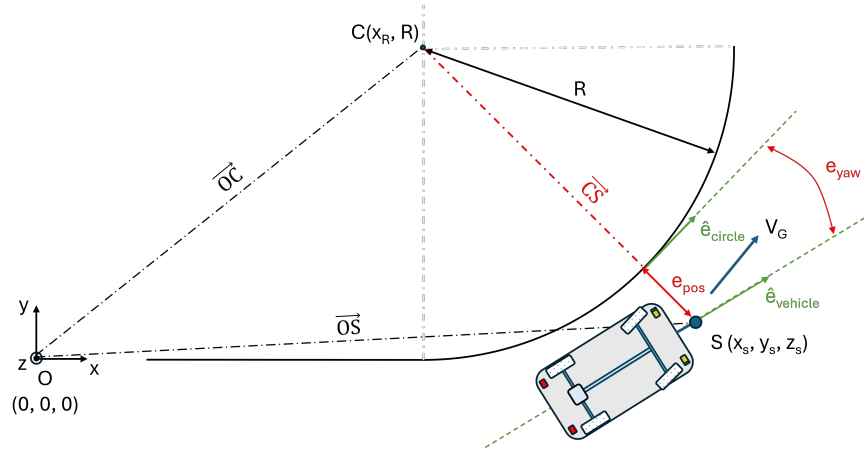
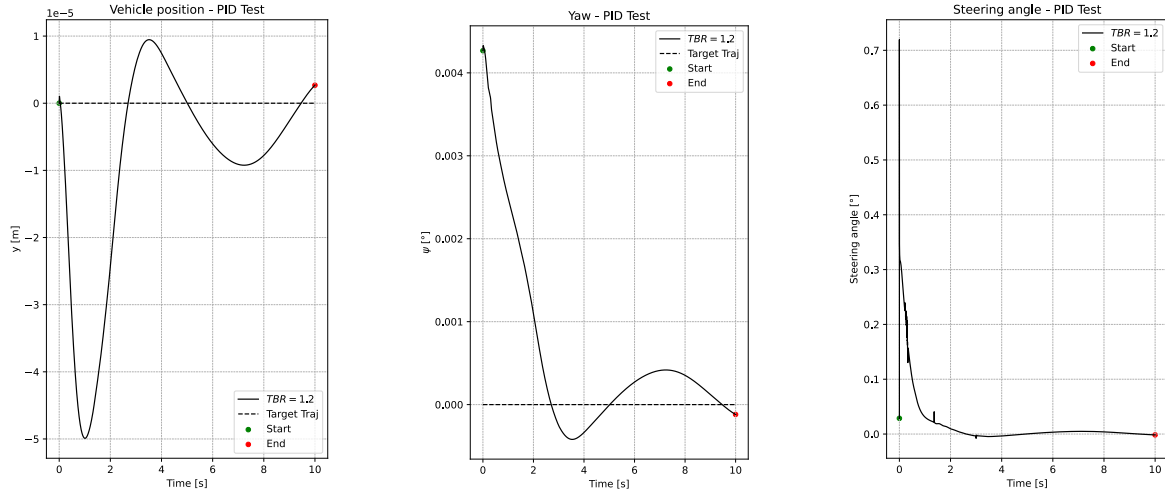


Figure 4.17: A sensor S positioned at 0.5 m in front of the car measures the position, the velocity and the acceleration of the vehicle in the origin frame (x , y and z).

The first trajectory is a simple straight line. In this condition, the position y of the vehicle measured by the sensor S must remain at $y = 0$ m (Figure 4.17). The desired yaw angle is also set to 0° (Figure 4.16). As we can see in Figure 4.18, the y -position does not exceed $5e^{-5}$ m. Because the time-step integration is 10^{-4} s, the PD controller can not correct the y -position any further.



(a) The vehicle tries to stay on the axis $y = 0$ during the acceleration step. The y-displacement ends near 0° .

(b) The engine acceleration can induce yaw and yaw moment. The PD reduces this effect.

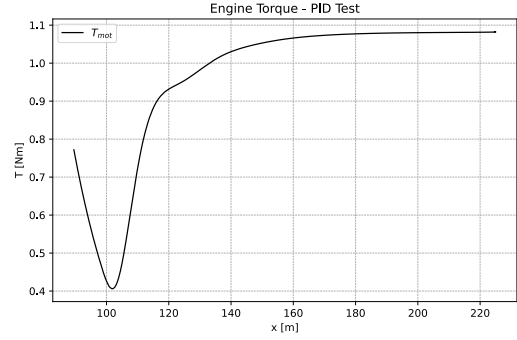
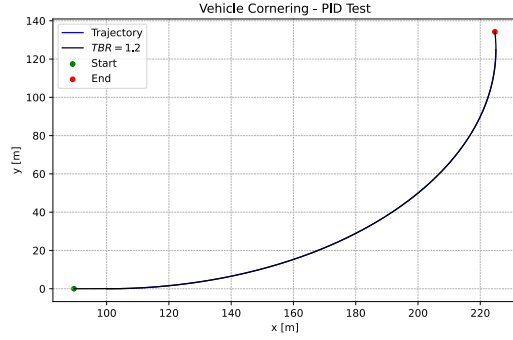
(c) The PD response adjusts the steering input to correct the vehicle's trajectory. The steering wheel ends near 0° .

Figure 4.18: During the 10-second simulation, the driver corrects the trajectory relative to $y = 0$. As one can see, the difference between the start and the end of the simulation is kept as small as possible to avoid torque bias caused by speed differences resulting from car deviation (in the absence of a PD controller).

One of the most interesting situations to compare the vehicles is cornering. Therefore, it remains attractive to use fixed trajectory for both vehicles (one with an open differential and the other with a *Torsen*). Figure 4.17 shows the key vectors employed to compute the vehicle's position. The error calculation steps can be found in Appendix B.4. In the cornering situation, the first error e_{lat} to consider is the distance between the vehicle's position and the reference circle (representing a turn of a radius R). The second error e_{yaw} to verify is the angle between the car's orientation and the circle tangent. These errors generate the necessary steering wheel correction:

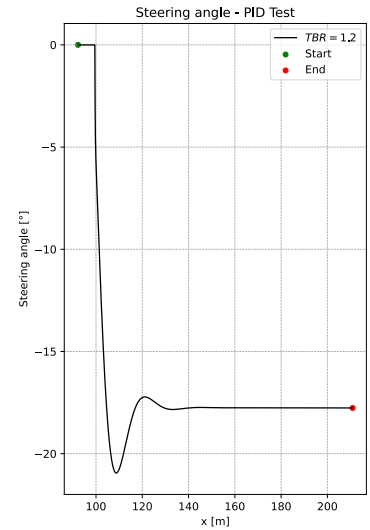
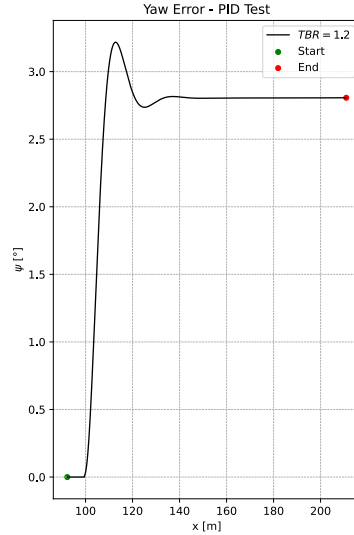
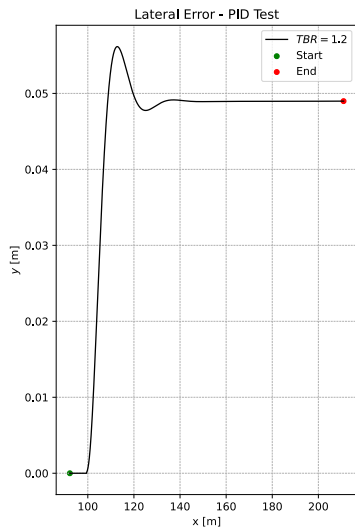
$$\delta = K_{P,lat} \cdot e_{lat} + K_{D,lat} \cdot \dot{e}_{lat} + K_{P,yaw} \cdot e_{yaw} + K_{D,yaw} \cdot \dot{e}_{lat} \quad (4.9)$$

For a corner of $R = 125$ m and a constant velocity of 50 km/h, the maximal lateral error reaches 0.0575 m and the maximal yaw error is about 3° (Figure 4.20). These errors are considered as acceptable, as the lateral error represents a difference of 0.046% between the targeted and measured trajectories. The position of the car closely follows the fixed cornering trajectory (Figure 4.19a). As front lateral forces in cornering produce drag forces, the torque must be adjusted to maintain the target velocity during cornering (Figure 4.19b). The PD parameters are adjusted to closely follow the target trajectory while enabling the vehicle to exhibit understeered or oversteered behaviour.



- (a) The vehicle firmly tracks the targeted trajectory. (b) The engine adjusts the velocity in the cornering in order to maintain a constant velocity.

Figure 4.19: The PD controllers correct the trajectory and the velocity as quickly as possible, while remaining flexible enough to allow the understeered/oversteered behaviour caused by the *Torsen* differential.



- (a) The small error at the end of the cornering seems reasonable compared to the radius. (b) The yaw error will be compensated by the steering wheel. (c) The steering wheel takes both lateral and yaw errors into account.

Figure 4.20: The turn begins after 100 metres to allow the car to reach a constant velocity of 50 km/h. The errors mitigated by the PD controller remain within acceptable limits relative to the characteristics of the turn.

Chapter 5

Differentials Validation

This chapter seeks to prove and to compare in similar road situations, the operational efficiency of the perfect open differential (*OD*), locked differential (*LD*), the open differential with friction (*Open*) and the *Torsen* differential with a *TBR* of 3.0. For this purpose, the vehicle initially starts from rest and accelerates to reach a speed of 50 km/h. Then, at $x = 100$ m, the driver steers the vehicle around a corner at a constant steering wheel angle δ of 30° . Following this steering input, an engine torque of 100 Nm is applied. For clarity, the straight line and the cornering phases are treated separately. The situation is illustrated in Figure 5.1.

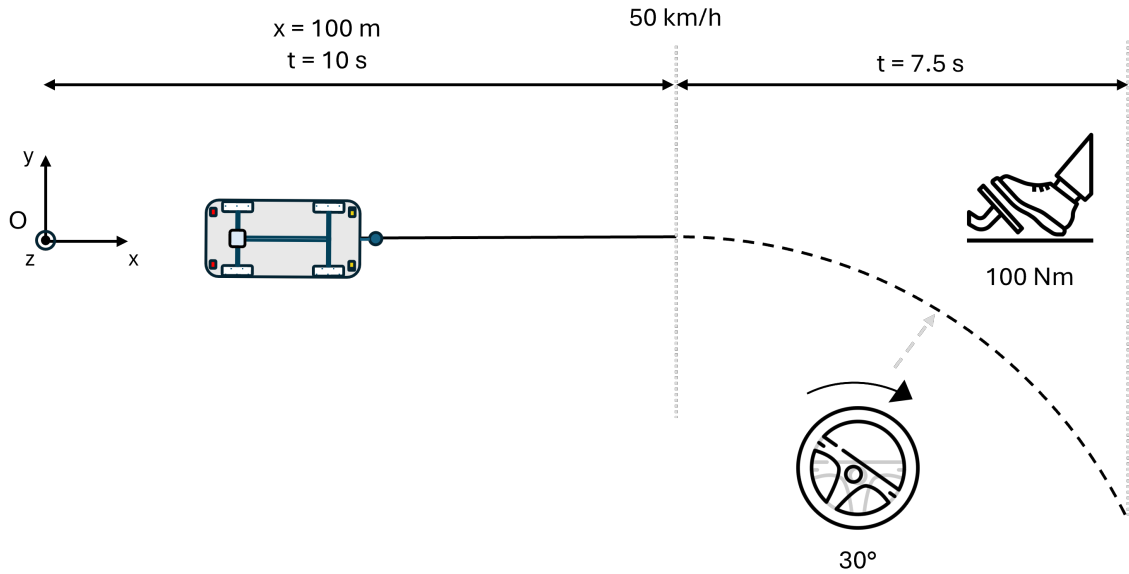


Figure 5.1: After travelling in a straight line of 100 m, the driver turns the steering wheel to 30° to the right, applying an engine torque of 100 Nm ($\approx 30\%$ of the accelerating pedal). The vehicle then stops after 7.5 seconds of cornering.

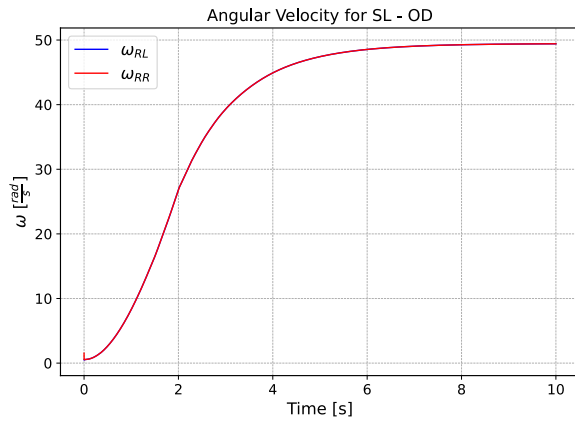
5.1 Straight Line

To achieve 50 km/h before starting the turn, the vehicle follows the same trajectory and engine torque as in the test used in Section 4 (Figures 4.16 and 4.18). In this context,

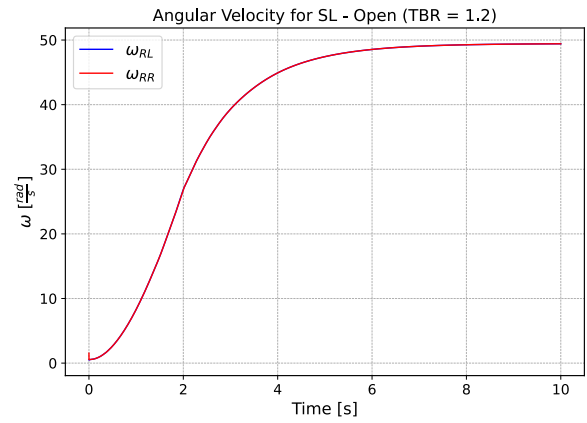
the torque distribution and the wheel angular velocities will be studied. This will confirm the differential laws established in the previous chapters.

5.1.1 Open Differentials

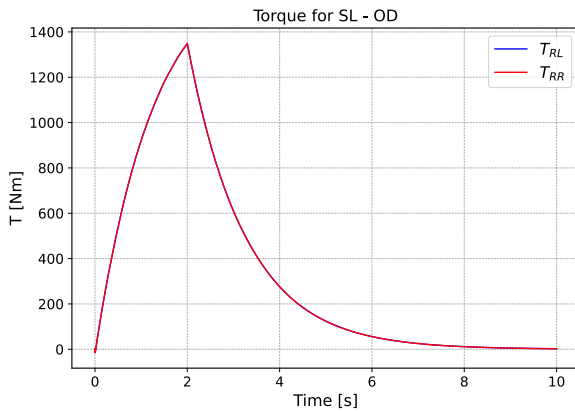
The straight line (*SL*) situation for the frictionless open differential *OD* and the open differential with friction (*Open*) highlights the same behaviour in terms of angular velocity and torque repartition. As explained in the chapter 4, the OD operates based on the velocity constraint, whereas the open differential with friction manages the *Torsen* law distribution with a *TBR* of 1.2. The vehicle remains on its trajectory under normal road conditions. The contact between the left and right tyres and the road stays the same. Thus, the torque is distributed equally to both wheels (Figures 5.2c and 5.2d). Figures 5.2a and 5.2b show that the angular velocity between the wheels is the same for both differentials.



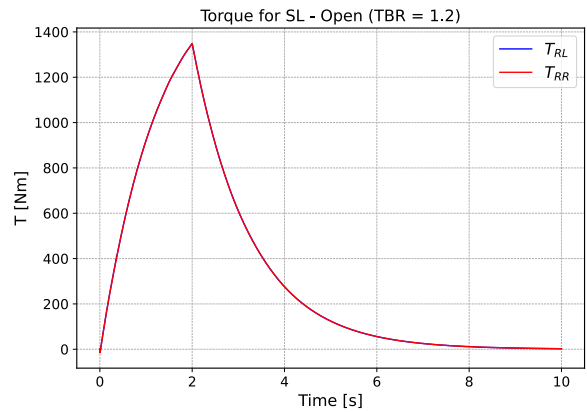
(a) Because the vehicle follows the straight line, the wheels rotate at the same angular velocity.



(b) The *Open* differential with the *Torsen* law behaves like an OD.



(c) As predicted in the previous chapter, the torque is exactly the same for both wheels.



(d) The torque split between both wheels follow the torque distribution of the *OD*.

Figure 5.2: As a first verification point, the behaviour of both open differentials (frictionless and friction) in a straight line is the same.

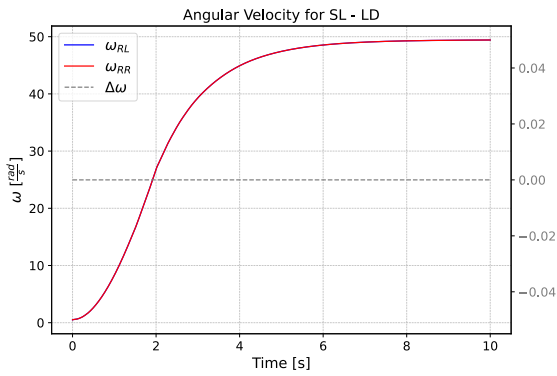
Although the first part of the simulation is the same for both types of differentials, it is worth mentioning that the open differential exhibits the same behaviour as the OD (as

discussed in section 3.5.2). These reactions seem obvious when considering the torque split equations demonstrated in the chapter 3 and the road conditions.

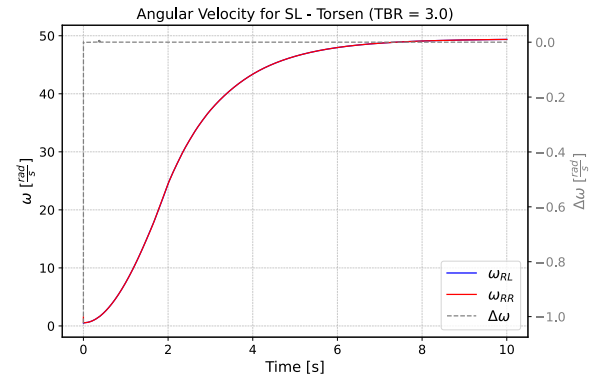
5.1.2 Locked and *Torsen* Differentials

Figure 5.3 displays the key features of the locked and the *Torsen* differentials. In this context, the equations 3.22 and 3.23 for the *locked* differential, as well as the torque and speed differences between the two rear wheels for the *Torsen* differential, illustrate how each type of differential behaves:

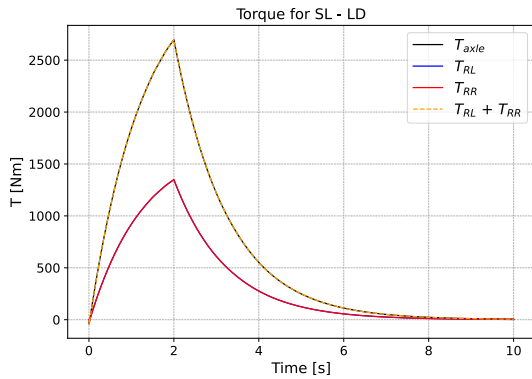
- For the *locked* differential, the graphs 5.3a and 5.3c show that no rotational speed difference is created ($\Delta\omega = 0$) throughout the entire straight line. This is consistent with expectations: as there is no difference in grip between the left and the right wheels, the torque is not distributed preferentially to one side. The sum of the two torques is equal to the multiplied engine torque, i.e. $T_m \cdot \tau_{reduction}$.



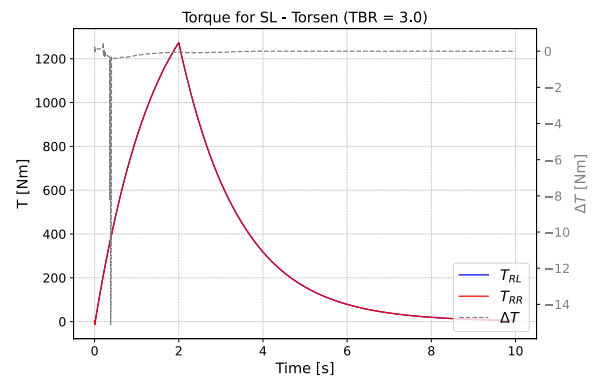
(a) The locked differential imposes $\Delta\omega = 0$.



(b) A difference between initial conditions and the PID engine torque creates a small $\Delta\omega$ at the beginning.



(c) The orange curve respects the relation 3.23. The torque applied to both wheels is the same in a straight line.



(d) The ΔT (expression 3.20) arises from the small $\Delta\omega$ generated. Because the speed difference is small, the torque distribution stays in the first part of the linear decoupling.

Figure 5.3: The torque and the angular velocity distributions validate all the design steps discovered in the section 3 and implemented in the section 4.

- For the *Torsen* differential, a very slight $\Delta\omega$ and a resulting ΔT can be spotted

(Figures 5.3b and 5.3d). This phenomenon, which does not affect the vehicle's behaviour, arises due to the sensitivity of the torque distribution to the *opening* moment ε of the *Torsen*. The position controller indirectly corrects these non-zero deltas when the vehicle is travelling in a straight line.

From a practical perspective, it seems inevitable that there will be slight differences between the left and right wheels in a *Torsen* differential. These differences can be attributed to the constant minor variations in surface adhesion, given that road conditions are rarely perfectly uniform along the entire route.

5.2 Cornering

The second part of the test involves cornering with an acceleration of $T_m = 100$ Nm. During this phase, the driver can either fix a constant steering angle or follow a turn with a specific radius R . First, the driver sets a constant steering angle δ of 30° , as Figure 5.1 illustrates.

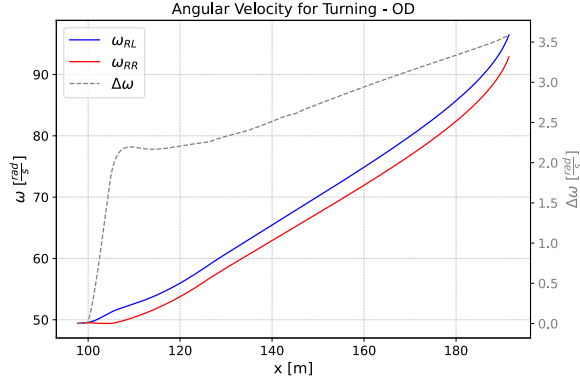
5.2.1 Constant Steering Angle

As for the straight line, the differential behaviour of each design will be studied in order to analyse and validate it.

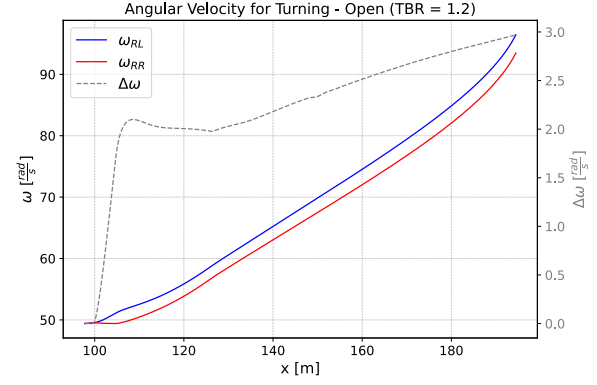
Open Differentials

As shown in Figure 5.4c, the torque difference ΔT between both wheels remains zero during all the cornering. The torque on each wheel is the same, in accordance with the equation 3.21. As the main function of the *OD* is the speed differentiation between the two wheels, the outer wheel (the left one, see Figure 4.9) rotates faster than the inner one (Figure 5.4a). Consequently, the angular speed difference $\Delta\omega$ increases progressively during the turn. This increase is caused by the vehicle acceleration applied by the driver, as the speed of the wheels rises.

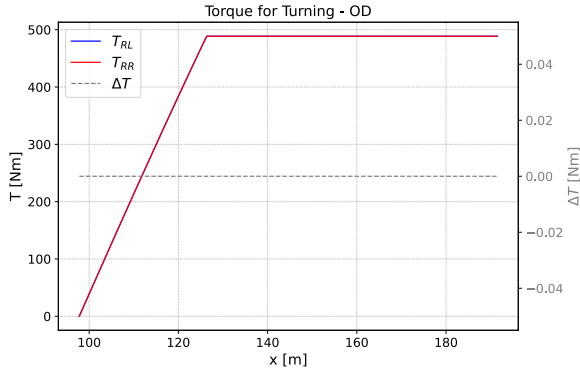
Due to the friction inside the open differential, its torque bias behaves like a *Torsen* differential law (Figure 5.4d). Therefore, for an engine torque T_m of 100 Nm, the torque balance respects the previously calculated ΔT (in chapter 4 for Figure 4.11b). Thus, following the *Torsen* mechanical steps detailed before (opening and opened differential), the engine torque is sufficient to induce the opening of the open differential. In Figure 5.4b, the $\Delta\omega$ is not zero. This enables the two wheels to travel at different speeds. Nevertheless, the value of $\Delta\omega$ is lower than for the *OD* because the torque bias applies to the inner wheel in the corner (right wheel in Figure 5.4d). This torque balance tends to slow down the outer wheel and to accelerate the inner one.



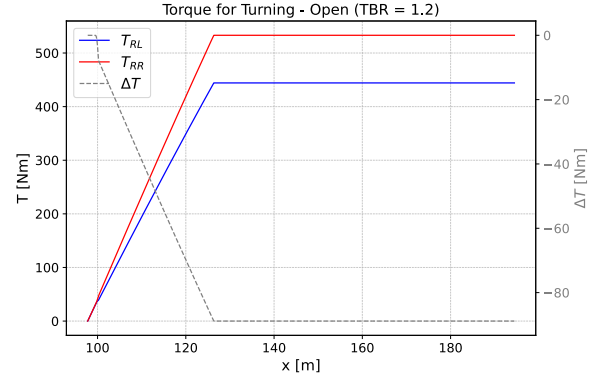
(a) The speed differentiation works as soon as the corner starts.



(b) The speed difference is lower than for the OD because of torque bias.



(c) The torque is split equally between both wheels due to the OD characteristic.



(d) The torque applied to the inner wheel (right wheel when cornering) is larger by an amount equal to ΔT compared to the outer wheel (the left wheel).

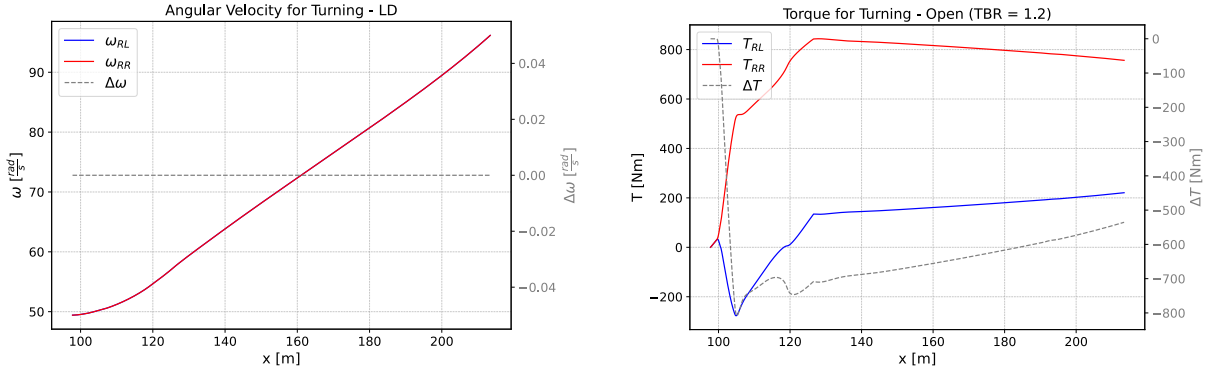
Figure 5.4: The torque and the speed differences start when the vehicle enters in the turn at $x = 100$ m (the measured TBR between the two wheels exceeds the fixed TBR of the open differential (1.2), see Appendix C.1). The open differential results in a lower speed difference $\Delta\omega$, due to the torque bias on the inner wheel.

Locked Differential

The locked differential imposes a null $\Delta\omega$ even during turning (Figure 5.5a). Consequently, because of the common rear axle for both wheels, the outer wheel slows down more than it would in an OD application. According to equation 3.23, the torque bias results in the application of the highest torque to the inner wheel, which has the best grip. Therefore, the negative torque between $x = 100$ m and $x = 120$ m in Figure 5.5b arises due to the constraint on the rotational velocity ($\Delta\omega = 0$). Despite this phenomenon, the sum of the torques remains equal to the torque applied to the axle (equivalent to T_m through the reductor).

The torque split between the two wheels depends on their longitudinal slip. Referring to the *Bakker* curve of the tyres (Figure 5.9), the longitudinal slip κ of the inner wheel is greater than that of the outer wheel. Since the engine torque T_m is low, the longitudinal slip κ for the inner wheel remains below the saturation peak. The inner wheel becomes the wheel

with the best grip. Because the *spool* imposes a common rotational velocity to the wheels, the inner wheel receives the largest applied torque. This compensates for the tendency of the corner to cause a difference in rotational velocity. The inner wheel longitudinal slip is thus greater than the outer wheel slip, resulting in a greater longitudinal force being developed by the inner wheel. This induces an understeered behaviour of the vehicle. As the lateral acceleration a_y increases during the rest of the cornering process (Figure 5.8a), so does lateral load transfer and the longitudinal force on the outer wheel. Figure 5.5b shows that the difference between the left and right torque decreases. An inversion of the torque balance could be expected if the lateral acceleration becomes sufficient. This point will be discussed in the chapter 6.



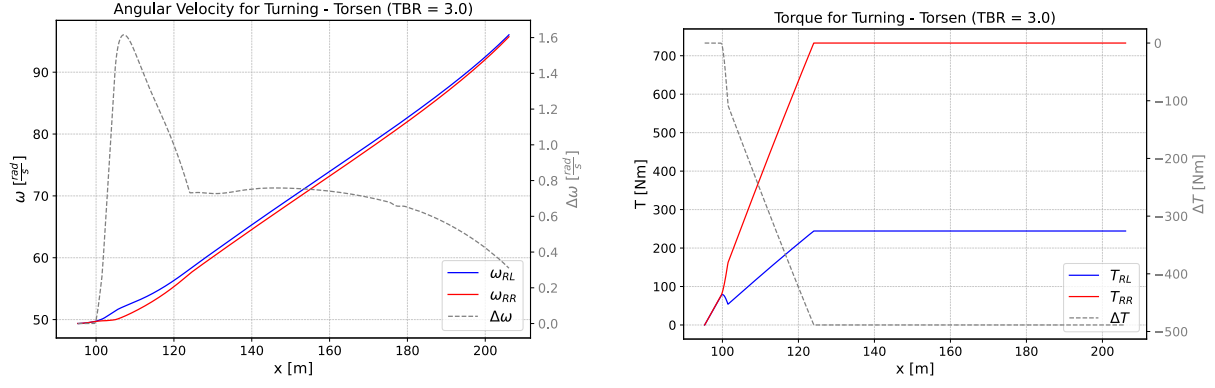
(a) The angular velocities stand equal.

(b) The left torque is negative to slow down the outer wheel at the beginning of the corner.

Figure 5.5: When the differential is locked, a single axle (spool) links the two wheels and imposes a common angular velocity. Consequently, the torque distribution between the wheels depends on the load transfer and the available grip.

Torsen Differential

Like the open differential, the *Torsen* differential with a *TBR* of 3.0 creates a torque bias as soon as the driver turns the steering wheel. As the measured *TBR* saturates to the fixed *TBR* (available in Appendix C.2), the *Torsen* differential opens. The initial phase of the turn exhibits a larger $\Delta\omega$, resulting in a significant speed difference between the wheels (Figure 5.6a). During the test, this speed difference gradually decreases. As with a locked differential, the generated torque difference slows down the outer wheel (its angular acceleration compared to the inner wheel decreases until the end of the test, see Appendix C.2). As illustrated in Figure 5.6b, the torque bias remains constant throughout the corner because the speed difference exceeds the opening limit ε . However, since the lateral acceleration is not sufficiently high, the resulting yaw moment still leads to an understeered behaviour (more torque is applied to the inner wheel than to the outer wheel).

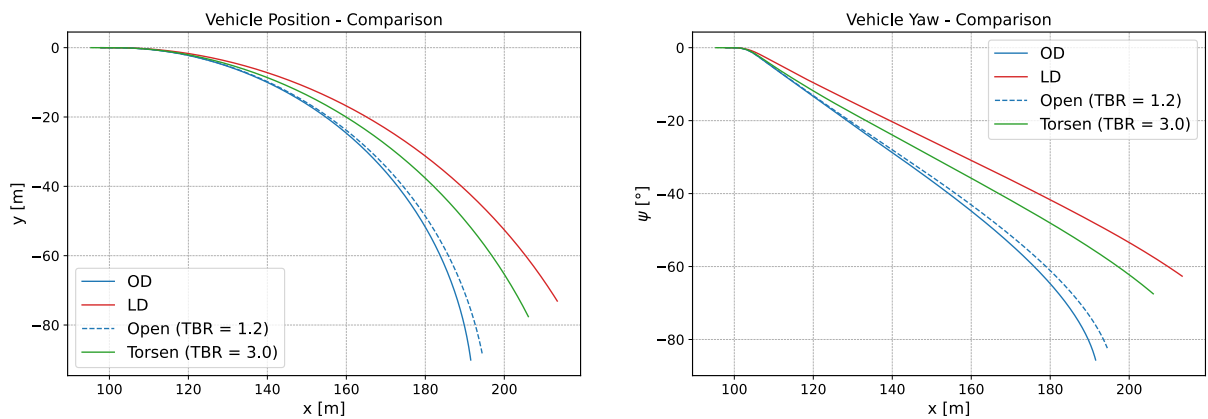


- (a) Due to the torque bias, the inner wheel accelerates and the outer wheel slows down, resulting in a decrease in speed difference.
- (b) Because a_y is insufficient, the torque balance ΔT is applied to the inner wheel.

Figure 5.6: The *Torsen* differential applies more torque to the inner wheel (right-hand wheel). This decreases the difference in wheel rotational velocity. The reaction of the *Torsen* differential lies between the open differential and the locked one, as a *limited-slip* differential must react.

Performances Comparison

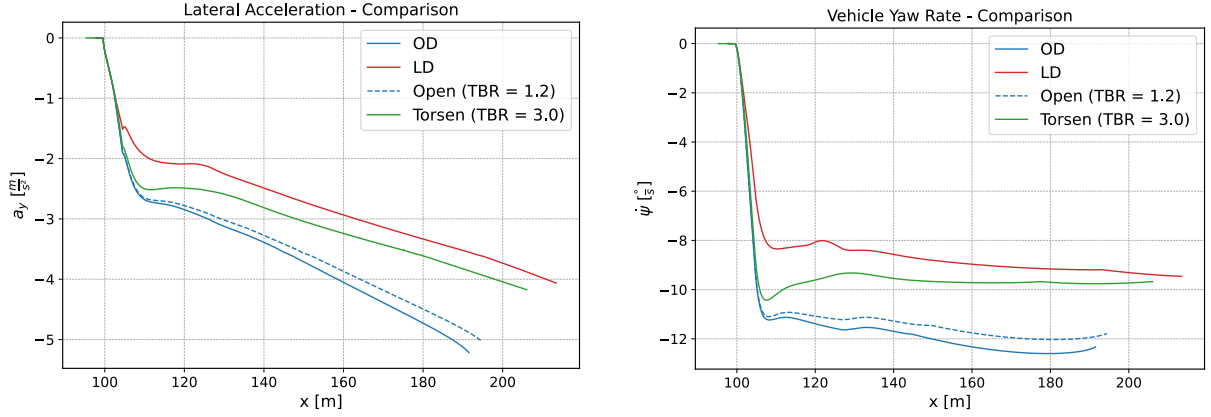
Each of the previously mentioned arguments, such as understeer yaw moment or lateral acceleration, provides a strong basis for comparing performance. Firstly, the position and the yaw indicate how the vehicle behaves when turning. As predicted, the more the locking effect between the two wheels, the more understeered the vehicle's behaviour becomes. The path of the car is larger for the locked differential than for the *Torsen*, the open differential and the OD (Figure 5.7a). In addition to position, vehicle yaw (rotation along the z-axis) is also affected in the same manner as the position. The locked differential results in lower yaw than for the OD (Figure 5.7b).



