

École polytechnique de Louvain

Topology optimization of high-performance passively levitated bearingless machines

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Abstract

Electrodynamic Thrust Self-Bearing Machines (EDTSBMs) achieve fully passive magnetic levitation by using the electrodynamic forces generated by the interaction between permanent magnets and armature windings. However, the stator and rotor topologies currently employed in such machines remain relatively conventional, leaving potential for performance improvement through more advanced designs. This thesis applies topology optimization methods for the design of new, more efficient stator and rotor topologies for EDTSBMs, with the optimization objective aimed exclusively on the machine's bearing function.

A first topology optimization was performed on the stator, using three density variables each representing the forward winding, the return winding, and the ferromagnetic material. Multiple stator design zones and rotor configurations were considered to assess the impact of ferromagnetic material on the bearing function. The results consistently showed that introducing ferromagnetic material into the stator provides no benefit, regardless of the configuration considered.

Ferromagnetic material was therefore excluded from further stator designs. Based on the optimized winding density distributions, feasible winding topologies were identified and optimized using parametric optimization, which confirmed that certain winding configurations significantly outperformed others in terms of bearing stiffness.

A second topology optimisation was defined on the rotor, by modelling the permanent magnets, the back iron yoke, and the ferromagnetic yoke surrounding the armature as independent density variables. The results demonstrated that the spatial arrangement of rotor components can be optimized to improve the electrodynamic stiffness of the machine. Feasible rotor topologies were then identified and validated through parametric optimization for each configuration studied.

Finally, a combined parametric optimization, which simultaneously optimized the geometric parameters of the stator and rotor, was performed. This combined approach demonstrated that simultaneously accounting for the interaction between the stator and the rotor resulted in a further improvement in bearing performance, beyond what is achievable through sequential and independent optimizations.

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Usage of Artificial Intelligence

The Artificial Intelligence, mainly Chat GPT and Claude, has been used to improve the quality of the writing in English.

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Chapter 1

Introduction

Magnetic Bearing

Magnetic bearings ensure the support of rotor using electromagnetic forces, thereby eliminating all mechanical contact between the rotor and the stator, unlike conventional ball or journal bearings. This absence of friction eliminates wear issues, reduces maintenance requirements, and extends the system's operational lifespan, while also making such bearings particularly suited to high- and ultra-high-speed machinery [1][2]. In addition, the absence of friction losses results in better energy efficiency compared to conventional solutions.

The majority of magnetic bearings used in industrial applications are Active Magnetic Bearings (AMBs), which consist of electromagnets whose current is actively regulated to generate an attractive force on a ferromagnetic rotor. However, since their operation relies on position sensors, power electronics and dedicated controllers, AMBs remain badly suited for applications requiring high compactness and reliability.

Passive Magnetic Bearings (PMBs) overcome these limitations by relying solely on passive physical phenomena (i.e. without any active control) and consist of Permanent Magnets (PMs). Although many topologies have been proposed, as indicated by Earnshaw's theorem [3], it is fundamentally impossible to achieve full magnetic levitation using only static magnetic fields.

Electrodynamic bearings (EDBs) are a type of PMB. In these bearings, the currents that generate the electromagnetic bearing forces are induced entirely by the change in magnetic flux detected by the windings, which is caused by the movement of the permanent magnet rotor. Based on the work of J.Van Verdegheem [4], they are preferably designed so that the conductors do not link any flux as the rotor is centred (known as the null-flux principle), thereby ensuring that no eddy currents and thus no losses are induced in the equilibrium position. EDBs can either support the radial degrees of freedom, in which case they are called Centring Electrodynamic Bearings, or support the axial degree of freedom of the rotor along its axis of rotation, in which case they are called Electrodynamic Thrust Bearings (EDTBs).

Electrodynamic Thrust Self-bearing Machines

Magnetic bearings occupy a significant part of the rotor's axial length at the expense of the drive section, which reduces the system's overall power density. One solution is to combine within a single structure, known as a self-bearing machines (SBMs) or bearingless machines, the motor or generator function with the rotor's magnetic suspension [5].

In [4], J. Van Verdegheem provides an overview of the state of the art regarding this machine topology, in which all machines use AMBs for magnetic rotor suspension. Some of the radial-flux topologies descriptions are taken from his work and summarized below.

Bauer et al. (2011) [6] introduced a thrust self-bearing machine based on a radial flux configuration. The rotor comprises three axially consecutive layers of surface-mounted permanent magnets on a back iron, as illustrated in Fig. 1.1(a) and (b): the middle layer generates a heteropolar radial field, while the top and bottom layers each produce a homopolar field with opposing radial polarisations. The stator consists of an armature winding composed of non-overlapping concentric air-core coils. As illustrated in Fig. 1.1(b), the axial levitation force is generated in the end-windings through the interaction of zero-sequence currents with the homopolar fields, while the driving torque is produced in the axial coil sections via conventional motor currents. Radial and tilt stabilisation are achieved through permanent magnet bearings located on both sides of the machine.

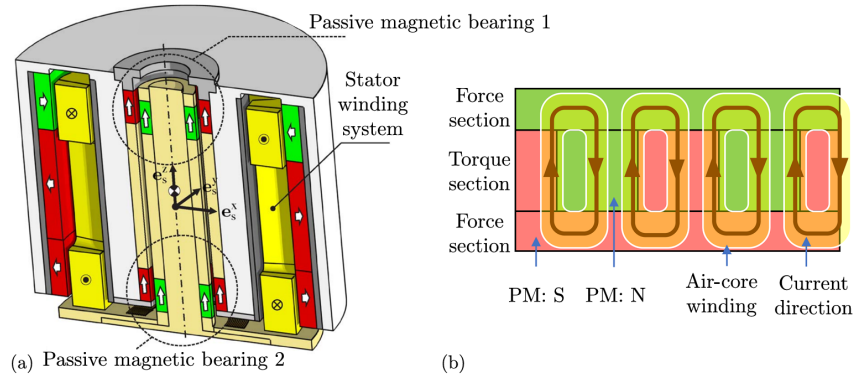


Figure 1.1: Machine described in [6]. (a) Cross-sectional view. (b) Axial force generation.

Qiang et al. (2016) proposed a structure composed of two separate radial flux slotless self-bearing motors [7], as depicted in Fig. 1.2(a). Each unit contains a ring-shaped permanent magnet with parallel magnetisation mounted on a back iron, as well as a three-phase toroidal armature winding whose coils are wound askew around the stator yoke. Radial and tilt motions are stabilised by permanent magnet centring bearings.

Sugimoto et al. (2014) presented a single-drive machine relying on a radial flux configuration [8], as illustrated in Fig. 1.3(a). The rotor consists of three layers of radially magnetised permanent magnets arranged with alternating azimuthal polarisations. The stator comprises a three-phase armature winding wound on ferromagnetic teeth forming a central laminated core, enclosed by ring yokes and upper and lower cores.

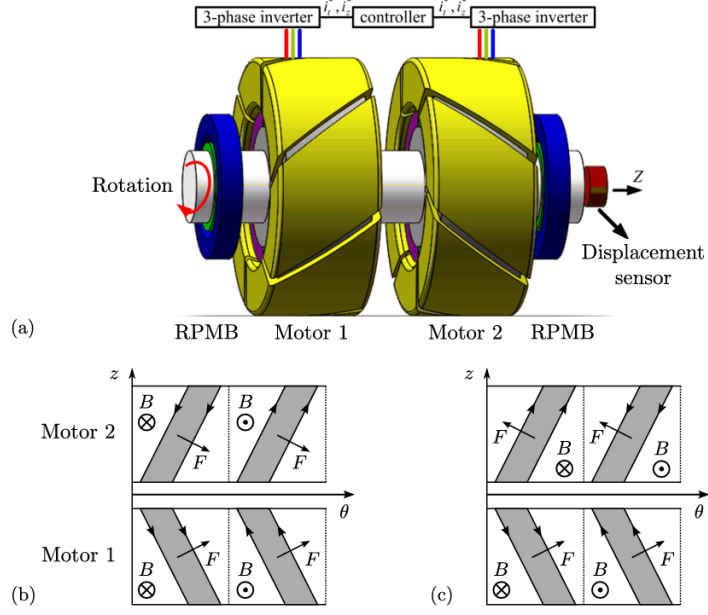


Figure 1.2: Thrust self-bearing machine introduced in [7]. (a) Overall view. (b) Torque generation. (c) Axial force generation.

The top and bottom layers, with opposing magnetisations, provide bias fluxes that combine with those arising from the armature winding. Repulsive centring permanent magnet bearings completes the rotor magnetic levitation.

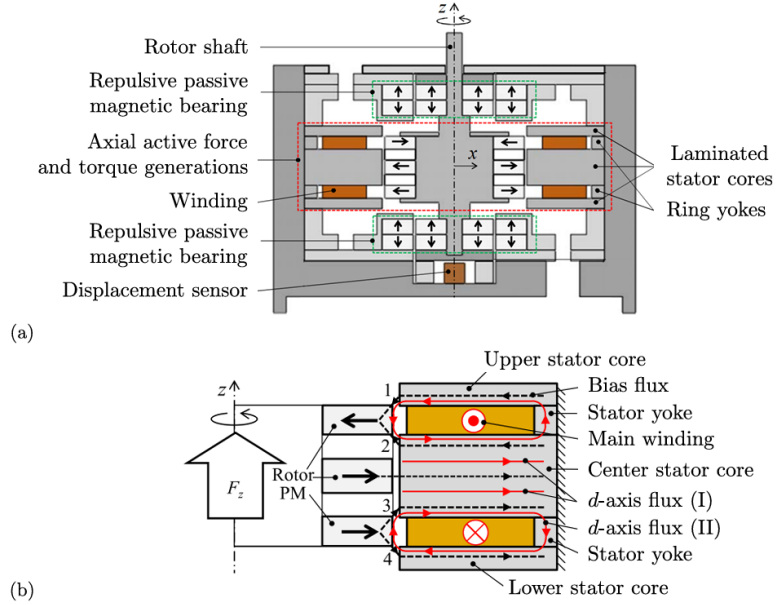


Figure 1.3: Machine described in [8]. (a) Cross-sectional view. (b) Axial force generation.

To reduce the number of actively stabilized degrees of freedom to 0, J. Van Verdegheem and B. Dehez (2021) proposed a fully passively levitated Electrodynamic Thrust Self-Bearing Machine (EDTSBM) [9], which simultaneously ensures the motor drive and the passive axial suspension of the rotor within a single winding structure, based on either an axial or radial flux topology, as illustrated in Fig. 1.4(a) and (b).

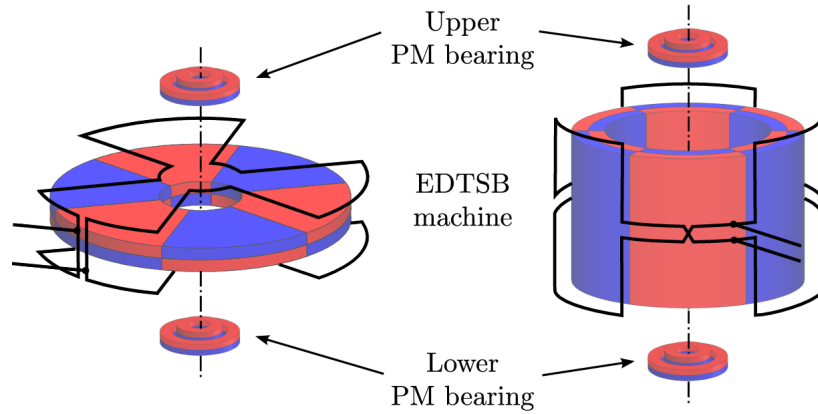


Figure 1.4: Fully passively levitated machine (only one phase). (a) Axial flux configuration. (b) Radial flux configuration [9].

The radial and tilt degrees of freedom are stabilized by two permanent magnet centring bearings located on both sides of the machine. The operating principle relies on the null-flux principle. When the rotor is axially centred, the flux linkages of the upper and lower windings of each phase are identical, so that the motor current I_M is distributed uniformly between them, producing only a driving torque. When the rotor deviates axially, the flux linkage asymmetry induces a passive suspension current I_S flowing in the short-circuited path formed by the series connection of both windings. This current interacts with the azimuthal PM magnetic field to generate an axial restoring electrodynamic force (and a dragging torque), while the motor current continues to provide only the driving torque, the two functions thus being decoupled. This operation principle is illustrated in Fig.1.5

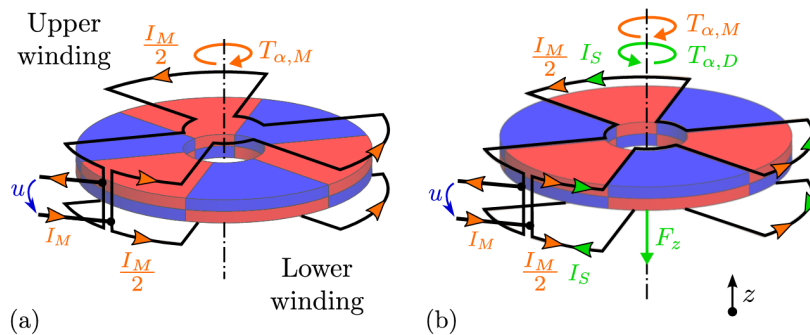


Figure 1.5: Electrodynamic thrust self-bearing machine operation. (a) Centred position ($z = 0$). (b) Off-centre position ($z = 0$) [9].

According to Earnshaw’s theorem [3], PM centring bearings introduce positive radial stiffness and destabilizing negative axial stiffness. This negative axial stiffness must therefore be compensated by the electrodynamic stiffness in order to circumvent Earnshaw’s law, thereby achieving a fully passively levitated machine.

Optimization Techniques

Optimization techniques are widely used in engineering design to determine the best trade-off between design objectives and design constraints. Optimization objectives vary widely: power density, torque, efficiency, mass, cost, etc. Optimization methods are generally divided into three main categories:

- **Parametric Optimization:** This involves optimizing a set of design parameters, such as geometric dimensions or material properties, while maintaining the overall topology of the design. This approach is suitable for problems where a reliable reference geometry already exists, but it consequently limits the exploration of the research space of possible topologies [10].
- **Shape Optimization:** Shape optimization extends the parametric optimization approach by allowing the boundaries of a design domain to be modified while preserving the topology. It thus offers greater flexibility than parametric optimization, although the search space remains constrained by the initial geometric configuration.
- **Topology Optimization (TO):** This is the most general approach: rather than modifying predefined parameters or boundaries, it determines the optimal distribution of one or more materials within a given design domain, without any prior assumptions about the resulting shape. The design space is discretized into a finite set of elements, and the optimization algorithm determines the optimal allocation of materials for each element.

Ideally, when searching for new topologies, the optimization process begins with topological optimization, followed by shape optimization, and finally parametric optimization.

Topology Optimization for Electrical Motors

Originally developed to solve engineering problems in solid mechanics, topological optimization has recently attracted growing interest in the design of electromagnetic devices [11]. In TO, the resulting structures may have complex geometries that are difficult to manufacture using conventional machining techniques. However, progress has been made in additive manufacturing technologies [12], and conductors printed on circuit boards (PCB) allow to implement more elaborate coil geometries [13].

TO problems are solved using two main algorithmic solvers: on the one hand, gradient-based methods generally converge more quickly but may converge to local optima. In particular, the Sparse Non-Linear Optimizer (SNOPT) solver demonstrates fast computation times and convergence compared to other gradient-based solvers [14].

On the other hand, gradient-free methods are more flexible because they do not require knowledge of the derivatives of the objective function with respect to the design variables,

but they require greater computational effort, particularly in the case of stochastic algorithms. Deterministic non-gradient methods (such as the Nelder-Mead Solver) can also be used [11].

Several TO methods have been proposed and applied to electromagnetic design.

The density-based methods, in particular the Solid Anisotropic Materials with Penalty (SIMP) method, are the most common formulation used to solve TO problems [15]. This technique represents the material property (e.g., permeability) as a continuous function. The optimization approach uses a gradient-based optimizer, which relies on the sensitivity of the objective and constraint functions to the material property within each element.

The Level Set Method (LSM), on the other hand, implicitly defines the interfaces between materials through the isocontours of a Level Set Function (LSF), thereby allowing the boundaries of materials to be clearly defined at the interfaces, as illustrated in Fig.1.6. Unlike density-based methods, which can produce intermediate material regions, LSMs inherently avoid such regions. They also accommodate topological changes (merging or splitting of material regions) during the optimization process without requiring explicit re-meshing. These properties make them particularly attractive for problems where sharp interface definition is critical to performance evaluation [16].

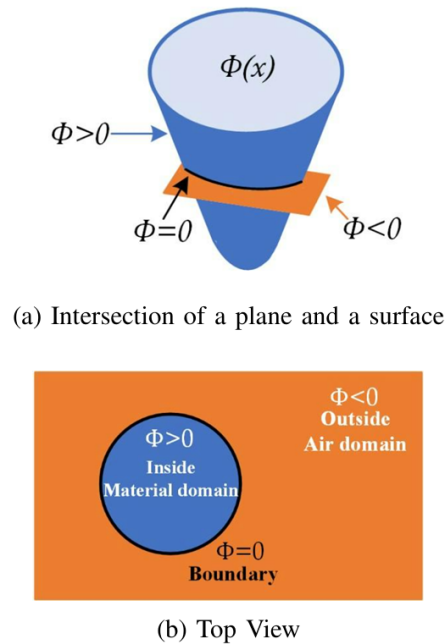


Figure 1.6: Material boundary representation using the level set method [16].

Another important distinction in TO problems concerns the number of materials considered. In uni-material TO, the algorithm distributes a single material within a domain. The design variable is thus reduced to a density indicating the presence or absence of this material (if the material is absent, the domain is typically filled with air). A representative application is the design of a lightweight rotor for a surface-mounted permanent magnet synchronous motor (PMSM) [17], where the density-based method was used to maximize the average output torque under a ferromagnetic volume constraint, demonstrating that

it is possible to achieve comparable electromagnetic performance with only 60% of the original rotor material, as illustrated in Fig. 1.7.

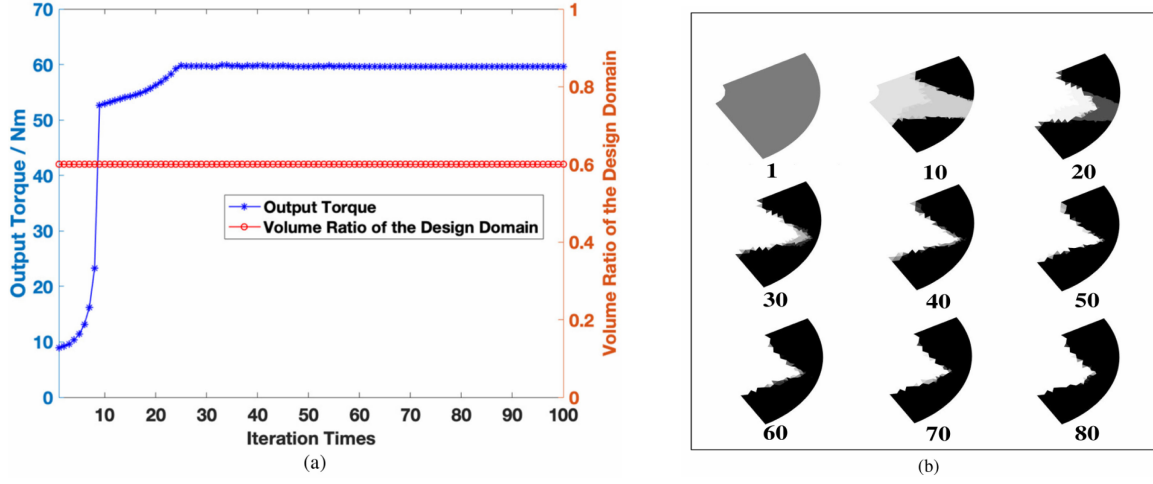


Figure 1.7: (a) Output torque and volume of the design domain iteration history, (b) rotor configurations at different iteration steps [17].

In multi-material TO, two or more distinct materials are distributed simultaneously within the design space. In the context of PMSMs [18], this allows the simultaneous optimization of the PM regions and the iron core, including the orientation of the PM field, see Fig. 1.8. The density-based SIMP method can be extended to multiple materials through a combination of multiple density variables, as described in Table 1.1.

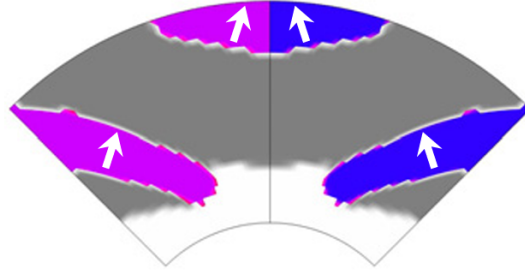


Figure 1.8: Design result, coloured regions represent PM blocks with different magnetization directions that are indicated by white-coloured arrows [18].

ρ_1	ρ_2	μ_r	$\bar{B}_r$	Material state
1	1	μ_{iron}	0	Iron
1	0	μ_{PM}	$\bar{B}_{\text{PM}}$	PM
0	1	μ_{air}	0	Air
0	0			

Table 1.1: Material states defined by design variables [18].

This multi-material TO implementation is also used in fields other than electric motors [19] [20].

Objectives of the Master Thesis

The stator and rotor topologies currently used in Electrodynamic Thrust Self-Bearing Machines (EDTSBMs) are relatively conventional, and therefore there is potential to improve performance by exploring more advanced topologies. However, designing these machines is a challenging task given the complex nature of the electromagnetic phenomena involved.

This thesis therefore aims to implement topological optimization methods to help in the design of new, more efficient topologies for EDTSBMs. The optimization will focus on the machine's bearing function rather than on performance criteria related to the motor function.

Two main objectives are being pursued. The first concerns the stator part of the machine, where the goal is to achieve an optimal winding topology, and where the impact of introducing ferromagnetic materials is being studied, as these materials can generate detent forces acting on the rotor and influence the electrodynamic stiffness. The second objective concerns the rotor part of the machine, for which the range of complex topologies described in the literature on Self-Bearing Machines (SBMs) is not very extensive. In this thesis, all topology optimization models will be based on the radial-flux EDTSBM configuration.

The thesis is structured as follows. First, the theoretical fundamentals necessary for defining and evaluating magnetic levitation performance indicators are established. Then, the optimization of the stator topology is presented, covering the optimization scenarios, the topological optimization model, and the results, as well as the parametric optimization aimed at deriving topologies based on these results. After, the optimization of the rotor topology is presented, following the same structure. Finally, a combined parametric study of the extracted stator and rotor topologies is conducted.

Chapter 2

Theoretical Fundamentals

This chapter presents the theoretical fundamentals necessary for introducing and understanding the topological optimization problems addressed in this thesis. It begins by presenting the electromagnetic principles governing the machine's stator function, and concludes with the concepts related to the chosen TO method.

2.1 Electromagnetic Force Characteristics

In order to define the bearing function as the objective function of topological optimization, the electromagnetic forces acting on the suspension system of the EDTSBM must first be characterised. The axial and radial electromagnetic forces can be derived from the co-magnetic energy with respect to the axial and radial positions:

$$F_z = \underbrace{\frac{\partial W_{cmag,0}(z)}{\partial z}}_{F_{z,d}} + \underbrace{\sum_{k=1}^N \frac{\partial \Phi_{k,0}(z)}{\partial z} i_k}_{F_{z,e}} + \underbrace{\sum_{k=1}^N \sum_{j=1}^N \frac{\partial L_{k,j}(z)}{\partial z} i_k i_j}_{F_{z,r}} \quad (2.1)$$

$$F_r = \underbrace{\frac{\partial W_{cmag,0}(r)}{\partial r}}_{F_{r,d}} + \underbrace{\sum_{k=1}^N \frac{\partial \Phi_{k,0}(r)}{\partial r} i_k}_{F_{r,e}} + \underbrace{\sum_{k=1}^N \sum_{j=1}^N \frac{\partial L_{k,j}(r)}{\partial r} i_k i_j}_{F_{r,r}} \quad (2.2)$$

Where F_z and F_r are the resulting axial and radial force, $W_{cmag,0}$ is the co-magnetic energy present in the bearing in absence of winding currents, $\Phi_{k,0}$ is the magnetic flux linked by the windings of phase k in the absence of currents, $L_{k,j}$ is the inductance coefficient linking the phase k with the phase j , and i_k and i_j are the currents flowing through the phase k and j .

