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A Fast Simulation-Driven Pre-Fabrication Workflow for Transmon-Based Superconducting Quantum Chips

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Abstract

Scalable fault-tolerant quantum computing requires hardware that faithfully translates designed Hamiltonians into physical devices. This goal remains challenged by fabrication-induced deviations in qubit frequencies, couplings, and coherence times. Superconducting transmons in cavity-QED architectures represent a leading platform, yet conventional design-build-test cycles impose prohibitive costs and delays due to their reliance on post-fabrication tuning and characterization. This thesis introduces a simulation-driven, pre fabrication workflow that links chip layout directly to circuit-level performance through the lumped-oscillator model (LOM). The conception flow start with analytically-derived resonator parameters that seed initial layout, from which capacitance and inductance values are extracted via Ansys Q3D for LOM-based quantization of qubit frequencies, anharmonicities, dispersive shifts, and coupling rates. Then, structured parametric sweeps over connector length, pad dimensions, pad gap, and Josephson inductance map the dominant geometric sensitivities governing coherence, frequency, and coupling strength. Complementary conformal-mapping calculations estimate external quality factor, photon lifetime, and linewidth for feedline coupling geometries. Additionally, HFSS eigenmode simulations validate preservation of the intended modal structure, establishing a robust starting point for forthcoming energy-participation-ratio (EPR) analysis. The resulting parameter response maps and tolerance windows provide actionable priors for design refinement, reduce reliance on iterative prototyping, and offer a transferable methodology for accelerating the development of scalable superconducting-qubit processors.

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Glossary

Term	Definition
Acronyms	
2D	2-dimensional
3D	3-dimensional
AC	Alternating current
API	Application programming interface
BCS	Bardeen-Cooper-Schrieffer (theory)
$\mathbb{C}$	Set of complex numbers
CAD	Computer aided design
CNOT	Controlled-NOT
CPB	Cooper pair box
CPW	Coplanar waveguide (resonator)
cQED	Cavity quantum electrodynamics
DC	Direct current
DOF	Degree of freedom
EM	Electromagnetic
EPR	Energy participation ratio
FEM	Finite element method
FWHM	Full width at half maximum
GDSII	Graphic data system II
GUI	Graphical user interface
HFSS	High frequency structure simulator
JC	Jaynes-Cummings
JJ	Josephson junction
LO	Local oscillator
LOM	Lumped oscillator model
$\mathbb{N}$	Set of natural numbers
NISQ	Noisy Intermediate-Scale Quantum (computing era)
PEC	Perfect electric conductor
PCB	Printed circuit board (trace)
Q3D	Quasi-static 3D extractor
QEC	Quantum error correction

Continued on next page

Term	Definition
QHO	Quantum harmonic oscillator
QND	Quantum non-demolition (measurement)
$\mathbb{R}$	Set of real numbers
RF	Radio frequency
RLC	Resistor - inductor - capacitor
RSA	Rivest-Shamir-Adleman (algorithm)
RWA	Rotating-wave approximation
SNR	Signal-to-noise ratio
SQUID	Superconducting quantum interference device
TEM	Transverse electromagnetic
TL	Transmission line
TLS	Two-level system (defects)
Transmon	Transmission line shunted plasma oscillation qubit
XMON	Cross-shaped transmon
$\mathbb{Z}$	Set of integer numbers
ZPF	Zero-point fluctuation
Greek Symbols	
α	Complex probability amplitude of the ground state $ 0\rangle$
α	Anharmonicity
β	Complex probability amplitude of the excited state $ 1\rangle$
β_l	Phase constant
Γ_1	Energy relaxation rate
Γ_P	Purcell decay rate
Γ_ϕ	Pure dephasing rate
γ	Global phase factor in quantum states
γ	Atomic spontaneous decay rate
γ_D	Damping probability
$\Delta(T)$	Temperature dependent superconducting energy gap
Δ	Qubit-resonator detuning
Δ	Lamb-Shift
ΔV_0	Zero-point voltage fluctuation
δ_α	Complex phase of α

Continued on next page

Term	Definition
δ_β	Complex phase of β
$\mathcal{E}_{\text{AD}}$	Amplitude damping channel
ϵ	Absolute permittivity of substrate
ϵ_0	Vacuum permittivity
ϵ_{eff}	Effective dielectric constant
ϵ_r	Relative permittivity (dielectric constant) of substrate
χ	Dispersive shift (cross-Kerr coefficient)
θ	Polar angle in Bloch sphere
κ	Decay rate
κ_c	Coupling decay rate
κ_i	Internal decay rate
λ	Axial phase in Bloch sphere
λ	Wavelength
μ	Chemical potential
φ	Azimuthal angle in Bloch sphere
φ^s	Gauge-invariant phase difference between superconductors
$\varphi_{mj}^{\text{ZPF}}$	Quantum zero-point fluctuation of the generalized flux (phase) across Josephson junction j due to eigenmode m
$\hat{\varphi}_J$	Generalized flux operator across the unique junction J
φ_q^{ZPF}	Quantum zero point fluctuations in qubit mode
φ_r^{ZPF}	Quantum zero point fluctuations in resonator mode
Φ	Magnetic flux
$\hat{\Phi}$	Magnetic flux operator
Φ_0	Magnetic flux quantum
Φ_{ext}	External magnetic flux
ϕ	Phase difference between probability amplitudes
ϕ^s	Global quantum phase of superconducting condensate
$\tilde{\phi}$	Shifted phase variable (SQUID)
ψ^s	Macroscopic quantum wavefunction of superconducting condensate
ρ	Density matrix
σ_x	Pauli-X matrix
σ_y	Pauli-Y matrix
σ_z	Pauli-Z matrix

Continued on next page

Term	Definition
σ^+	Qubit raising operator
σ^-	Qubit lowering operator
τ_γ	Photon decay rate
ω	Angular frequency
ω_p	Plasma frequency
ω_q	Qubit transition resonance frequency
ω_r	Resonator resonance frequency
ω_r^c	Coupled resonance frequency
ω_{01}	Qubit $ 0\rangle \leftrightarrow 1\rangle$ transition frequency
ω_{12}	Qubit $ 1\rangle \leftrightarrow 2\rangle$ transition frequency
Other Symbols	
$\hat{a}^\dagger$	Photon creation operator
$\hat{a}$	Photon annihilation operator
C_Σ	Total island capacitance
C_c	Coupling capacitance
C_g	Gate capacitance
C_J	Josephson junction capacitance
C_l	Capacitance per unit length
$\mathbf{C}_M$	Capacitance matrix
C_{ij}	Capacitance matrix element
C_{nm}^{eff}	Effective capacitance between the n-th and m-th subsystems
C_n	Capacitance of the n-th segment in lumped element model
C_r	Resonator capacitance
C_S	Shunt capacitance
C_{Sl}	Shunt capacitance per unit length
c_0	Speed of light in vacuum
c_l	Dimensionless per-unit coupling coefficient
$\mathbf{d}$	Atomic dipole moment
D	Coupling gap
$\mathbf{E}_0$	Vacuum electric field density
E_0	Kraus "no decay" operator
E_1	Kraus "decay" operator

Continued on next page

Term	Definition
E_c	Charging energy
E_J	Josephson energy
$\mathbf{E}_m$	Electric field of mode m
E_{total}	Total energy stored in the system
e	Elementary charge
f_0	Fundamental resonance frequency
f_{BE}	Bose-Einstein distribution function
f_m	m -th harmonic frequency
G_1	Conductance per unit length
G_n	Conductance of the n -th segment in lumped element model
g	Coupling factor
$\mathcal{H}$	Hilbert space
H	Classical Hamiltonian
H	Substrate thickness
H_{LC}	Classical LC Hamiltonian
$\hat{H}_{\text{LC}}$	Quantized LC Hamiltonian
$\hat{H}_{\text{CPB}}$	Cooper pair box Hamiltonian
$\mathbf{H}$	Magnetic field
$\mathbf{H}_m$	Magnetic field of mode m
h	Substrate thickness (CPW)
$\hbar$	Reduced Planck's constant
$\hat{I}$	Current operator
ΔI_0	Zero-point current fluctuation
I_c	Critical current
$I_{c,\text{eff}}$	Effective critical current (SQUID)
I_J	Current through the Josephson junction
I_L	Current through the inductor
I_R	Current through the resistor
$i_k(t)$	Current in the k -th branch at time t
$K(k)$	Complete elliptic integral of the first kind for modulus k
$\mathbf{k}$	Momentum
k_0	Geometric parameter ($= s/(s + 2w)$)
k'_0	Complementary modulus ($= \sqrt{1 - k_0^2}$)

Continued on next page

Term	Definition
k_1	Geometric parameter ($= \sinh(\pi s/(4h))/\sinh(\pi(s+2w)/(4h))$)
k'_1	Complementary modulus ($= \sqrt{1 - k_1^2}$)
k_B	Boltzmann constant
L	Length of transmission line
L_{nm}^{eff}	Effective inductance between the n-th and m-th subsystems
l_c	Connector length
l_r	Resonator length
L_J	Josephson inductance
L_k	Lindblad collapse operators
L_1	Inductance per unit length
L_r	Resonator inductance
n	Photon number (excitation manifold index)
n_g	Offset charge
n_s	Cooper pair density
$\hat{n}$	Charge number operator
$P_{\text{dissipated}}$	Dissipated power
p_{mj}	Energy participation ratio for Josephson junction j and mode m
Q	Total quality factor
Q_c	External coupling quality factor
Q_i	Internal quality factor
Q_L	Loaded quality factor
$\hat{Q}$	Charge operator
$\mathbf{Q}$	Vector of conductor charges
q	Filling factor (participation ratio)
$R_k(\alpha)$	Rotation gate by angle α about axis k
R_K	Resistance quantum: $h/(2e)$
R_1	Resistance per unit length
R_n	Normal-state resistance
S_{21}	Transmission coefficient
s	Gap width between center conductor and ground plane (CPW)
T_1	Energy relaxation time
T_2	Dephasing time
T_c	Critical temperature (superconductors)

Continued on next page

Term	Definition
T_ϕ	Pure dephasing time
t	Conductor film thickness (CPW)
t_g	Gate characteristic time
t_{transit}	Atom traversal time through cavity
$\hat{U}$	Quantum gate unitary operator
$\hat{U}^\dagger$	Conjugate transpose of $\hat{U}$
$\hat{V}$	Voltage operator
V_g	Gate voltage
V_{rms}	Root mean square average voltage across capacitor
$\mathbf{V}$	Vector of conductor potentials
v_0	Effective wave propagation speed in CPW
$v_k(t)$	Voltage in the k-th branch at time t
v_p	Phase velocity
w	Width of center conductor (CPW)
Z_0	Characteristic impedance of a TL
Z_r	Characteristic impedance of a parallel LC circuit
Z_{in}	Input impedance of a RLC
Z_{in}^c	Input impedance of a RLC with coupling
$\langle \cdot \rangle_m$	Expectation value in the state exciting mode m
$ 0\rangle$	Qubit ground state
$ 1\rangle$	Qubit excited state
$ c\rangle$	Control qubit state
$ e\rangle$	Qubit excited state
$ f\rangle$	Qubit second excited state
$ g\rangle$	Qubit ground state
$ n\rangle$	Fock state with n photons
$ t\rangle$	Target qubit state
$ \psi\rangle$	Qubit pure state
$ \psi\rangle_{\text{in}}$	Input quantum state
$ \psi\rangle_{\text{out}}$	Output quantum state
$ \psi\rangle_m$	Excited Fock state of mode m
$\uparrow$	Spin up
$\downarrow$	Spin down

1 Introduction

The conceptual foundations of quantum computing emerged in the early 1980s. Soviet mathematician Yuri Manin first theorized computation harnessing quantum physics in 1980. Shortly after, Richard Feynman independently proposed a basic quantum computer model for efficient simulation of quantum systems (1981 lecture, pub. 1982 [1]). Feynman’s work initiated the field [1], but it was to David Deutsch to formalize the concept a few years later by introducing the quantum Turing machine [2] and revealing that quantum computers would have some properties not reproducible on classical computers, including quantum parallelism [2]. The 1990’s witnessed transformative discoveries that demonstrated quantum computing’s practical potential. In 1994, Peter Shor shattered classical assumptions by showing that a quantum computer could factor large integers exponentially faster than the best-known classical algorithms [3]. This result threatened classical public-key cryptography (e.g., RSA) and showed quantum computing’s disruptive power. In 1997, Lov Grover provided a quadratic speedup for unstructured search problems [4].

While Shor’s and Grover’s algorithms established provable quantum advantage [3], their practical realization remained contingent on overcoming physical implementation barriers. Quantum systems face a fundamental predicament: they require isolation to maintain coherence, yet demand controlled interactions for operations and measurement, processes that inevitably introduce decoherence [4]. David DiVincenzo formalized this challenge by establishing five criteria for viable quantum computers [5]: (1) well-defined qubits with distinct and addressable quantum states; (2) high-fidelity initialization to a known state; (3) long coherence times to suppress decoherence from environmental noise; (4) universal gate operations (single- and multi-qubit gates) for arbitrary unitary control; and (5) high-precision measurement to resolve qubit states unambiguously. These criteria ensure controllability, coherence, and scalability, the latter being an implicit sixth requirement for integrating qubits into large-scale architectures.

Contemporary quantum computing operates in the noisy intermediate-scale quantum (NISQ) era [6, 7], defined by devices with insufficient qubit counts and error rates for fault-tolerant quantum error correction (QEC) [5]. Among physical platforms, superconducting circuits dominate due to lithographic scalability, nanosecond gate operations, and compatibility with microwave control infrastructure. The field of cavity quantum electrodynamics (cQED), which explores the coherent interaction between atoms and quantized radiation in high-finesse optical cavities, originated with its theoretical foundation [12]. Theoretical proposals to adapt cQED principles to solid-state systems emerged in the early 2000’s. Al-Saidi and Stroud pioneered the idea of coupling Josephson junctions (JJ), a nonlinear circuit element, to a single mode of a 3D microwave

¹While scholarly consensus affirms Manin’s conceptual priority, the cold war’s information barrier likely prevented Feynman’s awareness of this work, resulting in what most historians recognize as a parallel conceptual development.

²Quantum parallelism requires only a single processing unit, offering an exponential increase of computational power.

³Quantum advantage denotes scenarios where a quantum computer solves a problem provably faster than any classical computer.

⁴Decoherence occurs when a quantum system’s environment randomizes its phase relationships, destroying superpositions and entanglement and limiting how long quantum computations retain useful quantum behavior (coherence time).

⁵Quantum error correction (QEC) protects quantum information by encoding it non-locally across multiple physical qubits, enabling real-time detection and correction of errors without disturbing the encoded state. This allows fault-tolerant computation when physical error rates fall below a threshold [8–11].

cavity, leveraging the JJ’s tunable inductance to create “artificial atoms” [13]. The field advanced significantly with Blais et al theoretical proposal of a practical superconducting quantum system using a one-dimensional transmission line resonator to achieve strong light-matter coupling via the strong-coupling regime of cavity quantum electrodynamics in superconducting circuits [14]. This design was experimentally validated by Wallraff et al., who demonstrated strong coupling between a Cooper pair box (CPB) which is a charge qubit, and a coplanar waveguide (CPW) resonator, observing quantum energy oscillations [15]. Another pivotal advancement came in 2007 with the transmon qubit, introduced by Koch et al. [16]. By shunting the Cooper pair box with a large capacitor, the transmon architecture exponentially suppresses sensitivity to charge noise, extending coherence times, with state-of-art devices now exceeding 100 μs [17], [18].

The development of scalable superconducting quantum processors to achieve quantum advantage faces a fundamental challenge: bridging the gap between theoretically specified quantum behavior and physical device realization. Quantum algorithms require precisely engineered Hamiltonian that define critical properties. Translating these specifications into functional hardware remains profoundly non-trivial. Fabrication imperfections routinely introduce deviations between designed and achieved quantum behavior. Traditional design methodologies, reliant on iterative fabrication cycles to correct these deviations, become prohibitively costly and time-inefficient. This necessitates a paradigm shift toward computational solutions. Through high-fidelity electromagnetic (EM) simulation, we can model complex microwave interactions inherent in transmon architectures, optimizing resonator geometries, coupling strengths, and loss channels entirely *in silico*. These tools have thus become indispensable for predictive design of scalable quantum processors, directly enabling the methodology developed in this thesis.

In this thesis, we establish a robust, simulation-driven design methodology to optimize superconducting quantum hardware *in silico*. By performing critical validations and refinements before fabrication, the approach reduces reliance on physical prototyping, lowers development costs, accelerates the design cycle, and enhances predictability. The aim is to increase the likelihood that fabricated devices reproduce target quantum behavior. We achieve this through predictive parametric modeling with electromagnetic (EM) simulations that map how geometric parameters influence quantum performance metrics and enable target-driven optimization, in which critical quantum parameters guide automated layout refinements. To enable robust *in silico* optimization, we first lay out the theoretical foundations governing quantum behavior in superconducting circuits in Section 2, which is essential to turn EM simulations into predictive tools for target-driven optimization.

We start by Section 2.1 that establishes the fundamentals of quantum information. The qubit, the fundamental quantum unit, is introduced in Section 2.1.1. We formalize its state as a coherent superposition in a two-dimensional Hilbert space and representing it geometrically via the Bloch sphere. Key non-classical properties are highlighted: superposition enables parallel state representation, measurement induces probabilistic collapse, and interference amplifies computational pathways, enabling exponential state-space encoding. In Section 2.1.2, we present quantum gates as unitary operations, detailing single-qubit rotations (e.g., Pauli- X , Hadamard) and two-qubit gates creating entanglement (e.g., CNOT). Finally, Section 2.1.3 addresses decoherence, where environmental interactions degrade coherence via the longitudinal decoherence time (T_1) and transverse decoherence time (T_2). These timescales fundamentally bound quantum algorithm execution. Only T_1 , governed by electromagnetic losses, can be directly simulated in our frame-

work, making it an optimization focus in later chapters.

In Section 2.2, we establish the physics of superconducting qubit. First, in Section 2.2.1, we present them as engineered artificial atoms, where Josephson junctions provide nonlinear inductance to create the essential anharmonicity for quantum operations. Then, Section 2.2.2 establishes the quantum foundations of superconducting qubits, demonstrating how superconductivity enables macroscopic quantum coherence while suppressing dissipation. We develop the Josephson junction physics, deriving the critical parameters like Josephson energy and inductance. Section 2.2.3 details the state of the art direct-current superconducting quantum interference device (DC SQUID) configuration, created by embedding two junctions in a superconducting loop. It creates dynamic control of qubit frequencies via magnetic flux, which is critical for quantum processor calibration and operation. Finally, Section 2.2.4 presents the transmon qubit architecture, balancing Josephson and charging energies to achieve both noise protection and the anharmonicity required for quantum operations

Section 2.3 covers the resonator physics for superconducting quantum devices. We first introduce transmission line fundamentals in Section 2.3.1: distributed versus lumped-element models, 50 Ω impedance optimization for minimal loss, and resonant standing wave formation via solution of lossless telegrapher equations. In Section 2.3.2, we model transmission line resonators as RLC circuits to derive critical parameters like resonant frequency and quality factors. Then, we analyze the effects of capacitive coupling on resonance frequency, quality factors, and qubit readout performance. Section 2.3.3 presents the coplanar waveguide resonator used in this work, detailing existing configurations with analytical models. While neglecting nanoscale kinetic inductance, these models provide efficient initial parameter estimates that accelerate the optimization workflow, a practical necessity given time constraints that preclude comprehensive EM simulation of all design iterations.

Section 2.4 integrates the foundational concepts from previous sections to present the cavity quantum electrodynamics (cQED): a theoretical framework for coherent interactions between an artificial atom (qubit) and single photons in a resonator. Background is established in Section 2.4.1. The Jaynes-Cummings model is developed in Section 2.4.2, deriving its Hamiltonian and demonstrating how zero-detuning creates maximally entangled dressed states that prevent definite state occupation. Quantum information processing requires minimal entanglement, achieved in the dispersive regime (large detuning). Section 2.4.3 establishes dispersive readout mechanisms critical for quantum measurement.

Building on the theoretical foundations of Section 2, Section 3 establishes a simulation-driven framework for *in silico* validation of superconducting quantum processors. We combine analytical models for initial parameter selection with EM analysis for efficient parametric sensitivity studies, enabling physical insight and accelerated design iteration while reducing prototyping cycles.

A central challenge in superconducting quantum processor design lies in the reliable translation of a target quantum Hamiltonian, encoding the desired system behavior, into a functional physical device. Section 3.1 details the rigorous, iterative methodology developed to bridge this gap. This multi-stage workflow integrates theoretical specification, computational design, EM simulation, quantum analysis, fabrication and experimental validation.

This work focuses exclusively on the pre-fabrication design optimization of superconducting quantum processors. Section 3.2 details a computationally rigorous methodology for translating a target system Hamiltonian into an optimized physical layout prior to fabrication, that will be presented in subsequent sections. Central to this is an iterative simulation loop that refines the device design by repeatedly comparing predicted quantum parameters against target specifications. Discrepancies drive layout modifications and re-simulation until convergence is achieved. This closed-loop approach minimizes reliance on costly post-fabrication tuning by ensuring performance metrics are met at the design stage.

Section 3.3 defines the design specifications for the superconducting quantum processor under study. We adopt a canonical architecture comprising a transmon qubit capacitively coupled to a quarter-wave resonator, augmented by a feedline for readout via inductive coupling to the resonator. This system offers a practical testbed for studying key quantum effects while remaining simple to simulate. Given the absence of a predefined application-specific Hamiltonian, our methodology prioritizes the specification of key target quantum parameters. These include the Josephson-to-charging energy ratio, qubit and resonator frequencies, anharmonicity, and coherence time. This parameter-centric approach ensures the design captures core quantum properties while remaining agnostic to a precise final Hamiltonian form.

The device layout methodology, implemented using the open-source qiskit-metal framework [19], is presented in Section 3.4. This python-based platform enables programmatic quantum circuit design through its API, executed via jupyter notebook [20], while providing an interactive graphical interface for visualization and real-time adjustments. The tool automates generation of physical layouts.

Section 3.5 outlines the EM simulation and quantum parameter extraction methodology. Two parallel workflows are used: In Section 3.6, Ansys HFSS performs high-fidelity finite-element eigenmode simulations on the initial and refined layouts, yielding resonant frequencies, EM field distributions, and mode profiles. For completeness, we also present a theoretical description of the energy participation ratio (EPR) workflow in HFSS, intended as part of the full design process for this thesis but not executed here; its inclusion situates the implemented methods within the broader planned framework. In Section 3.7, Ansys Q3D extractor performs quasi-static capacitance extraction on the device layout and derive the capacitance matrix of the circuit. Then, the lumped-oscillator model (LOM) analysis is used to derive the system’s effective Hamiltonian.

Section 3.8 validates the chip design methodology through systematic parameter studies. Parametric sensitivity analysis is performed by sweeping one geometric parameter at a time (e.g., resonator coupling, qubit pad geometry, junction inductance) while holding others fixed, using LOM as the primary analysis engine. This allows us to quantify each parameter’s influence on the relevant quantum properties and to map the design’s response surface efficiently prior to fabrication.

This introduction has outlined a simulation-driven, pre-fabrication workflow for superconducting quantum processors and shown its validation via LOM-based parametric studies. To complete the loop, EPR analysis must now be performed to drive automated geometry optimization toward the target parameters. Integrating EPR into this framework is the next critical step, after which the optimized layout will be ready for fabrication and experimental validation.

2 Superconducting quantum circuits

The control and readout infrastructure for superconducting qubits requires precise coordination of classical and quantum components across thermal gradients. This section outlines the canonical signal pathway model, with technical descriptions adapted primarily from the cryogenic engineering frameworks of Refs. [21] and [22].

A quantum computer is fundamentally a hybrid system where classical electronics precisely control and interrogate the quantum state of a qubit. At its core, quantum computation manipulates the wavefunction of the qubit through carefully engineered energy landscapes. The system's total energy, quantified by its Hamiltonian, governs the wavefunction's temporal evolution. This Hamiltonian is dynamically shaped by applying precisely calibrated EM pulses, typically in the microwave domain for superconducting systems. These pulses must exhibit exact characteristics: specific frequency, amplitude, phase, and precise duration. Classical electronics generate microwave pulses that perform three distinct quantum operations: initialization, manipulation, and readout. These functions correspond to fundamental requirements for quantum computation:

1. **Initialization pulses:** prepare qubits in the computational ground state (see DiVicenzo criteria 2). This requires cryogenic temperatures of 10-20 mK, maintained by a dilution refrigerator. At these temperatures, thermal energy is smaller than typical superconducting qubit energy gaps, ensuring that spontaneous relaxation to the ground state dominates over thermal excitation.
2. **Manipulation pulses** implement quantum gates by dynamically engineering the system's Hamiltonian (see Section 2.1.2). These precisely shaped microwave pulses drive controlled transitions between quantum states.
3. **Readout pulses** perform quantum state measurement through microwave resonator coupled to the qubit. This approach implements cavity quantum electrodynamics (cQED), (see Section 2.4), where the resonator frequency is intentionally offset from the qubit's operating frequency. When a microwave pulse probes the resonator, the qubit state (ground or excited) induces a measurable shift in the resonator's response frequency. This shift modifies either the phase (timing) or amplitude (strength) of the reflected pulse, encoding the quantum state information. Crucially, this constitutes quantum non-demolition (QND) readout: the measurement process preserves the quantum state while extracting its information.

Figure 1 presents the complete signal pathway for a superconducting quantum processor. We trace this pathway following the pulse's journey from generation to detection:

1. **Signal creation:** At room temperature (300 K), classical computers coordinate with specialized electronics generate control pulses with precisely engineered characteristics critical for quantum operations: phase coherence, temporal shape, nanosecond-scale duration, controlled amplitude, and specific frequency. Since superconducting qubits operate in the microwave domain (typically 4-8 GHz), these signals are up-converted to the appropriate frequency range using mixers [6] and local oscillators (LOs) [7].

⁶Mixers are electronic components that combine signals to create new frequencies.

⁷Local oscillators are stable microwave sources that provide reference frequencies.

2. **Signal attenuation:** As signals descend through cryogenic stages, they pass through a series of attenuators, which are passive components reducing signal power while preserving waveform fidelity. This reduces pulse power to single-photon levels required for quantum control and suppresses thermal noise.
3. **Chip-level quantum interactions:** These attenuated single-photon pulses enter the chip and interact with qubits via the shared resonator (blue color in Fig. 1). In our design, both control and readout operations occur through a shared resonator that is capacitively coupled to the qubit (orange color in Fig. 1). This architecture simplifies chip layout by eliminating separate control/readout paths. During control phases, microwave pulses directly manipulate qubit states via the resonator. During readout phases, the pulses inside the resonator experience state-dependent frequency shifts due to the coupling between the qubit and the resonator, that encode information about the state of the qubit in their reflected phase/amplitude. Before emerging from the chip, the readout signal usually goes through a Purcell filter (green colour in Fig. 1, integrated to suppress qubit decay through this shared channel while preserving measurement fidelity).
4. **Signal amplification:** Readout signals emerge from the quantum processor too weak for room-temperature detection. They undergo successive amplification at each temperature (10 mK $\rightarrow$ 3 K $\rightarrow$ 300 K). This preserves the quantum signal-to-noise ratio (SNR)⁸, which will enables reliable discrimination between quantum states at room temperature electronics.
5. **Signal detection:** Finally, amplified signals are down-converted to baseband frequencies using mixers and LOs. Room-temperature electronics digitize these signals, enabling classical electronics to distinguish the qubit states through statistical processing of the modulated pulse characteristics.

⁸SNR quantifies the strength of a desired signal relative to background noise. The cryogenic amplification chain preserves SNR by amplifying the quantum signal before noise dominates at higher temperatures.

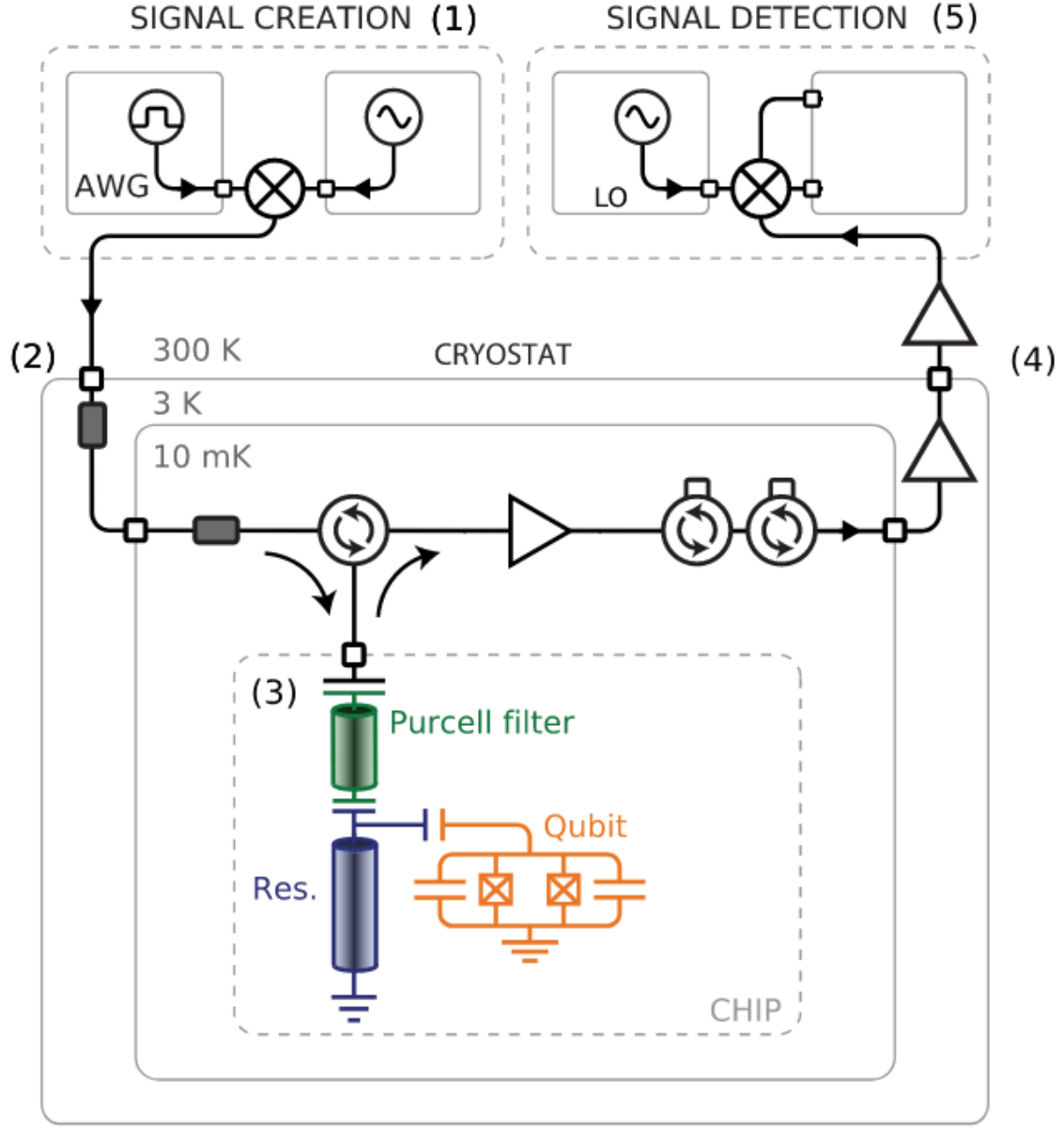


Figure 1: Complete signal pathway for a superconducting quantum processor, adapted from Ref. [22]. The full signal chain traces microwave pulses from room-temperature generation (300 K) to quantum-state detection. The chip is outlined with dashed lines. Key stages: (1) Pulse synthesis/up-conversion to 4–8 GHz, (2) Cryogenic attenuation to single-photon levels, (3) Qubit control/readout via shared resonator (blue) coupled to the qubit (orange), with Purcell filter (green) suppressing decoherence, (4) Amplification across temperature stages (10 mK $\rightarrow$ 3 K $\rightarrow$ 300 K) preserving SNR, and (5) Down-conversion/digitization for state discrimination. The shared resonator architecture simplifies on-chip routing.