- (a) The higher the locking effect, the larger is the turn.
- (b) The locking effect reduces the car's yaw position.

Figure 5.7: The behaviour of the vehicle in the cornering is affected by the locking between the two wheels.

Next, both the lateral acceleration a_y and the yaw rate $\dot{\psi}$ decrease as the locking effect increases. The locking effect between the two wheels limits the lateral acceleration, inducing a smaller lateral force (Figure 5.8a). This results in a more understeered car. However, the yaw rates between differential types tend to be closer as the vehicle corners (Figure 5.8b). When the user accelerates while maintaining the same steering wheel angle, lateral acceleration increases as the load transfers. Therefore, understeered moment effect decreases and could change to a more oversteered moment. This potentially profitable effect is discussed in more detail in Section 6.



(a) The lateral acceleration is smaller due to the torque distribution when the locking effect increases at the same position in the corner.

(b) The lateral load transfer affects the yaw rate. It remains constant for the *Torsen* while the absolute value for the locked increases.

Figure 5.8: The yaw rate and the lateral acceleration both decrease as the locking effect increases. This has an impact on the vehicle's cornering performance.

For example, Figure 5.9 highlights the difference in understeered moment between the *Torsen* and the locked differentials. At the x -position of 140 metres into the turn, the longitudinal forces modelled with the *Bakker* tyre curves are outlined at this particular moment. As previously described, the longitudinal slip κ of the inner wheel is greater for the locked differential than for the *Torsen* one, resulting in a larger understeered moment, as indicated by the green arrow. The yaw moment, as calculated using the equation given in 3.24 is detailed in Appendix C.2.

Bakker's curves illustrate why the yaw moment and the torque distribution could reverse. Since the longitudinal force F_{long} is a non-linear function of the longitudinal slip ratio κ , the longitudinal force generated by the outer wheel $F_{x,RL}$ could surpass that generated by the inner wheel $F_{x,RR}$. This phenomenon arises because the longitudinal slip of the outer wheel increases as the wheel velocities increase (due to an increase of engine torque T_m). Because of the load transfer to the outer wheel, the normal force F_n is greater on the left side, and the rear left tyre can generate a higher longitudinal force than the inner wheel. In the case of a *Locked* differential, the understeered moment produced by the difference in the longitudinal force is greater because more torque is applied to the inner wheel. With a *Torsen* differential, which is limited by its *TBR*, the torque applied to the inner wheel is lower. The saturation limit of longitudinal slip κ_{lim} on the inner wheel could be reached if the torque applied to this wheel is too high. This is because the maximum longitudinal force on the inner wheel is lower than on the outer wheel. This could reverse

the torque distribution, with the outer wheel becoming the one with the best grip (close to, but before, the saturation limit). This point will be discussed in Section 6.

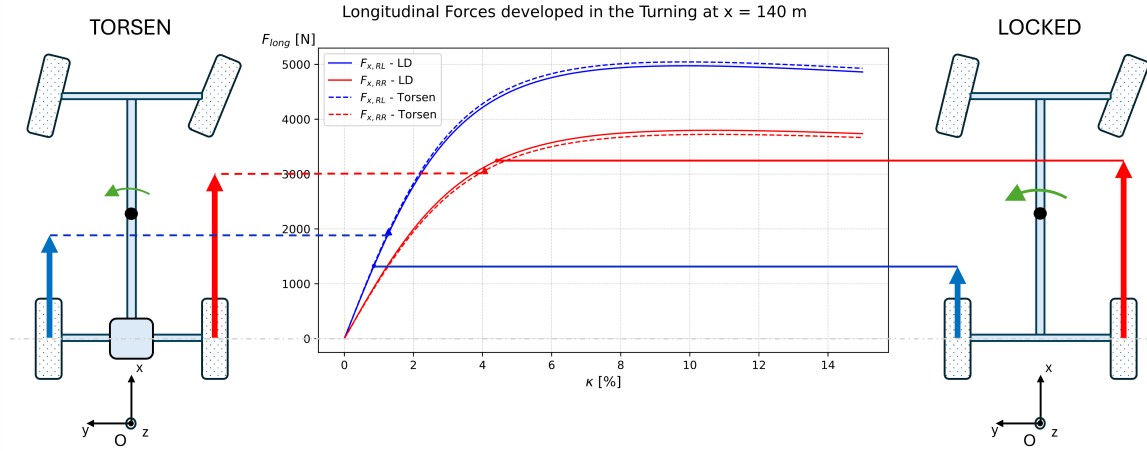


Figure 5.9: The analysed values highlight the vehicle's behaviour at $x = 140$ m. The yaw moment (green arrow) is larger for the locked differential because the difference in longitudinal forces (related to the torque bias) is bigger. The *Torsen* differential experiences a greater load on the left side than the locked differential because a_y is greater.

5.2.2 Driver Reaction on a Fixed Trajectory

The last noteworthy point concerns how the driver reacts to each type of differential implemented. The turning situation is modified by setting a constant trajectory for each car to follow. The PD controller tracks a left curve with a fixed radius of $R = 125$ m, as Figure 5.10 describes.

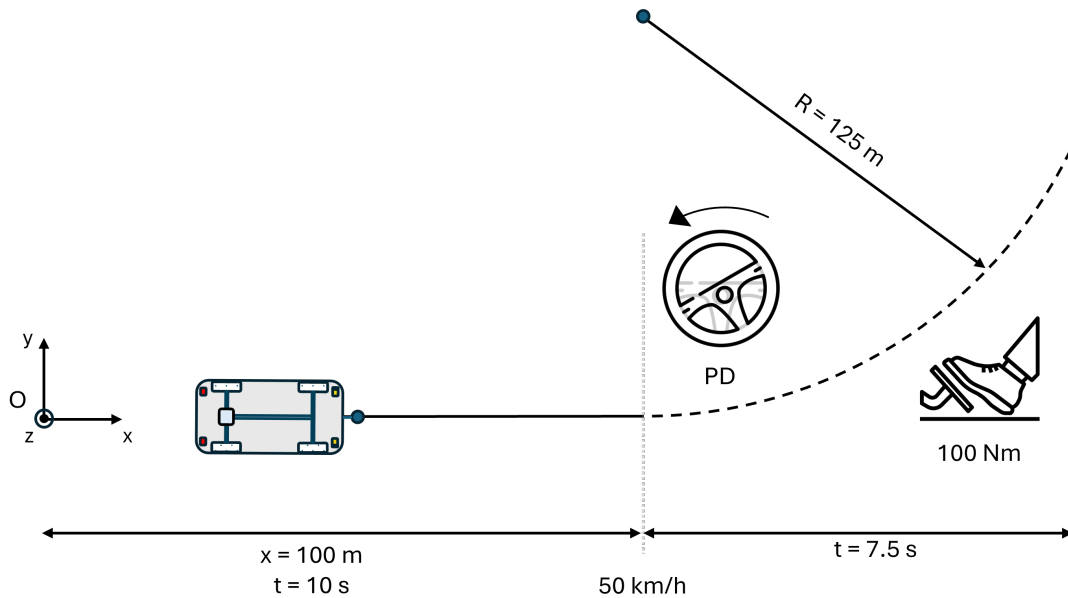
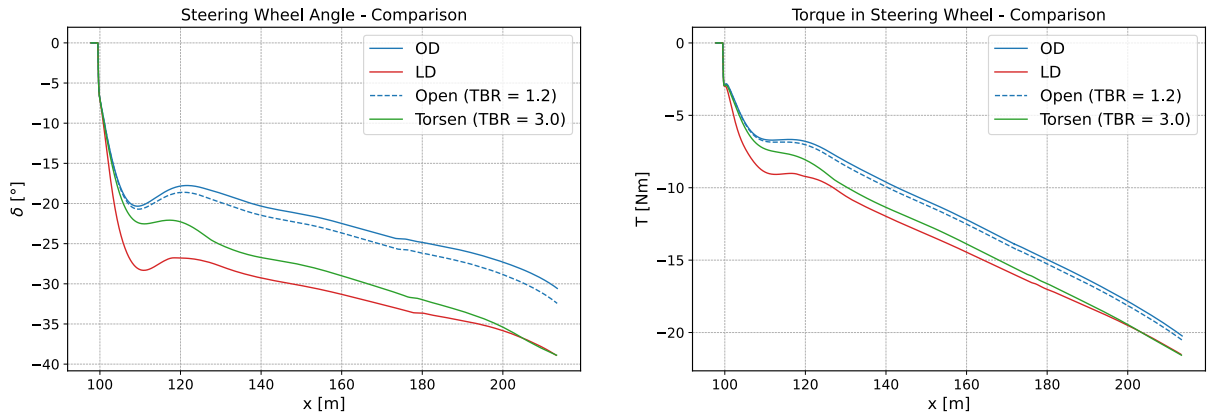


Figure 5.10: After a straight line of 100 m, the driver trails a corner with a radius $R = 125$ m. The PD controller adapts the steering wheel to follow the trajectory while the driver injects a engine torque of 100 Nm. This time, the driver turns to the left.

The steering angle applied by the driver and the amount of torque in the steering wheel confirm the previous results. The vehicle exhibits a more understeered behaviour when the locking effect increases. Therefore, the driver must apply a greater steering angle to follow the intended trajectory closely (Figures 5.11a). This induces a greater difficulty to turn the steering wheel, as the torque in the steering wheel increases with the TBR (Figure 5.11b). Multiple sources (like [26]) cite a maximum steering wheel torque of ≈ 10 Nm for an inexperienced driver, and around 15-20 Nm for a Formula 1 driver. Consequently, based on the value produced by the locked differential compared to the open differentials, we can conclude that manoeuvring the car becomes significantly more challenging during low- or medium-speed cornering due to the torque bias.

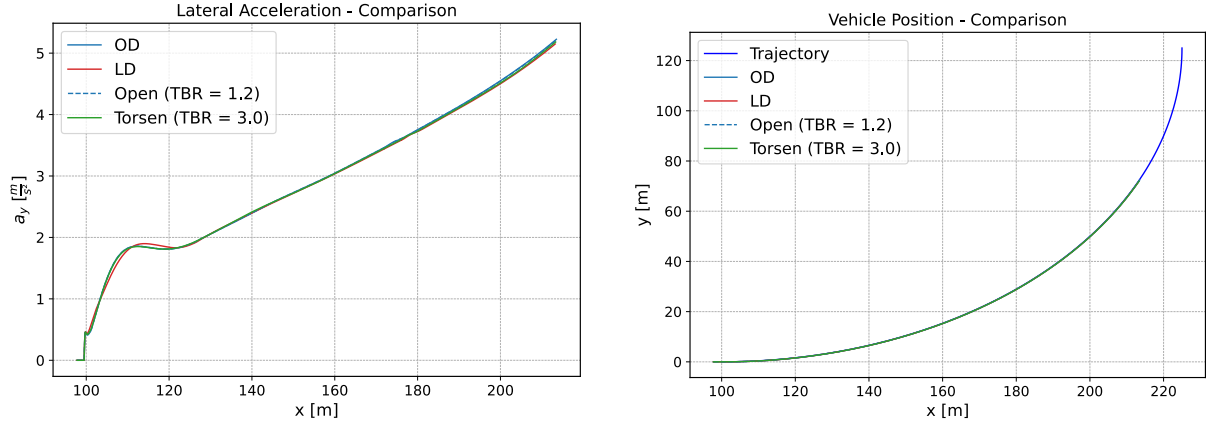


(a) As the locking effect increases, the yaw moment becomes more understeered. The driver steps up the steering wheel angle to follow the fixed curve.

(b) Due to the understeering, the torque in the steering wheel is higher for a locked differential. It becomes more and more difficult to trail the curve.

Figure 5.11: The driver's reactions reveals how easy or difficult the car is to drive when the locking effect varies. As mentioned in the section 3, the manoeuvrability of the locked differential is more difficult than that of the open differential.

The end of the test reveals a surprising but insightful result. During the corner, the lateral acceleration a_y and the position of the different vehicles remain almost identical (Figures 5.12a and 5.12b). Nevertheless, the curves corresponding to the locked and the *Torsen* differentials for the steering wheel angle and torque intersect at the end of the test. The driver's torque decreases due to the inversion of the yaw moment's sign. This phenomenon is discussed in more detail in the section on cornering performance in chapter 6.



(a) Because the vehicles respect the trajectory and accelerate with the same engine torque, the lateral acceleration is almost equal between all the differentials.

(b) The cars trail closely the fixed curve thanks to the PD controller on the steering wheel.

Figure 5.12: Even if the vehicles have the same lateral acceleration and trajectory, the driver differently manages his reaction on the steering wheel to corner depending on the differential used.

5.3 Conclusion on the Influence of Differential Types on Vehicle Handling

The two situations discussed establish a strong correlation between the implementation of the multibody models and the theoretical insights detailed in the chapter 3. Thus, it is clearly demonstrated that the locking effect significantly influences the driver's ability to maintain a desired trajectory. Preliminary observations suggest that the *Torsen* differential requires greater steering effort to follow an identical path compared to an open differential. Subsequent chapters will provide a deeper analysis of the conditions under which the *Torsen* differential offers advantages over the open differential.

Chapter 6

Cornering Performance

This chapter aims to develop and to explore in more detail the differences in cornering behaviour between the open and the *Torsen* differentials. As it was partially introduced in the section 5, the driver's reaction varies depending on the equipped differential, the targeted trajectory, and the applied engine torque. First, the situation of the fixed curve with an engine torque of 100 Nm will be extended (Figure 5.10). Then, on this curve, other longitudinal acceleration rates will be tested. This will be followed by an analysis of the timing gain in a corner with specific input torque. The section about coasting will conclude the chapter. A reminder of the situation and the definitions of the wheels can be found in Figure 6.1.

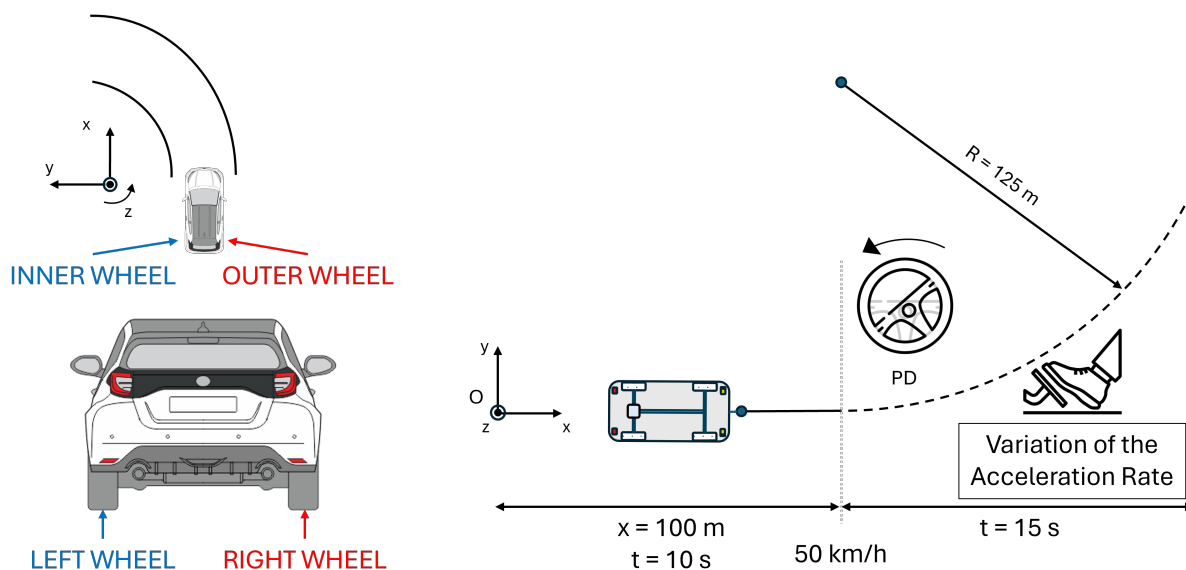
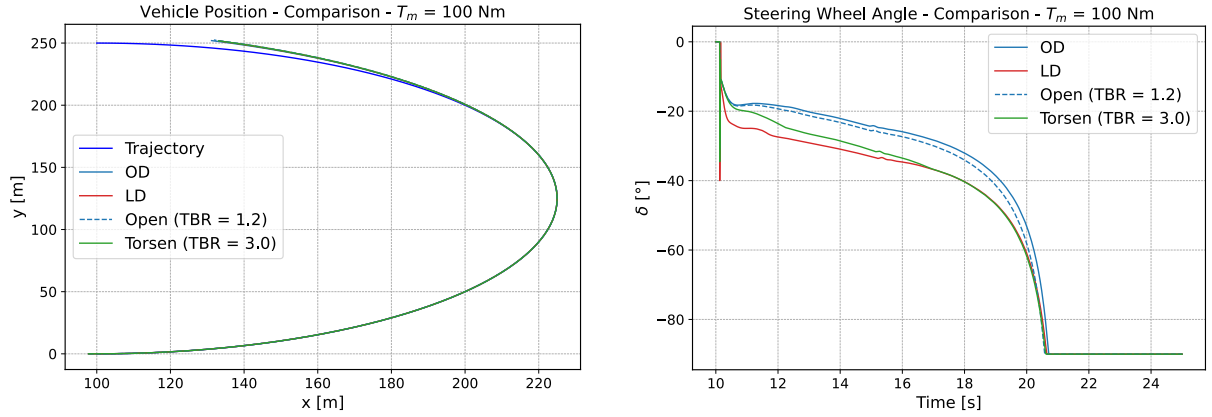


Figure 6.1: As a reminder, the situation involves a left turn with a constant radius of $R = 125\text{ m}$. This means that the left wheel is on the inside of the corner (the inner wheel), while the right wheel is on the outside (the outer wheel).

6.1 Preface of Cornering Behaviour

As discussed in the final part of Subsection 5.2.2, the influence of the locking effect and the lateral load transfer on the torque bias appeared at the end of the test. Therefore,

it would be worthwhile to extend this test in order to observe the evolution of the driver's reaction and the vehicle's handling. The corner is transformed into a 180° turn (Figure 6.2a). Firstly, none of the tested differentials hold the trajectory throughout the entire corner. The understeered yaw moment becomes too large compared to the steering wheel angle injected by the driver. Observing the evolution of the steering angle in Figure 6.2b, the PD controller's response (i.e. the driver's) increases until it reaches the maximum imposed value ($\delta_{lim} = 90^\circ$) at $t \approx 20.5$ s. The driver can no longer turn the vehicle to follow the target trajectory.



(a) All the vehicles deviate from the intended trajectory. The understeered moment is too significant for the driver to be able to compensate for it.

(b) The PD controller is limited to $\pm 90^\circ$ because it is considered as the biggest 'standard' steering wheel angle possible.

Figure 6.2: The driver can not longer resist to the understeered vehicle behaviour. The vehicle deviates from the desired trajectory when δ_{lim} is reached. As the locking effect increases, the car becomes increasingly understeered and requires a greater steering angle.

In sum, it is evident that the locking effect and the level of engine torque affect the vehicle's handling. Therefore, the vehicle's performance when cornering varies according to the longitudinal acceleration a_x and the lateral acceleration a_y , both of which are affected by the engine torque T_m .

6.2 Influence of Engine Torque

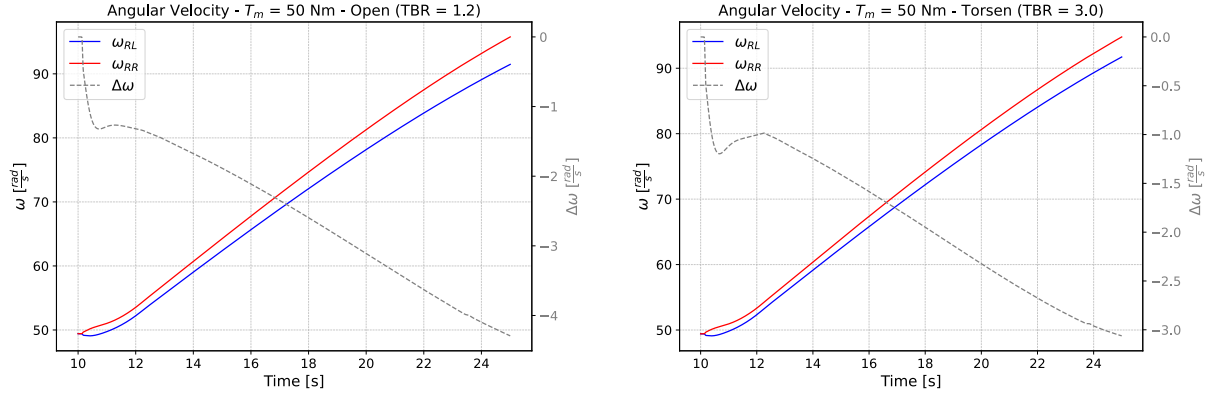
The car's tendency to understeer or oversteer is influenced by the engine torque. The following points show how the torque bias acts differently when the pedal torque varies. This analysis will focus on open and *Torsen* differentials ¹.

6.2.1 Low Acceleration Rate

The acceleration rate, modified by the engine torque, affects the rate of change of u in the lateral load transfer, as discussed in Section 2.3. Before analysing the case with $T_m =$

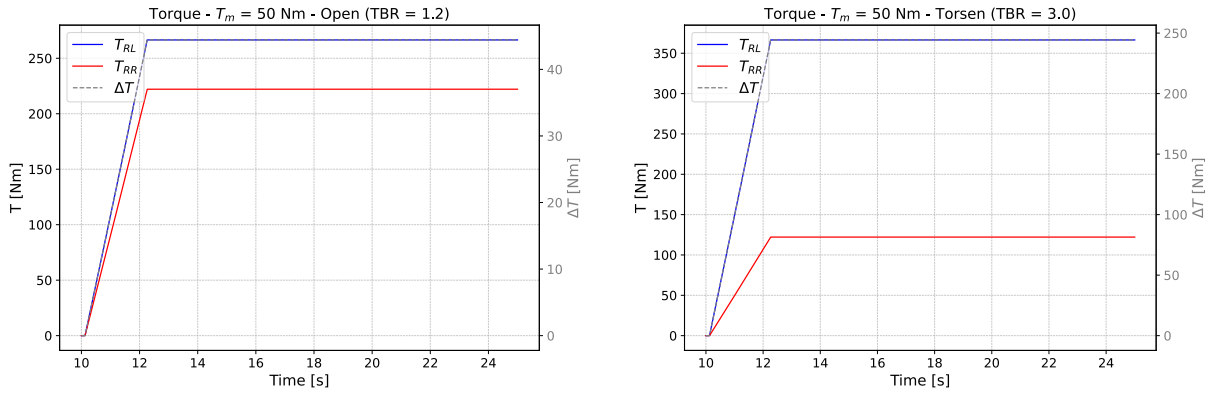
¹The results will be presented as a function of time to eliminate ambiguities in the plots caused by the 180-degree turn, in cases where the data are referenced to the longitudinal displacement x .

100 Nm in depth, it is important to first examine the impact of a smaller engine torque on vehicle handling. Therefore, the driver first applies an engine torque of $T_m = 50$ Nm.



(a) The outer wheel is the faster one. The torque balance will favour the left wheel (inner wheel).

(b) The $\Delta\omega$ is lower compared to the open differential.



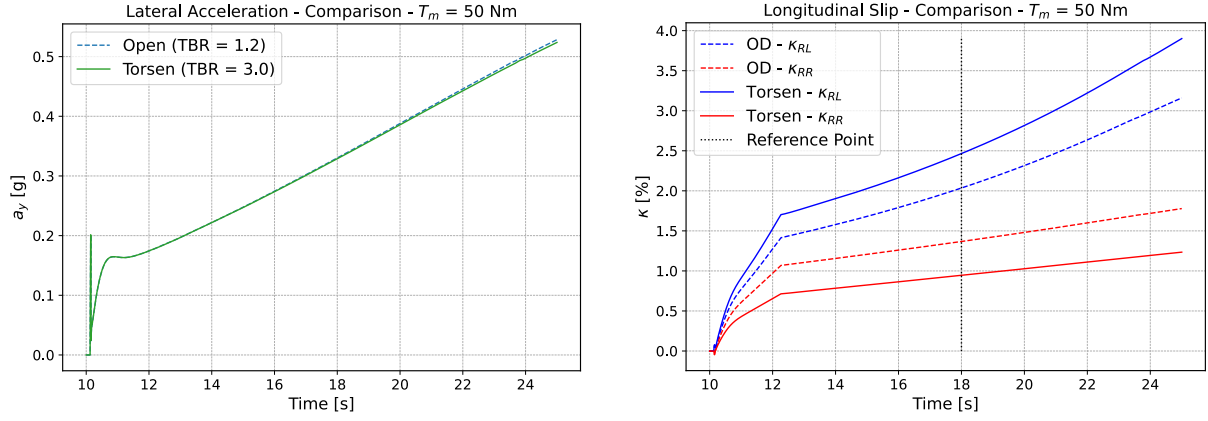
(c) Because the TBR is low with an open differential, the difference between both wheels is smaller than for the *Torsen* because the TBR is larger.

(d) The locking effect of 50% induces a large amount of torque on the outer wheel. This slows down the outer wheel while accelerating the inner one.

Figure 6.3: At low engine torque, the torque is distributed to the inner wheel in order to reduce the difference in rotational speed between the two wheels. For the same corner, the *Torsen* differential results in a more understeered behaviour due to the greater TBR .

The open differential allows a higher speed difference $\Delta\omega$ compared to the *Torsen* differential (Figures 6.3a and 6.3b). This variation is produced by the difference in the torque bias: compared to the open differential, the *Torsen* injects more torque on the inner wheel, reducing the speed difference. Moreover, this induces a larger understeer yaw moment. The torque distributions along the wheels are presented in Figures 6.3c and 6.3d.

The lateral acceleration a_y grows when the driver turns the steering wheel because the longitudinal acceleration $\dot{u}$ increases (Figure 6.4a). The value of a_y is slightly higher for the open differential due to the *Torsen* differential's tendency to understeer.



(a) Lateral acceleration increases with engine torque during cornering.. (b) κ is larger for the inner wheel in the *Torsen* case and lower for the outer wheel.

Figure 6.4: The lateral acceleration a_y is almost identical for both differentials; it is slightly lower for the *Torsen* differential. Like the longitudinal slip κ of the wheels, a_y increases as the velocity in the corner increases. As illustrated in Figure 5.9, the difference in longitudinal slips, which is motivated by the difference in longitudinal forces, leads to a greater understeer moment with the *Torsen* differential.

The torque bias of the *Torsen* differential applies more torque to the inner wheel because it is the slowest (c.f.r chapter 3). Therefore, as the *TBR* is larger for the *Torsen* differential, the torque on the left wheel (the inner wheel) is larger than that on the left wheel of the open differential. Consequently, the longitudinal slip of the inner wheel (left wheel) is higher for the *Torsen* differential compared to the open differential (Figure 6.5). This difference leads to increased energy dissipation in the tyres (Figure 6.4b). To compare the behaviour of the two differentials, we will use the instant $t = 18$ s as a reference point. At this moment, the power dissipated by the left and right tyres can be computed as follows:

$$P_{dissipative} = \vec{F} \cdot \vec{V}_m = F_{long}V_{m,x} + F_{lat}V_{m,y} \quad (6.1)$$

With $V_{m,x}$ the longitudinal velocity of the material contact point between the tyre and the ground, and $V_{m,y}$ its lateral component. For the reference point, we find:

Dissipative Powers (t = 18s) - $T_m = 50$ Nm						
$\backslash$	CBV [m/s]	κ [%]	F_{long} [N]	$ \alpha $ [°]	F_{lat} [N]	P_{dis} [kW]
Open, RL	26.31	2.04	1774.3	1.9	1107.3	- 1.461
<i>Torsen</i> , RL	26.17	2.47	2058.4	1.78	953.9	- 1.608
Open, RR	26.31	1.36	2061.4	1.87	1357.9	- 1.475
<i>Torsen</i> , RR	26.17	0.95	1468.7	1.75	1305.5	- 1.091

Table 6.1

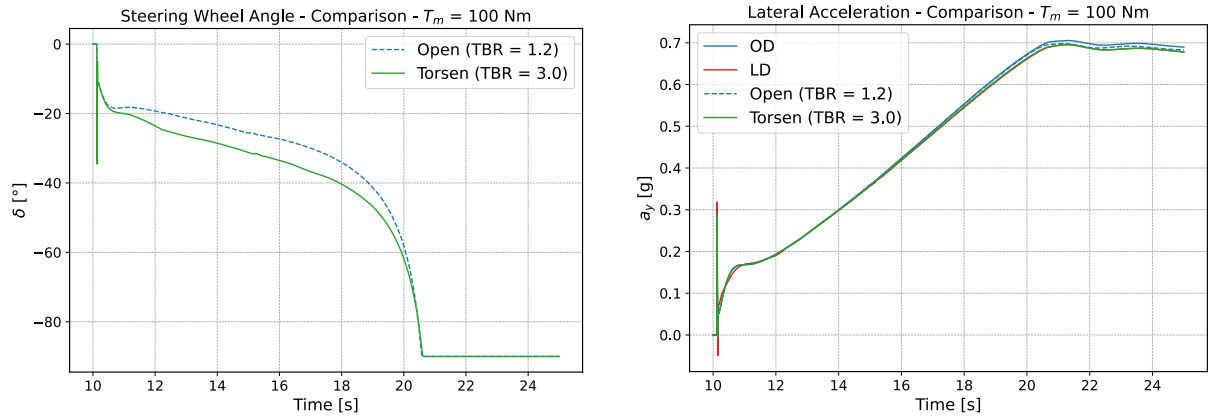
In this case of a low acceleration rate, the *Torsen* differential induces higher tyre wear on the inner wheel due to the increased dissipative power (Table 7.2). Although the power is lower on the outer wheel for the *Torsen* differential, excessive power dissipation can

still lead to premature tyre wear. This phenomenon is directly related to the impact of the spool on the manoeuvrability, as discussed in Section 3.3.3 ². Consequently, when cornering at low speed, the increased locking effect is no longer advantageous due to the accelerated tyre wear and the induced understeer (the lateral forces are lower for the *Torsen* differential). The driver needs to apply greater steering effort (the steering angle and the steering wheel torque are shown in Figures D.1a and D.1b). It remains noteworthy to increase the level of engine torque and to check the evolution of the driver's reaction. This will demonstrate why the *Torsen* differential favours torque balance on the inner wheel during low- or mid-speed cornering.

6.2.2 Transition Acceleration Rate

As introduced in the *Differentials Validation* section, the extended test for the engine torque of 100 Nm reveals different behaviour perspective between the *Torsen* and open differentials. In this case, two different scenarios emerge for both differentials.

First, since the driver applies more torque to the differential (nearly 1000 Nm), the additional braking effect on the outer wheel tends to reduce the amount of $\Delta\omega$ between the two wheels (Figures 6.6a and 6.6b). For the open differential (Figure 6.6c), the torque bias remains constant during cornering. The outer wheel *RR* remains the faster one and the positive ΔT is applied to the inner wheel. However, the effect varies for the *Torsen* differential: after $t = 17$ s, the torque level on the inner wheel decreases while that on the outer wheel increases (Figure 6.6d).

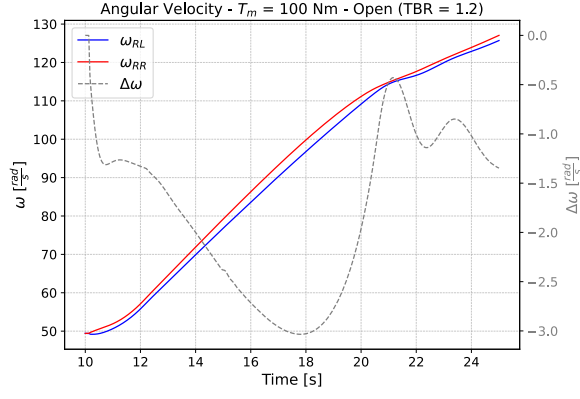


(a) The steering angle is greater for the *Torsen* differential. This is because the *Torsen* differential exhibits more understeered behaviour due to the torque balance.

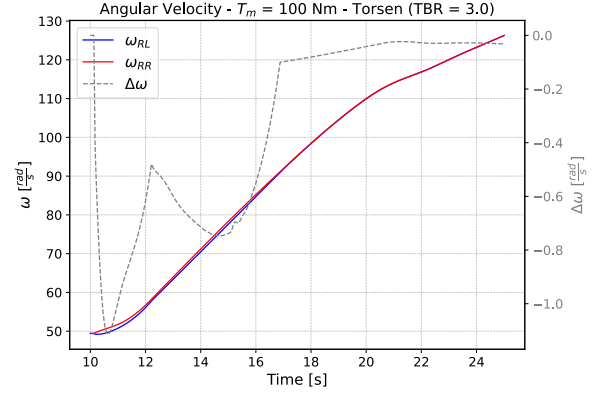
(b) The lateral acceleration achieves 0.7 g for the OD when the driver reaction reaches δ_{lim} . The a_y is a bit smaller with a higher locking effect in the differential.

Figure 6.5: The lateral acceleration a_y is saturated. The driver can not turn the steering wheel any further to counteract the understeering behaviour because δ_{lim} is reached.

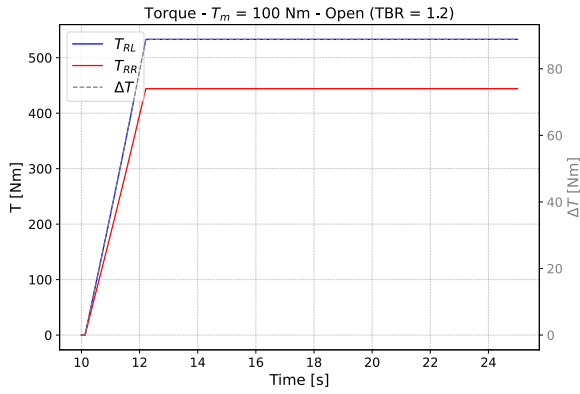
²The expression developed is a very basic method for calculating tyre power loss. In fact, tyre viscosity, elasticity or pressure are parameters quoted in many documents studying tyre wear, such as [25]. Nevertheless, the expression 6.1 allows a first comparison point between open and *Torsen* differentials with respect to tyre wear.



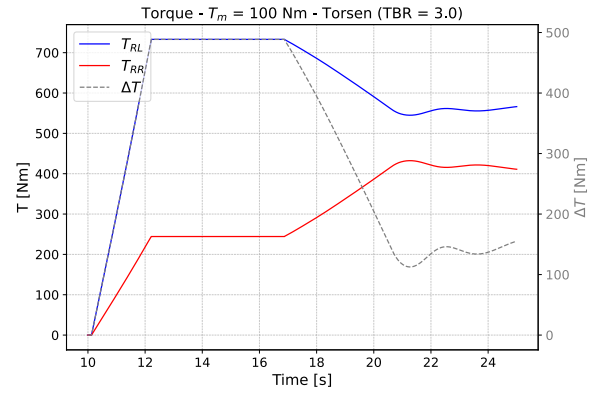
(a) The outer wheel stays the faster one, but the speed difference between both wheels is reducing.



(b) With the amount of the engine torque, the *Torsen* tends to impose a common rotational velocity to the wheels.