The electromagnetic force can thus be decomposed into the sum of three contributions:

- **Detent force** $F_{z,d}/F_{r,d}$: arising from the attraction between permanent magnets and ferromagnetic materials respectively attached to the rotor and the stator, and vice versa.
- **Electrodynamic force** $F_{z,e}/F_{r,e}$: arising from the interaction between the permanent flux linked by the upper and lower windings and the currents flowing through

them. It should be noted that for the machine considered in this thesis, the radial component $F_{r,e}$ does not exist.

- **Reluctant force** $F_{z,r}/F_{r,r}$: arising from the variation of the inductance coefficients with the rotor position.

Each of these force components can either stabilize or destabilize the rotor's equilibrium. For an axial or radial displacement, a force directed toward the rotor's nominal position produces positive (stabilizing) stiffness, while a force that amplifies the displacement produces negative (destabilizing) stiffness. The stiffness components associated with each force contribution are defined below.

Detent Stiffness

Under the assumption of small rotor displacements, the axial and radial detent force $F_{z,d}$ and $F_{r,d}$ can be linearised, and are therefore modelled through the corresponding stiffnesses $k_{z,d}$ and $k_{r,d}$:

$$F_{z,d} = -k_{z,d}z \iff k_{z,d} = -\frac{F_{z,d}}{z} \quad (2.3)$$

$$F_{r,d} = -k_{r,d}r \iff k_{r,d} = -\frac{F_{r,d}}{r} \quad (2.4)$$

The physical origin of these stiffnesses is illustrated in Figure 2.1. In the centred position, the magnetic flux generated by the permanent magnets flows symmetrically through the rotor and the stator. When the rotor undergoes an axial displacement, the permeance of the air gap decreases, which reduces the magnetic flux flowing between the rotor and the stator. Since the detent force acts to maximize the magnetic flux, the rotor is driven towards its centred position, resulting in positive axial detent stiffness. When the rotor undergoes a radial disturbance, the air gap permeance increases on the side toward which the rotor has moved, concentrating the flux at that location and producing a net force in the direction of displacement, which results in negative radial detent stiffness. The opposite behaviour can be observed in the case of a stator structure where the iron is placed above and below the rotor rather than left and right.

This is consistent with the fact that free, stable levitation cannot be achieved using ferromagnetic materials alone [21].

Electrodynamic Stiffness

The following development is based on the axial model derived by J. Van Verdegheem in [4]. Under quasi-static conditions ($\ddot{z} = 0$) and at constant spin speed ω , the electrodynamic force exerted by the thrust bearing is:

$$F_{z,e} = -z \left(\frac{K_z^2 N}{2L_c} \right) \frac{\omega^2}{\omega^2 + \left(\frac{1}{p} \frac{R}{L_c} \right)^2}. \quad (2.5)$$

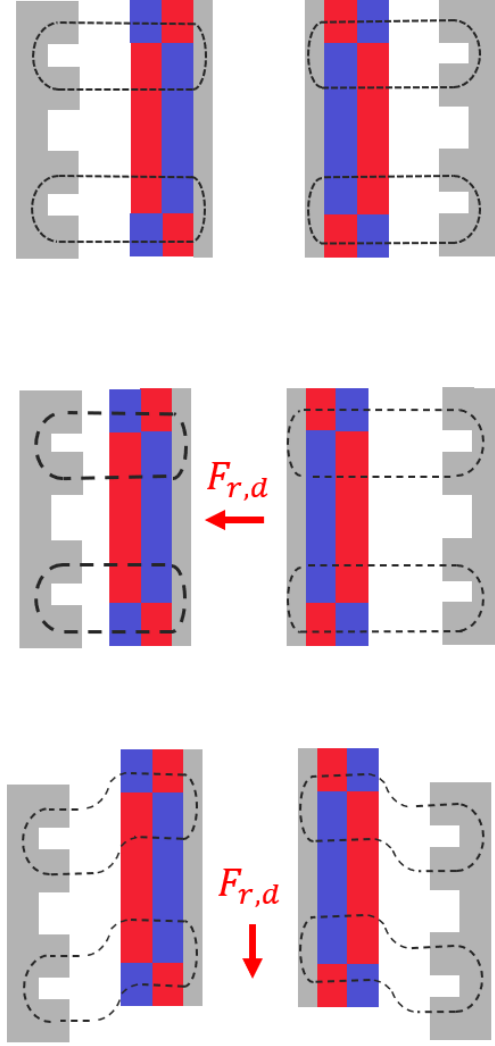


Figure 2.1: Illustration of the detent force for axial and radial rotor displacements. This illustration is inspired by [22].

Where K_z is the flux gradient, defined as the proportionality coefficient between the flux amplitude arising from the permanent magnets linked by the winding and the axial displacement z :

$$K_z = \frac{\partial \Phi(z)}{\partial z} \quad (2.6)$$

For the radial machine topology studied in this thesis, this gradient results from the variation of the surface area intercepting the flux ($\Phi = \int_S \vec{B} dS$) as a function of axial displacement, unlike in axial machines where it results from the decrease in magnetic field strength with distance from the permanent magnets. L_c is the cyclic inductance, defined as follows:

$$L_c = \sum_{k=1}^N L_{1,k} \cos \left((k-1) \frac{2\pi}{N} \right) \quad (2.7)$$

The electrodynamic force acts to restore the rotor to its nominal axial position, which generates a positive (stabilising) stiffness $k_{z,e}$:

$$k_{z,e} = -\frac{F_{z,e}}{z} = \left(\frac{K_z^2 N}{2L_c} \right) \frac{\omega^2}{\omega^2 + \left(\frac{1}{p} \frac{R}{L_c} \right)^2}. \quad (2.8)$$

This stiffness is independent of the rotor axial position z and increases with the spin speed ω , as illustrated in Fig. 2.2. The electrodynamic stiffness is characterised through two coefficients, namely the rotor spin speed ω_e related to the electrical pole, defined as:

$$\omega_e = \frac{1}{p} \frac{R}{L_c} \quad (2.9)$$

and the asymptotic (infinite spin speed) stiffness:

$$k_{z,e}^\infty = \frac{K_z^2 N}{2L_c} \quad (2.10)$$

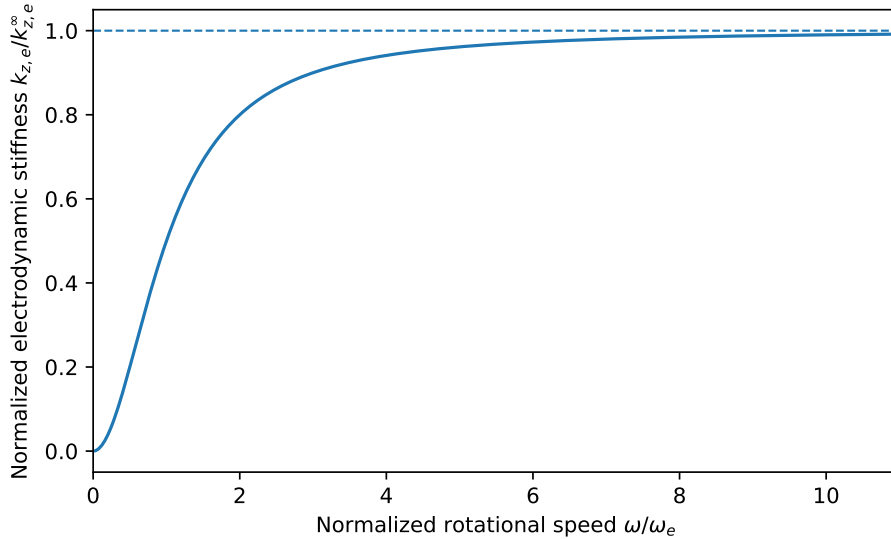


Figure 2.2: Electrodynamic stiffness $k_{z,e}$ as a function of spin speed ω .

Reluctant Stiffness

In the implementation of topological optimization methods aimed at maximizing the bearing function of an EDTSBM, contributions related to reluctance forces are not taken into account. This decision is based on the conclusions presented in [23], in which the authors demonstrate that, for passively levitated self-bearing machines, the contribution of reluctance stiffness remains negligible compared to the dominant electrodynamic component.

2.2 Electromagnetic Stiffness Budget

The objective of this thesis is to maximise the bearing function of the EDTSBM machine, which is equivalent to simultaneously maximise the stiffness in all directions. The total bearing stiffness is defined as the sum of all stiffness contributions.

$$\sum_i k_i = k_{z,e} + k_{z,d} + 2k_{r,d} \quad (3D \text{ model}) \quad (2.11)$$

$$\sum_i k_i = k_{z,e} + k_{z,d} + k_{r,d} \quad (2D \text{ model}) \quad (2.12)$$

By Earnshaw's theorem [3], stable passive levitation cannot be achieved using permanent magnets alone, which means that adding a permanent magnet centring bearing or thrust bearing introduces a positive stiffness in one direction at the cost of a negative stiffness in the other:

$$k_z + 2k_r = 0 \quad (3D), \quad k_z + k_r = 0 \quad (2D) \quad (2.13)$$

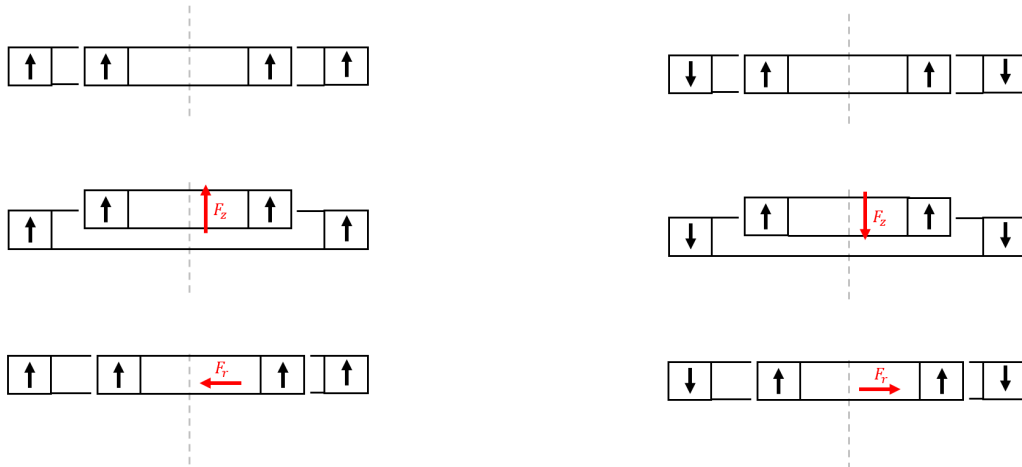
Since the electrodynamic bearing provides a “free” positive axial stiffness $k_{z,e}$ that is not subject to this constraint, it can compensate for the negative axial stiffness introduced by a radial centring bearing, thereby enabling fully passive levitation in all directions, taking into account the fact that permanent magnet centring bearings also help ensuring tilt stabilization when placed on both sides.

A central question of this thesis is therefore to assess whether the introduction of a ferromagnetic material into the stator, which introduces detent stiffness components and affects the electrodynamic stiffness by altering the magnetic field distribution perceived by the windings, can improve the total stiffness of the bearings $\sum_i k_i$.

Passive Stabilisation Elements

In configurations where the sum of all stiffness is positive but one of the stiffness components remains negative, it is possible to introduce a permanent magnet corrective element. Two types are considered, as illustrated in Figure 2.3:

- Permanent magnet centring bearing, used when $\sum k_r \leq 0$, since it introduces positive radial stiffness at the expense of negative axial stiffness, as shown in Fig.2.3a.
- Permanent magnet thrust bearing, used when $\sum k_z \leq 0$, since it introduces positive axial stiffness at the expense of negative radial stiffness, as shown in Fig.2.3b.



(a) Permanent magnet centring bearing

(b) Permanent magnet thrust bearing

Figure 2.3: Permanent magnet corrective elements. Other configurations of permanent magnet radial and thrust bearings are available in [24].

When these elements are added, Earnshaw's theorem imposes the relation (Eq. 2.13) between their axial and radial stiffness contributions, which must be taken into account in the overall stiffness evaluation during the complete design of the machine. However, this topic is beyond the scope of this thesis.

2.3 SIMP Topology Optimization Method

Problem Formulation

In mathematical terms, the topological optimization (TO) problem aims to determine the optimal material distribution within a design domain Ω by solving the following optimization problem [25]:

$$\min_{\theta} \mathcal{F}(\mathbf{u}(\theta), \theta) \quad (2.14)$$

subject to:

$$g_i(\theta) \leq 0, \quad i = 1, \dots, k \quad (2.15)$$

$$h_j(\theta) \leq 0, \quad j = 1, \dots, l \quad (2.16)$$

$$0 \leq \theta(\mathbf{x}) \leq 1, \quad \forall \mathbf{x} \in \Omega \quad (2.17)$$

where $\mathcal{F}$ is the objective function, $\theta(\mathbf{x})$ is the density variable field, bounded between 0 (absence of material) and 1 (material fully present), $\mathbf{u}(\theta)$ is the state variable (i.e., the solution of the governing physics dependant on the density variable field), g_j are the inequality constraints (on the total allowable mass usage for example), and h_j are the equality constraints. These densities are then used to modulate some properties (e.g. magnetic permeability).

SIMP Method

In topological optimization, intermediate density values (i.e. $\theta \in (0, 1)$) do not correspond to a feasible solution. In order to steer the solution towards discrete distributions (0 and 1), the SIMP (Solid Isotropic Material with Penalty) method, introduced by Bendsøe and Sigmund [26], is used. This method penalises intermediate densities by raising them to a power $p > 1$, which reduces the material property attributed to the intermediate values relative to the material as a whole, thereby making these values less attractive to the solver.

However, the effect of the penalty exponent p on the convergence behaviour must be taken into account. If p is too low, intermediate densities remain competitive and the solution may fail to converge to a discrete topology. Conversely, if p is set too high from the outset, the solver may converge too early to extreme density values in the most sensitive regions before the less sensitive regions have had a chance to evolve, which may lead the solution into a local minimum. An intermediate value of p therefore ensures that the sensitive regions guide the overall topology whilst allowing the rest of the domain to follow. This behaviour is illustrated in Figure 2.4.

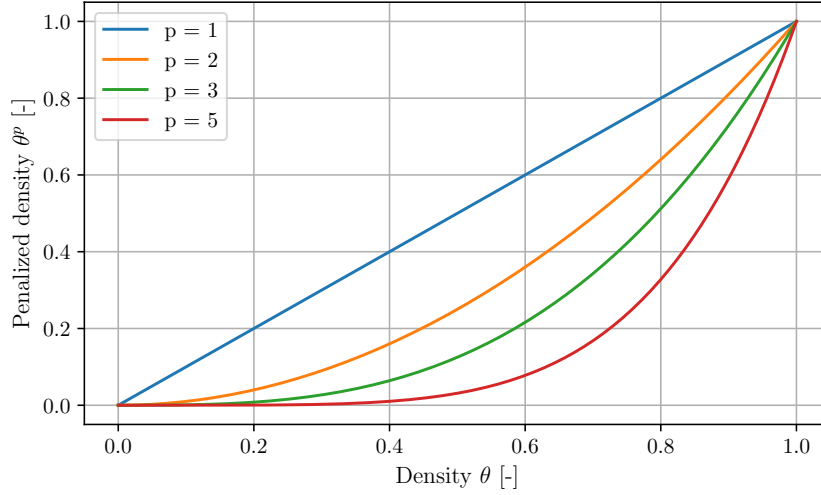


Figure 2.4: Effect of the SIMP penalisation exponent p on the density.

In this thesis, the topology optimization method employed is therefore the density-based SIMP method, implemented in *COMSOL Multiphysics®* [27]. The Sparse Nonlinear OPTimizer (SNOPT) gradient-based solver is used, which offers fast computation times and convergence compared to other gradient-based solvers, as discussed in Chapter 1.

In addition, *COMSOL Multiphysics®*’s “Density Model” feature supports smoothing using a Helmholtz filter [28]. Without smoothing, the topology-optimized solution may exhibit mesh-dependent results and checkerboard patterns.

COMSOL Multiphysics®

COMSOL Multiphysics® [27] is a simulation platform including both fully coupled multiphysics and single-physics modelling capabilities. Its Model Builder includes all of the steps in the modeling workflow, from defining geometries, material properties, and the physics that describe specific phenomena to performing computations and evaluating the results. The Optimization Module, an add-on to *COMSOL Multiphysics®* [29], is based on the Finite Element Method (FEM) and provides tools for parameter, shape, and topology optimization, as well as parameter estimation. It can be used together with other modules to optimize devices and processes that involve phenomena such as electromagnetics, structural mechanics, etc.

The use of the finite element method (FEM) in the *COMSOL Multiphysics®* model offers several advantages in this optimization context. Firstly, it allows for the accurate modelling of complex and irregular geometries, which is particularly relevant given the objective of finding optimized topologies. Secondly, it provides highly accurate solutions and also allows for easy visualization of the results, which facilitates the interpretation of the topological optimization results (making it easier to determine where to place which component).

2.4 Material Properties

The topology optimization models implemented in *COMSOL Multiphysics*® assign distinct material properties to each component of the rotor and stator. The materials used for the permanent magnets, the ferromagnetic yokes, and the armature windings are described below.

Rotor Iron Yokes

The rotor ferromagnetic yokes are modelled using the *Soft Iron (Without Losses)* material available in *COMSOL Multiphysics*®. This material can be defined by a nonlinear B – H magnetisation curve, as shown in Fig. 2.5, which allows for magnetic saturation and thus provides a more realistic representation of the magnetic field distribution. The model neglects magnetic losses, but this simplification is common practice in magnetostatic finite element models, where simulations are performed under steady-state conditions. Thermal effects on the B – H characteristic are also not taken into account.

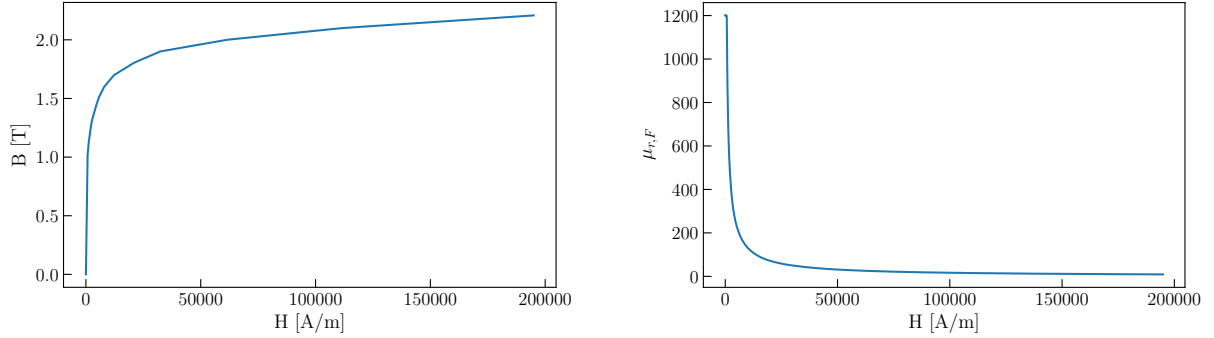


Figure 2.5: (a) B – H and (b) μ_r – H curves for Soft Iron (Without Losses) as implemented in *COMSOL Multiphysics*®. It should be noted that the relative permeability (at unsaturated conditions) of the ferromagnetic material ($\mu_{r,F}$) is 1200.

Permanent Magnets

The permanent magnets are modelled using a remanent flux density magnetisation model, in which the constitutive B – H relation is governed by the recoil permeability and the remanent flux density norm:

$$\mathbf{B} = \mu_0 \mu_{rec} \mathbf{H} + \|\mathbf{B}_r\| \frac{\mathbf{e}}{\|\mathbf{e}\|} \quad (2.18)$$

where μ_{rec} is the recoil permeability, $\|\mathbf{B}_r\|$ is the remanent flux density norm, and $\mathbf{e}/\|\mathbf{e}\|$ is the unit vector defining the magnetisation direction of the permanent magnet.

In the topology optimization model, the N50 grade of sintered NdFeB is used as the material for permanent magnets, with $\mu_{rec} = 1.05$ and $\|\mathbf{B}_r\| = 1.41$ [T].

Armature Windings

In *COMSOL Multiphysics*® models, the winding regions are made of copper and are assigned the same magnetic properties as air. This approximation, which is common in the modelling of electromagnetic machines, is justified by the fact that the windings do

not contribute to the magnetic circuit. Their role in the model is limited to defining the regions through which current flows. The current flowing through the winding is defined as:

$$I = \int_{S_{eff}} \mathbf{J} dS = \int_S \|J\| \cdot N_w \cdot \mu_{f.f.} dS \quad (2.19)$$

where $\|J\|$ is the current density norm, N_w is the number of turns per coil, $\mu_{f.f.}$ is the copper fill factor, and S is the cross-sectional area of the conductor.

However, as it will be noted in the next chapter, the asymptotic electrodynamic stiffness is independent of the current magnitude, and therefore independent of the $\|J\|$ and N_w parameters. The fill factor $\mu_{f.f.}$ is the only winding parameter that influences the topological results, as it influences the winding resistance and therefore affects the electric pole ω_e , which determines the spin speed dependence of the electrodynamic stiffness as defined in Eq. 2.8. A copper fill factor of $\mu_{f.f.} = 0.5$ is chosen, based on [30].

The complete set of material parameters used in the models is summarised in Table 2.1.

Parameter	Symbol	Value
<i>Ferromagnetic yoke (Soft Iron)</i>		
Relative permeability (unsaturated)	$\mu_{r,F}$	1200
Relative reluctivity (unsaturated)	$\nu_{r,F}$	$\frac{1}{1200}$
<i>Permanent magnet (N50 Sintered NdFeB)</i>		
Recoil permeability	μ_{rec}	1.05
Remanent flux density norm	$\ \mathbf{B}_r\ $	1.41 T
<i>Armature winding (Copper)</i>		
Current density norm	$\ J\ $	1 A/mm ²
Number of turns	N_w	12
Copper fill factor	$\mu_{f.f.}$	0.5
Copper resistivity	ρ	$1.667 \times 10^{-8} \Omega \cdot \text{m}$

Table 2.1: Material parameters used in the *COMSOL Multiphysics*® topology optimization models.

Chapter 3

Stator - Topology Optimization

The main objective of this chapter is to apply topological optimization to the design of stator structures for Electrodynamic Thrust Self-Bearing Machines (EDTSBM). More specifically, the goal is to determine whether the introduction of a ferromagnetic material into the stator, or the spatial arrangement of the stator windings, can improve the bearing function of these machines.

The chapter is organized as follows. Section 3.1 defines the optimization scenarios under consideration, covering all rotor configurations, stator design regions, and objective functions examined. Section 3.2 details the topological optimization model as implemented in *COMSOL Multiphysics*®. Section 3.3 presents and interprets the optimization results for all scenarios. Section 3.4 describes a parametric validation procedure conducted to verify that the optimal topologies can be correctly extracted. Finally, section 3.5 presents a second formulation of the topology optimization model, closer to the multi-material models as indicated in the introduction, which serves as a basis for comparison and validation of the first model.

It should be noted that the machine studied throughout this thesis is the radial-flux EDTSBM, and more specifically that the topology optimization model is based on the configuration where the upper and lower coil assemblies are connected independently, forming figure-eight shaped null-flux coils, as illustrated by its three-dimensional representation in Figure 3.1. It should be noted that the optimization methods developed in this thesis remain adaptable to other radial-flux EDTSBM configurations presented in [4].