2.1 Quantum information fundamentals

2.1.1 Qubit state and representations

To understand the qubit, let's start by presenting the classical bit. Classical bits form the fundamental building blocks of digital computing, encoding information as binary states (0 or 1). Physically realized through devices like transistors, bits rely on controlled switching between these discrete states to perform deterministic logic operations. The qubit is the quantum analogue of the classical bit, but as we are going to see, the quantum nature of the qubit unlocks a fundamentally different and sometimes more efficient way to represent, process, and transmit information. A qubit is a two-level quantum system with Hilbert space $\mathcal{H} \cong \mathbb{C}^2$. Its general pure state $|\psi\rangle$ can be represented by a coherent superposition of the basis states:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad \alpha, \beta \in \mathbb{C}, \quad (1)$$

where $|0\rangle$ and $|1\rangle$ form the computational basis:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (2)$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (3)$$

When we measure this qubit in the standard basis, the Born rule quantifies the probability that a measurement of a quantum system will give a given result. For a qubit state, the probability of observing outcome $|0\rangle$ is $P(0) = |\alpha|^2$, while the probability of observing outcome $|1\rangle$ is $P(1) = |\beta|^2$. Unlike classical bits, which exist deterministically as 0 or 1, a qubit state prior to measurement is a coherent superposition of all computable states simultaneously. This allows a single qubit to represent a continuum of states between $|0\rangle$ and $|1\rangle$. More crucially, n qubits in superposition can represent 2^n simultaneous states. This exponential scaling is the foundation of quantum parallelism.⁹

Under measurement, this superposition irreversibly collapses to a single basis state destroying any phase information and leaving the system in a classical-like definite state. Unlike the smooth, deterministic changes described by the Schrödinger equation, collapse is a non-unitary, stochastic process that cannot be undone. Therefore, quantum algorithms must be designed so that the correct answer has a very high probability (ideally close to 1) after the final step. This is achieved by carefully manipulating superpositions to engineer constructive interference for the right answer and destructive interference for wrong answers.

Since the complex numbers α and β represent probabilities of having a certain outcome, they must satisfy the normalization constrain:

$$|\alpha|^2 + |\beta|^2 = 1. \quad (4)$$

⁹Quantum parallelism: A quantum operation applied to a register of n qubits in superposition acts simultaneously on all 2^n states encoded in that superposition. This is the core mechanism allowing quantum algorithms to evaluate a function $f(x)$ for many different inputs x in one computational step.

Since α and β are complex numbers, they have a magnitude and a phase. We have $\alpha = |\alpha|e^{i\delta_\alpha}$ and $\beta = |\beta|e^{i\delta_\beta}$, where δ_α and δ_β represent the phase angles of the complex coefficients α and β . This introduces four real parameters, resulting in four degrees of freedom (DOFs) in Eq. [1](#). However, normalization of the state vector (Eq. [4](#)) reduces the DOFs to 3. By substituting:

$$|\alpha| = \cos \frac{\theta}{2}, \quad (5)$$

$$|\beta| = \sin \frac{\theta}{2}, \quad (6)$$

into Eq. [1](#), the pure state $|\psi\rangle$ becomes:

$$\begin{aligned} |\psi\rangle &= \cos \frac{\theta}{2} e^{i\delta_\alpha} |0\rangle + \sin \frac{\theta}{2} e^{i\delta_\beta} |1\rangle, \\ &= e^{i(\delta_\alpha + \delta_\beta)/2} \left(\cos \frac{\theta}{2} e^{i(\delta_\alpha - \delta_\beta)/2} |0\rangle + \sin \frac{\theta}{2} e^{i(-\delta_\alpha + \delta_\beta)/2} |1\rangle \right), \\ &= e^{i\gamma} \left(\cos \frac{\theta}{2} e^{-i\phi/2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi/2} |1\rangle \right), \end{aligned} \quad (7)$$

where $\gamma = (\delta_\alpha + \delta_\beta)/2$ and $\phi = \delta_\beta - \delta_\alpha$. The global phase $e^{i\gamma}$ is physically unobservable, as it leaves the expectation value of any observable invariant. Discarding $e^{i\gamma}$, the state simplifies to two parameters, $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi)$, which define the polar and azimuthal angles, respectively. This allows the pure state $|\psi\rangle$ of the qubit to be parameterized on a unit sphere, called the Bloch sphere (Fig. [2](#)). While this provides a 3D geometric representation, the qubit state inherently resides in a 2D complex Hilbert space. Each point on the Bloch sphere represents an infinite number of states $|\psi\rangle_i$ differing only by their global phases $e^{i\gamma_i}$. The Bloch sphere thus bridges quantum mechanics and geometric intuition, despite the qubit non-classical nature [\[21\]](#).

2.1.2 Quantum gates operations

Having established the mathematical description of the qubit we now confront a critical question: How do we actively process quantum information? This is where quantum gates enter as the fundamental tools of quantum computation and information manipulation. A qubit in superposition holds potential computational power, but this power remains inert without dynamic control. Quantum gates are the operators that perform active manipulations on qubit states. They are the quantum analogue of classical logic gates (like AND, OR, NOT), but with capabilities uniquely enabled by quantum mechanics.

A quantum gate is a reversible operation that transforms an input state $|\psi\rangle_{\text{in}}$ into an output state $|\psi\rangle_{\text{out}}$. Mathematically, a quantum gate is represented by a unitary matrix U :

$$|\psi\rangle_{\text{out}} = U|\psi\rangle_{\text{in}}, \quad (8)$$

where U satisfies the unitary condition $U^\dagger U = I$. Here, $U^\dagger$ denotes the Hermitian conjugate (conjugate transpose) of U . This unitarity guarantees reversibility: applying $U^\dagger$ on $|\psi\rangle_{\text{out}}$ recovers the original state $|\psi\rangle_{\text{in}}$. For an n -qubit system, U is a $2^n \times 2^n$ matrix, preserving the norm of the state vector and maintaining the total probability of outcomes as 1.

2.1.2.1. Single-qubit gates

Single-qubit gates performs fundamental unitary operations, acting exclusively on individual qubits, with each gate fully specified by four real parameters. When disregarding the global phase factor (which induces no observable effects due to phase invariance in quantum measurements), any arbitrary single-qubit operation admits a decomposition into three sequential rotations using the Z-Y-Z Euler angles parameterization. This canonical form expresses the gate as $U = e^{i\gamma} R_z(\lambda) R_y(\theta) R_z(\varphi)$, where $R_k(\alpha) = \exp(-i\alpha\sigma_k/2)$ denotes a rotation generated by Pauli matrices σ_k about axis k by angle α . The corresponding matrix representation in the computational basis $\{|0\rangle, |1\rangle\}$ is explicitly given by:

$$U = e^{i\gamma} \begin{pmatrix} \cos \frac{\theta}{2} & -e^{i\lambda} \sin \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} & e^{i(\lambda+\varphi)} \cos \frac{\theta}{2} \end{pmatrix}, \quad (9)$$

with $\theta \in [0, \pi]$ controlling the major rotation amplitude, $\varphi, \lambda \in [0, 2\pi)$ defining axial phases, and γ representing the physically inconsequential global phase. Geometrically, the parameters $(\theta, \varphi, \lambda)$ uniquely parameterize rotations on the Bloch sphere, as represented in Fig. 2, while alternative Euler sequences (e.g., X-Y-X) provide equivalent complete representations. This decomposition shows that all single-qubit gates correspond to rotations implementable through Hamiltonian evolutions $U = \exp(-iHt)$, with H being a traceless 2×2 Hermitian matrix.

As for example, the Pauli- X gate (analogous to a classical NOT gate) rotates the state by π radians about the x -axis, flipping $|0\rangle$ and $|1\rangle$:

$$X = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad X|0\rangle = |1\rangle, \quad X|1\rangle = |0\rangle. \quad (10)$$

Similarly, the Hadamard gate H , which creates superpositions, corresponds to a rotation placing the state on the equator of the Bloch sphere:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}. \quad (11)$$

2.1.2.2. Two-qubit gates

The primary quantum role of two-qubit gates is to generate entanglement, a non-classical correlation where qubits lose their individual identities and become described by a single global state. Entanglement is the essential resource for many quantum applications. The CNOT (controlled-NOT) gate entangle qubits by conditioning the state of one qubit (target) on another (control). For two qubits $|c\rangle|t\rangle$, the CNOT gate applies X to the target qubit $|t\rangle$ only if $|c\rangle = |1\rangle$:

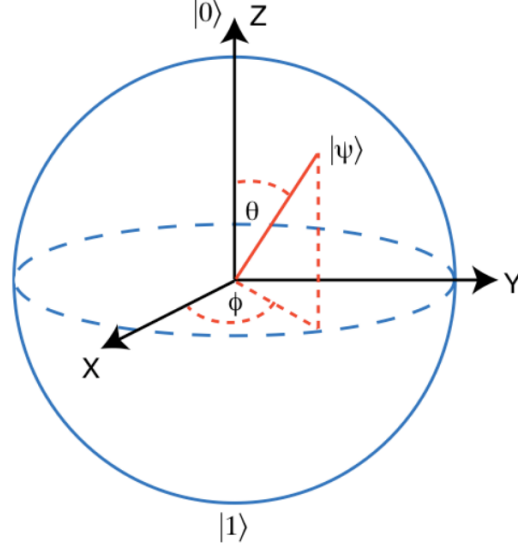


Figure 2: Bloch sphere in the real 3D space, adapted from Ref. [23]. The state vector of the system $|\psi\rangle$ (in red) is projected onto the surface of the Bloch sphere. The polar and azimuthal angles θ and ϕ are shown. The ground state corresponds to the point $|0\rangle$ on the sphere, and the excited state to the point $|1\rangle$

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \text{CNOT} |10\rangle = |11\rangle. \quad (12)$$

A specific, finite set of quantum gates is called universal if any possible any unitary transformation can be approximated arbitrarily well using only gates from that set in a quantum circuit. For instance, we can use $\{e^{i\alpha I}, R_x(\theta_x), R_y(\theta_y), R_z(\theta_z)\}$ as a universal set of quantum gates to form arbitrary single-qubit gates. Universality guarantees that a quantum computer built with this gate set is, in principle, capable of performing arbitrary single-qubit gates, satisfying DiVincenzo criterion 4.

2.1.3 Decoherence dynamics

Having established quantum gates as unitary operations, quantum computation demands that gates maintain coherent superposition, entanglement, and precise phase relationships. However, real-world quantum systems interact with their environment, causing decoherence. This process rapidly degrades quantum states by entangling qubits with environmental noise, transforming pure quantum states into classical mixtures and destroying phase coherence and entanglement. Crucially, this degradation occurs within characteristic coherence times. Those are finite durations (typically microseconds) during which qubits retain their quantum properties before environmental interactions overwhelm them. Consequently, implemented gates deviate from ideal unitarity, and quantum information is irreversibly lost unless computations complete within these coherence

times.

Decoherence imposes fundamental constraints on quantum computation, primarily governed by two distinct timescales: the energy relaxation time T_1 (quantifying energy loss to the environment) and the coherence time T_2 (measuring quantum state preservation). This work focuses on optimizing T_1 through electromagnetic design because:

- T_1 is geometrically determined through dielectric absorption, inductive losses, and radiative decay. All of them are quantifiable via electromagnetic simulations of the device layouts.
- T_2 depends critically on low-frequency noise processes (flux, charge, and critical current fluctuations with $1/f$ spectra) that lack spatial signatures in electromagnetic simulations and require experimental characterization.

By targeting T_1 through geometric optimization during design, we preemptively mitigate dominant energy-loss decoherence pathways, while acknowledging T_2 optimization necessitates post-fabrication techniques.

While the Schrödinger equation could theoretically model these effects, the vast environmental degrees of freedom make this impractical¹⁰. Instead, the Lindblad master equation [24] provides a tractable framework for open quantum systems:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{ L_k^\dagger L_k, \rho \} \right), \quad (13)$$

where L_k are collapse operators describing decoherence channels, and $\rho = \text{Tr}(|\Psi\rangle\langle\Psi|)$ the reduced density matrix.

The energy relaxation time T_1 , or longitudinal decoherence time, is the characteristic timescale for an excited qubit state to spontaneously decay to its ground state due to energy dissipation to the environment. On the Bloch sphere, this collapses the state toward the north pole at a rate $\Gamma_1 = 1/T_1$ (Fig. 2). In this formalism, T_1 is modeled by the amplitude damping channel $\mathcal{E}_{\text{AD}}$:

$$\mathcal{E}_{\text{AD}}(\rho) = E_0 \rho E_0^\dagger + E_1 \rho E_1^\dagger, \quad (14)$$

with $E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}$, $E_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$, and $\gamma = 1 - e^{-t/T_1}$.

The pure dephasing time, T_ϕ , represents phase damping, where environmental noise (e.g., charge or flux noise) disrupts the phase relationship between superposition states without energy loss, collapsing the Bloch vector toward the equatorial plane at rate $\Gamma_\phi = 1/T_\phi$ (Fig. 2). The dephasing time T_ϕ is modeled by the phase damping channel $\mathcal{E}_{\text{PD}}$:

¹⁰The Schrödinger equation governs closed quantum systems, requiring a full wavefunction $|\Psi\rangle$ describing both the system (e.g., transmon) and its environment (e.g., phonon/defect modes). This leads to a Hilbert space dimension exponentially larger than the system's alone. Time-evolving $|\Psi\rangle$ is computationally intractable and experimentally unwarranted since we cannot measure environmental states. The Lindblad master equation avoids this by evolving only the system's reduced density matrix, incorporating environmental effects via phenomenological collapse operators (L_k) and dissipation rates. This allow to have a scalable model for decoherence.

$$\mathcal{E}_{\text{PD}}(\rho) = \rho_{00} |0\rangle \langle 0| + \rho_{11} |1\rangle \langle 1| + e^{-t/T_2}(\rho_{01} |0\rangle \langle 1| + \rho_{10} |1\rangle \langle 0|). \quad (15)$$

The dephasing time T_2 includes contributions from both energy relaxation and pure dephasing. The timescales satisfy $T_2 \leq 2T_1$, with T_ϕ and T_2 defined by:

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}. \quad (16)$$

As a result, quantum gates must operate on timescales $t_G \ll T_1, T_2$ to minimize decoherence errors.

2.2 The superconducting qubit

2.2.1 Artificial atoms

A variety of quantum computing architectures exist for qubit implementations, including superconducting circuits [25], trapped ions [26], quantum dots [27], photonic systems [28], and, more recently, topological platforms including systems hosting Majorana fermions [29]. Among these, superconducting qubits [16], [30] constructed with superconducting qubits are circuits stand out due to their scalability and compatibility with modern nanofabrication techniques. Therefore, it is this type of qubit that will be the focus of this thesis. Importantly, superconducting qubits are fabricated as artificial atoms. This is crucial for the application in quantum computing, as will be seen in Section 2.2.4.

Natural atoms are systems that exhibit discrete energy levels with non-uniform spacing, where the energy required for a transition between adjacent states (e.g., $\hbar\omega_{01}$ for the $|0\rangle \rightarrow |1\rangle$ transition) is distinct from that of higher transitions (e.g., $\hbar\omega_{12}$ for the $|1\rangle \rightarrow |2\rangle$ transition). This property is crucial since it enables selective excitation of specific transitions by using external drives. In contrast, the LC circuit (shown in Fig. 3(a)) has a quantized Hamiltonian that corresponds to a quantum harmonic oscillator (QHO) with a parabolic potential (see Fig. 3(b)) with equidistant energy levels ($\hbar\omega_r$, with ω_r the resonant frequency of the circuit, detailed in Section 2.3). Such uniformity in level spacing renders the system unsuitable for qubit operation, as applying a microwave drive at the transition frequency resonantly couples all transitions, leading to uncontrolled population of higher energy states and resulting in leakage errors and decoherence.

To address this limitation, superconducting qubits engineer anharmonicity by replacing the linear inductance of the LC circuit with the nonlinear inductance of a superconducting element: the Josephson junction [31] (see Fig. 3(c)). The resulting qubit, known as transmon, act as an artificial atom. When operated below the critical current, the non-linear inductance of the Josephson junction introduces a nonlinear Hamiltonian, that deforms the quadratic potential into a cosined-shaped potential (see Fig. 3(d)). The finite depth of the potential well causes higher energy levels to converge. The resulting anharmonicity creates non degenerate transition frequencies $\hbar\omega_{01} \neq \hbar\omega_{12}$ that allow for robust isolation of the computational subspace for qubit control and operation.

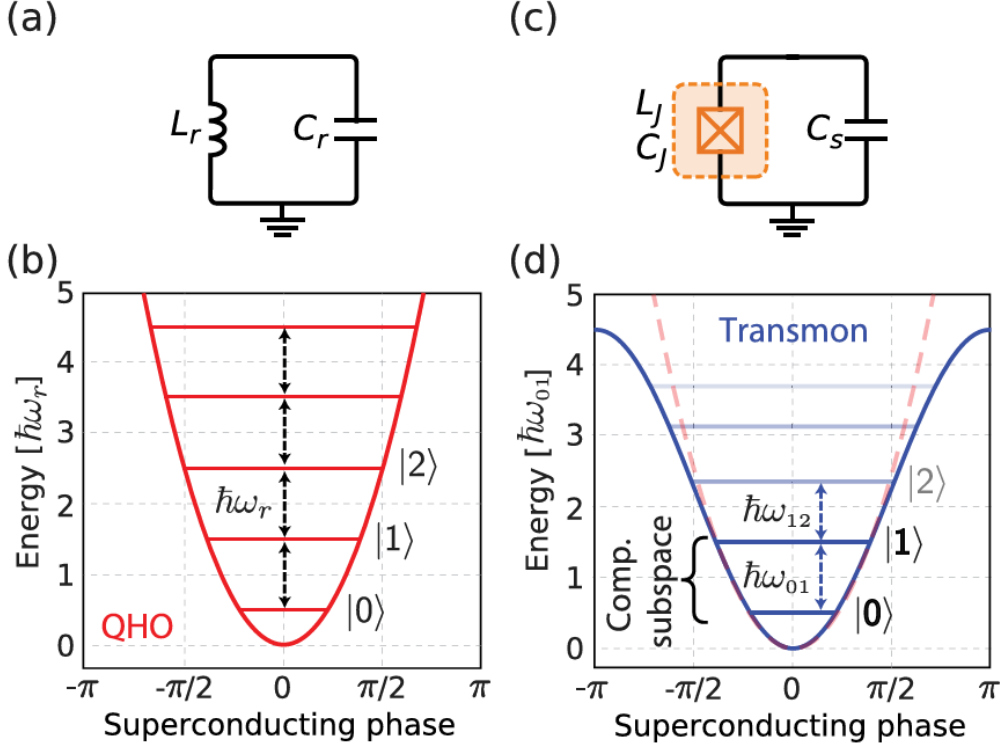


Figure 3: Comparison between LC and transmon circuit and potential, adapted from Ref. [22] (a) LC oscillator circuit with parallel inductance L_r and capacitance C_r . (b) Parabolic potential of the QHO (solid red), exhibiting uniformly spaced energy levels separated by $\hbar\omega_r$. (c) Transmon qubit circuit, substituting the linear inductor with a Josephson junction with non linear inductance L_J and capacitance C_J , represented by the Josephson subcircuit in the dashed orange box, with a shunted capacitance, C_S , in parallel. (d) Cosine-shaped potential (solid blue) versus quadratic potential (dashed red). The transmon potential has anharmonic energy spacings.

2.2.2 Josephson junction and superconductivity

2.2.2.1. Phase coherence in the superconducting state

Electrons in a superconductor form Cooper pairs—bound states with opposite momentum ($\mathbf{k}, -\mathbf{k}$) and opposite spin ($\uparrow, \downarrow$) through an effective attractive interaction mediated by quantized lattice vibrations (phonons), as described by the Bardeen-Cooper-Schrieffer (BCS) theory [32]. This pairing transforms electrons—fermions following the Fermi-Dirac statistics and the Pauli exclusion principle, which prohibits two fermions from occupying identical quantum states—into composite bosons. Therefore, Cooper pairs obey the Bose-Einstein statistics, where the expected occupation number in an energy state is given by the distribution function f_{BE} :

$$f_{BE}(E) = \frac{1}{e^{\frac{E-\mu}{k_B T}} - 1}, \quad (17)$$

where k_B is the Boltzmann constant, μ is the chemical potential and no degeneracy is assumed. Below the critical temperature T_c , the chemical potential μ aligns with the ground state energy E_0 . The Bose-Einstein distribution simplifies for $T \rightarrow 0$ because for $E > \mu$, $f_{BE} \sim 0$. This implies

that all Cooper pairs condense into the lowest-energy state, forming a phase-coherent ensemble where all electrons share the same ground state wavefunction:

$$\psi^s = \sqrt{n_s} e^{i\phi^s}, \quad (18)$$

where ϕ^s is the global quantum phase and n_s the Cooper pair density. It is this macroscopic phase coherence described in Eq. 18 that enables remarkable quantum phenomena: Superconductivity creates uniform phase coherence across the entire circuit, creating a single quantum ground state. The system then behaves as a macroscopic quantum entity with discrete energy levels, analogous to artificial atoms. Moreover, the phase rigidity, enforced by flux quantization and the Bose-Einstein condensate of Cooper pairs, suppresses local phase fluctuations through an energy barrier. This inherent protection of phase stability is crucial for maintaining quantum coherence in circuits.

2.2.2.2. Physical description of the Josephson junction

Physically, a Josephson junction (JJ) is a heterostructure comprising two superconducting electrodes separated by a thin insulating barrier (typically 1–3 nm thick), forming a Superconductor-Insulator-Superconductor (SIS) trilayer as shown in Fig. 4 (a). Conventionally, aluminum (Al) is used for both superconducting electrodes due to its well-characterized superconductivity below its critical temperature $T_c \approx 1.2\text{ K}$. The barrier consists of nonstoichiometric aluminum oxide (AlO_x) formed through controlled oxidation, where the amorphous AlO_x layer exhibits structural defects and localized states that permit coherent tunneling of Cooper pairs. As depicted in Fig. 4 (b), this Al/ AlO_x /Al trilayer is deposited on a high-resistivity substrate (typically intrinsic silicon), which minimizes microwave dielectric losses and parasitic capacitance critical for preserving quantum coherence in superconducting circuits.

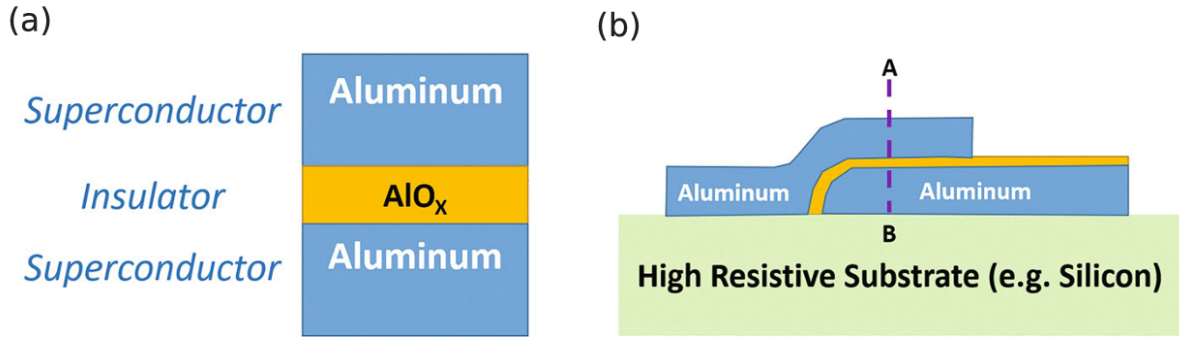


Figure 4: Physical structure of a Josephson junction, adapted from Ref. 21. (a) Josephson junction Al/ AlO_x /Al trilayer heterostructure. (b) Profile view of a Josephson junction fabricated on a highly resistive substrate. The A-B cutline corresponds to the cross section shown on the left.

When two superconducting regions (A and B) are weakly coupled via an insulator, their macroscopic wavefunctions $\psi_A^s = \sqrt{n_s} e^{i\phi_A^s}$ and $\psi_B^s = \sqrt{n_s} e^{i\phi_B^s}$ overlap. As a result, below T_c , quantum tunneling of Cooper pairs across the barrier occurs, without dissipation, giving rise to a supercurrent. The Josephson equations, derived in Ref. 33, govern this behavior:

$$I_J = I_c \sin \varphi^s, \quad (19)$$

$$V = \frac{\Phi_0}{2\pi} \frac{d\varphi^s}{dt}, \quad (20)$$

where I_c is the critical current, the maximum supercurrent the junction can sustain without resistance, and $\varphi^s = \phi_A^s - \phi_B^s$ is the gauge-invariant phase difference between the superconductors, i.e., the difference between the phases of the wavefunctions in region A and region B, and $\Phi_0 = \frac{h}{2e}$ is the magnetic flux quantum, with $e = 1.60217663410 \times 10^{-19}$ C the elementary charge. The first Josephson equation is also called the DC Josephson Effect (Eq. 19), it shows that a direct supercurrent I_J flows without dissipation through the junction in the absence of an applied voltage, determined solely by the phase difference across the junction. The second equation is the AC Josephson Effect (Eq. 20), it shows that when a constant voltage V is applied across a JJ, it induces a time-dependent phase evolution, generating an oscillating supercurrent through the junction.

2.2.2.3. Critical parameters of the Josephson junction

Now, let's derive the critical parameters for the JJ. According to the Tellegen's theorem, for any closed electrical circuit, the instantaneous total absorbed power satisfies:

$$\sum_k v_k(t) i_k(t) = 0, \quad (21)$$

where $v_k(t)$ and $i_k(t)$ represent the voltage and current in the k -th branch. This implies energy conservation within the system. The energy conservation leads directly to the Hamiltonian formulation through temporal integration:

$$H = \int_{-\infty}^t \sum_k v_k(t') i_k(t') dt', \quad (22)$$

This fundamental relationship holds for any circuit topology. For a JJ, the circuit has one single branch, therefore, the energy dynamics in a Josephson junction can be expressed through the integral relationship:

$$H = \int_{-\infty}^t dt' (i_j v_j), \quad (23)$$

$$= -E_J \cos \varphi^s, \quad (24)$$

where the second equality is found by injecting Eq. 19 and Eq. 20 into the integral. We then found the Josephson energy:

$$E_J \equiv \frac{\Phi_0 I_c}{2\pi}. \quad (25)$$

This term is at the origin of the cosine potential of the transmon (see Fig. 3(d)). By differentiating Eq. 19 with respect to time and substituting Eq. 20 into it, one finds:

$$\frac{dI_J}{dt} = I_c \frac{2\pi V}{\Phi_0} \cos \varphi^s. \quad (26)$$

As the Josephson inductance L_J is defined through the voltage-current relation:

$$V = L_J \frac{dI_J}{dt}, \quad (27)$$

substituting Eq. 26 into Eq. 27 gives:

$$L_J = \frac{\Phi_0}{2\pi I_c \cos \varphi^s}. \quad (28)$$

Like regular inductors which can store energy, a JJ also stores energy through the inertial energy of its charge carriers. As can be seen in Eq. 28, the cosine in the denominator of this kinetic inductance shows that it is nonlinear and depends on φ^s . This non-linearity is one of the key factors that will allow us to exploit the Josephson junction for quantum information.

We have found two interdependent parameters governing the behavior of the JJ: the Josephson energy E_J and the Josephson inductance L_J . These parameters are critical for transmon qubit optimization: as will be seen in Section 2.2.4, E_J determines the qubit frequency and anharmonicity, while L_J sets the nonlinear response which is essential for quantum control.

Both E_J and L_J depends on the critical current I_c . The critical current is fundamentally linked to the junction's normal-state resistance R_n , a measurable quantity at room temperature that reflects electron tunneling through the insulating barrier in the non-superconducting state. This relationship is formalized by the Ambegaokar-Baratoff formula [34]:

$$R_n = \frac{\pi \Delta(T)}{2e} \coth \frac{\Delta(T)}{2k_B T}, \quad (29)$$

where $\Delta(T)$ is the temperature-dependent superconducting energy gap of the electrode. In the superconducting regime, where thermal excitations are negligible, the equation simplifies to:

$$R_n = \frac{\pi \Delta}{2I_c e}, \quad (30)$$

$$= \frac{\pi^2 L_J \Delta}{\Phi_0 e}. \quad (31)$$

This formula has some important design applications, as increasing the cross-sectional area of the junction creates additional parallel tunneling pathways, effectively reducing R_n and increasing I_c . Similarly, thinning the insulating barrier enhances the tunneling probability, further lowering R_n and increasing I_c [21].

This device characteristics are essential in practice for physically manufacturing devices with specific Josephson energy (E_J) and inductance (L_J). However, it is not a part of our simulation

methodology. Instead, our simulations treat E_J and L_J directly as fundamental input parameters. The theoretical basis for simulating transmon behavior using these parameters is provided in Section 2.2.4.

2.2.3 SQUID-based frequency tuning

A direct-current superconducting quantum interference device (DC SQUID) consists of two Josephson junctions (JJs) connected in parallel within a superconducting loop, as shown in Fig. 5. The loop is mounted in parallel with a large shunt capacitance C_S (discussed in the next Section). It serves as a flux-tunable nonlinear element in transmon qubits, enabling *in situ* frequency control for superconducting quantum processors (QPUs).

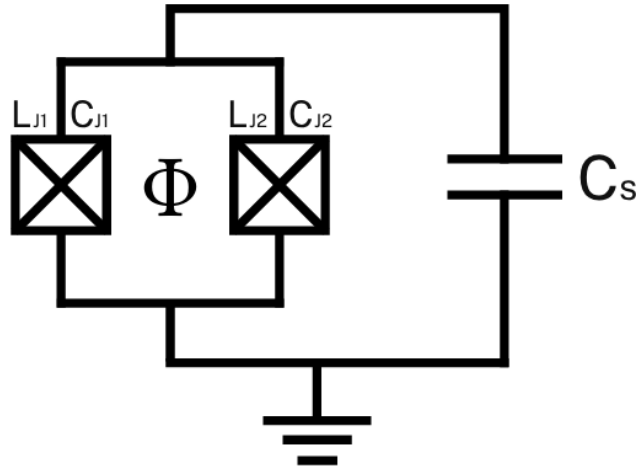


Figure 5: Lumped-element model of a DC SQUID, adapted from Ref. [35]. Magnetic flux Φ threads the loop containing two Josephson junctions: Junction 1 (E_{J1} , L_{J1}) and Junction 2 (E_{J2} , L_{J2}), connected in parallel with capacitance C_S . Flux modulation of the effective Josephson energy enables qubit frequency tunability.

The macroscopic quantum coherence of superconductors requires the superconducting wavefunction (Eq. 18) to remain single-valued after traversing the loop. This imposes flux quantization: the total magnetic flux Φ in the loop is quantized in units of the flux quantum Φ_0 :

$$\Phi = n\Phi_0, \quad n \in \mathbb{N}. \quad (32)$$

For an externally applied flux Φ_{ext} , where Φ_{ext} includes contributions from external magnetic fields and persistent currents, flux quantization constrains the phase differences φ_1^s and φ_2^s across the two JJs:

$$\varphi_1^s - \varphi_2^s = 2\pi \frac{\Phi_{\text{ext}}}{\Phi_0} \mod 2\pi. \quad (33)$$

For symmetric JJs ($I_{c,1} = I_{c,2} = I_c$), the total supercurrent is the sum of the currents through each junction:

$$I = I_c(\sin \varphi_1^s + \sin \varphi_2^s). \quad (34)$$

Substituting the flux quantization condition (Eq. 33) and applying trigonometric identities, the supercurrent becomes:

$$\begin{aligned} I &= 2I_c \sin \left(\varphi_1^s - \pi \frac{\Phi_{ext}}{\Phi_0} \right) \cos \left(\pi \frac{\Phi_{ext}}{\Phi_0} \right), \\ &\equiv I_{c,\text{eff}}(\Phi_{ext}) \sin \tilde{\phi}, \end{aligned} \quad (35)$$

where $I_{c,\text{eff}}(\Phi_{ext}) = 2I_c |\cos(\frac{\pi\Phi_{ext}}{\Phi_0})|$ is the flux-modulated critical current, and $\tilde{\phi} = \varphi_1^s - \pi \frac{\Phi_{ext}}{\Phi_0}$ is a shifted phase variable. The effective Josephson energy $E_{J,\text{eff}}$ and Josephson inductance L_J exhibit periodic modulation:

$$E_{J,\text{eff}} = 2E_J \left| \cos \left(\pi \frac{\Phi_{ext}}{\Phi_0} \right) \right|, \quad (36)$$

$$(37)$$

$$L_J(\Phi) = \frac{\Phi_0}{2\pi I_{c,\text{eff}}} = \frac{\Phi_0}{4\pi I_c |\cos(\pi\Phi_{ext}/\Phi_0)|}, \quad (38)$$

with $L_{J0} = \Phi_0/(4\pi I_c)$ being the effective inductance of the SQUID when no external magnetic flux threads the loop. The expression for L_J is valid for $|\frac{\Phi_{ext}}{\Phi_0} - n| < 1/2$, $n \in \mathbb{Z}$. This periodic dependence enables detection of single-flux-quantum changes and dynamic control of superconducting circuits.