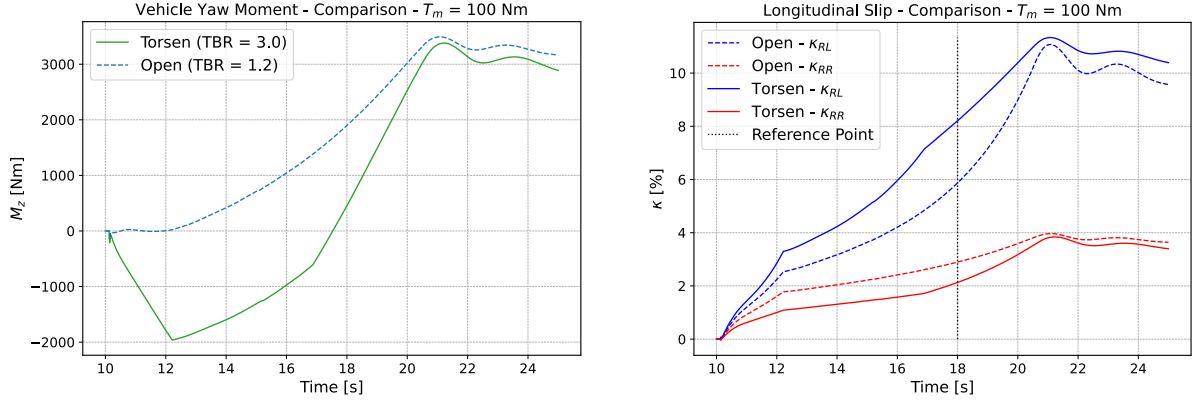
(c) The torque balance between both wheels stays constant as the $\Delta\omega$ does not change in sign.



(d) At $t = 16s$, the torque splitting starts to change since the maximum force tends to be on the outer wheel.

Figure 6.6: The inversion of the *Torsen* balance starts to be triggered at the end to the test. Nevertheless, the positive torque balance is still being applied to the inner wheel.

Currently, the driver can not counteract the understeering behaviour caused by the torque bias using steering input alone because the limit on the steering wheel angle δ_{lim} is achieved (Figure 6.5a). Therefore, the car achieves its maximum lateral acceleration a_y around 0.7 g (Figure 6.5b). This value is similar for all four of the tested vehicles, although it is slightly higher when the locking effect decreases (open differential). Therefore, it is interesting to compare the yaw moment with the torque bias ratio, focusing on the *open*, the *Torsen* and the locked differentials (Figure 6.7a).



(a) The yaw moments are related with the longitudinal forces. It remains positive for the open while it tends to be more negative when the locking effect increases.

(b) The longitudinal slip tends to saturate when a_y reaches the maximal value. The $\Delta\kappa$ remains larger for the *Torsen*.

Figure 6.7: When the longitudinal slips stop increasing, the longitudinal forces remain constant and the yaw moment is constant. The yaw moment stops to increase when the lateral acceleration a_y saturates. The *Torsen* differential induces a higher longitudinal slip difference than the open because of the torque distribution.

The primary reason for the understeered behaviour is the longitudinal forces distribution. As explained previously, the yaw moment induced by the longitudinal forces distribution affects the vehicle's handling. The yaw moment varies in sign because of the non-linearity of the longitudinal and lateral forces distribution of the tyres. As discussed in Section 2.1.2, when the longitudinal slip of the inner wheel becomes too high and exceeds the saturation limit, the longitudinal force F_{long} saturates. In cornering, the load transfer is higher on the outer wheel and induces a higher longitudinal force peak. Therefore, an inversion moment occurs when the longitudinal force on the outer wheel exceeds that on the inner wheel. This moment arises because the longitudinal slip of the outer wheel grows as the car's longitudinal velocity increases. This effect is less pronounced for the open differential because the difference in longitudinal slip between the two wheels is smaller.

The second reason is the torque balance of the *Torsen* differential. The *Torsen* torque distribution favours the torque to the wheel with the best grip. As Figure 6.8 shows, at the reference moment ($t = 18$ s), the longitudinal slip κ for both wheels remains below the saturation limit. Therefore, the left wheel (the inner one) has more grip than the right wheel (the outer wheel) because its longitudinal slip is closer to the saturation limit. The left wheel (the inner wheel) can therefore develop more longitudinal force than the right wheel (the outer wheel). Thus, the *Torsen* differential applies more torque to the inner wheel in this case.

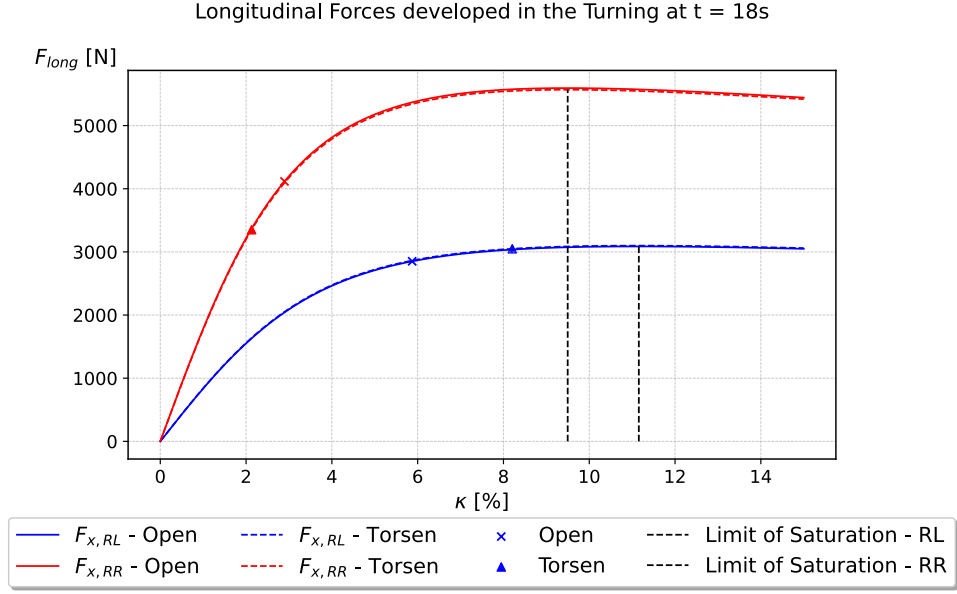


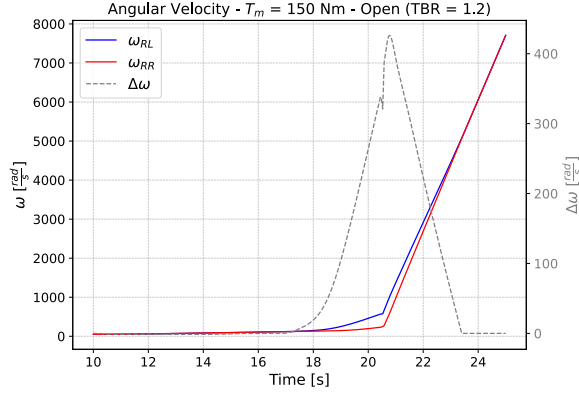
Figure 6.8: Due to the lateral load transfer, the longitudinal force becomes higher on the right wheel (the outer wheel of the corner). However, the *Torsen* applies more torque to left wheel because the longitudinal slip of this wheel is closer to the saturation limit (the best grip point).

Therefore, the lateral load transfer induced by the lateral acceleration a_y , which results from the engine torque and the steering angle (or the turning radius R), plays a key role in the torque distribution between the inner and outer wheels. The *Torsen* torque distribution tends to reverse at the end of the corner (Figure 6.6d). The understeered tendency produced by the torque distribution decreases. However, the engine torque is insufficient to trigger an inversion of the torque transfer because the left wheel maintains the best grip. The driver has already reached the maximum steering angle δ_{lim} and the yaw rate saturates (Figure D.3). Therefore, he is unable to produce additional lateral acceleration a_y through the steering angle to induce oversteer.

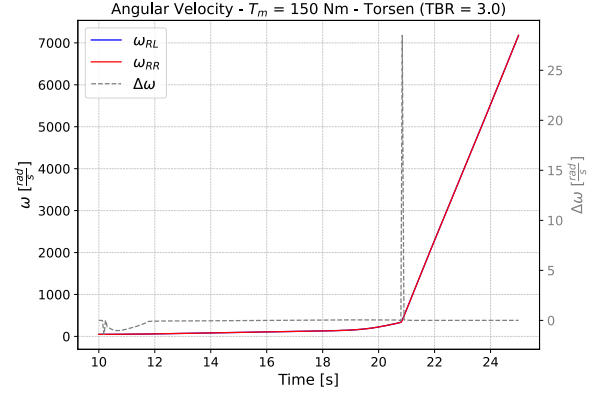
In this case, we observe the positive effect of the *Torsen* differential: the inversion of the torque balance. Nonetheless, the engine torque remains insufficient to highlight this effect as a positive feature when cornering.

6.2.3 High Acceleration Cornering

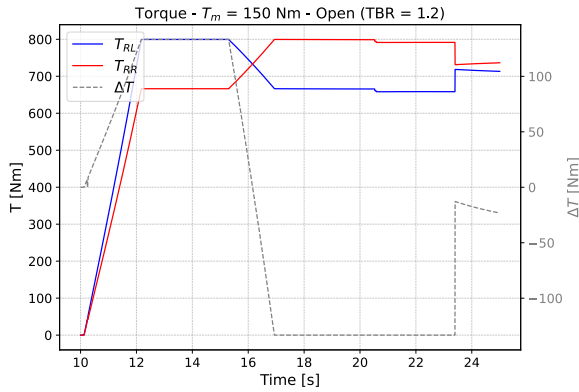
The driver begins cornering with an engine torque of $T_m = 150$ Nm. The torque balance inversion occurs slightly earlier for the open differential than for the *Torsen* one (Figures 6.9c and 6.9d). This generates an oversteering effect rather than an understeering one. Given the *TBR* coefficient of the *Torsen*, this moment variation is more pronounced. In this case, the variation in torque bias can be spotted in Figure 6.9d. This is because the measured *TBR* is not fully reached under the applied engine torque.



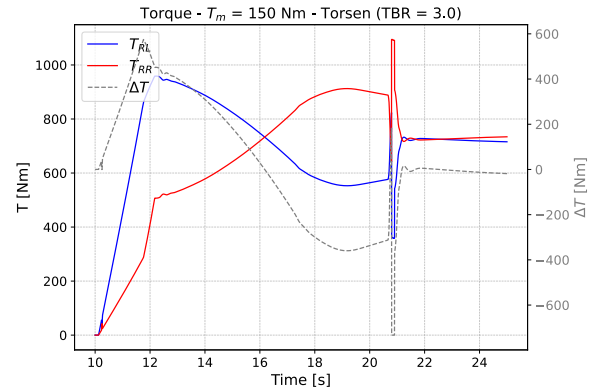
(a) The open differential accepts a rotational speed differences until both wheels lose grip and sharply accelerate.



(b) The $\Delta\omega$ becomes non-null when the wheels slip saturates.



(c) The inversion of torque bias is triggered by the load transfer and the lateral acceleration. It creates an oversteering moment useful for the driver.

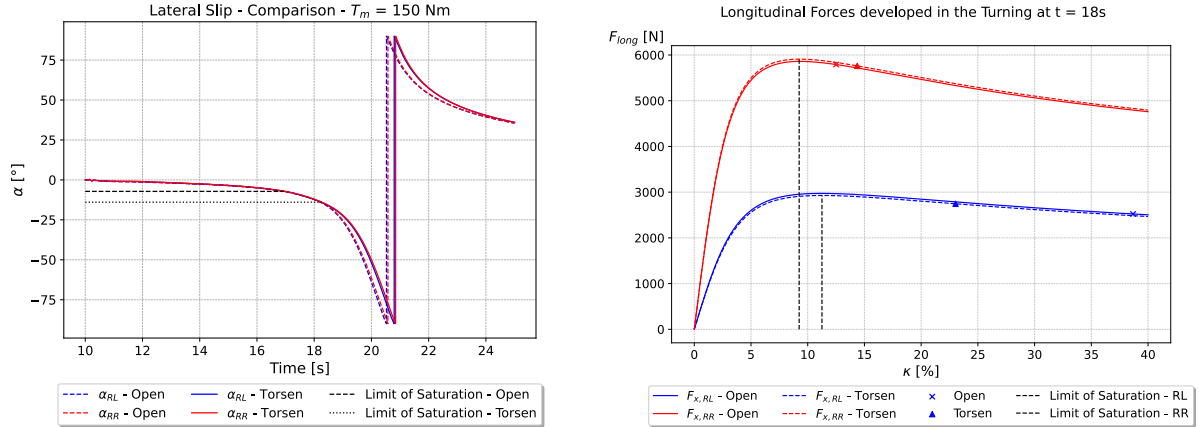


(d) The inversion of torque balance occurs later than for the open differential, but the amount of oversteering moment induced by the torque balance is larger because of the TBR .

Figure 6.9: With a sufficient engine torque, the *Torsen* offers a positive effect with the oversteering moment, before the vehicle loses grip and spin.

The angular velocities of the open differential and the *Torsen* differential allow the introduction of a first comparison point (Figure 6.9a and 6.9b). We can see that the open differential triggers the speed difference between wheels at $t = 17$ s. It is induced because the measured TBR overcomes the fixed TBR of the open differential. Unlike the engine torque $T_m = 50$ and 100 Nm at $t = 17$ s, it is the inner wheel *RL* that rotates faster than the outer wheel. The positive torque balance on the right wheel attempts to reduce the speed difference. As previously mentioned, the load transfer induces longitudinal slippage of the inner wheel for both differentials. However, while the *Torsen* differential can apply torque to the slower wheel with better grip, the low amount of friction inside the *open* differential limits the torque bias. Since it can no longer transfer additional torque to the outer wheel with the best grip, the torque applied to the inner wheel exceeds the maximum that can be transmitted by the tyre. The saturation limit is exceeded. The wheel then accelerates, causing the tyre to skid (Figure 6.10b). Additionally, it is evident that the inner wheel of the open differential is slipping before the one of the *Torsen* differential (Figure 6.10a). The lateral slip saturation (α_{max}) has been exceeded. Therefore, the driver

of a vehicle with an open differential will lose control before the driver of a vehicle with a *Torsen* differential, for the same amount of engine torque.



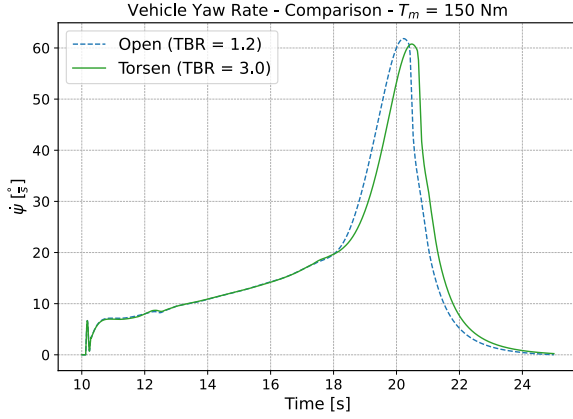
(a) Lateral slip saturation occurs later with the *Torsen* differential. Therefore, grip is enhanced when cornering, enabling the driver to hold the turn for longer before losing control.

(b) Compared to the *Torsen* differential, the tyre for the open differential on the left wheel (the inner one) skids a lot more. The tyre cannot transfer more force and dissipates a lot of energy.

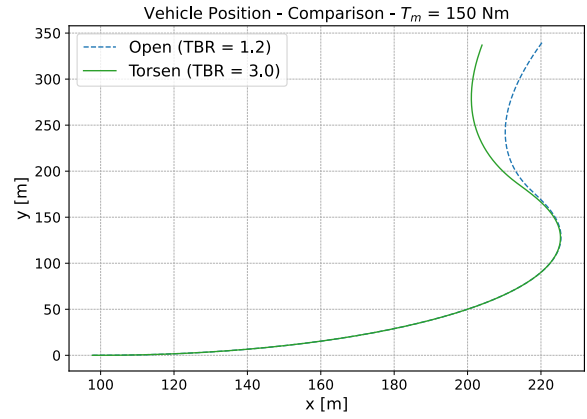
Figure 6.10: The lateral saturation slip limit is reached later in the case of the *Torsen* differential. The open longitudinal slip extends further than with the *Torsen* differential, which reduces the longitudinal force that the tyre can transmit to the ground.

The speed difference continues to increase until $t = 20$ s, at which point the rotational speeds rise sharply (Figure 6.9a and 6.9b). At this moment, the vehicles lose grip due to excessive oversteer. Consequently, the rear of the car steps out, causing it to 'spin' more than intended. This effect occurs later with the *Torsen* differential. As shown in Figure 6.9d, the *Torsen* differential does not allow a rotational speed difference until the fixed *TBR* is reached (when $\Delta T = -750$ Nm near $t = 21$ s). Although both vehicles deviate completely from the intended path due to excessive oversteer (Figure 6.11b), it is useful to analyse where these moments occur for each differential. The loss of grip is expected, as the scenario involves constant longitudinal acceleration without considering the vehicle's dynamic response.

The vehicle's handling can be verified using the yaw rate $\dot{\psi}$. The acceleration along the z-axis grows sharply when the loss of grip is triggered (Figure 6.11a). As intended by the analyses of longitudinal and lateral slips, the vehicle with the *Torsen* accepts a larger yaw rate before losing grip and spinning; the position of the yaw rate peak shifts right as the locking effect increases.



(a) The yaw rate peak arrives later for the *Torsen* differential. It admits a longer derive time before yaw rate peak comes.



(b) The oversteering handling is highlighted with the small difference between both curves near $x = 220\text{m}$.

Figure 6.11: The yaw rate peak causes both vehicles to lose grip and deviate from their intended path. This peak occurs later for the *Torsen*. A car with a *Torsen* differential can hold a corner for longer than a car with an open differential and a high acceleration rate.

Before the vehicles spin, the torque balance induced by the *Torsen* law distribution proves why the *Torsen* is more advantageous than the open differential (Figure 6.9d). With the inversion of the torque bias, the variation in the torque balance is beneficial for maintaining grip in high-speed corners compared to the open option. The *Torsen* limits the lateral slip. The tyres tend to work in their optimal area of performance (closer to the saturation limit), as can be seen in Figure 6.10b. Hence, under the same turning conditions, the *Torsen* allows a smaller *drift angle* β (Figure 6.12). Thanks to a better grip and higher cornering forces (induced by smaller lateral slip angles), he spends less time in curves.

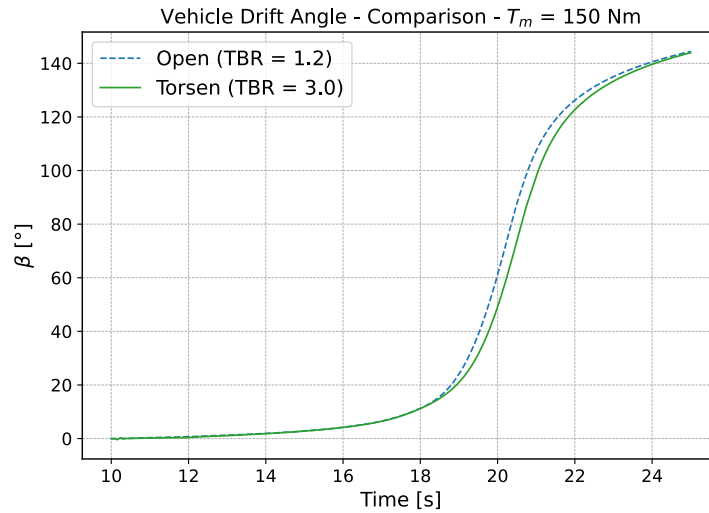
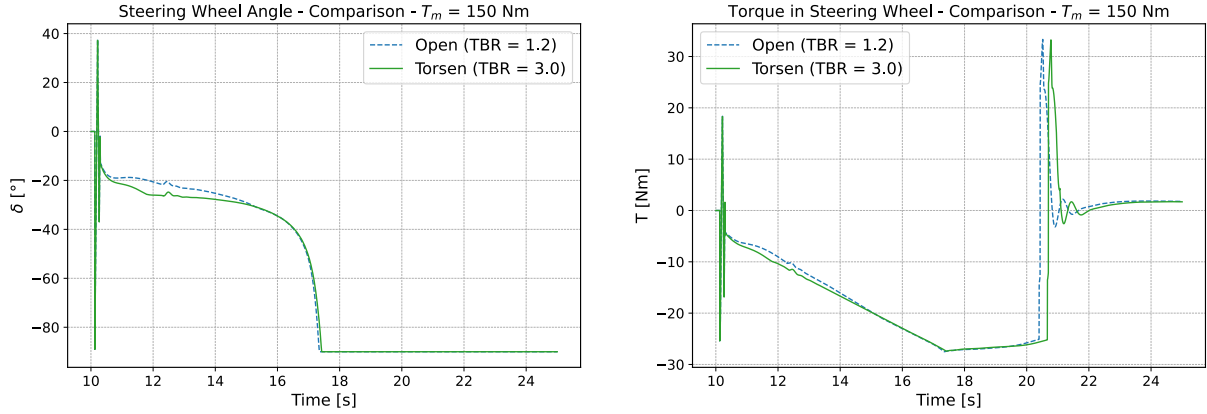


Figure 6.12: The *Torsen* differential provides the driver with more driving facilities thanks to the drift angle shift between the two differentials. The *Torsen* differential results in a lower drift angle for the same position in the corner.

6.2.4 Cornering Ability

With a smaller drift angle, the driver has greater control over aligning the vehicle with its intended trajectory. This reduces the required steering wheel angle correction (Figure 6.13). Before the steering wheel angle reaches the δ_{lim} (90°), the difficulty of the driver's reaction reverses. At $t = 16$ s, the steering angle δ needed for the *Torsen* differential crosses the curve for the open differential. The correction for the latter is stronger than that for the *Torsen*. Therefore, as can be seen in Figure 6.13, the steering effort required for the open differential increases.



(a) The δ for the open differential crosses the curve for the *Torsen*. The steering angle needed becomes stronger for the open.

(b) The steering torque increases for both differentials until δ reaches δ_{lim} . After $t = 16$ s, the steering torque for the open differential becomes slightly higher than that for the *Torsen*.

Figure 6.13: The required driver's steering action varies during the turn. The *Torsen* differential becomes weaker at high-speed corners. The peak in steering torque at $t = 20$ s occurs because the car is spinning.

For high-speed driving, the *Torsen* offers more cornering stability. This can be highlighted by the relationship between the steering angle and the lateral acceleration a_y . As discussed in the theoretical section with Figure 2.21, the steering wheel angle response varies according to the differential used. Depending on the understeered or oversteered behaviour (influenced by the engine torque, as explained), the steering response varies. In the following example, an engine torque of $T_m = 200$ Nm was applied to induce an oversteered car. This amount of torque allows for a quicker driver response because oversteering arises earlier than with an engine torque of $T_m = 150$ Nm.

For the area 1, only the torque balance between the two wheels may help to align the vehicle in the cornering. This is why, for the same level of steering angle, the lateral acceleration rises during cornering (Figure 6.14).

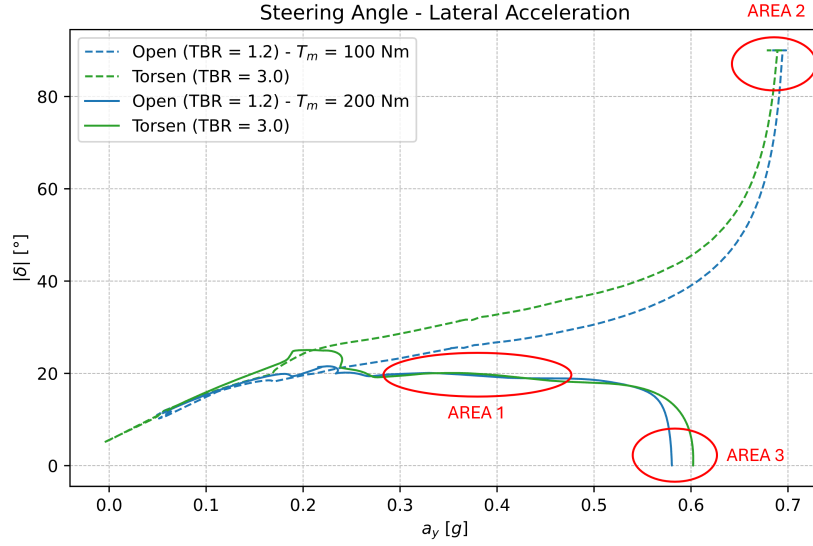


Figure 6.14: Three different areas highlight variations in the driver steering response. This depends on the differential type and the amount of the injected engine torque. The PD controller parameters play a key role in the transition zone at $a_y = 0.2g$.

Area 3 shows the moment when the steering angle decreases while maintaining the same level of lateral acceleration a_y . Thanks to the positive effect of the torque balance, the driver reduces the initially applied steering wheel angle δ . Since the effect is more pronounced with a *Torsen* differential, a vehicle with this type of differential will achieve a greater lateral acceleration with the same steering wheel angle. The curves end before the driver needs to countersteer. As discussed in the theoretical section and shown in Figure 2.18b, an excessive oversteering moment results in a yaw angle that exceeds that required for the cornering path. This forces the driver to steer in the opposite direction to maintain control of the vehicle's trajectory. With the higher lateral acceleration offered by the *Torsen*, the driver enhances the lateral forces F_y during cornering compared to the open differential. When cornering at high velocity, the vehicle keeps more grip because the lateral and longitudinal slips remain within the range of values that produce the maximal forces (Figure 6.10b).

Area 2 displays the understeering phenomenon, as demonstrated by the low and middle acceleration rate tests ($T_m = 50$ Nm and 100 Nm). With a *Torsen* differential, the driver needs to turn the steering wheel further in order to maintain the same lateral acceleration as with an open differential. This explains why the transition part of Figure 6.14 before the second area is larger for the *Torsen* differential ($a_y \simeq 0.2g$). Before achieving the oversteering moment (the inversion of the torque bias), the vehicle exhibits an understeered behaviour induced by the positive torque balance on the inner wheel. The variation in path can easily be seen by zooming in on the car's position during the test. The trajectory with the *Torsen* begins to be larger than the open differential (Figure 6.15). Once the torque balance inversion occurs, the trajectory of the open differential crosses that of the *Torsen*. This variation in torque balance enables the vehicle to follow a tighter curve at the same level of engine torque and steering angle. As a result, the path length through the turn is reduced, enabling the driver to save valuable time compared to an open differential.

Therefore, in sport car competition such as *WRC* or *Formula 1*, where drivers need to be as fast as possible, the *Torsen* differential is the best choice in terms of mechanical performance and vehicle handling.

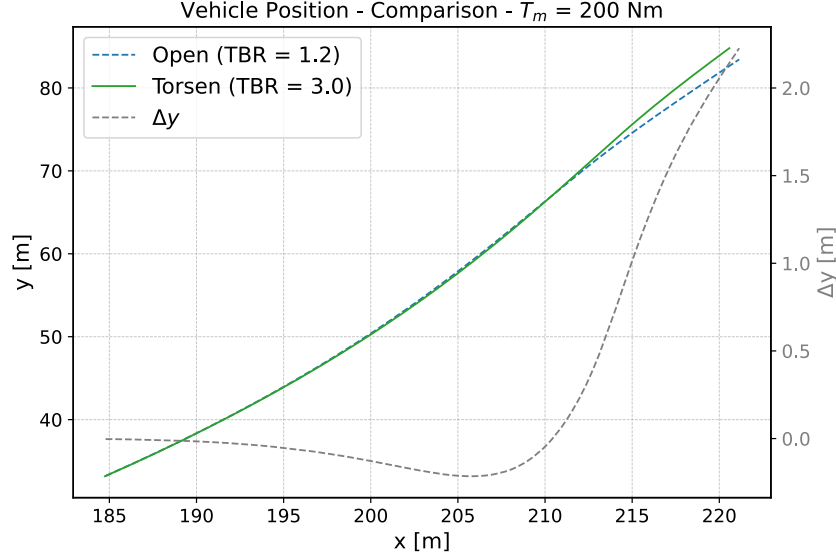


Figure 6.15: The *Torsen* differential curve tightens, enabling the driver to complete the corner more quickly. Understeer appears reduced because of the PD control on the trajectory. The Δy is defined as $\Delta y = y_{Torsen} - y_{open}$

Regarding low-rate acceleration, the locking effect resulted in excessive tyre wear and seemed less advantageous. However, with a higher engine torque in the corner, the tendency reverses. Comparing the results in Table 6.1 with the dissipative power for the high engine torque (Table 6.2), the *Torsen* reduces the tyre wear and becomes the most tyre-preserving option at high speeds. For the example, the data at $x = 185$ m (Figure 6.15) show the following results:

Dissipative Powers ($x = 185$) - $T_m = 200Nm$						
$\backslash$	CBV [m/s]	κ [%]	F_{long} [N]	$ \alpha $ [°]	F_{lat} [N]	P_{dis} [kW]
Open, RL	27.89	39.96	2947.6	7.91	916.4	-35.874
<i>Torsen</i> , RL	27.94	20.37	3327.2	7.46	1490.3	-24.052
Open, RR	27.89	12.26	5752.1	7.76	3248.3	-32.164
<i>Torsen</i> , RR	27.94	13.13	5756.5	7.34	3018.1	-32.069

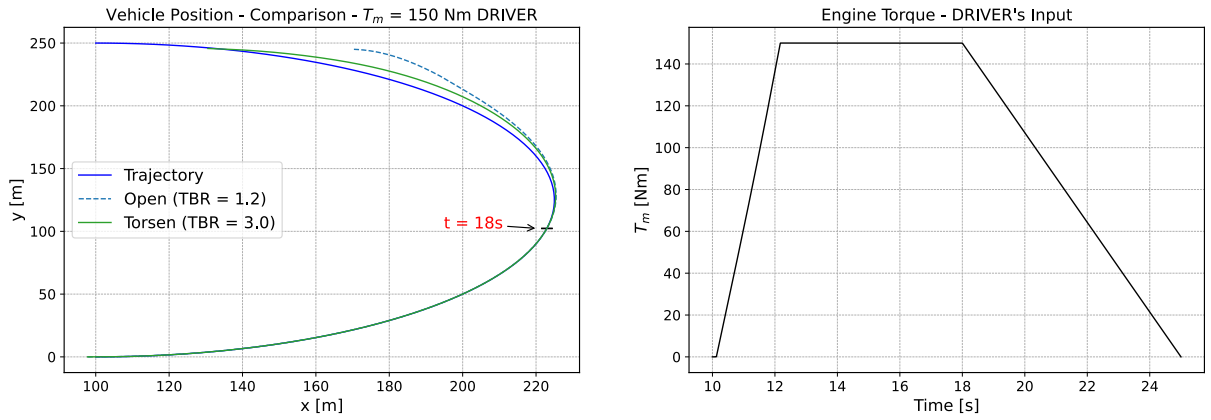
Table 6.2

At this particular point, the vehicle with an open differential is slipping completely and losing grip. The value of the longitudinal force on the left tyre is significantly lower than that for the *Torsen* differential. The longitudinal slip ratio κ for the left wheel is significantly higher with an open differential than with a *Torsen*. Therefore, the values for the dissipative power P_{dis} shift to huge amounts. Nevertheless, the *Torsen* differential

seems to prevent tyre wear during high-speed cornering and provides better cornering performance in terms of forces and torque repartition between the two wheels and a smaller drift angle. This allows the vehicle to recover velocity in the corner, as Table 6.2 shows (*CBV* means Carbody Velocity).

6.3 Driver Cornering Influence

From the points developed previously, the driver's actions with regard to pedal torque and steering wheel input clearly influence the vehicle's handling. Engine torque (T_m) and steering wheel angle (δ) play a key role in lateral load transfer and torque distribution. The vehicle's performance in curve can be advantageous for a standard driver from a tyre preservation point of view, as well as for a more sporty driver. The *Torsen* differential considerably improves speed and therefore cornering time, compared with an open differential. The amount of time saved depends mainly on how the driver handles the vehicle when cornering. Therefore, it is instructive to compare the performance of the two vehicles on a common curve with a modification to the torque during the turn ($R = 125$ m). The engine torque that the driver applied at the start of the turn is reduced during the half-turn (180° turn). This reduction occurs at $t = 18$ s, when the torque transfer has been reversed (Figure 6.16a). The engine torque curve used for the two differentials is shown in Figure 6.16b. The driver starts the corner with $T_m = 150$ Nm. At $t = 18$ s, he reduces the engine torque value to reach $T_m = 0$ Nm when the simulation ends at $t = 25$ s. The objective is to compare vehicle performance while preventing the car from spinning.



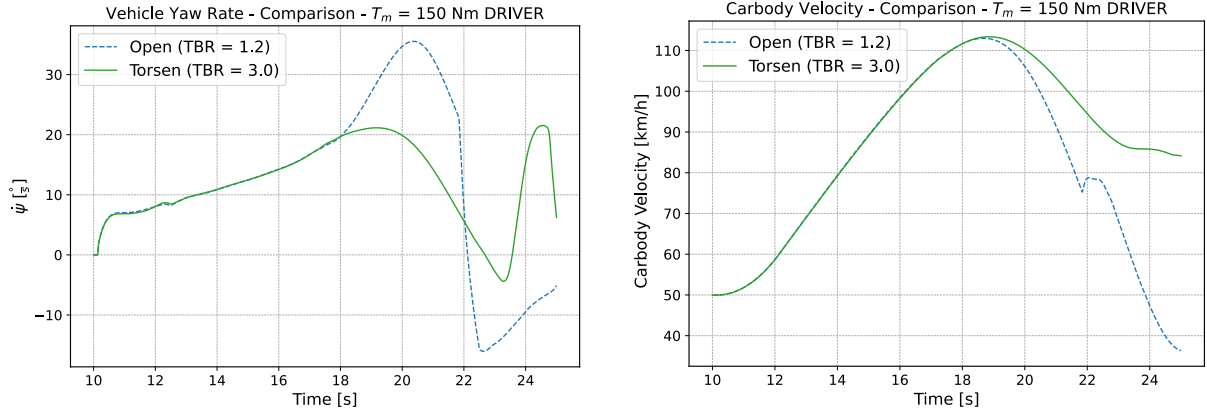
(a) The *Torsen* differential stays closer to the targeted trajectory than an open differential. This is because the *Torsen* offers a supplementary oversteered moment through its torque distribution.

(b) The driver reduces the engine's torque T_m when turning in order to prevent the car from spinning and the driver from losing control.

Figure 6.16: To avoid losing control of the car, the driver reduces the level of engine torque once torque inversion is reached at $t = 18$ s.

The *Torsen* differential allows the driver to recover the trajectory, whereas the vehicle with an open differential leaves the corner (Figure 6.16a). In reality, the driver with the open differential loses control of his vehicle as the yaw rate peaks (Figure 6.17a). As torque is transferred more to the outer wheel (Figures D.4a and D.4b in Appendix D.4), the *Torsen*

differential provides an improved oversteered moment. Compared to an open differential, the drift angle β decreases when cornering (Figure 6.18a). This improves the vehicle's orientation in the corner, enabling the driver with the *Torsen* differential to go faster. Therefore, as shown in Figure 6.17b, the velocity of the *Torsen* vehicle is higher, enabling the driver to save valuable time by avoiding excessive grip losses when taking the same corner.