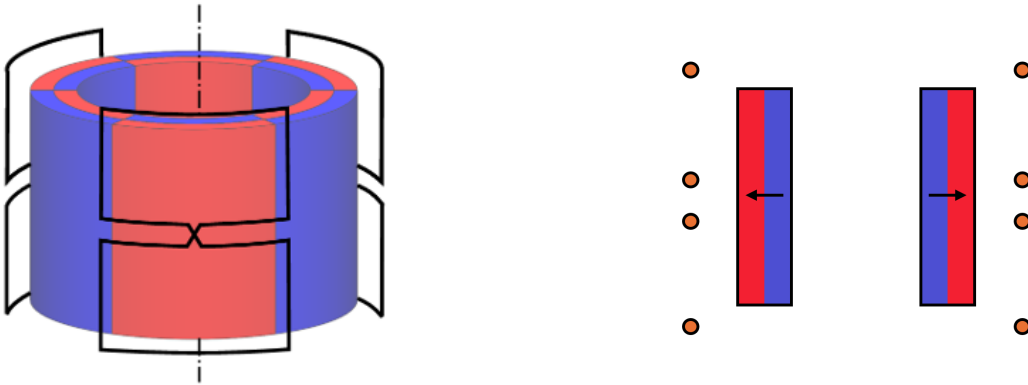


Figure 3.1: Three-dimensional representation of the radial-flux EDTSBM considered [4] and the two-dimensional representation based on it.

3.1 Stator - Optimization Scenarios

This subsection presents the various scenarios considered for the stator topology optimization. The study of a wide range of configurations is primarily aimed at assessing the influence on the bearing function of adding ferromagnetic material. Another objective is to determine the winding and iron spatial distributions that best suit each configuration, as well as to determine whether it is possible to identify a recurring optimal topology among all the configurations.

Four rotor configurations are being considered, as the arrangement of the permanent magnets (PM) directly determines the orientation and distribution of the magnetic field in the air. Iron yoke are incorporated into all rotor configurations to achieve a more realistic machine structure. They serve both as the mechanical support for the permanent magnets and as a low-reluctance return path for the magnetic field. The resulting configurations are illustrated in Fig. 3.2.

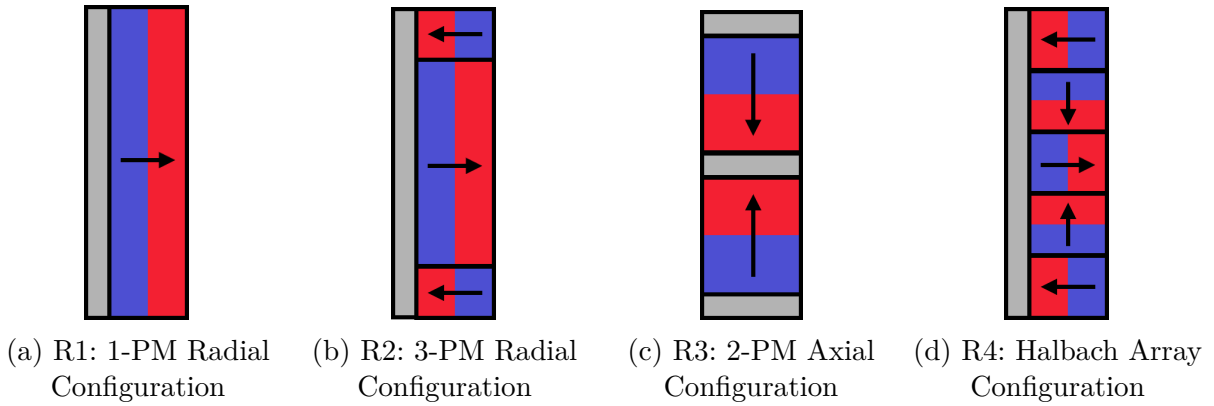
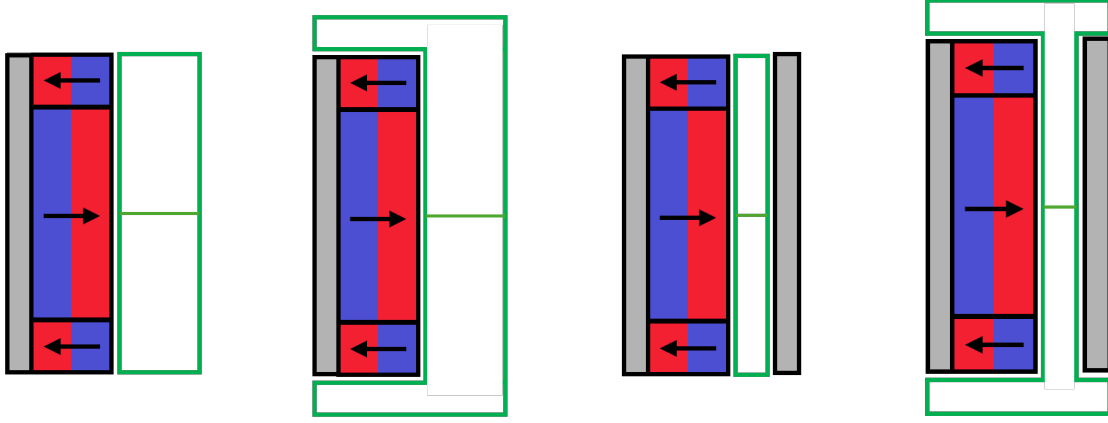


Figure 3.2: Each rotor configuration has its own magnetic field distribution and orientation.

Four stator design zones have been defined, as shown by the green boxes in Fig. 3.3. These zones were selected in order to investigate whether the introduction of a rotor ferromagnetic yoke surrounding the armature winding, or the extension of the design zone above the rotor (while keeping a constant air gap around the rotor), has a significant effect on the optimized stator topology. It is important to note that each winding zone is divided into lower and upper windings that are symmetrical, in order to correspond to the radial-flux machine considered for this thesis.

Configurations, in which the winding extends above the rotor, are based on two principles: that the ferromagnetic material in this area is capable of generating a positive radial detent stiffness, and that the windings interact with a stronger magnetic field in this area. It should be noted, however, that these stator zones are considered in a 2D model, where the heteropolar component of the magnetic field does not appear, unlike the axial and radial components, which are well defined. This means that the field lines cannot close in the radial plane (despite the alternating polarity of the machine in its 3D configuration). It will therefore be important to bear in mind the absence of the heteropolar component of the magnetic field when critically analysing the results.



(a) S1 - Wide winding zone, no iron yoke (b) S2 - Winding zone **S1** extended above the rotor, no iron yoke (c) S3 - Narrow winding zone, with iron yoke (d) S4 - Winding zone **S3** extended above the rotor, with iron yoke

Figure 3.3: The optimization scenarios for each rotor configuration include four stator design zones, as illustrated here for the R2 rotor configuration.

Furthermore, for each rotor-stator configuration, three distinct objective functions are considered:

- **OBJ1**: Maximization of the total stiffness, with ferromagnetic material (soft iron) and windings as design variables.
- **OBJ2**: Maximization of the total stiffness, with windings only (no soft iron).
- **OBJ3**: Maximization of the electrodynamic stiffness alone, with soft iron and windings.

It should be noted that, in the absence of ferromagnetic material in the stator, the total stiffness and the electrodynamic stiffness are identical, since the contributions related to the detent stiffness are null in the absence of iron in the stator.

These three objective functions enable a rigorous assessment of the contribution of ferromagnetic materials to the total stiffness of the bearing for every rotor-stator configuration, as well as to the electrodynamic stiffness component.

3.2 Stator - Topology Optimization Model

In the present work, the Topology Optimization functionality of the *COMSOL Multiphysics*® Optimization Module is used. The implementation of the Topology Optimization Model is detailed in the following subsections, covering the density field definitions, parameter and variable estimation, geometry, physics formulation and its coupling to the optimization densities, as well as the formulation of the objective functions.

Density Definitions and Parameters

As described in Section 2.3, topology optimization is defined by densities that serve as control variables for the optimization problem. Three penalised density fields θ^p are defined

in Table 3.1, each representing a distinct material component within the design domain (Ω_S):

Model Component	Penalised Density
Ferromagnetic material	$\theta_F^p \in [0, 1]$
Forward winding	$\theta_{W+}^p \in [0, 1]$
Return winding	$\theta_{W-}^p \in [0, 1]$

Table 3.1: Definition of the three density fields used in the topology optimization model.

These densities are then used to modulate both the current distribution and the relative magnetic reluctivity within the stator. Since two materials cannot physically coexist in the same location, an exclusion penalty must be taken into account throughout the design domain (Ω_S) in the formulation of the objective function. The initial values of the density fields and the SIMP exponent used throughout the optimization are summarized in Table 3.2.

Parameter	Symbol	Value
Ferromagnetic material initial density	$\theta_{F,0}^p$	0.5
Forward winding initial density	$\theta_{W+,0}^p$	0.55
Return winding initial density	$\theta_{W-,0}^p$	0.45
SIMP penalisation exponent	p	5

Table 3.2: Density initialisation values and SIMP exponent used in the optimization model.

The ferromagnetic density is initialised at 0.5, a neutral starting point. The forward and return winding densities are intentionally initialised at distinct values (0.55 and 0.45, respectively) to ensure that the net current is non-zero at initialisation (see Eq.(3.10)). A net current of zero would result in ‘division by zero’ errors when calculating physical quantities such as the cyclic inductance (L_c). The same penalty exponent $p = 5$ is applied to all three density fields, thereby ensuring consistent penalty behaviour for all components of the model.

Geometry and Parameter Estimation

The model is formulated in two dimensions, as a three-dimensional topology optimization would result in excessive computational costs. This two-dimensional representation implies that the model represents a single phase and a single pole pair.

Five physical quantities are required to evaluate the stiffness components defined in Section 2.1: the flux gradient K_z , the cyclic inductance L_c , the winding resistance R , the axial holding force $F_{z,d}$, and the radial holding force $F_{r,d}$. Since these parameters require distinct steady-state simulation setups to be calculated, four isolated magnetic field zones are defined within the geometry. These four magnetic zones are listed below and illustrated in Fig. 3.4:

- **Configuration (1):** $\Delta z = 0$, $\Delta r = 0$, $\mathbf{J}_e$ *on*, $\mathbf{B}_{r,\text{PM}}$ *on*
- **Configuration (2):** $\Delta z = \varepsilon_z$, $\Delta r = 0$, $\mathbf{J}_e$ *off*, $\mathbf{B}_{r,\text{PM}}$ *on*
- **Configuration (3):** $\Delta z = 0$, $\Delta r = \varepsilon_r$, $\mathbf{J}_e$ *off*, $\mathbf{B}_{r,\text{PM}}$ *on*
- **Configuration (4):** $\Delta z = 0$, $\Delta r = 0$, $\mathbf{J}_e$ *on*, $\mathbf{B}_{r,\text{PM}}$ *off*

where Δz and Δr denote the axial and radial rotor displacements from the centred position, $\mathbf{J}_e$ indicates whether current is flowing through the windings, and $\mathbf{B}_{r,\text{PM}}$ indicates whether the permanent magnet remanent magnetisation is active. All four configurations are solved at each optimization iteration, with the density field updated accordingly.

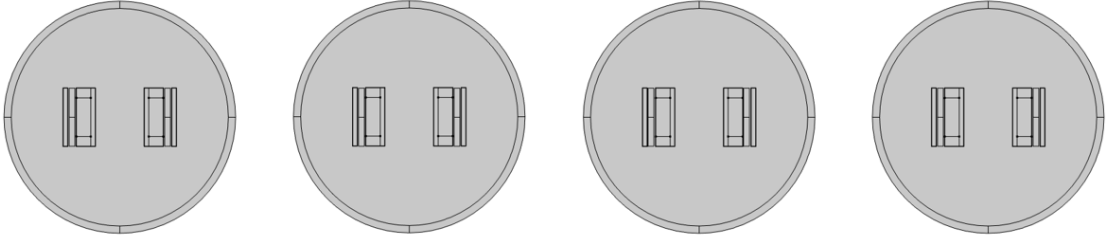


Figure 3.4: Four isolated magnetic field zones required to compute K_z , L_c , R , $F_{z,d}$, and $F_{r,d}$.

Magnetic zone (1) isolates the electrodynamic contribution, thereby allowing the flux gradient K_z to be calculated. Indeed, at the nominal position ($\Delta z = 0$, $\Delta r = 0$), the machine geometry exhibits both axial and radial symmetry as the rotor is perfectly centred. Under these symmetries, the inductance coefficients and the magnetic co-energy $W_{\text{cmag},0}$ are at an equilibrium point with respect to both z and r , and their partial derivatives are thus zero, thereby eliminating the reluctance and detent terms.

It should be noted that, by the null-flux principle, no net electrodynamic force should arise at $\Delta z = 0$ either, since the suspension current is not induced at the centred position. However, under the assumption that K_z is constant for small axial displacements, meaning that the second-order variation with respect to the axial position of the magnetic flux linked by the windings is neglected, K_z can be evaluated at $\Delta z = 0$ by fictitiously injecting a current I into the windings and computing the resulting force. As demonstrated in Section 2.1, the axial electrodynamic force then reduces, for a single-phase machine ($N = 1$), to:

$$F_z = \sum_{k=1}^N \frac{\partial \Phi_{k,0}(z)}{\partial z} i_k \xrightarrow{N=1} F_z = \frac{\partial \Phi_0(z)}{\partial z} I \quad (3.1)$$

from which the flux gradient is directly identified as:

$$K_z = \frac{F_z}{I} \quad (3.2)$$

Since K_z is defined as the ratio of the electrodynamic force to the current, and since both of these quantities scale linearly with the current intensity, K_z is independent of the current intensity and therefore of the parameters that define it: the current density $\|J\|$, the number of turns N_w , and the fill factor $\mu_{f.f.}$. The only factor that influences K_z is the spatial distribution of the current within the design domain. This spatial distribution is precisely what is controlled by the forward and return winding density variables.

The magnetic forces acting on the rotor are computed in *COMSOL Multiphysics*® by integrating the Maxwell stress tensor over the outer surface of the rotor domain (Ω_R):

$$\mathbf{F} = \oint_{\partial\Omega_R} \mathbf{n} \cdot \mathbf{T} d\Gamma \quad (3.3)$$

Magnetic zones (2) and (3) eliminate both the contributions from electrodynamic forces and those from reluctance forces by setting the current flowing through the windings to zero. The remaining force therefore stems purely from detent effects, which allows us to extract the axial and radial detent stiffnesses $k_{z,d}$ and $k_{r,d}$ respectively from configurations (2) and (3).

Magnetic zone (4) is used to compute the cyclic inductance L_c . The permanent magnet remanent magnetisation is deactivated, and a current I is injected simultaneously into the upper and lower winding regions to capture both the self-inductance and the mutual inductance between them. The quantity L_c corresponds to the inductance of either the upper or the lower winding considered individually (not their sum). Furthermore, since the winding resistance is evaluated only over the stator design domain Ω_S , which represents either the left or the right half of the stator in the two-dimensional representation, only one quarter of the total magnetic energy stored in the system is associated with L_c . The inductance is therefore identified as follows:

$$\frac{1}{4}W_m = \frac{1}{2}L_c I^2 \quad \Rightarrow \quad L_c = \frac{W_m}{2I^2} \quad (3.4)$$

By the same reasoning as for K_z , the cyclic inductance L_c is also independent of the current magnitude, since the stored magnetic energy is proportional to I^2 and the I^2 term cancels out in the expression above.

The winding resistance per unit depth is independent of the rotor position and is therefore evaluated once, regardless of the configuration, from the density distribution using Pouillet's law:

$$R = \frac{\rho}{S} = \frac{\rho}{\mu_{f.f.} \int_{\Omega_S} \frac{|\theta_{W+}^p - \theta_{W-}^p|}{2} d\Omega_S} \quad (3.5)$$

where ρ is the electrical resistivity of copper, $\mu_{f.f.}$ is the copper fill factor, and Ω_S is the stator design domain. Unlike K_z and L_c , the resistance R depends on $\mu_{f.f.}$, which therefore directly affects the electric pole ω_e and consequently the electrodynamic stiffness at finite speed.

Since all computations are performed in two dimensions, all physical quantities are expressed per unit depth. For the reader's convenience, all reported values are multiplied by a reference depth of 1 m.

Geometry Components and Symmetry Reduction

The density variables are defined on a single stator design domain Ω_S and are then mapped into the winding regions of all four magnetic zones by translation. When the upper and lower winding regions are related by orthogonal symmetry, the density fields are inverted accordingly. Infinite Element Domains are used at the outer boundaries to enforce the correct far-field behaviour, as illustrated in Fig. 3.5.

Geometric symmetry is exploited to reduce the extent of each of the four magnetic zones, thereby decreasing the total number of mesh elements and the associated computational cost. The symmetry conditions applicable to each configuration are detailed in Appendix A.1. The main geometric parameters of the model are presented in Table A.1 in Appendix A.2. Since this thesis focuses on the topology optimization methodology rather than on improving an existing machine, these parameters were chosen to be representative of the machine class studied.

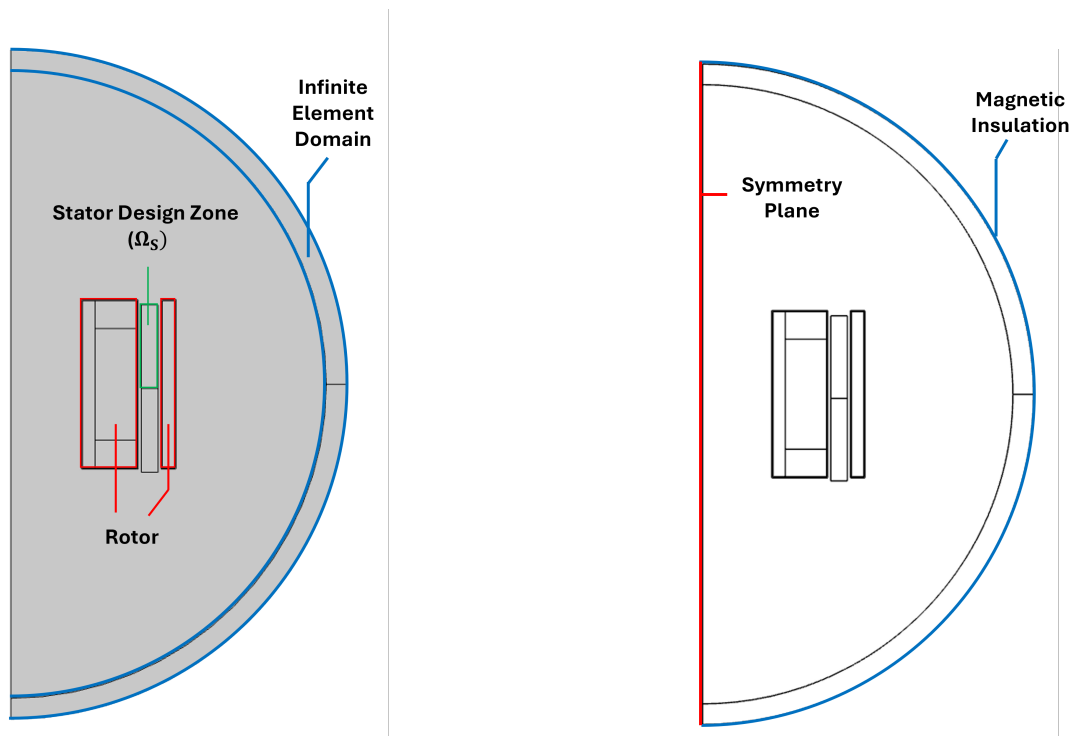


Figure 3.5: **Magnetic zone (2)**. The R2–S3 configuration is illustrated, with its components: rotor, stator design domain Ω_S , and Infinite Element Domains. The radial symmetry ($\Delta r = 0$) halves the domain and reduces the mesh element count.

Physics

The *Magnetic Fields* physics interface in *COMSOL Multiphysics*® is used. The governing equations are derived from Maxwell's equations, expressed in terms of the magnetic field

strength $\mathbf{H}$, the magnetic flux density $\mathbf{B}$, and the magnetic potential vector $\mathbf{A}$:

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad \mathbf{B} = \nabla \times \mathbf{A} \quad (3.6)$$

where the current density $\mathbf{J}$ is given by:

$$\mathbf{J} = \sigma \mathbf{E} + \sigma \mathbf{v} \times \mathbf{B} + \mathbf{J}_e \quad (3.7)$$

In a magnetostatic simulation without an induced electric field, the current density $\mathbf{J}$ reduces to the external contribution $\mathbf{J}_e$ alone, that is, the current flowing through the windings. Two boundary conditions are applied over the domain, as illustrated in Fig. 3.5: magnetic insulation on the outer boundary, and a symmetry condition where applicable:

$$\mathbf{n} \times \mathbf{A} = 0 \quad (\text{magnetic insulation}), \quad \mathbf{n} \cdot \mathbf{B} = 0 \quad (\text{symmetry}) \quad (3.8)$$

Two electromagnetic quantities are modified by the topology optimization density fields. The first is the external current density $\mathbf{J}_e$, which models the contribution of the armature windings. It is expressed as:

$$\mathbf{J}_e = \|J\| \cdot N_w \cdot \mu_{f.f.} \cdot (\theta_{W_+}^p - \theta_{W_-}^p) \quad (3.9)$$

When the difference of $\theta_{W_+}^p - \theta_{W_-}^p$ equals +1, the material behaves as a forward winding, having a positive $\mathbf{J}_e$ and when the difference equals -1, the material behaves as a return winding. When the difference is zero, the material is either air or iron. The effective current per winding turn can then be obtained by integration over the design domain Ω_S :

$$I = \int_{\Omega_S} \|J\| \cdot \mu_{f.f.} \cdot \frac{|\theta_{W_+}^p - \theta_{W_-}^p|}{2} d\Omega_S \quad (3.10)$$

This formulation highlights the importance of initializing $\theta_{W_+}^p$ and $\theta_{W_-}^p$ to distinct values: if both densities are equal throughout the entire domain, $I = 0$, which leads to numerical issues (division by 0 errors) in derived quantities such as the inductance.

Since the current can be positive or negative based on the definition of the considered stator configuration (with the forward and return windings defined within the same design zone), this explains the factor $\frac{1}{2}$ in the formula above. Furthermore, physical consistency requires that the net forward and return currents be equal, which is enforced through the following equality constraint:

$$\int_{\Omega_S} \|J\| \cdot \mu_{f.f.} \cdot \theta_{W_+}^p d\Omega_S = \int_{\Omega_S} \|J\| \cdot \mu_{f.f.} \cdot \theta_{W_-}^p d\Omega_S \iff \int_{\Omega_S} \theta_{W_+}^p d\Omega = \int_{\Omega_S} \theta_{W_-}^p d\Omega_S \quad (3.11)$$

The second quantity affected by the densities is the relative magnetic reluctivity ν_r , which is interpolated between the air value ($\nu_{r,air} = 1$) and the ferromagnetic value $\nu_{r,F}$ according to the iron density θ_F^p :

$$\nu_r = \nu_{r,air} + (\nu_{r,F} - \nu_{r,air}) \theta_F^p, \quad \mathbf{B} = \mu_0 \frac{1}{\nu_r} \mathbf{H} \quad (3.12)$$

Relative reluctivity is used instead of relative permeability to achieve faster convergence in *COMSOL Multiphysics*®. The components that make up the rotor (rotor iron yokes, permanent magnets) are modelled using the materials selected in Section 2.4 and based on the rotor configuration (R1, R2, R3, or R4).

Objective Functions

Three objective functions are defined, as described in the optimization scenarios presented in section 3.1. Since the ferromagnetic material and the winding material cannot physically coexist in the same location, a penalty term must be included in any objective function that includes the iron density variable θ_F^p . The coexistence constraint between θ_F^p and each winding density is enforced using the following normalized penalty coefficients:

$$P_1(\theta_F^p, \theta_{W_+}^p) = \frac{\int_{\Omega_S} \max(0, \theta_F^p + \theta_{W_+}^p - 1) d\Omega_S}{\int_{\Omega_S} d\Omega_S} \quad (3.13)$$

$$P_2(\theta_F^p, \theta_{W_-}^p) = \frac{\int_{\Omega_S} \max(0, \theta_F^p + \theta_{W_-}^p - 1) d\Omega_S}{\int_{\Omega_S} d\Omega_S} \quad (3.14)$$

P_1 and P_2 are dimensionless quantities that are equal to zero when there is no overlap between the density fields of the iron and the winding, and that increase in proportion to the degree of overlap. A penalty parameter ($\lambda = 10^4 \text{ N/mm}$) is used to make the penalty coefficients compatible with stiffness terms. Its value is chosen sufficiently large to effectively prevent any overlap of materials, without being so high as to dominate the objective function and hinder convergence toward an optimal topology

It should be noted that no penalty is required to prevent the overlap of the forward and reverse winding current densities, since the superposition of $\theta_{W_+}^p$ and $\theta_{W_-}^p$ at the same point results in the cancellation of the external current density $\mathbf{J}_e$, which means that either air or iron is present.