SQUIDs provide exceptional flux sensitivity and frequency tunability, enabling key applications in superconducting qubits and quantum sensing. While their flux-dependent $E_{J,\text{eff}}$ and L_J allow for in situ frequency control and noise suppression at sweet spots, they also introduce sensitivity to magnetic noise and crosstalk. Despite these challenges, SQUIDs remain essential components of today state of the art quantum circuits, offering unmatched versatility despite their operational complexities.

2.2.4 Transmon qubit design

Having established the Josephson junction physics in Section 2.2.2, we now introduce the practical qubit implementation that will be used in our simulations. This section presents the transmon qubit. The transmon qubit was developed to weaken charge noise susceptibility in the CPB. It is a charge-insensitive design derived from the Cooper-pair box (CPB) that specifically targets charge noise susceptibility, a dominant decoherence mechanism in quantum processors. The CPB architecture implements the artificial atom concept through its nonlinear potential, but requires critical modifications to achieve viable coherence times.

2.2.4.1. The Cooper-pair box

The Cooper-pair box (CPB) architecture is shown in Fig. 6:

- Its physical implementation is represented in Fig. 6 (a): A superconducting island (in green) is connected to a reservoir via a Josephson junction (in orange). The island is electrostatically biased by a gate voltage V_g through the gate capacitance C_g .
- Its circuit representation is shown in Fig. 6 (b): The island (in green) is connected to the ground via the JJ and the stray capacitance C_s in parallel. On the other side, it is connected to the voltage source V_g via the gate capacitance C_g .

The gate voltage V_g controls the Cooper pair occupation on the island. Qubit manipulation occurs via microwave excitation at the $|0\rangle \rightarrow |1\rangle$ transition frequency. While state-of-the-art implementations typically incorporate SQUIDs for flux tuning (Section 2.2.3), this work utilizes single-junction transmons to isolate fundamental behavior.

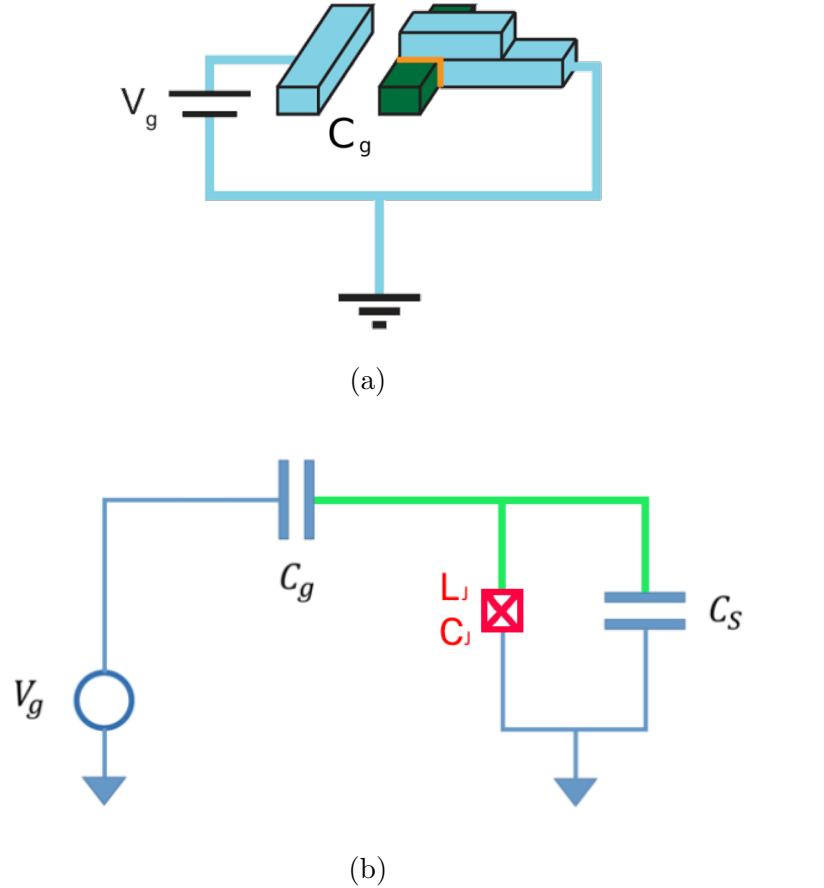


Figure 6: Cooper-pair box architecture. **(a)** Physical structure of a Cooper-pair box, adapted from Ref. [25]: The superconducting island (green) is coupled to a reservoir via a Josephson junction (orange). A gate voltage V_g biases the island through the gate capacitance C_g . **(b)** Equivalent circuit diagram of the CPB, adapted from Ref. [21]. The Josephson junction with critical parameters (E_J, L_J) and shunt capacitance C_s are connected in parallel between the island and ground. The island is coupled to the gate voltage V_g via C_g .

The total Hamiltonian of the system is similar to the one of the quantized LC oscillator, derived from Eq. [120](#) from Annex [B.1](#), recalled here for convenience:

$$\hat{H}_{\text{LC}} = \frac{1}{2}C\hat{V}^2 + \frac{1}{2}L\hat{I}^2. \quad (39)$$

The difference now is that the inductance term is replaced by the non-linear inductance from the Josephson junction. In an ideal superconducting qubit, the only voltage applied on the coupling capacitor is from the microwave source. However, in practice, there is always a DC voltage which is present from the external environment. This DC voltage causes an offset charge on the shunt capacitor, that affect the charge present in our qubit and requires the Hamiltonian to be modified. To achieve that goal, we first go from the charge basis $\hat{Q}$ to the charge number basis $\hat{n} = \hat{Q}/2e$, where $2e$ represents the charge of a Cooper pair and $\hat{n}$ is the quantized charge, which is the number of excess Cooper pairs in the island:

$$\frac{\hat{Q}}{2C} = \frac{(2e\hat{n})^2}{2C_\Sigma} = 4E_C\hat{n}^2. \quad (40)$$

$E_C = e^2/2C_\Sigma$ is the charging energy for a single electron, and $C_\Sigma = C_g + C_J + C_S$ is the total island capacitance. To account for the charge buildup on the shunt capacitor, we subtract the gate-induced offset charge $n_g = C_g V_g/2e$ from the quantized charge $\hat{n}$, mainly:

$$\hat{H}_{\text{CPB}} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\left(\frac{2\pi\hat{\Phi}}{\Phi_0}\right). \quad (41)$$

2.2.4.2. Energy and Cooper-pair tunneling

The resulting Hamiltonian describes the CPB, which is also known as the charge base qubit due to the dependence of the Hamiltonian on offset charge present on the shunt capacitor. This formulation reveals the two competing energy scales governing quantum processor performance: the Josephson energy E_J that sets the tunneling rate through the cosine potential, while the offset charge n_g introduces sensitivity to charge noise that limits coherence times through low-frequency fluctuations. The total Hamiltonian of the system have great similarities with the one of the quantized LC oscillator (Annex [B](#)). We recall the equation (Eq. [116](#)) here for clarity:

$$\hat{H}_{\text{LC}} = \frac{\hat{Q}^2}{2C} + \frac{1}{2}C\omega_r^2\hat{\Phi}^2, \quad (42)$$

where $\omega_r = 1/\sqrt{LC}$. Similarly, the CPB Hamiltonian is the sum of an electrostatic term (charge operator) and a kinetic term (flux operator). The first term in Eq. [41](#) is the electrostatic energy; it depends on the polarization charge, which represents the preferred amount of excess island charge (in electron). The Josephson Junction creates a second kinetic energy term. Josephson coupling enables Cooper pair tunneling through the barrier, i.e., to change the charge state of the qubit.

2.2.4.3. Reducing charge noise susceptibility

The shape of the energy bands and composition of the wavefunctions can be expressed purely in terms of the ratio E_J/E_C , with the characteristic tunneling energy E_J being much smaller than the charging energy, such that $E_J \ll 4E_C$. This define the charge qubit regime in which the qubit

state can be accurately expressed in terms of the nearest two charge states.

In the charge qubit regime, the most important effect, shown in Fig. 7, is that the Josephson coupling lifts the degeneracy present when the Cooper-pair box is electrostatically frustrated at $n_g = 1$. At this bias condition, the ground and excited eigenstates are superpositions of charge states $(|0\rangle \pm |1\rangle)/\sqrt{2}$ with the energy difference between the ground and the excited state being equal to E_J . When the energy difference between the ground and the excited state is bigger than $k_B T$, there will be no thermal population of the first (or any higher lying) excited state. By operating qubit around those optimal points, known as “sweet-spots” ($n_g = 1$), the transition energy becomes insensitive to first order fluctuations of n_g [25].

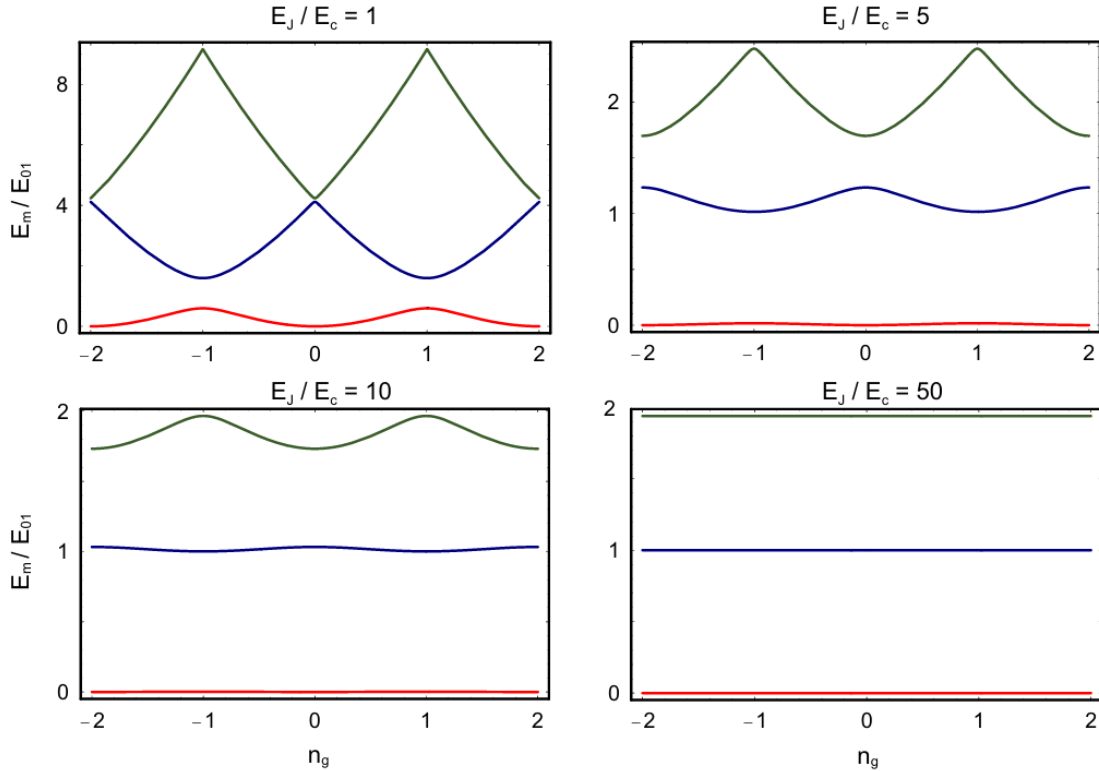


Figure 7: CPB energy bands of the first three energy levels E_m , $m = \{1, 2, 3\}$, as function of the gate-induced offset charge n_g at different E_J/E_C ratios, adapted from Ref. [25]. The charge dispersion decrease exponentially with increasing E_J/E_C , reducing the dependence of the CPB on charge fluctuations. Increasing E_J/E_C also reduce anharmonicity. At $E_J/E_C \approx 50$, the charge dispersion can be neglected and the levels resemble a harmonic oscillator.

2.2.4.4. Transmon qubit

The transmon qubit was developed to weaken charge noise susceptibility in the CPB. The crucial modification distinguishing the transmon from the CPB is a shunting connection of the two superconductors via a large capacitance C_S , accompanied by an increase in the gate capacitance C_g . This modification reduces the charging energy E_C by increasing the total island capacitance C_Σ . Unlike the charge qubit, the transmon is designed to operate in a regime of significantly increased ratio of Josephson energy E_J and charging energy E_C , called the transmon regime: $E_J/E_C > 50$.

The transmon exploits the fact that charge dispersion decreases exponentially with $\sqrt{E_J/E_C}$, while anharmonicity α decays algebraically. It is because the anharmonicity is defined as:

$$\alpha \equiv \omega_{12} - \omega_{01} \approx -\frac{E_C}{\hbar} + \mathcal{O}\left(\frac{E_C^2}{E_J\hbar}\right). \quad (43)$$

The anharmonicity expression derives from the transmon's energy spectrum, where the $|0\rangle \rightarrow |1\rangle$ transition energy is:

$$\hbar\omega_{01} \approx \sqrt{8E_JE_C} - E_C, \quad (44)$$

This allow the transmon to achieve drastically reduced charge noise sensitivity, enhanced coherence times, and increased qubit-photon coupling strength ($|\mathbf{d}| \propto (E_J/E_C)^{1/4}$, see Section 2.4), while retaining sufficient anharmonicity for selective control ($|\alpha|/2\pi \sim 200\text{--}300$ MHz). The parameter E_J (Josephson energy) will serve as key optimization variables in subsequent device simulations. The inverse relationship $E_J \propto 1/L_J$ further links these energies to the junction's effective inductance L_J .

2.3 Transmission line resonator

2.3.1 Transmission line

Transmission lines (TL) are structures designed to guide and contain electromagnetic (EM) waves with minimal losses, they also enable high-fidelity microwave signal propagation in superconducting quantum processors by delivering control pulses to qubits and routing readout signals to exterior. In microwave engineering, where the length of the line is smaller than the wavelength, a carefully designed TL is needed. An adapted TL with characteristic impedance $Z_0 = 50 \, \Omega$ and length L is shown in Fig. 8 (a). The distributed element construction, consisting of an infinite series small lumped elements, is shown in Fig. 8 (b). It has the same impedance per unit length as the transmission line:

$$Z_0 = \sqrt{\frac{R_l + j\omega L_l}{G_l + j\omega C_l}}, \quad (45)$$

where L_l , R_l and C_l are the inductance, resistance and capacitance per unit length. Note that R_l can usually be associated with conductor loss, while the conductance per unit length G_l represented by a shunt resistor separating the two conductors can be thought of as leakage in the dielectric. Around its resonance frequency, the TL can be represented by an infinite series of parallel RLC oscillators, called the lumped element model, as shown in Fig. 8 (c).

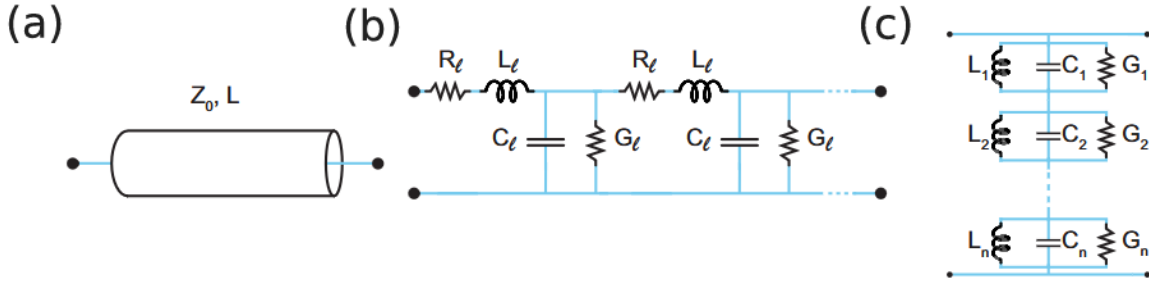


Figure 8: Transmission line models, adapted from Ref. [25]. **(a)** Schema of a TL with characteristic impedance Z_0 and length L . **(b)** Distributed element construction of a TL, which correctly predict the DC impedance. **(c)** Lumped-element circuit model of the TL, where the n -th parallel section has inductance L_n , capacitance C_n , and conductance G_n .

From the telegrapher's equations, we derive two decoupled partial differential equations (see Annex C for full derivation): one for voltage $v(x, t)$ and one for current $i(x, t)$. For superconducting qubits, which maintain superconductivity below the critical temperature, the lossless line approximation is well-justified. We model this by neglecting R_l and G_l . Differentiating the voltage equation with respect to x and combining with the current equation yields:

$$\frac{\partial^2 v(x, t)}{\partial t^2} - c^2 \frac{\partial^2 v(x, t)}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^2 i(x, t)}{\partial t^2} - c^2 \frac{\partial^2 i(x, t)}{\partial x^2} = 0, \quad (46)$$

where $c = 1/\sqrt{L_l C_l}$ is the wave propagation speed. The symmetric current equation is obtained by differentiating the current telegrapher's equation with respect to x and applying the voltage relationship. The L_l and C_l parameters directly determine the phase velocity:

$$v_p = \frac{1}{\sqrt{L_1 C_1}}, \quad (47)$$

and thus the wavelength $\lambda = 2\pi v_p/\omega$ and the phase constant $\beta_l = \omega\sqrt{L_1 C_1}$. At resonance, the transmission line length and phase constant β_l satisfy a specific condition that causes constructive interference between forward and backward waves. The resulting standing wave pattern emerges from solutions to the telegrapher's equations. In this pattern, energy oscillates between magnetic energy stored in distributed inductance and electric energy stored in distributed capacitance. For transmon qubits, this standing wave is critical: its electric field antinodes (points of maximum amplitude) provide locations where qubits are placed to maximize coupling strength, enabling high-fidelity quantum operations. These distributed oscillations are functionally identical to those in a lumped RLC circuit.

2.3.2 Resonator Q-factors and coupling

As we have seen in Section 2.3.1, a transmission line close to resonance can be modeled as an infinite series of parallel RLC resonators. This equivalence is essential for quantifying critical resonator parameters, particularly the angular resonance frequency and quality factors presented below, which fundamentally govern device performance. In the following section, we will first analyze an isolated single RLC circuit operating at its fundamental mode to derive and interpret these key parameters.

2.3.2.1. RLC resonator

The input impedance of a RLC circuit is given by [36], recalled in Eq. 48:

$$Z_{\text{in}}(\omega) = \left(R + \frac{1}{i\omega L} + i\omega C \right)^{-1}. \quad (48)$$

The angular resonance frequency ω_r is the frequency for which the input impedance is purely real. Since the resistance in Eq. 48 is real, this condition translates to:

$$\frac{1}{i\omega_r L} + i\omega_r C = 0, \quad (49)$$

$$\omega_r^2 = \frac{1}{LC}. \quad (50)$$

Since the frequency is positive:

$$\omega_r = \frac{1}{\sqrt{LC}}. \quad (51)$$

2.3.2.2. Quality factor

The internal quality factor of a resonant circuit is a dimensionless quantity that measures its efficiency in storing and dissipating energy. It quantifies the number of oscillations a system can sustain before it reaches equilibrium. The quality factor is inversely proportional to the decay rate κ at which the system loses energy. At its resonance frequency:

$$Q_i = \frac{\omega_r}{2\kappa} = \omega_r RC, \quad (52)$$

where the second equality is derived from $1/2\kappa = RC$. The internal quality factor can also be defined as the ratio of the total energy stored in the system to the power dissipated per cycle, multiplied by the resonant angular frequency. At resonance:

$$Q_i = \frac{E_{\text{total}}}{P_{\text{dissipated}}} \omega_r \quad (53)$$

The energy stored in the circuit can be expressed in terms of the root mean square (rms) voltage V_{rms} across the capacitor or the current I_L through the inductor. On resonance, the electric energy inside the capacitor is equal to the magnetic energy inside the inductor. As a result, the total energy is:

$$E_{\text{total}} = CV_{\text{rms}}^2 = LI_L^2. \quad (54)$$

The dissipated power is found through Ohm's law:

$$P_{\text{dissipated}} = I_R V_{\text{rms}} = \frac{V_{\text{rms}}^2}{R}, \quad (55)$$

where I_R is the current through the resistor. When a resonator is coupled to an external circuit, see Fig. 9, the total input impedance $Z_{\text{in}}(\omega)$ is modified by the coupling capacitance C_c . As a consequence, the frequency and quality factor are both modified by the coupling of the system to the environment. This is described by Eq. 56:

$$Z_{\text{in}}^c(\omega) = \frac{1}{i\omega C_c} + Z_{\text{in}}. \quad (56)$$

Where $Z_r(\omega)$ is the intrinsic impedance of the resonator and C_c is the coupling capacitance. The external coupling influences both the resonance frequency and the quality factor.

The total quality factor Q is the combined effect of both the internal quality factor Q_i and the external coupling quality factor Q_c . The total quality factor is given by the parallel combination of these two factors:

$$Q = \left(\frac{1}{Q_i} + \frac{1}{Q_c} \right)^{-1}. \quad (57)$$

The total quality factor Q quantifies resonator energy loss. The photon lifetime τ_γ is related to Q by Eq. 58:

$$\tau_\gamma = \frac{Q}{\omega_r}, \quad (58)$$

while the decay rate is given by:

$$\kappa = \frac{\omega_r}{Q}. \quad (59)$$

Higher Q yields longer photon lifetimes and narrower linewidth. This total Q also combines internal losses (Q_i) from intrinsic defects and external coupling (Q_c) through controlled photon leakage to measurement circuitry. The external quality factor Q_c , specifically determines leakage rate: low Q_c (strong coupling) enables fast leakage for quick measurements but reduces photon lifetime and broadens linewidth, while high Q_c (weak coupling) minimizes leakage to preserve coherence. Consequently, over-coupled designs ($Q_c \ll Q_i$) maximize leakage for fast readout applications like qubit measurement, whereas under-coupled resonators ($Q_c \gg Q_i$) minimize leakage to achieve high coherence for quantum memory implementations.

For small coupling capacitance C_c , the external quality factor Q_c can be approximated as:

$$Q_c \approx \frac{2\pi}{2\omega_r Z_0 C_c^2}. \quad (60)$$

When coupled to an external circuit via capacitance C_c , the resonance frequency is shifted from its original uncoupled value (given by Eq. [51](#)) to a new coupled resonance frequency:

$$\omega_r^c = \frac{1}{\sqrt{L(C + 2C_c)}}. \quad (61)$$

In the regime of small coupling ($C \gg C_c$), the impact on the fundamental frequency is negligible. The physical length of a superconducting resonator is the key parameter that governs its fundamental resonance frequency.

2.3.2.3. Coupling factor and readout

The on-resonance transmission coefficient, or S-parameter S_{21} , is often used to characterize the response of the resonator in a coupled circuit. It describes how much of the input signal is transmitted through the coupled resonator, taking into account both the internal impedance and the effects of the coupling circuit [11](#). By probing the transmission spectrum (detailed in Section [2.4.3](#)), the resonance frequency extraction relies on the analysis of the transmission spectrum. The on-resonance transmission coefficient S_{21} is defined by [37](#):

$$|S_{21}| = \frac{1}{1 + g}, \quad (62)$$

where g is the coupling factor that quantifies the coherent energy exchange between the resonator mode and a coupled quantum system (e.g., a qubit). It is defined as:

$$g = \frac{Q_i}{Q_c}, \quad (63)$$

The total quality factor depends g through the following relation:

¹¹Non-demolition techniques allow quantum state measurement without destroying the state during readout. Among them, dispersive readout works by coupling a qubit to this resonator so the qubit's state slightly shifts the resonator's frequency.

$$Q = \frac{Q_i}{1 + g}. \quad (64)$$

This reveals a fundamental design conflict: maximizing Q (minimizing linewidth) requires minimizing g , while achieving measurable resonance dips ($|S_{21}| \ll 1$) demands maximizing g . The resolution depends on application. For device characterization (extracting Q_i , ω_r) the critical coupling $g = 1$ optimizes sensitivity, while for qubit readout, over-coupling ($g \gg 1$) prioritizes speed despite Q reduction.

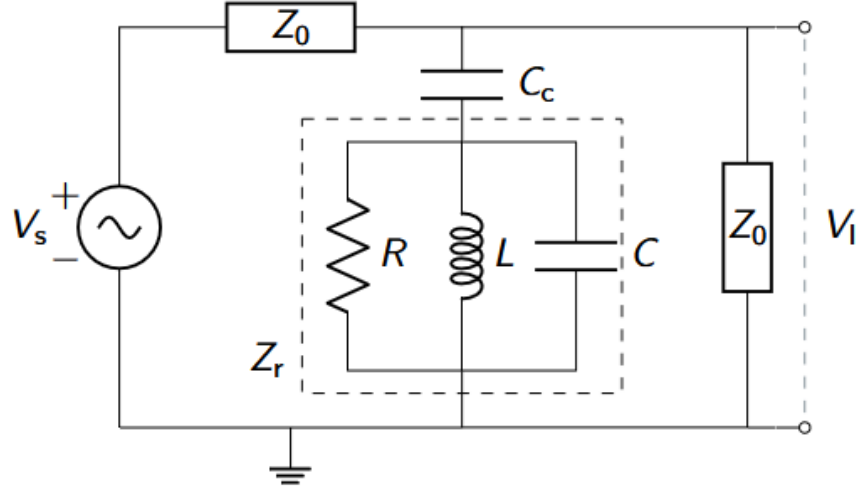


Figure 9: Circuit diagram of a lumped-element parallel RLC circuit connected via a coupling capacitance C_c to a voltage source via a TL of impedance Z_0 . Image adapted from Ref. [36].

2.3.3 Coplanar waveguide resonator

There exist a variety of transmission line designs. For this work, we have chosen the coplanar waveguide (CPW) configuration, the most used for superconducting qubit applications.

2.3.3.1. Description

The CPW has a very simple structure: a central conductive line surrounded by two neighboring ground planes on top of a dielectric substrate, as seen in Fig. [10]. The center conductor has width w and is separated from the ground plane by a distance s on each side. We consider that the CPW is of constant thickness. Both conductors are typically deposited on a low-loss dielectric substrate of absolute permittivity $\epsilon = \epsilon_r \epsilon_0$ and thickness much larger than the characteristic dimensions of the CPW; ϵ_0 being the free space permittivity and ϵ_r the substrate dielectric constant.

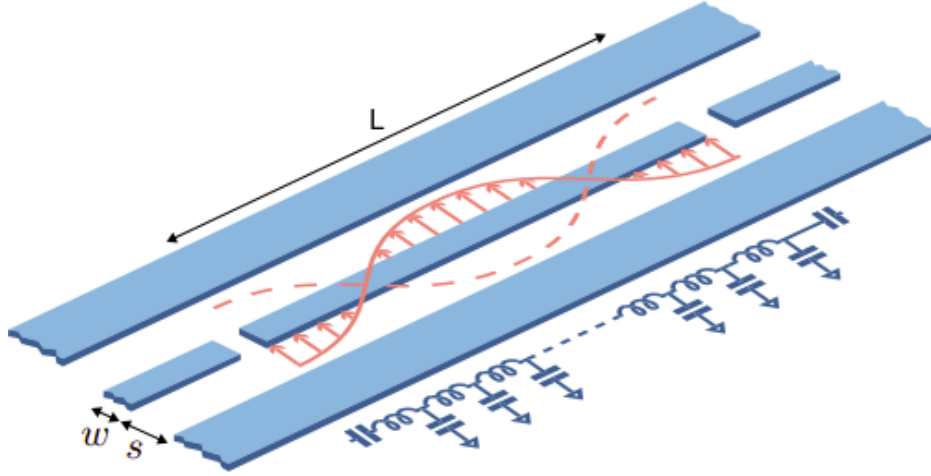


Figure 10: Schematic representation of a $\frac{\lambda}{2}$ resonator, adapted from Ref. [24]. The TL has length L , center conductor width w and separation from ground plane s . The electric field is represented in red and the corresponding distributed model is shown below.

This CPW design offers many advantages. The 2D geometry makes the fabrication process simple, avoiding complex multi-layer fabrication techniques. All components (qubit, resonator, control lines,...) are lithographically patterned on a single chip and the design is compatible with standard processes like photolithography and electron-beam lithography. As seen in Fig. [11](a), the electromagnetic field is concentrated in the gaps between the center conductor and the ground planes, diminishing dielectric losses in the substrate. A CPW resonator confines electromagnetic waves by defining specific boundary conditions at its endpoints. When both ends are left open (achieved by creating gaps between parts of the center conductor), standing waves are formed with current nodes (minima) and voltage antinodes (maxima) at each termination. This configuration, known as a $\lambda/2$ resonator and shown in Fig. [10], exhibits a fundamental resonance frequency :

$$f_0 = \frac{v_0}{2L}, \quad (65)$$

where v_0 is the effective wave propagation speed in the CPW (typically $v_0 \approx 1.3 \times 10^8$ m/s for common substrates) and L is the physical resonator length. Higher-order harmonics occur at integer multiples, as shown in Fig. [11](b):

$$f_m = (m+1)f_0, \quad m = 0, 1, 2, \dots \quad (66)$$

When one end is shorted to ground (creating a voltage node, $V = 0$, and current antinode), the system becomes a $\lambda/4$ resonator with fundamental frequency:

$$f_0 = \frac{v_0}{4L}. \quad (67)$$

This is the configuration we will use for our simulation cell, the grounded end then enforces a voltage minimum while allowing maximum current flow. Both resonator types enable precise frequency engineering through dimensional control (L) and material selection ($v_0 = c_0/\sqrt{\epsilon_{\text{eff}}(f,t)}$, where $\epsilon_{\text{eff}}(f,t)$ is the substrate-dependent effective dielectric constant, function of the frequency and the thickness).

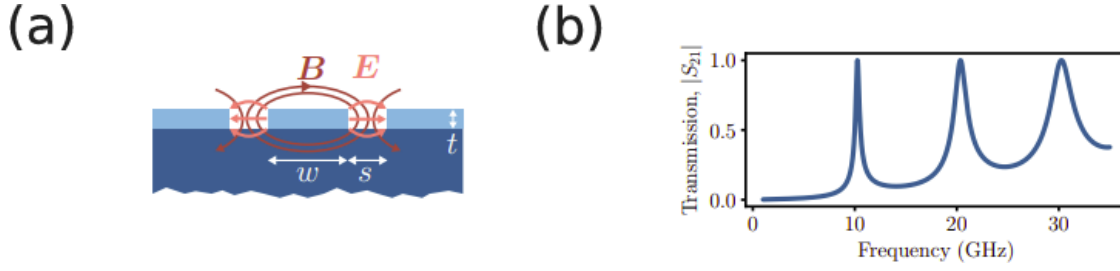


Figure 11: Image adapted from Ref. [24]. **(a)** Cross-section of a CPW showing EM field distributions: electric field (E) and magnetic field (B). Key dimensions are represented: center conductor width w , gap width s , and metal thickness t . **(b)** Magnitude of the transmission coefficient $|S_{21}|$ versus frequency (GHz). The peaks correspond to resonant modes of the system.