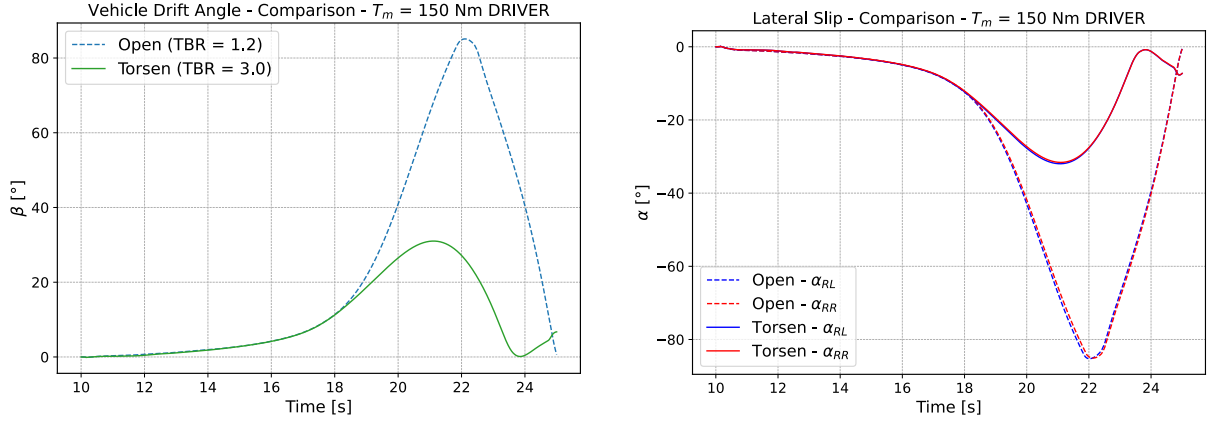


(a) The yaw rate peak for the open differential causes the driver to lose control. The driver of the vehicle with the *Torsen* differential keeps the control in the corner.

(b) Thanks to the *Torsen* differential's enhanced handling, the vehicle's velocity in the curve is improved compared to the open differential.

Figure 6.17: *Torsen* differential allows for higher velocity in corners. Moreover, the driver of an open differential loses control due to the yaw rate peak. Therefore, the *Torsen* differential offers a significant advantage over the open differential.

The lateral slip angle of the rear tyres is lower with a *Torsen* differential thanks to the optimised torque balance (Figure 6.18b). With the *TBR* level in the *Torsen* differential, more torque is applied to the wheel with the best grip (i.e. the outer wheel). Thus, the drift angle β is reduced (Figure 6.18a). Therefore, the lateral forces in the cornering in the case of the *Torsen* differential are greater. The vehicle drifts less and is more stable when exiting the curve. For the same corner, the *Torsen* offers valuable timing savings because the vehicle handling performance is improved.



(a) The drift angle is lower for the *Torsen* differential. The vehicle drifts less when turning and the driver can maintain more speed. This enhances the vehicle's orientation and improves stability when cornering.

(b) Because the lateral slip angle for the rear wheels is lower for the *Torsen* differential and closer to the saturation limit, the *Torsen*'s lateral force is improved compared to an open differential.

Figure 6.18: The open differential has a greater lateral slip angle and drift angle than the *Torsen* differential. Hence, the *Torsen* differential will provide superior cornering performance.

Ultimately, the time saved by the *Torsen* differential on curves can be computed. In the test, the vehicles travelled around the turn for 15 seconds before the simulation ended. The vehicle with the open differential travelled 160.66 metres around the bend, whereas the vehicle with the *Torsen* differential travelled 192.431 metres. The time taken by the *Torsen* vehicle to cover an equivalent distance to that covered by the vehicle with the open differential helps us to calculate the time saved when cornering:

$$t_{Open, 160.66m} = 15s \quad \rightarrow \quad t_{Torsen, 160.66m} = 13.63s$$

$$\Delta t = 15 - 13.63 = 1.37s \quad \text{Time saved by the } Torsen$$

In addition to improving cornering stability, the *Torsen* vehicle achieves a time saving of 1.37 seconds in this situation. This significant time saving could be particularly beneficial in competitive events such as the *WRC* or *Formula 1*. These savings are considerable when compared with the qualifying time differences mentioned in the introduction to this Master's thesis. The global representation of the situation with the time saved is presented in Figure 6.19.

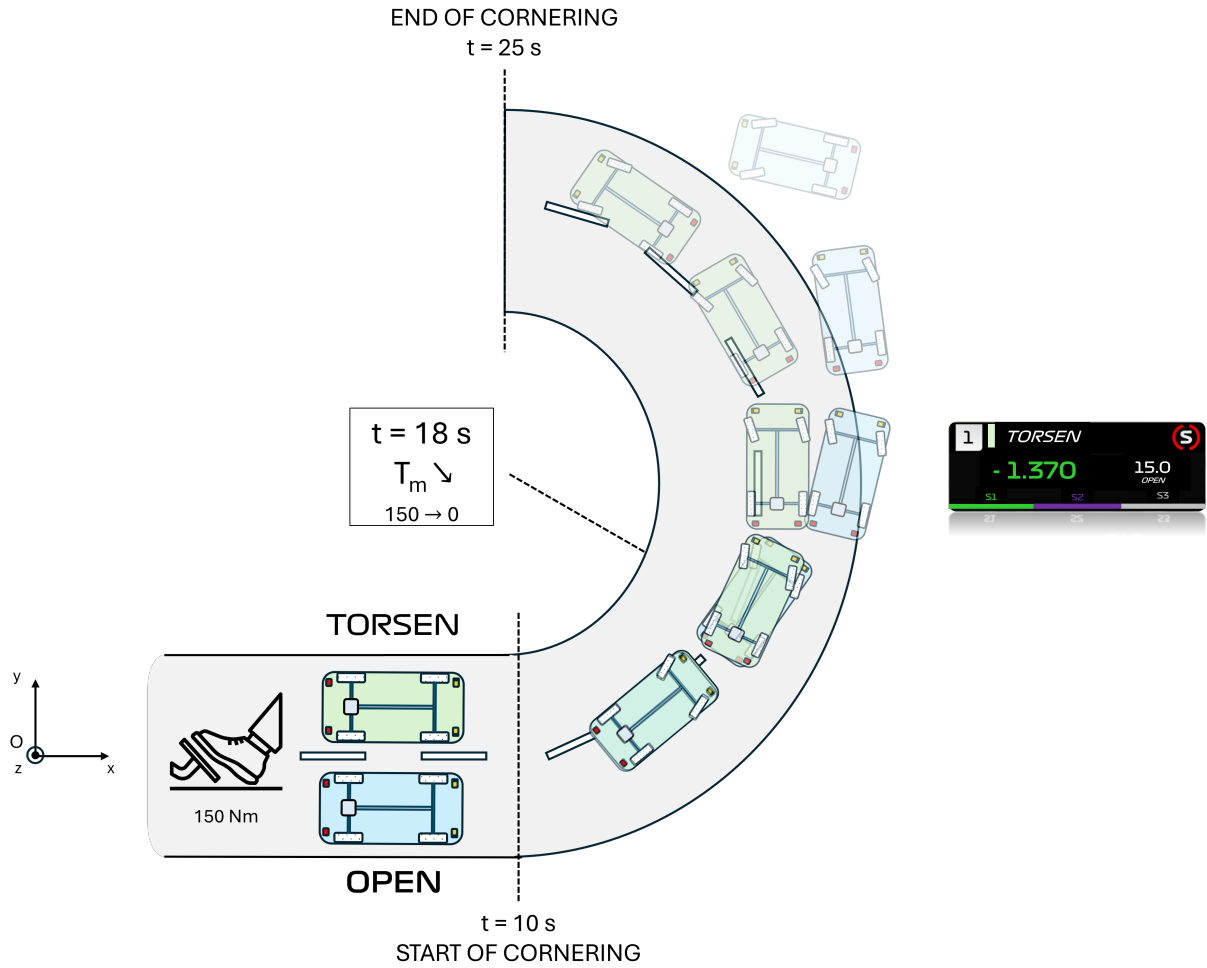


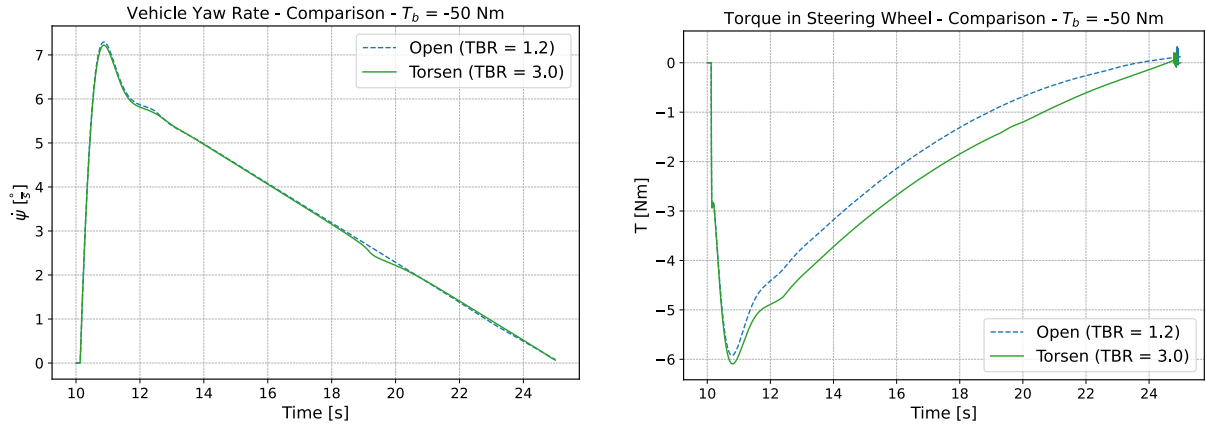
Figure 6.19: The global situation and the benefits of the *Torsen* differential are presented. Compared to an open differential, a vehicle with a *Torsen* differential can recover its intended trajectory at a higher velocity. While the driver with the open differential loses control of his car (the blue car), the driver with the *Torsen* differential (the green car) saves 1.37 seconds and is able to reorient his car at the end of the turn for the next stage.

6.4 Throttle-off Reaction

The last situation to discuss is the coast mode cornering. In this case, the driver applies a braking torque of $T_b = T_m = -50$ Nm. The goal is to test a "throttle-off" situation with a negative torque simulating the engine braking. This torque is injected into the differential.

According to the negative *Torsen* law torque distribution (Figure 3.23), the torque balance is applied to the fastest wheel (Figures 6.21c and 6.21d). Therefore, the supplementary braking torque distributed by the torque bias reduces the angular speed difference $\Delta\omega$ between the two wheels. By comparing Figures 6.21a and 6.21b, the *Torsen* differential reduces the rotational velocity of the fastest wheel more quickly due to the variation in *TBR*. The negative torque repartition induces understeer. This phenomenon is more pronounced with the *Torsen* differential compared to an open differential. The driver must therefore apply more torque to the steering wheel to maintain the trajectory (Figure

6.20b). Nevertheless, the vehicle remains on the curve (see Appendix D.5).

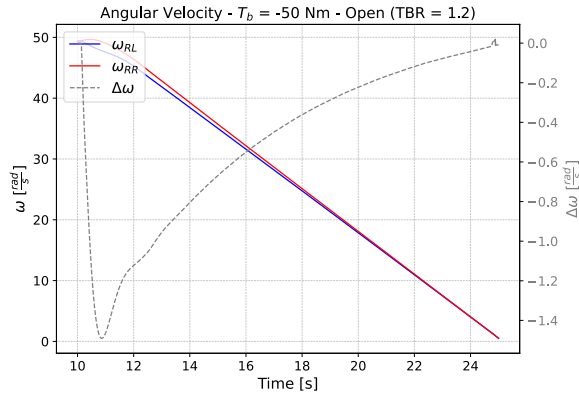


(a) The yaw rate for the *Torsen* differential is slightly lower than for the open differential. This remains noteworthy for the stability of the vehicle.

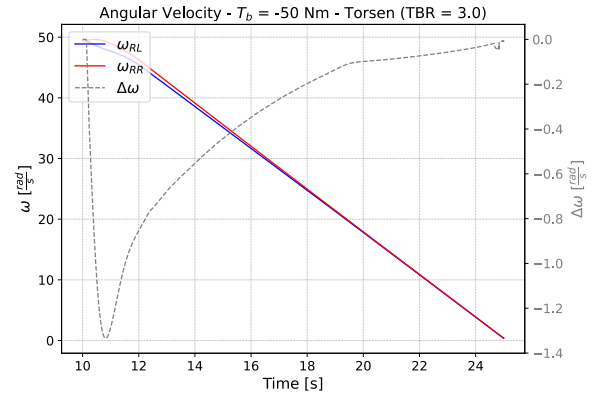
(b) As intended, the driver steering torque is higher in the case of the *Torsen*. Therefore, the curve is more difficult for him to follow.

Figure 6.20: Although the supplementary negative torque balance on the faster wheel has a positive effect on the vehicle handling, the driver's reaction is stronger with the *Torsen* differential.

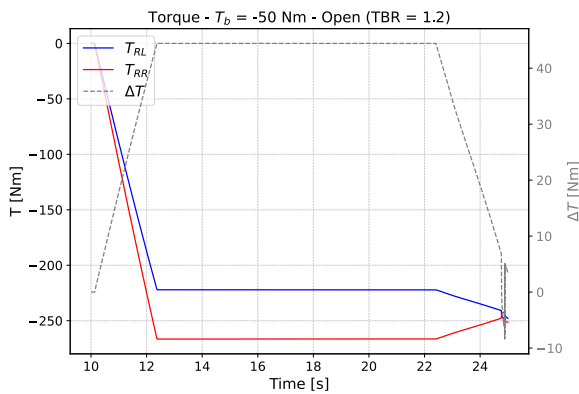
The enhanced understeered moment induced by the *Torsen* differential results in a variation in the yaw rate. In Figure 6.20a, the *Torsen* possesses a smaller yaw rate than the open differential. In a throttle-off situation, the *Torsen* works like a *yaw damper* and improves the cornering stability when the driver stops accelerating. The effect is particularly evident in Figure D.6a when a throttle-off situation with a constant steering wheel angle is tested. It can also be seen that the understeer of the *Torsen* differential pushes the vehicle out of the corner. Thanks to the locking effect, the stability of the vehicle is therefore enhanced.



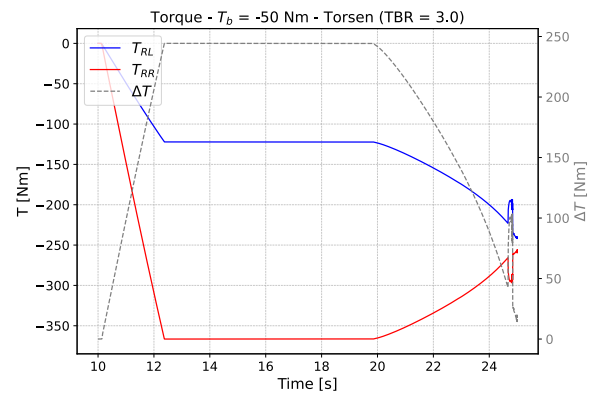
(a) The braking torque reduces the wheel speeds and also the speed difference between the wheels.



(b) The braking effect on the faster wheel is stronger due to the larger locking effect.



(c) Because the speed between both wheels is reducing, the torque's law goes throw the slope of the linear *Torsen* law.



(d) The torque applied on the outer wheel (fastest one) is higher in absolute value. The torque bias creates the understeered moment of the *Torsen*.

Figure 6.21: The torque bias reduces the angular speed difference and induces the previously presented understeered behaviour. The last part of the torque distribution change because the wheel speeds difference decreases.

At the end, the stabilisation effect is positive until the vehicle's understeering becomes too large for the driver to compensate for with steering wheel input. Therefore, as explained in the last part of Section 3.5.1, the *TBR* of driving or coasting mode can be tuned to match the desired behaviour of the vehicle as specified by the car manufacturer or racing team for their vehicles. Another positive aspect of the locking effect is the power loss during the throttle-off. Due to the friction, there is a certain amount of power dissipated into the differential. The friction power W_f remains larger for the *Torsen* differential (Figure 6.22). This power can be recovered in hybrid or electric vehicles to recharge the batteries. Under *WRC* regulations, this is particularly relevant as the cars are required to be hybrid and equipped with batteries.

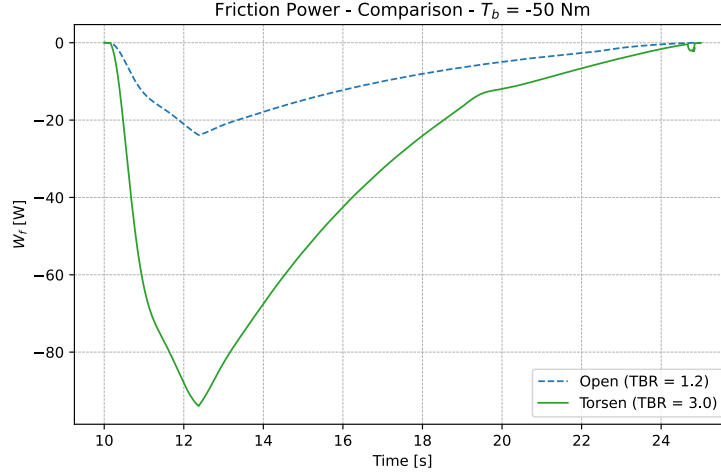


Figure 6.22: The *Torsen* differential produces more friction power thanks to its higher locking effect. This can be converted into energy for the batteries in case of hybrid or electric cars. The expression 3.7 is used.

6.5 Conclusion on Vehicle Cornering Performance

In conclusion, this chapter highlights two distinct performance windows in which one differential appears to be more effective than the other. In a standard driving situation involving low acceleration cornering, the driver needs to apply less torque to the steering wheel with the open differential to achieve the same performance as with the *Torsen*. Furthermore, the tyres are more preserved. Nevertheless, at middle or high acceleration cornering, the tendency is reversed: the *Torsen* differential provides the vehicle with more grip, better performance and handling. The driver can push the car to its limits while better limiting tyre wear during the cornering. The advantages conferred by the *Torsen* differential over the open differential appear to make it suited to sports cars or motorsport applications rather than to average drivers.

Chapter 7

Split- μ Test: Open VS *Torsen*

The *Split* - μ test is designed to evaluate the *Torsen* and the open differentials in conditions involving sudden surface changes in the road surface. It simulates real-world scenarios in which a driver might encounter a puddle or a patch of ice. Beyond daily driving situations, it also mimics the challenging surface transitions encountered in rally racing, where drivers must adapt to changing grip levels to optimise cornering times and trajectories. To simulate these conditions, the *RWD* vehicle is placed on a straight track. The driver applies various levels of engine torque. Between 4 and 6 seconds into the test, the left rear wheel encounters a low-friction surface, either **wet asphalt** or **ice**. The PD controller replicates the driver's reaction, maintaining a straight trajectory in response to these sudden grip losses. Figure 7.1 summarises the situation steps.

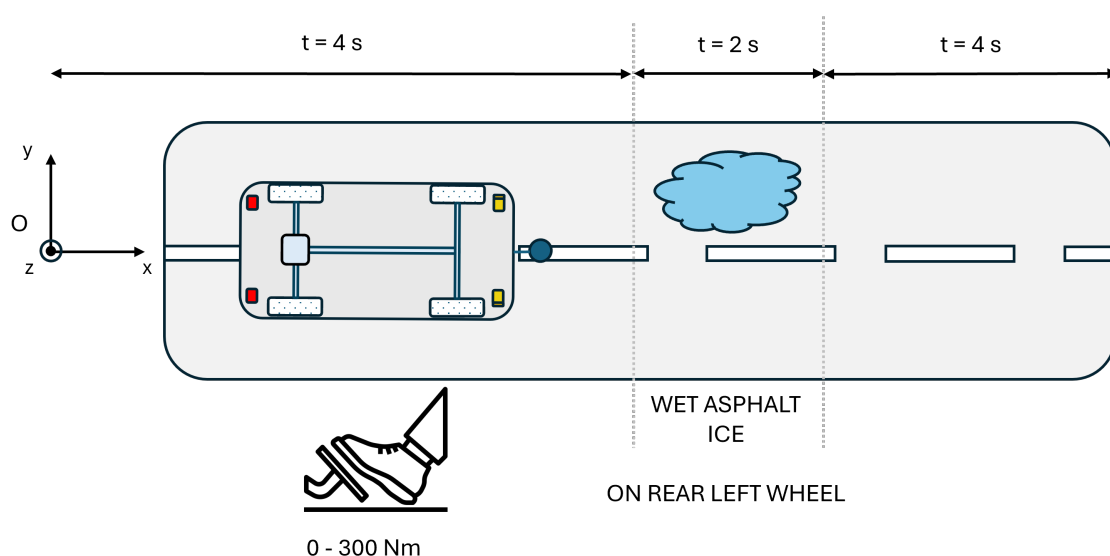


Figure 7.1: The driver starts from rest and accelerates during the 10 seconds of the simulation. After 4 seconds, there appears a patch of wet asphalt or snowed/iced road on the rear left wheel for 2 seconds. The end of the test allows us to observe the driver's reaction.

To simulate a puddle on the road, the *Bakker's* function for the rear left tyre is modified for two seconds during the test. The coefficient D is divided by the coefficient k . Therefore, the friction coefficients between the left rear tyre and the road are as follows:

Friction coefficients on the different asphalts			
$\backslash$	μ_{DRY}	μ_{WET}	μ_{ICE}
Longitudinal	1.048	0.524	0.2096
Lateral	0.912	0.456	0.182

Table 7.1

A constant engine torque application is preferred to a constant velocity start. The main purpose of the test is to focus on the torque distribution between the two wheels of the rear axle, and to compare the driver's reactions between the two differentials.

7.1 On Wet Road

The driver starts from rest and applies a certain amount of engine torque T_m during the straight line. The analysis focuses on the open differential ($TBR = 1.2$) and the *Torsen* differential ($TBR = 3.0$). Three selected engine torques ($T_m = 50, 140$ and 190 Nm) are used in order to compare the differentials and the driver's reaction to the same input data.

As a quick reminder, the difference between the open differential and the *Torsen* differential lies in the locking effect amount (TBR). In the *Split - μ* test, the TBR is a multiplication factor of the lowest available torque (as explained in Section 3.5.3). Figure 7.2 represents a global view of the *Torsen* law distribution application in the case of low surface friction on the left tyre:

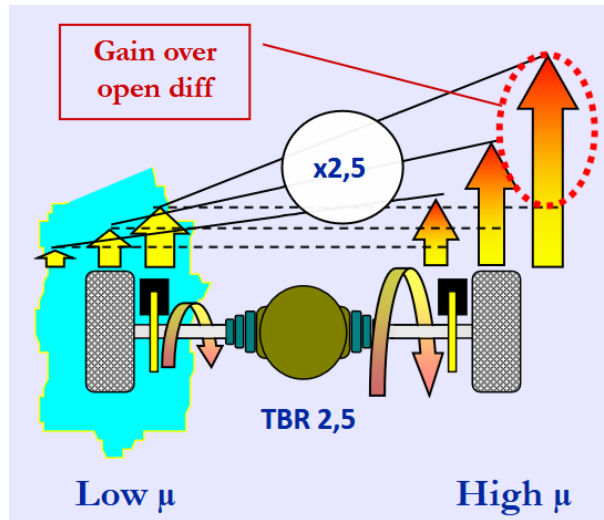


Figure 7.2: The locking effect or TBR plays the role of torque multiplier between the two wheels. Inspired from JTEKT *Torsen* source.

7.1.1 Low Acceleration Rate

The first torque tested on the water is $T_m = 50$ Nm. This represents an acceleration rate of ± 15 %. The driver applies the torque linearly during 2 seconds at the beginning

of the simulation to reach the target torque engine.

Firstly, as we can observe in Figures 7.4c and 7.4d, a torque balance between the two wheels appears as soon as the rear left wheel encounters the wet asphalt. This torque bias is caused by the lower friction coefficient between the left tyre and the ground. Because the friction is lower, the available friction torque on this wheel is lower and the effect of the *Torsen* law takes effect. As introduced in Section 3.5.3, the locking effect inside the differential aims to transfer the torque value available on the faster wheel, multiplied by the *TBR*, to the slower wheel (with the higher available friction). The *Torsen* differential favours the wheel with the best grip.

In this case, because the friction coefficients ratio and the measured *TBR* (the torque ratio directly measured on the wheels) are lower than the fixed *TBR*, the open and the *Torsen* differentials stay closed (Figure 7.3). The torque balance values are induced by the small rotational speed differences between $t = 4$ and 6 s (Figures 7.4a and 7.4b). The mechanical relations inside the differential and the torque balance prevent the left wheel from skidding on the wet surface. Therefore, the vehicle continues to have grip on both wheels. The ratio between the friction coefficients is:

$$k = \frac{\mu_{high}}{\mu_{low}} = \frac{1.048}{0.524} = \frac{0.912}{0.456} = 2.0; \quad TBR_{Open} = 1.2; \quad TBR_{Torsen} = 3.0;$$

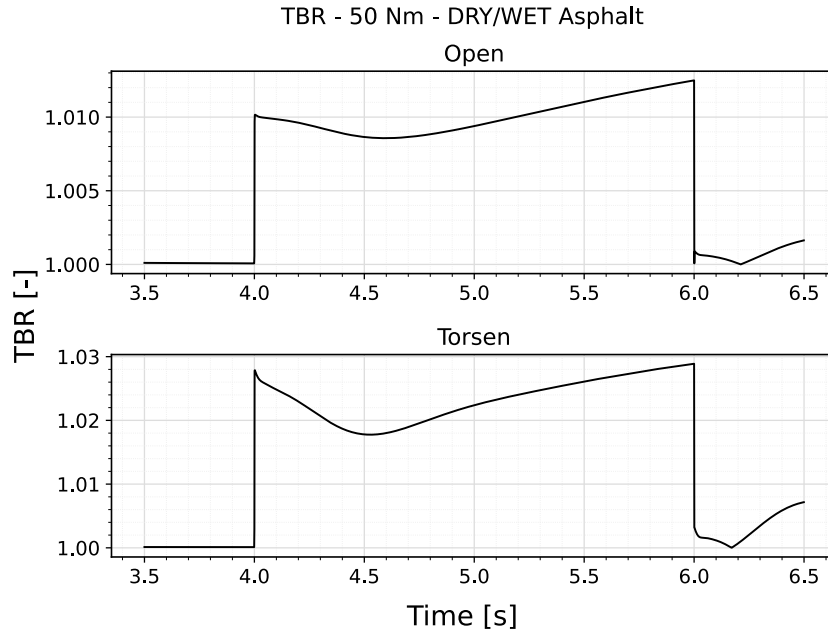
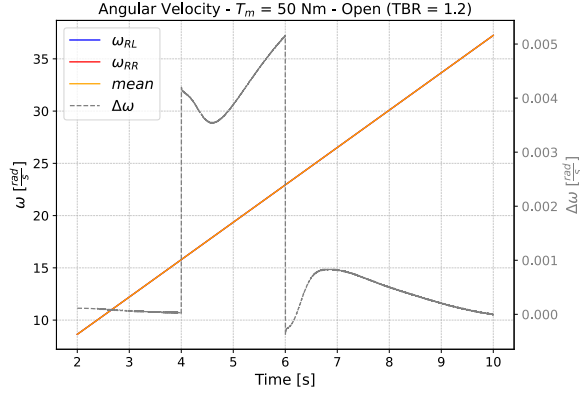
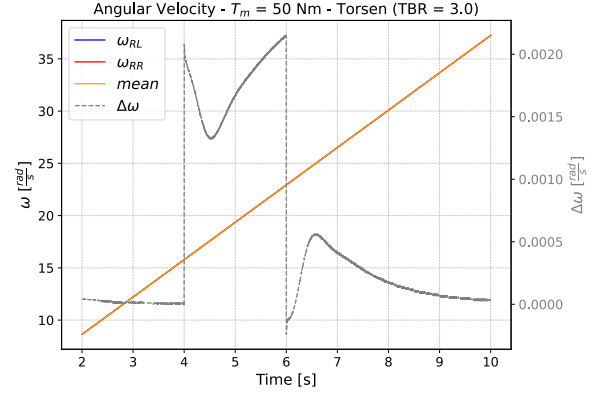


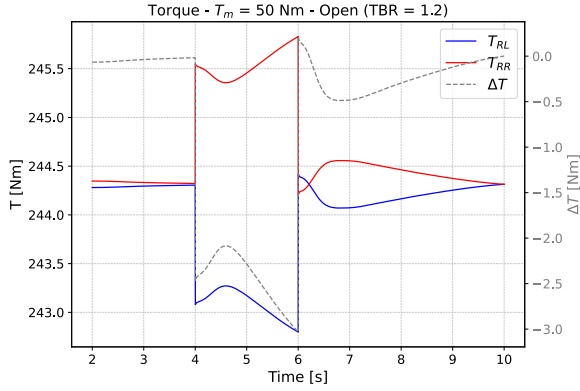
Figure 7.3: The measured TBR does not overcome the fixed *TBR* of the *Torsen* and the open differential. The differentials remain *closed*. Therefore, the rotational velocities of the wheels remain the same.



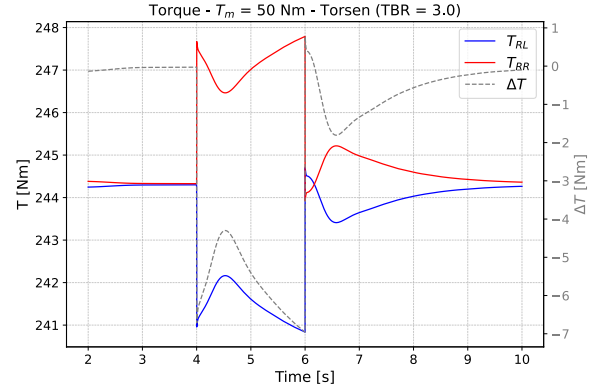
(a) The $\Delta\omega$ remain low along the test even during the contact with the wet asphalt because the differentials stay closed.



(b) Because the fixed TBR or the *Torsen* is higher than the open one, the wheels have almost the same rotational speed.



(c) The torque bias between both wheels is applying on the left rear wheel. After the wet asphalt, there is a small torque inversion produced by the angular acceleration of the wheels.



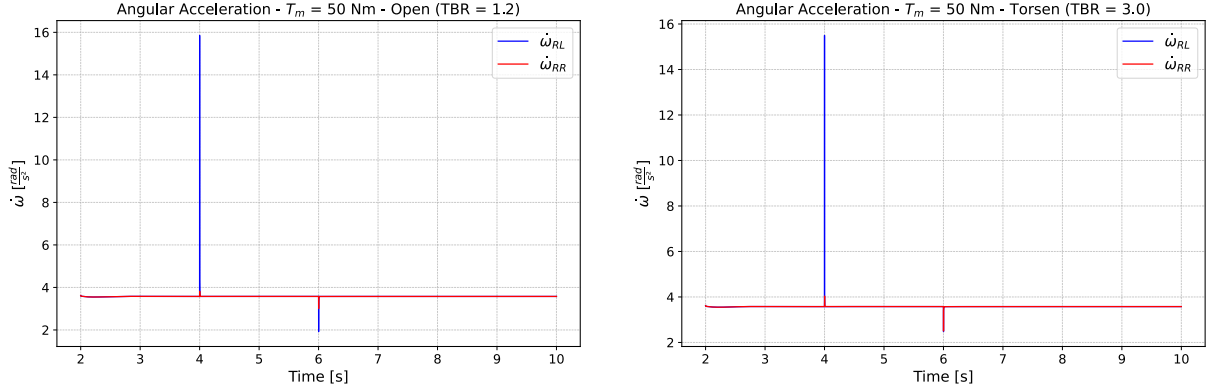
(d) Because the TBR is higher, the torque on the right wheel is higher than for the open differential.

Figure 7.4: After the puddle, the wheel's speed and the torque balance between both wheels tend to stabilize.

When the vehicles exit the puddle (at $t = 6$ s), a small inversion of the torque bias between the wheels appears (Figures 7.4c and 7.4d). This induces a variation of $\Delta\omega$ (Figures 7.4a and 7.4b). This inversion occurs when the left rear wheel recovers more grip as it returns to dry asphalt with a higher friction coefficient. At the moment of the "new contact", the longitudinal slip ratio κ of the left wheel decreases. Due to the elasticity of the tyre, there is a slight peak in its angular acceleration at the outlet of the puddle (Figures 7.5a and 7.5b). In fact, the *Torsen* differential principle tends to slow down the wheel's rotational speed difference.

Because of the lower friction coefficient between the left wheel and the ground, the left wheel longitudinal slip is higher than that of the right wheel (Figure 7.6). Therefore, $\Delta\omega$ is positive during the contact with the water. Immediately after the wet asphalt patch, the torque balance reverses on the left and the right wheels to reduce the value of $\Delta\omega$. Due to the TBR level, this phenomenon is more pronounced for the *Torsen* differential (Figures 7.4c and 7.4d). The rotational velocity difference $\Delta\omega$ remains lower with the

Torsen differential because more torque is applied to the slowest wheel (i.e. the wheel with the best grip *RR*), as observable in Figures 7.4a and 7.4b.



(a) The angular accelerations of the wheels possess peaks at the enter and the outlet of the puddle. The peak for the left wheel at the end of the wet asphalt is higher.

(b) Due to the different torque repartition in the *Torsen* differential, the peaks of angular accelerations are smaller than the open differential.

Figure 7.5: The angular accelerations of the wheels affect the velocity and thus the torque distribution in the *Torsen*.

Figure 7.6 compares the longitudinal slip ratio κ for both differentials. The difference in longitudinal slips between the *Torsen* differential wheels is smaller than that between the open differential wheels. However, this tendency reverses at the puddle's outlet due to the higher torque transfer delivered by the *Torsen* differential, as demonstrated previously. This difference remains small compared to the wet asphalt patch.

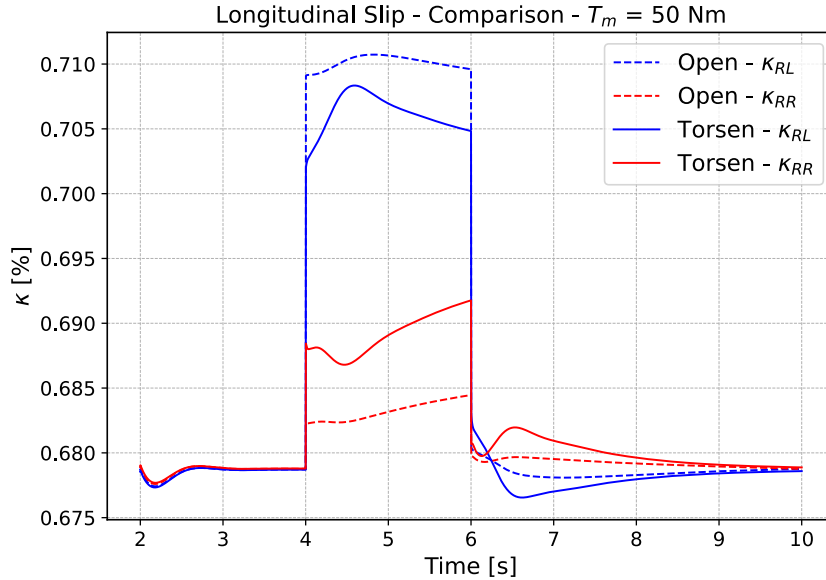


Figure 7.6: At the beginning of the puddle, the slip of the left wheel increases more than for the right wheel due to the change of the friction coefficient. The *Bakker's* peak curve decreases and the maximum longitudinal slips shift. The difference between κ is more pronounced for the open differential.