OBJ1 maximises the total bearing stiffness over all design variables, including the iron density:

$$\text{OBJ1 : } \max_{\theta_F^p, \theta_{W_+}^p, \theta_{W_-}^p} \left[\sum_i k_i(\theta_F^p, \theta_{W_+}^p, \theta_{W_-}^p) - \lambda [P_1(\theta_F^p, \theta_{W_+}^p) + P_2(\theta_F^p, \theta_{W_-}^p)] \right] \quad (3.15)$$

where $\sum_i k_i = k_{z,e} + k_{z,d} + k_{r,d}$.

OBJ2 maximises the total bearing stiffness using winding densities only, with no ferromagnetic material. In the absence of iron, there is no need for penalty coefficients, and there is also no detent stiffness, given their definition.

$$\text{OBJ2 : } \max_{\theta_{W_+}^p, \theta_{W_-}^p} \left[\sum_i k_i(\theta_{W_+}^p, \theta_{W_-}^p) \right] = \max_{\theta_{W_+}^p, \theta_{W_-}^p} [k_{z,e}(\theta_{W_+}^p, \theta_{W_-}^p)] \quad (3.16)$$

OBJ3 maximises the electrodynamic stiffness alone, over all design variables including the iron density. The detent stiffness contributions are thus excluded from the objective.

$$\text{OBJ3 : } \max_{\theta_F^p, \theta_{W_+}^p, \theta_{W_-}^p} \left[k_{z,e}(\theta_F^p, \theta_{W_+}^p, \theta_{W_-}^p) - \lambda [P_1(\theta_F^p, \theta_{W_+}^p) + P_2(\theta_F^p, \theta_{W_-}^p)] \right] \quad (3.17)$$

Additional notes on the Stator Topology Optimization Model

To improve the accuracy of the Maxwell stress tensor integration used to calculate forces, an “air box” is defined around the rotor surface. This layer ensures that the mesh is sufficiently fine in the region where the surface integral is evaluated, thereby reducing numerical error in the calculated forces.

In the topological optimization model, since the electrodynamic stiffness depends on the machine’s spin speed (as shown in Figure 2.2), a spin speed of $\omega = 2000 \text{ rad/s} \approx 19100 \text{ rpm}$ was chosen arbitrarily.

3.3 Stator - Topology Optimization Results

This section presents the results of the stator topology optimization. For each rotor configuration, the stiffness values obtained for all stator design zones and all objective functions are presented in a dedicated subsection, including optimized density plots. A summary of the results for all configurations is provided at the end of this section.

To ensure a fair comparison among all configurations, the detent forces ($F_{z,d}$ and $F_{r,d}$) calculated in the total absence of ferromagnetic material (without density variables for iron) were subtracted from those calculated with ferromagnetic material to cancel out the mesh-dependent parasitic forces resulting from the integration of the Maxwell stress tensor. This correction ensures that the detent stiffness is exactly zero when no ferromagnetic material is present in the stator.

3.3.1 Rotor Configuration R1

Table 3.3 reports the stiffness results for rotor configuration R1, across all stator design zones and objective functions.

Stator	OBJ1				OBJ2	OBJ3	
	$\sum_i k_i$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$k_{z,e}$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$k_{z,d}$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$k_{r,d}$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$\sum_i k_i = k_{z,e}$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$\sum_i k_i$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$k_{z,e}$ $\left[\frac{\text{N}}{\text{mm}}\right]$
S1	40.504	41.074	0.042	−0.612	41.006	−961.36	186.86
S2	897.46	902.02	−4.009	−0.548	901.69	−4346.4	1369.3
S3	46.438	47.174	−0.146	−0.590	47.066	−929.47	152.13
S4	898.33	903.15	−4.340	−0.483	902.94	−4348.8	1173.9

Table 3.3: Stiffness components for all stator configurations and objective functions (rotor configuration R1)

Several observations can be drawn from these results. First, the total bearing stiffness obtained with OBJ2 is slightly greater than that obtained with OBJ1 for all stator configurations. If the use of a ferromagnetic material had a beneficial effect on the sum of the stiffnesses, OBJ1 would have yielded a value strictly greater than that of OBJ2 in at least one of the stator configurations. This is not the case, indicating that adding the iron

density variable does not improve the total bearing stiffness for this rotor configuration.

Second, the detent stiffness components $k_{z,d}$ and $k_{r,d}$ are non-zero for the objective function OBJ1, whereas one would expect the iron to completely disappear from the optimal topology and thus for these stiffnesses to be zero. This is explained by the fact that the optimization stops before the iron density has fully converged to zero. For example, for the R1-S1 configuration, at the end of the simulation, the iron density averaged over the domain is $\bar{\theta}_F^p = 0.0010001$, which corresponds to an average relative reluctivity of $\bar{\nu}_r \approx 0.999$ over the design area. Although small, this residual reluctivity is sufficient to produce negative detent stiffness values via the integration of the Maxwell stress tensor. This behaviour is present in all rotor configurations and will not be discussed further.

Third, the OBJ3 optimization function shows that the ferromagnetic material in the stator can significantly improve the electrodynamic stiffness $k_{z,e}$ when it is the sole optimization objective. However, this gain is accompanied by significant negative detent stiffnesses that dominate the total bearing stiffness, resulting in strongly negative values for $\sum_i k_i$. Iron therefore has a significant negative effect on the overall stiffness of the bearing, despite its positive influence on the electrodynamic component when considered in isolation.

The best total stiffness for configuration rotor R1 is achieved with stator design zone S4. The corresponding optimized topologies for all three objective functions of this stator configuration are shown in Figs. 3.6–3.8.

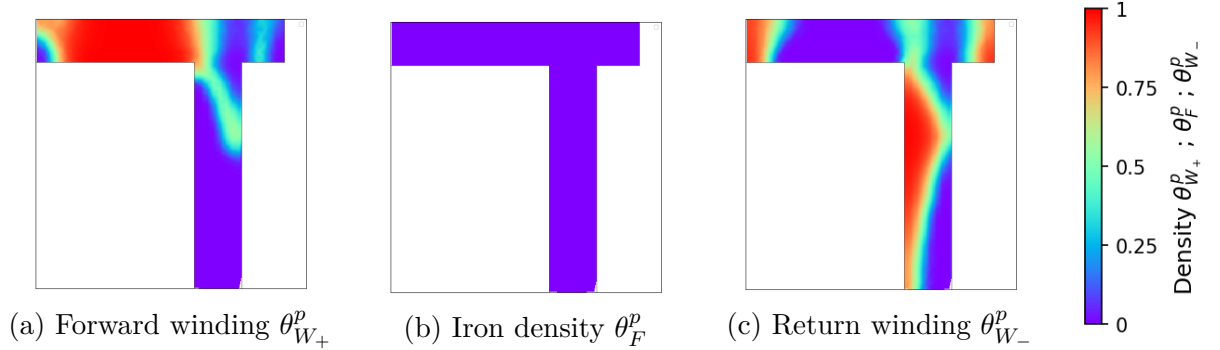


Figure 3.6: Optimized topology under OBJ1, configuration R1-S4. The iron density is visually negligible across the entire domain, which is consistent with the numerical results.

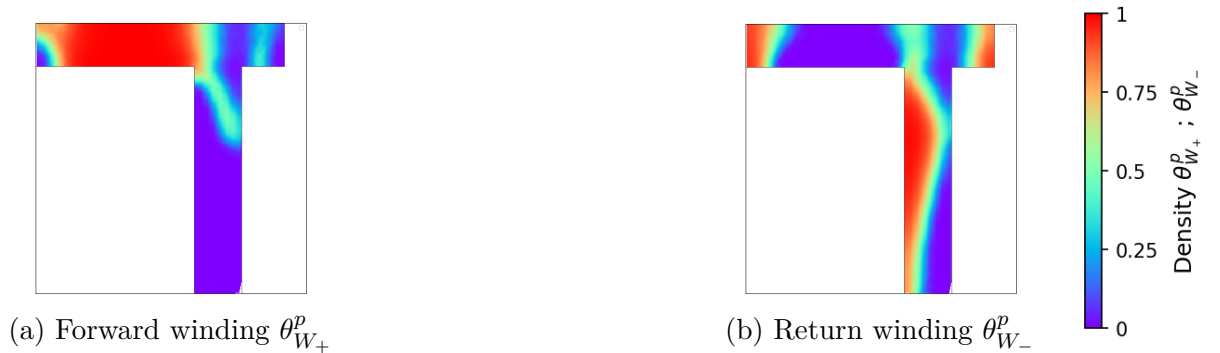


Figure 3.7: Optimized topology under OBJ2, configuration R1-S4.

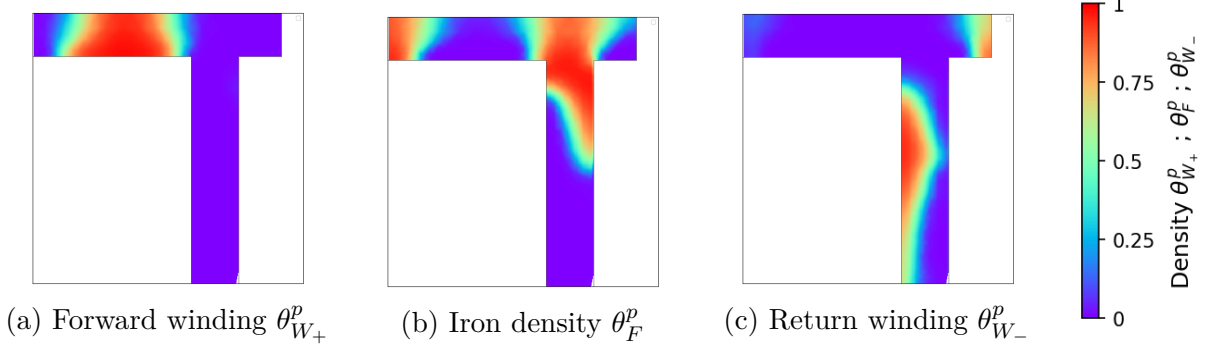


Figure 3.8: Stator optimized topology under OBJ3, configuration R1-S4.

For the topological results of the objective functions OBJ1 and OBJ2, a topological feature is that the forward windings are concentrated axially above the rotor, in the region where the magnetic field lines close radially. In the two-dimensional model, this arrangement optimizes the electrodynamic force generated by the windings through the product of the currents and the magnetic field (i.e., the Lorentz force).

Indeed, the orientation of the magnetic field lines is radial at the axial ends of the magnet, thereby producing a strong radial magnetic field component that the windings can utilize efficiently. It should be noted, however, that this result must be interpreted with caution in the context of a real three-dimensional machine.

In fact, in a 3D configuration, the heteropolar component of the magnetic field introduces magnetic field components that do not close in the same way as in the 2D model. The optimal winding placement identified here may therefore not be translated directly into a three-dimensional design, and the stiffness values obtained should be considered indicative rather than a realistic estimate.

Furthermore, in the context of the OBJ3 objective function, the iron is distributed around the winding zones, acting as a magnetic flux concentrator that strengthens the magnetic field with which the armature windings interact. This behaviour is consistent with the objective of maximizing the electrodynamic stiffness $k_{z,e}$ in isolation, at the expense of contributions from the detent stiffness.

The final estimated parameters (K_z , L_c , R , $k_{z,e}^\infty$, ω_e) obtained from the second objective function (OBJ2), are presented in Appendix A.3.

3.3.2 Rotor Configuration R2

Table 3.4 reports the stiffness results for rotor configuration R2, across all stator design zones and objective functions. In this rotor configuration, The same observations can be made as for the numerical results obtained in the section 3.3.1. The corresponding optimized topologies for all three objective functions of the configuration R2-S4 are shown in Figs. 3.9–3.11.

Stator	OBJ1		OBJ2	OBJ3	
	$\sum_i k_i$	$k_{z,e}$		$\sum_i k_i$	$k_{z,e}$
	$\left[\frac{N}{mm}\right]$	$\left[\frac{N}{mm}\right]$		$\left[\frac{N}{mm}\right]$	$\left[\frac{N}{mm}\right]$
S1	696.15	701.55	701.29	-5006.2	1966.9
S2	722.02	728.11	728.43	-7288.0	2576.6
S3	1116.0	1121.0	1118.7	-4289.8	1883.6
S4	1154.1	1160.4	1157.6	-9878.2	3178.1

Table 3.4: Stiffness components for all stator configurations and objective functions (rotor configuration R2)

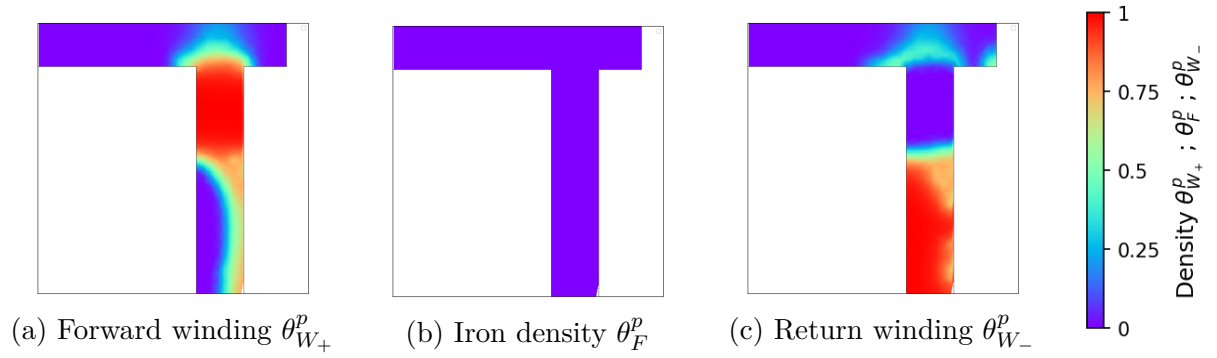


Figure 3.9: Optimized topology under OBJ1, configuration R2-S4.

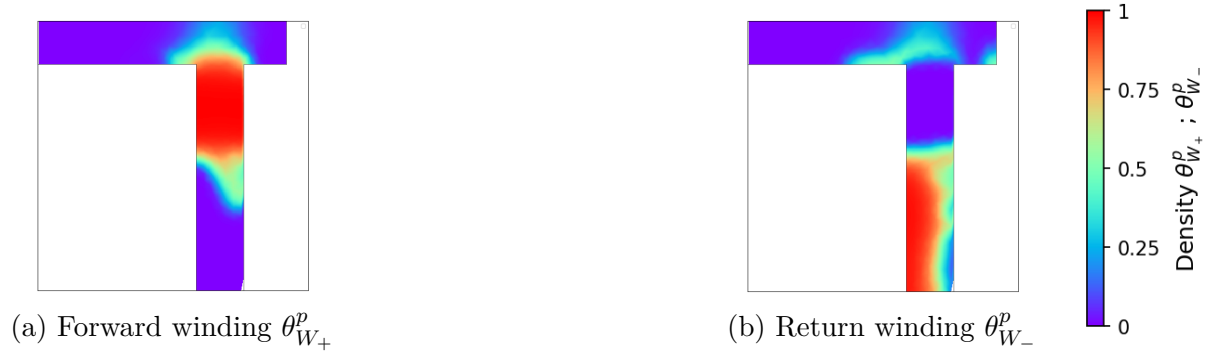


Figure 3.10: Optimized topology under OBJ2, configuration R2-S4.

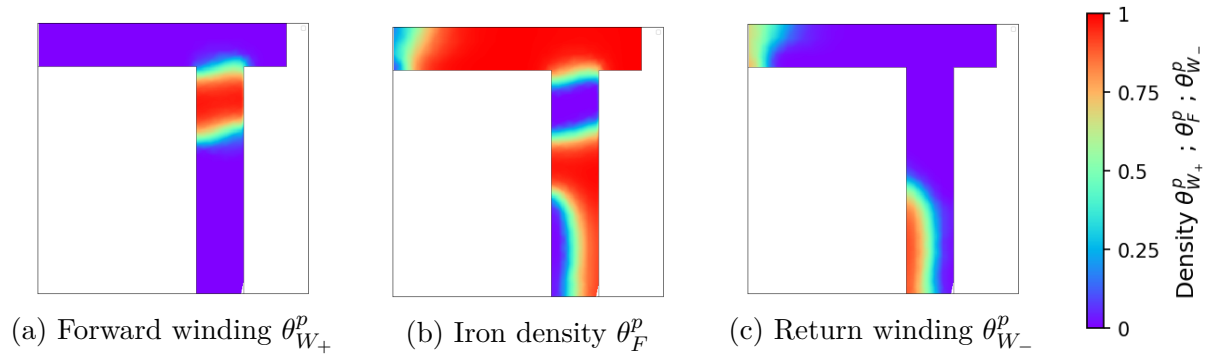


Figure 3.11: Optimized topology under OBJ3, configuration R2-S4.

With regard to the topological results for the R2-S4 configuration, the forward and return windings are located lower down. This is because the magnetic field lines close lower down due to the three permanent magnet configuration. Furthermore, thanks to this magnet arrangement, the heteropolar component is actually less significant for the 3D machine in this rotor configuration. These results should always be viewed critically, but the topologies already appear more realistic in this case.

The final estimated parameters (K_z , L_c , R , $k_{z,e}^\infty$, ω_e) obtained from the second objective function (OBJ2), are presented in Appendix A.3.

3.3.3 Rotor Configuration R3

Table 3.5 reports the stiffness results for rotor configuration R3, across all stator design zones and objective functions.

Stator	OBJ1		OBJ2	OBJ3	
	$\sum_i k_i$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$k_{z,e}$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$\sum_i k_i = k_{z,e}$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$\sum_i k_i$ $\left[\frac{\text{N}}{\text{mm}}\right]$	$k_{z,e}$ $\left[\frac{\text{N}}{\text{mm}}\right]$
S1	820.48	825.98	825.36	-7558.5	3425.9
S2	851.85	858.01	858.78	-13320	5953.1
S3	1508.8	1519.1	1512.0	-5949.8	3093.4
S4	1543.72	1550.9	1550.2	-14983	6508.4

Table 3.5: Stiffness components for all stator configurations and objective functions (rotor configuration R3).

The corresponding optimized topologies for all three objective functions of the configuration R3-S4 are shown in Figs. 3.12–3.14.

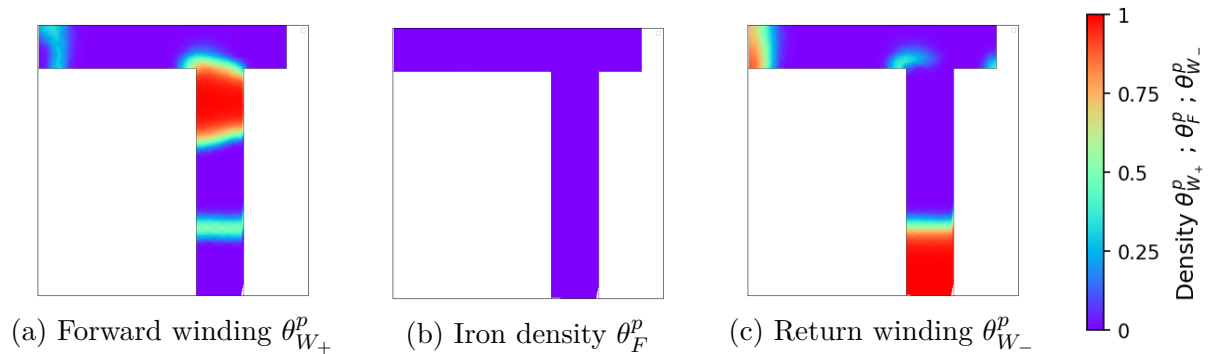


Figure 3.12: Optimized topology under OBJ1, configuration R3-S4.

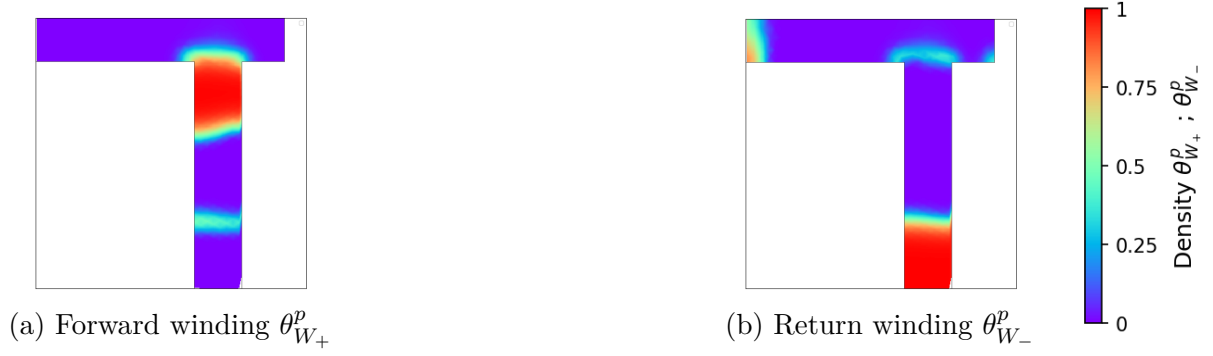


Figure 3.13: Optimized topology under OBJ2, configuration R3-S4.

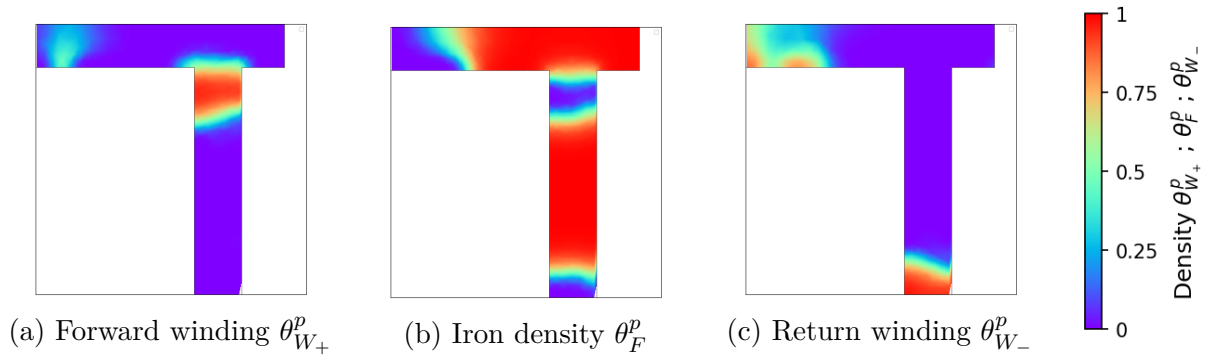


Figure 3.14: Optimized topology under OBJ3, configuration R3-S4.

For the numerical results and the results of the topological optimization, the same observations can be made as in the previous sections 3.3.1 and 3.3.2

The final estimated parameters (K_z , L_c , R , $k_{z,e}^\infty$, ω_e) obtained from the second objective function (OBJ2), are presented in Appendix A.3.

3.3.4 Rotor Configuration R4

Table 3.6 reports the stiffness results for rotor configuration R4, across all stator design zones and objective functions.

Stator	OBJ1		OBJ2	OBJ3	
	$\sum_i k_i$ $\left[\frac{N}{mm}\right]$	$k_{z,e}$ $\left[\frac{N}{mm}\right]$		$\sum_i k_i$ $\left[\frac{N}{mm}\right]$	$k_{z,e}$ $\left[\frac{N}{mm}\right]$
S1	1344.9	1353.3	1351.7	-8770.1	4553.3
S2	1537.1	1547.4	1545.4	-17212	6206.0
S3	2031.1	2038.6	2038.2	-6871.9	3757.6
S4	2300.8	2310.4	2308.8	-9142.8	5052.8

Table 3.6: Stiffness components for all stator configurations and objective functions (rotor configuration R4)

The corresponding optimized topologies for all three objective functions of the configuration R4-S4 are shown in Figs. 3.15–3.17.