2.3.3.2. Critical parameters

Finding the correct values for the critical parameters of the resonator like characteristic impedance or capacitance is not a trivial problem. Fortunately for us, some values for the CPW have been found using an extended computer aided design (CAD) formalism [38]. A concise summary is also provided in Appendix A of Ref. [24]. Following Ref. [38], the shunt capacitance per unit length for a conventional CPW (zero thickness conductor) is given by:

$$C_{\text{Sl}} = 2\epsilon_0(\epsilon_r - 1) \frac{K(k_1)}{K(k'_1)} + 4\epsilon_0 \frac{K(k_0)}{K(k'_0)}, \quad (68)$$

where $K(k)$ is the complete elliptic integral of the first kind for a modulus k (where $0 \leq k \leq 1$):

$$K(k) = F\left(\frac{\pi}{2}, k\right) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}. \quad (69)$$

The geometric parameters are defined as:

$$k_0 = \frac{s}{s + 2w}, \quad (70)$$

$$k'_0 = \sqrt{1 - k_0^2}, \quad (71)$$

$$k_1 = \frac{\sinh\left(\frac{\pi s}{4H}\right)}{\sinh\left(\frac{\pi(s+2w)}{4H}\right)}, \quad (72)$$

$$k'_1 = \sqrt{1 - k_1^2}, \quad (73)$$

where H is the substrate thickness. The zero-thickness condition holds when the film thickness is much smaller than the sum of the center conductor width and twice the gap width ($t \ll s + 2w$). The total transmission line capacitance is:

$$C = C_{\text{Sl}} \times L. \quad (74)$$

Adapted transmission lines typically used in RF systems, with characteristic impedance of $50 \, \Omega$, minimize the signal reflections and power losses. The characteristic impedance, calculated in Ref. [39], is:

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_{\text{eff}}(f, t)}} \frac{K(k_0)}{K(k'_0)}. \quad (75)$$

To find the characteristic impedance, the frequency dependent effective dielectric constant, is required. The effective dielectric constant for zero-thickness is derived through:

$$\epsilon_{\text{eff}}(f, 0) = 1 + q(\epsilon_r - 1), \quad (76)$$

$$q = \frac{1}{2} \frac{K(k_1)K(k'_0)}{K(k'_1)K(k_0)}, \quad (77)$$

where q is the filling factor or participation ratio, that represent the fraction of electric field energy concentrated in the substrate rather than in the air. Although it is possible to use the formulas above to analytically derive the impedance, this method ignores the potential impact of the kinetic inductance of the superconducting material, so should only be used provided the kinetic inductance is much lower than the external inductance, i.e. if the geometries of the circuit are all larger than the coherence length and penetration depth, the kinetic inductance can be practically ignored. This approximation typically fails for transmon qubit since narrow Josephson junctions (widths ~ 100 nm) exhibit significant kinetic inductance, requiring specialized modeling beyond Eq. [75](#). The analytical expressions employed in this section provides valuable qualitative insight into parameter dependencies and reasonable first-order approximations of resonator behavior. While final validation requires EM simulations to account for kinetic inductance and boundary effects, these analytical solutions provide essential guidance to speed-up the design workflow when time is limited.

2.4 Cavity quantum electrodynamics

2.4.1 Background

The readout mechanism in superconducting qubits interrogates the qubit state through a microwave drive coupled to a high-Q resonator. Such systems require full quantum field treatment and cannot rely on semiclassical approximations. Consequently, the electromagnetic field must be quantized through second quantization, promoting the classical field to operators. A rigorous description of this process is provided by Mandl and Shaw [40] and will not be detailed here. Cavity quantum electrodynamics (cQED) provides the theoretical framework for coherent interaction between discrete quantum emitters (e.g., atoms) and quantized EM modes confined within a high-Q cavity. Let's consider the prototypical optical cQED system, shown in Fig. 12a. We isolate a single EM cavity mode inside the cavity, with decay rate κ . Then, a two-level atom is introduced, traversing the cavity in a transition time t_{transit} and atomic spontaneous decay rate γ . Resulting from interaction between the oscillating electric field $\mathbf{E}_0$ in the EM wave and the atomic dipole moment $\mathbf{d}$, a light-matter coupling strength $g = \mathbf{d}\mathbf{E}_0/\hbar$ appears. As illustrated in Fig. 12b, the system is modeled as a lumped-element circuit comprising a resonator and a transmon qubit capacitively coupled via C_g . The resonator, characterized by inductance L_r and capacitance C_r , behaves as a harmonic oscillator when quantized, analogous to a single EM mode in an optical cavity. The transmon qubit, acting as an artificial atom, consists of a Josephson junction with energy E_J and a shunt capacitance C_S . This configuration yields a weak anharmonicity α . Under cryogenic conditions ($k_B T \ll \hbar\omega_q$), the transmon's anharmonicity α energetically isolates its two lowest levels ($|g\rangle, |e\rangle$), permitting a two-level approximation. Capacitive coupling mediates a coherent qubit-resonator interaction of strength g . To maintain the two-level approximation during dynamics, the system operates in the regime $|\alpha| \gg \kappa, \gamma$, where anharmonicity exceeds dissipation rates, suppressing leakage to higher states [24]. This coupled system constitutes a circuit quantum electrodynamics (cQED) implementation..

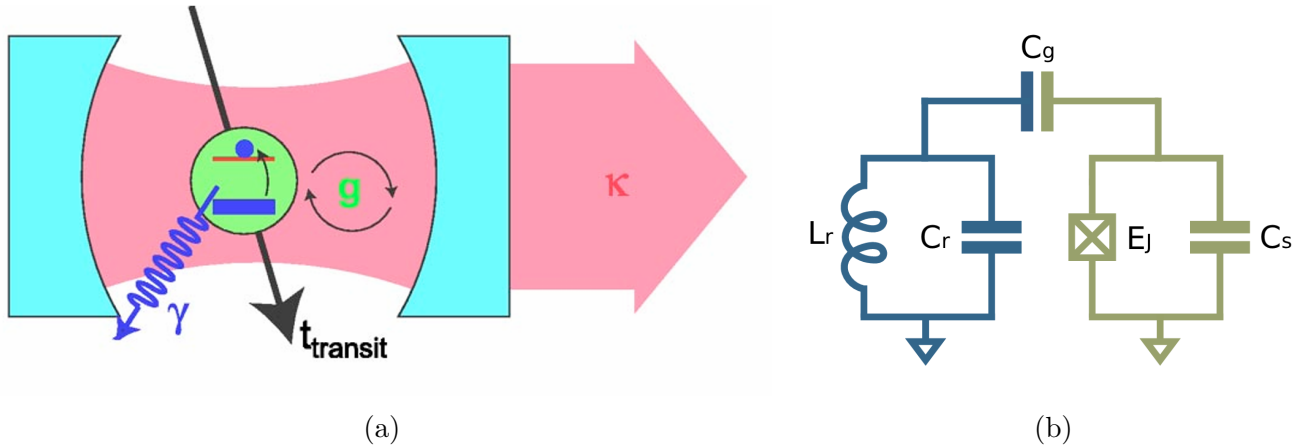


Figure 12: **(a)** Optical cQED, adapted from Ref. [14]: A single atom (green sphere) traverses an optical resonator formed by mirrors (blue), interacting with a quantized EM field (red). The atom-cavity coupling strength g mediates coherent energy exchange, with dissipation from atomic spontaneous emission (γ) and cavity photon leakage (κ). **(b)** CQED realization, adapted from Ref. [24]: Lumped-element model of a transmon qubit (green; Josephson energy E_J) capacitively coupled via C_g to an LC resonator (blue; inductance L_r , capacitance C_r). This architecture produces qubit-resonator interaction through the capacitor C_g .

2.4.2 Jaynes-Cummings model

A theoretical model describing the coherent behavior of a two-level atom interacting with a single quantized mode in an optical cavity via dipole coupling is the Jaynes-Cummings model [12]. The Hamiltonian that describes the full system is:

$$\hat{H}_{\text{JC}} = \underbrace{\hbar\omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right)}_{\text{cavity}} + \underbrace{\frac{\hbar\omega_q}{2} \hat{\sigma}_z}_{\text{qubit}} + \underbrace{\hbar g (\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+)}_{\text{dipole coupling}}, \quad (78)$$

The first term represents the energy of the quantized EM field, described by Fock states $|n\rangle$ with photon occupation numbers $n \in \{0, 1, 2, \dots\}$. The second term describes the transmon qubit modeled as a two-state system, with quantum states denoted by the basis $\{|g\rangle, |e\rangle\}$ representing the ground and excited states, respectively, as illustrated in Fig. 13. The energy difference between $|g\rangle$ and $|e\rangle$ is $\hbar\omega_q$. The third term represents a dipole interaction where the system can absorb ($\hat{\sigma}^+ \hat{a}$) or emit ($\hat{a}^\dagger \hat{\sigma}^-$) photons at rate g , where $\hat{\sigma}_z \equiv |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli-Z operator in the qubit basis, and $\hat{\sigma}^+$, $\hat{\sigma}^-$ are the raising and lowering operators satisfying $\hat{\sigma}^+ = |e\rangle\langle g|$, $\hat{\sigma}^- = |g\rangle\langle e|$. The JC Hamiltonian admits exact analytical solutions that reveal the formation of dressed states (hybridized energy eigenstates arising from the coherent interaction between the qubit and resonator). At resonance, when the detuning $\Delta = \omega_q - \omega_r = 0$, these dressed states exhibit key quantum features illustrated in Fig. 13. The coupling term lifts the degeneracy of bare states within each excitation manifold. For states sharing the same total excitation number n , the energy splitting is $2g\sqrt{n}$.

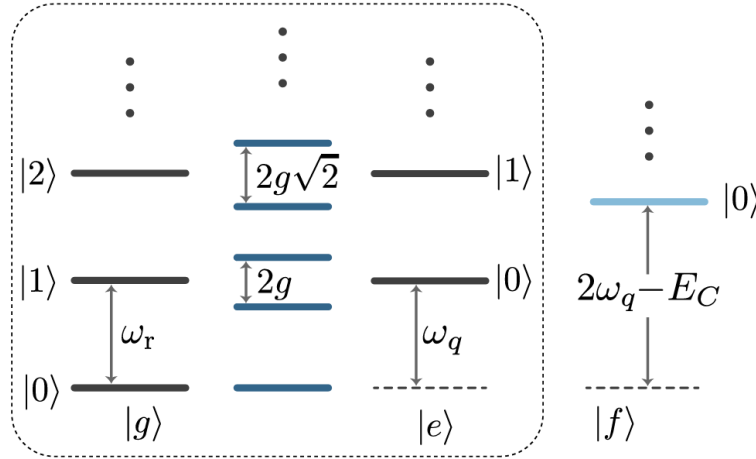


Figure 13: In the resonant JC system ($\omega_q = \omega_r$). The left panel contrasts uncoupled eigenstates (gray) with dipole-coupled dressed states (blue). The dipole interaction lifts degeneracies within each n -excitation manifold, producing energy splittings of $2g\sqrt{n}$. The transmon's level $|f\rangle$ (light blue, external to main panel) further perturbs dressed states in manifolds with $n \geq 2$ [24].

The Jaynes-Cummings model gives the formula for the coupling factor g :

$$g = \omega_r \frac{C_g}{C_\Sigma} \left(\frac{E_J}{2E_C} \right)^{\frac{1}{4}} \sqrt{\frac{\pi Z_0}{R_K}}. \quad (79)$$

C_Σ is the total qubit capacitance, C_g the qubit-CPW coupling capacitance and $R_K = \frac{h}{2e}$ the resistance quantum.

2.4.3 Qubit readout: principles and dispersive regime

Performing measurements at zero detuning is fundamentally problematic in cQED since the dressed states are maximally entangled. The coupling will cause the system to oscillate between states $|g, n\rangle$ and $|e, n-1\rangle$ due to successive absorptions (emission) of photons by the qubit. The qubit is, by itself, never in a well-defined state; i.e., the reduced state of the qubit found by tracing over the resonator is not pure. For quantum information processing, it is thus far more practical to work in the dispersive regime where the qubit-resonator detuning is large with respect to the coupling strength, ($\hbar\Delta \gg g$). In this case, the coherent exchange of quanta between the two systems described by the interaction term in the JC Hamiltonian is not resonant, and interactions take place only via virtual photon processes. The qubit and resonator being only weakly entangled, and the JC Hamiltonian can be obtained from second-order perturbation theory [24]:

$$\hat{H} = \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar\omega_q \frac{\sigma_z}{2} + \hbar\chi\sigma_z \hat{a}^\dagger \hat{a}. \quad (80)$$

Comparing this equation with Eq. 78, we see that both of them have a resonator energy term due to the photon number and a qubit frequency term. The atom and cavity still interact through the dispersive shift term $\hbar\chi\sigma_z \hat{a}^\dagger \hat{a}$ in Eq. 80. Because it commutes with the rest of the Hamiltonian it conserves both the photon number and the atom state, but shifts the energies of both the atom and the photons. The effective qubit frequency and the effective resonator frequency are shifted compared to the bare frequencies. The qubit frequency is shifted to $\omega'_q = \omega_q \pm \chi_{01}$ while the resonator frequency is shifted to $\omega'_r = \omega_r \pm \chi_{12}$, depending on the state of the atom, see Fig. 14.

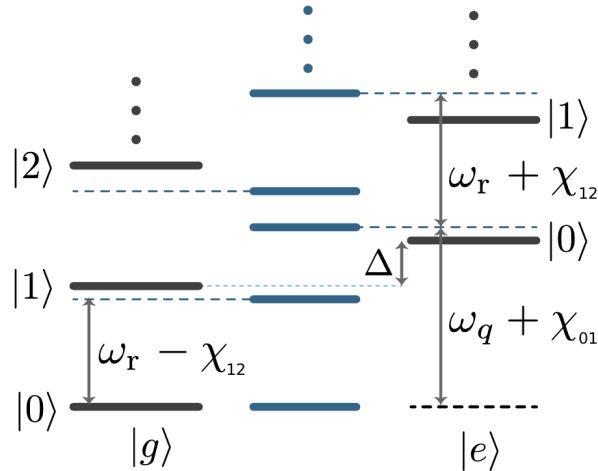


Figure 14: Energy levels in the dispersive regime: uncoupled eigenstates (gray) and coupled dressed states (blue). The dressed states energies are shifted in opposite directions by a constant that depends on the transmon state. As a result, the energy of the transition from state $|g\rangle$ to state $|e\rangle$ depends on the qubit state [24].

We describe the cross Kerr, also called effective dispersive shift, as:

$$\chi = \chi_{01} - \frac{\chi_{12}}{2}, \quad (81)$$

$$= -2 \frac{g^2}{\Delta} \frac{\alpha}{\Delta - \alpha}. \quad (82)$$

This atom state dependent cavity shift can be used to perform a quantum non-demolition (QND) measurement of the atom state. A microwave probe at frequency $\omega_p \approx \omega_r \pm \chi$ is injected through a transmission line coupled to the resonator. The qubit state ($|g\rangle$ or $|e\rangle$) shifts the resonator's resonant frequency by $\pm\chi_{12}$, altering the amplitude and phase of the transmitted or reflected probe signal. Homodyne detection measures the quadrature of the output field, mapping the qubit state to a classical microwave signal. This protocol is QND because the dispersive Hamiltonian $\hat{H}_{\text{disp}} = \hbar(\omega_r + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a}$ commutes with $\hat{\sigma}_z$, preserving the measured eigenstates during readout. Consequently, repeated measurements can be performed without perturbation, enabling error correction and fidelity enhancement in quantum computing protocols.

3 Quantum hardware design

3.1 Quantum hardware design and fabrication

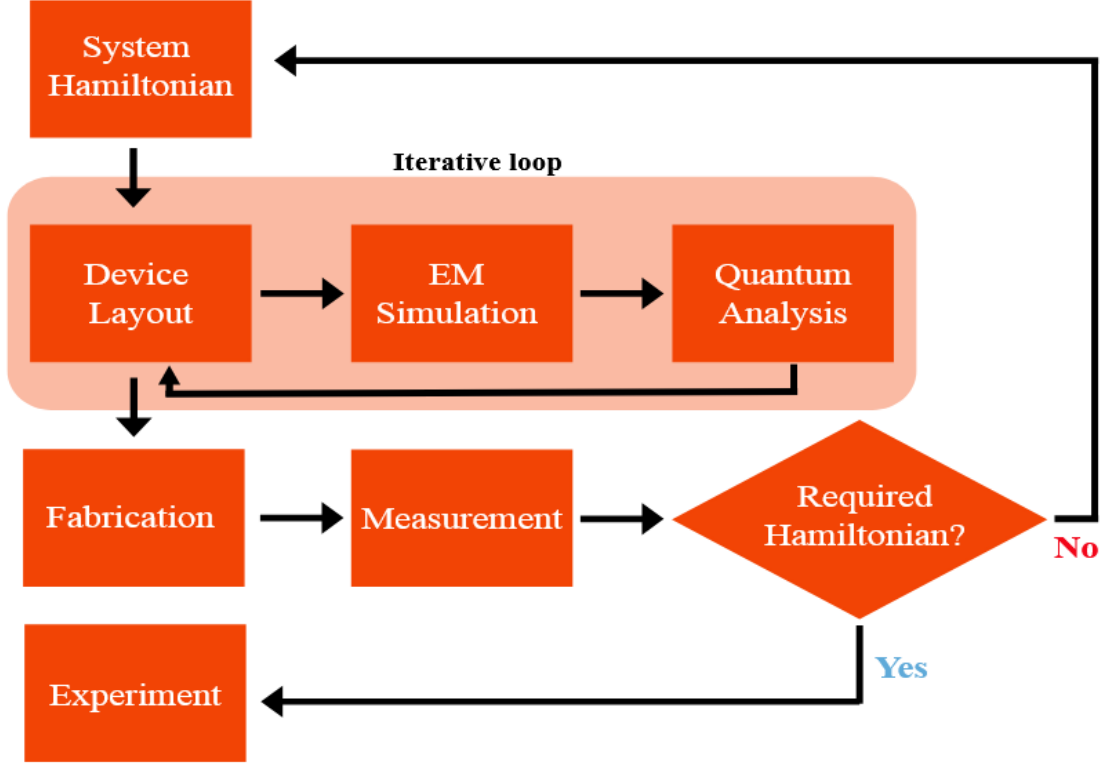


Figure 15: Quantum hardware design workflow, from Hamiltonian design to operation [41]: **1. Hamiltonian:** Define target Hamiltonian and circuit parameters. **2. Layout:** Generate physical device layout. **3. Simulation:** Compute EM fields using the appropriate tool. **4. Analysis:** Extract Hamiltonian from simulation data. **5. Validation:** Compare parameters to targets: if mismatching is observed, refine layout and simulate, iteratively; if matching is observed, proceed to fabrication. **6. Fabrication:** Manufacture physical device. **7. Measurement:** Characterization of the physical device: compare results to target parameters; refine layout until criteria are met.

The central challenge in superconducting quantum chip design is translating desired quantum behavior, encoded in the target Hamiltonian of the system, into a physical realization that reproduce the desired properties. This process demands an iterative, multi-stage workflow integrating theory, simulation, fabrication, and device validation. Below, we present this rigorous methodology [41].

The core workflow, shown in Fig. 15, begins with defining a target Hamiltonian that encodes the desired quantum behavior of our system. The initial design phase involves selecting the number and type of quantum elements (e.g., transmon qubits, $\lambda/4$ CPW,...), with a final application in mind. In most cases, the exact Hamiltonian form may be unknown, and parametric approach are often used, where key target parameters are specified to achieve the desired quantum properties. From those design specifications, an initial device layout is generated using specialized

tools (e.g., qiskit-metal [19], KQCCircuits [42]). This layout undergoes electromagnetic (EM) simulation, a critical step that models the circuit’s EM fields to predict parameters like coupling strengths, mode frequencies, and anharmonicity. Subsequent quantum analysis extracts the effective Hamiltonian from these EM results. If the Hamiltonian parameters are out of tolerance, the layout is refined and simulated again, and the simulation-analysis loop repeats (see iterative loop in Fig. 15). This iterative cycle optimizes performance prior to fabrication, minimizing costly experimental iterations. The layout, once finalized, is exported for fabrication, enabling GDSII mask design, scripting-based customization, and cleanroom compatibility. Post-fabrication, rigorous measurements assess whether the device achieves its target metrics: Hamiltonian parameter fidelity, suppressed crosstalk, appropriate resonator linewidth and coherence times. Discrepancies here may necessitate revisiting the design loop, and going through the fabrication process again.

3.2 Simulation workflow

While Fig. 15 depicts the complete quantum hardware lifecycle, this work focuses exclusively on the pre-fabrication optimization loop (red rectangle), ranging from target Hamiltonian specification to target parameter convergence, with fabrication and measurement constituting natural extensions beyond this thesis’ scope. For our chip design, we used a standardized simulation workflow that integrates the open-source quantum design tool qiskit-metal (v0.1.5) [19] with the industry-standard electromagnetic solvers Ansys electronics desktop 2023 R1 [43] for precise simulation. The simulation workflow tested in this work, depicted in Fig. 16, will be detailed in the following subsections. The important steps are briefly covered here:

1. **Design specifications** (Section 3.3 — **Status: Implemented**): We defined the quantum processor architecture by selecting quantum elements. We chose to analyze a two-pad transmon qubit capacitively coupled to a $\lambda/4$ resonator. We added a feedline inductively coupled to the resonator which has contact to the environment through launchpads. This chip is complex enough to allow us to investigate a wide range of behaviors while being simple enough to have a small simulation time. Since we don’t have a precise application in mind, the exact Hamiltonian form may be unknown, we decided to select approximate values of key parameters to achieve some desired properties. Target circuit parameters were selected: the ratio of Josephson energy over the charging energy (E_J/E_C), anharmonicity (α), coherence time T_1 , resonator frequency ω_r and qubit frequency ω_q .
2. **Device layout** (Section 3.4 — **Status: Implemented**): The initial superconducting quantum-chip layout in this thesis was generated using the qiskit-metal framework (v0.1.5) [19]. Prior to layout generation, analytical design tools were applied to determine baseline values for parameters that are otherwise difficult to gauge (such as CPW resonator dimensions), providing a physically meaningful starting point. The layout and its components (e.g., transmons, resonators, feedline) were instantiated programmatically in a jupyter notebook [20] via the qiskit-metal API using these analytically derived parameters. The software then produced the physical design, defining geometric primitives, layer assignments, and material properties. A synchronized graphical user interface (GUI) allowed visual inspection and interactive placement adjustments; any edits made in the GUI were immediately propagated back to the underlying model in the notebook environment. This step delivered a complete initial layout ready for subsequent EM validation.

3. **EM Simulations and quantum analysis** (Section 3.5): Once the initial layout was finalized, we exported the design to a 3D finite-element method (FEM) solver for physical simulations. We used Ansys electronics desktop 2023 R1 [43], a FEM solver and performed high-fidelity EM modeling. In Ansys, we derived the quantum-level parameters using two different simulation methods:
 - (a) **EPR analysis** (Section 3.6 — **Status: Partially executed**): We performed the HFSS eigenmode analysis using Ansys high frequency structure simulator (HFSS), obtaining for each targeted mode m the complex eigenfrequency ω_m and the corresponding field distributions $\mathbf{E}_m(\mathbf{r})$ and $\mathbf{H}_m(\mathbf{r})$. The Energy Participation Ratio (EPR) method [44] was implemented using the pyEPR library (v0.9.0) [45] to compute mode participations and extract circuit parameters such as qubit anharmonicity α , dispersive shifts χ , and coupling rates g . While the EPR simulations were successfully executed and the code is provided in the Annexe A, the resulting data were not critically analyzed due to time constraints. As such, no EPR-derived quantities are reported or used in the interpretation of results. The inclusion of EPR in the modeling pipeline nonetheless demonstrates readiness for future integration and complements the LOM-based approach adopted in this thesis.
 - (b) **LOM analysis** (Section 3.7 — **Status: Implemented**): We used Ansys Q3D extractor, a quasi-static EM solver, to compute the Maxwell capacitance matrix $\mathbf{C}_M$ for the full set of conductors, obtained by partitioning the device layout into distinct parts. This matrix was then used in the lumped-oscillator model (LOM) analysis [46] to reconstruct the circuit Hamiltonian and extract key quantum parameters.
4. **Design validation**: We systematically verify specifications through two stages:
 - (a) **Parametric sensitivity analysis** (Section 3.8 — **Status: Implemented**): We characterized the impact of individual design parameters (e.g., qubit pad size, resonator length, Josephson inductance) on the Hamiltonian parameters using an initial layout. By sweeping one parameter at a time and applying LOM analysis to quantify the resulting changes in chip behavior, we established how each parameter influences other parameters within the circuit.
 - (b) **Iterative optimization loop** (Section 3.9 — **Status: Proposed, not executed**): The intended next step is to return to the initial layout and compare simulated parameters from the LOM analysis against the target specifications. If discrepancies are observed, geometric parameters in the jupyter code or GUI would be modified, guided by the sensitivity analysis results. The design would then be re-simulated in Ansys (pyEPR and/or LOM) and re-validated, repeating this closed loop until all design-specification criteria are met.
5. **Fabrication output** (Section 3.9 — **Status: Proposed, not executed**): In the documented workflow, once the quantum device design met all specifications, the finalized chip layout would be exported in GDSII format (the industry standard for integrated-circuit photomasks) to KLayout [47]¹². Final checks would then be performed before submitting the files to the cleanroom for fabrication.

¹²KLayout is an open-source layout viewer and editor commonly used for semiconductor mask design and verification in cleanroom fabrication workflows.

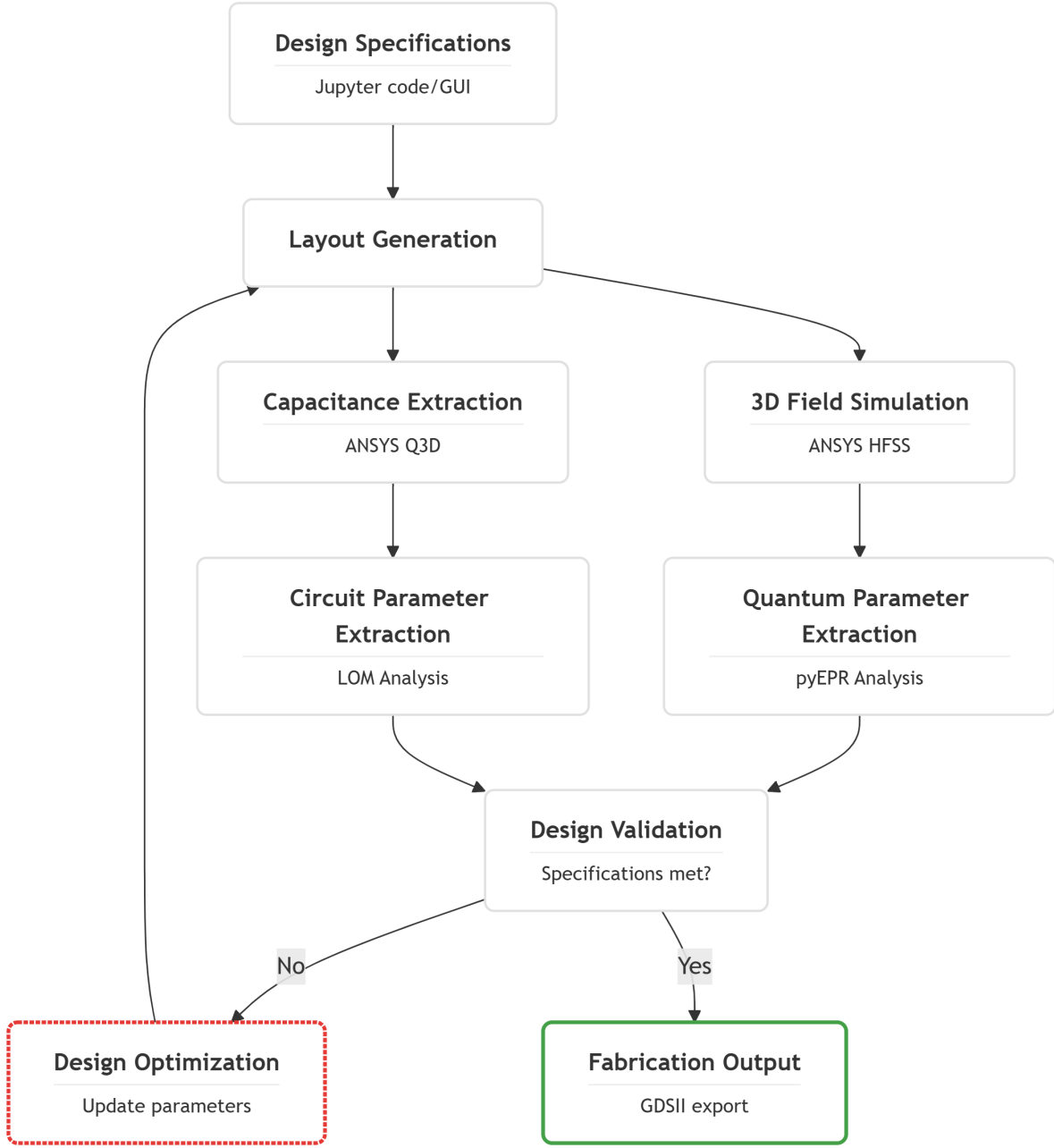


Figure 16: Integrated optimization workflow. The steps are detailed in the following subsections: design specifications (Sec. 3.3), layout generation (Sec. 3.4), Ansys simulation paths (Sec. 3.5), design validation (Sec. 3.8).

3.3 Design specifications

We now turn to the concrete design specifications that define our target Hamiltonian. In this subsection, we detail the key quantum parameters that our pre-fabrication process aims to realize:

1. Ratio of the Josephson energy over the charging energy: Target $(E_J/E_C) > 50$. Ensures operation in the transmon regime, suppressing charge noise sensitivity to enhance coherence times. This comes at the cost of reduced anharmonicity.

2. Anharmonicity: Target absolute anharmonicity $|\alpha|/2\pi = 200\text{-}300$ MHz [21], [48]. Higher values of anharmonicity would improve two-level system approximation and control selectivity. Ideally, anharmonicity should be of the order of 10% of ω_q [21]. However, fabrication constrains limit achievable values.
3. Coherence time: Target energy relaxation time $T_1 \approx 50$ μs , maximizing qubit lifetime for quantum operations.
4. Detuning: The resonator frequency is set to $\omega_r/2\pi = 7$ GHz, while the qubit frequency targets $\omega_q/2\pi \approx 4.5\text{-}5.5$ GHz, yielding a detuning $\Delta/2\pi \equiv |\omega_r - \omega_q|/2\pi \approx 1.5\text{-}2.5$ GHz. This strategic frequency separation ($|\Delta| \gg g, \kappa$, where g is the qubit-resonator coupling strength and κ the resonator decay rate) ensures operation in the dispersive regime (Sec. 2.4.3), which provides three critical advantages:
 - (a) Quantum non-demolition (QND) readout: The qubit state induces a dispersive shift $\chi \approx g^2/\Delta$ in the resonator frequency, enabling measurement without disturbing the qubit state.
 - (b) Suppressed energy exchange: The large detuning prevents resonant photon transfer ($\Delta \gg g$), freezing direct qubit-resonator energy exchange.
 - (c) Suppressed Purcell decay: By satisfying $|\Delta| \gg \kappa$, the qubit's relaxation rate via the resonator channel $\Gamma_P \approx (g^2/\Delta^2)\kappa$ is exponentially reduced.

3.4 Layout generation with qiskit-metal

3.4.1 Full chip description

Fig. 17 presents the initial layout of the superconducting quantum circuit implemented in qiskit-metal, with critical dimensions annotated by their corresponding design variables. The circuit employs a square transmon pocket geometry with 650 μm width and height (`pocket_width`, `pocket_height`). Within this pocket, the transmon is capacitively coupled to a CPW $\lambda/4$ resonator. The resonator length `resonator_length` = 4.258 mm (shorted coupling end to open qubit end), with a voltage node at the grounded end. Inductive coupling to the feedline (see Fig. 19 for details) is provided by a parallel section of length 200 μm (`coupling_length`). For clarity, the feedline in Fig. 17 is truncated. Launch pads at both ends provide wire-bond connections to printed circuit board (PCB) traces. In dispersive readout, off-resonant tones propagate with minimal reflection, while near-resonant response is qubit-state dependent. In this initial design, the dimensions of the resonator and coupling zone are set analytically (see Section for derivations), whereas remaining layout dimensions are nominal qiskit-metal settings to be refined by EM simulations.