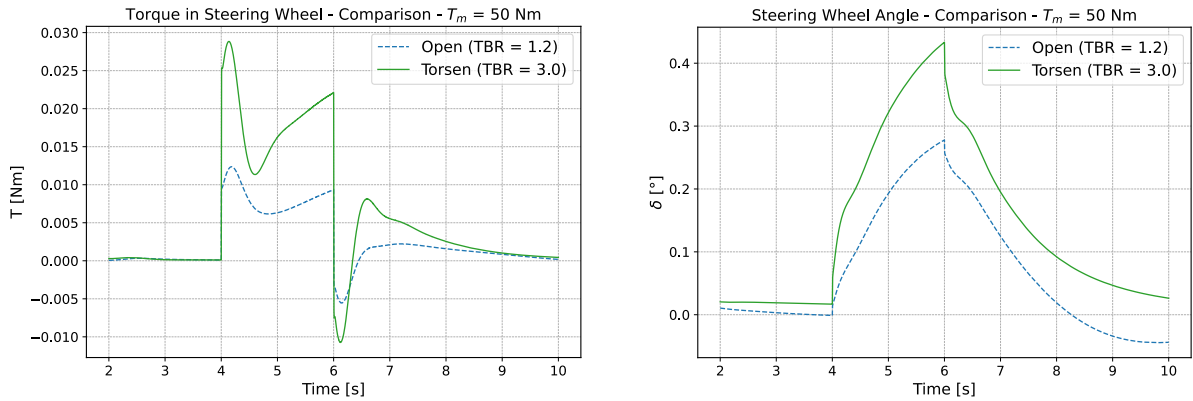
This difference in longitudinal slip could be advantageous in terms of tyre wear. The dissipative power P_{dis} of the tyres affects the tyre wear and should be minimised to prevent one tyre from wearing out prematurely. In the case of a straight line, with the expression 6.1 at the middle of the puddle ($t = 5s$), we find:

Dissipative Powers ($t = 5s$) - $T_m = 50Nm$				
$\backslash$	$CBV [m/s]$	$\kappa [\%]$	$F_{long} [N]$	$P_{dis} [W]$
Open, RL	5.38	0.710	858.98	- 32.89
<i>Torsen</i> , RL	5.38	0.706	855.0	- 32.57
Open, RR	5.38	0.684	869.18	- 31.92
<i>Torsen</i> , RR	5.38	0.69	876.37	- 32.48

Table 7.2

As can be seen in the previous table, the power dissipated P_{dis} by the left tyre of the open differential is the higher one, because of the value of the wheel's longitudinal slip. Nonetheless, the variation between the two wheels remains very small for the two differentials.

The last point to check is how the driver reacts when the vehicle encounters the wet asphalt. The driver's corrections remain minor due to the limited torque difference between the left and right wheels. Nevertheless, it becomes clear in Figures 7.7a and 7.7b that the *Torsen* differential requires a greater steering wheel angle and steering torque than the open differential. This additional effort is needed for the driver to maintain a straight path during acceleration and prevent deviation caused by the water patch. Furthermore, the torque balance inversion forces the driver to apply a steering torque of the opposite in sign. To maintain the corrected steering angle and compensate for the vehicle's deviation, the driver varies the amount of torque applied to the left or right.



(a) The torque in the steering wheel is higher for the *Torsen* differential, as the driver reacts stronger due to the torque bias between the two wheels.

(b) The driver corrects more the trajectory with the *Torsen* differential because of the higher torque bias.

Figure 7.7: The *Torsen* differential requires more efforts from the driver.

At low acceleration rate, the *Torsen* differential requires the driver to make greater efforts

to maintain a straight line trajectory. Despite this, the *Torsen* offers less tyre wear and enhances the grip between the vehicle and the road surface.

7.1.2 Transition Acceleration Rate

The second acceleration rate tested reveals a difference between the open differential and the *Torsen* one. With an engine torque of $T_m = 140$ Nm, the measured *TBR* of the open differential exceeds its fixed *TBR*. Therefore, the locking torque capability of the differential is fully exploited. The differential opens to allow wheel speed differentiation.

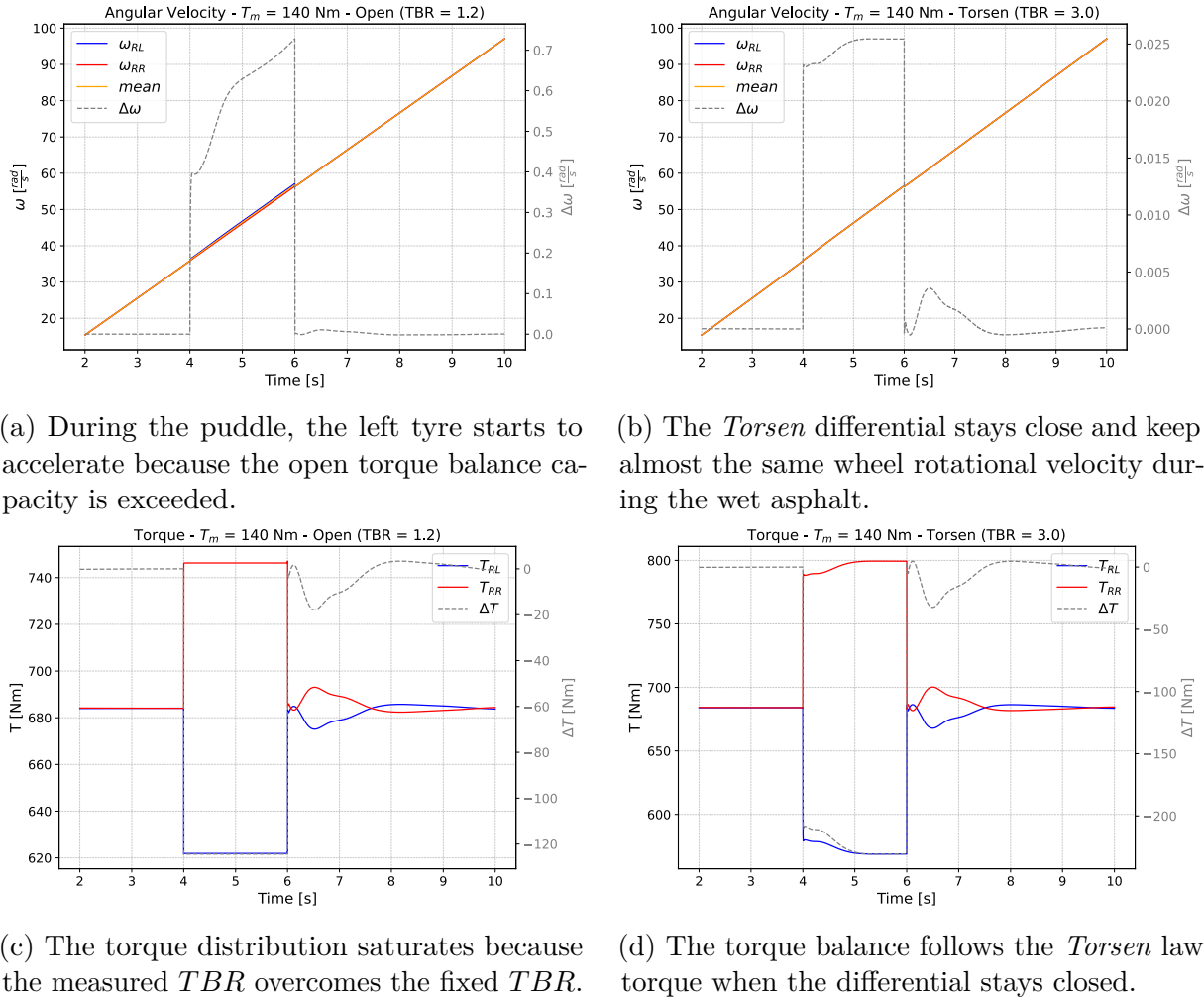
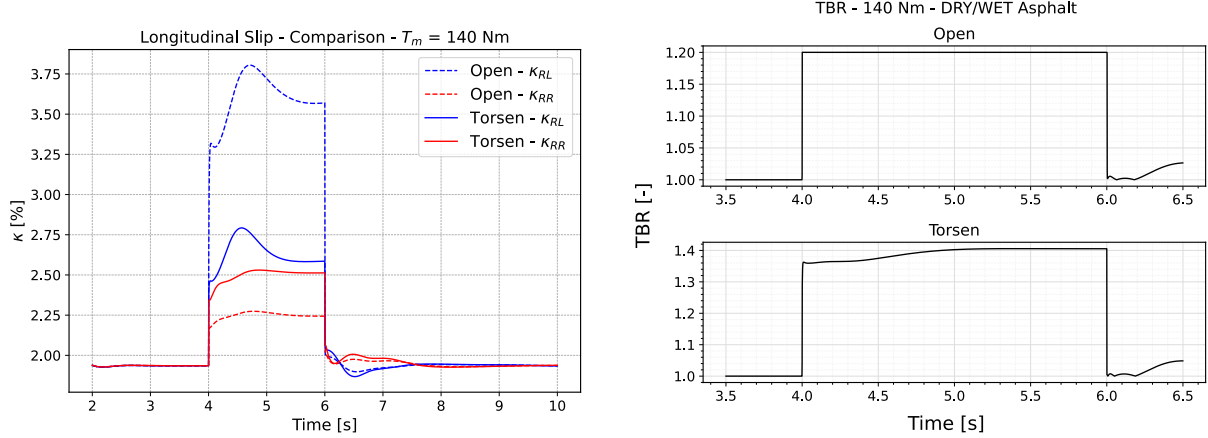


Figure 7.8: The open differential is opened while the *Torsen* differential still tends to lock the wheel speeds together. The torque inversion at the puddle outlet helps to readjust the same rotational speed velocity because the friction coefficient returns to being identical for both tyres.

The measured *TBR* exceeds 1.2 for the open differential but remains below 2 for the *Torsen* differential (Figure 7.9b). Therefore, the open differential permits a speed difference between the two rear wheels. The left wheel begins to rotate faster than the right one. Thanks to the differential's mechanical construction, the relation 3.1 is respected (the orange curve in the Figures 7.8a and 7.8b). This prevents the angular velocity of the

left wheel from increasing excessively. As intended, the longitudinal slip of the left wheel κ_{RL} for the open differential begins to increase when the wheel is in the puddle. The open differential slip is much higher than the left wheel slip for the *Torsen* differential (Figure 7.9a). Conversely, the difference in slips for the *Torsen* stays low due to the torque distribution between the two wheels (the torque balance helps to limit the wheel velocity difference and slippage on the water). The torque distribution for the open differential is saturated because of the $TBR_{measured}$, as Figure 7.8c presents.



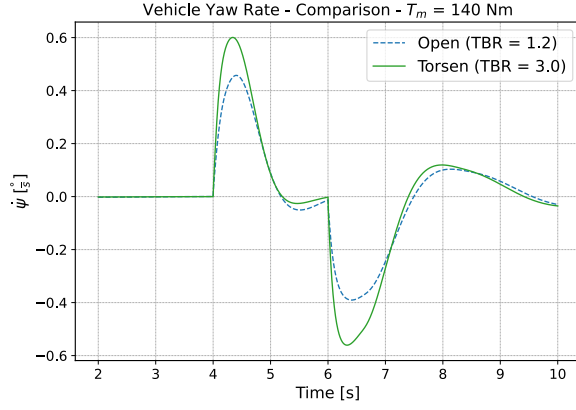
(a) The slips of both differentials indicate that the left wheel of the open differential starts to accelerate, but not the *Torsen*.

(b) The TBR for the open differential exceeds the fixed one (saturated on the above graph) but remains below 2 for the *Torsen*.

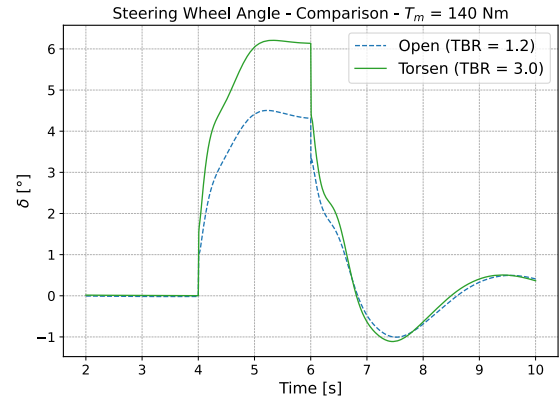
Figure 7.9: For the open differential, the longitudinal slip of the left wheel is higher on wet asphalt than for the *Torsen* differential. The open differential's full locking capacity is reached, but not that of the *Torsen* differential.

The longitudinal slip increases when the left wheel of the open differential comes into contact with wet asphalt. It remains below the saturation limit κ_{lim} , before total tyre skidding occurs. Here, as $\kappa_{lim} \pm 5.10$ % and the $\kappa_{Open,max,RL} \pm 3.81$ %, the tyre retains the possibility to recover grip when the vehicle is fully on dry asphalt. The maximum longitudinal slip stays close to the saturation limit. Hence, the tyre's dissipative power remains the highest for the open differential.

Once again, because the *Torsen* differential offers a higher torque difference between the two wheels (Figure 7.8d), the yaw moment produced on the car body induces a stronger driver's steering wheel correction. The yaw rate variation presented in Figure 7.10a is thus greater when the vehicle with the *Torsen* differential drives through the puddle. Once the yaw rate has stabilised at around $t = 5$ s, the open differential surpasses the *Torsen* differential. The steering wheel angle becomes higher for the *Torsen* differential when the vehicle is on the wet asphalt (Figure 7.10b).



(a) During the puddle, the yaw rate is affected by the higher torque on the right wheel. The driver proportionally reacts on the steering wheel.

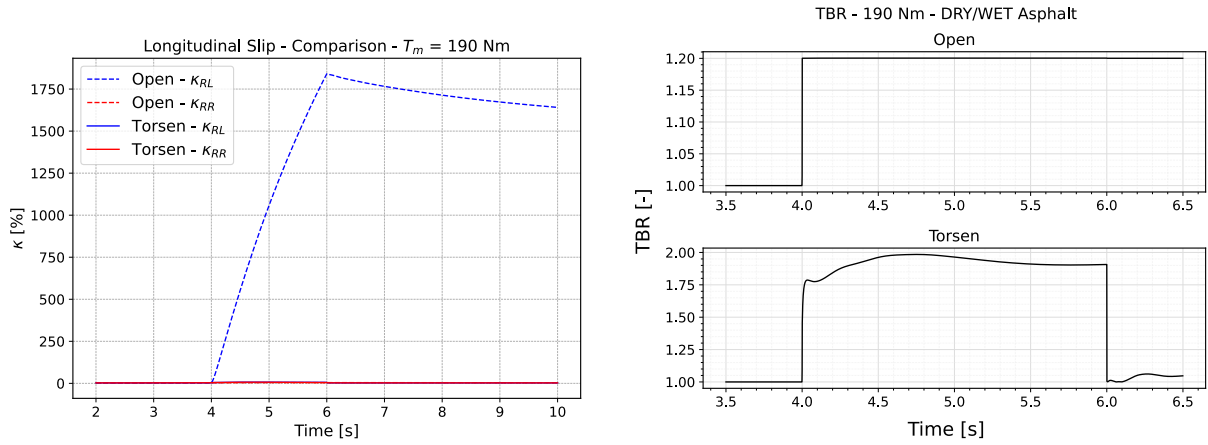


(b) The driver compensates more the trajectory with the *Torsen* differential due to the higher torque bias.

Figure 7.10: The *Torsen* differential requires greater steering input from the driver with $T_m = 140$ Nm.

7.1.3 High Acceleration Rate

The engine torque of $T_m = 190$ Nm affects the handling of the *Torsen* and the open differentials. The measured *TBR* indicates that the *Torsen* differential is approaching friction ratio limit. For the open differential, the *TBR* remains saturated at 1.2 (Figure 7.11b). The locking capacity of the open differential remains exceeded. This allows speed differentiation, as is the case with the transition acceleration rate. Nevertheless, the rotational velocity of the left wheel of the open differential increases sharply as soon as the vehicle enters the wet asphalt, in contrast to the *Torsen* differential. (Figures 7.12a and 7.12b). Regarding the longitudinal slip ratio of this wheel (Figure 7.11a), it is clear that it reaches excessive levels, indicating a full skid. The longitudinal slip κ is well beyond the saturation limit.

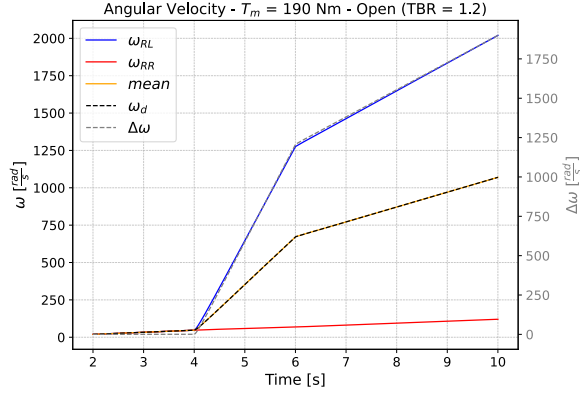


(a) The slips of both differentials indicate that the left wheel for the open differential is in total slipping situation but not for the *Torsen*.

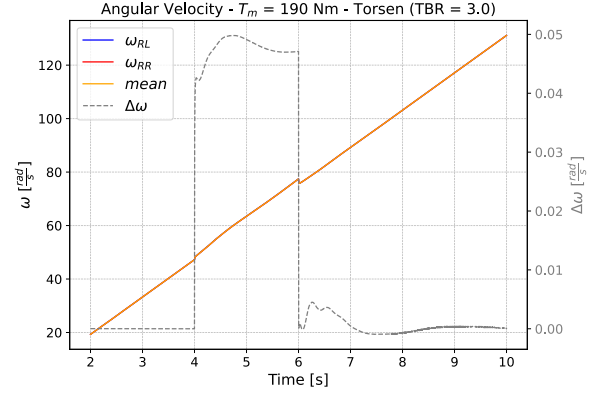
(b) The *TBR* for the open differential is greater than the fixed one (which is saturated in the above graph), but remains below 2.

Figure 7.11: The measured *TBR* exceeds the fixed *TBR* for the open differential. Consequently, the left wheel begins to skid on the wet surface. This does not happen with the *Torsen* differential.

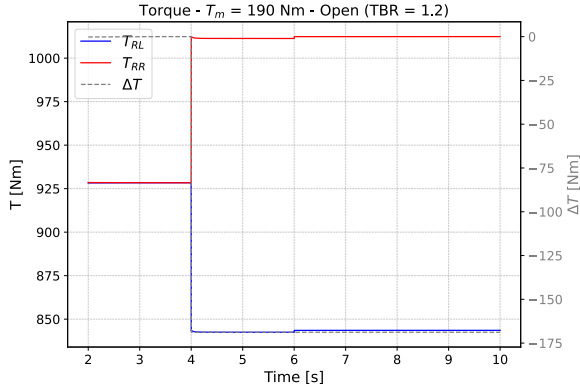
Despite the loss of grip, the left wheel continues to accelerate because of the limitation of the torque balance in the open differential (the angular acceleration of the wheels is available in Appendix E.1). This sharp increase in speed is caused by the excessive torque still being sent to the low-grip wheel (Figure 7.12c). Unlike the *Torsen* differential, the higher *TBR* allows a more favourable torque distribution. It is not limited by its locking effect. As a result, the torque sent to the left wheel does not exceed the maximum transmittable torque under the reduced grip condition, preventing the left wheel from skidding (Figures 7.12b and 7.12d). Nevertheless, the mechanical links in the differentials limit the rotational velocity of the left wheel in the case of the open differential. The left wheel cannot exceed twice the angular velocity of the differential, as the dotted line in Figure 7.12a shows.



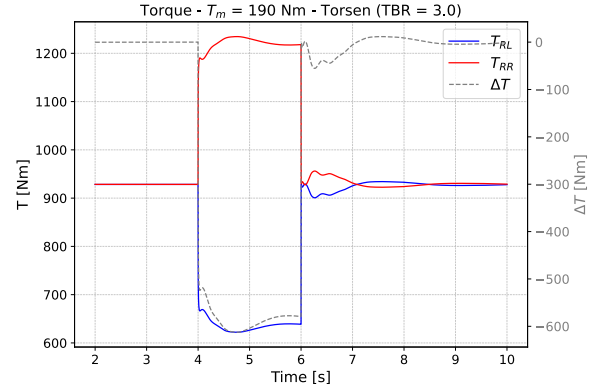
(a) The left wheel velocity rises sharply because the tyre has lost grip and can not transmit all the torque to the ground.



(b) The TBR of 3.0 allows the *Torsen* to keep a small $\Delta\omega$ and so limit the slip of the left wheel. This enables the vehicle to maintain grip on the ground.



(c) As the wheel left speed does not decrease, the biggest part of torque distribution sustains on the right wheel.



(d) The torque distribution persists stable after the puddle. The locking effect permits a higher torque transmission on the ground.

Figure 7.12: Compared to the *Torsen* vehicle, which maintains grip on wet asphalt, the left wheel of the open differential skids. This limits the amount of torque that the driver can transmit to the ground.

In this case, the grip loss of the open differential confers a supplementary feature to the *Torsen* differential. The dissipative power P_{dis} for the open differential tyres reaches huge values as expected due to the longitudinal slip value of the left wheel κ_{RL} (Table 7.3). Moreover, the carbody velocity of the *Torsen* differential vehicle shows a net difference in terms of torque transmission between both differentials. The *Torsen* differential transfers more torque to the ground, preventing excessive speed loss when the car encounters a change in the coefficient of friction of the surface it is travelling on.

Dissipative Powers ($t = 5s$) - $T_m = 190Nm$					
$\backslash$	$CBV [m/s]$	$CBV [km/h]$	$\kappa [\%]$	$F_{long} [N]$	$P_{dis} [kW]$
Open, RL	15.54	55.95	1058.54	1215.28	-199.749
<i>Torsen</i> , RL	16.51	59.44	6.78	2318.54	-2.721
Open, RR	15.54	55.94	4.68	4207.46	-3.408
<i>Torsen</i> , RR	16.51	59.44	6.59	4608.68	-5.149

Table 7.3

In this experiment, we controlled for the effect of the wet asphalt in terms of timing. Therefore, the distance that the vehicle travels on a wet surface varies between both differentials. In fact, the wet path length for the open differential is 31.08 m and 33.02 m for the *Torsen*. The length of the water patch crossed by the *Torsen* differential vehicle can be applied to the passage time and speed of the open differential (2 seconds at 15.54 m/s). Indeed, it is possible to estimate the time gained by the *Torsen*-equipped vehicle compared to the one with the open differential (Figure 7.13).

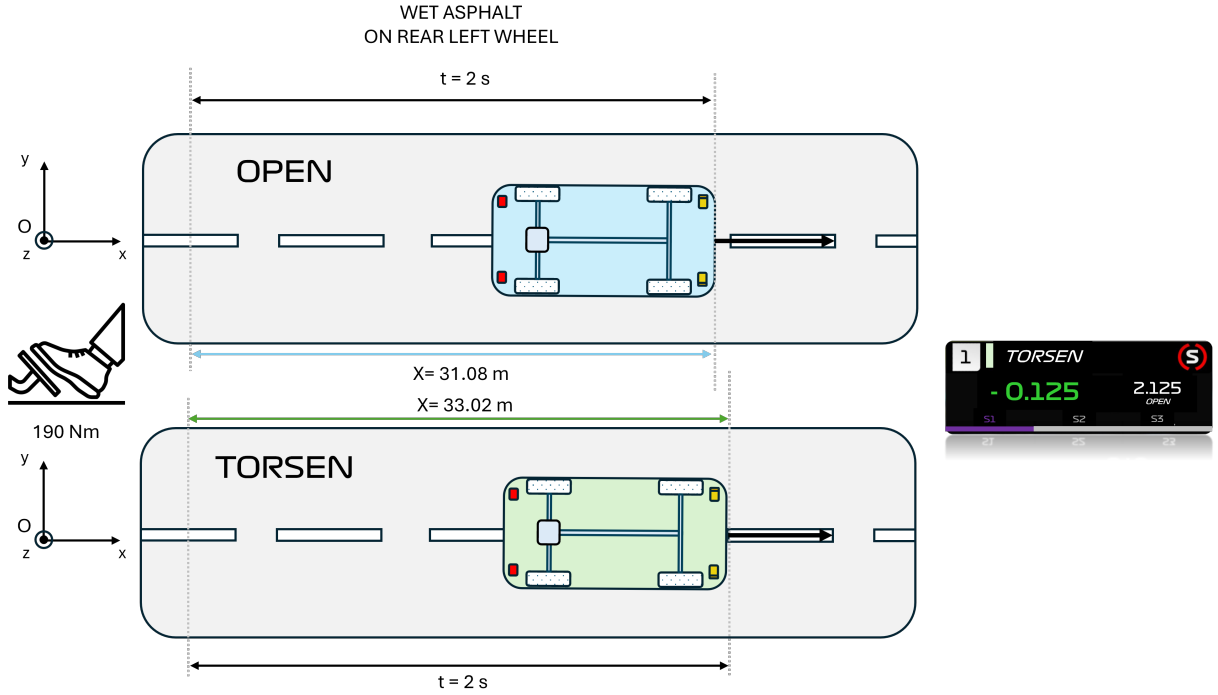


Figure 7.13: The *Torsen* differential recovers 0.125 seconds compared to the open differential during the wet asphalt path.

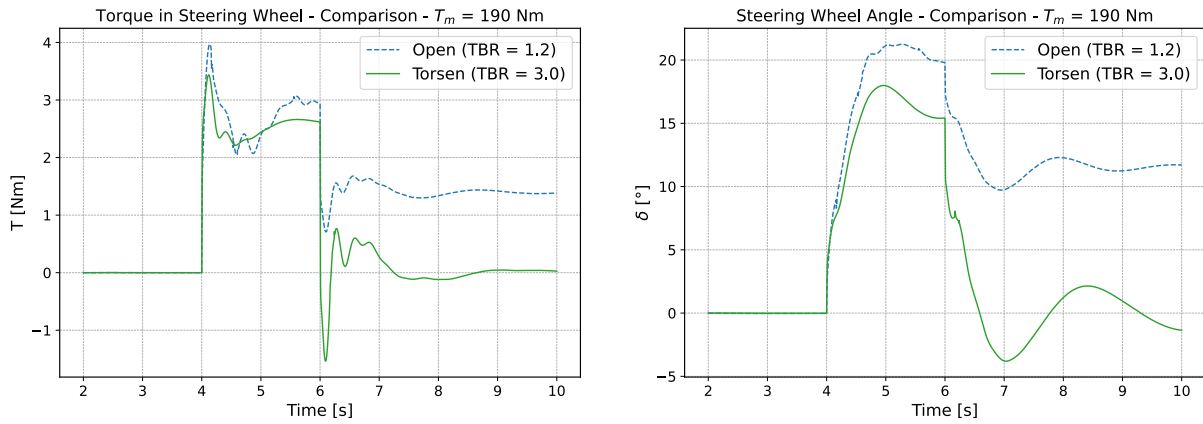
For the 2 seconds of wet asphalt, we need to know how long it takes for the open differential to track the same path as the *Torsen* vehicle. Therefore, the timing and the difference between the two vehicles are:

$$t_{Open, 31.08m} = 2s \quad \rightarrow \quad t_{Open, 33.02m} = 2 \cdot \frac{33.02}{31.08} = 2.125s$$

$$\Delta t = 2.125 - 2 = 0.125s \quad \text{Time saved by the } Torsen.$$

In addition to reducing excessive tyre wear on low-friction surfaces by preventing longitudinal wheel slip under high engine torque, the *Torsen* differential enables valuable time savings on challenging terrain. This is made possible by its more effective torque distribution, which favours the wheel with the best grip. These time savings can be advantageous in *Formula 1*, but even more in rally racing, where drivers frequently encounter road sections with varying surface conditions.

In terms of the driver actions, the *Torsen* differential allows a reduction in steering wheel angle. Even when the steering angle decreases (Figure 7.14b), the torque in Figure 7.14a needed for the correction of the trajectory remains nearly the same for both differentials (slightly higher for the open one). The main difference is the reaction after the wet path: with the open differential, the driver must continuously apply a certain level of steering wheel angle in order to keep the vehicle on a straight trajectory.



(a) The torque in the steering wheel decreases after the wet patch for the *Torsen* differential. The difference in steering torque between the two differentials remains almost the same throughout the wet asphalt.

(b) The driver must correct the trajectory more with the open differential during and after the wet asphalt patch. This is induced by the poor torque distribution.

Figure 7.14: With a higher engine torque, the difference in driver's effort is reversed. The *Torsen* requires less steering wheel angle correction. The torque on the steering wheel remains almost the same for both vehicles during the patch. Afterwards, the driver has to maintain a constant torque in the steering wheel for the open differential, but not for the *Torsen* one.

Clearly, the level of engine torque and the locking effect of the differentials influence the vehicle's handling and the driver's reaction.

7.1.4 Influence of The *TBR* and The Engine Torque

As we have seen in previous subsections, the engine torque and the locking effect influence the vehicle behaviour in the event of a variation in coefficient of friction. Therefore, a global parametric analysis is needed, with an engine torque T_m ranging from 0 to 300 Nm in steps of 10 Nm, and a fixed *TBR* between 1.2 and 5.0 is needed. The range of fixed *TBR* tested is $TBR_{fixed} = [1.2; 2.0; 3.0; 4.0; 5.0]$. This will highlight the difference in driver

reaction between an open differential and a *Torsen* differential. Table 7.4 summarises the level of engine torque and longitudinal acceleration at which the differential opens.

Steering Wheel Angle

As introduced with the three engine torques tested, the driver's action on the steering wheel is affected by the level of engine torque and the locking effect. Figure 7.15 shows the maximum steering wheel angle applied by the driver during the 2 seconds on the wet patch. The dotted line displays the limit at which the vehicle yaw exceeds 90°: at this point, it is considered that the driver is no longer able to recover the vehicle's trajectory using the steering wheel. Therefore, the car spins because of the loss of grip at the rear axle. The dashed line highlights the opening torque of each *TBR*. As previously demonstrated, the measured *TBR* of the open and *Torsen* differential could exceed the fixed *TBR*. At this particular moment, the differential is opening and the difference in wheel speed is being triggered.

Opening Torque for Each <i>TBR</i>		
<i>TBR</i>	T_m [Nm]	a_x [$\frac{m}{s^2}$]
1.2	120	2.394
2.0	200	3.974
3.0	250	4.982
4.0	-	-
5.0	-	-

Table 7.4

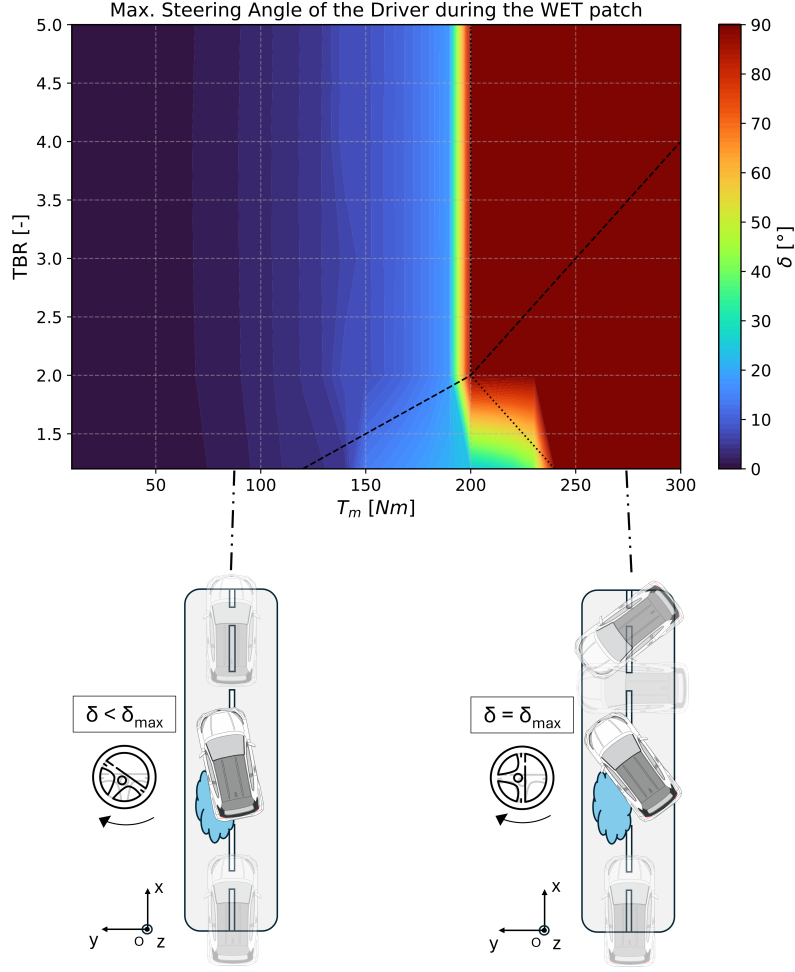


Figure 7.15: The maximum δ is achieved for the same engine torque for all TBR above 2.0. The maximum steering angle is $\delta_{max} = 90^\circ$.

The maximum steering angle applied by the driver is fixed at $\delta_{max} = 90^\circ$. This limit is reached at $T_m = 200$ Nm for all values of TBR above 2.0 (Figure 7.15). Between $TBR = 1.2$ and $TBR = 2.0$, the maximum engine torque before the car spins drops from 240 Nm ($TBR = 1.2$) to 200 Nm ($TBR = 2.0$). Therefore, with an engine torque below 240 Nm, the driver can always correct the trajectory using the steering wheel when the vehicle is equipped with a low TBR differential (i.e. an open differential). The steering wheel angle δ grows as the motor torque rises because of the torque distribution. As the TBR rises, the driver's reaction remains similar. If the engine torque reaches $T_m = 200$ Nm and the TBR is above 2.0, the additional torque on the right wheel is too great for the driver to prevent the car from spinning. Therefore, the steering wheel angle saturates for the *Torsen* differentials **before** the open differential.

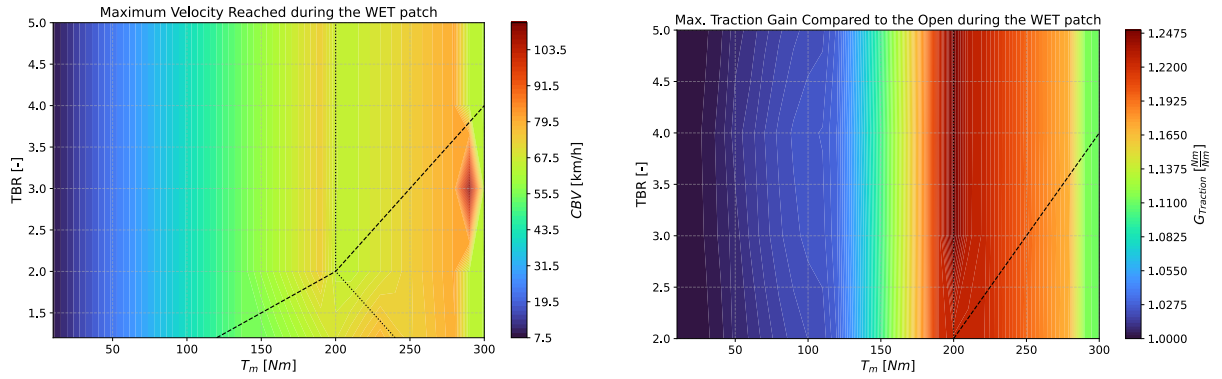
Traction Gain

The main objective of the *Torsen* differential is to transmit more torque to the wheel with the best grip. Therefore, it is interesting to measure the *traction gain* offered by a *Torsen* differential compared with an open differential ($TBR = 1.2$). The traction gain is defined

as follows:

$$G_{Traction} = \max \begin{cases} \frac{T_{rl,Torsen}}{T_{rl,Open}} & \text{if } \Delta T \geq 0 \\ \frac{T_{rr,Torsen}}{T_{rr,Open}} & \text{else} \end{cases} \quad (7.1)$$

The maximum traction gain measured during the wet patch achieves its maximum value before the driver loses the vehicle's control (Figure 7.16b). This increase in traction is reflected in the car's speed (Figure 7.16a). The greater the locking effect, the more torque the driver can transfer to the right-hand wheel. As more torque is transferred to the ground, the vehicle's speed increases. The difference between each TBR for each torque becomes apparent once the yaw limit is reached. Between 150 Nm and 200 Nm, the vehicle speed increases slightly with a TBR greater than 1.2. However, it remains unchanged when TBR exceeds 2.0. In the case of a wet asphalt patch, the TBR has a smaller impact on $G_{Traction}$ than the engine torque T_m .