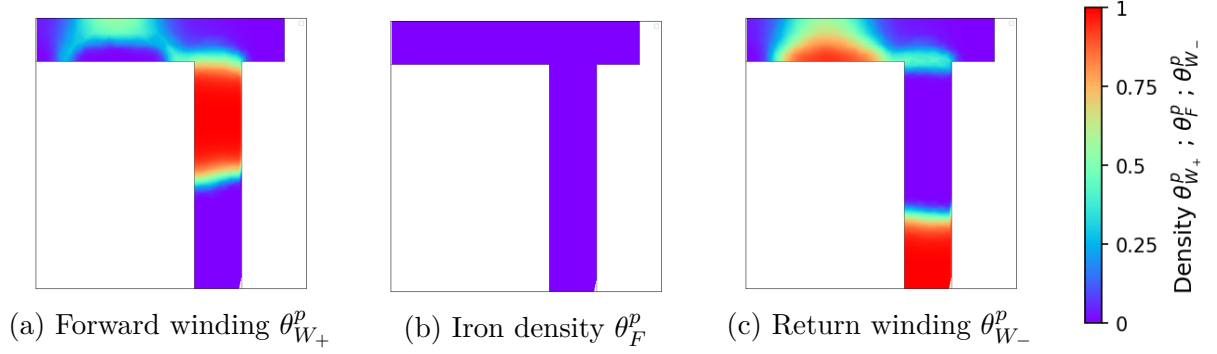


Figure 3.15: Optimized topology under OBJ1, configuration R4-S4.

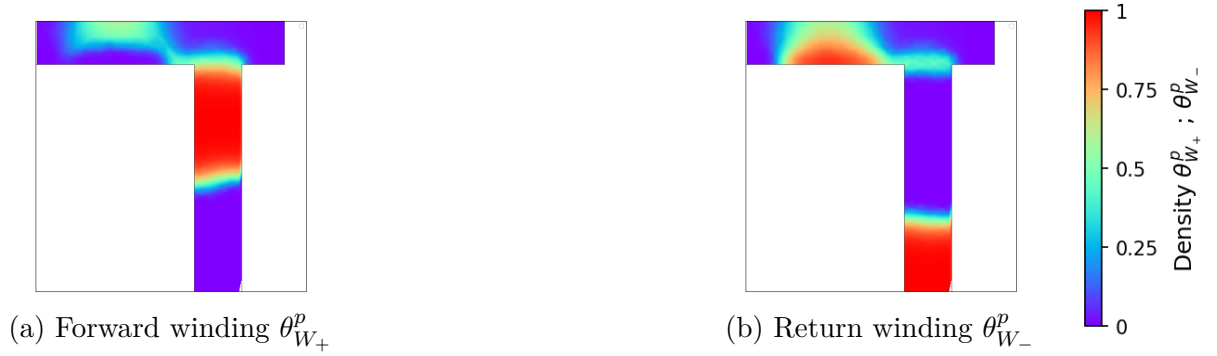


Figure 3.16: Optimized topology under OBJ2, configuration R4-S4.

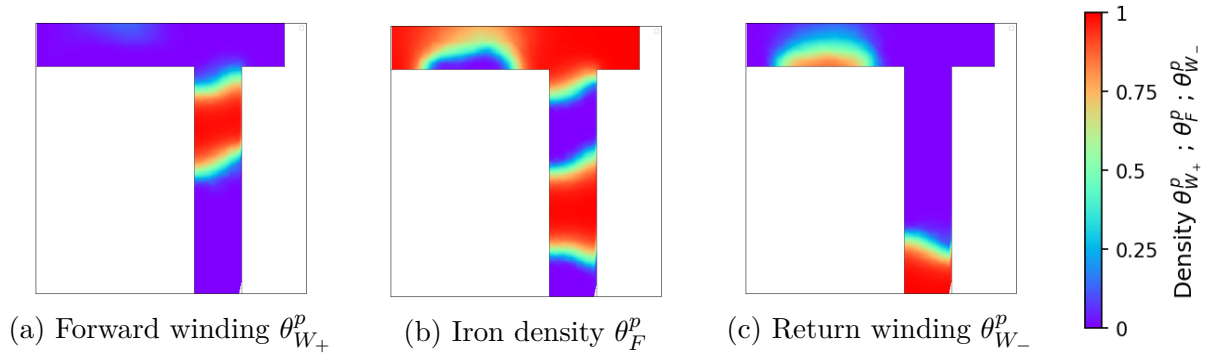


Figure 3.17: Optimized topology under OBJ3, configuration R4-S4.

For the numerical results and the results of the topological optimization, the same observations can be made as in the previous sections 3.3.1 and 3.3.2. Additionally, it is worth noting that for the R4 rotor configuration, two different winding zones were identified for the return windings (θ_{W-}^p). It should be noted, however, that this does not correspond to the structure of the machine under study (particularly due to its figure-eight shaped null-flux coils). Consequently, if a topology were to be chosen, the lower region of the return winding would be selected.

The final estimated parameters (K_z , L_c , R , $k_{z,e}^\infty$, ω_e) obtained from the second objective function (OBJ2), are presented in Appendix A.3.

3.3.5 Summary of Results

Table 3.7 reports the electrodynamic stiffness $k_{z,e}$ obtained under optimization function OBJ2 for all rotor–stator configurations. It should be noted that the values obtained correspond to a 2D model with a reference depth of 1 m. Therefore, these values are more indicative than accurate estimates of reality.

	S1	S2	S3	S4
	$\left[\frac{\text{N}}{\text{mm}} \right]$	$\left[\frac{\text{N}}{\text{mm}} \right]$	$\left[\frac{\text{N}}{\text{mm}} \right]$	$\left[\frac{\text{N}}{\text{mm}} \right]$
R1	41.006	901.69	47.066	902.94
R2	701.29	728.43	1118.7	1157.6
R3	825.36	858.78	1512.0	1550.2
R4	1351.7	1545.4	2038.2	2308.8

Table 3.7: Electrodynamic stiffness $k_{z,e}$ under optimization function OBJ2 for all rotor–stator configurations.

These results reveal several trends. First, expanding the stator design zone ($\text{S1} \rightarrow \text{S2}$ and $\text{S3} \rightarrow \text{S4}$) consistently increases the objective function value $k_{z,e}$ across all rotor configurations. This is consistent with what is expected from the implementation of topological optimization: greater design freedom allows the optimizer to find a better solution, which confirms that the model has been implemented correctly.

Then, configurations S3 and S4 consistently outperform S1 and S2 for all rotor configurations. This is attributed to the ferromagnetic yoke surrounding the armature windings in stator configurations S3 and S4, which concentrates the magnetic field near the armature and thereby increases the interaction of the PM magnetic field with the windings. Although S4 has numerically higher stiffness values than S3, its winding regions extend axially above the rotor where the magnetic flux closes radially in the two-dimensional representation. As noted in section 3.3.1, this placement uses a magnetic field path that does not exist in the same form in a three-dimensional machine, where the heteropolar field component alters the magnetic circuit. The stiffness values obtained for S4 are therefore likely overestimated. For this reason, the stator configuration S3 is considered the most physically representative and is preferred for topology extraction.

The optimized winding topologies for all rotor–stator combinations under OBJ2 objective function are presented in Table 3.8. In all cases, the forward and reverse winding regions are clearly separated in space. Their arrangement maximizes the positive contribution of the forward and return windings to the electrodynamic force, consistent with a Lorentz-type interaction between the winding current and the radial component of the permanent magnetic field. These topologies form the basis for the manufacturable geometry extraction discussed in the following section.

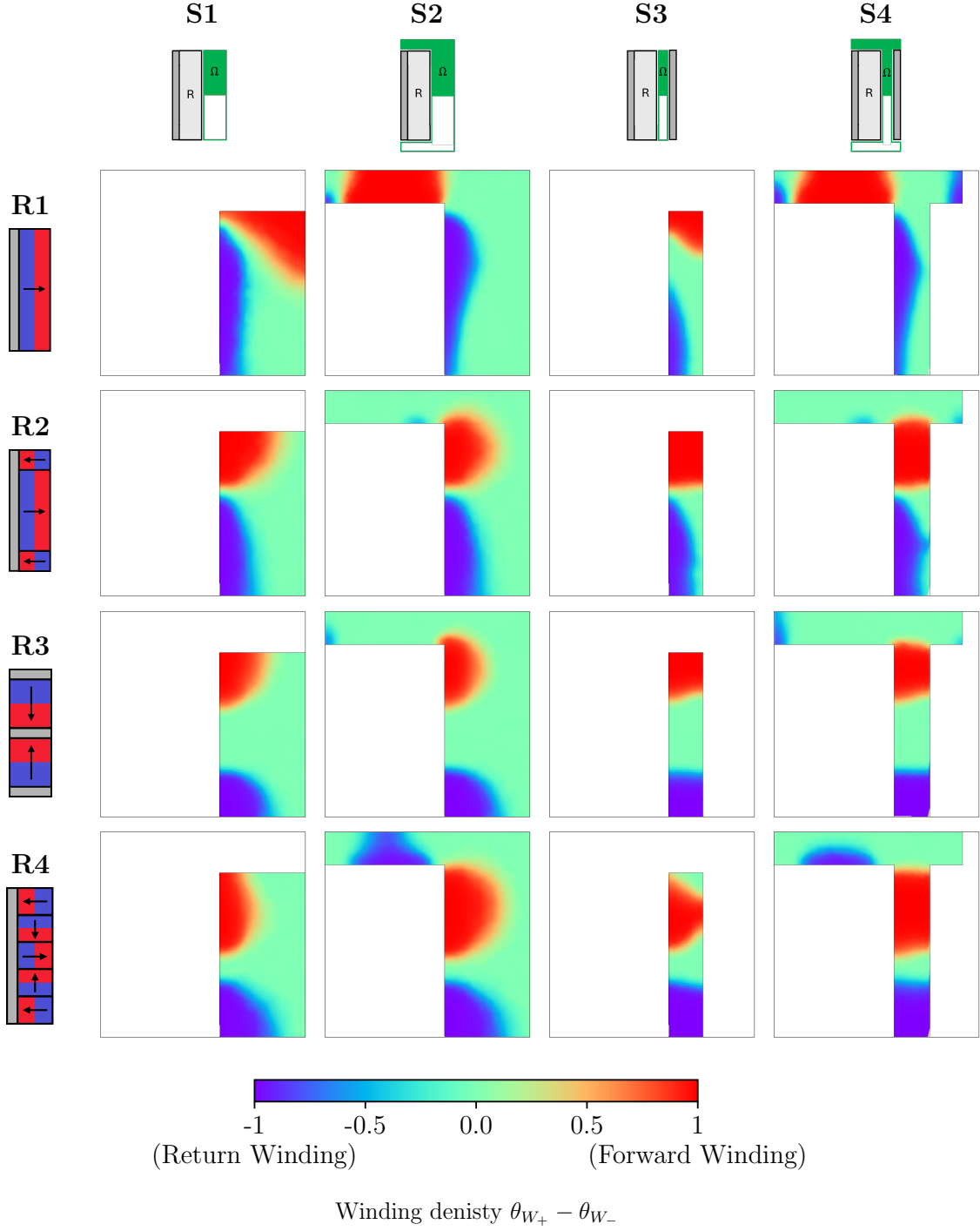


Table 3.8: Topological optimization results for rotor-stator configurations

3.4 Stator - Parametric Optimization Model

Topology Extraction and Design Choice

Based on the numerical stiffness results and the topological results obtained and presented in the previous section, the S3 stator configuration (narrow winding zone with a rotor iron yoke surrounding the armature) was selected as the basis for topological extraction, so that it could be parametrically optimized.

To ensure an affordable and easy-to-manufacture topology, the geometry derived from the optimization results consists of two rectangular winding cross-sections, representing the forward winding (W_+) and the reverse winding (W_-), each defined by a width, a height, and the coordinates of its center. The geometric parameters are illustrated in Figure 3.18. Although further simplification is possible, for example by imposing a square cross-section, this thesis focuses more on the search for new, higher-performance topologies.

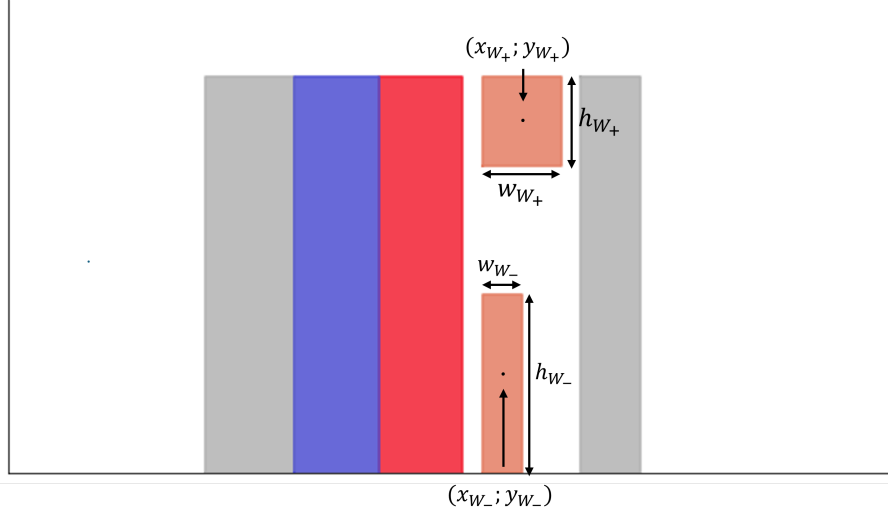


Figure 3.18: Parametric representation of the extracted S3 stator topology for rotor configuration R1.

Problem Formulation

Parametric optimization uses the same electromagnetic model as topological optimization, but the density fields have been replaced by geometric parameters as control variables. Since no ferromagnetic material is considered in the stator, the parametric optimization problem is formulated as:

$$\max_{\mathbf{p}} \quad k_{z,e}(\mathbf{p}) \quad (3.18)$$

where the vector of control variables is:

$$\mathbf{p} = (w_{W_+}, w_{W_-}, h_{W_-}, x_{W_+}, y_{W_+}, x_{W_-}, y_{W_-}) \in \mathbb{R}_+^7 \quad (3.19)$$

The control variables are subject to certain geometric constraints that ensure that the winding zones remain within the stator design region (Ω_S) and do not overlap.

$$x_{W_+} - \frac{1}{2}w_{W_+} \geq x_{S,\min}, \quad x_{W_+} + \frac{1}{2}w_{W_+} \leq x_{S,\max} \quad (3.20)$$

$$x_{W_-} - \frac{1}{2}w_{W_-} \geq x_{S,\min}, \quad x_{W_-} + \frac{1}{2}w_{W_-} \leq x_{S,\max} \quad (3.21)$$

$$y_{W_-} - \frac{1}{2}h_{W_-} \geq y_{S,\min}, \quad y_{W_+} + \frac{1}{2}h_{W_+} \leq y_{S,\max} \quad (3.22)$$

$$y_{W_-} + \frac{1}{2}h_{W_-} \leq y_{W_+} - \frac{1}{2}h_{W_+} \quad (\text{no overlap between } W_+ \text{ and } W_-) \quad (3.23)$$

where $x_{\Omega_S,\min}$, $x_{\Omega_S,\max}$, $y_{\Omega_S,\min}$, and $y_{\Omega_S,\max}$ define the boundaries of the stator design domain Ω_S . Since physical consistency requires that the net forward and reverse currents

be equal, the height of the forward winding depends on the other three dimensional parameters:

$$h_{W_+} = \frac{h_{W_-} \cdot w_{W_-}}{w_{W_+}} \quad (3.24)$$

This ensures that the cross-sectional areas of W_+ and W_- remain equal throughout the optimization. The geometry is illustrated schematically in Fig. 3.18.

As the control variables directly modify the geometry and thus the mesh, gradient-based methods cannot be used. The solver used in parametric optimizations is therefore the Nelder-Mead non-derivative solver.

Results

The winding geometry before and after parametric optimization is shown in Figs. 3.19 and 3.20, respectively. The initial and optimal values of all control variables are provided in Tables A.3– A.6 in Appendix A.3.

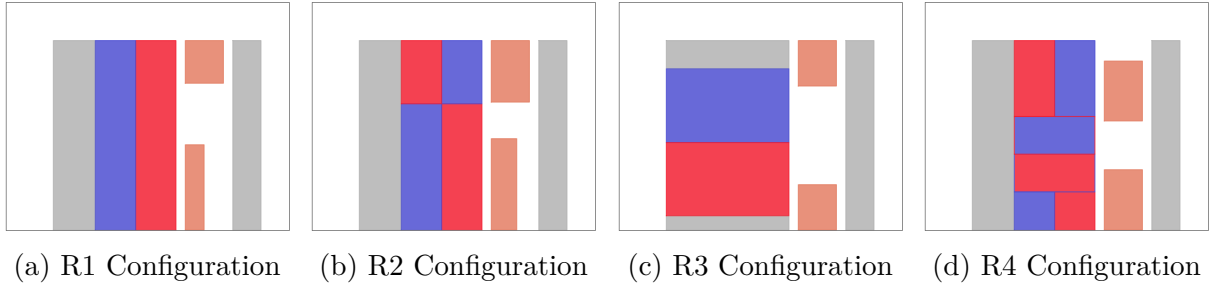


Figure 3.19: Windings parametric disposition before parametric optimization, extracted from results available in Table 3.8.

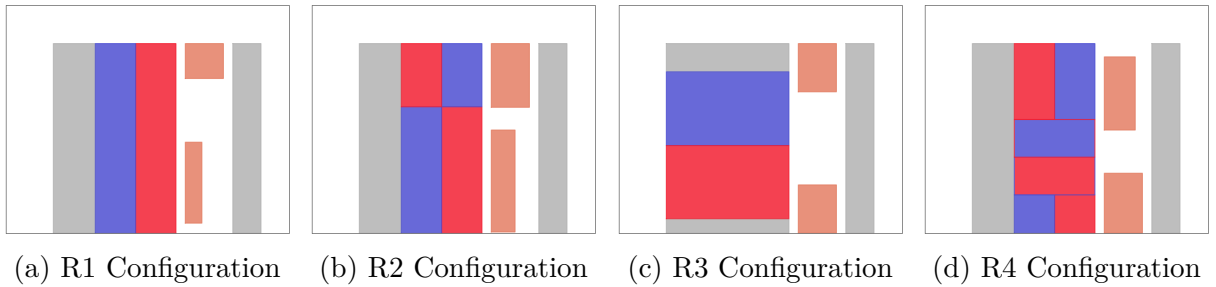


Figure 3.20: Windings parametric disposition after parametric optimization

Table 3.9 reports the electrodynamic stiffness at each stator optimization step, expressed both in absolute value and as a relative improvement with respect to the topology optimization result.

	R1–S3	R2–S3	R3–S3	R4–S3
$k_{z,e}^{\text{topo}} \left[\frac{\text{N}}{\text{mm}} \right]$	47.066	1118.7	1512.0	2038.2
$k_{z,e}^{\text{param}} \left[\frac{\text{N}}{\text{mm}} \right]$	46.270	1129.1	1505.4	2058.7
Relative improvement [%]	−1.69	+0.93	−0.44	+1.01

Table 3.9: Electrodynamic stiffness before and after parametric optimisation of the extracted S3 stator topology, for all rotor configurations.

These results show that a rectangular cross-section of the winding provides a close approximation to the optimal topology identified by the topology optimization model. The small variations in stiffness observed from one optimization model to another ($\leq \pm 2\%$) are consistent with this interpretation: configurations whose optimized density distributions were already close to rectangular shapes exhibit the smallest deviations between the two optimization steps.

The extracted topologies can thus be considered good starting points for the rotor topology optimization as well as for the combined stator-rotor parametric optimization, presented respectively in Chapters 4 and 5.

3.5 Stator - Alternative Model: Two-Variable Density Formulation

As noted in Chapter 1, several topological optimization formulations found in the literature represent component distributions as the product of two density variables rather than by assigning a density to each component. In this approach, each combination of the two density values corresponds to a distinct component of the model. Compared to the first model developed in section 3.2, only the density definitions and their coupling with the physics differ. All other aspects of the model (geometry, mesh, simulation configurations, and parameter extraction) remain unchanged.

Density Definitions

Two penalised density variables, θ_1^p and θ_2^p , are defined over the design domain Ω_S , each taking values between 0 and 1. Rather than assigning a density to each component of the model, the four possibilities are encoded by the combination of these two variables, as summarised in Table 3.10.

Model Component	θ_1^p	θ_2^p
Ferromagnetic material	1	1
Forward winding	1	0
Return winding	0	1
Air	0	0

Table 3.10: The combination of different variables results in different model components.

The initial values for the densities and the SIMP penalty exponent are given in Table 3.11. The two densities are arbitrarily initialised to distinct values in order to avoid a net current of zero during the first iteration, the implications of which are discussed below. The penalty exponent $p = 5$ is applied to both density fields, thereby ensuring consistent penalty behaviour for all components of the model.

Parameter	Symbol	Value
Density 1 initial value	$\theta_{1,0}^p$	0.45
Density 2 initial value	$\theta_{2,0}^p$	0.55
SIMP penalisation exponent	p	5

Table 3.11: Density initialisation values and SIMP exponent for the second stator topology optimization model.

Coupling with the Density Fields

Three electromagnetic quantities are influenced by the density fields θ_1^p and θ_2^p . The external current density $\mathbf{J}_e$, which corresponds to the contribution of the armature winding, is expressed as:

$$\mathbf{J}_e = \|J\| \cdot N_w \cdot \mu_{f.f.} \cdot (\theta_1^p \cdot (1 - \theta_2^p) - (1 - \theta_1^p) \cdot \theta_2^p) \quad (3.25)$$

The external current density is positive in forward winding regions ($\theta_1^p \rightarrow 1, \theta_2^p \rightarrow 0$), negative in return winding regions ($\theta_1^p \rightarrow 0, \theta_2^p \rightarrow 1$), and is null in both air and ferromagnetic regions. The effective current per winding turn can then be obtained by integration over the design domain Ω_S :

$$I = \int_{\Omega_S} \|J\| \cdot \mu_{f.f.} \cdot \frac{|\theta_1^p \cdot (1 - \theta_2^p) - (1 - \theta_1^p) \cdot \theta_2^p|}{2} d\Omega_S \quad (3.26)$$

This formulation highlights the importance of initializing θ_1^p and θ_2^p to distinct values: if both densities are equal throughout the entire domain, $I = 0$, which leads to numerical issues (division by 0 errors) in derived quantities such as the inductance. Since the current can be positive or negative based on the definition of the considered stator configuration (with the forward and return windings defined within the same design zone), this explains

the factor $\frac{1}{2}$ in the formula above. Furthermore, physical consistency requires that the net forward and return currents be equal, which is enforced through the following equality constraint:

$$\int_{\Omega_S} \theta_1^p \cdot (1 - \theta_2^p) d\Omega_S = \int_{\Omega_S} (1 - \theta_1^p) \cdot \theta_2^p d\Omega_S \quad (3.27)$$

The local magnetic reluctivity is interpolated between the air value and the ferromagnetic value ν_F using the product $\theta_1^p \cdot \theta_2^p$, which equals 1 only in ferromagnetic regions ($\theta_1^p \rightarrow 1, \theta_2^p \rightarrow 1$):

$$\nu_r = 1 + (\nu_{r,F} - 1) \cdot \theta_1^p \cdot \theta_2^p, \quad \mathbf{B} = \frac{\mu_0}{\nu_r} \mathbf{H}. \quad (3.28)$$

Finally, the winding resistance per unit depth is evaluated from the effective winding cross-section using Pouillet's law:

$$R = \frac{\rho}{S} = \frac{\rho}{\mu_{f.f.} \cdot \int_{\Omega_S} \frac{|\theta_1^p \cdot (1 - \theta_2^p) - (1 - \theta_1^p) \cdot \theta_2^p|}{2} d\Omega_S} \quad (3.29)$$

Objective functions

Similar to the first model, objective functions are defined, as described in the optimization scenarios presented in section 3.1. Because of the new implementation of the density variables, and particularly by the multiplication of the densities, the ferromagnetic material and the winding material cannot mathematically coexist in the same location, as requested physically. This means, therefore, that the penalty coefficients are not required.

The two optimization functions that can be reused from the first model are therefore as follows:

OBJ1 maximises the total bearing stiffness over all design variables, including the iron density:

$$\text{OBJ1} : \max_{\theta_1^p, \theta_2^p} \left[\sum_i k_i(\theta_1^p, \theta_2^p) \right] \quad (3.30)$$

where $\sum_i k_i = k_{z,e} + k_{z,d} + k_{r,d}$.