The quantum circuit is fabricated on a high-resistivity silicon substrate ($H = 0.75$ mm and intrinsic relative permittivity $\epsilon_r = 11.7$). In EM simulations, conductive features (transmon qubit, coplanar waveguides, launchpads) were modeled as perfect electric conductors (PEC), allowing material-independent analysis at this stage. In practice, these could be implemented using niobium films or alternatives such as aluminum or titanium nitride, subject to process compatibility. Since the available fabrication process at the university is not yet defined and no material property study was carried out in this work, no specific metallisation is assumed. The ground plane would employ

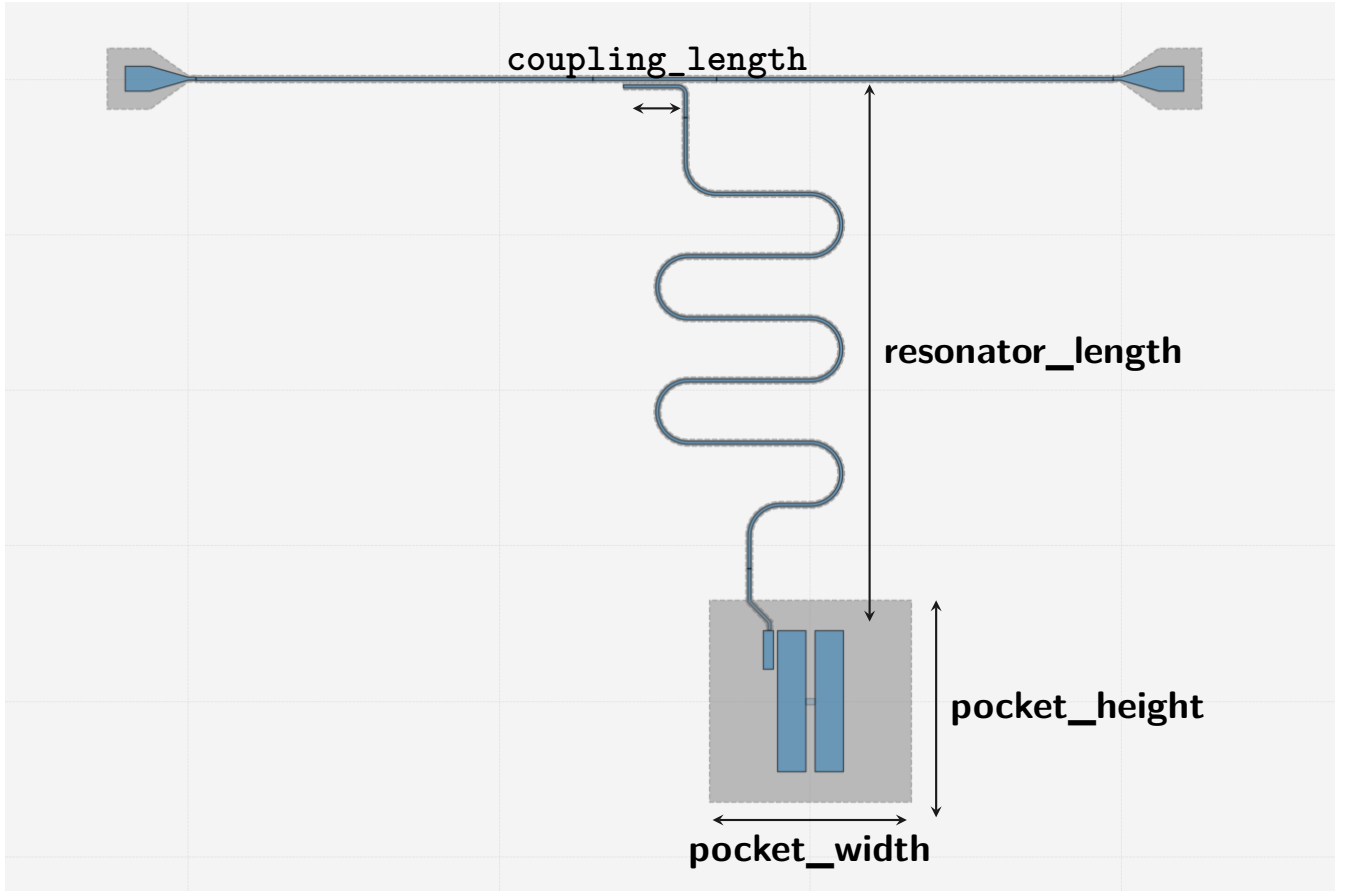


Figure 17: Initial layout of the superconducting quantum chip rendered in the qiskit-metal GUI, showing the transmon qubit in its metallic pocket, the CPW $\lambda/4$ resonator, feedline with parallel coupling section, launch pads, and annotated design variables (`pocket_width`, `pocket_height`, `resonator_length`, `coupling_length`). Color coding: blue = conductor, grey = exposed substrate, light grey = ground plane.

the same metallisation in fabrication. In the layout, blue regions denote patterned conductors, grey regions indicate exposed substrate, and light grey regions represent the ground plane.

3.4.2 Transmon layout

Fig. 18 shows the transmon qubit. It possesses two large capacitor pads separated by a gap, creating a dipole-like structure. These pads establish the dominant shunt capacitance C_S , which sets the qubit's charging energy and suppresses sensitivity to charge noise. Capacitive coupling to the resonator is provided by a narrow connector, characterized by the geometric coupling capacitance C_g , enabling photon-mediated interactions. The capacitor pads are linked by a JJ, typically implemented as an Al/AlO_x/Al trilayer tunnel barrier. The JJ is modeled as a lumped element with nonlinear inductance $L_J = 10$ nH and junction capacitance $C_J = 2$ fF. The qubit is embedded within a metallic pocket region (parameters `pocket_width` and `pocket_height`), which reduces parasitic capacitance to the ground plane while maintaining controlled coupling via the pad edges.

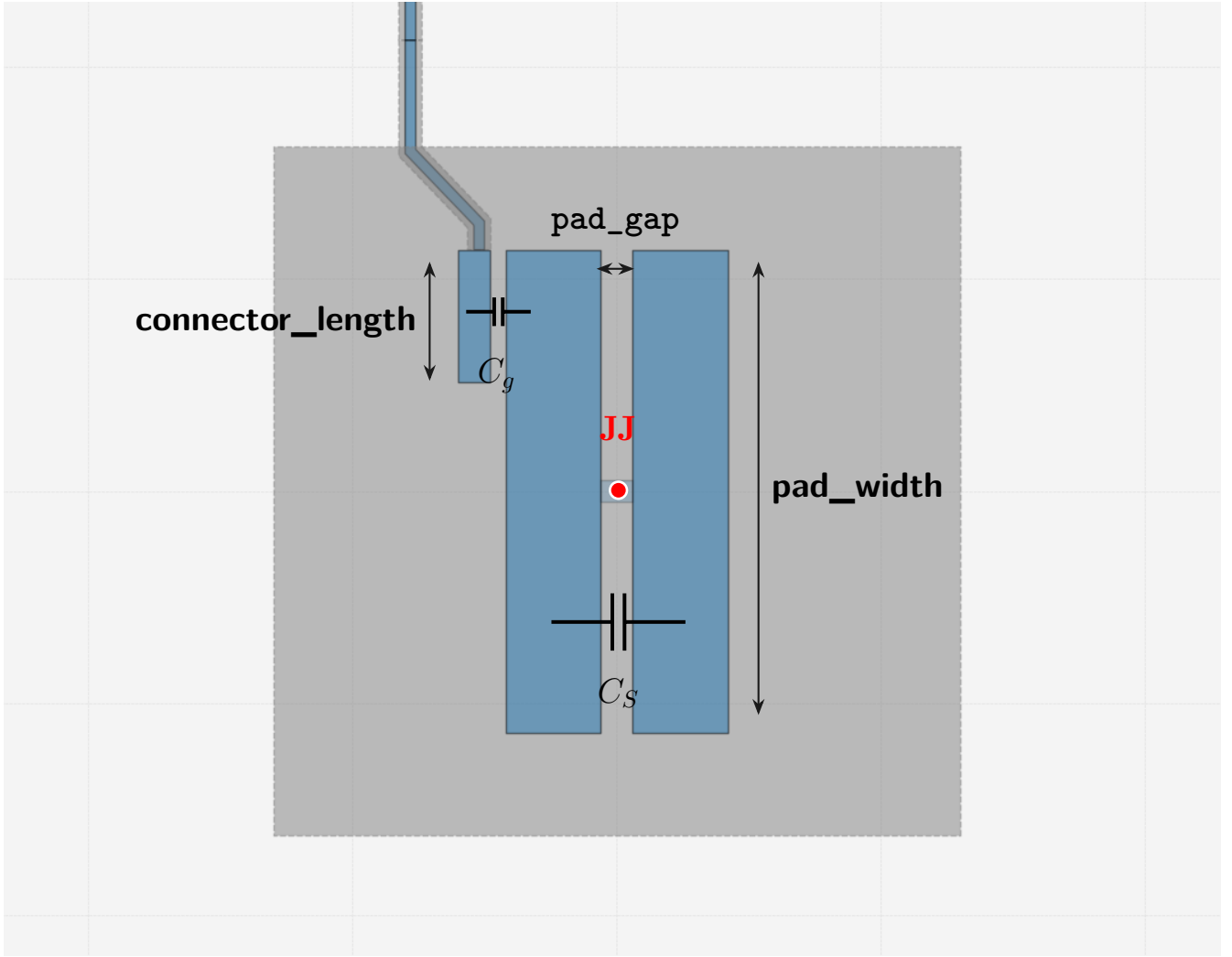


Figure 18: Layout of the transmon qubit showing two capacitor pads separated by `pad_gap` and connected via a JJ (red). Capacitive coupling to the resonator is set by `connector_length`, defining the geometric capacitance C_g . The shunt capacitance C_s arises from the pad geometry. The qubit resides in a metallic pocket defined by `pocket_width` and `pocket_height`, which controls parasitic coupling.

3.4.3 Feedline and qubit coupling

Fig. 19 shows the feedline–resonator coupling region, consisting of a CPW center conductor of width w separated from the ground plane by a gap s , over a parallel coupling length (parameter `coupling_length`) with a coupling gap D to the adjacent feedline. At this end, the resonator is shorted to the ground plane, forming a voltage node (current antinode) that yields predominantly magnetic-field (mutual inductive) coupling to the feedline. At the opposite, open-circuited end (Fig. 18), the resonator is coupled capacitively to the transmon qubit via a designed coupling capacitance. In this initial design, all coupling-zone parameters (w , s , `coupling_length`, and D) are set from analytical calculations, prior to EM simulations (see Section for derivations). This configuration implements a $\lambda/4$ resonator with inductive coupling to the feedline at the grounded end and capacitive coupling to the qubit at the open end.

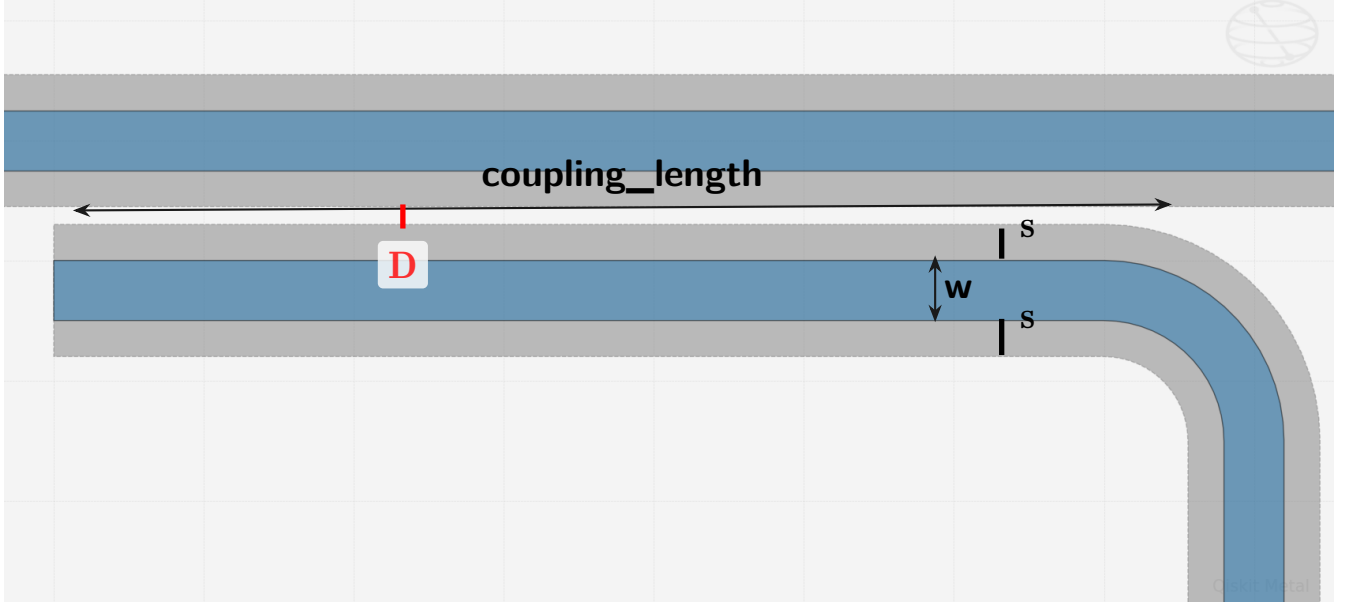


Figure 19: Layout of the feedline–resonator coupling zone, with annotated design variables: center conductor width w , ground plane separation s , `coupling_length`, and coupling gap D .

3.5 Electromagnetic simulations

3.5.1 Ansys

EM simulations are a central tool for verifying that the physical layout implements the intended Hamiltonian, identifying unwanted modes or parasitic couplings, and optimising the design prior to fabrication to minimise costly iteration cycles. In this work, we used commercial Ansys solvers to extract EM field solutions and circuit parameters directly from the device geometry, employing both full-wave and quasi-static formulations. In the full-wave case (HFSS), Maxwell’s equations are solved in the frequency domain for each eigenmode m , while in the quasi-static case (Q3D) the Laplace problem is solved to obtain capacitance or inductance matrices between conductors. In both cases, the finite-element method (FEM) discretises the geometry, approximates fields within each element, and iteratively refines the mesh in high-field regions until target quantities (e.g., mode frequencies) converge within a set tolerance.

The solver choice is matched to the parameter of interest. HFSS Eigenmode analysis provides resonance frequencies and modal field distributions of the coupled qubit–resonator structure, enabling precise identification of modes relevant to the Hamiltonian. Q3D Extractor computes the capacitance matrix between named nets in the layout, essential for constructing the linear part of the quantized model and subsequent LOM analysis; both of which are central to this thesis. For the devices studied here, the Josephson junction is modeled as a parallel inductance L_J and capacitance C_J , forming a lumped RLC boundary between the electrodes (Ref. [49]). This incorporates the junction’s linear response into the EM model without resolving its microscopic geometry.

Driven (scattering) simulations were not performed, as the design process focused on internal circuit parameters rather than broadband transmission. Scattering analysis, which characterizes how energy is reflected or transmitted through ports, is important for evaluating impedance matching,

packaging effects^[13], and frequency-dependent coupling to external circuitry. These aspects, while relevant for full system integration, fall outside the scope of this work. Our eigenmode model assumes perfect conductors, lossless dielectrics, and purely reactive elements, forming a closed and ideal system with infinite quality factors by construction. External coupling parameters will instead be obtained using complementary analytical estimates (see Section 3.8), ensuring that the absence of driven results does not compromise the extraction of the key Hamiltonian coefficients.

3.6 Energy participation ratio analysis

3.6.1 Theoretical foundations

The energy participation ratio (EPR) method provides a first-principles approach to circuit quantization by reformulating the Hamiltonian derivation as an EM energy partitioning analysis. In the general case of multiple Josephson junctions, the EPR p_{mj} of junction j in eigenmode m is defined as the fraction of the inductive energy of mode m that resides in junction j when only mode m is excited^[44]^[14]. This ratio is the key quantity that bridges classical and quantum circuit analysis and plays a primary role in constructing the system's many-body Hamiltonian. For the transmon-resonator system studied here, there is a *single* Josephson junction, which we label with the uppercase index J . In this case, the full Hamiltonian reads:

$$\hat{H}_{\text{full}} = \underbrace{\sum_m \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m}_{\hat{H}_{\text{lin}}} - \underbrace{E_J \left(\cos \hat{\phi}_J - \frac{\hat{\phi}_J^2}{2} \right)}_{\hat{H}_{\text{nl}}}, \quad m \in \{r, q\}. \quad (83)$$

The linear term $\hat{H}_{\text{lin}}$ contains the harmonic response of the junction-resonator structure with eigenfrequencies ω_m , where $\hat{a}_m$ and $\hat{a}_m^\dagger$ are the annihilation and creation operators of photons in mode m . The nonlinear term $\hat{H}_{\text{nl}}$ captures the non-quadratic part of the Josephson potential: the Josephson energy E_J ; the reduced phase operator across the unique junction $\hat{\phi}_J$. The quadratic contribution is subtracted to avoid double counting the junction's linear inductive response. $\hat{\phi}_J$ expands in the normal-mode basis as:

$$\hat{\phi}_J = \sum_m \varphi_m^{\text{ZPF}} (\hat{a}_m + \hat{a}_m^\dagger) = \varphi_q^{\text{ZPF}} (\hat{a}_q + \hat{a}_q^\dagger) + \varphi_r^{\text{ZPF}} (\hat{a}_r + \hat{a}_r^\dagger), \quad (84)$$

where φ_q^{ZPF} and φ_r^{ZPF} are the zero-point phase fluctuation amplitudes of the junction in the qubit and resonator modes, respectively. In the general multimode/multijunction case, the zero-point phase amplitudes relate to the EPRs via:

$$\varphi_{mj}^{\text{ZPF}} = \sqrt{\frac{p_{mj} \hbar\omega_m}{2E_J}}. \quad (85)$$

¹³Packaging effects are alterations of a device's electrical or EM behavior caused by the housing, interconnects, and mounting structures, which add parasitic inductance, capacitance, and resistance, potentially detuning resonances or changing impedance.

¹⁴It should not be confused with the filling factor q of the CPW resonator (Eq. 77). The filling factor q represents the fraction of the total EM energy of a mode stored in a specific dielectric region and quantifies sensitivity to dielectric losses, whereas p_{mj} represents the fraction of the total inductive energy stored in a specific Josephson junction for a given mode.

In its most general form, the participation ratio p_{mj} of junction j in eigenmode m is given by

$$p_{mj} = \frac{\text{inductive energy stored in junction } j}{\text{total inductive energy stored in mode } m} \quad (86)$$

$$= \frac{\langle \psi_m | \frac{1}{2} E_J \hat{\phi}_j^2 | \psi_m \rangle}{\langle \psi_m | \frac{1}{2} \hat{H}_{\text{lin}} | \psi_m \rangle}, \quad (87)$$

where $|\psi_m\rangle$ denotes the excited Fock state of mode m ; $\hat{\phi}_j$ is the reduced phase operator across junction j ; and $\hat{H}_{\text{lin}}$ is the linearized Hamiltonian of the system. From this definition, the following constraints hold:

$$0 \leq \sum_j p_{mj} \leq 1 \quad \text{for each mode } m, \quad \sum_m p_{mj} = 1 \quad \text{for each junction } j. \quad (88)$$

For a single-junction circuit ($j = J$), as in this work, the participation ratio will be written $p_{mJ} \equiv p_m$. Solving $\hat{H}_{\text{full}}$ corresponds to determining the unknown parameters $\omega_r, \omega_q, \varphi_r^{\text{ZPF}}, \varphi_q^{\text{ZPF}}$. To this end, we perform Ansys HFSS eigenmode analysis to identify the qubit and resonator eigenmodes and their frequencies and field distributions. These eigenfields provide the necessary information to estimate the participation ratios and, in turn, the zero-point phase amplitudes (Eq. 85). In practice, the eigenmode solutions are used to extract these quantities, which then serve as the foundation for obtaining the effective circuit parameters discussed next. This energy partitioning enables the derivation of key quantum parameters through:

$$\alpha_m = \frac{\partial^2 E_m}{\partial n_m^2} = p_m^2 \frac{\hbar \omega_m^2}{8 E_J}, \quad (\text{anharmonicity}), \quad (89)$$

$$\chi_{mn} = \frac{\partial^2 E_m}{\partial n_m \partial n_n} = p_m p_n \frac{\hbar \omega_m \omega_n}{4 E_J}, \quad (\text{dispersive shift}), \quad (90)$$

$$\Delta_m = \frac{1}{2} \sum_{n=1}^M \chi_{mn}, \quad (\text{Lamb shift}). \quad (91)$$

These expressions are evaluated for each mode using the participation ratios and eigenfrequencies obtained from the eigenmode analysis. Under the rotating-wave approximation in the dispersive regime, the effective Hamiltonian reads:

$$\hat{H}_{\text{eff}} = \sum_m (\omega_m - \Delta_m) \hat{n}_m - \frac{1}{2} \sum_m \alpha_m \hat{n}_m (\hat{n}_m - 1) - \sum_{m \neq n} \chi_{mn} \hat{n}_m \hat{n}_n, \quad (92)$$

where $\hat{n}_m = \hat{a}_m^\dagger \hat{a}_m$ is the number operator. This form makes explicit that Δ_m , α_m , and χ_{mn} enter as frequency shifts with negative sign by convention.

In summary, the EPR workflow proceeds as:

1. Perform Ansys HFSS eigenmode simulation to compute ω_m and the field distributions $\mathbf{E}_m, \mathbf{H}_m$.
2. Evaluate inductive participation ratios p_m from the eigenmode solutions.

3. Obtain φ_m^{ZPF} via Eq. [85](#).
4. Derive α_m , χ_{mn} and Δ_m from Eqs. [89](#), [90](#) and [91](#).
5. Construct full Hamiltonian with Eq. [92](#).

3.6.2 Eigenmode analysis

We determined and validated the fundamental EM modes of the quantum chip using HFSS eigenmode analysis on the initial layout. The extracted mode frequencies defined the linear part of the system Hamiltonian, while the corresponding electric-field distributions were examined visually to verify that energy was localized in the expected regions (e.g., qubit capacitor gap, resonator structure) and free from spurious artifacts. This qualitative inspection, combined with frequency data, confirmed that the layout supported the intended mode structure. We also characterized the adaptive-meshing convergence behavior and identified the lowest-order physical modes relevant to the design.

The HFSS eigenmode solver was configured for a linearized representation of the device, modeling the JJ as an inductance $L_J = 10 \text{ nH}$ in parallel with a capacitance $C_J = 2 \text{ fF}$. The solver returned a discrete set of eigenfrequencies $\{\omega_m\}$ and associated electric-field maps $\{\mathbf{E}_m(\mathbf{r})\}$ for the passive structure. Under this assumption, the two lowest-order modes were identified from their spatial field profiles as the qubit and resonator modes. They provided the frequencies inputs for the linear Hamiltonian in Eq. [83](#). Nonlinear contributions from the Josephson element are beyond this thesis scope, the corresponding EPR calculation is implemented in the code provided in Annex [A](#). To ensure accurate eigenfrequencies, we employed HFSS adaptive meshing starting from a coarse grid. After each pass, the solver recalculated the modes, identified high-error regions such as capacitor gaps and coupling interfaces, and refined the mesh locally. Convergence was monitored via the maximum relative eigenfrequency change between passes. A 0.5% stability threshold was enforced on the real frequency, with at least one additional pass after reaching the threshold to guard against false convergence.

The resulting convergence trends are shown in Fig. [20](#). In the top panel, the qubit-like (blue), resonator-like (orange), and feedline-like (green) modes exhibit monotonic frequency stabilization with increasing mesh density. The 0.5% criterion was first met at pass 11, and by pass 18 all three modes varied by less than 0.1%. The bottom panel correlates this behavior with a rapid early drop in the maximum relative frequency shift $\max(\Delta f/f)$ (red, logarithmic scale) and a stepwise increase in the total number of tetrahedral elements (blue, right axis) from approximately 5×10^4 to 8×10^5 , reflecting targeted refinement in high-field regions. The converged spectrum (Fig. [20](#)) yields a qubit mode at $\omega_q/2\pi = 6.18 \text{ GHz}$, a resonator mode at $\omega_r/2\pi = 6.59 \text{ GHz}$, and a feedline mode at $\omega_f/2\pi = 6.95 \text{ GHz}$. Their spatial field profiles confirm the modal identities: the qubit mode concentrates at the JJ and capacitor gap, the resonator mode displays a $\lambda/4$ pattern with an antinode at the open end and node toward ground (Fig. [21](#)), and the feedline mode shows fields at both launchpads from fringing/impedance transitions, weaker in the central section, with blue indicating low-field regions (Fig. [22](#)).

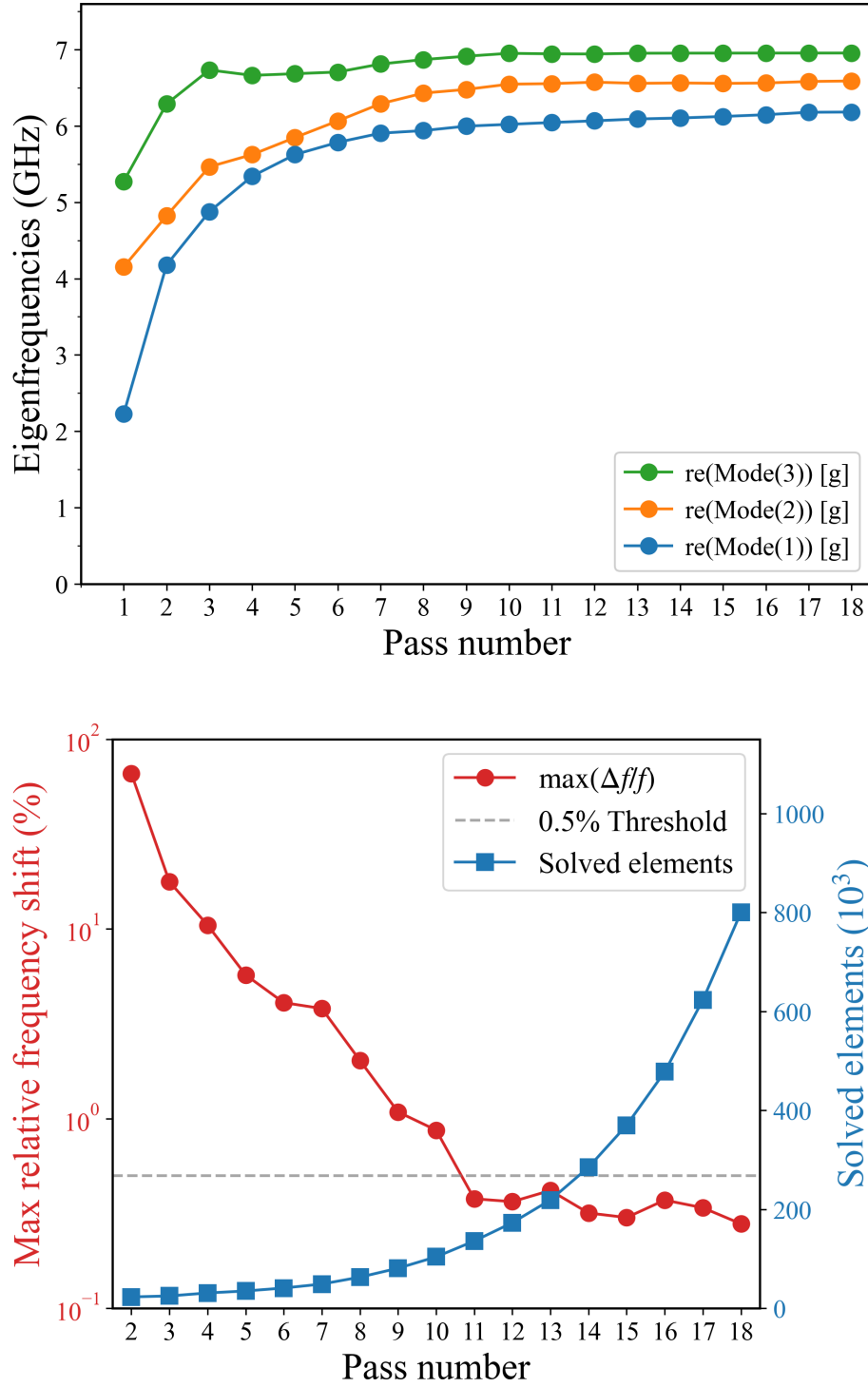


Figure 20: Convergence of the HFSS eigenmode analysis with 18 adaptive passes and a 0.5% stability threshold. **(top)** Eigenfrequencies of the qubit-like (blue), resonator-like (orange), and feedline-like (green) modes versus pass number. **(bottom)** Maximum relative eigenfrequency shift $\max(\Delta f/f)$ (red, logarithmic scale) and total tetrahedral elements (blue, in thousands) versus pass number. The 0.5% threshold is first met at pass 11, and the frequency variation is below 0.1% by pass 18. The mesh grows from approximately 5×10^4 to 8×10^5 elements due to targeted refinement in high-field regions.

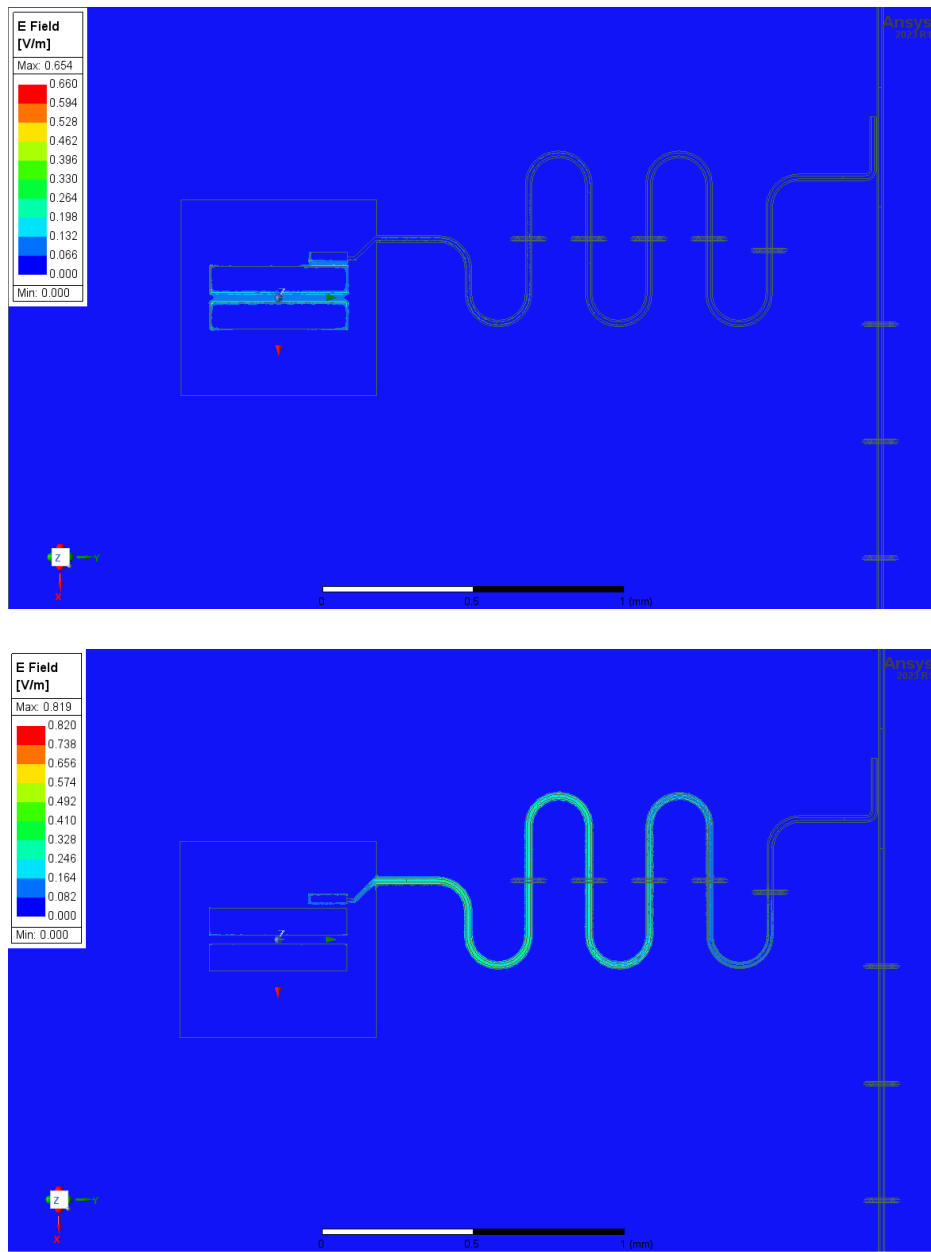


Figure 21: Electric field magnitude ($|\mathbf{E}|$) distributions from eigenmode simulations in Ansys HFSS. Blue regions indicate zones with negligible electric field (e.g., substrate). Simulations used 18-pass adaptive meshing with 0.5% frequency convergence threshold. **(top)** Qubit mode concentration at JJ and between capacitance plates. **(bottom)** Resonator mode.