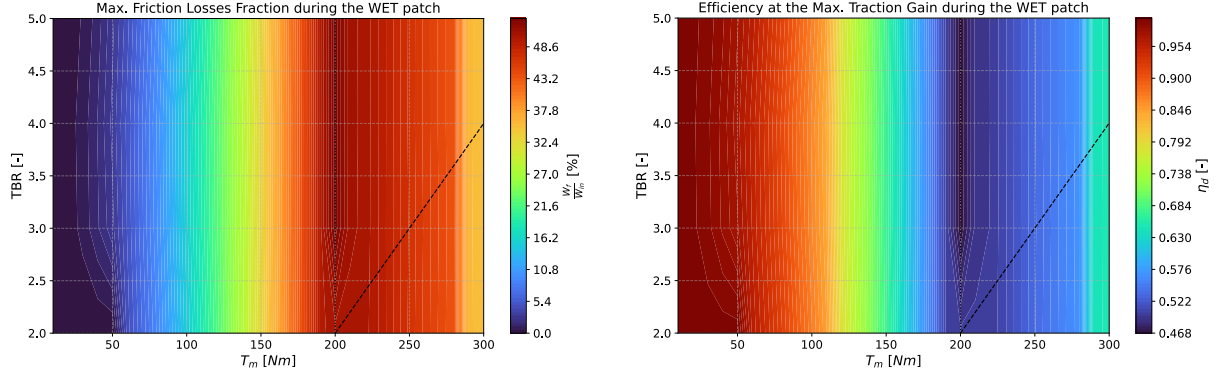
(a) The TBR slightly affects the speed of the vehicle on the water. The difference is only noticeable as you approach the vehicle's yaw limit.

(b) The evolution of the traction gain through the engine torques remains constant for each TBR . The minimum TBR of a *Torsen* is considered to be 2.0.

Figure 7.16: The gain in traction slightly affects the vehicle's velocity. In this case, there is no particular advantage in having a higher locking effect. The vehicle's speed and traction mainly depend on engine torque (T_m), rather than TBR .

Losses and Internal Efficiency

As described in Section 3, the locking effect is produced by gear friction. Referring to equation 3.2.2, the power loss and the internal efficiency of the differentials are observable in Figure 7.17. Figure 7.17a shows the ratio $\frac{W_f}{W_{in}}$ to highlight the difference ratio between the power loss in the differential W_f and the injected power W_{in} by the driver (with the engine torque). The efficiency η_d presented in Figure 7.17b is computed using equation 3.8. All the values are taken at the maximum traction gain $G_{Traction}$ moment.



(a) Friction losses reach 50 % when the yaw exceeds the fixed limit at $T_m = 200$ Nm. (b) When the engine torque is low and with a low TBR in the differential, the efficiency seems to be the best.

Figure 7.17: As expected, the efficiency graph seems exactly the opposite of the loss graph. The higher $\frac{W_f}{W_{in}}$, the lower the efficiency. The results depends solely on engine torque. An increase in TBR has little influence.

In Figure 7.17a, the TBR does not affect the amount of power lost on wet asphalt. The efficiency remains unchanged according to the TBR for each motor torque. As none of the tested TBR values allow the differential to be opened, there is no impact for any of the stand-out engine torques. Friction losses grow as the engine torque rises. Efficiency is at its worst when the driver can no longer keep the vehicle on its trajectory at $T_m = 200$ Nm.

Global Gain

Finally, it seems noteworthy to relate the losses in the differential to the gain it offers. Thus, to balance the internal efficiency with the traction gain, the *global gain* G_{tot} can be defined as:

$$G_{tot} = \eta_d G_{Traction} \quad (7.2)$$

As with the previous graphs, the overall result in Figure 7.18 shows that there is no significant influence of TBR on the overall gain when the car is on wet asphalt. However, the area 1 suggests that a TBR between 2.0 and 2.5 offers the greatest possible gain. Overall, the total gain is between 0.5 and 1, which demonstrates the advantage of a *Torsen* differential over an open differential in terms of grip. However, the gain values decrease with increasing engine torque. The influence of the locking effect will be much more noticeable when the vehicle is on ice.

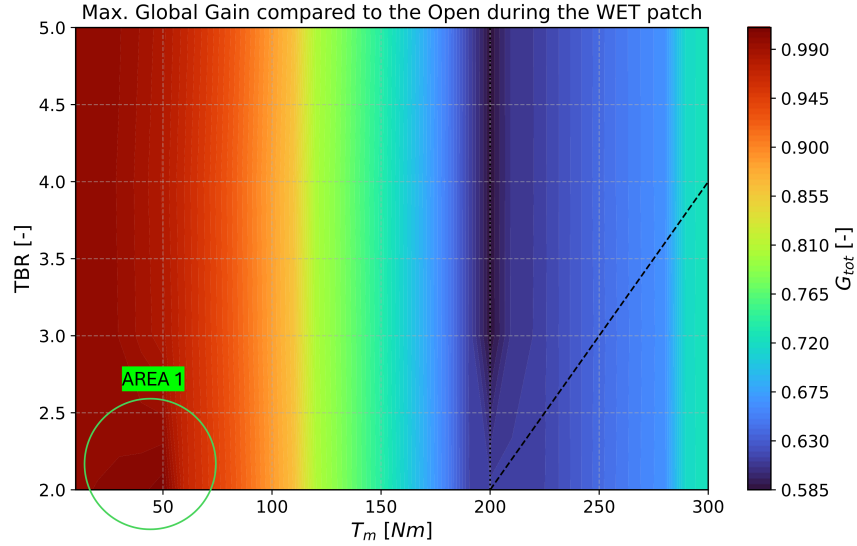


Figure 7.18: The area 1 displays where the total gain is the highest. The best gain is at low engine torque with a TBR between 2.0 and 2.5.

7.2 On Ice Road

In the same situation as the wet asphalt, the puddle is changed by a path of ice. In this case, the driver's maximum steering angle is fixed at $\delta_{lim} = 45^\circ$. The friction coefficients are presented in Table 7.1. The influence of the TBR and the engine torque T_m on the driver's reaction and the vehicle handling will be established after detailing two engine torques. For the ice, the friction coefficient ratio becomes:

$$k = \frac{\mu_{high}}{\mu_{low}} = \frac{1.048}{0.2096} = \frac{0.912}{0.182} = 5.0; \quad TBR_{Open} = 1.2; \quad TBR_{Torsen} = 3.0;$$

7.2.1 Low Acceleration Rate

On the ice, the initial torque applied is $T_m = 60$ Nm. This selected torque causes the opening of the open differential. On the ice patch, the engine torques at which the differentials open are well below the values shown in Table 7.4. The TBR of the open differential exceeds the fixed TBR (Figure 7.19). Therefore, the differential opens and the full locking capacity of the open differential is effective. For the *Torsen* differential, the differential remains closed because the TBR stays below 3.0.

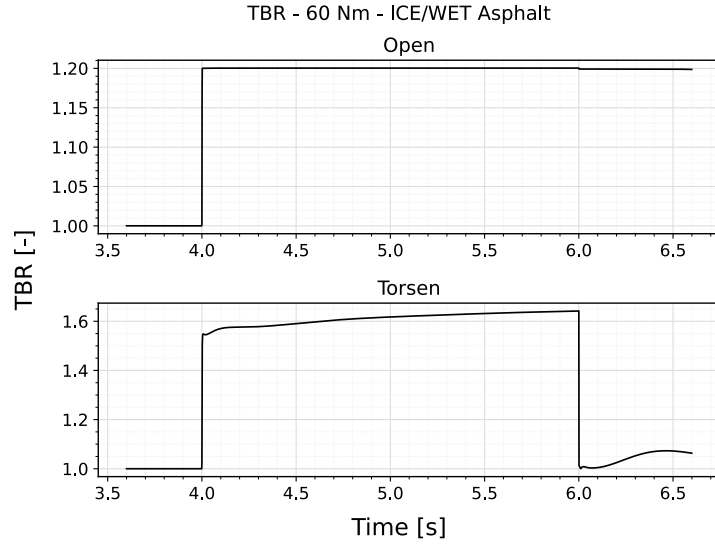
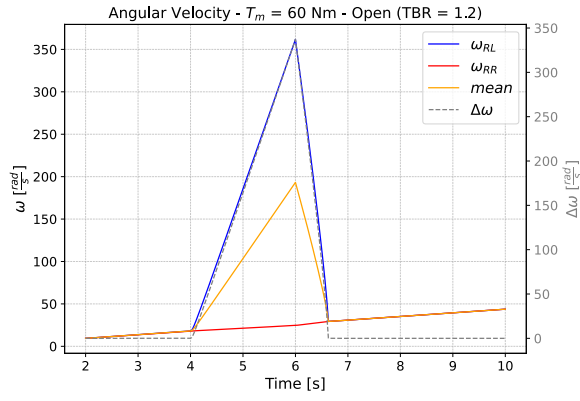


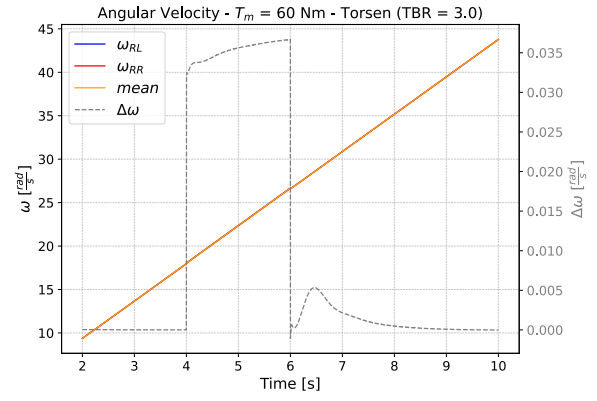
Figure 7.19: The measured TBR for the open differential saturates at the fixed TBR value. The *Torsen* differential will remain closed.

Consequently, the open differential opens and the left wheel on the ice patch rotates faster than the right wheel (Figure 7.20a). The left wheel begins to slip and accelerate due to the injected torque by the differential. At $T_m = 60$ Nm, the torque balance on the left wheel for the open differential becomes too significant. It exceeds what the tyre can transmit to the ground due to the fixed TBR . As the value is 1.2, the ΔT between the two wheels stays small, limiting the torque balance on the wheel with the best friction coefficient. Therefore, the wheel accelerates and slips. With the *Torsen* differential, the torque bias between the two wheels (Figure 7.20d) permits a smaller torque on the left wheel compared to the open differential (Figure 7.20c). This prevents left-wheel skid, and the $\Delta\omega$ stays small (Figure 7.20b). As in the case of wet asphalt, the torque balance remains after the patch of ice. The delta torque continues to have a positive effect on the right wheel. Consequently, the left wheel slows down sharply when it returns to the dry asphalt (Figure 7.20a). This decrease is due to the low peak of the left wheel's angular acceleration (available in Appendix E.3). The differential can rebalance the wheel speeds.

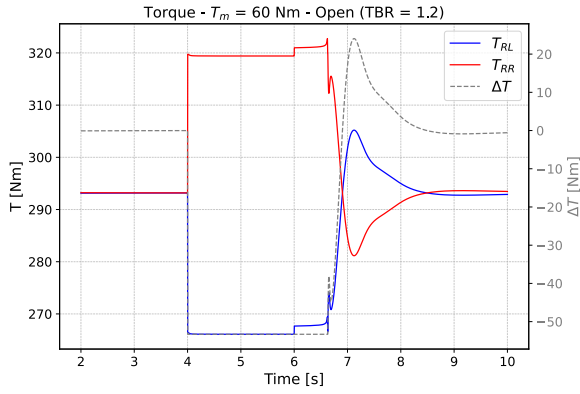
This slip phenomenon, which occurs at low engine torque for the open differential, induces premature tyre wear. In Figure 7.21, the longitudinal slip κ for the left wheel of the open differential becomes extreme as soon as the vehicle touches the ice patch. This extreme value of the longitudinal slip causes premature tyre wear. At this low level of engine torque, the *Torsen* differential is more effective at preserving tyres during acceleration.



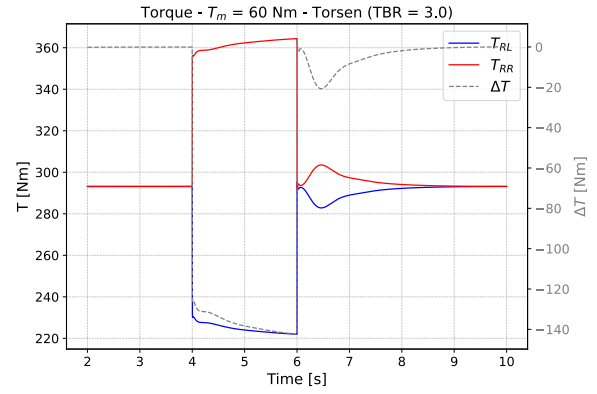
(a) At the beginning of the ice patch, the left wheel accelerates. By mechanical construction, the mean velocity is equal to the differential rotational speed.



(b) The *Torsen* differential prevents the left wheel from slipping. The torque balance stays in the slop of the *Torsen* linear torque distribution.



(c) At the end of the ice patch, the positive torque balance remains on the right wheel. Meanwhile, the speed of the left wheel decreases as it recovers grip. Thanks to the positive ΔT , the speed of the right wheel catches up with the speed of the left wheel.



(d) The torque distribution is not saturated to its maximum value. After the patch, the small ΔT allows the wheel to recover the same rotational velocity.

Figure 7.20: After the ice, the speed of the wheels and the torque balance between them tend to stabilise for the *Torsen*. With the open differential, the left wheel skids and the vehicle loses grip.

For example, at the midpoint of the ice patch ($t = 5$ s), we observe that:

Dissipative Powers ($t = 5$ s) - $T_m = 60Nm$					
$\backslash$	CBV [m/s]	CBV [km/h]	κ [%]	F_{long} [N]	P_{dis} [kW]
Open, RL	5.91	21.28	778.85	447.97	-20.618
<i>Torsen</i> , RL	6.18	22.25	1.26	817.98	-0.067
Open, RR	5.91	21.28	1.06	1324.42	-0.090
<i>Torsen</i> , RR	6.18	21.28	1.09	1367.93	-0.093

Table 7.5

As shown in Figure 7.21, the dissipative power for the left tyre of the open differential is huge because of the significant slip ratio. The values for the *Torsen* stay reasonable thanks to the better torque balance between the wheels.

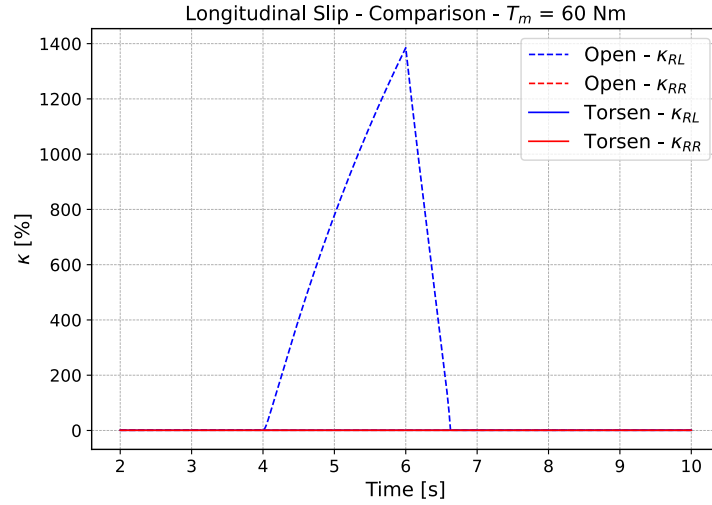
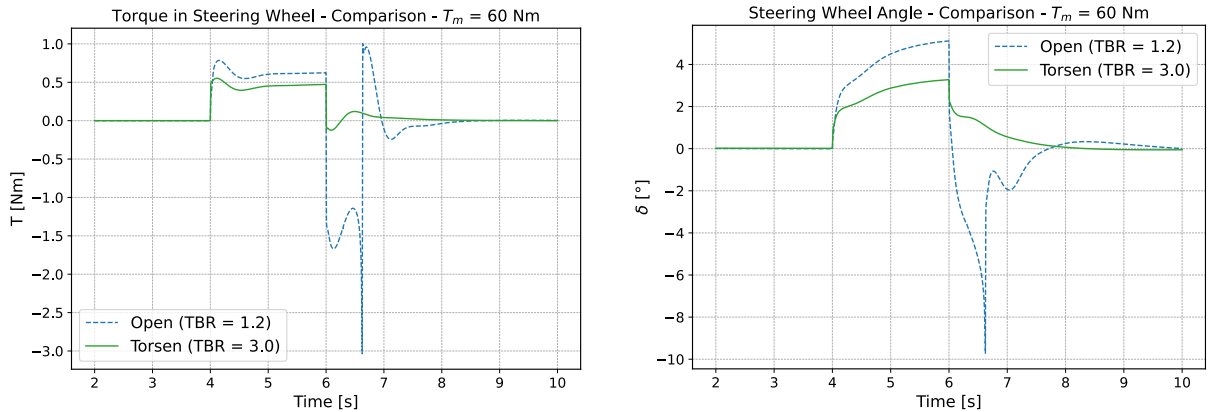


Figure 7.21: The left wheel of the open differential is skidding. The left-hand torque is too high.

In terms of the driver's reaction, the *Torsen* differential reduces the amount of the steering wheel correction required (Figure 7.22b). Since the differential is not open, the driver needs to apply less effort to maintain a straight line (Figure 7.22a). After the ice patch, the driver with the open differential must readjust the steering angle, as the torque distribution does not become neutral (Figure 7.20c). This affects the yaw and the yaw rate of the vehicle. The PD controller (i.e. the driver) reacts by reducing these parameters to zero (see Appendix E.2, for straight line trajectory).



(a) Due to the longitudinal slip peak of the left wheel, the open differential requires the driver to apply more steering torque to maintain the straight line trajectory.

(b) The steering wheel angle δ decreases at the outlet of the ice patch for the *Torsen*, while the driver with the open differential continues to correct the trajectory.

Figure 7.22: As the vehicle with the *Torsen* has more grip, the driver finds it easier to steer at low engine torque than with an open differential.

As for the wet patch, the *Torsen* differential therefore wins time on ice. For an engine torque T_m of 60 Nm over a period of 2 seconds on ice, the path length for the open differential is 11.82 m whereas it is 12.36 m for the *Torsen*. The global situation is presented in Figure 7.23. Applying the length for the *Torsen* to the velocity of the open differential, the time gained for ice contact is:

$$t_{Open,11.82m} = 2s \quad \rightarrow \quad t_{Open,12.36m} = 2 \cdot \frac{12.36}{11.82} = 2.091s$$

$$\Delta t = 2.091 - 2 = 0.091s \quad \text{Time saved by the } Torsen.$$

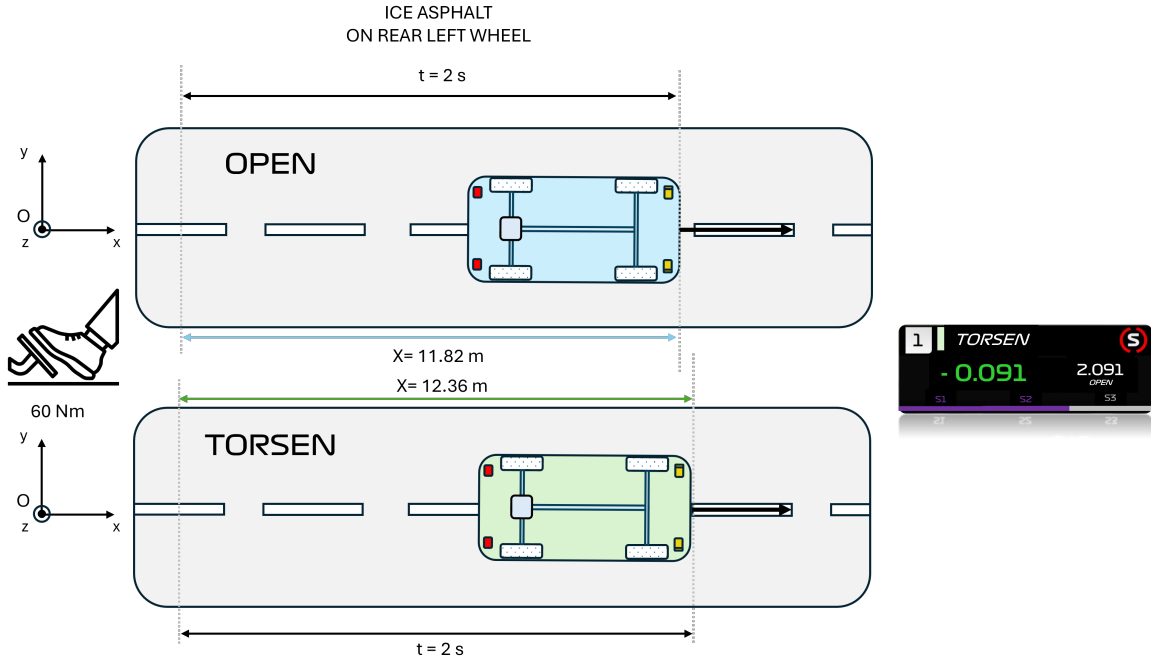


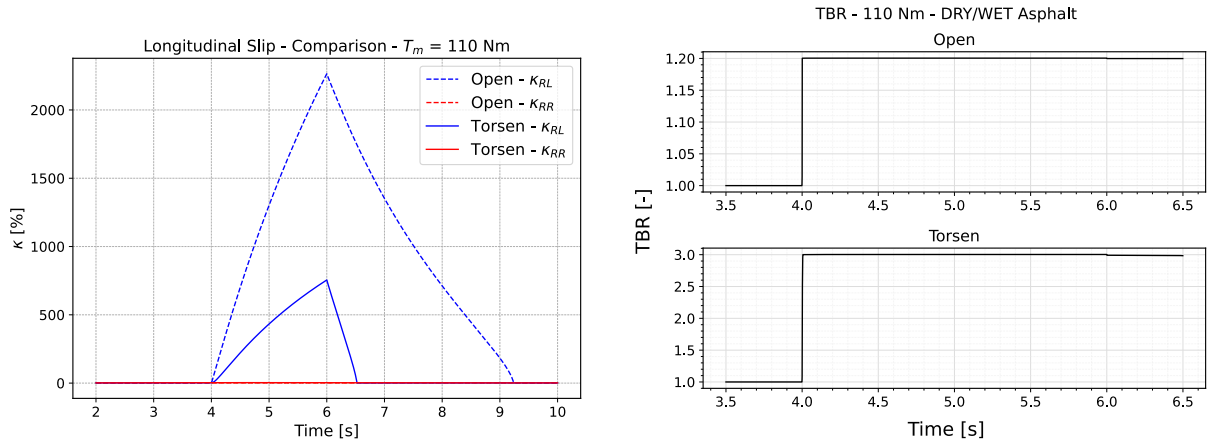
Figure 7.23: The *Torsen* differential recovers 0.091 second compared to the open differential during the ice path.

7.2.2 Transition Acceleration Rate

When the driver applies more torque, the *Torsen* differential opens. With an engine torque of $T_m = 110$ Nm, both differentials open, and the driver's reaction varies.

First, the measured *TBR* exceeds the fixed value for the open and the *Torsen* differentials (Figure 7.24b). Therefore, the full locking capacity of both differentials is reached, resulting in slippage of the left wheel in both cases (Figure 7.24a). As the measured *TBR* exceeds the fixed *TBR*, the longitudinal slip κ_{RL} of the left wheel of the open differential rises sharply. The longitudinal slip κ_{RL} for the *Torsen* differential is also rising, with a smaller peak value than the open differential. As for $T_m = 60$ Nm, the left wheel's slip increases the left wheel rotational velocity as soon as the vehicle enters the ice patch. The left wheel of the *Torsen* differential is also slipping. As shown in Figures 7.26a and 7.26b, the increase in speed of the left wheel is smaller for the *Torsen* than for an open differential. This advantage is once again conferred by the better torque distribution between the wheels (Figures 7.26c and 7.26d). Less torque is transmitted to the left wheel with lower

grip in the case of the *Torsen* differential. Thus, the left wheel slips more slowly than in the case of the open differential. This reduces its angular velocity compared to the open differential case.



(a) The peak in longitudinal slip is lower for the *Torsen* than for the open differential. After the ice patch, grip is recovered by the left wheel of both differentials and the longitudinal slip κ_{RL} decreases.

(b) The *TBR* for both differentials exceeds the fixed value and becomes saturated. The engine torque T_m becomes too high for the fixed *TBR* to distribute it between the wheels without inducing differential opening.

Figure 7.24: Because the measured *TBR* exceeds the fixed *TBR* for both differentials, the left wheel of each differential starts to skid.

As the *Torsen* transmits more torque to the right wheel (the one with the highest friction coefficient), the total amount of torque on the ground increases. Consequently, the vehicle's longitudinal acceleration a_x is affected. As intended, the longitudinal acceleration of the *Torsen* vehicle is higher on ice than that of the open vehicle because more engine torque is transmitted to the ground instead of accelerating the skidding wheel (Figure 7.25). Therefore, as for $T_m = 60$ Nm, the longitudinal velocity of the *Torsen* remains higher on the ice than that of the vehicle with the open differential. The time gain is presented in Figure 7.27.

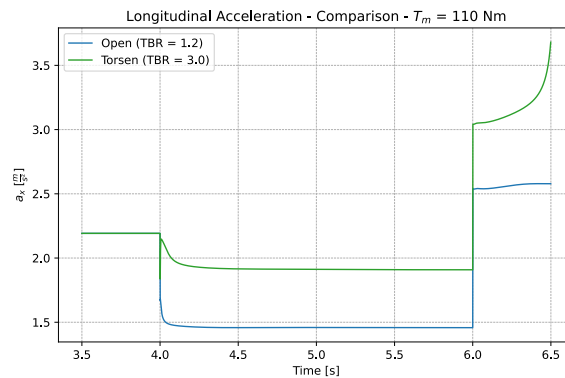
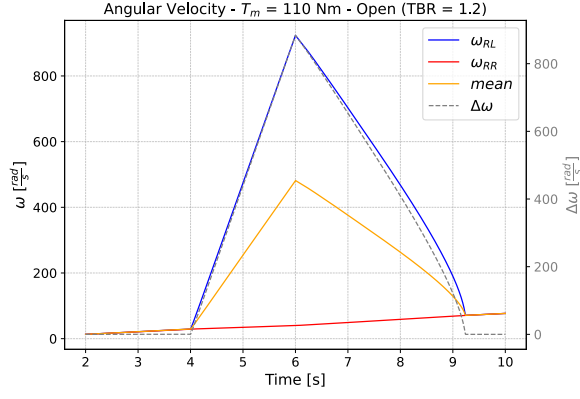
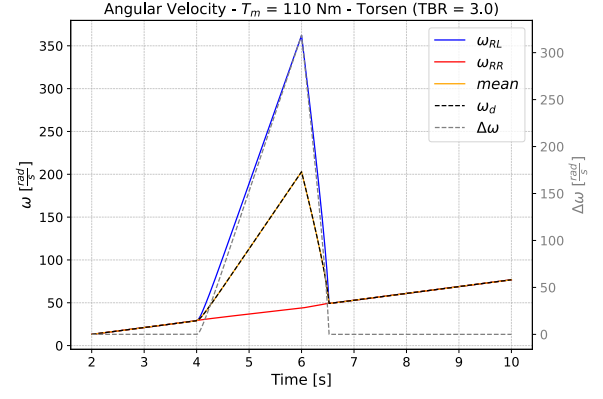


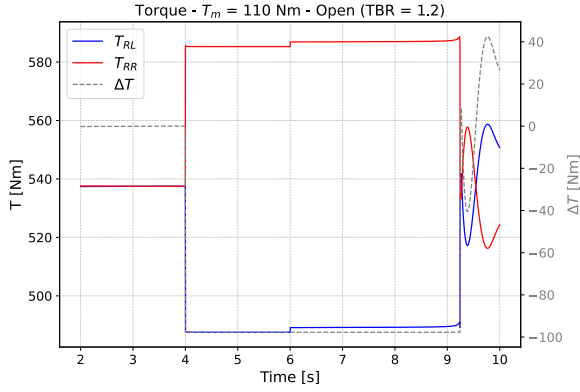
Figure 7.25: The *Torsen*'s longitudinal acceleration remains higher than that of the open differential. Thanks to the increased torque on the wheel with the best grip, the vehicle achieves a higher longitudinal velocity, saving the driver time.



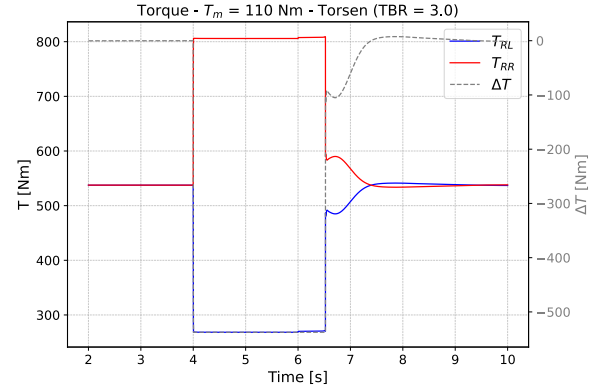
(a) When the vehicle emerges from the ice, the left wheel velocity reduces as it recovers grip. The timing for the reduction increases as the peak value is higher.



(b) The left wheel velocity rises because it is slipping. By mechanical construction, the mean velocity and the differential angular velocity remains the same.



(c) As for $T_m = 60$ Nm, ΔT achieves its maximum value directly on the ice patch. Since left wheel recovers grip, the left torque at the exit of the patch becomes lower than the limit on the ground. Thus, the left wheel decelerates.



(d) As for the open differential, the positive ΔT on the right wheel allow the wheels to recover the same rotational speed. Because less torque is applied to the left wheel by the *TBR*, the left wheel slip is reduced.

Figure 7.26: For $T_m = 110$ Nm, both differentials open, causing the left wheel to slip. The *Torsen* differential remains the best choice for ideal torque distribution.

As a result, with an engine torque $T_m = 110$ Nm and 2 seconds of ice, the path length for the open differential is 18.9 m, whereas it is 19.82 m for the *Torsen*. Relating this length to the velocity of the open differential reveals the time gained through the ice contact (Figure 7.27):

$$t_{Open, 18.9m} = 2s \quad \rightarrow \quad t_{Open, 19.82m} = 2 \cdot \frac{19.82}{18.9} = 2.097s$$

$$\Delta t = 2.097 - 2 = 0.097s \quad \text{Time saved by the } Torsen.$$

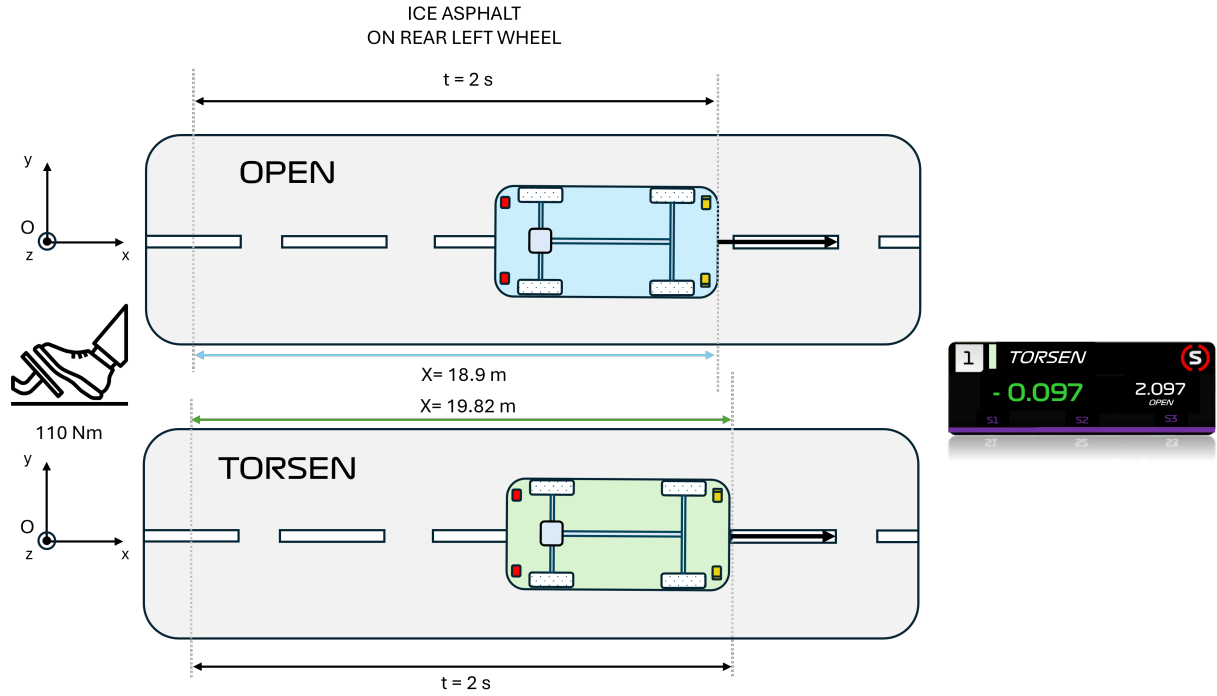


Figure 7.27: The *Torsen* differential recovers 0.097 second compared to the open differential during the ice patch at $T_m = 110$ Nm.

7.2.3 Influence of The *TBR* and The Engine Torque

With the two engine torques tested, the influence of the *TBR* and the engine torque on the vehicle handling is demonstrated. As a result, a parametric analysis similar to that for wet asphalt allows the differences between open and *Torsen* differentials to be highlighted.

Steering Wheel Angle

Firstly, the engine torques at which the differentials open are lower than those for wet asphalt. For the ice case, Table 7.6 shows the torque and the longitudinal acceleration a_x for each *TBR* tested. All the differentials open in this situation. The higher the *TBR*, the greater the torque required to open the differential.

Exactly as for the wet case, the following Figures present the two black lines for the yaw exceeding 90° and the opening torques. For the ice patch, the steering wheel angle limit is fixed to $\delta_{lim} = 45^\circ$.