OBJ3 maximises the electrodynamic stiffness alone. The detent stiffness contributions are thus excluded from the objective.

$$\text{OBJ3} : \max_{\theta_1^p, \theta_2^p} [k_{z,e}(\theta_1^p, \theta_2^p)] \quad (3.31)$$

Stator - Alternative Model: Results

To compare the two topological optimization models, the R4 rotor–S3 stator configuration will be considered for the objective functions OBJ1 and OBJ3. The values obtained for these two objective functions are presented in Table 3.12, and the optimized topologies

are shown in Figs.3.21 and 3.22, obtained for the objective functions OBJ1 and OBJ3, respectively.

Objective Function	First Model	Alternative Model
OBJ1 : $\sum_i k_i$ [N/mm]	2031.1	2012.3
OBJ3 : $k_{z,e}$ [N/mm]	3757.6	3841.7

Table 3.12: Comparison of the objective function values obtained with the two topology optimization models for the R4–S3 configuration.

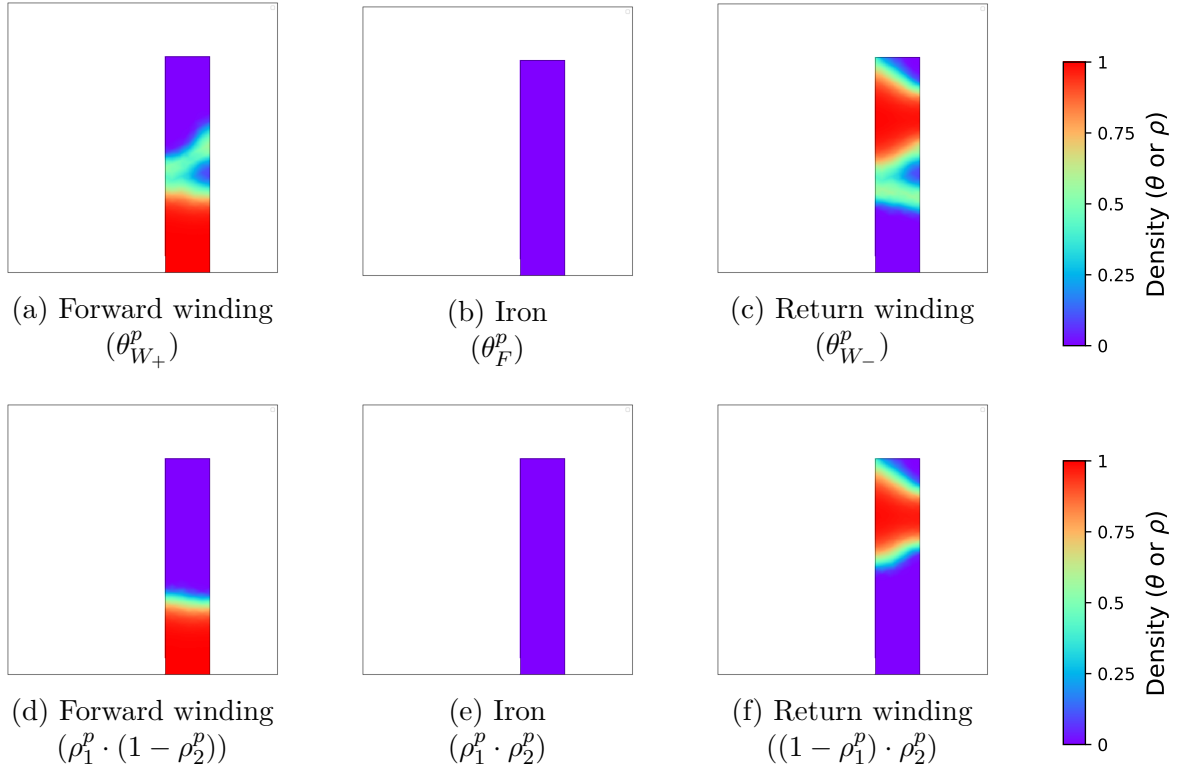


Figure 3.21: Comparison of optimized topologies obtained with OBJ1 for configuration R3–S4.

What is immediately apparent when reviewing these topological and numerical results is that the two models provide very similar results. In reality, the only difference lies in the fact that, in cases where materials cannot coexist, such as with forward and return windings and the ferromagnetic material in the context of the OBJ3 objective function, the alternative model is better at separating the components. This is why this topological optimization model achieves a better result for this objective function.

The reason the first model implemented gives better results for objective function OBJ1 is that the coexistence of forward and return windings in certain regions, as illustrated by the areas of intermediate density in Figures 3.21 (a) and (c)—allows the transition zones between the air and the winding to be more abrupt

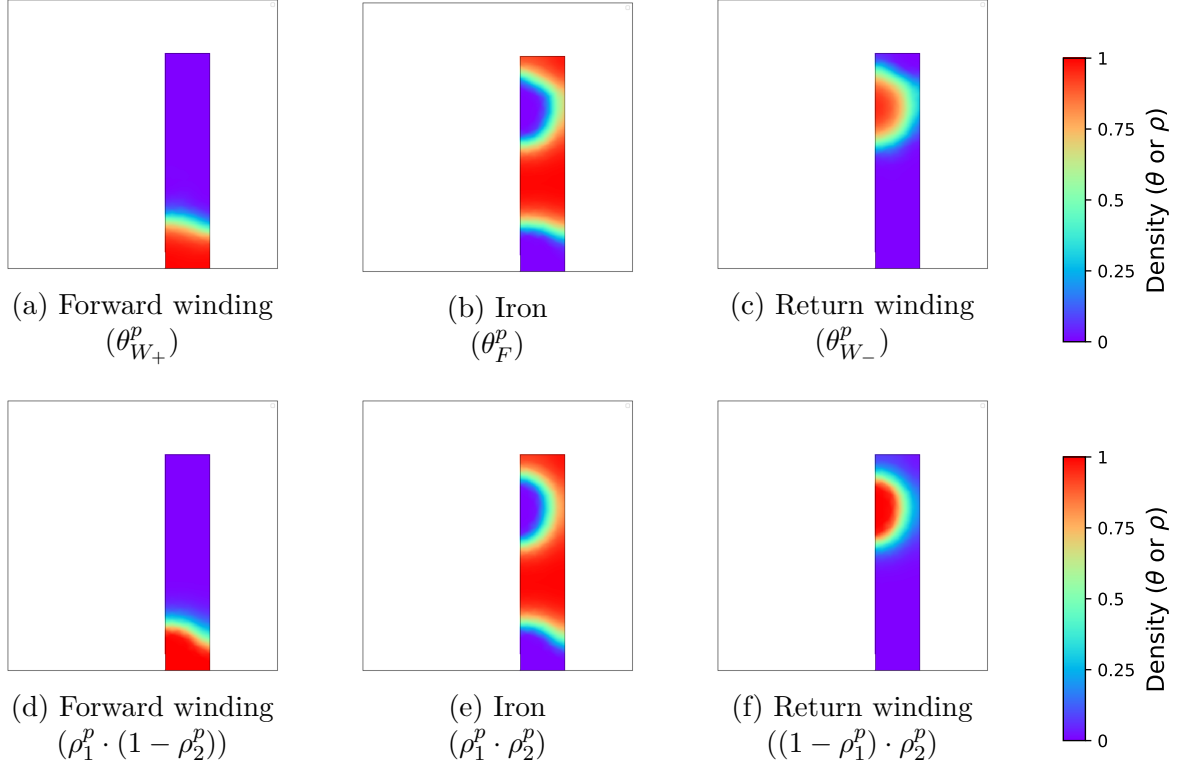


Figure 3.22: Comparison of optimized topologies obtained with OBJ1 for configuration R3-S4.

To conclude this section, it should be noted that other 2-density variable implementation are also possible for the stator topology optimization, as presented in Table 3.13:

Model Component	θ_1^p	θ_2^p
Forward winding	1	1
Ferromagnetic material	1	0
Return winding	0	1
Air	0	0

Table 3.13: The combination of different variables results in different model components.

Chapter 4

Rotor - Topology Optimisation

As discussed in Chapter 1, the range of rotor topologies studied in the literature remains limited. This chapter explores new rotor topologies through topology optimisation, with the aim of identifying configurations that improve the bearing function of Electrodynamic Thrust Self-Bearing Machines (EDTSB).

The chapter is organised as follows: Section 4.1 defines the optimisation scenarios considered, including the rotor configurations and the fixed stator topologies derived from the previous chapter. Section 4.2 details the topology optimisation model as implemented in COMSOL Multiphysics®, covering the density definitions, physics formulation and geometry. Section 4.3 presents and interprets the optimisation results for all configurations.

4.1 Rotor - Topology Optimization Scenarios

As shown by the sum of stiffness values presented in the stator topology optimization results (Section 3.3), the rotor configuration has a significant influence on bearing performance. This stems directly from the fact that the arrangement of the permanent magnets determines the orientation and spatial distribution of the magnetic flux in the air gap, which determines the flux linked by the upper and lower armature windings. Given that the literature offers only a limited number of rotor topologies, it is worth investigating whether other configurations might improve the bearing performance of the machine.

Based on the conclusions drawn in Chapter 3 regarding optimal stator topologies, the winding topologies identified and optimized are adopted as the starting point for rotor optimisation. Ferromagnetic materials are excluded from the stator, which eliminates any contribution from detent stiffness and ensures that the bearing function relies only on electrodynamic stiffness. Passive stabilisation in all other directions is provided by permanent magnet centring devices, as described in Section 2.2.

Three rotor configurations are considered, as illustrated in Figure 4.1. The Halbach-type configuration (R4 in Section 3.1) is not considered here due to its specific geometry and the resulting complexity it would introduce into a topological optimization model. It should also be noted that these three configurations are symmetric with respect to the central axial plane and that therefore, similar to the stator topological optimization, the rotor design area can be limited to the upper or lower part of the rotor. Furthermore, the iron yoke that serve both as the mechanical support for the permanent magnets and as

a low-reluctance return path for the magnetic flux, will have to be implemented in the Rotor Topology Optimization Model. The topology of the rotor iron yoke surrounding the armature windings will also be investigated.

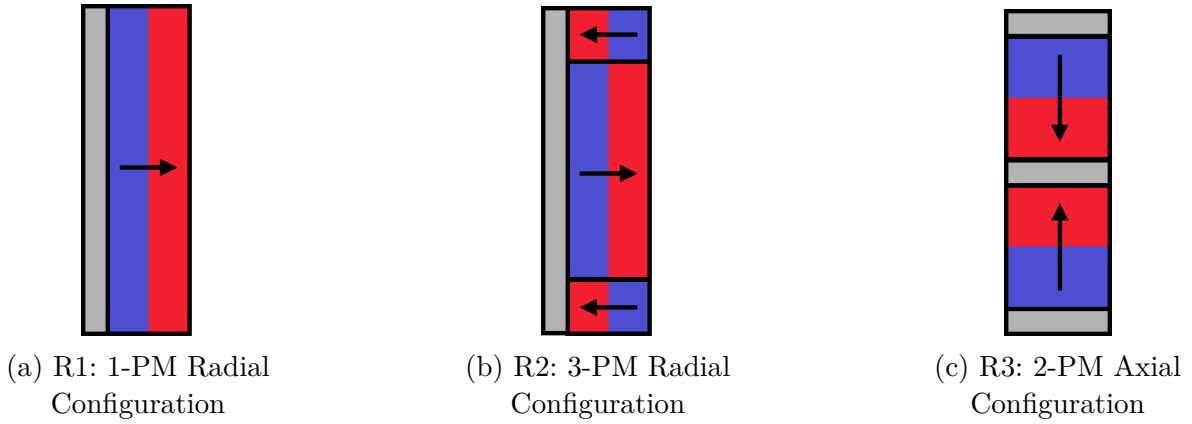


Figure 4.1: The three rotor configurations considered for rotor topology optimisation.

The fixed stator winding topologies, extracted from the stator topology optimisation results of Chapter 3, are shown in Fig. 4.2 for each of the three rotor configurations.

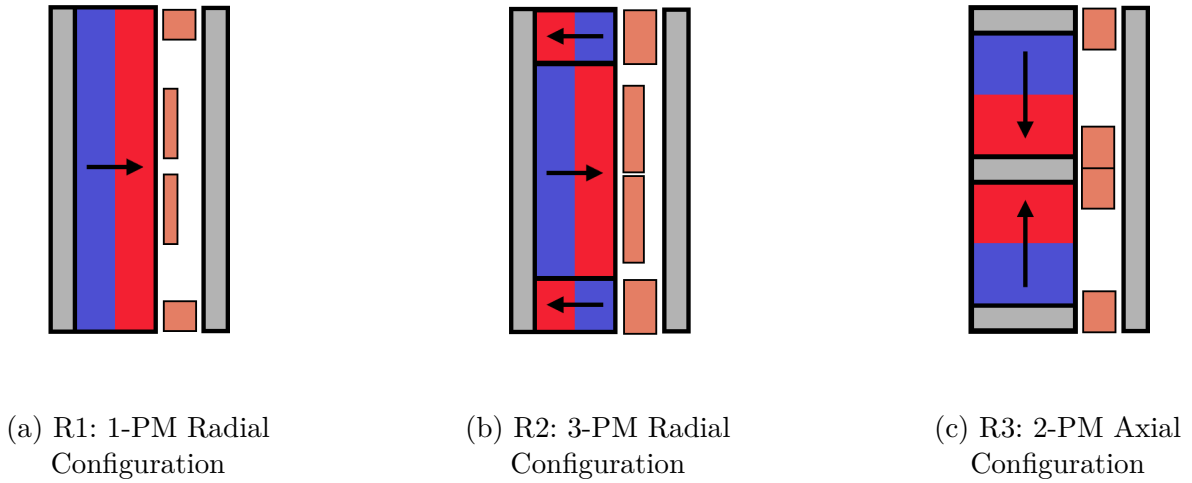


Figure 4.2: Fixed stator winding topologies used in the rotor optimisation model, derived from the stator topology optimisation results. The topology of the rotor iron yoke surrounding the armature will also be investigated.

4.2 Rotor - Topology Optimisation Model

The topological optimisation feature of the COMSOL Multiphysics® optimisation module is used. The rotor topology optimization model is presented in detail here, including definitions of the density field, the geometry, the physics model and its coupling to the optimisation densities, as well as the adaptation of the objective function to this model.

Density Definitions and Parameters

As described in Section 2.3, the optimisation is governed by penalised density fields serving as control variables. The number and nature of these fields depend on the rotor configuration considered.

For configurations R1 and R3, two model components are present in the rotor: the iron yoke and the permanent magnet. Two density fields are therefore defined in Table 4.1.

Model Component	Penalised Density
Ferromagnetic yoke	$\theta_F^p \in [0, 1]$
Permanent magnet	$\theta_{PM}^p \in [0, 1]$

Table 4.1: Density fields for rotor configurations R1 and R3.

The R2 configuration requires two distinct PM polarities ($\rightarrow$ and $\leftarrow$), and therefore three density fields, following the same logic used to distinguish the forward windings from the return windings in the first stator topological optimization model. These densities are defined in Table 4.2.

Model Component	Penalised Density
Ferromagnetic yoke	$\theta_F^p \in [0, 1]$
Permanent magnet ($\rightarrow$)	$\theta_{PM,\rightarrow}^p \in [0, 1]$
Permanent magnet ($\leftarrow$)	$\theta_{PM,\leftarrow}^p \in [0, 1]$

Table 4.2: Density fields for rotor configuration R2.

All density fields are initialised at 0.5, a neutral starting point, and the same SIMP penalisation exponent $p = 5$ is applied uniformly to all fields, ensuring consistent penalisation behaviour across all components. The full set of initialisation parameters is summarised in Table 4.3.

Parameter	Symbol	Value
SIMP penalisation exponent	p	5
<i>Configurations R1 and R3</i>		
Ferromagnetic yoke initial density	$\theta_{F,0}^p$	0.5
Permanent magnet initial density	$\theta_{PM,0}^p$	0.5
<i>Configuration R2</i>		
Ferromagnetic yoke initial density	$\theta_{F,0}^p$	0.5
Permanent magnet ($\rightarrow$) initial density	$\theta_{PM,\rightarrow,0}^p$	0.5
Permanent magnet ($\leftarrow$) initial density	$\theta_{PM,\leftarrow,0}^p$	0.5

Table 4.3: Density initial values and SIMP exponent for the rotor optimisation model.

Geometry and Parameters Estimation

The model is formulated in two dimensions because three-dimensional topological optimization would result in excessive computational costs. This choice of dimensions for a radial-flux machine means that the model represents a single phase and a single pole pair.

Since the stator contains no ferromagnetic material, the detent stiffnesses are zero. Therefore, it is not necessary to calculate the radial and axial detent forces. Thus, only two magnetic zones are required to calculate the three physical quantities used to evaluate the electrodynamic stiffness: the flux gradient K_z , the cyclic inductance L_c , and the winding resistance R .

- **Magnetic zone (1):** $\Delta z = 0$, $\Delta r = 0$, $\mathbf{J}_e$ on, $\mathbf{B}_{r,PM}$ on
- **Magnetic zone (2):** $\Delta z = 0$, $\Delta r = 0$, $\mathbf{J}_e$ on, $\mathbf{B}_{r,PM}$ off

where Δz and Δr denote the axial and radial displacements of the rotor relative to the central position, $\mathbf{J}_e$ indicates whether current is flowing through the windings or not, and $\mathbf{B}_{r,PM}$ indicates whether the permanent magnet remanent magnetisation is active. The procedure for estimating the physical quantities follows the same steps as in the stator topology optimization model (Section 3.2). The only difference is that the winding resistance per unit depth is calculated directly from the winding's cross-sectional area S , since there is no density field in the winding region:

$$R = \frac{\rho}{S} \quad (4.1)$$

where ρ is the electrical resistivity of the conductor.

Since all calculations are performed in two dimensions, all physical quantities are expressed per unit of depth. For the reader's understanding, all values given are multiplied by a reference depth of 1 m.

The available symmetries of the various configurations, which were previously identified in the stator optimization model, are used to reduce the computational domain. The density variables are defined on a single design domain Ω_R (see Fig. 4.3) and mapped into all other rotor zones. Infinite element domains are applied to the outer boundaries to ensure correct far-field behaviour. It should be noted that the key parameters defining the geometry are taken from the stator's topological optimization model.

Physics

The *Magnetic Fields* physics interface in COMSOL Multiphysics® is used. The governing equations are derived from Maxwell's equations, expressed in terms of the magnetic field strength $\mathbf{H}$, the magnetic flux density $\mathbf{B}$, and the magnetic potential vector $\mathbf{A}$:

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad \mathbf{B} = \nabla \times \mathbf{A} \quad (4.2)$$

In a magnetostatic simulation without an induced electric field, the current density $\mathbf{J}$ reduces to the external contribution $\mathbf{J}_e$ alone, that is, the current flowing through the

windings, which is defined with opposite signs in the forward and reverse winding regions:

$$\mathbf{J}_e = +\|J\| \cdot N_w \cdot \mu_{f.f.} \quad (\text{forward winding}), \quad \mathbf{J}_e = -\|J\| \cdot N_w \cdot \mu_{f.f.} \quad (\text{return winding}) \quad (4.3)$$

Two boundary conditions are applied over the domain, as illustrated in Fig. 4.3: magnetic insulation on the outer boundary, and a symmetry condition where applicable:

$$\mathbf{n} \times \mathbf{A} = 0 \quad (\text{magnetic insulation}), \quad \mathbf{n} \cdot \mathbf{B} = 0 \quad (\text{symmetry}) \quad (4.4)$$

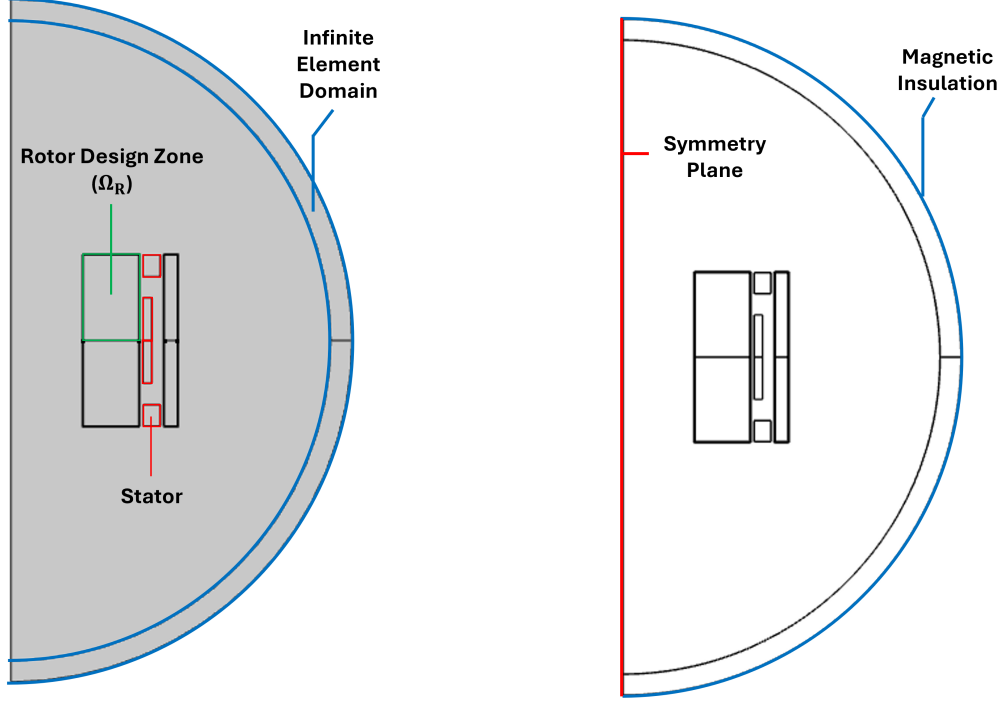


Figure 4.3: **Magnetic zone (1)**: is illustrated, with its components: rotor design domain Ω_R , fixed stator windings, and Infinite Element Domains. The $r = 0$ symmetry conditions reduce the domain by a factor of two, lowering the total mesh element count

Coupling with Densities

As explained in Section 2.4, the permanent magnet used in the model is defined by a linear constitutive relation:

$$\mathbf{B} = \mu_0 \mu_{\text{rec}} \mathbf{H} + \|B_r\|_{\text{eff}} \frac{\mathbf{e}}{\|\mathbf{e}\|} \quad (4.5)$$

where μ_{rec} is the recoil permeability, $\|B_r\|_{\text{eff}}$ is the remanent flux density norm, and $\frac{\mathbf{e}}{\|\mathbf{e}\|}$ is the magnetisation direction. However, the remanent flux density norm should be interpolated from the density fields to match the rotor optimization scenarios as follows.

For configurations R1 and R3, a single PM polarity is present in the rotor design domain:

$$\|B_r\|_{\text{eff}} = \|B_r\| \cdot \theta_{PM}^p. \quad (4.6)$$

The magnetisation direction is prescribed as $(\hat{r}, \hat{z}) = (1, 0)$ for R1, and respectively as $(0, -1)$ and $(0, +1)$ for the upper and lower halves of the rotor in R3.

For configuration R2, two opposing PM polarities coexist within the design domain. The remanent flux density norm is expressed as:

$$\|B_r\|_{\text{eff}} = \|B_r\| \cdot (\theta_{PM,\rightarrow}^p - \theta_{PM,\leftarrow}^p), \quad (4.7)$$

with a fixed magnetisation direction of $(\hat{r}, \hat{z}) = (1, 0)$. When the difference of $(\theta_{PM,\rightarrow}^p - \theta_{PM,\leftarrow}^p)$ equals $+1$, the material behaves as a PM magnetised in the positive r -direction. When it equals -1 , the material behaves as a PM magnetised in the negative r -direction. When the difference is zero, the location is occupied by either air or iron.