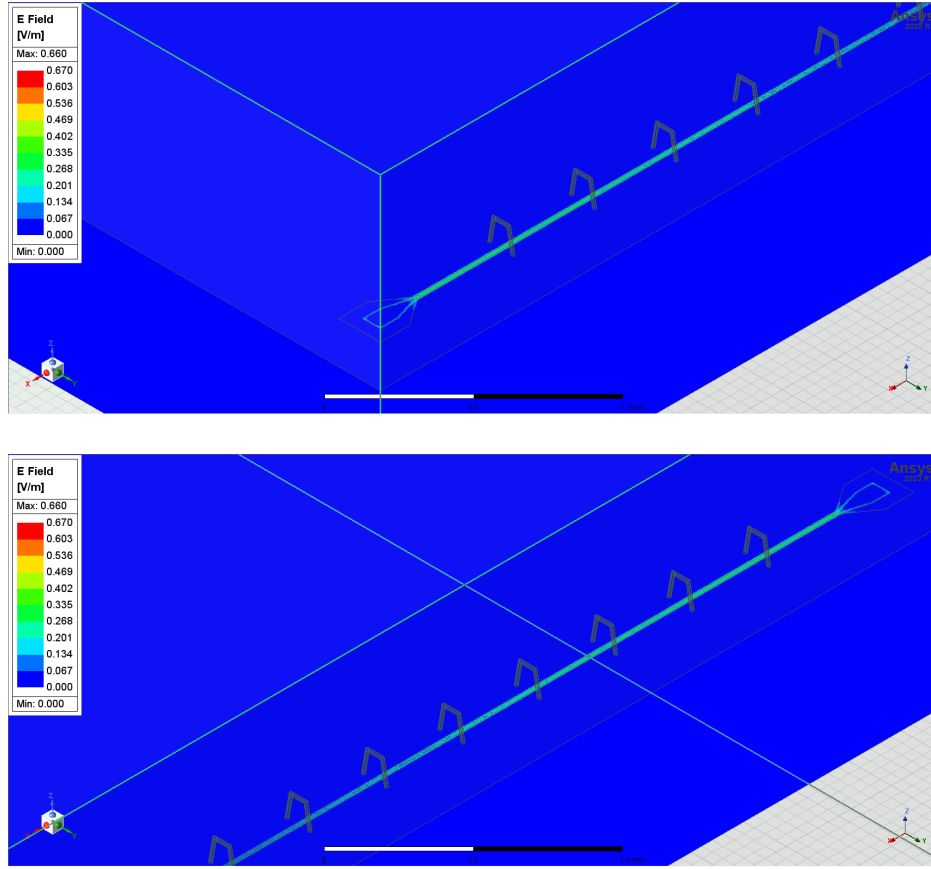


Figure 22: Electric field magnitude ($|\mathbf{E}|$) distributions from eigenmode simulations in Ansys HFSS for the feedline mode. Blue regions indicate zones with negligible electric field (e.g., substrate). Simulations used 18-pass adaptive meshing and 0.5% frequency convergence threshold. The electric field is concentrated at both ends of the feedline: **(top)** the left launchpad is shown, **(bottom)** the right launchpad is displayed.

3.7 Lumped oscillator model analysis

The lumped oscillator model (LOM) analysis represents the quantum device layout as a circuit graph with every component (capacitors, inductors, Josephson junctions) approximated as discrete lumped elements. This method avoids spatial discretization of Maxwell's equations, achieving significant improvement in computational efficiency compared to full-wave EM simulations [46], [50].

The LOM analysis partitions the quantum chip layout into modular subsystems that exhibit weakly correlated dynamics. In isolation, each subsystem is described by a bare Hamiltonian with well-characterized parameters. Those parameters can be derived from first-principles analytical models or numerical electromagnetic simulations. When integrated into a composite circuit, however, mutual interactions mediated by coupling elements induce loading effects on each component: the effective parameters of each subsystem become dressed by the Hamiltonian dynamics of its coupled neighbors. For a given subsystem interacting with K neighbors, the full Hamiltonian of the composite system in the quasi-lumped approximation is expressed as:

$$\hat{H}_{\text{full}} = \hat{H}_0 + \sum_{n=1}^K \hat{H}_n + \sum_{n=0}^{K-1} \sum_{m=n+1}^K \hat{H}_{nm}, \quad (93)$$

where $\hat{H}_0$ is the dressed Hamiltonian of the primary subsystem, $\hat{H}_n$ denotes the dressed Hamiltonian of the n -th neighboring subsystem, and $\hat{H}_{nm}$ accounts for the interaction between n -th and m -th subsystems. This dressed interaction Hamiltonian is described by:

$$\hat{H}_{nm} = \frac{\hat{Q}_n \hat{Q}_m}{C_{nm}^{\text{eff}}} + \frac{\hat{\Phi}_n \hat{\Phi}_m}{L_{nm}^{\text{eff}}}, \quad (94)$$

where C_{nm}^{eff} and L_{nm}^{eff} are the effective capacitance and inductance between the n -th and m -th subsystems. $\hat{Q}_n$ and $\hat{\Phi}_n$ correspond to the generalized charge and magnetic flux operators of the n -th subsystem. The effective Hamiltonian in Eq. 93 depends on accurate extraction of the inter-node capacitances and inductances C_{nm}^{eff} and L_{nm}^{eff} , through Eq. 94. This is achieved by constructing the Maxwell capacitance matrix, that encodes every capacitive interaction in the device.

3.7.1 Capacitance matrix theory

The Maxwell capacitance matrix $\mathbf{C}_M$ is a relationship which gives the relations between the voltages and charges on a set of conductors: $\mathbf{Q} = \mathbf{C}_M \mathbf{V}$. Thus, the information about the linear part of the circuit of the n -th cell can be organized in the cell's geometric capacitance. The capacitance matrix $\mathbf{C}_M$ for a cell with n nodes takes the form:

$$C_M = \begin{matrix} & \begin{matrix} n_1 & n_2 & n_3 & \cdots & n_n \end{matrix} \\ \begin{matrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_n \end{matrix} & \begin{pmatrix} C_{1\Sigma} & -C_{12} & -C_{13} & \cdots & -C_{1n} \\ & C_{2\Sigma} & -C_{23} & \cdots & -C_{2n} \\ & & C_{3\Sigma} & \cdots & -C_{3n} \\ & & & \ddots & \vdots \\ & & & & C_{n\Sigma} \end{pmatrix} \end{matrix}, \quad (95)$$

The Maxwell capacitance matrix $\mathbf{C}_M$ constitutes a symmetric square matrix of dimension n , its off-diagonal elements $-C_{ij}$ ($i \neq j$) correspond to the negative mutual capacitance between conductors i and j . The diagonal terms $C_{i\Sigma}$ represent the self-capacitance of conductor i relative to the circuit datum, an idealized reference conductor at infinity with zero potential. The diagonal terms $C_{i\Sigma}$ represent the self-capacitance of the i -th node C_{ii} plus the value of all coupled capacitances; i.e., $C_{i\Sigma} = \sum_{j=1}^n C_{ij}$.

For a grounded transmon, the total capacitance C_Σ corresponds directly to the capacitance between the qubit island and the ground. In contrast, a floating transmon comprising two isolated superconducting islands connected by a Josephson junction exhibits distinct electrostatic behavior. Here, the dominant shunt capacitance C_S between the two islands provides the primary capacitive pathway. Crucially, nearby circuit elements (particularly resonators) enable electrostatic screening.

When charges move on the qubit islands, induced voltage adjustments in coupled components partially cancel the qubit’s electric field. This screening effect reduces the energy required for charge transitions, thereby lowering the effective qubit capacitance compared to a grounded architecture with identical geometry. The precise determination of C_Σ requires rigorous Lagrangian analysis of the lumped-element circuit model. Derivation lies outside the present scope but can be found in Ref. [35].

3.7.2 Capacitance matrix extraction and LOM quantization

In order to derive the capacitance matrix, the device layout is partitioned into strictly disjoint geometric modules called cells. As illustrated in Fig. 23, a representative cell integrates elements (or nodes) from multiple subsystems: Lower pad and Upper pad (transmon qubit subsystem), Read-out (CPW resonator subsystem), with Ground plane serving as the reference node. Therefore, a cell partition and system partition constitute distinct hierarchical layers in the composite circuit representation. Cells define EM simulation boundaries for parameter extraction, while subsystems are functional groupings of circuit elements (such as resonator and its connector), that form the basic building blocks of the quantum model. These groupings represent dressed components and are represented by a dressed Hamiltonian: their properties have been renormalized to account for interactions with neighboring elements. Subsystems emerge naturally when translating the device’s quasi-lumped circuit schematic into quantum operators, and they directly define the terms $(\hat{H}_0, \hat{H}_{nm})$ in the system’s Hamiltonian.

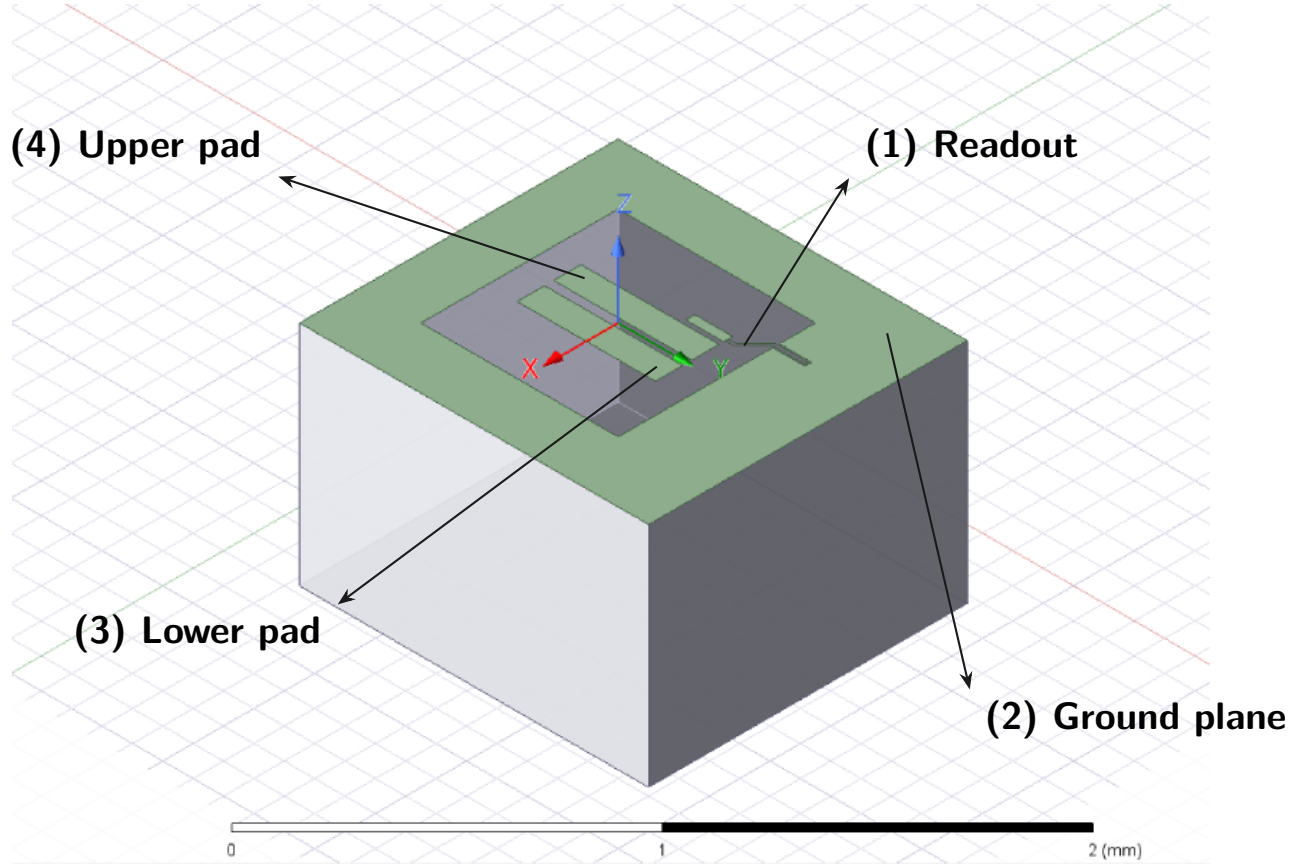


Figure 23: Q3D cell model from Ansys Q3D, showing nodal configuration. The axis is centered at the JJ. **Node 1:** Readout (resonator and connector). **Node 2:** Ground plane. **Node 3:** Lower pad. **Node 4:** Upper pad.

For each cell, the Maxwell capacitance matrix $\mathbf{C}_M$ (4×4) is derived through quasi-static EM simulations in Ansys Q3D extractor. This extraction workflow is automated within qiskit-metal, which interfaces directly with Ansys Q3D to streamline both EM rendering and subsequent LOM parameter extraction through scripted workflows. As for the EPR analysis, the workflow utilizes specific parameters to control numerical accuracy and physical modeling. We need to control adaptive mesh refinement iterations by setting the number of passes in qiskit-metal for the capacitance matrix extraction and the LOM analysis. The resulting capacitance matrix of our transmon system, displayed programmatically within qiskit-metal, is shown in Fig. 24.

	a_connector_pad_Q1	ground_main_plane	pad_bot_Q1	pad_top_Q1
a_connector_pad_Q1	49.02775	-33.12267	-1.64763	-13.42038
ground_main_plane	-33.12267	181.14775	-47.66444	-42.93328
pad_bot_Q1	-1.64763	-47.66444	87.86347	-35.11140
pad_top_Q1	-13.42038	-42.93328	-35.11140	94.66415

Figure 24: Capacitance matrix $\mathbf{C}_M$ (units: fF) extracted via qiskit-metal. Node correspondence: **Node 1:** a_connector_pad_Q1 $\rightarrow$ Readout, **Node 2:** ground_main_plane $\rightarrow$ Ground plane. **Node 3:** pad_bot_Q1 $\rightarrow$ Lower pad, **Node 4:** pad_top_Q1 $\rightarrow$ Upper pad, diagonal elements: $C_{i\Sigma}$, off-diagonal elements: $-|C_{ij}|$.

Our Q3D cell contains four nodes:

- pad_bot_Q1 and pad_top_Q1: Qubit pads (Upper/Lower pad in Fig. 23).
- a_connector_pad_Q1: Readout resonator node (Readout in Fig. 23).
- ground_main_plane: Reference node (Ground plane in Fig. 23).

The matrix captures mutual capacitances C_{ij} (off-diagonal elements) and self-capacitances $C_{i\Sigma}$ (diagonal elements), essential for Hamiltonian construction. Finally, Fig. 25 presents the Lumped Oscillator Model (LOM) analysis results. Prior to Hamiltonian diagonalization, the bare subsystem parameters must be defined as inputs for the dressing process mediated by the capacitance matrix:

- $L_j = 10 \text{ nH}$: Josephson inductance governing nonlinearity.
- $C_j = 2 \text{ fF}$: Junction capacitance setting anharmonicity.
- $\text{freq_readout} = 7.0 \text{ GHz}$: Target fundamental mode frequency for the readout resonator.

These parameters establish the undressed subsystem characteristics before interaction-induced renormalization. The optimization methodology for L_j and C_j selection will be systematically addressed in Sec. 3.8.

```

#LOM analysis

c2.setup.junctions=Dict(Lj=10, Cj=2)
c2.setup.freq_readout = 7.0
c2.setup.freq_bus = []

c2.run_lom()
c2.lumped_oscillator_all

[1, 2] [3]
Predicted Values

Transmon Properties
f_Q 5.559665 [GHz]
EC 260.191293 [MHz]
EJ 16.339560 [GHz]
alpha -293.424313 [MHz]
dispersion 1.241781 [KHz]
Lq 9.995968 [nH]
Cq 74.446104 [fF]
T1 32.193522 [us]

**Coupling Properties**

tcqbus1 -9.681739 [fF]
gbus1_in_MHz -136.722398 [MHz]
chi_bus1 -4.464125 [MHz]
1/T1bus1 4943.694598 [Hz]
T1bus1 32.193522 [us]
Bus-Bus Couplings

```

Figure 25: LOM analysis of a transmon qubit with junction parameters $LJ = L_j = 10$ nH, $CJ = C_j = 2$ fF and readout resonator frequency. `frequency_readout` = $\omega_r = 7.0$ GHz. Then, `c2.run_lom()` execute the LOM analysis code to compute the chip parameters. `c2.lumped_oscillator_all` access LOM result dictionary and display the simulation outputs: Transmon properties include: qubit transition frequency $f_Q = \omega_q/2\pi = 5.56$ GHz, charging energy $EC = E_C = e^2/(2C_\Sigma) = 260.2$ MHz, Josephson energy $EJ = E_J = (\Phi_0/2\pi)^2/L_j = 16.34$ GHz, anharmonicity $\alpha = \alpha = \omega_{12} - \omega_{01} = -293.4$ MHz, higher-order dispersion `dispersion` = 1.24 kHz, total inductance $Lq \approx L_j = 9.996$ nH, total capacitance $Cq = C_\Sigma = 74.45$ fF, and energy relaxation time $T1 = 32.19$ μ s. Coupling to the readout resonator (Bus1) shows: coupling capacitance $tcqbus1 = C_g = -9.68$ fF, coupling strength $gbus1_in_MHz = g/2\pi = -136.7$ MHz, dispersive shift $\chi_bus1 = \chi/2\pi = -g^2/\Delta = -4.46$ MHz (where $\Delta = \omega_q - \omega_r < 0$), Purcell decay rate $1/T1bus1 = \Gamma_{\text{Purcell}} = 4.94$ kHz, and relaxation time $T1bus1 = T_1^{\text{bus1}} = 32.19$ μ s ($\approx T_1$, confirming Purcell-limited decay). The empty Bus-Bus couplings indicates no direct resonator-resonator interactions, consistent with the single-resonator configuration.

3.8 Design validation methodology

Here we present our design validation methodology (Step 4 of our simulation workflow, see Section 3.2). The goal is to understand how each parameter individually modify the chip behavior (and therefore understand the chip physics) before being able to optimize it. This will allow to reduce physical prototyping cycles through *in silico* validation. Specifically, this section documents:

1. **Initial parameter selection:** Analytical models were employed to determined geometric parameters for the CPW resonator to establish the initial device layout that was presented in Section 3.4. Key calculations focused on designing TL elements to achieve $50\ \Omega$ impedance matching, which sets the signal-to-ground spacing $s = 5.8\ \mu\text{m}$, the center conductor width $w = 10\ \mu\text{m}$ and the effective relative permittivity for the fundamental CPW mode $\epsilon_{\text{eff}} = 6.331$. Target fundamental resonance frequency $\omega_r/2\pi = 7\ \text{GHz}$ sets the physical resonator length $l_r = 4.258\ \text{mm}$. These initial dimensions for the resonator dimensions significantly reduced the EM validation search space by providing approximate values for the chip dimensions.
2. **Parametric sensitivity analysis:** Then, the qiskit-metal generated layout was used to systematically characterize the geometric dependencies. Through controlled parameter sweeps, we quantified how specific design variations impact Hamiltonian parameters. The list of tested parameters is represented in Tab. 2.

Table 2: List of tested design parameters in the qubit simulation study.

Design component	Parameter swept
Resonator-feedline coupling	Coupling length and gap dimensions
Qubit geometry	Pad width and pad gap
Connector	Connector length
Josephson junction	Josephson inductance

The parameters on the resonator-feedline coupling where extracted using an analytical model developed in Ref. [51]. For the other design parameters, capacitance matrices were extracted from electrostatic simulations and Hamiltonian parameters were computed via Lumped Oscillator Model (LOM) analysis (9-passes adaptive meshing). Parameter sweeps were performed by iteratively testing values of a single parameter and then resetting the layout before testing the next parameter. The LOM computed Hamiltonian parameters via linear algebra operations on the capacitance matrix, avoiding the computationally intensive spatial discretization and iterative solution of the vector wave equation required by full-wave EM simulators (e.g., Ansys HFSS for EPR analysis). This enabled rapid exploration of parameter spaces that would otherwise be prohibitively time-consuming. This methodology synthesizes and systematizes techniques established in prior works [37], [48], [52].

3. **Iterative optimisation loop:** Although the implementation of a fully automated iterative optimization process lies beyond the scope of this thesis, the sensitivity data obtained from the parametric analysis constitutes a critical foundation for future target-driven design refinement. As discussed in Section 3.9, such data could be used to guide layout refinement using the EPR analysis.

This validation methodology delivers: (i) physical insight into parameter influence on device behavior; (ii) computationally tractable parameter space exploration via LOM efficiency; and (iii) foundation for targeted optimization. While full-wave EPR analysis remains essential for final validation (Step 4.b), LOM provides rapid first-order characterization that identifies critical parameter sensitivities and accelerates design iteration cycles. This hybrid approach significantly reduces physical prototyping requirements while providing actionable parameter guidance for subsequent high-accuracy verification.

3.8.1 Initiation of the resonator dimensions

The resonator and the feedline must be meticulously designed to achieve target scattering parameters, notably a characteristic impedance of $Z_0 = 50 \, \Omega$. Prior to EM simulations, analytical calculations are employed to obtain initial estimates of geometric parameters, enabling rapid design space exploration and identification of approximate solutions. Using an analytical calculator for CPW resonators [53], Z_0 is derived from substrate properties and CPW geometry by solving Eq. [75]. The equation is recalled here:

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_{\text{eff}}(\omega, t)}} \frac{K(k_0)}{K(k'_0)}, \quad (96)$$

where $\epsilon_{\text{eff}}(\omega, t)$ is the effective relative permittivity of the CPW structure, which depends on the angular frequency ω and conductor thickness t . For a fixed operational angular frequency and fixed conductor thickness $t \approx 0$, $\epsilon_{\text{eff}}(\omega, 0)$ reduces to a function of geometry and substrate properties. As shown in Eq. [76], $\epsilon_{\text{eff}}(\omega, 0)$ depends on the substrate's intrinsic relative permittivity ϵ_r and on the geometric parameters (See Fig. [10]): center conductor width w , signal-to-ground spacing s , and substrate height H (via Eq. [77] and [70]–[73]). Given a silicon substrate of height $H = 0.75 \, \text{mm}$ and intrinsic relative permittivity $\epsilon_r = 11.7$, iterative adjustment of w and s were performed manually to minimize $|Z_0 - 50 \, \Omega|$. This optimization yielded $s = 5.8 \, \mu\text{m}$ and $w = 10 \, \mu\text{m}$, resulting in $Z_0 = 50.017 \, \Omega$ and an effective relative permittivity for the fundamental CPW mode of $\epsilon_{\text{eff}} = 6.331$. This ϵ_{eff} represents the specific value of $\epsilon_{\text{eff}}(\omega, t)$ evaluated at the design angular frequency ω , thickness $t \approx 0$, and optimized geometry.

The physical length l_r of a CPW resonator is a critical design parameter for achieving the target resonator frequency $f_r = \omega_r/2\pi = 7 \, \text{GHz}$. This length is determined by the relationship of Eq. [97]:

$$l_r = \frac{c_0}{4f_r\sqrt{\epsilon_{\text{eff}}}}. \quad (97)$$

Substituting $c_0 = 3 \times 10^8 \, \text{m/s}$, the speed of light in vacuum, and $\epsilon_{\text{eff}} = 6.331$ found previously, we obtain $l_r = 4.258 \, \text{mm}$. As evident from Eq. [97], l_r exhibits an inverse proportionality ω_r . Interestingly, the resonator-qubit coupling strength magnitude $|g|$ scales with the vacuum electric field density $\mathbf{E}_0$ at the resonator open end, where:

$$\mathbf{E}_0 \propto \sqrt{\frac{\hbar\omega_r}{C_{\text{sl}}l_r}}, \quad (98)$$

$$g = \frac{\mathbf{d} \cdot \mathbf{E}_0}{\hbar}, \quad (99)$$

$\mathbf{d}$ denoting the qubit dipole moment and C_{S1} the shunt capacitance per unit length (Eq. 68). The coupling magnitude satisfies $|g| \propto l_r^{-1/2}$ due to enhanced field confinement in shorter resonators, which concentrate electromagnetic energy in reduced volumes. This inverse-square-root dependence highlights the critical design trade-off between resonator size and coupling efficiency.

The geometric parameters defining the CPW structure are now fully constrained by two independent design criteria. The characteristic impedance requirement $Z_0 = 50 \, \Omega$ determines the transverse dimensions: signal-to-ground spacing $s = 5.8 \, \mu\text{m}$ and center conductor width $w = 10 \, \mu\text{m}$. While the target fundamental resonance frequency $\omega_r/2\pi = 7 \, \text{GHz}$ sets the physical resonator length $l_r = 4.258 \, \text{mm}$. This length critically influences the resonator-qubit coupling strength g through vacuum electric field density scaling, establishing a fundamental size-coupling trade-off. These parameters define the initial device layout configuration in the qiskit-metal tool, represented in Fig. 17.

3.8.2 Parametric sensitivity analysis

3.8.2.1. Connector length

After selecting the geometric parameters for the resonator, the newly found initial device layout is imported into Ansys Q3D extractor. We thus performed a discrete sweep of the connector length (see Fig. 18) l_c across the range $50 \, \mu\text{m}$ to $225 \, \mu\text{m}$ in increments of $25 \, \mu\text{m}$, generating 8 distinct geometric variants, while holding other parameters constant. For each l_c value, we computed the full capacitance matrix $\mathbf{C}_{\mathbf{M}}$ and, in a second time, we performed a LOM analysis with 9-passes adaptive meshing to extract the Hamiltonian parameters. The most important results are shown in Fig. 26.

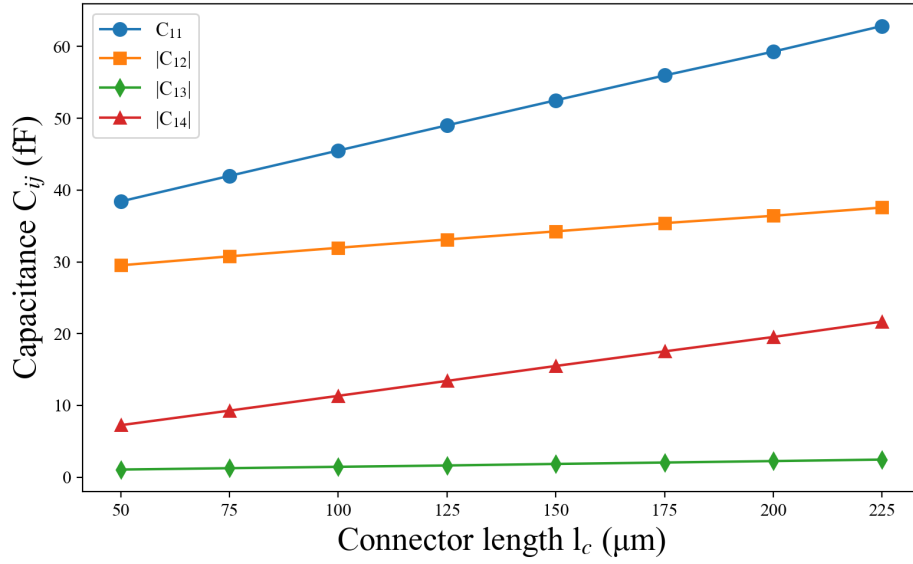
Figure 26a presents the evolution of the capacitance matrix elements with increasing connector length. Following the notations from Fig. 24, those elements are: the connector self-capacitance (C_{11}), the connector-ground coupling ($|C_{12}|$), the mutual capacitance between the connector and the lower pad ($|C_{13}|$) and the upper pad ($|C_{14}|$). Elements exhibit approximately linear scaling with l_c , consistent with increased capacitive surface area. In contrast, $|C_{13}|$ demonstrates minimal dependence on l_c , which is likely due to the connector not being in direct proximity to the lower pad and therefore being almost not affected by changes in connector.

Fig. 26b presents the evolution of resonator-qubit coupling strength $|g|$ and dispersive shift $|\chi|$ with connector length l_c , extracted through LOM analysis. The observed increase in $|g|$ arises from enhanced capacitive coupling due to a larger coupling capacitance C_g . Extending the connector length l_c expands the capacitive area, directly increasing C_g . Since $C_g \ll C_S$ (where C_S is the dominant shunt capacitance), the total qubit capacitance $C_{\Sigma} = C_g + C_J + C_S$ remains approximately constant. Consequently, the ratio $\beta = C_g/C_{\Sigma}$ in Eq. 79 increases proportionally to C_g . Results showed an increase in β of 15.88%, consistent with the measured $|g|$ enhancement of 15.77%. This confirms that $|g|$ scales with β when C_{Σ} is preserved. The near-constancy of C_{Σ} maintains stable charging energy $E_C = e^2/(2C_{\Sigma})$ and ratio E_J/E_C , resulting in negligible changes to charge noise sensitivity and anharmonicity $|\alpha|$. The increase in $|\chi|$ follows Eq. 82 through its dominant g^2 dependence. Assuming constant Δ and $|\alpha|$, g^2 increased by 33.98%, consistent with the observed $|\chi|$ enhancement of 32.72%. The minor discrepancy ($\sim 1.26\%$) arises from concurrent variations in Δ (increasing from 1.438 GHz to 1.444 GHz) and $|\alpha|$ (decreasing from 293.72 MHz

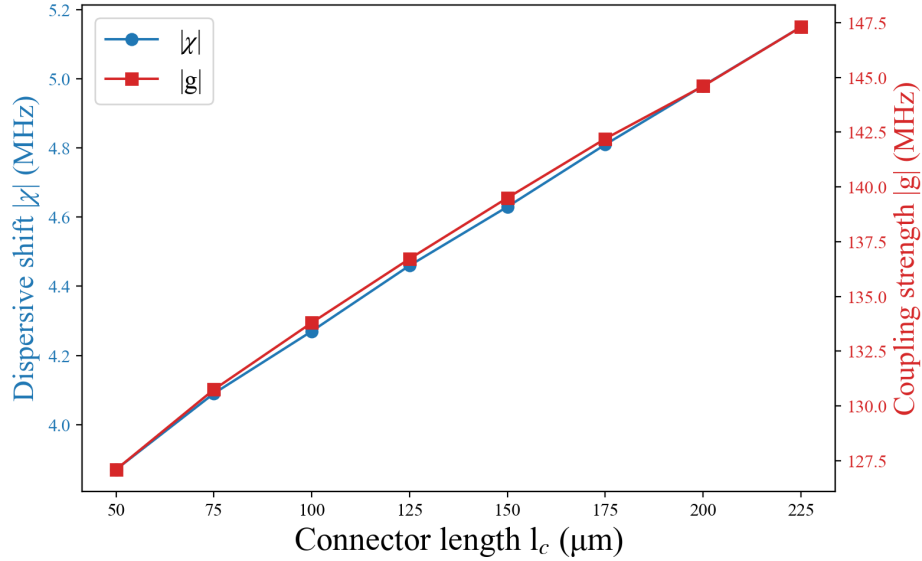
to 293.17 MHz). Accounting for these changes yields a predicted $|\chi|$ enhancement of 32.87%. The increase in the dispersive shift $|\chi|$ enables faster readout by enhancing state discrimination, but simultaneously amplifies the Purcell decay channel (see Sec. 2.4) that reduces the qubit coherence time. We observed a decrease of T_1 by 24.71%. This fundamental tradeoff arises because both $|\chi| \propto g^2/\Delta$ and $\Gamma_P \propto (g/\Delta)^2$ scale with the qubit-resonator interaction strength.

Finally, while geometrically distinct from the resonator length l_r , the connector length l_c induces analogous effects on the system's electromagnetic mode structure¹⁵. We also observed a marginal reduction in qubit frequency ω_q due to increased capacitive loading (see Eq. 61). These perturbations remain negligible due to the small dimensions of the connector.

¹⁵In the strong coupling regime, the coupler-resonator form a dressed subsystem with a unique mode. l_c contributes to the effective electrical length. Increasing l_c effectively extends this dressed mode's spatial extent, similar to increasing a resonator length.



(a)



(b)

Figure 26: Parametric analysis of the connector length l_c from 50 μm to 225 μm by 25 μm steps. **(a)** Capacitance matrix elements versus l_c : C_{11} (connector self-capacitance), $|C_{12}|$ (connector-ground coupling), $|C_{13}|$ (connector-lower pad), $|C_{14}|$ (connector-upper pad). **(b)** Hamiltonian parameters extracted from Ansys Q3D LOM analysis (9-pass adaptive meshing): dispersive shift $|\chi|$ and coupling strength $|g|$ versus connector length l_c .

3.8.2.2. Coupling strength

The coupling zone between the CPW resonator and the feedline (represented in Fig. 19) modify the resonator boundary conditions, introducing corrections to standard frequency models. The total quality factor (Q) decomposes as:

$$\frac{1}{Q} = \frac{1}{Q_i} + \frac{1}{Q_c}, \quad (100)$$

where Q_i represents intrinsic dielectric and conductor losses, while Q_c quantifies radiative losses through the feedline. When $Q_i \gg Q_c$, coupling losses dominate and $Q \approx Q_c$. This condition typically holds for high-quality-factor superconducting cavities, where internal dissipation is negligible. This approximation will be employed in the subsequent analysis. To quantify the influence of the coupling geometry on the chip behavior, we used the conformal mapping technique established by Besedin and Menushenkov [51], implemented through a free online calculator. Using this software, we analytically computed quality factors from geometric parameters and material inputs. The coupling region geometry is shown in Fig. 27).