Opening Torque for Each TBR		
TBR	T_m [Nm]	a_x [$\frac{m}{s^2}$]
1.2	60	1.195
2.0	80	1.594
3.0	100	1.994
4.0	120	2.394
5.0	130	2.593

Table 7.6

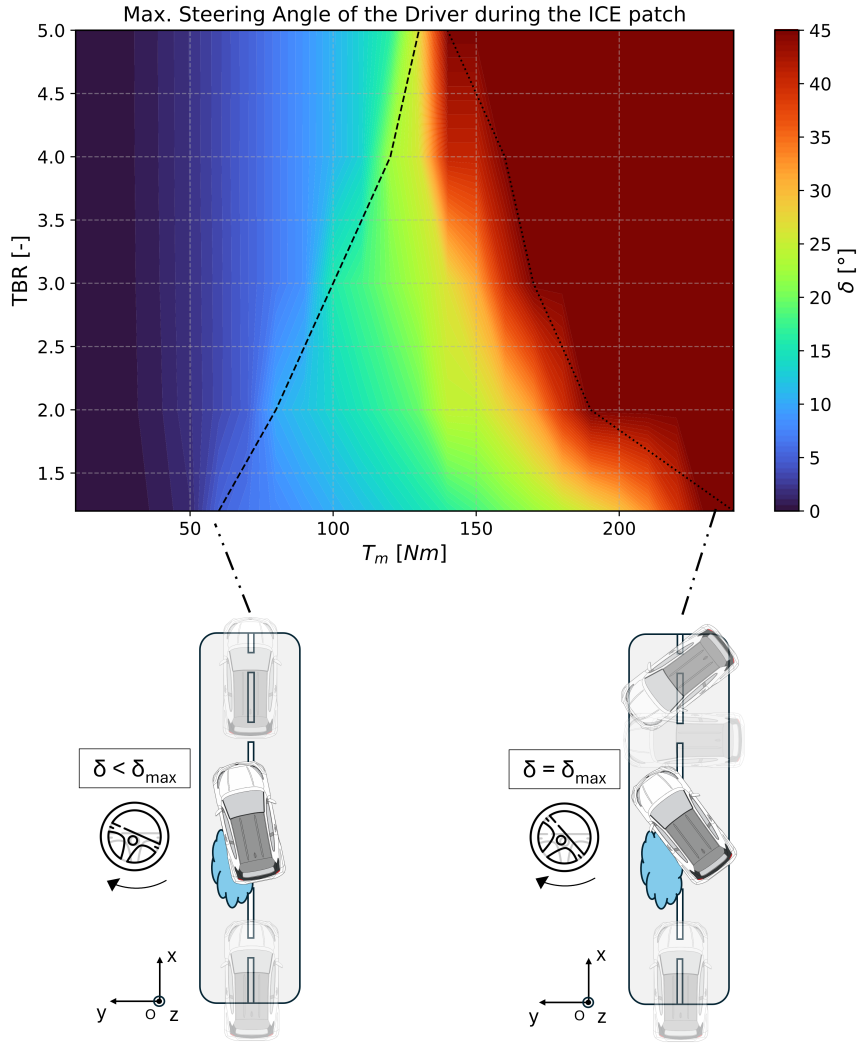


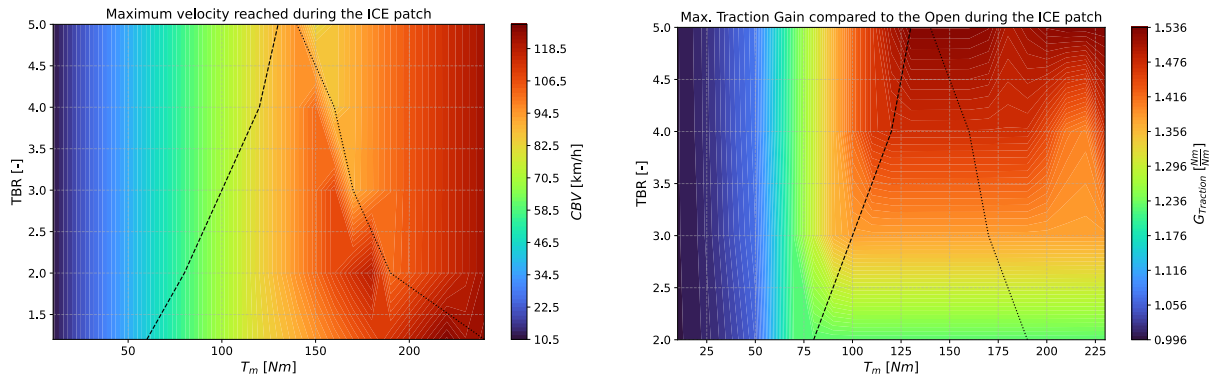
Figure 7.28: The maximum steering wheel angle applied by the driver on the ice patch is shown. Unlike on water, the locking effect has a greater influence on driver behaviour. As the TBR increases, the δ_{max} is reached at a more reliable T_m .

The driver has to correct more the trajectory using the steering angle when the TBR rises (Figure 7.28). A zone is formed between the two lines: for the dashed line (with low engine torque), the higher the TBR , the less the driver has to correct the trajectory using

the steering wheel (as examined for $T_m = 60$ Nm). However, this trend is reversed when the engine torque increases, close to the dotted line: the higher the TBR , the more the driver has to correct the trajectory. The limit value for steering wheel angle is reached at a lower engine torque with a higher TBR . As a result, the differential opening considerably impacts the driver's reaction. At high TBR , more engine torque is required to open the differential, resulting in a weaker PD controller correction. As soon as a differential with a higher locking effect opens, the driver must correct the trajectory much more due to the greater torque balance between the two wheels.

Traction Gain

Regarding the equation 7.1, the traction gain on the ice patch is observable in Figure 7.29b. At low engine torques, TBR has no effect on traction gain. However, as T_m approaches 80 Nm, a higher locking effect (TBR) results in a higher traction gain. Two distinct zones then emerge. at $T_m = 80$ Nm and from $TBR = 2.5$, the gain values become relevant: a high TBR between 80 and 175 Nm results in greater torque being transmitted to the ground than with an open differential. Below $TBR = 2.5$, the traction advantage is less dependant on the engine torque, and the impact of TBR is slightly reduced.



(a) A higher TBR gives a greater longitudinal speed. This is limited by the yaw line. Therefore, a compromise must be found for TBR in order to optimise the performance of the vehicle.

(b) The maximum traction gain is achievable with a $TBR = 5.0$ between $T_m = 125$ Nm and 150 Nm. Below $TBR = 3.0$, the influence of TBR on $G_{traction}$ is slightly reduced.

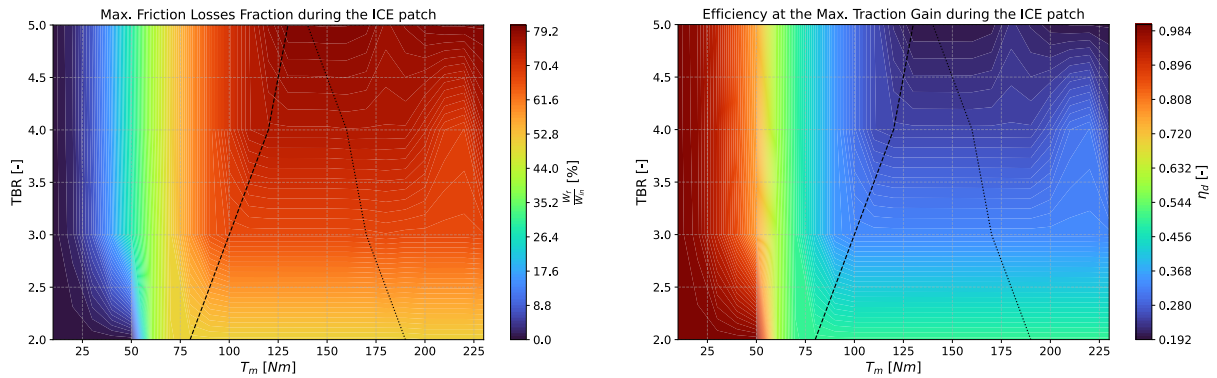
Figure 7.29: The choice of the fixed TBR in the differential will affect the vehicle's performance on the ice. A threshold between the car's maximum velocity (maximum time saved) and traction gain in tricky situations exists. Although a TBR of 5.0 offers the largest maximum results, the optimal engine torque window is reduced compared to the optimal utilisation window for a smaller TBR .

This advantage is reflected in the speed of the car on ice in Figure 7.29a, before the yaw of the vehicle exceeds 90° . The maximum velocity reached for each TBR tends to be close to the dotted line. The higher the TBR , the higher the speed at a given engine torque. However, since the yaw limit is reached at a lower engine torques when the locking effect is greater, the vehicle's maximum speed is reached at a TBR of around 2.0, between $T_m = 150$ Nm and 200 Nm (Figure 7.29a). Therefore, even if a higher TBR gives a greater gain in traction, this does not necessarily improved the performance of the vehicle. There

is a trade-off between achieving a high level of traction gain (for example, to optimise the torque on one of the wheels in order to get out of a tricky situation) and the vehicle's performance (to save as much time as possible on ice).

Losses and Internal Efficiency

As expected, the loss ratio in Figure 7.30a and the internal efficiency in Figure 7.30b follow the shape of the traction plot in Figure 7.29b. The previously highlighted window for the $TBR = 5.0$ is not actually optimal in terms of efficiency and internal losses. Consequently, the correlation between traction gain and internal efficiency will show the optimal TBR threshold for each engine torque.



(a) Friction losses peak at $TBR = 5.0$ before the yaw limit at $T_m = 140$ Nm. Therefore, the optimum traction gain zone highlighted becomes the worst in terms of losses.

(b) The range below $TBR = 3.0$ shows a better efficiency than at high TBR for the same engine torque. As with the traction gain plot, the efficiency does not vary with T_m once the opening limit (dotted line) is exceeded.

Figure 7.30: Efficiency is the highest at low TBR and low engine torques. Before the opening limit is reached, it is only influenced by the engine torque. When the dotted line is crossed, TBR takes precedence over the influence of differential efficiency and, therefore, internal losses.

Global Gain

The expression 7.2 in the case of the ice patch is observable in Figure 7.31. The global gain highlights the optimal TBR choice for the tested engine torque range. In this figure, several interesting areas stand out.

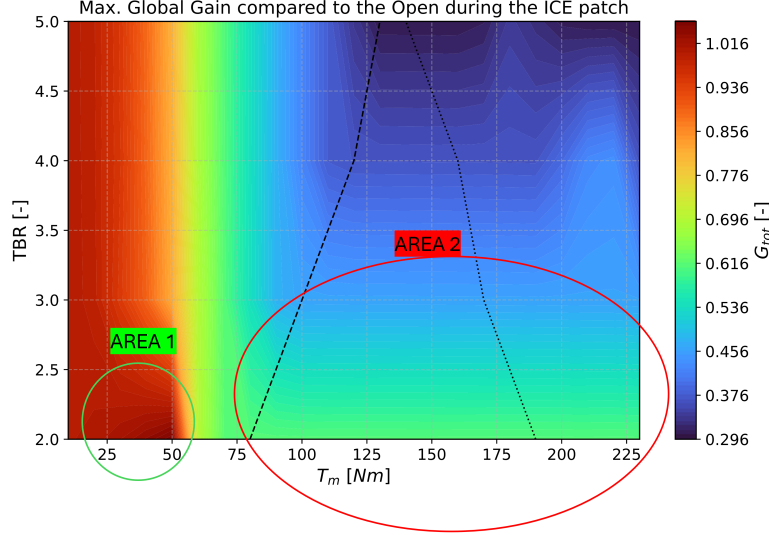


Figure 7.31: Area 1 shows the best overall gain. It approaches 1.0 because the efficiency at low engine torque and low TBR is very high. Area 2 highlights the optimal threshold of G_{tot} for the largest engine torque range.

Firstly, Area 1 shows the TBR and the engine torque at which the overall gain of the *Torsen* compared to the open differential is felt. Between 0 and 50 Nm, a TBR close to 2.0 yields the most significant gain. In this area, the TBR has little influence on the value of the gain, as it is largely dependent on the engine torque (as do the efficiency and the traction gain). Next, the dashed line clearly shows that for each TBR , the opening of the differential marks the transition to a regime in which the TBR mainly influences the *Torsen* gain compared to the open differential. According to this line, it appears that for a TBR above 3.0, the gain becomes less than 0.5: a high TBR differential is less and less profitable for higher engine torques. Conversely, for a TBR between 2.0 and 3.0 (Area 2), the locking effect of the differential is highly advantageous compared to an open differential. Therefore, the compromise to be made when choosing the TBR value will lie between these two values. The objective is to optimise the use of the *Torsen* compared to the open differential over a wider range of engine torque. This range of TBR values is confirmed by the choices already made by JTEKT *Torsen* for its products. For example, the new *Alpha Romeo Junior* SUV has a TBR of 2.13 in drive mode and 2.03 in coast mode. For the *Lancia L21 HF*, the driving mode TBR is 2.28.

7.3 Conclusion on the Performance of *Torsen* and Open Differentials on Low-Grip Surfaces

In conclusion, this chapter highlights different operational areas. The first is at low engine torque, where a *Torsen* differential with a low TBR is more advantageous than a differential with a higher TBR . In fact, it provides a slightly better traction for the same level of steering wheel angle correction. This small area disappears at higher engine torque. Although the steering wheel correction with a *Torsen* differential is greater because of the amount of the torque balance between the two wheels, the torque bias confers an

overall advantage in tyre conservation compared to the open differential. This preserves the tyre's wear on a low-friction coefficient surface. However, a vehicle equipped with an open differential allows higher level of engine torque before to spin. This level decreases as the TBR increases between 1.2 and 2.0. Nevertheless, the value of engine torque before the vehicle spins remains high (≈ 60 % of the maximum acceleration rate). This level seems too high to be reached by the average driver. Therefore, the *Torsen* differential offers benefits to a wide range of drivers on wet surface, as the global grip index G_{tot} confirms.

Moreover, the range of engine torque at which an open differential is advantageous remains small compared to the range at which a *Torsen* differential with a higher TBR is advantageous. With higher engine torques on water and, in particular, on snow, the locking effect provides improved driver control, better tyre conservation, and enhanced overall performance (i.e. time saved when travelling the same road). Conversely, if the TBR is too high (above 3.0), the driver has to react more strongly, making the vehicle harder to control at lower engine torques (the engine torque for which the yaw limit is reached decreases as the locking effect grows). Therefore, when designing the *Torsen* differential, a threshold must be established to optimise the engine torque range at which the gain with a *Torsen* differential is most effective. Clearly, a TBR between 2.0 and 3.0 is optimal for the tested engine torque range.

Chapter 8

Conclusion

This Master’s thesis is the result of a close and productive collaboration with the company *JTEKT Torsen*, originally based on a personal project proposal. The main objective was to conduct a comparative dynamic analysis of the performance of vehicles equipped with either an open or a limited-slip differential (*Torsen*). These comparisons were carried out using multi-body simulations with the *Robotran* software under various driving scenarios.

The study started with a review of the theoretical principles necessary for understanding vehicle dynamics and differential systems. A thorough review of the differential variants, including open, locked, speed-sensing LSD and torque-sensing LSD, emphasised the distinct benefits offered by the *Torsen* design. These include its fast response, compatibility with ABS systems and improved torque distribution between wheels under varying grip conditions.

The simulation models developed for this thesis provided insights into the influence of the differential type on straight-line and cornering behaviour. In low-torque situations, open differentials were shown to reduce tyre wear and improve driver facilities. However, as soon as higher engine torques were applied, the *Torsen* differential demonstrated clear benefits in terms of cornering speed, vehicle stability and overall handling performance. These advantages are particularly relevant in motorsport contexts, where even marginal gains can be crucial.

The final part of the study focused on more extreme driving scenarios, such as asymmetric grip conditions (i.e. wet asphalt or icy surfaces). The analysis highlights that in terms of traction gain, the *Torsen* remains preferable to an open differential. While more effort may be required from the driver, depending on the engine torque applied, the *Torsen* differential offers improvements in terms of vehicle control, tyre preservation and vehicle performance. The surface on which the vehicle is driven, the *TBR* and the amount of engine torque applied by the driver all influence the conditions in which the *Torsen* is most effective. Therefore, there is a threshold to be reached when designing a *Torsen* in order to maximise its advantages for all types of drivers and on all types of surfaces.

Overall, the results emphasise the importance of both the torque level and the locking effect on vehicle dynamics and driver response. This confirms the relevance of *Torsen* differential for enhancing both performance in motorsport and safety in everyday driving. While the

objectives of the Master's thesis were met, there are several promising improvements for future research. These could include analysing vehicle behaviour during throttle-off and cornering in *Split* – μ conditions or conducting standardised tests, such as the Moose Test. Additionally, refining the driver control model by incorporating steering angle, engine torque and locking effect feedback could improve the understanding of the interaction between human inputs and mechanical response.

Despite the limitations in scope and time, this work provides a solid foundation and contributes meaningful insights into the comparative performance of open and *Torsen*-type differentials.

Appendix A

The Differential

A.1 Velocity Difference in Cornering

Thanks to the *Ackermann* principle, the importance of the differential aspect is clear. In the case of a right turn, taking the linear velocity of each wheel and considering the angular cornering velocity of the entire car body as ω , one can write:

$$V_{outer} = V_{left} = \omega \cdot R_{RL} \quad \& \quad V_{inner} = V_{right} = \omega \cdot R_{RR}$$

Because $R_{RL} > R_{RR}$ as described in Figure A.1, the velocity of the outer wheel V_{outer} will automatically be larger than V_{inner} . Therefore, taking the global cornering radius of the car R , the reason why ω_{outer} is larger than ω_{inner} appears:

$$V = \omega_{wheel,i} \cdot R_{wheel,i} \quad \rightarrow \quad \omega_{RL} = \frac{V_{outer}}{R} > \frac{V_{inner}}{R} = \omega_{RR}$$

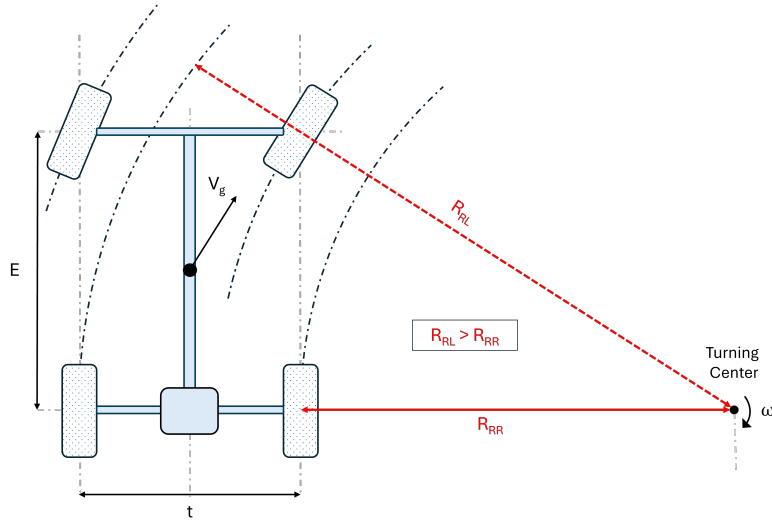


Figure A.1: Each wheel owns its radius of curvature inducing a different rotational velocity for each of them.

A.2 Transmission Variant in a Vehicle

The three types of transmission are presented in Figure A.2. In the case of a 4WD vehicle, a transfer box (torque converter) is required to split the torque between the front and rear axles.

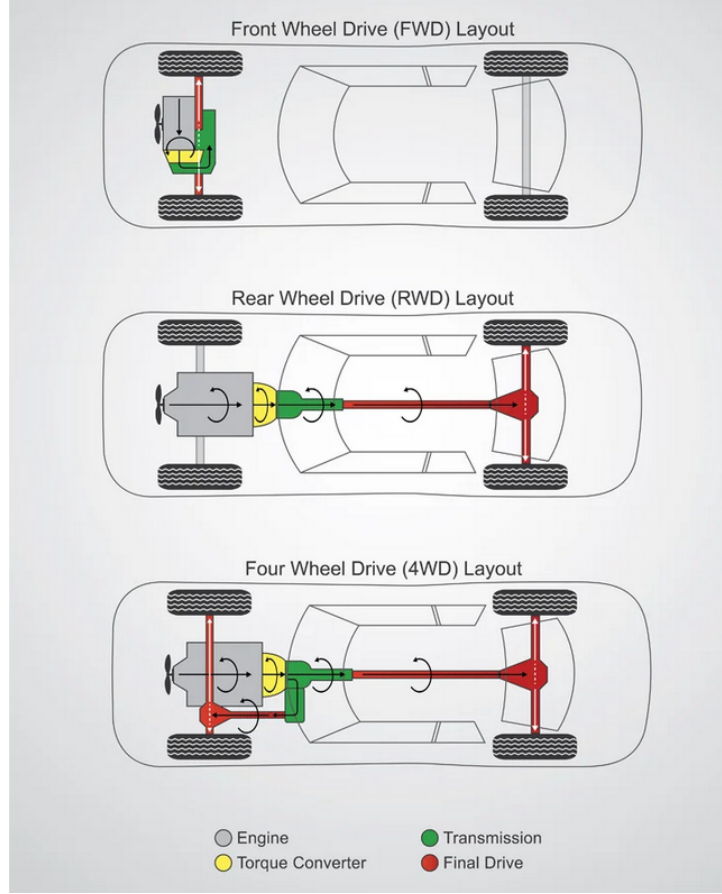


Figure A.2: Different types of transmission found in typical cars. The torque converter is the transfer box in the case of 4WD vehicle [38].

A.3 Longitudinal Slip Difference for a Locked Differential

The difference in the longitudinal slip ratio (κ) of the left and right wheels when the differential is locked causes tyre wear. When the wheel velocities are the same, we can write for traction mode [14]:

$$\kappa_l = \frac{\omega_d R_{wheel} - V_{x,l}}{\omega_d R_{wheel}} > 0 \quad \& \quad \kappa_r = \frac{\omega_d R_{wheel} - V_{x,r}}{\omega_d R_{wheel}} > 0 \quad (\text{A.1})$$

Where R_{wheel} means the wheel radius. Compared to the definition in the theoretical section, we impose a positive sign of the longitudinal slip κ . The difference between right

and left longitudinal slip is denoted $\Delta\kappa$:

$$\Delta\kappa = \kappa_r - \kappa_l = \frac{V_{x,l} - V_{x,r}}{\omega_d R} \quad (\text{A.2})$$

The longitudinal velocity can be transformed with the yaw velocity of the car body in cornering $\dot{\psi}$. It states how fast the vehicle turns around the turning centre, i.e. the IRC.

$$\Delta\kappa = \frac{\dot{\psi} R_{RL} - \dot{\psi} R_{RR}}{\omega_d R} \quad (\text{A.3})$$

Taking R_{RL} the radius of curvature of the rear left wheel and R_{RR} the radius of curvature of the rear right wheel (see Figure A.1). During a right turn, the longitudinal slip on the right wheel is greater than that on the left because R_{RL} is greater than R_{RR} .

A.4 Link Between *TBR* and ε_d

Figure A.3 shows the relation given in equation 3.10. The locking effect ε_d saturates as the *TBR* increases.

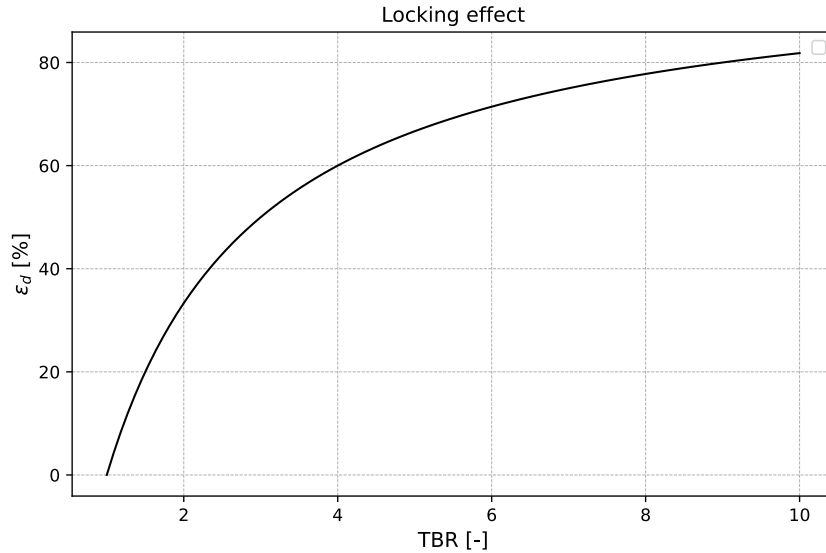


Figure A.3: Plot of the relation 4.18. The locking effect becomes saturated when the *TBR* increases.

A.5 Gear Relations in a *Torsen* Type B

Different handling situations involve different gear movements inside the *Torsen* Type B differential:

- In straight line, with equal surface friction coefficient: the *Torsen* reacts exactly like an open differential. As the *Torsen* is a torque-sensing device, it detects when one wheel loses grip and splits the torque accordingly, before wheel speed differentiation occurs. The worm wheels only rotate on the worm gear but not on their own shaft. They act as keys.

- In low speed cornering: as described earlier, the outer wheel in turning rotates faster than the inner wheel. Therefore, the *Torsen* differential will distribute more torque to the slower wheel (the wheel having the better traction). The friction inside the differential tends to prevent the inner wheel to rotate slower than the outer wheel. In the case of a low speed cornering, this additional torque will apply on the inner wheel. The total amount of ΔT will depend only on the input torque and the *TBR*. In turning, sun gears drive the element gears. They rotate on itself in opposite direction. Taking a left turn as a basic example: the left side gear will drive its element gears. By the element gear shafts, the right element gears are activated (in type A, this trigger action is provoked by the spur gears relations between element gears). Between right element gears and right side gear, there is a locking effect corresponding to the *Torsen TBR*.

It is important to notice that speed differentiation of wheels is only allowed when the available torque on each wheel exceeds the fixed *TBR* of the *Torsen* (because of the inside fixed gear friction). As it is observable in Figure 3.23, the torque differentiation will immediately appear before $\Delta\omega \neq 0$. Thus, the vehicle will follow an understeered behaviour compared to a vehicle equipped with an open differential[8, 34].

- In high speed cornering: as explained in the section 3.2.3, the outer wheel will become the slower one because of the load transfer produced by the lateral acceleration. *Torsen* differential will send more torque to the outer wheel **before** the inner wheel begins to spin. Accordingly, with a sufficient amount of input driver's torque, the vehicle will have an oversteered moment compared to the open differential case. This improves the lateral stability and enhances the handling control of the driver. Finally, the driver is able to control his trajectory not only through the steering wheel but also by adjusting the amount of torque he applies, allowing for understeered and oversteered management. This behaviour could be surprising: "we have to apply more torque to turn more sharply". This characteristic seems not easy for standard driver as we have the feeling to coast/to brake when we go too fast in a turning. This is one of the reasons why most of the cars tend to be understeered and are equipped with an open differential: for a standard driver, it is more naturally to decelerate or brake when the vehicle left the turning trajectory rather than accelerating.

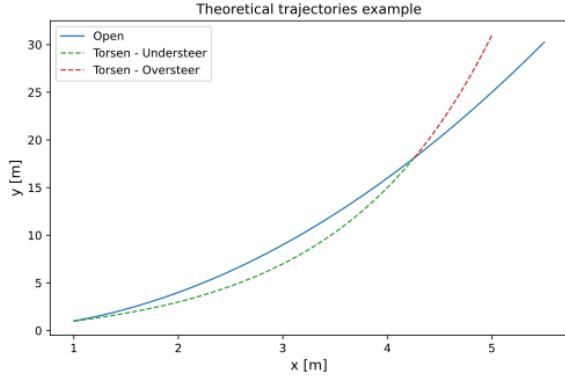


Figure A.4: Torsen produces an overtseered moment that can be advantageous for advanced/experienced drivers when engine torque increases.

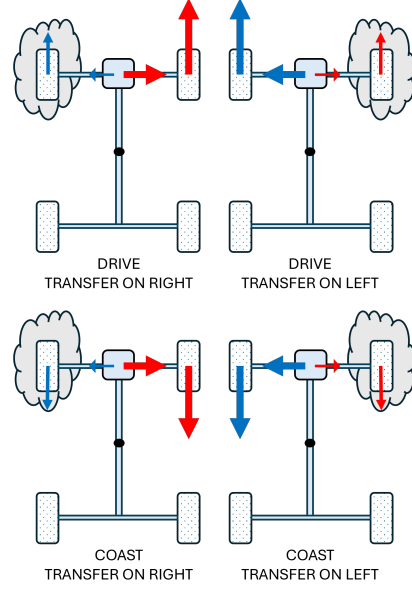


Figure A.5: Torque transfer for the 4 modes of *Torsen* type B utilisation. The clouds represent a low surface's friction coefficient.

- In case of losing wheel/ground contact: the mechanism torque distribution which occurred in the *Torsen* depends on the available traction on both wheels. Like the turning situations, when a wheel is on a surface with lower friction coefficient μ , the *Torsen* tends to prevent wheel spin if the input driver torque is higher than the torque available on the wheels. Hence, this limited-slip differential will inject more torque to the wheel with a better traction. The torque transfer occurs like in turning, with the locking effect created by the internal friction. Before the slip of wheel with the worst traction occurs, the worm wheel of this wheel drives its analogous worm gear. By the worm gear/worm wheel mechanism, the worm gears of the second wheel tries to drive the side gear. Therefore, there is a locking effect related to the *TBR* value of the differential. We can see the torque balance in the four modes in Figure A.5 [31, 16].
- Coast mode: when the driver lifts off his foot of the torque pedal, there is a small negative torque injected inside the differential, due to the engine braking. As the *Torsen* differential tries to limit the speed differentiation between wheels, the torque balance will occur on the outer wheel in cornering. Thus, the vehicle gets an understeered moment because of the small negative torque injected on the outer wheel.

Appendix B

Multi-Body Verification

B.1 Engine Torque Curve

Currently, the JTEKT *Torsen* company designs *Torsen* differentials for electrical vehicles, like previously mentioned the *Alfa Romeo Junior*. Therefore, the vehicle model is adapted with an existing electrical motor with a specific and known torque curve, like presented in the Figure B.1. In order to determine all the values for each engine speed, an interpolation was executed on special torque points.

5.2.3 E-Motor characteristics

Max power: 175 kW
Max torque: 345 N.m
Torque curve:

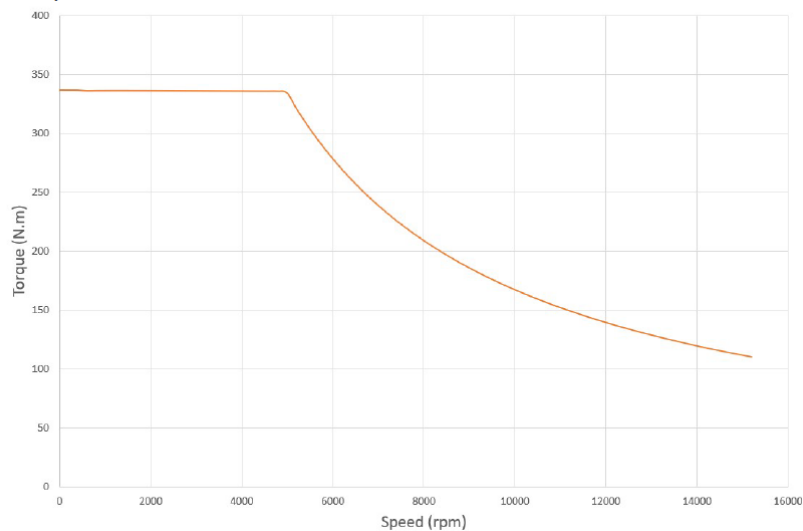


Figure B.1: The torque characteristic provided by *JTEKT* is interpolated to find all the points of the engine torque in function of the engine speed.

B.2 Tyres Coefficients

The coefficients for tyres in the *Bakker-Pacejka* model depend on the normal load F_n applied to the wheel. In the expression 2.4, the coefficients are:

$$C_{long} = 1.65$$

$$D_{long} = \frac{(-21.3F_n^2 + 1144.0F_n)}{k}$$

$$E_{long} = -0.006F_n^2 + 0.056F_n + 0.486$$

$$K_{long} = (49.6F_n^2 + 226.0F_n) \exp(0.069F_n)$$

$$B_{long} = \frac{K_{long}}{C_{long}D_{long}}$$

$$C_{lat} = 1.3$$

$$D_{lat} = \frac{(-22.1F_n^2 + 1011.0F_n)}{k}$$

$$E_{lat} = -0.354F_n + 0.707$$

$$K_{lat} = (1078.0 \sin(1.82 \arctan(0.208F_n)))$$

$$B_{lat} = \frac{K_{lat}}{C_{lat}D_{lat}}$$

B.3 Torque Distribution on Rear Wheels

Hyperbolic Function Distribution

Compared with the Figure 4.11b, the torque distribution offered by the hyperbolic tangent function reacts as the linear decoupling function. The green, orange, and purple points aim to follow the *Torsen* mechanical decomposition (opening) during all the process of the torque law distribution, detailed in the Figure 4.13a.

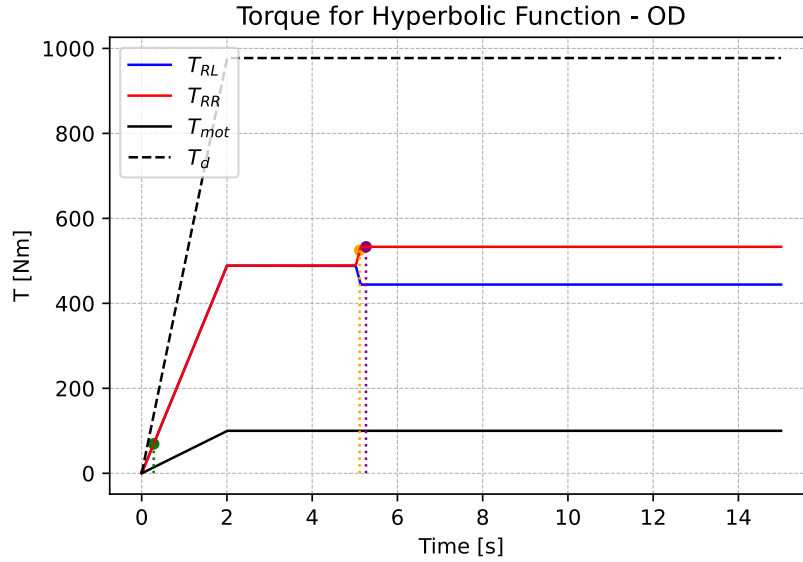


Figure B.3: The torque distribution on the rear axle for the hyperbolic tangent function operates as the linear decoupling in Figure 4.11b.

Bakker's Function Distribution

As we can immediately see on the following plot, a stiff problem induces perturbation for the torque wheels distribution. As soon as the simulation starts, there is a torque

difference which does not stabilize during the simulation. It provokes a wrong vehicle behaviour compared to the linear decoupling in the same situation, presented in Figure 4.11b.

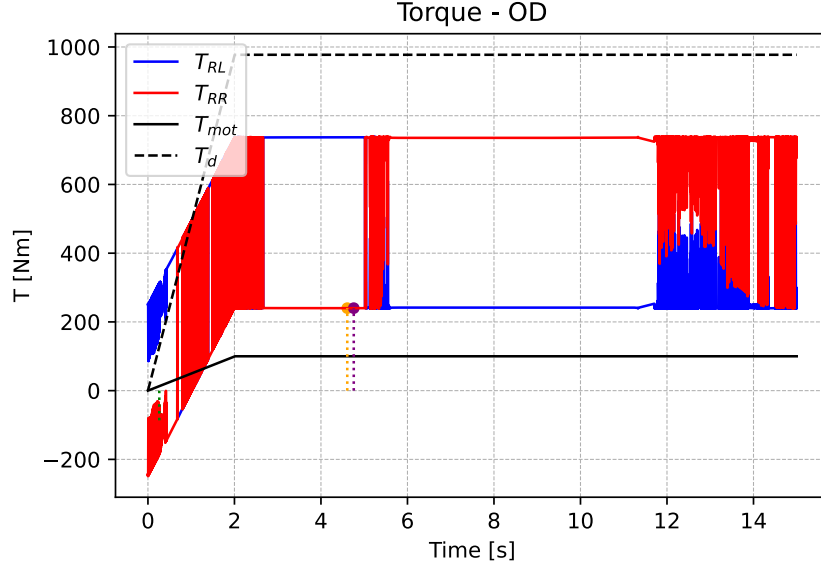


Figure B.4: The stiff problem induces instabilities in the numerical steps and disturbs the torque balance on the rear wheels.

B.4 Trajectory Errors Calculation Steps

The error in radial position takes form with:

$$\text{The distance } CS \text{ is } \|\vec{CS}\|: \quad \vec{CS} = \vec{OS} - \vec{OC} \quad (\text{B.1})$$

$$\text{with:} \quad \vec{OS} = \vec{S}_P = \begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} \quad \& \quad \vec{OC} = \begin{pmatrix} x_R \\ R \\ 0 \end{pmatrix}$$

$$\text{Thus, the lateral error is:} \quad e_{lat} = \|\vec{CS}\| - R$$

The PD controller on the trajectory measures and corrects the yaw error compared to the targeted angle:

$$\text{The circle tangent is defined as:} \quad \hat{e}_{circle} = \begin{pmatrix} \frac{-CS[1]}{\|\vec{CS}\|} \\ \frac{CS[0]}{\|\vec{CS}\|} \\ \frac{CS[2]}{\|\vec{CS}\|} \end{pmatrix} \quad (\text{B.2})$$

The yaw error is the angle of the following scalar product, using the rotational matrix R_{ij} between the car and the original frame xyz :

$$e_{yaw} = \arccos(\hat{e}_{vehicle} \cdot \hat{e}_{circle}) \quad \text{with} \quad \hat{e}_{vehicle} = \begin{pmatrix} R_{11} \\ R_{12} \\ R_{13} \end{pmatrix}$$

The final output of the PD controller acting on the steering wheel takes the following form:

$$\delta = K_{P,lat} \cdot e_{lat} + K_{D,lat} \cdot \dot{e}_{lat} + K_{P,yaw} \cdot e_{yaw} + K_{D,yaw} \cdot \dot{e}_{lat} \quad (\text{B.3})$$

Appendix C

Additional Results for Validation of Differentials

C.1 Validation of OD and Open Differential

The $TBR_{measured}$ exceeds the fixed TBR of the open differential. For the OD , since there is no friction inside the differential, the $TBR_{measured}$ remains equal to 1.0.