To take into account the ferromagnetic materials in the rotor design region, the relative magnetic reluctivity ν_r is interpolated between the air value ($\nu_{r,\text{air}} = 1$) and the local ferromagnetic value $\nu_{r,F}$ as a function of the iron density θ_F^p :

$$\nu_r = \nu_{r,\text{air}} + (\nu_{r,F} - \nu_{r,\text{air}}) \theta_F^p, \quad \mathbf{B} = \mu_0 \frac{1}{\nu_r} \mathbf{H} \quad (4.8)$$

However, in order to avoid overestimating the contribution of iron to the magnetic circuit in this model, the material saturation is taken into account, and the reluctivity value $\nu_{r,F}$ is therefore not treated as a constant but varies as a function of the local magnetic field norm $\|\mathbf{B}\|$.

The following expression for the reluctivity of the ferromagnetic material is therefore used to account for the material's saturation:

$$\nu_{r,F}(\|\mathbf{B}\|) = \left[\nu_{r,\text{air}} + \frac{\left(\frac{1}{\nu_{r,F,0}} \right) - 1}{1 + \left(\frac{\|\mathbf{B}\|}{B_0} \right)^n} \right]^{-1} \quad (4.9)$$

where $\nu_{r,F,0}$ is the relative reluctivity of the ferromagnetic material without saturation, B_0 and n are two parameters used to fit the nonlinear B - H curve of the *Soft Iron* material, as illustrated in Fig. 4.4. Although magnetic saturation using this formula does not exactly correspond to the saturation part of the B - H curve used for ferromagnetic materials, this expression ensures that as $\|\mathbf{B}\|$ approaches 0, the value of $\nu_{r,F}$ is $\nu_{r,F,0}$, and that as $\|\mathbf{B}\| \gg B_0$, the value of $\nu_{r,F}$ is $\nu_{r,\text{air}}$ (required behavior when the material reaches saturation). The relative reluctivity of the permanent magnet material is close to unity and is therefore treated as equivalent to air in the reluctivity interpolation.

Objective Functions

Two objective functions are defined, as configuration R1 and R3 and configuration R2 uses two different sets of densities variables. Since the ferromagnetic material and the permanent magnet material cannot physically coexist in the same location, a penalty term must be included in both objective functions. This coexistence constraint between θ_F^p and each permanent magnet density is enforced using the following normalized penalty coefficients:

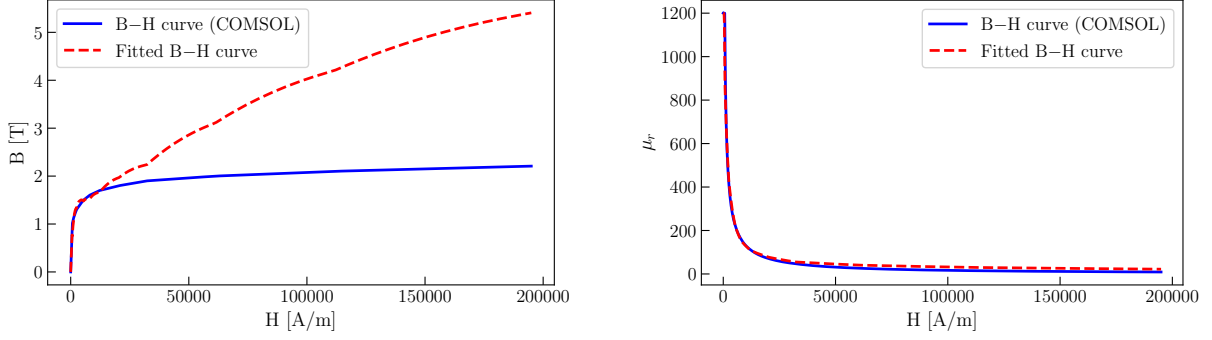


Figure 4.4: B – H curves for Soft Iron (Without Losses) as implemented in COMSOL together with the fitted model. The fitted parameters are: $B_0 = 1.1870$ [T] and $n = 6.4821$

$$P_0(\theta_F^p, \theta_{PM}^p) = \frac{\int_{\Omega_R} \max(0, \theta_F^p + \theta_{PM}^p - 1) d\Omega_R}{\int_{\Omega_R} d\Omega_R} \quad (4.10)$$

$$P_1(\theta_F^p, \theta_{PM,\rightarrow}^p) = \frac{\int_{\Omega_R} \max(0, \theta_F^p + \theta_{PM,\rightarrow}^p - 1) d\Omega_R}{\int_{\Omega_R} d\Omega_R} \quad (4.11)$$

$$P_2(\theta_F^p, \theta_{PM,\leftarrow}^p) = \frac{\int_{\Omega_R} \max(0, \theta_F^p + \theta_{PM,\leftarrow}^p - 1) d\Omega_R}{\int_{\Omega_R} d\Omega_R} \quad (4.12)$$

P_0 , P_1 and P_2 are dimensionless quantities that are equal to zero when there is no overlap between the density fields of the iron and the PM, and that increase in proportion to the degree of overlap. The penalty coefficient ($\lambda = 10^4$ N/mm) has the unit N/m, which makes it compatible with stiffness terms. Its value is chosen sufficiently large to effectively prevent any overlap of materials, without being so high as to dominate the objective function and hinder convergence toward an optimal topology.

It should be noted that no penalty is required to prevent the overlap of the two PM densities, since the superposition of $\theta_{PM,\rightarrow}^p$ and $\theta_{PM,\leftarrow}^p$ at the same point results in the cancellation of the PM remanent flux, which means that either air or iron is present.

The following objective function maximises the total stiffness over all densities for configuration R1 and R3:

$$\text{OBJ} : \max_{\theta_F, \theta_{PM}} \left[\sum_i k_i(\theta_F, \theta_{PM}) - \lambda P_0(\theta_F, \theta_{PM}) \right] \quad (4.13)$$

where $\sum_i k_i = k_{z,e}$ (No ferromagnetic material at the stator).

The following objective function maximises the total bearing stiffness over all densities for configuration R2:

$$\text{OBJ : } \max_{\theta_F, \theta_{PM, \rightarrow}, \theta_{PM, \leftarrow}} \left[\sum_i k_i(\theta_F, \theta_{PM, \rightarrow}, \theta_{PM, \leftarrow}) - \lambda [P_1(\theta_F, \theta_{PM, \rightarrow}) + P_2(\theta_F, \theta_{PM, \leftarrow})] \right] \quad (4.14)$$

4.3 Rotor - Topology Optimization Results

This section presents the results of the rotor topology optimization. A specific subsection is devoted to each rotor configuration considered in the optimization scenarios, following a consistent structure. The optimized density fields are first presented and interpreted, and then a manufacturable topology is extracted. A parametric optimization is then performed on this extracted topology. The final stiffness values obtained for each optimal rotor topology are also provided, in order to observe the improvement made at each stage.

4.3.1 Results: Rotor Configuration R1

Topology Optimisation Results

The optimised density fields obtained after topology optimisation are shown in Fig. 4.5.

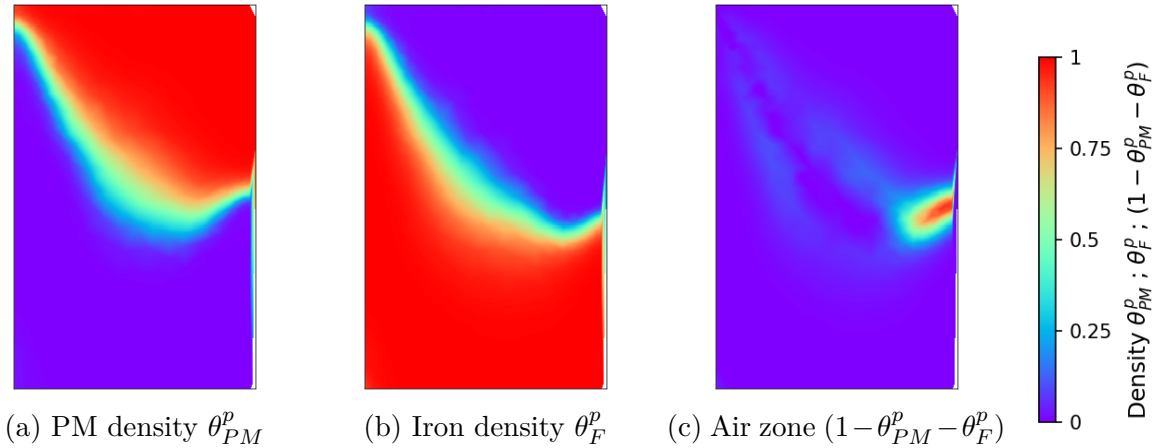


Figure 4.5: Optimised rotor density fields, configuration R1.

The permanent magnet and the ferromagnetic material occupy two distinct regions in space, confirming that the penalty term introduced into the objective function effectively prevents the coexistence of the materials. The PM is located in the upper part of the design zone, where the ferromagnetic material completes the design zone by surrounding the PM. It is worth noting the presence of a small air gap located at the centre of the right end of the design zone.

Topology Extraction

Based on the optimized density distribution, we obtain the following manufacturable topology: a permanent magnet with a rectangular cross-section (i.e., a rectangle in the 2D model), surrounded by a ferromagnetic yoke that occupies the remainder of the design space. Due to its small size, the air gap is neglected. This topology is parameterized by two geometric parameters, h_{PM} and w_{PM} , which denote the height and width of the magnet's cross-section, respectively, as illustrated in Figure 4.6.

Parametric Optimization Model

Parametric optimization uses the same objective function as topological optimization, but the density fields are replaced by geometric parameters as control variables. The parametric optimisation problem is formulated as:

$$\max_{\mathbf{p}} \quad \sum_i k_i(\mathbf{p}) = k_{z,e}(\mathbf{p}) \quad (4.15)$$

where the vector of control variables is:

$$\mathbf{p} = (h_{PM}, w_{PM}) \in \mathbb{R}_+^2 \quad (4.16)$$

subject to the geometric constraint:

$$0 < h_{PM} < h_R, \quad 0 < w_{PM} < w_R \quad (\text{PM within rotor design domain}) \quad (4.17)$$

The initial and optimal values of the parameters are provided in Table B.1 in Appendix B. The geometry before and after parametric optimization is illustrated in Figure 4.6.

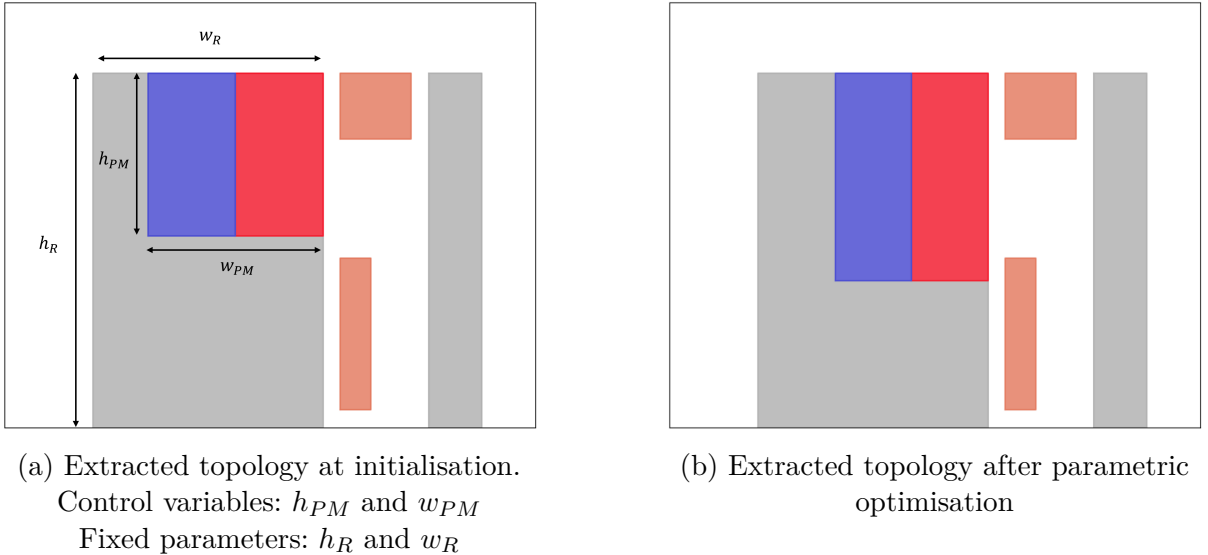


Figure 4.6: Parametric optimisation of the extracted rotor topology, configuration R1.

The electrodynamic stiffness values obtained following topological and parametric optimization will be discussed in Section 4.3.4.

4.3.2 Results: Rotor Configuration R2

Topology Optimisation Results

The optimised density fields obtained after topology optimisation are shown in Fig. 4.7. The absolute value used to illustrate the topological result of the air zone serves to highlight the areas where the cancellation of the residual PM remanent flux occurs.

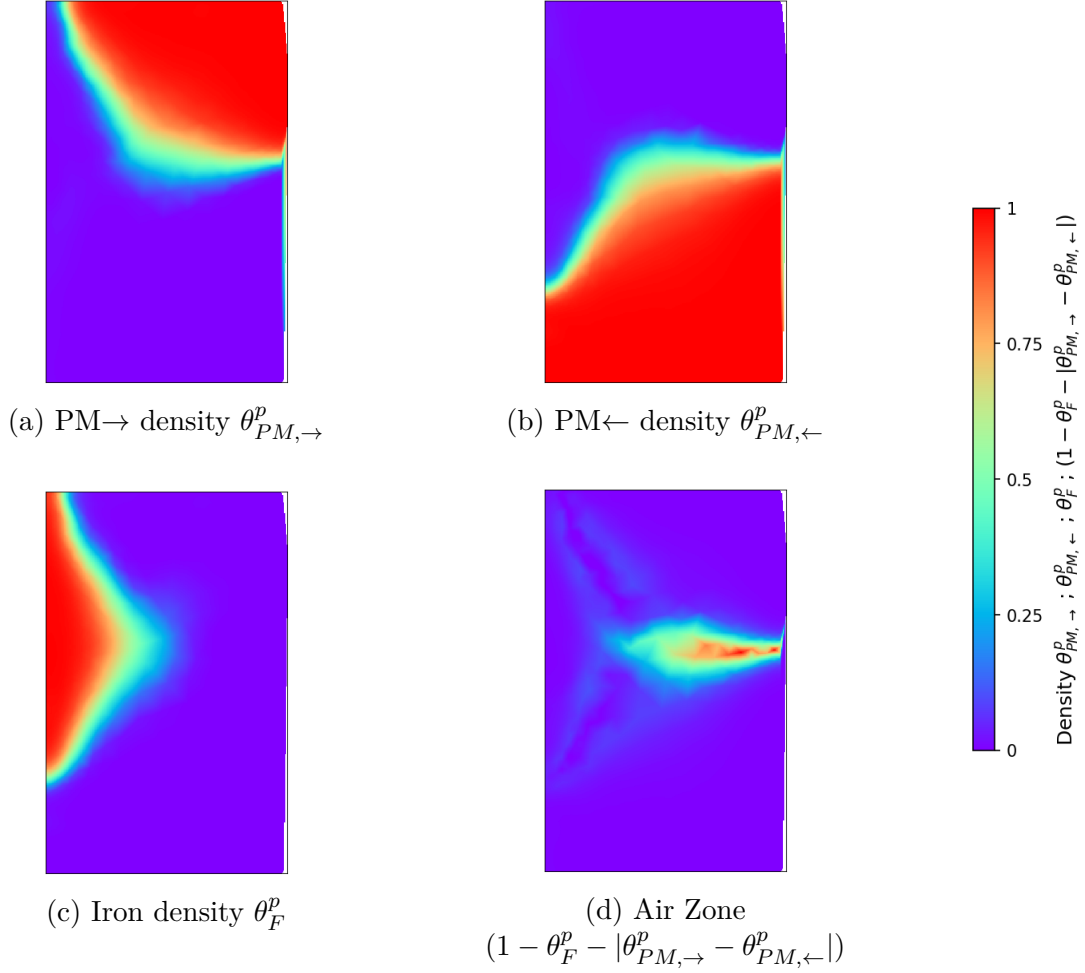


Figure 4.7: Optimised rotor density fields, configuration R2.

The two permanent magnets with two opposite magnetization direction and the ferromagnetic material occupy clear distinct regions in space. The PM magnetised in the positive r -direction is located in the upper right corner of the design zone, whereas the PM magnetised in the negative r -direction is located in the lower right corner. The ferromagnetic material lies at the back of the two PMs in a triangular shape. Additionally, the air zone appearing in Fig.4.7 results from the transition zone between the densities of the two polarities of the permanent magnets

Topology Extraction

Based on the optimized density distribution, we obtain the following manufacturable topology: two permanent magnet with opposite polarities with rectangular cross-sections, backed by a triangular shaped ferromagnetic yoke. To simplify manufacturing, the two

PMs with different magnetization directions are considered to be symmetric. This topology is thus parametrized by one geometric parameter, h_Δ , which refer to the height of the triangle-shaped back iron as illustrated in Fig. 4.8(a).

Parametric Optimization Model

Parametric optimization model uses the same objective function as topological optimization, but the design zones with density fields are replaced by geometries depending on control variables. The parametric optimization problem is formulated as:

$$\max_p \sum_i k_i(p) = k_{z,e}(p) \quad (4.18)$$

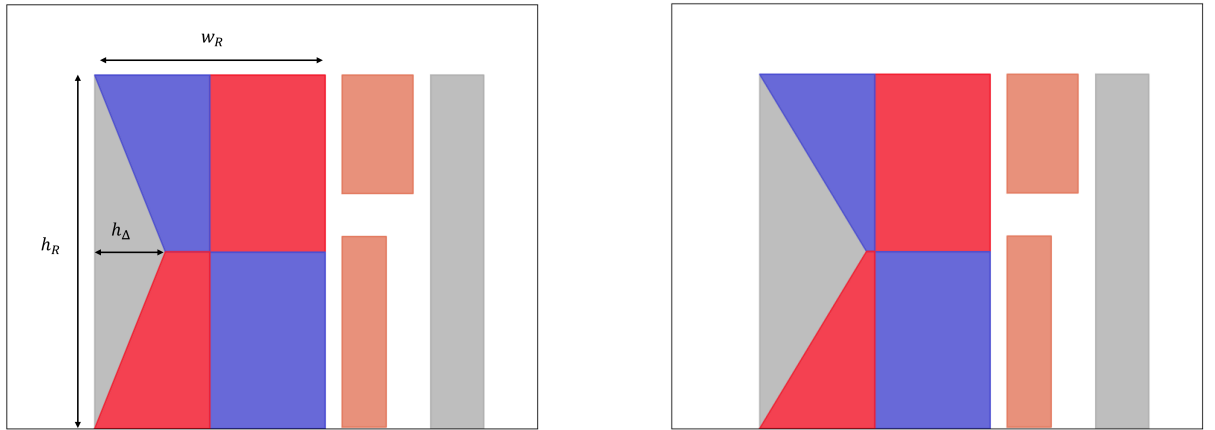
where the control variable is:

$$p = h_\Delta \in \mathbb{R}_+ \quad (4.19)$$

subject to the geometric constraint:

$$0 < h_\Delta \leq w_R \quad (\text{PM within rotor design domain}) \quad (4.20)$$

The initial and optimal values of the parameters are provided in Table B.2 in Appendix B. The geometry before and after parametric optimization is illustrated in Figure 4.8.



(a) Extracted topology at initialisation. Control parameters: $h_{PM,\rightarrow}$, $h_{PM,\leftarrow}$, w_{IR1} and w_{IR2}

(b) Extracted topology after parametric optimisation.

Figure 4.8: Parametric optimisation of the extracted rotor topology, configuration R1.

The electrodynamic stiffness values obtained following topological and parametric optimization will be discussed in section 4.3.4.

4.3.3 Results: Rotor Configuration R3

Topology Optimisation Results

The optimised density fields obtained after topology optimisation are shown in Fig. 4.9.

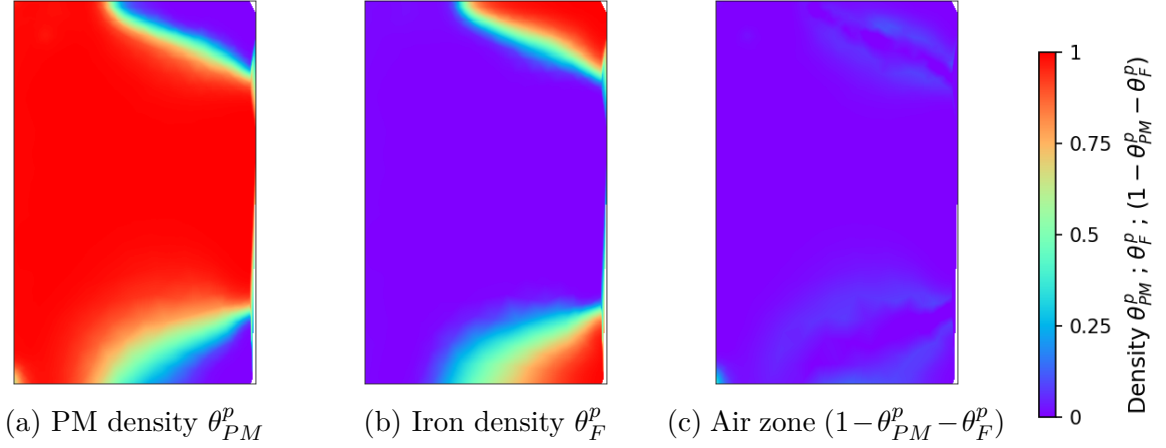


Figure 4.9: Optimised rotor density fields, configuration R3.

The permanent magnet and the ferromagnetic material occupy two spatially distinct regions. The ferromagnetic material is located at the upper and lower right corners of the rotor design domain, while the permanent magnet fills the central region. However, this triangular ferromagnetic topologies raises a practical manufacturability problem. Indeed, the R3 rotor configuration uses two permanent magnets with opposing axial magnetisation directions $((0, +1)$ and $(0, -1)$). Without iron yokes to separate and support them, their assembly would not be feasible. For this reason, rectangular-section iron yokes at the upper and lower boundaries of the design domain are chosen for the topology extraction.

Topology Extraction

The extracted manufacturable topology consists of two rectangular iron yokes located at the upper and lower boundaries of the rotor design domain, with the permanent magnet that occupies the remainder of the design space. This topology is parametrised by two geometric variables: h_{F1} , the height of the upper iron yoke, and h_{F2} , the height of the lower iron yoke, as illustrated in Fig. 4.10.

Parametric Optimisation

Parametric optimization uses the same objective function as topological optimization, but the density fields are replaced by geometric parameters as control variables. The parametric optimisation problem is formulated as:

$$\max_{\mathbf{p}} k_{z,e}(\mathbf{p}) \quad (4.21)$$

where the vector of control variables is:

$$\mathbf{p} = (h_{F1}, h_{F2}) \in \mathbb{R}_+^2 \quad (4.22)$$

subject to the geometric constraint:

$$0 < h_{F1} + h_{F2} < h_R \quad (\text{Rotor iron yokes within design domain}) \quad (4.23)$$

The initial and optimal values of the parameters are provided in Table ?? in Appendix B. The geometry before and after parametric optimization is illustrated in Figure 4.10.

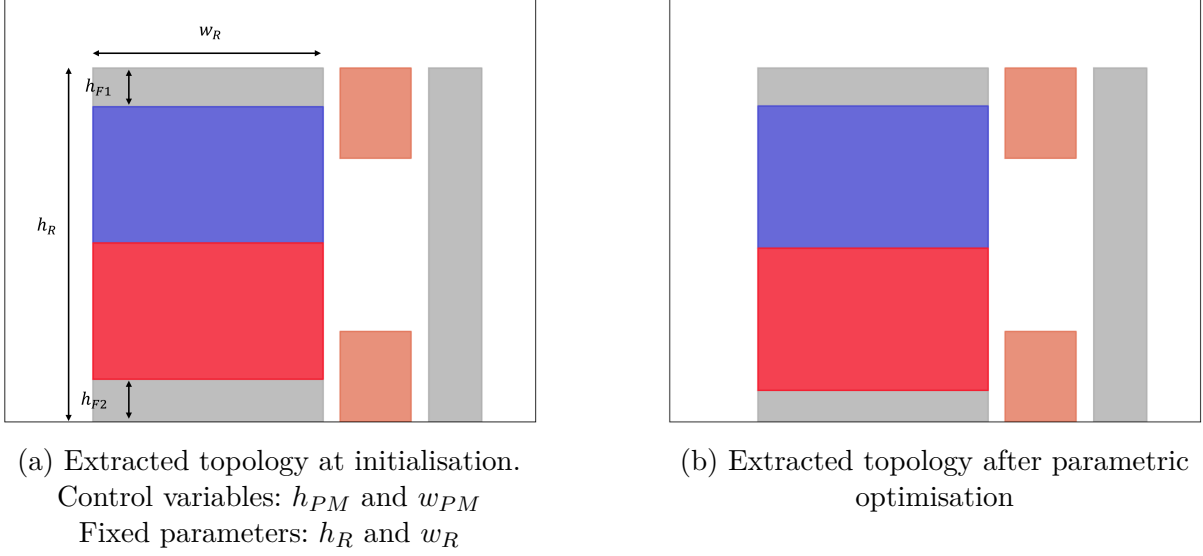


Figure 4.10: Parametric optimisation of the extracted rotor topology, configuration R3.