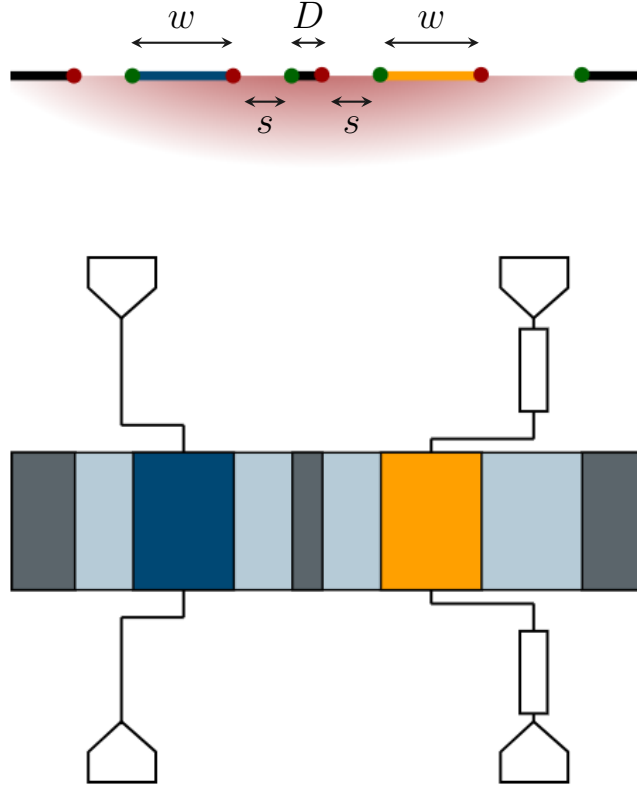


Figure 27: Coupling zone geometry for a CPW resonator coupling to a feedline, from Ref. [51]. **(top)** Cross-sectional view. **(bottom)** Top-down view. The images show conductor classification: ground plane (gray), feedline (blue), and resonator (yellow). Critical junctions are defined by branching points (red/green markers). The dimensions are: $s = 5.8 \mu\text{m}$, $w = 10 \mu\text{m}$ and $l_r = 4.258 \text{ mm}$.

The online calculator allows the selection of the CPW geometry; we chose the optimal geometric parameters for the resonator, as described in Section 3.8.1. $s = 5.8 \mu\text{m}$, $w = 10 \mu\text{m}$ and $l_r = 4.258 \text{ mm}$. Those parameters are then used to generate analytical calculations to compute the resonator-feedline parameters. The coupling strength is governed by two geometric parameters, which are shown in Fig. 19: the coupling gap dimension D (capacitive coupling) and the coupler segment length L_c (inductive coupling length). To characterize parametric sensitivity, we performed orthogonal numerical sweeps:

1. With fixed $L_c = 0.2 \text{ mm}$, $D \in [1.0, 8.0] \mu\text{m}$ is swept in $1.0 \mu\text{m}$ steps.

2. With fixed $D = 3.0 \text{ } \mu\text{m}$, $L_c \in [0.1, 0.5] \text{ mm}$ is swept in 0.05 mm steps.

According to Ref. [51], a uniform TL coupler with matched ports has an external quality factor that can be approximated by

$$\frac{1}{Q_c} \propto c_l^2 \sin^2\left(\frac{2\pi f_0 L_c}{v_p}\right), \quad (101)$$

where c_l is the dimensionless per-unit coupling coefficient, f_0 the bare resonance frequency, and v_p the phase velocity in the coupler. This distributed-element formulation differs from the lumped-element expression previously used in Section [60]:

$$Q_c \approx \frac{2\pi}{2\omega_r Z_0 C_c^2} \propto \frac{1}{C_c^2}, \quad (102)$$

which models the coupling as a single capacitance C_c into a matched load. The lumped approach neglects both the finite electrical length of the coupler and the explicit $\sin^2$ dependence on L_c . In the small coupling limit:

$$\frac{2\pi f_0 L_c}{v_p} \ll 1, \quad (103)$$

and the trigonometric term can be linearized, and both models reduce to the same scaling law $Q_c \propto 1/L_c^2$. The quality factor obtained with the Besedin–Menushenkov analysis is plotted in Fig. [28].

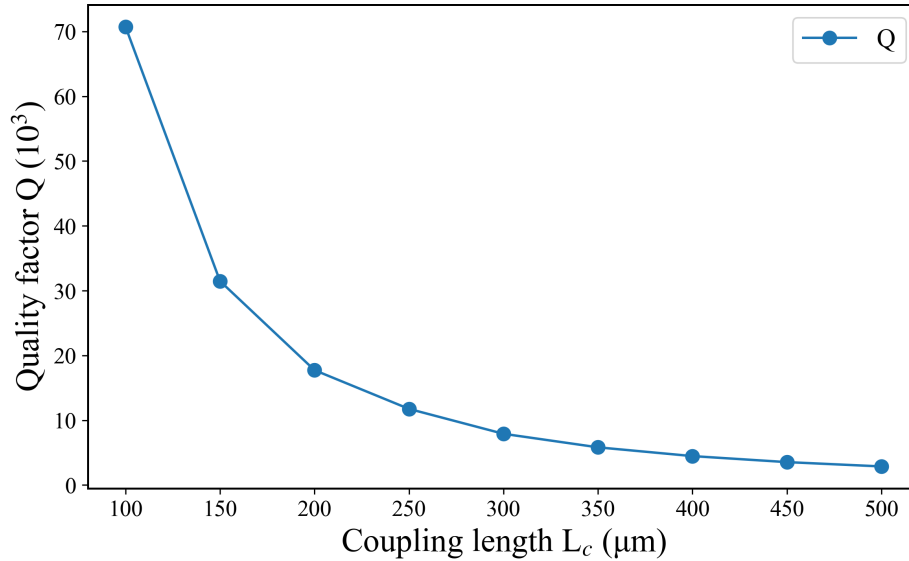


Figure 28: Inverse quadratic scaling of quality factor Q versus coupler length L_c ($D = 3.0 \text{ } \mu\text{m}$ fixed).

The quality factor Q is related to the decay rate κ by:

$$\kappa = \frac{\omega_r}{Q}, \quad (104)$$

From Fig. [29], the quadratic growth $\kappa/2\pi \propto L_c^2$ confirms:

$$\kappa \propto \frac{1}{Q} \propto L_c^2, \quad (105)$$

where $\kappa/2\pi$ is the resonator linewidth (or full-width-at-half-maximum). The evolution of the resonator linewidth (in MHz) is shown in Fig. 29 and shows good agreement with the theory.

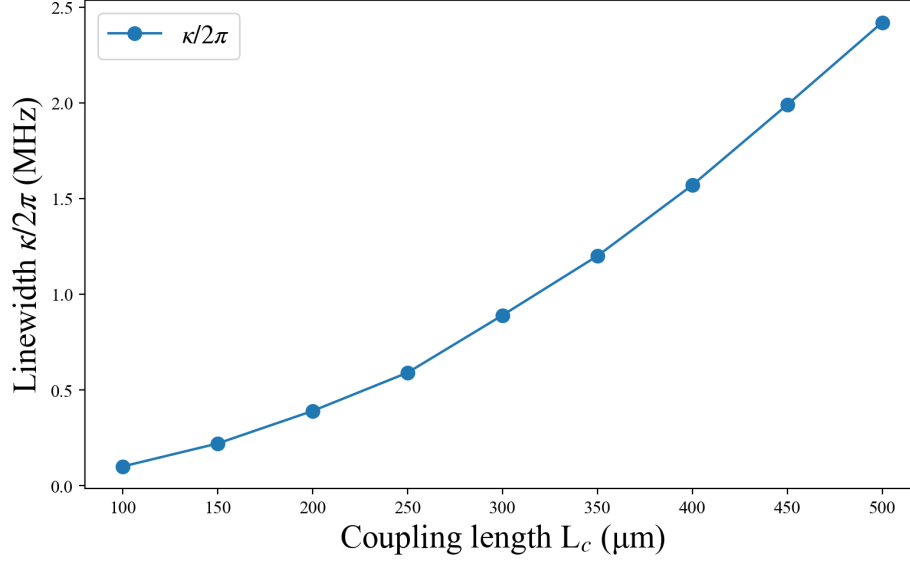


Figure 29: Quadratic scaling of linewidth κ versus coupler length L_c ($D = 3.0 \mu\text{m}$ fixed).

Finally, we observed the photon lifetime τ_γ evolution with L_c . Results are shown in Fig. 30 and compared with theoretical expectations. Photon lifetime derives from the decay rate (Eq. 58):

$$\tau_\gamma = \frac{1}{\kappa} = \frac{Q}{\omega_r}. \quad (106)$$

We can see the inverse quadratic dependence on L_c . Again, the results are in good agreement with the theory.

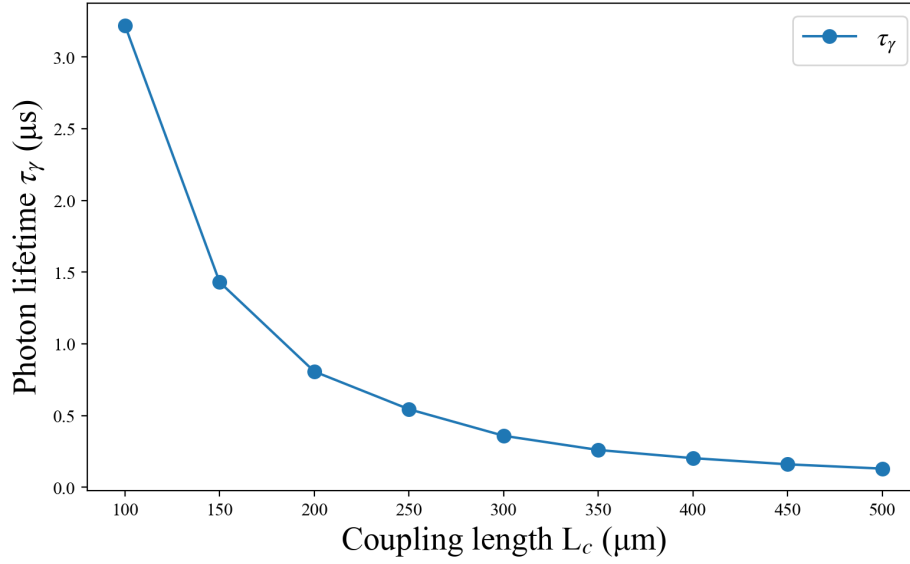


Figure 30: Quadratic scaling of photon lifetime τ_γ versus coupler length L_c ($D = 3.0 \mu\text{m}$ fixed).

The coupling gap D directly determines the coupling capacitance C_c between the resonator and the feedline, with $C_c \propto 1/D$ under constant geometry conditions. A larger gap D reduces C_c , thereby weakening the resonator-feedline coupling. This relationship is confirmed in Fig. 31, showing that the total quality factor Q increases with D due to diminished capacitive coupling.

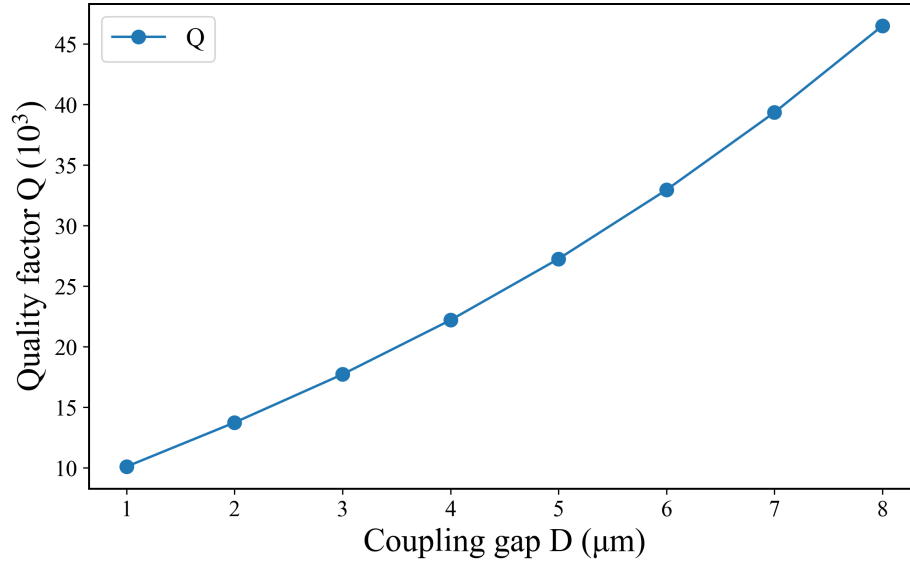


Figure 31: Quadratic scaling of quality factor Q versus coupling gap D ($L_c = 0.2 \text{ mm}$ fixed).

As shown in Fig. 32, the resonator linewidth $\kappa/2\pi$ (in MHz) narrows with increasing D , reflecting the reduction in energy decay rate. This occurs because the coupling-dependent decay rate κ_c dominates the total decay rate $\kappa = \kappa_i + \kappa_c$, where $\kappa_c \propto C_c^2 \propto 1/D^2$ according to Eq. 60.

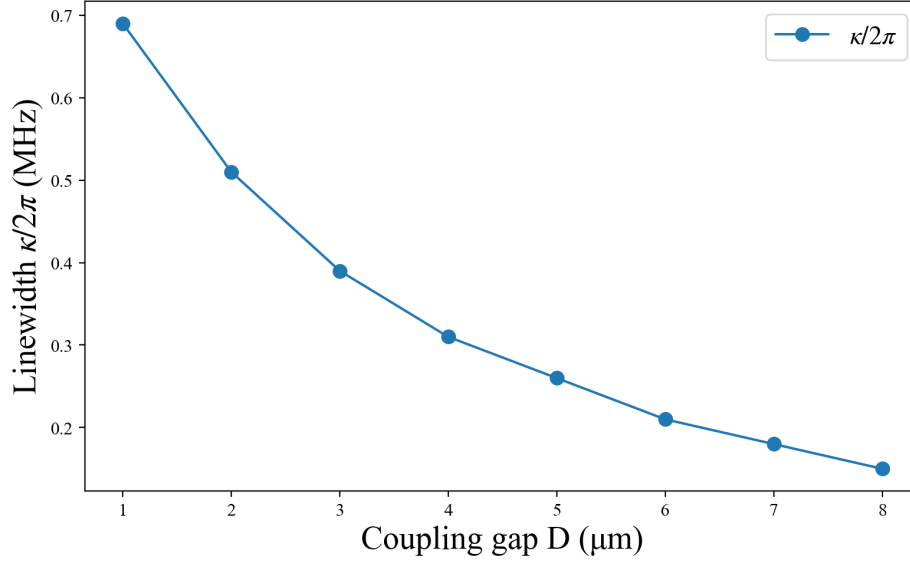


Figure 32: Quadratic scaling of linewidth $\kappa/2\pi$ versus coupling gap D ($L_c = 0.2$ mm fixed).

The reduced radiative losses consequently increase the photon lifetime τ_γ , following Eq. 106. Fig. 33 demonstrates this inverse relationship between τ and κ . The D -dependence of Q (Fig. 31) follows directly from the capacitive coupling mechanism expressed in Eq. 60, where $Q \approx Q_c$ when internal losses are negligible, as established previously.

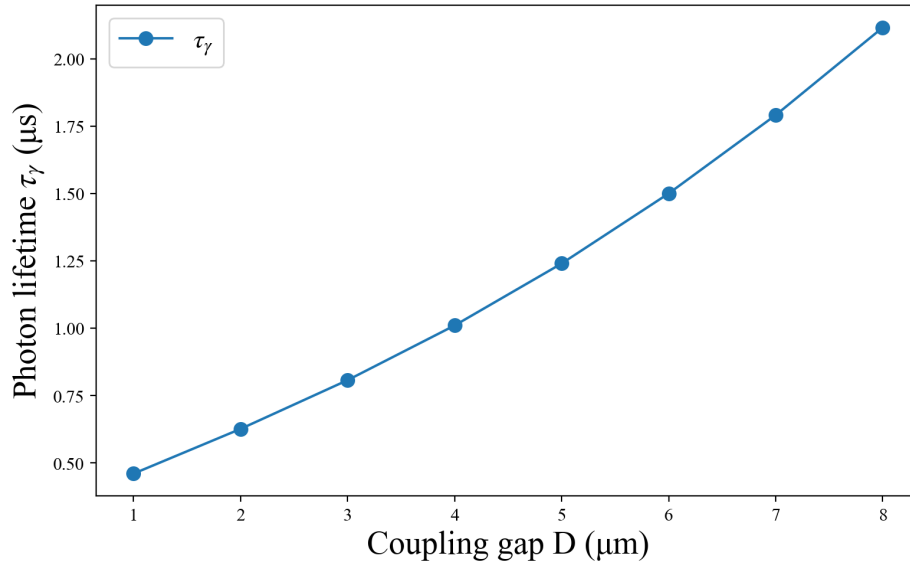


Figure 33: Quadratic scaling of photon lifetime τ_γ versus coupling gap D ($L_c = 0.2$ mm fixed).

3.8.3 Pad dimensions

We systematically varied the transmon's pad width (shown in Fig. 18) from 200 μm to 600 μm by 100 μm increments. The dominant effects are shown in Fig. 34. In Fig. 34a, we observe that both anharmonicity and coupling strength decrease with increasing the pad width. Fig. 34b shows an increase in qubit capacitance C_Σ and a decrease in qubit frequency $\omega_q/2\pi$ with increasing the pad width. Indeed, increasing the pad dimensions increases the total qubit capacitance C_Σ

due to enhanced capacitive area. This change in capacitance area directly increases the shunt capacitance C_S between the two pads, which dominates the qubit capacitance C_Σ . This increase in C_Σ reduces the charging energy E_C , subsequently lowering the qubit transition frequency ω_q and the anharmonicity $|\alpha|$ ¹⁶, through the following relations:

$$\omega_q \approx \sqrt{8E_CE_J - E_C}, \quad (107)$$

$$\alpha = \omega_{01} - \omega_{12} \approx -\frac{E_C}{\hbar}. \quad (108)$$

Finally, the qubit-resonator coupling strength $|g|$ diminishes due to reduced spatial overlap integral between the qubit dipole moment $\mathbf{d}$ and resonator vacuum electric field $\mathbf{E}_0$. Increasing the capacitive pads suppresses charge dispersion through increased E_J/E_C , enhancing T_1 coherence times from 0.8 to 114 μs as pad width grows from 200 to 600 μm . This improvement relies thus on two critical tradeoffs: (i) $|\alpha|$ decreases from 554 to 218 MHz, increasing state leakage errors, and (ii) $|g|$ decreases from 157 to 116 MHz, limiting gate and readout speeds.

¹⁶ α is inherently negative for transmon, when we say that anharmonicity decreases, we refer to α becoming less negative (closer to zero).

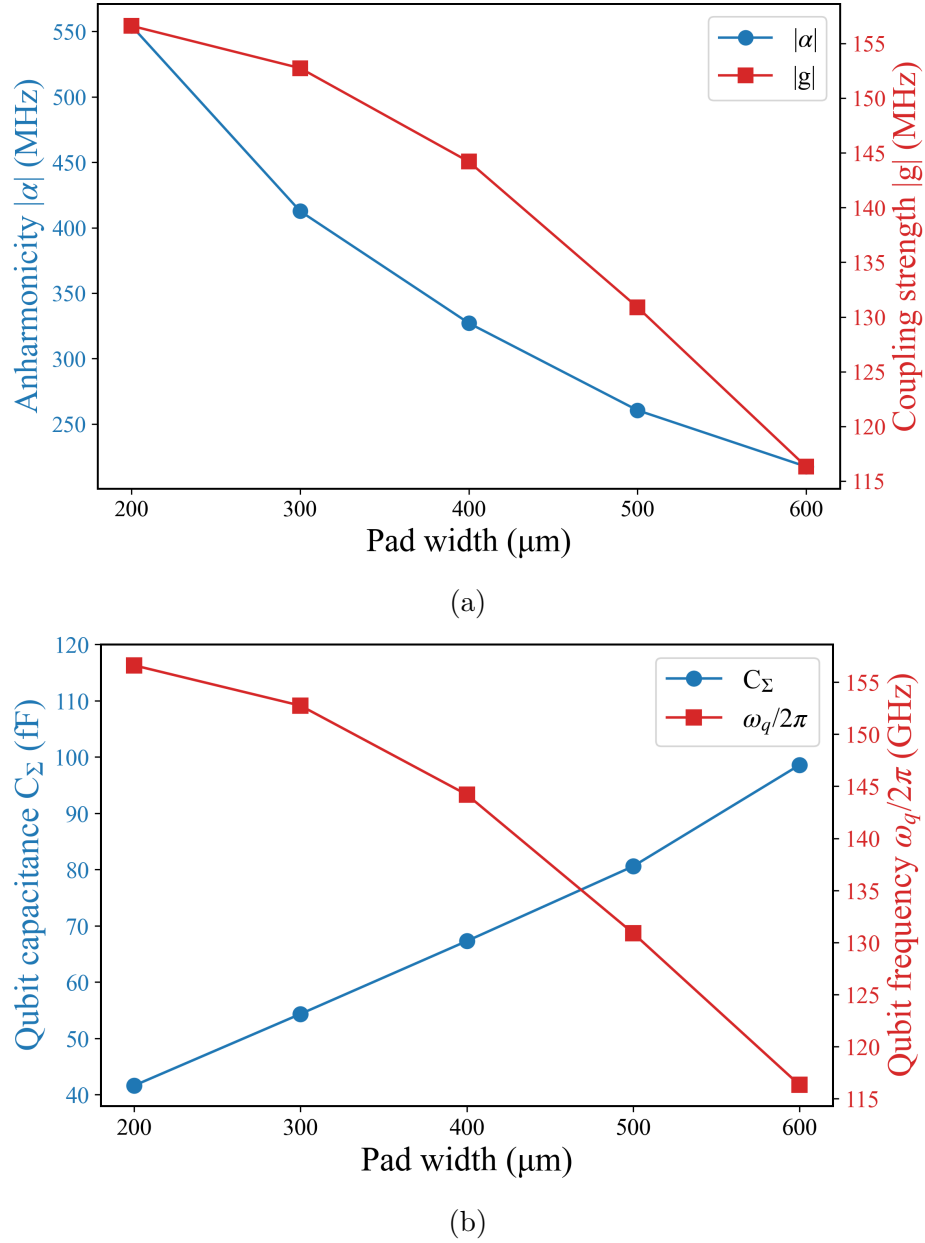
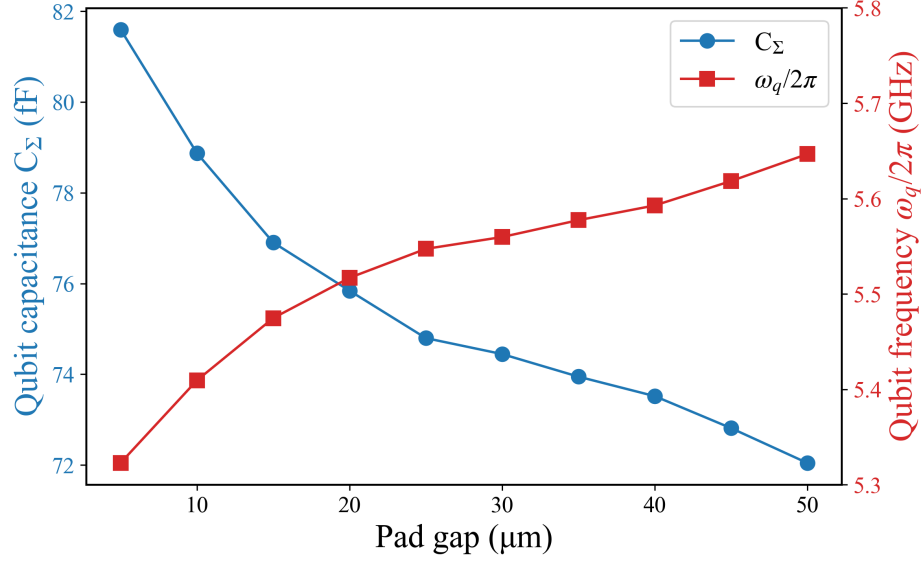


Figure 34: Parametric analysis of pad width from 200 to 600 μm by 100 μm steps. Hamiltonian parameters extracted from Ansys Q3D LOM analysis (9-pass adaptive meshing). **(a)** Decrease in anharmonicity $|\alpha|$ and coupling strength $|g|$ with increasing pad width. **(b)** Increase in qubit capacitance C_Σ and qubit frequency $\omega_q/2\pi$ reduction with increasing pad width.

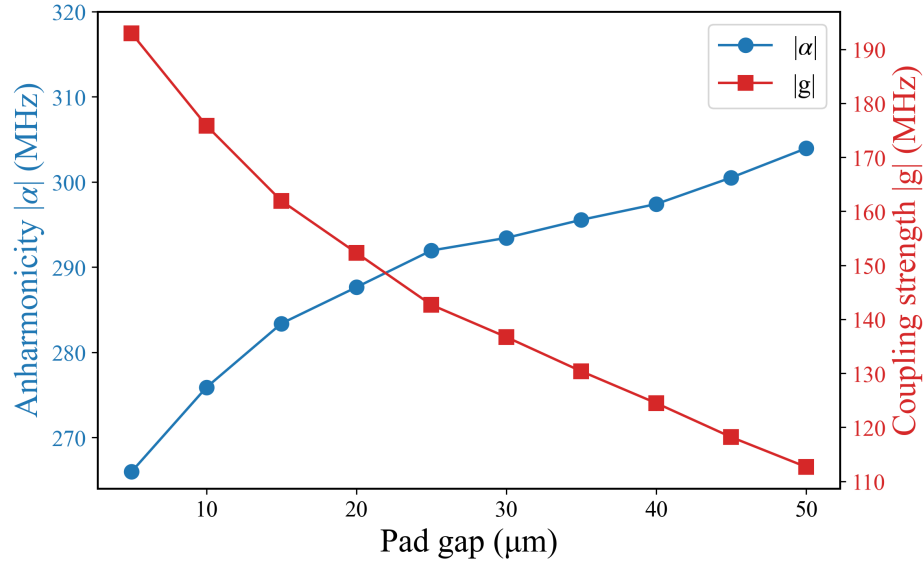
3.8.4 Pad gap

Next, we investigated the influence of the gap between the transmon's pads (shown in Fig. 18) on the chip properties. The results are shown in Fig. 35. Increasing the distance between the transmon pads reduces the capacitance C_S between the pads, thereby decreasing the total qubit capacitance C_Σ . As established in the precedent section, this reduction in C_Σ increases the qubit transition frequency ω_q (Fig. 35a) and the anharmonicity $|\alpha|$ (Fig. 35b). However, smaller pad gaps intensify electric field magnitudes in the dielectric substrate and interfaces. This elevates interaction with defects in the material, increasing dielectric loss and reducing coherence times.

Increasing the distance between the transmon pads reduces the shunt capacitance C_S between the transmon pads ($62.8 \rightarrow 27.2$ fF) and the total capacitance C_Σ ($81.6 \rightarrow 72.0$ fF). This elevates the qubit frequency ω_q ($5.32 \rightarrow 5.65$ GHz) and anharmonicity $|\alpha|$ ($266 \rightarrow 305$ MHz) as predicted. However, larger gap weakens the qubit-resonator coupling $|g|$ ($193 \rightarrow 113$ MHz) due to reduced field overlap. This also reduces dielectric loss, increasing T_1 from 23 ($5 \mu\text{m}$) to 41 μs ($50 \mu\text{m}$).



(a)



(b)

Figure 35: Parametric analysis of pad gap from 5 to 50 μm by 5 μm steps. Hamiltonian parameters extracted from Ansys Q3D LOM analysis (9-pass adaptive meshing). **(a)** Increase in qubit frequency $\omega_q/2\pi$ and decrease in qubit capacitance C_Σ with increasing pad gap. **(b)** Increase in anharmonicity $|\alpha|$ and decrease in coupling strength $|g|$ with increasing pad gap.

3.8.5 Josephson inductance

The effects of the JJ on the qubit properties has been addressed. The Josephson inductance L_J is a fundamental parameter determined by the junction's geometry and materials. In our EM simulations using Ansys, we treat L_J as a direct input to the lumped element model, while maintaining fixed physical dimensions. This approach allows systematic investigation of L_J 's impact on transmon properties, as shown in Fig. 36. The Josephson energy E_J scales inversely with L_J :

$$E_J = \frac{\Phi_0^2}{4\pi^2 L_J}. \quad (109)$$

Increasing L_J reduces E_J , which decreases the qubit frequency ω_q (Fig. 36a). Simultaneously, the coupling strength g diminishes (Fig. 36b) due to reduced dipole matrix elements:

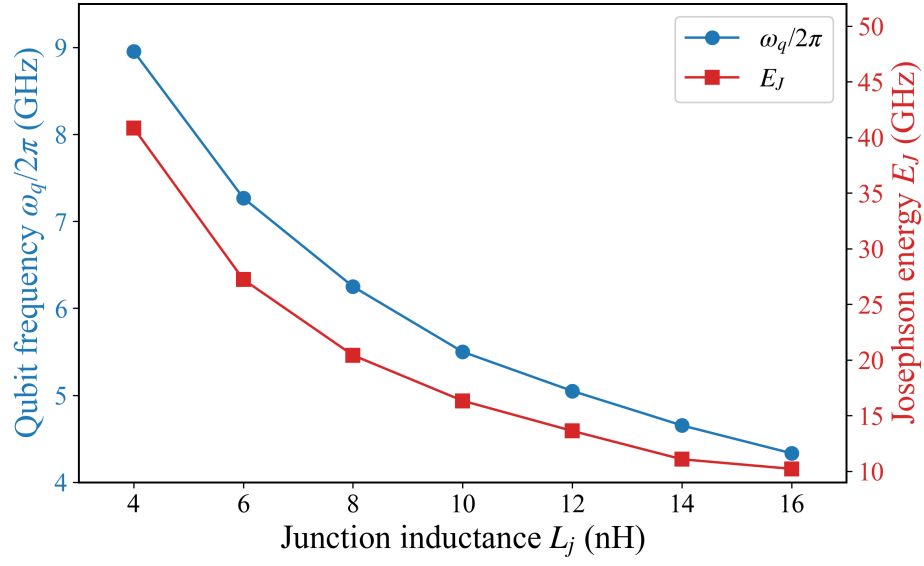
$$g \propto \left(\frac{E_C}{E_J}\right)^{\frac{1}{4}} \mathbf{E}_0. \quad (110)$$

The anharmonicity α exhibits a complex dependence on E_J (Fig. 36b), deviating from the asymptotic transmon approximation $\alpha \approx -E_C$. As derived from Mathieu equation analysis:

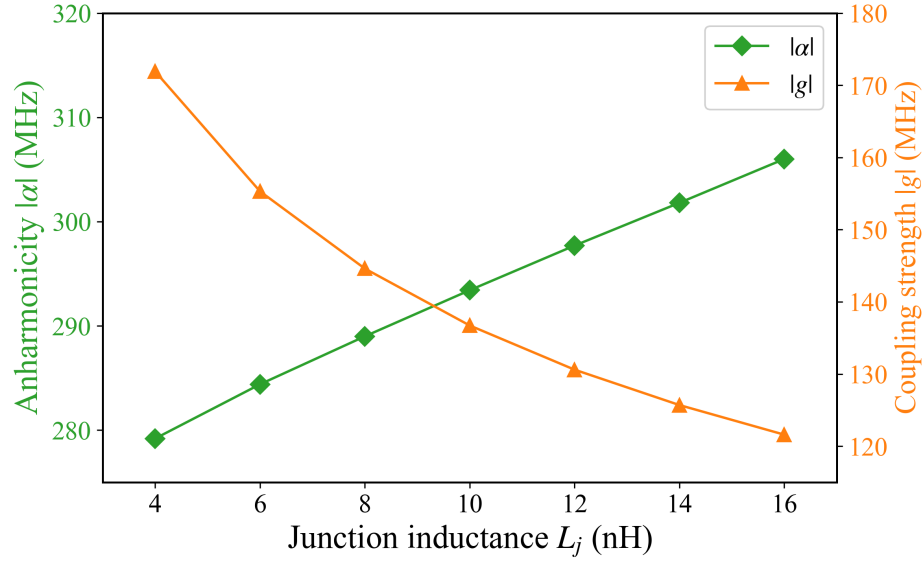
$$\alpha \approx -E_C + \frac{\sqrt{2}}{4} E_C \left(\frac{E_C}{E_J}\right)^{\frac{1}{2}} + \mathcal{O}\left(\frac{E_C^2}{E_J}\right), \quad (111)$$

explaining the +31% increase in $|\alpha|$ as L_J increases from 4 to 16 nH. The systematic variation of L_J reveals critical tradeoffs for qubit optimization:

- **Frequency vs. anharmonicity:** Lower L_J achieves higher operating frequencies but sacrifices anharmonicity ($\omega_q/2\pi = 8.96$ GHz and $|\alpha| = 279$ MHz at 4 nH). Higher L_J boosts anharmonicity but reduces frequency ($\omega_q/2\pi = 4.33$ GHz and $|\alpha| = 366$ MHz at 16 nH).
- **Coupling vs. coherence:** Strong coupling at low L_J enables faster gates but correlates with reduced T_1 ($T_1 = 23$ μ s and $g \approx 172$ MHz at 4 nH). Higher L_J greatly improves T_1 but weakens coupling strength ($g = 122$ MHz and $T_1 = 187$ μ s at 16 nH), requiring longer gate operations.
- **Sweet spot identification:** The $L_J = 8$ -12 nH range emerges as optimal for 5-6 GHz operation, balancing ~ 290 MHz anharmonicity, ~ 140 MHz coupling strength, and ~ 70 μ s coherence. This regime maintains $E_J/E_C \approx 50 - 80$, staying within the transmon regime while avoiding excessive dispersion.