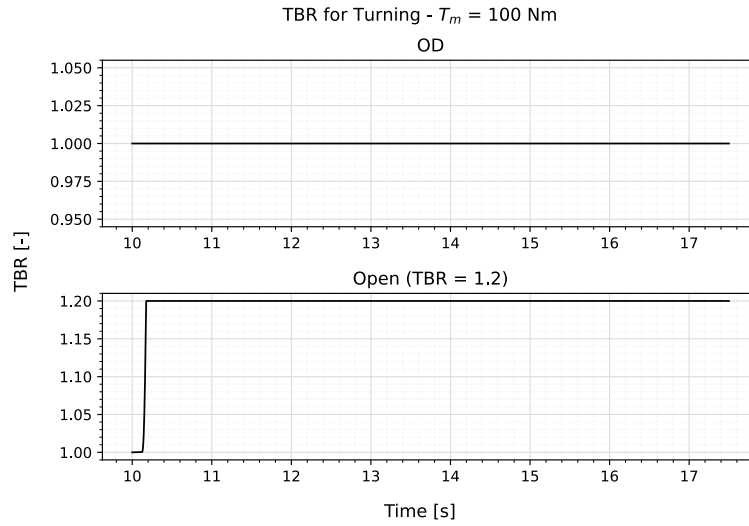


Figure C.1: Once the $TBR_{measured}$ reaches the fixed TBR , the open differential opens.

C.2 Validation of Locked and *Torsen* differentials

Measured TBR

The $TBR_{measured}$ is calculated as follows:

$$TBR_{measured} = \begin{cases} \frac{T_{rl}}{T_{rr}} & \text{if } T_{rl} > T_{rr} \\ \frac{T_{rr}}{T_{rl}} & \text{else} \end{cases} \quad (C.1)$$

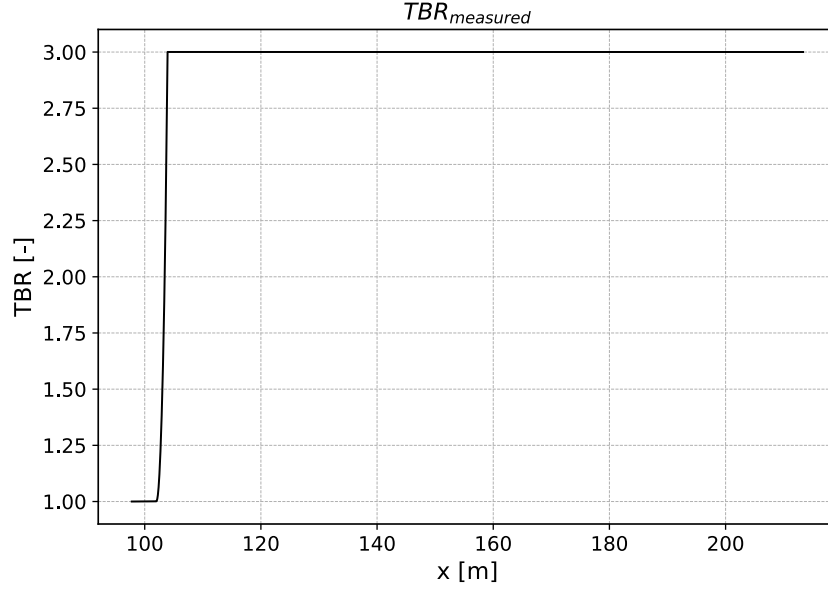


Figure C.2: Once the $TBR_{measured}$ reaches the fixed TBR , the *Torsen* differential opens.

Angular acceleration

The angular acceleration of the outer wheel is a bit smaller compared to the angular acceleration of the inner wheel. Therefore, the speed of the outer wheel tends to increase but at a slower rate compared to the inner wheel's speed during the turn.

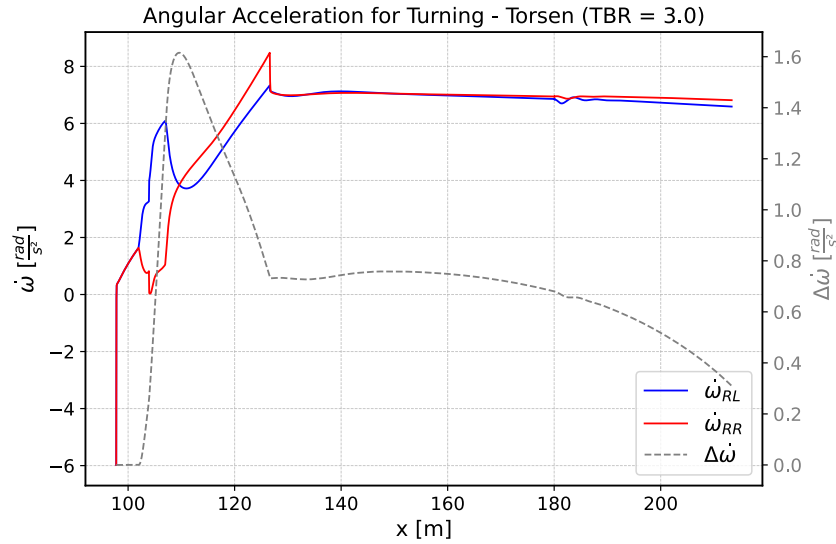


Figure C.3: The velocity of the outer wheel decreases due to its smaller angular acceleration compared to the inner wheel.

Yaw moment

The understeer yaw moment for the locked differential is higher than for the *Torsen* one. Nevertheless, the difference decreases due to the lateral load transfer with the rising of the lateral acceleration a_y .

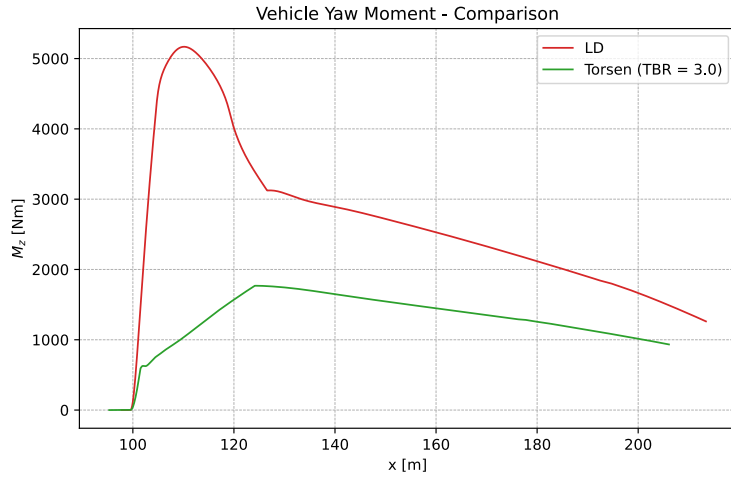


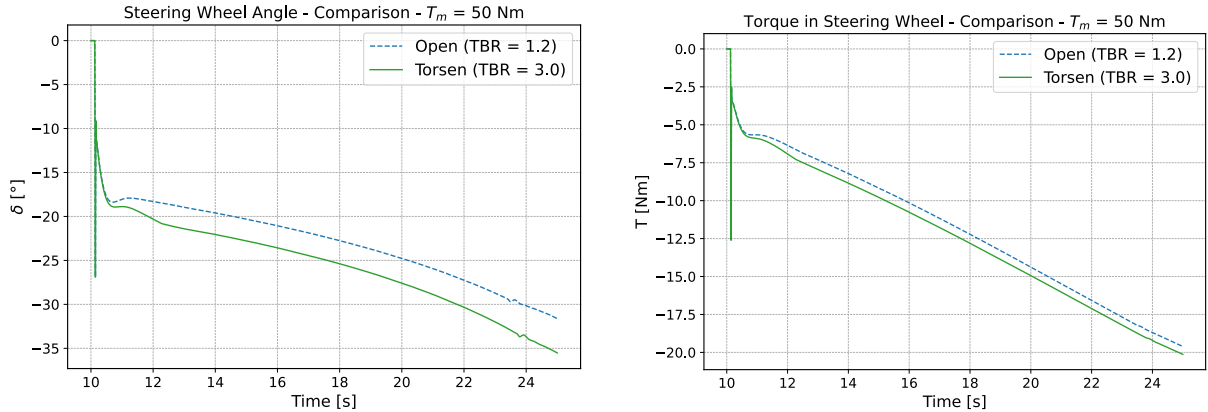
Figure C.4: The yaw moment can be calculated knowing the *Bakker* longitudinal forces on the rear wheels.

Appendix D

Additional Results for Cornering Behaviour Differences

D.1 Driver's Reaction with $T_m = 50$ Nm

The driver's reaction needed to follow the trajectory is greater with a *Torsen* differential. This larger effort is the consequence of the torque balance. For an equivalent corner, it is more difficult for the driver to follow the trajectory, as we can observe in Figures D.1a and D.1b.



(a) The steering angle is greater for the *Torsen* to maintain the same path than the open differential.

(b) The *Torsen* vehicle is more understeered than the open one. The driver has to apply more torque in the steering wheel to keep the same trajectory.

Figure D.1: The driver's effort is higher with a *Torsen* differential at low engine torque.

D.2 Lateral Slip

The lateral acceleration producing the lateral forces stays nearly the same for all the differentials. It is a bit larger for the open differential due to the more understeering handling of the *Torsen* differential.

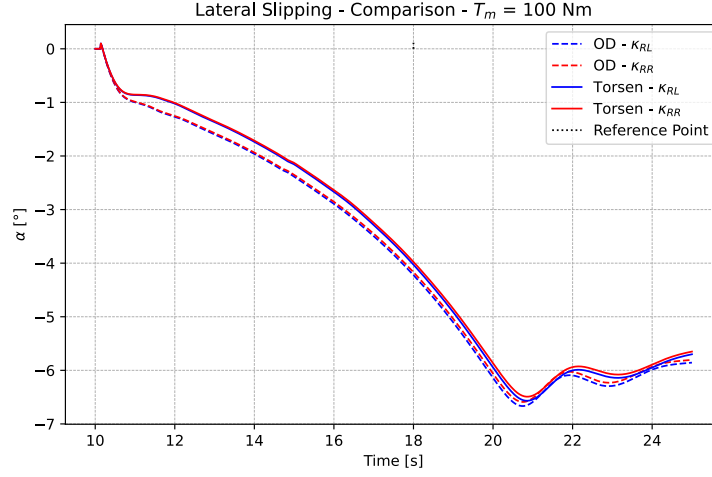


Figure D.2: The lateral slip for the open differential persists larger than for the *Torsen* because of the more understeering behaviour of the latter.

D.3 Yaw Rate of The Vehicles

The yaw rate tends to be the same until the *Torsen* starts to trigger the inversion of torque bias. The difference will be more pronounced with an engine torque of $T_m = 150$ Nm.

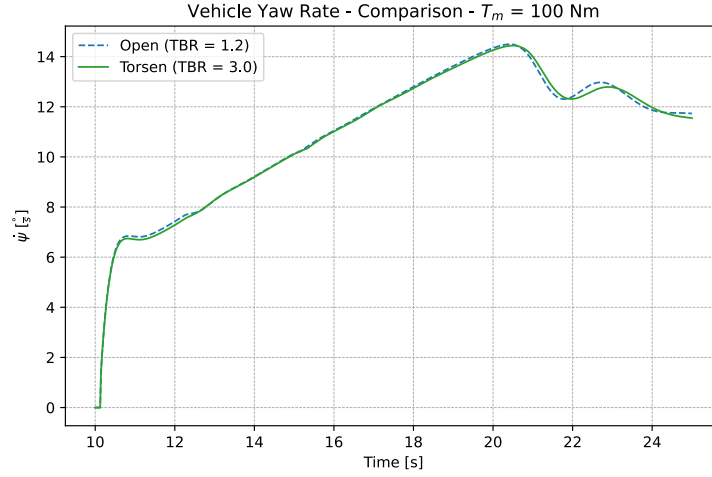
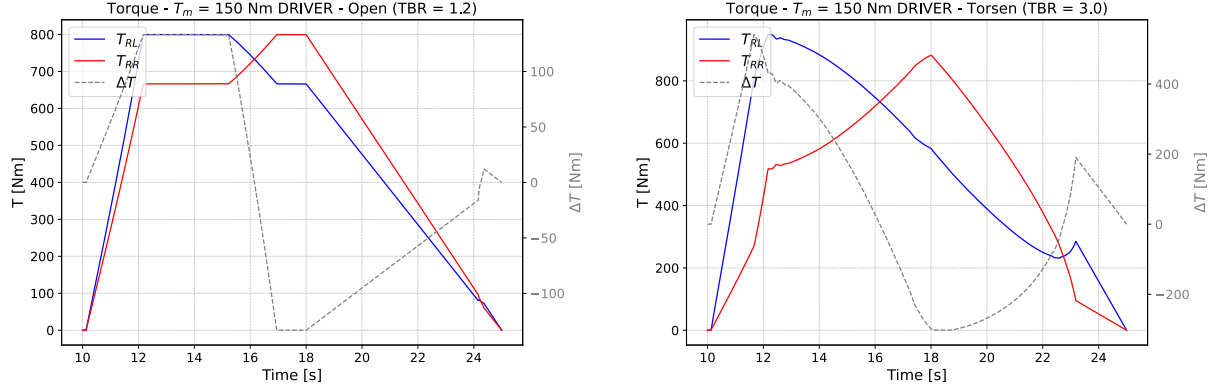


Figure D.3: The yaw rate, describing the vehicle tendency to align itself with the trajectory, stays nearly the same for both differentials.

D.4 Torque Distribution for Driver Cornering Influence

The torque distribution produces a more favourable oversteering moment in the case of the *Torsen* differential. The driver with the open differential can not stay close to the trajectory and lose control of his car.



(a) There is insufficient torque distribution on the outer wheel to create sufficient oversteering moment when cornering.

(b) The torque applied on the outer wheel (fastest one) is higher. The torque bias creates a more pronounced oversteering moment.

Figure D.4: Torque balance inversion occurs as soon as the outer wheel provides the best grip. As T_m decreases, the load transfer is reduced, meaning that the inner wheel becomes the wheel with the best grip by the end of the turn. Consequently, the positive torque balance is redistributed to the inner wheel at the end of the test.

D.5 Vehicle's Position in Coast Situation

The engine braking induces a more understeered moment for the *Torsen* differential, but both vehicles stay on the corner.

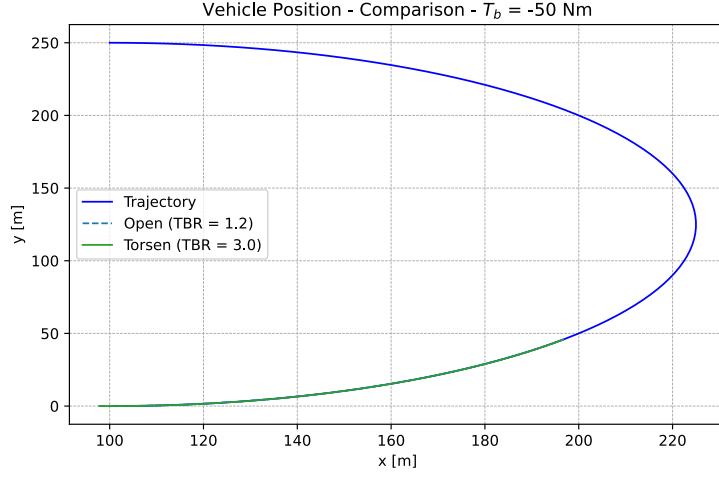
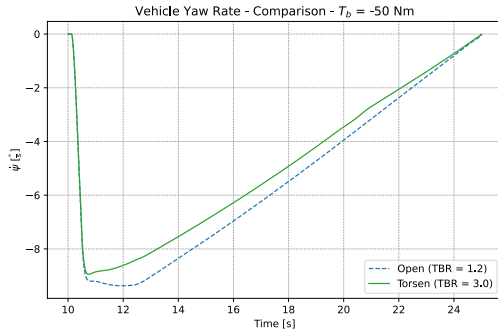
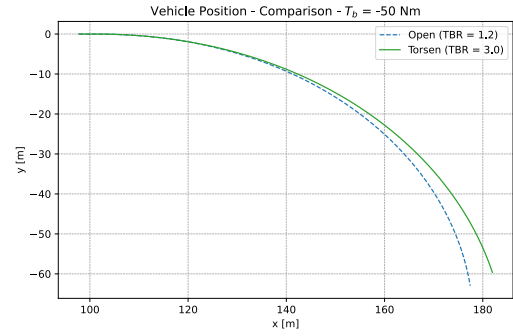


Figure D.5: The vehicles follow strongly the trajectory.

With a constant steering angle of 30° , the yaw rate reduction of the *Torsen* differential is clearly highlighted (Figure D.6a). This results in a more understeered vehicle, as the position in Figure D.6b proves.



(a) The yaw rate reduction proves the yaw damper effect of the *Torsen* differential.



(b) The *Torsen* vehicle is more understeered than the open one with the same steering angle and engine torque.

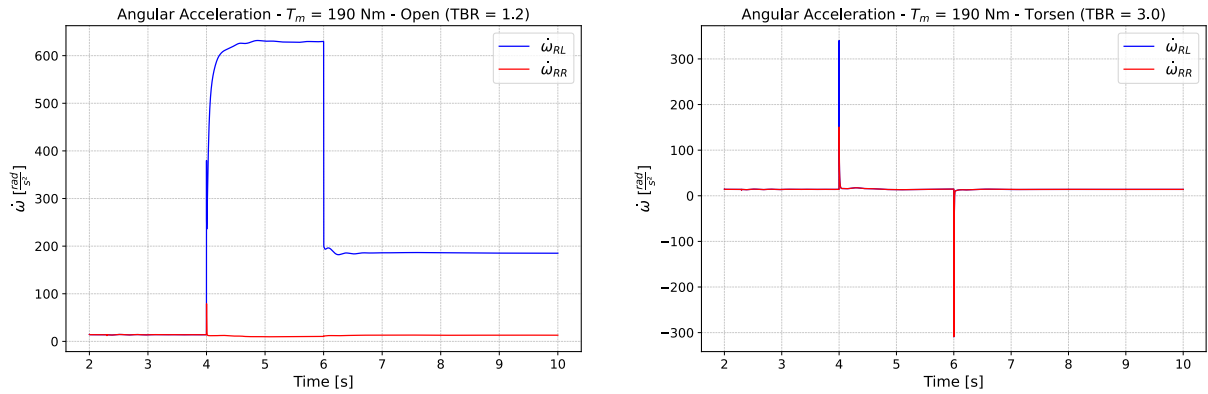
Figure D.6: The effect of the throttle-off situation induces more stability in the cornering.

Appendix E

Additional Results for *Split* – μ Test

E.1 Angular Acceleration on Wet Asphalt

) The angular acceleration of the left wheel increases sharply when the vehicle enters the puddle. Linked to the amount of torque injected on the left wheel, it explains why the velocity of this wheel continues to grow during the patch of water:



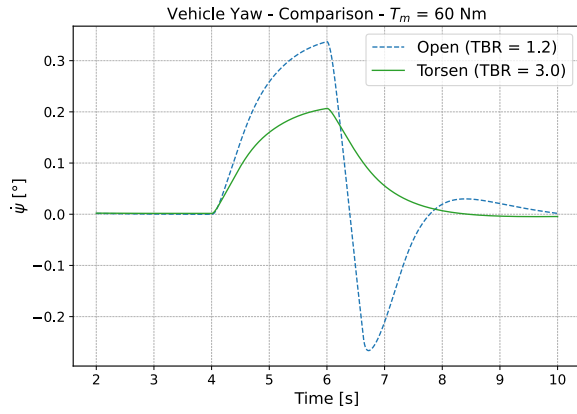
(a) The angular accelerations of the wheels reaches a huge value as soon as the wheel touches the water. This acceleration is induced by the left torque injected by the differential exceeding the friction torque available on the wheel.

(b) Due to the different torque repartition in the *Torsen* differential, the peaks of angular accelerations are smaller than the open differential.

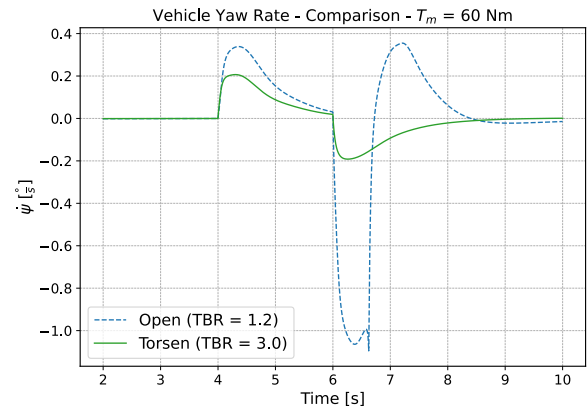
Figure E.1: The angular accelerations of the wheels affect the velocity and thus the torque distribution in the *Torsen*.

E.2 Yaw and Yaw Rate on Wet Asphalt

The PD controller response depends on the yaw and the yaw rate. Because these parameters are influenced by the torque distribution on the rear wheels, the driver reaction will vary in function of the differential used.



(a) The yaw still remains higher for the *Torsen* due to the torque distribution.



(b) The yaw rate is higher for the open differential because the left wheel is skidding. The driver reaction is easier with the *Torsen* differential.

Figure E.2: The angular accelerations of the wheels affect the velocity and thus the torque distribution in the *Torsen*.

E.3 Angular Acceleration on Ice Asphalt

The angular acceleration of the left wheel increases sharply when the vehicle enters the ice patch. Linked to the amount of torque injected on the left wheel, it explains why the velocity of this wheel continues to grow during the ice. When the vehicle comes out the ice, the left wheel recovers grip and its rotational velocity drops. This is why the angular acceleration of the left wheel becomes negative.

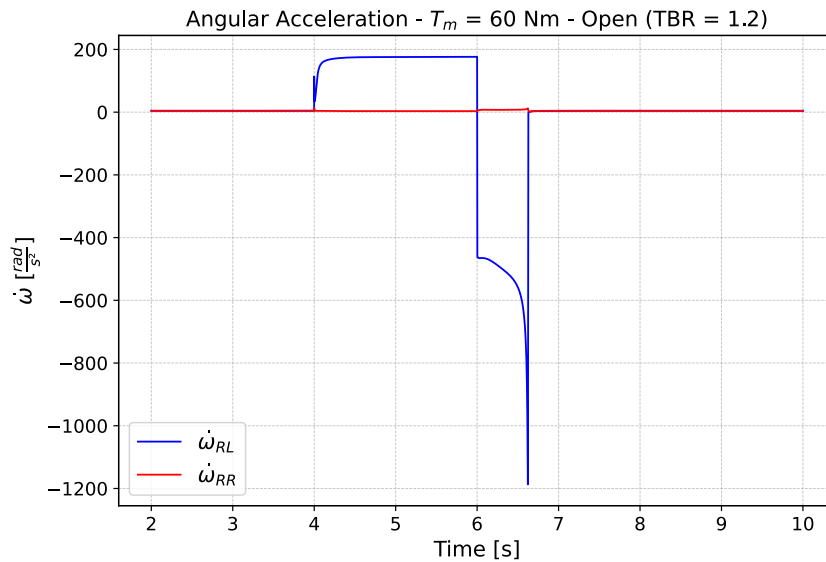


Figure E.3: Due to the amount of engine torque in the left wheel, the angular velocity rises sharply when the vehicle is on the ice. It becomes negative when both wheels return to dry asphalt.

Bibliography

- [1] Aaron Black. *Worm Gears Explained*. en. URL: <https://www.machinerylubrication.com/Read/1080/worm-gears> (visited on 05/23/2025).
- [2] Bohner, Max, Foscher, Richard, and Gscheidle, Rolf. *Technologie des véhicules à moteur*. Français. Europa LEHMITTEL. 27è. Verlag Europa-Lehrmittel, 2004. ISBN: 3-8085-2221-6.
- [3] *Brush Model - an overview | ScienceDirect Topics*. URL: <https://www.sciencedirect.com/topics/engineering/brush-model> (visited on 05/12/2025).
- [4] *Cornering Force - an overview | ScienceDirect Topics*. URL: <https://www.sciencedirect.com/topics/engineering/cornering-force> (visited on 03/19/2025).
- [5] *Différentiel (mécanique)*. fr. Page Version ID: 216800258. July 2024. URL: [https://fr.wikipedia.org/w/index.php?title=Diff%C3%A9rentiel_\(m%C3%A9canique\)&oldid=216800258](https://fr.wikipedia.org/w/index.php?title=Diff%C3%A9rentiel_(m%C3%A9canique)&oldid=216800258) (visited on 11/17/2024).
- [6] Docquier N., Poncelet A., and Fisette P. *ROBOTRAN: a powerful symbolic gnerator of multibody models*. 2013. (Visited on 02/01/2025).
- [7] Eaton. *What is a differential? Eaton explains the types, importance, and how it works*. URL: <http://site-340394.bcvp0rtal.com/detail/videos/differentials/video/6162818326001/what-is-a-differential-eaton-explains-the-types-importance-and-how-it-works.?autoStart=true&page=1> (visited on 11/29/2024).
- [8] Bernard C. Egnaczak. “The New Torsen II Traction Technology”. In: Mar. 1994, p. 940736. DOI: 10.4271/940736. URL: <https://www.sae.org/content/940736/> (visited on 10/27/2024).
- [9] *F1 - The Official Home of Formula 1® Racing*. en. URL: <https://www.formula1.com> (visited on 05/19/2025).
- [10] Paul Fisette and Jean-Claude Samin. “Road vehiclus: wheel/ground model”. In: *MBS - Reference Book*.
- [11] François BLASE and Steve MATOUT. *Automotive Handling on Wet Surface : Dynamic Analysis and Tyre Grip Sensitivity*. 2023.
- [12] *Friction coefficients for varying types of road surfaces in...* en. URL: https://www.researchgate.net/figure/Friction-coefficients-for-varying-types-of-road-surfaces-in-satisfactory-condition_tbl1_330012787 (visited on 03/20/2025).

- [13] Hans Fromm. “Berechnung des Schlupfes beim Rollen deformierbarer Scheiben”. en. In: *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik* 7.1 (1927). _eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1002/zamm.19270070106>. pp. 27–58. ISSN: 1521-4001. DOI: 10.1002/zamm.19270070106. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/zamm.19270070106> (visited on 05/12/2025).
- [14] M. Gadola, D. Chindamo, and Basilio Lenzo. “Revisiting the mechanical limited-slip differential for high-performance and race car applications”. en. In: *Engineering Letters* 29.3 (Aug. 2021). Number: 3 Publisher: International Association of Engineers, pp. 824–839. ISSN: 1816-093X. URL: <https://shura.shu.ac.uk/30090/> (visited on 10/05/2024).
- [15] T.K. Garrett, K. Newton, and W. Steeds. “Chapter 31. The differential”. In: Dec. 2000. ISBN: 978-0-7506-4449-5. DOI: 10.1016/B978-075064449-5/50032-8.
- [16] Geoffrey Virlez. “Multibody Modelling of Mechanical Transmission Systems in Vehicle Dynamics”. Anglais. Ingénieur Civil Mécanicien. Liège: ULiège, Aerospace and Mechanical Engineering Department, June 2014. (Visited on 03/11/2025).
- [17] Gleason Corporation. *[PDF] The development of a differential for the improvement of traction control / Semantic Scholar*. 1988. URL: <https://www.semanticscholar.org/paper/The-development-of-a-differential-for-the-of-Chocholek/43823bae6340d123b97203887cb893dc32aa699e> (visited on 10/27/2024).
- [18] Graphics Unleashed. *Toyota Yaris GR 2020 Car Template*. 2020. URL: <https://www.pinterest.com/pin/996632592520237176/>.
- [19] Massimo Guiggiani. “Handling of Road Cars”. en. In: *The Science of Vehicle Dynamics: Handling, Braking, and Ride of Road and Race Cars*. Ed. by Massimo Guiggiani. Cham: Springer International Publishing, 2023, pp. 239–353. ISBN: 978-3-031-06461-6. DOI: 10.1007/978-3-031-06461-6_7. URL: https://doi.org/10.1007/978-3-031-06461-6_7 (visited on 11/18/2024).
- [20] Massimo Guiggiani. *The Science of Vehicle Dynamics: Handling, Braking, and Ride of Road and Race Cars*. en. Cham: Springer International Publishing, 2023. ISBN: 978-3-031-06460-9 978-3-031-06461-6. DOI: 10.1007/978-3-031-06461-6. URL: <https://link.springer.com/10.1007/978-3-031-06461-6> (visited on 12/03/2024).
- [21] Massimo Guiggiani. “Vehicle Model for Handling and Performance”. en. In: *The Science of Vehicle Dynamics: Handling, Braking, and Ride of Road and Race Cars*. Ed. by Massimo Guiggiani. Cham: Springer International Publishing, 2023, pp. 67–176. ISBN: 978-3-031-06461-6. DOI: 10.1007/978-3-031-06461-6_3. URL: https://doi.org/10.1007/978-3-031-06461-6_3 (visited on 11/18/2024).
- [22] indiamart. *Planetary Gearbox, For Industrial*. Feb. 2022. URL: <https://www.indiamart.com/proddetail/planetary-gearbox-23653642712.html> (visited on 12/29/2024).
- [23] Innovation Discoveries. *Torsen Limited Slip Differential*. English. Jan. 2020. URL: <https://innovationdiscoveries.space/torsen-limited-slip-differential/> (visited on 03/09/2025).

- [24] K. Lorenz, C. Dietrich, and E. Donges. *Influence of a limited slip differential on the traction and handling of rear axle driven cars, part 1 and 2*. 1986. (Visited on 02/01/2025).
- [25] Bronisław Andrzej Kolator, Michał JANULIN, and Oleksandr Vrublevskiy. “EFFECT OF A TORSSEN DIFFERENTIAL MECHANISM ON CAR TYRE WEAR”. In: *Tribologia* 280 (Aug. 2018), pp. 31–37. DOI: 10.5604/01.3001.0012.7482.
- [26] MacIntyre. *How close to real life is the steering forces we have in iRacing?* Community. Nov. 2022. URL: <https://community.granitedevices.com/t/how-close-to-real-life-is-the-steering-forces-we-have-in-iracing/10267> (visited on 04/28/2025).
- [27] MathWorks. *Differential as a planetary bevel gear*. URL: <https://www.mathworks.com/help/vdynblks/ref/opendifferential.html> (visited on 12/29/2024).
- [28] S.K. Mohan. “Tribology of differentials and traction control devices”. en. In: *Tribology and Dynamics of Engine and Powertrain*. Elsevier, 2010, pp. 735–772. ISBN: 978-1-84569-361-9. DOI: 10.1533/9781845699932.2.735. URL: <https://linkinghub.elsevier.com/retrieve/pii/B9781845693619500247> (visited on 10/05/2024).
- [29] P. Fisette and JC. Samin. “Road vehicles: wheel/ground model”. In: *Mulibody System Dynamics, Reference Book*. EPL, UCLouvain. Vehicle System Dynamics. Louvain-la-Neuve. (Visited on 09/15/2024).
- [30] Hans B. Pacejka. “Chapter 4 - Semi-Empirical Tire Models”. In: *Tire and Vehicle Dynamics (Third Edition)*. Ed. by Hans B. Pacejka. Oxford: Butterworth-Heinemann, Jan. 2012, pp. 149–209. ISBN: 978-0-08-097016-5. DOI: 10.1016/B978-0-08-097016-5.00004-8. URL: <https://www.sciencedirect.com/science/article/pii/B9780080970165000048> (visited on 03/19/2025).
- [31] XJ Auto Parts. *Torsen B-type Limited Slip Differential Working Principle*. en-US. Aug. 2024. URL: <https://www.xjxparts.com/torsen-b-type-limited-slip-differential-working-principle/> (visited on 03/11/2025).
- [32] Pierre DUYSINX. *INTRODUCTION A LA MECANIQUE DES PNEUMATIQUES - Partim 2: Forces latérales Pierre DUYSINX LTAS - Ingénierie des Véhicules Terrestres Année ...* fr. 2014. URL: <https://fr.readkong.com/page/introduction-a-la-mecanique-des-pneumatiques-9298417> (visited on 05/11/2025).
- [33] Nicolas Poulet. *Séminaire Torsen - VSD*. Slides Séminaires. (Visited on 10/12/2024).
- [34] Amit Suhane, R. S. Rana, and Rajesh Purohit. “Prospects of Torsen Differential in Four Wheel Drive Automobile Transmission System”. In: *Materials Today: Proceedings*. 7th International Conference of Materials Processing and Characterization, March 17-19, 2017 5.2, Part 1 (Jan. 2018), pp. 4036–4045. ISSN: 2214-7853. DOI: 10.1016/j.matpr.2017.11.664. URL: <https://www.sciencedirect.com/science/article/pii/S2214785317329371> (visited on 10/05/2024).
- [35] *Thierry Neuville’s decade-long quest ends: World Rally Champion at last!* en. Nov. 2024. URL: <https://www.redbull.com/int-en/thierry-neuville-wrc-2024-world-champion> (visited on 05/19/2025).
- [36] Thomas D. Gillespie. *Fundamentals of Vehicle Dynamics: Thomas D. Gillespie / PDF / Suspension (Vehicle) / Tire*. fr. 1992. URL: <https://www.scribd.com/document/200548726/vd1> (visited on 03/24/2025).

- [37] Timon Alferink. *Plated limited slip differential explained*Plated limited slip differential explained. KMP Drivetrain Solutions. Sept. 2023. URL: <https://www.kmpdrivetrain.com/differentials/plated-limited-slip-differential-explained/> (visited on 02/25/2024).
- [38] tripmeandaman.com. *Front-Wheel Drive vs. Rear-Wheel Drive / Pros & Cons*. URL: <https://tripmeandaman.com/front-wheel-drive-vs-rear-wheel-drive-pros-cons/>.
- [39] Ugears. *MAQUETTE MÉCANIQUE - LE DIFFÉRENTIEL*. (Visited on 11/17/2024).
- [40] *Understeer & Oversteer. What It Is And How To Recover From It. / GDL Automotive / Mechanic Hornsby & Warriewood*. en-AU. Section: GDL Auto Blog. Apr. 2023. URL: <https://gdlauto.com.au/understeer-oversteer-what-it-is-and-how-to-recover-from-it/> (visited on 03/29/2025).
- [41] Geoffrey Virlez et al. *Simulation of Differentials in Four-Wheel Drive Vehicles Using Multibody Dynamics*. Vol. 4. Journal Abbreviation: Proceedings of the ASME Design Engineering Technical Conference Publication Title: Proceedings of the ASME Design Engineering Technical Conference. Jan. 2011. DOI: 10.1115/DETC2011-48313.
- [42] Vivek Shah. *Viscous-coupling differential*. URL: <https://www.carexpert.com.au/car-news/differentials-explained> (visited on 02/20/2025).

UNIVERSITÉ CATHOLIQUE DE LOUVAIN
École polytechnique de Louvain

Rue Archimède, 1 bte L6.11.01, 1348 Louvain-la-Neuve, Belgique | www.uclouvain.be/epl