4.3.4 Summary of Results

Table 4.4 reports the electrodynamic stiffness at each optimization step of the rotor topology optimization section, for all rotor configurations, together with the relative improvement at each step.

	Stator	Rotor	Rotor	Rotor vs Stator
	Param. Opt.	Topology Opt.	Param. Opt.	improvement
	$\left[\frac{N}{mm}\right]$	$\left[\frac{N}{mm}\right]$	$\left[\frac{N}{mm}\right]$	$[\%]$
R1–S3	46.270	430.72	338.71	+632%
R2–S3	1129.1	1320.6	1357.0	+20.2%
R3–S3	1505.4	1525.8	1513.0	+0.50%
Relative improvement: rotor topology optimization $\rightarrow$ rotor parametric optimization				
R1–S3		430.72	338.71	–21.4%
R2–S3		1320.6	1357.0	+2.76%
R3–S3		1525.8	1513.0	–0.84%

Table 4.4: Electrodynamic stiffness at each optimisation step for all rotor configurations.

4.4 Rotor - Iron Yoke Surrounding the Armature: Topology Optimisation

In the rotor optimisation scenarios defined in Section 4.1, it was noted that the topology of the rotor iron yoke surrounding the armature should also be investigated. However, in the topology optimisation models implemented in Section 4.2, this yoke was assigned a fixed

reference geometry: a rectangle of width 0.3 cm and height 4 cm in the two-dimensional model.

To address this, the model of Section 4.2 is extended by introducing an additional density variable ($\theta_{F,aY}$) representing the ferromagnetic material within a supplementary design domain corresponding to the defined rectangle of the rotor yoke surrounding the armature. This brings the total number of density variables to three for rotor configurations R1 and R3, and to four for configuration R2. The optimised density distributions obtained for this additional design zone are shown in Fig. 4.11.

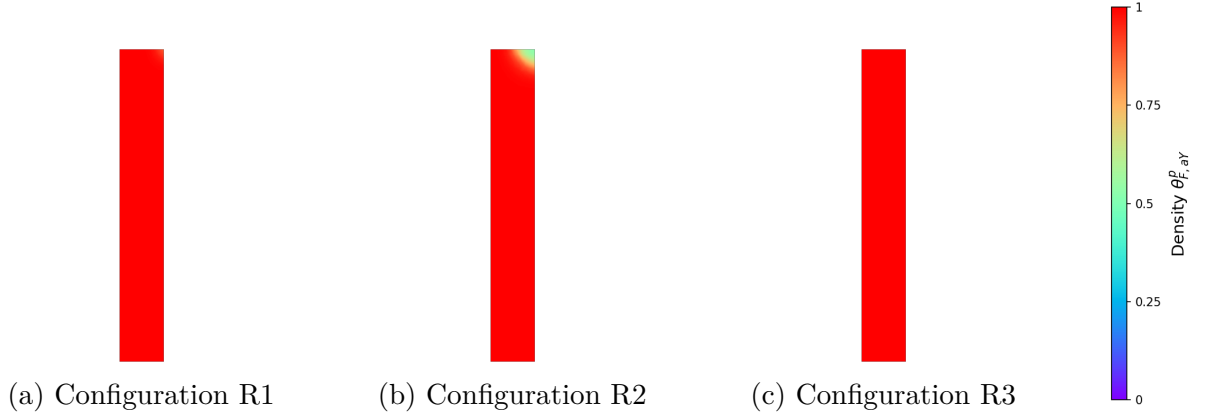


Figure 4.11: Optimised topology of the rotor ferromagnetic yoke surrounding the armature for configurations R1, R2, and R3. The iron density fills the entire available design zone in all cases, confirming that no topology distinct from the reference geometry is identified.

In all three configurations, the iron density variable distributes uniformly across the entire design zone, indicating that the optimiser finds no benefit in removing material from this region. The reference rectangular topology therefore already constitutes the optimal geometry for the rotor yoke surrounding the armature.

Chapter 5

Combined Stator–Rotor Parametric Optimization

The previous two chapters have demonstrated that manufacturable stator and rotor topologies can be derived from topology optimization results. In particular, it was established that introducing ferromagnetic material into the stator provides no benefit to the total bearing stiffness, such that the contributions to stiffness due to the detent effect are null and the total stiffness is reduced to electrodynamic stiffness alone. At the same time, new rotor topologies have been identified to improve electrodynamic stiffness by producing more favourable orientations and distributions of the magnetic field in the airgap.

Performing a simultaneous stator-rotor topology optimization is not feasible, as the large number of density variables leads to significant convergence difficulties. However, once feasible geometries have been identified independently for both the stator and the rotor, a combined parametric optimization becomes feasible. This chapter therefore presents a parametric optimization in which the stator winding geometry and the rotor permanent magnet and iron yoke geometry are optimized simultaneously, using the topologies extracted in Chapters 3 and 4 as the starting point.

The chapter is organised as follows. Section 5.1 defines the optimisation scenarios and presents the combined stator–rotor topologies used as initial configurations.

5.1 Optimisation Scenarios

The stator topology optimization demonstrated that the S3 configuration, being the narrow winding zone, with a rotor iron yoke surrounding the armature, offers the best results in terms of electrodynamic stiffness for the machine considered, while remaining physically representative of a three-dimensional configuration. Rectangular winding cross-sections were extracted from this stator topology and then optimised parametrically optimized for each rotor configuration, resulting in an optimal winding arrangement for each case.

The rotor topology optimization, performed using the stator topologies derived above as fixed inputs, demonstrated that ferromagnetic material and permanent magnets can be optimally arranged within the rotor design domain for configurations R1, R2, and R3. These rotor topologies were extracted and optimised parametrically. The resulting combined stator–rotor configurations, which serve as the starting point for the this chapter,

are illustrated in Fig. 5.1.

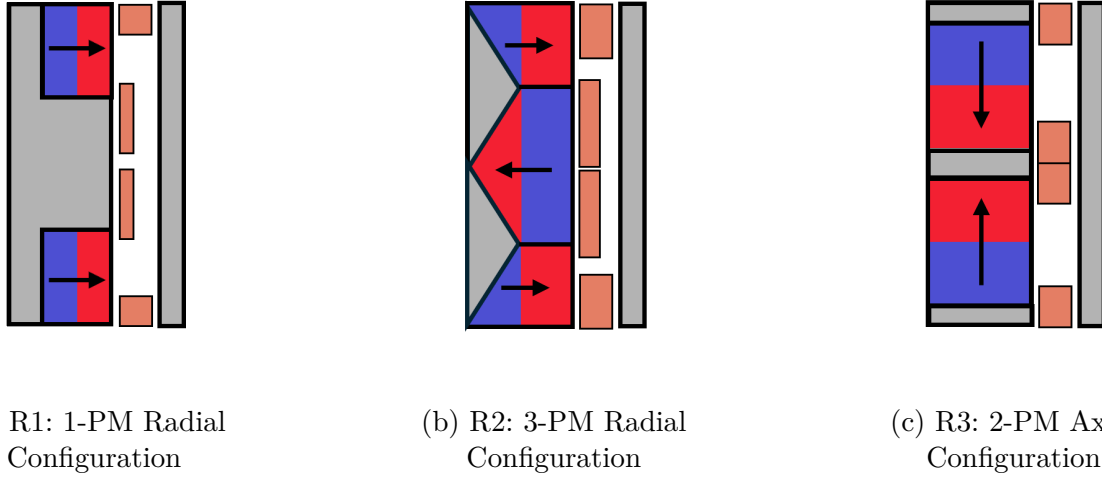


Figure 5.1: Combined stator-rotor starting topologies for the parametric optimisation, derived from the stator and rotor topology optimisation results of Chapters 3 and 4.

A combined parametric optimization is performed for each of these three configurations to determine whether the combined optimization of all stator and rotor topology parameters offers a further improvement in electrodynamic stiffness compared to the stiffness results achieved by the previous optimizations performed independently.

5.2 Parametric Optimization Models

5.2.1 Configuration Rotor R1– Stator S3

The parametric model combines the stator and rotor parametric models developed in Chapters 3 and 4. The rotor design domain is divided into two material zones: a permanent magnet region and a ferromagnetic yoke filling the remainder of the domain. The shape of the permanent magnet within this domain is controlled by two parameters, h_{PM} and w_{PM} , which serve as the rotor control variables.

The stator winding geometry comprises two rectangular winding cross-sections: the forward winding (W_+) and the return winding (W_-), each defined by four parameters: a width and a height (w_{W_+} , h_{W_+} , w_{W_-} , h_{W_-}) and the coordinates of its centre (x_{W_+} , y_{W_+} , x_{W_-} , y_{W_-}). The initial parametric configuration is illustrated in Fig. 5.2(a).

Since no ferromagnetic material is considered in the stator, the parametric optimization problem is formulated as:

$$\max_{\mathbf{p}} k_{z,e}(\mathbf{p}) \quad (5.1)$$

where the vector of control variables is:

$$\mathbf{p} = (w_{PM}, h_{PM}, w_{W_+}, w_{W_-}, h_{W_+}, h_{W_-}, x_{W_+}, y_{W_+}, x_{W_-}, y_{W_-}) \in \mathbb{R}_+^9 \quad (5.2)$$

The control variables are subject to certain geometric constraints that ensure that the winding zones remain within the stator design region (Ω_S) and do not overlap, but also

that the rotor components stay within the rotor design region (Ω_R).

$$0 < w_{PM} < w_R, \quad 0 < h_{PM} < h_R \quad (\text{PM within rotor design domain}) \quad (5.3)$$

$$x_{W_+} - \frac{1}{2}w_{W_+} \geq x_{S,\min}, \quad x_{W_+} + \frac{1}{2}w_{W_+} \leq x_{S,\max} \quad (5.4)$$

$$x_{W_-} - \frac{1}{2}w_{W_-} \geq x_{S,\min}, \quad x_{W_-} + \frac{1}{2}w_{W_-} \leq x_{S,\max} \quad (5.5)$$

$$y_{W_-} - \frac{1}{2}h_{W_-} \geq y_{S,\min}, \quad y_{W_+} + \frac{1}{2}h_{W_+} \leq y_{S,\max} \quad (5.6)$$

$$y_{W_-} + \frac{1}{2}h_{W_-} \leq y_{W_+} - \frac{1}{2}h_{W_+} \quad (\text{no overlap between } W_+ \text{ and } W_-) \quad (5.7)$$

where $x_{\Omega_S,\min}$, $x_{\Omega_S,\max}$, $y_{\Omega_S,\min}$, and $y_{\Omega_S,\max}$ define the boundaries of the stator design domain Ω_S , and w_R and h_R define the dimensions of the rotor design domain Ω_R . In order to maintain equality between the cross-sectional areas of the forward and return windings, which is necessary for the net forward and return currents to be equal, the height of W_+ is not an independent variable, but is derived from:

$$h_{W_+} = \frac{h_{W_-} \cdot w_{W_-}}{w_{W_+}} \quad (5.8)$$

Results

The combined stator-rotor geometry before and after parametric optimisation is shown in Fig. 5.2.

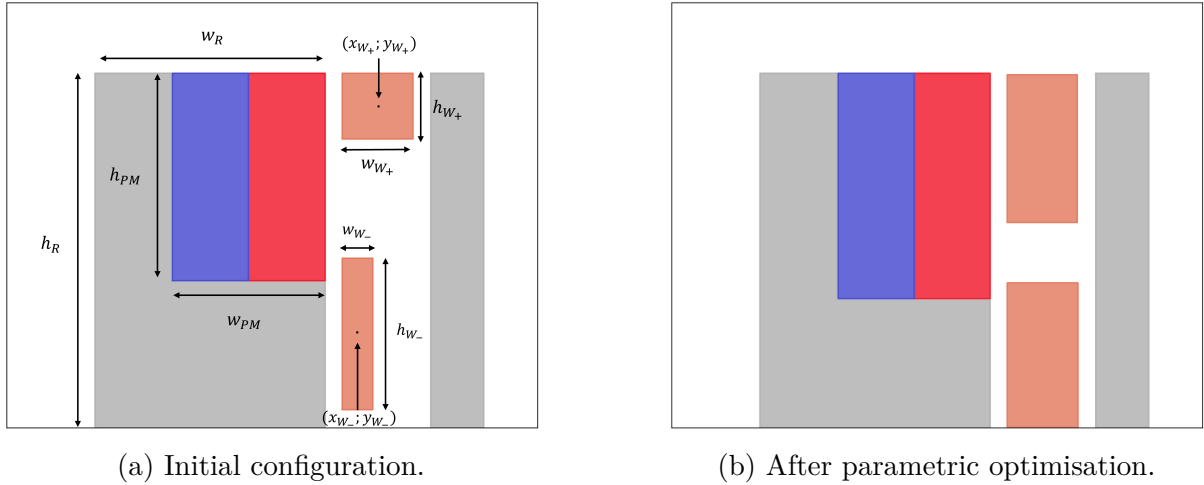


Figure 5.2: Combined stator-rotor parametric optimisation, configuration R1-S3.

Chapter 6

Conclusion and Future Work

This thesis begins with the observation that the stator and rotor topologies currently used in the Electrodynamical Thrust Self-Bearing Machines (EDTSBMs) are relatively conventional. The thesis objective was therefore to apply topological optimization methods to facilitate the design of new, more efficient topologies for EDTSBMs, focusing on performance criteria related to the machine's bearing function. The difficulty lies in the fact that, given the complex nature of the phenomena involved, the design of these machines is a challenging task, particularly when the presence of a magnetic circuit is taken into account.

One of the primary objectives of this thesis was to evaluate whether the impact of ferromagnetic materials was beneficial for the bearing function of the machine under study, as these materials can generate detent forces acting on the rotor and influence the magnetic circuit, and thus the electrodynamic stiffness.

To this end, a topological optimization method was applied to the stator, using three different material densities for each component of the self-bearing passively levitated radial-flux machine: the forward winding, the return winding, and the ferromagnetic material. To properly assess the impact of the ferromagnetic material, several stator and rotor configurations were defined in the optimization scenarios. Based on the results from all these scenarios, it was observed that the inclusion of the ferromagnetic material offered no benefit in this type of radial-flux machine, since its detent effects, which introduce a negative total stiffness, contributed to a reduction in the machine's bearing function. It was therefore concluded that the ferromagnetic material should no longer be considered in the search for new topologies.

In this context, clear topological results were observed for the armature windings. From there, winding topologies with rectangular cross-sections, which are easier to manufacture, were identified and optimized through parametrization. This demonstrated that some winding configurations are more interesting to improving the machine's bearing function than others.

The second objective of this thesis focused on the rotor section of the machine, for which the range of complex topologies for self-balancing machines (SBMs) are not very extensive. A second topological optimization was performed to model the effects of rotor components, such as the back iron yoke of the rotor, the permanent magnets, and the fer-

romagnetic yoke surrounding the windings. Here too, the topological results showed that the rotor components could be arranged in a way that improves the machine's bearing function. A parametric optimization was therefore performed on the topology extracted for each configuration to validate these topological results.

Finally, while simultaneous optimization of the rotor and stator seemed too complex, a parametric optimization combining the geometric parameters of the topologies extracted from both the stator and the rotor showed that by combining the optimization while taking both parts of the machine into account, the bearing function could perform even better.

Future Work

To further advance the search for new topologies aimed at helping in the design of new, more efficient topologies for EDTSBMs, an area for improvement would be to take into account both the bearing function and the motor function of the machine, which also depend on the configuration of the rotor and stator. To achieve this, a parametric optimization incorporating the topologies identified in this thesis could be performed.

Implementing a 3D topological optimization model could better reflect the 3D characteristics of the magnetic field, particularly the heteropolar component of the field, but appears to be computationally too costly. A good compromise would therefore be to incorporate the topological results discovered in this thesis to perform 3D parametric optimization.

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Appendix A

Stator Topology Optimization - Additional Informations

A.1 Symmetry

The symmetry reductions applied to each configuration are as follows.

- In configuration (1), the force is computed by integrating the Maxwell stress tensor over the complete outer surface of the rotor. The axial symmetry ($\Delta z = 0$) therefore cannot be exploited for force computation, and only the radial symmetry ($\Delta r = 0$) is applied, halving the domain. The axial force calculated in COMSOL is therefore multiplied by a factor of 2:

$$F_z = 2 \cdot F_{z, COMSOL} \quad (\text{A.1})$$

- In configurations (2), only $\Delta r = 0$ holds, so the radial symmetry is applied. The axial force calculated in COMSOL is therefore multiplied by a factor of 2:

$$F_z = 2 \cdot F_{z, COMSOL} \quad (\text{A.2})$$

- In configurations (3), only $\Delta z = 0$ holds, so the axial symmetry is applied. The radial force calculated in COMSOL is therefore multiplied by a factor of 2:

$$F_r = 2 \cdot F_{r, COMSOL} \quad (\text{A.3})$$

- In configuration (4), both $\Delta z = 0$ and $\Delta r = 0$. However, since the permanent magnet is deactivated, the only magnetic field produced is that generated by the injected current, which is then axially antisymmetric about $z = 0$ and radially symmetric about $r = 0$. The total magnetic energy integrated in COMSOL is therefore multiplied by a factor of 4:

$$W_m = 4 \cdot W_{m, COMSOL} \quad (\text{A.4})$$

A.2 Stator Geometry Parameters

The main geometrical parameters used for the stator topology optimization are summarized in the table below. It should be noted that these parameters are chosen arbitrarily and can be adjusted for different types of radial flux machines

Parameter	Value
Rotor inner radius	1.7 cm
Rotor back-iron yoke width	0.3 cm
Rotor outer radius	3.0 cm
Rotor height	4.0 cm
Rotor iron yoke surrounding the armature width (S2 & S4 only)	0.3 cm
Air box around the rotor	0.5 mm
Inner airgap	0.1 cm
Outer airgap (S2 & S4 only)	0.1 cm
Infinite Element Domain inner radius	7.5 cm
Infinite Element Domain outer radius	8.0 cm
Axial displacement ε_z	0.3 mm
Radial displacement ε_r	0.3 mm
Stator design zone S1	Rectangle 1.0 cm $\times$ 2.0 cm
Stator design zone S2	Union: 1.0 cm $\times$ 2.1 cm and 2.4 cm $\times$ 0.4 cm
Stator design zone S3	Rectangle 0.4 cm $\times$ 2.0 cm
Stator design zone S4	Union: 0.4 cm $\times$ 2.1 cm and 2.2 cm $\times$ 0.4 cm

Table A.1: Main geometric parameters of the stator topology optimisation model.

A.3 Stator Geometry Parameters

Rotor	Stator	K_z [$\frac{\text{Wb}}{\text{m}}$]	L_c [μH]	R [$\text{m}\Omega$]	$k_{z,e,\infty}$ [$\frac{\text{N}}{\text{m}}$]	ω_{pe} [$\frac{\text{rad}}{\text{s}}$]
R1	S1	2.9619	62.377	105.48	7.0322×10^4	1691.1
	S2	14.393	80.852	104.90	1.2811×10^6	1297.4
	S3	5.4204	183.16	307.38	8.0206×10^4	1678.3
	S4	14.950	91.328	108.86	1.2237×10^6	1192.0
R2	S1	13.657	88.769	125.28	1.0505×10^6	1411.4
	S2	13.317	85.923	110.94	1.0320×10^6	1291.2
	S3	21.027	142.64	177.12	1.5499×10^6	1241.7
	S4	19.744	128.61	143.03	1.5156×10^6	1112.2
R3	S1	17.745	131.23	176.77	1.1997×10^6	1347.0
	S2	17.192	123.08	155.32	1.2007×10^6	1262.0
	S3	29.147	218.73	233.26	1.9419×10^6	1066.4
	S4	26.029	167.40	185.02	2.0236×10^6	1105.3
R4	S1	20.062	99.533	140.17	2.0219×10^6	1408.3
	S2	15.920	51.308	79.370	2.4700×10^6	1546.9
	S3	29.580	161.06	185.79	2.7162×10^6	1153.5
	S4	24.226	82.261	121.47	3.5673×10^6	1476.7

Table A.2: Electromagnetic quantities obtained for all Rotor–Stator configurations considering objective function OBJ2.

A.4 Stator - Parametric Optimization: Additional Results

Symbol	Initial value	Optimised value
<i>Forward winding (W_+)</i>		
w_{W_+}	0.400 cm	0.400 cm
h_{W_+}	0.450 cm	0.371 cm
x_{W_+}	3.300 cm	3.300 cm
y_{W_+}	1.775 cm	1.815 cm
<i>Return winding (W_-)</i>		
w_{W_-}	0.200 cm	0.173 cm
h_{W_-}	0.900 cm	0.855 cm
x_{W_-}	3.200 cm	3.187 cm
y_{W_-}	0.450 cm	0.530 cm

Table A.3: Initial and optimised stator winding parameters, configuration R1–S3.

Symbol	Initial value	Optimised value
<i>Forward winding (W_+)</i>		
w_{W_+}	0.400 cm	0.400 cm
h_{W_+}	0.651 cm	0.668 cm
x_{W_+}	3.300 cm	3.300 cm
y_{W_+}	1.675 cm	1.666 cm
<i>Return winding (W_-)</i>		
w_{W_-}	0.270 cm	0.249 cm
h_{W_-}	0.965 cm	1.078 cm
x_{W_-}	3.235 cm	3.224 cm
y_{W_-}	0.483 cm	0.546 cm

Table A.4: Initial and optimised stator winding parameters, configuration R2–S3.

Symbol	Initial value	Optimised value
<i>Forward winding (W_+)</i>		
w_{W_+}	0.400 cm	0.400 cm
h_{W_+}	0.480 cm	0.509 cm
x_{W_+}	3.300 cm	3.300 cm
y_{W_+}	1.760 cm	1.745 cm
<i>Return winding (W_-)</i>		
w_{W_-}	0.400 cm	0.400 cm
h_{W_-}	0.480 cm	0.509 cm
x_{W_-}	3.300 cm	3.300 cm
y_{W_-}	0.240 cm	0.255 cm

Table A.5: Initial and optimised stator winding parameters, configuration R3–S3.

Symbol	Initial value	Optimised value
<i>Forward winding (W_+)</i>		
w_{W_+}	0.400 cm	0.328 cm
h_{W_+}	0.635 cm	0.771 cm
x_{W_+}	3.300 cm	3.264 cm
y_{W_+}	1.468 cm	1.473 cm
<i>Return winding (W_-)</i>		
w_{W_-}	0.400 cm	0.633 cm
h_{W_-}	0.635 cm	3.100 cm
x_{W_-}	3.300 cm	3.416 cm
y_{W_-}	0.318 cm	1.550 cm

Table A.6: Initial and optimised stator winding parameters, configuration R4–S3.

Appendix B

Rotor Results: Detailed Parameters

Parameter	Initial value	Optimised value
h_{PM}	0.9185 cm	1.1721 cm
w_{PM}	0.9865 cm	0.8601 cm

Table B.1: Initial and optimised values of the geometric parameters used in the parametric optimisation for rotor configuration R1.

Parameter	Initial value	Optimised value
h_{Δ}	0.4 cm	0.6044 cm

Table B.2: Initial and optimised values of the geometric parameter used in the parametric optimisation for rotor configuration R2.

Parameter	Initial value	Optimised value
h_{F1}	0.22 cm	0.2165 cm
h_{F2}	0.24 cm	0.1749 cm

Table B.3: Initial and optimised values of the geometric parameters used in the parametric optimisation for rotor configuration R3.

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