(a)



(b)

Figure 36: Parametric analysis of Josephson inductance L_J from 4 to 16 nH by 2 nH steps. Hamiltonian parameters extracted from Ansys Q3D LOM analysis (9-pass adaptive meshing). **(a)** Decrease in qubit frequency $\omega_q/2\pi$ and Josephson energy E_J with increasing L_J . **(b)** Increase in anharmonicity $|\alpha|$ and increase in coupling strength $|g|$ with increasing L_J .

3.9 Conclusion

This work has proposed a fast simulation-driven, pre-fabrication workflow for the design of superconducting quantum chips. The methodology integrates analytical modeling for initial parameters selection with electromagnetic analysis to perform efficient, insight-driven parametric sensitivity studies.

Initial resonator geometries were specified analytically to meet target microwave characteristics, giving high-quality starting parameters for the initial layout that substantially narrowed the electromagnetic simulation search space. For rapid design-space exploration, we performed lumped-oscillator model (LOM) simulations, using Ansys Q3D to supply the required parameter extractions; sensitivity analyses revealed fundamental dependencies: resonator-qubit coupling, coherence, anharmonicity, and readout performance across connector length, pad dimensions, pad gap, and Josephson inductance. Separately, conformal-mapping calculations predicted how coupling geometry determines external quality factor, photon lifetime, and linewidth, quantities not directly accessible via the LOM. Additionally, high-fidelity HFSS eigenmode simulations validated the fundamental EM modes of the target transmon-resonator-feedline architecture, confirming that the intended modal structure was preserved and establishing a robust starting point for subsequent energy participation ratio (EPR) analysis. Although EPR simulations were not executed within the scope of this work, the infrastructure for their application has been prepared and verified, ensuring readiness for the next design-refinement stage.

We have obtained parameter-response trends that captures the physical mechanisms behind each geometric sensitivity tested, providing informed priors that accelerate convergence toward target quantum parameters. By integrating pre-existing analytical and EM tools into a coherent, programmatic pipeline, we deliver a theoretical description and tested implementability of the prefabrication workflow for superconducting quantum-chip design. This framework quantitatively maps trade-offs among coupling, coherence, and anharmonicity to enable data-driven decisions over heuristics, predicts performance sensitivities upfront to mitigate fabrication risks, improve yield, and offers a scalable approach that other groups can adopt to shorten the qubit-resonator development cycle.

Our current study still has several limitations. First, the energy-participation-ratio methodology has not yet been embedded in an iterative design loop to converge on optimal chip geometry and capture coupling-induced effects. Second, the Ansys simulations assume perfect conductors and lossless dielectrics, neglecting both material dissipation and packaging-related parasitic effects. Finally, we have not performed driven, scattering-parameter analyses, leaving broadband impedance matching, resonator Q-factor under realistic port loading, and potential box-mode resonances unverified.

The parametric sensitivity analysis with LOM facilitates rapid screening of numerous geometries due to its lower computational cost compared to energy participation ratio (EPR) analysis. However, LOM assumes a fixed resonator frequency and models only the qubit-resonator subsystem. While analytical tools were used to study resonator-feedline coupling, these calculations are approximate and neglect critical phenomena such as kinetic inductance and spurious modes. Consequently, full-wave electromagnetic simulations (e.g., Ansys HFSS) coupled with EPR analysis are essential for robust quantum hardware design. Unlike LOM, EPR models the complete chip geometry, including the feedline coupling, and self-consistently computes qubit frequencies. LOM

parameter sweeps provide optimized starting points for EPR simulations. After comparing EPR results with LOM predictions, we can iteratively refine parameters toward target specifications. This hybrid approach, accelerated by prior LOM analysis, efficiently navigates the design space.

Incorporating driven, scattering-parameter simulations will be a critical enhancement of our workflow. By exciting the validated eigenmodes with realistic port boundary conditions and sweeping across the operational band, S-parameter analysis will reveal broadband impedance matching, packaging parasitic, and frequency-dependent external Q-factors that lie beyond the scope of our current eigenmode and analytical estimates. This step will not only validate and refine the CPW feedline launch and coupler geometry, but also provide accurate loading conditions for the subsequent energy-participation-ratio calculations. In doing so, it explicitly bridges the current gap between idealized, closed-boundary eigenmode analysis and realistic, port-driven device characterization—linking the internal Hamiltonian parameters that describe the device’s intrinsic physics to the real-world port behavior observed under experimental drive. Together, these driven simulations will complete the loop and ensure that both intrinsic and extrinsic device characteristics are fully optimized before fabrication.

Finally, the natural next step will be to advance through physical fabrication and characterization of optimized transmon devices. This experimental phase will enable critical investigation of manufacturing parameter dependencies. Specifically, it will allow to systematically vary Josephson junction fabrication parameters (cross-sectional area and barrier thickness) and correlate these with room-temperature electrical measurements of junction resistance using the Ambegaokar-Baratoff formula. Since this resistance directly determines the junction’s quantum-critical superconducting properties, we could establish empirical relationships between manufacturing variables and resulting quantum Hamiltonian parameters (E_J , L_J , ω_q). Unlike purely *in silico* approaches, this methodology will allow to quantify real-world disorder effects, providing actionable process tolerances for scalable quantum processor production. Now is the moment to confront our tolerances with real chips: fabricate, measure, and refine, to transform our insights into a reality.

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A Quantum processor implementation

```

1  #!/usr/bin/env python
2  # coding: utf-8
3
4
5
6  """
7  QUANTUM PROCESSOR DESIGN, SIMULATION, AND ANALYSIS WORKFLOW
8  Master Thesis Annex: Full Implementation Code
9
10 Purpose: This integrated pipeline enables end-to-end design, electromagnetic
11          simulation, and quantum characterization of superconducting quantum
12          processors.
13 It combines:
14 1. Parametric layout generation (Qiskit Metal)
15 2. Finite-element EM simulation (Ansys HFSS/Q3D)
16 3. Quantum parameter extraction (EPR/LOM analysis)
17
18 Workflow Overview:
19 A. Quantum Processor Layout Generation
20 B. Single-Qubit Eigenmode Simulation & EPR Analysis
21 C. Full-Chip Eigenmode Simulation & EPR Analysis
22 D. Capacitance Extraction & Lumped Oscillator Model
23
24 Author: Burignat Yohan
25 Advisor: Sorin Melinte
26
27 Date: 17/07/2025
28 Institution: UCLouvain
29 """
30
31 # Environment Initialization =====
32
33 # Enable automatic reloading of modules when modifications are made
34 get_ipython().run_line_magic('load_ext', 'autoreload')
35 get_ipython().run_line_magic('autoreload', '2')
36
37 import qiskit_metal as metal
38 from qiskit_metal import designs, draw
39 from qiskit_metal import MetalGUI, Dict, open_docs, Headings
40
41 # Import electromagnetic analysis toolkit
42 import pyEPR as epr # Energy-participation ratio computation
43
44 # qiskit_metal.renderers.renderer_ansys: The Python package path for Ansys
45 # integration
46 # ansys_renderer: The specific module containing Ansys interface code
47 # QAnsysRenderer: The core class that bridges Qiskit Metal with Ansys
48 # simulation software
49 from qiskit_metal.renderers.renderer_ansys.ansys_renderer import QAnsysRenderer
50 QAnsysRenderer.default_options
51
52 # Initialize Quantum Design Environment =====

```

```

51
52 # DesignPlanar: 2D circuit architecture for superconducting qubits
53 design = designs.DesignPlanar()
54
55 # Launch Metal GUI: allow to see, edit manually and simulate the QDesign
56 gui = MetalGUI(design)
57
58
59 # Quantum Component Library Import =====
60 # Import specific quantum circuit elements from Qiskit Metal's library
61
62 # Qubit components:
63 from qiskit_metal.qlibrary.qubits.transmon_pocket_cl import TransmonPocketCL #
64     Flux-tunable transmon
65 from qiskit_metal.qlibrary.qubits.transmon_pocket_cl import TransmonPocket #
66     Fixed-frequency transmon
67
68 # Waveguide Routing:
69 from qiskit_metal.qlibrary.tlines.straight_path import RouteStraight #
70     Direct CPW connections
71 from qiskit_metal.qlibrary.tlines.meandered import RouteMeander #
72     Compact resonator routing
73 from qiskit_metal.qlibrary.tlines.anchored_path import RouteAnchors #
74     Custom path definitions
75 from qiskit_metal.qlibrary.tlines.pathfinder import RoutePathfinder #
76     Auto-routing for complex layouts
77 from qiskit_metal.qlibrary.couplers.coupled_line_tee import CoupledLineTee #
78     Coupling with TL
79
80 # for pins:
81 from qiskit_metal.qlibrary.terminations.open_to_ground import OpenToGround
82     # Open end
83 from qiskit_metal.qlibrary.terminations.short_to_ground import ShortToGround
84     # Short end
85
86 # Control/Readout Interfaces:
87 from qiskit_metal.qlibrary.terminations.launchpad_wb import LaunchpadWirebond
88     # Wirebond pads
89 from qiskit_metal.qlibrary.terminations.launchpad_wb_coupled import
90     LaunchpadWirebondCoupled # Coupled line launch
91
92
93 # =====
94 # A. QUANTUM PROCESSOR LAYOUT GENERATION
95 # =====
96 """
97 Objective: Create parametric model of transmon qubit coupled to a readout
98     resonator. The resonator is coupled to the exterior via a feedline
99 Components:
100 - TransmonPocket: Flux-tunable transmon qubit
101 - CoupledLineTee: Coupling zone between 2 CPW
102 - RouteMeander: CPW bus waveguide
103 - LaunchpadWirebond: launchpads
104 """

```

```

95 # Enable design overwrite to allow iterative refinement without object
    collisions
96 # Critical for parametric optimization workflows (e.g., sweeping gap sizes)
design.overwrite_enabled = True
97
98
99 # Transmon Qubit Instantiation
100 Q1 = TransmonPocket(design, 'Q1', options = dict(
101     pos_x = '0mm',
102     pos_y='0mm',
103     orientation='90',
104     connection_pads = dict(
105         a = dict(
106             connector_location = '180',
107             claw_length = '95um'),
108         )
109     )
110 )
111
112 # Render and verify layout
113 gui.rebuild()
114 gui.autoscale()
115 gui.screenshot()
116
117
118
119 # RESONATOR-COUPLED FEEDLINE SUBSYSTEM
    =====
120
121 design.overwrite_enabled = True
122
123 # Add a  $\lambda/4$  resonator coupled to a Feedline
124 TQ1 = CoupledLineTee(design, 'TQ1', options=dict(
125     pos_x=' -0.5mm',
126     pos_y='2mm',
127     coupling_length='200um', # Sets capacitive coupling strength
128     open_termination=False   # Shorted end ( $\lambda/4$  resonator)
129 )
130 )
131
132
133 gui.rebuild()
134 gui.autoscale()
135 gui.screenshot()
136
137
138
139 # BUS CONNECTION =====
140
141 design.overwrite_enabled = True
142
143 # Meandered Bus Waveguide
144 bus1 = RouteMeander(design, 'Bus1', options = dict(
145     total_length='4.1mm',      # total length of bus - coupling length
146     hfss_wire_bonds = True,    # Critical for accurate parasitic modeling
147     fillet='99um',             # Prevent radiative loss
148     pin_inputs=Dict(

```

```

149         start_pin=Dict(component='Q1', pin='a'),
150         end_pin=Dict(component='TQ1', pin='second_end')
151     )
152 ))
153
154 gui.rebuild()
155 gui.autoscale()
156 gui.screenshot()
157
158
159
160 # CONTROL AND READOUT INTERFACE =====
161
162 design.overwrite_enabled = True
163
164 # Wirebond Launch Pads
165 launch_TL_L = LaunchpadWirebond(design, 'launch_TL_L', options = dict(pos_x =
166     '-2mm', pos_y = '2mm'))
167 launch_TL_R = LaunchpadWirebond(design, 'launch_TL_R', options = dict(pos_x =
168     '1mm', pos_y = '2mm', orientation='180'))
169 launch_Phase = LaunchpadWirebond(design, 'launch_Phase', options = dict(pos_x =
170     '0mm', pos_y = '-2mm', orientation='90'))
171
172 # Feedline Waveguide Routing
173
174 # Right feedline segment
175 ops_oR = Dict(
176     hfss_wire_bonds = True, # Include wirebonds in HFSS simulation
177     pin_inputs=Dict(
178         start_pin=Dict(
179             component='TQ1', pin='prime_end'),
180         end_pin=Dict(
181             component='launch_TL_R', pin='tie'))
182 )
183 cpw_openRight = RouteStraight(design, 'cpw_openRight', options = ops_oR)
184
185 # Left feedline segment
186 ops_oL = Dict(
187     hfss_wire_bonds = True,
188     pin_inputs = Dict(
189         start_pin = Dict(component = 'TQ1', pin = 'prime_start'),
190         end_pin = Dict(component = 'launch_TL_L', pin = 'tie'))
191 )
192 cpw_openLeft = RouteStraight(design, 'cpw_openLeft', options = ops_oL)
193
194 gui.rebuild()
195 gui.autoscale()
196 gui.screenshot()
197
198 # =====
199 # B. SINGLE-QUBIT EIGENMODE SIMULATION & EPR ANALYSIS
200 # =====
201 """

```

```

202 Objective: Characterize fundamental qubit properties:
203 - Qubit frequency ( $\omega_q$ )
204 - Anharmonicity ( $\alpha$ )
205 - Junction participation ( $p_j$ )
206 - Dielectric loss limit ( $T1_{diel}$ )
207 """
208 from qiskit_metal.analyses.quantization import EPRanalysis
209
210
211 # Initialize EPR analysis with Ansys HFSS eigenmode solver
212 eig_qb = EPRanalysis(design, "hfss") # 'hfss' specifies the EM simulation
    engine
213
214 # Access the HFSS renderer interface
215 hfss = eig_qb.sim.renderer
216
217 # Launch Ansys Electronics Desktop session
218 hfss.start()
219
220 # Activate or create eigenmode simulation design
221 hfss.activate_ansys_design("TransmonQubit", 'eigenmode') # 'eigenmode':
    Simulates resonant modes without external excitation
222
223
224 # QUBIT EIGENMODE SIMULATION AND EPR ANALYSIS
    =====
225
226 # Render qubit geometry in Ansys (only Q1 component, no additional layers)
227 hfss.render_design(['Q1'], [])
228
229 # Simulation configuration
230 setup = hfss.pinfo.setup
231 setup.nmodes = 1 # Calculate only fundamental mode (qubit transition)
232 setup.passes = 10 # Adaptive meshing refinement iterations
233
234 # Josephson junction parametrization
235 pinfo = hfss.pinfo
236 pinfo.design.set_variable('Lj', '10 nH') # Josephson inductance
237 pinfo.design.set_variable('Cj', '2 fF') # Junction capacitance
238
239 # Execute eigenmode simulation
240 setup.analyze() # Runs finite-element EM simulation
241
242
243 # SIMULATION CONVERGENCE ANALYSIS =====
244 eig_qb.sim.convergence_t, eig_qb.sim.convergence_f, _ = hfss.get_convergences()
245 eig_qb.sim.plot_convergences() # Visualize solution stability vs. mesh
    refinement
246
247
248 # ELECTROMAGNETIC FIELD VISUALIZATION =====
249 hfss.modeler._modeler.ShowWindow() # Display Ansys GUI
250 hfss.plot_fields('main') # Plot E-field of fundamental mode
251 hfss.save_screenshot() # Document field distribution
252
253 # Clean up previous field plots

```

```

254 hfss.plot_ansys_delete(['Mag_E1']) # Remove temporary field visualizations
255
256
257 # JUNCTION DEFINITION FOR QUANTUM ANALYSIS =====
258 pinfo = hfss.pinfo
259 pinfo.junctions['jj'] = {
260     'Lj_variable': 'Lj', # Links EM model to quantum parameter
261     'rect': 'JJ_rect_Lj_Q1_rect_jj', # Ansys object name for junction
262     'line': 'JJ_Lj_Q1_rect_jj_', # Integration path across junction
263     'Cj_variable': 'Cj', # Junction capacitance variable
264 }
265 pinfo.validate_junction_info() # Verify geometric-to-quantum mapping
266
267 # Specify lossy dielectric regions (critical for T1 prediction)
268 pinfo.dissipative['dielectrics_bulk'] = ['main'] # 'main' = substrate region
269
270
271 # ENERGY PARTICIPATION COMPUTATION =====
272 eprd = epr.DistributedAnalysis(pinfo) # Initialize EPR calculator
273
274 # Compute electric energy distribution
275  $\mathcal{E}_{elec}$  = eprd.calc_energy_electric() # Total electric energy
276  $\mathcal{E}_{elec\_substrate}$  = eprd.calc_energy_electric(None, 'main') # Substrate
    contribution
277  $\mathcal{E}_{mag}$  = eprd.calc_energy_magnetic() # Total magnetic energy
278
279
280 # HAMILTONIAN PARAMETERS EXTRACTION =====
281 eprd.do_EPR_analysis() # Core EPR computation
282
283
284 # =====
285 # FULL-CHIP EIGENMODE SIMULATION AND ANALYSIS
286 # =====
287 # Purpose: Characterize multi-component quantum processor
288
289 from qiskit_metal.analyses.quantization import EPRanalysis
290
291 # Clean previous simulation data from Ansys workspace
292 hfss.clean_active_design()
293
294 # Create new eigenmode simulation for complete processor
295 hfss.activate_ansys_design("Chip1", 'eigenmode')
296 """
297 'Chip1': New design container for full-system analysis
298 'eigenmode': Simulates natural resonant modes without external ports
299 """
300
301 # Render entire quantum processor geometry
302 hfss.render_design() # Processes all components in the design
303 hfss.save_screenshot()
304
305
306 # MULTI-MODE SIMULATION CONFIGURATION =====
307 setup = hfss.pinfo.setup
308 setup.passes = 18 # Adaptive meshing refinement cycles

```

```

309 setup.n_modes = 3          # Calculate first three eigenmodes
310
311 print(f"""
312 Full-Chip Simulation Parameters:
313     Modes calculated: {setup.n_modes} (Qubit mode + resonator mode + Feedline
314     mode)
315     Mesh refinement passes: {setup.passes}
316     Frequency convergence threshold: {setup.delta_f}%
317 """)
318 # Apply Josephson junction parameters globally
319 pinfo = hfss.pinfo
320 pinfo.design.set_variable('Lj', '10 nH') # Junction inductance
321 pinfo.design.set_variable('Cj', '2 fF')  # Junction capacitance
322
323 # Execute multi-mode simulation
324 setup.analyze() # Runs full EM simulation for all specified modes
325
326
327 # SOLUTION CONVERGENCE VERIFICATION =====
328 eig_qb.sim.convergence_t, eig_qb.sim.convergence_f, _ = hfss.get_convergences()
329 eig_qb.sim.plot_convergences()
330
331 """
332 Critical for full-chip simulations:
333     - Ensures mesh independence across complex geometry
334     - Verifies solution stability for all three modes
335 """
336
337
338 # MODAL FIELD ANALYSIS =====
339 # Mode 1: Fundamental qubit mode
340 hfss.set_mode(1, "Setup") # Select first eigenmode
341 hfss.modeler._modeler.ShowWindow()
342 hfss.plot_fields('main') # Visualize mode 1 E-field distribution
343 hfss.save_screenshot()   # Display field distribution
344 hfss.clear_fields(['Mag_E1']) # Clean temporary data
345
346
347 # Mode 2: Resonator mode
348 hfss.set_mode(2, "Setup")
349 hfss.modeler._modeler.ShowWindow()
350 hfss.plot_fields('main') # Visualize mode 2 E-field distribution
351 hfss.save_screenshot()
352 hfss.clear_fields(['Mag_E2'])
353
354
355
356 # Mode 3: Feedline mode
357 hfss.set_mode(3, "Setup")
358 hfss.modeler._modeler.ShowWindow()
359 hfss.plot_fields('main') # Visualize mode 3 E-field distribution
360 hfss.save_screenshot()
361 hfss.clear_fields(['Mag_E3'])
362
363

```

```

364
365
366 # EPR ANALYSIS =====
367 pinfo = hfss.pinfo
368
369 # Map junction properties for EPR calculation
370 pinfo.junctions['jj'] = {
371     'Lj_variable': 'Lj',
372     'rect': 'JJ_rect_Lj_Q1_rect_jj',
373     'line': 'JJ_Lj_Q1_rect_jj_',
374     'Cj_variable': 'Cj'
375 }
376
377 pinfo.validate_junction_info() # Check validity names of variables and objects
378
379 # Specify loss mechanisms (critical for T1 prediction)
380 pinfo.dissipative['dielectrics_bulk'] = ['main'] # Substrate dielectric loss
381
382 # Initialize distributed EPR analysis
383 eprd = epr.DistributedAnalysis(pinfo)
384
385 # Compute full-system energy distributions
386  $\mathcal{E}_{elec}$  = eprd.calc_energy_electric() # Total electric energy
387  $\mathcal{E}_{elec\_substrate}$  = eprd.calc_energy_electric(None, 'main') # Substrate
    contribution
388  $\mathcal{E}_{mag}$  = eprd.calc_energy_magnetic() # Total magnetic energy
389
390 print(f"""
391 Full-Chip Energy Analysis:
392     Total E-field energy = { $\mathcal{E}_{elec}$ :.4e} J
393     Substrate E-energy    = { $\mathcal{E}_{elec\_substrate}$ :.4e} J
394     Substrate EPR        = { $\mathcal{E}_{elec\_substrate} / \mathcal{E}_{elec} * 100$  :.1f}%
395
396     Total B-field energy = { $\mathcal{E}_{mag}$ :.4e} J
397 """)
398
399 # Execute comprehensive EPR analysis
400 eprd.do_EPR_analysis() # Computes participation matrices
401
402 # SIMULATION CLEANUP =====
403 hfss.clean_active_design() # Remove temporary simulation files
404 hfss.stop() # Gracefully terminate Ansys session
405
406
407
408 # =====
409 # LUMPED OSCILLATOR MODEL (LOM) AND CAPACITANCE MATRIX ANALYSIS
410 # =====
411 # Purpose: Quantitatively characterize quantum circuit parameters using:
412 #           1. Capacitance matrix extraction (via Ansys Q3D)
413 #           2. Lumped Oscillator Model (LOM) for quantum parameter prediction
414 # Theory: LOM approximates the circuit as a network of lumped elements to
    compute:
415 #           - Qubit frequencies
416 #           - Anharmonicities
417 #           - Coupling strengths

```

```

418 #             (Compared to EPR: Faster but less accurate for complex geometries)
419
420 from qiskit_metal.analyses.quantization import LOManalysis
421
422 # Initialize LOM analysis with Q3D renderer for capacitance extraction
423 c2 = LOManalysis(design, "q3d") # 'q3d' specifies Ansys Q3D as EM solver
424
425 # Access the Q3D renderer interface
426 q3d = c2.sim.renderer
427
428
429 # CAPACITANCE MATRIX EXTRACTION WORKFLOW =====
430 q3d.start() # Launch Ansys Electronics Desktop with Q3D
431
432 # Create or activate capacitive simulation design
433 q3d.activate_ansys_design("TransmonResonator_q3d", 'capacitive')
434
435 """
436 'capacitive': Simulation type for capacitance extraction
437 Design name 'TransmonResonator_q3d' will appear in Ansys project manager
438 """
439
440 # Render design components in Q3D (empty lists = render all components)
441 q3d.render_design(['Q1'], [('Q1', 'a')])
442
443 # Execute capacitance simulation
444 q3d.analyze_setup("Setup") # "Setup" is the default simulation configuration
445
446 # Retrieve capacitance matrix results
447 c2.sim.capacitance_matrix, c2.sim.units = q3d.get_capacitance_matrix()
448
449 # Optional: Get convergence data across simulation passes
450 c2.sim.capacitance_all_passes, _ = q3d.get_capacitance_all_passes()
451 """
452 Useful for:
453 - Verifying solution convergence
454 - Identifying mesh sensitivity
455 """
456
457
458 # LUMPED OSCILLATOR MODEL ANALYSIS =====
459 # Configure Josephson junction parameters
460 c2.setup.junctions = Dict(
461     Lj = 10, # Josephson inductance (nanohenry)
462     Cj = 2   # Junction capacitance (femtofarad)
463 )
464
465 # Set resonator frequencies (critical for coupling calculations)
466 c2.setup.freq_readout = 7.0 # Readout resonator frequency (GHz)
467 c2.setup.freq_bus = []      # Frequencies of other bus resonators (none here)
468
469 # Execute LOM analysis to compute quantum parameters
470 c2.run_lom()
471
472 # Access LOM results dictionary
473 results = c2.lumped_oscillator_all

```

```
474  
475 # clean Q3D design  
476 q3d.clean_active_design()  
477  
478 # terminate Q3D session  
479 q3d.stop()
```

B LC resonator's Hamiltonian

A LC resonator is an electrical circuit characterized by an inductance L and a parallel-connected capacitance C with a resonance frequency $\omega_r = 1/\sqrt{LC}$. In the case of superconducting qubit operating at temperature around ~ 10 mK, we can rely on the assumption that there is no dissipation of energy due to resistance, and work with such a simple LC circuit. The classical Hamiltonian represents the total energy of this system, given by the sum of its charging and inductive energy:

$$H_{\text{LC}} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}, \quad (112)$$

where Q is the charge on the capacitor and Φ the flux on the inductor. The Hamiltonian H_{LC} reads as:

$$H_{\text{LC}} = \frac{Q^2}{2C} + \frac{1}{2}C\omega_r^2\Phi^2. \quad (113)$$

This form is similar to the one of an harmonic oscillator with coordinate Φ , conjugate momentum Q , and mass C .

B.1 Quantized LC resonator

In the precedent sections, quantities have been calculated in a classical manner. To go from a classical scheme to a quantum scheme, like the one of cavity QED, we will have to derive the quantum Hamiltonian of our circuit. We will proceed by working from the classical Lagrangian described above. Using the method of general analogy (Dirac [54]), we use the relation between the classical operators – they are canonically conjugate and therefore they have a Poisson bracket of one – to derive the commutation relation between the analogous quantum operators. For the conjugate pair (Φ, Q) satisfying the Poisson bracket:

$$\{\Phi, Q\}_{\text{PB}} = 1, \quad (114)$$

the corresponding non-commuting observables $\hat{\Phi}$ and $\hat{Q}$ obey the commutation relation:

$$[\hat{\Phi}, \hat{Q}] = i\hbar. \quad (115)$$

The quantized Hamiltonian simply reads:

$$\hat{H}_{\text{LC}} = \frac{\hat{Q}^2}{2C} + \frac{1}{2}C\omega_r^2\hat{\Phi}^2. \quad (116)$$

We recognize the QHO. The ladder operators $\hat{a}$ (annihilation) and $\hat{a}^\dagger$ (creation) are defined through the canonical flux and charge operators:

$$\hat{a}^\dagger = \sqrt{\frac{1}{2\hbar Z_r}}(\hat{\Phi} - iZ_r\hat{Q}), \quad \hat{a} = \sqrt{\frac{1}{2\hbar Z_r}}(\hat{\Phi} + iZ_r\hat{Q}), \quad (117)$$

where Z_r is the characteristic impedance of the resonator. Writing the Hamiltonian in terms of ladder operators we get:

$$\hat{H}_{LC} = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right), \quad (118)$$

which is a QHO of a single mode. The creation operator $\hat{a}^\dagger$ increases the excitation number by one quantum, corresponding to a coherent excitation of the resonator's flux-charge phase space. Conversely, the annihilation operator $\hat{a}$ reduces the excitation number by one quantum. These discrete excitations manifest physically as single photons of frequency ω_r within the circuit. Consequently, the eigenenergy spectrum exhibits equidistant levels with spacing $\hbar\omega_r$, characteristic of harmonic potentials, as visualized in Fig. 3(b):

$$E_n = \hbar\omega_r \left(n + \frac{1}{2} \right), \quad n = 0, 1, \dots \quad (119)$$

B.2 Cavity quantum electrodynamics

The quantum Hamiltonian of the LC resonator uses $\hat{Q} = C\hat{V}$ and $\hat{\Phi} = L\hat{I}$, where $\hat{V}$ is the voltage operator and $\hat{I}$ is the current operator:

$$\hat{H}_{LC} = \frac{1}{2}C\hat{V}^2 + \frac{1}{2}L\hat{I}^2. \quad (120)$$

In a quantum system, the Heisenberg uncertainty principle prohibits conjugate variables (e.g., flux $\hat{\Phi}$ and charge $\hat{Q}$) from attaining simultaneous definite values, even at the ground state of the system. By taking the zero-point energy of our Hamiltonian, we can find the zero-point fluctuations (ZPF): the zero-point voltage ΔV_0 and the zero-point current ΔI_0 , derived from the quantum mechanical expectation values of the voltage and current. Following the example in Ref. [24] of a typical lumped-element LC resonator, with $L \sim 0.8$ nH and $C \sim 0.4$ pF and a characteristic impedance $Z_r \sim 50$ Ω , the zero-point voltage and current are:

$$\Delta V_0 = \sqrt{\langle 0 | \hat{V}^2 | 0 \rangle} = \sqrt{\frac{\hbar\omega_r}{2C}} \sim 1 \text{ } \mu\text{V}, \quad (121)$$

$$\Delta I_0 = \sqrt{\langle 0 | \hat{I}^2 | 0 \rangle} = \sqrt{\frac{\hbar\omega_r}{2L}} \sim 5.5 \text{ nA}. \quad (122)$$

These vacuum fluctuations represent irreducible quantum noise, manifesting as nonzero voltage oscillations in the ground state. In circuit QED, these fluctuations generate large electric fields that enable strong electric-dipole interactions with artificial atoms like transmons. Crucially, superconducting resonators exhibit exceptionally large vacuum fluctuations compared to other systems. The characteristic impedance $Z_r = \sqrt{L/C}$ governs the ZPF magnitude through the relations:

$$\Delta V_0 \propto \sqrt{Z_r}, \quad \Delta I_0 \propto 1/\sqrt{Z_r}. \quad (123)$$

This is critical for engineering for the following reasons. First, ΔV_0 directly determines the qubit-resonator coupling strength g . For transmons, $g = d \cdot \Delta V_0 / \hbar$, where d is the transition dipole

moment. In a quantum non-demolition measurement, the ZPF scale sets the dispersive shift:

$$\chi = \frac{g^2}{\Delta} \propto (\Delta V_0)^2, \quad (124)$$

where $\Delta = |\omega_q - \omega_r|$ is the qubit-resonator detuning. Larger ZPF enhances χ , improving readout fidelity by increasing the qubit-state-dependent frequency shift. However, this simultaneously increases the Purcell decay rate $\Gamma_P = \kappa(g/\Delta)^2$, creating a design trade-off between readout performance and qubit coherence. At finite temperatures, thermal fluctuations compete with quantum effects. To preserve quantum behavior, we require:

$$\hbar\omega_r \gg k_B T \quad (\text{e.g., } : \omega_r/2\pi = 5 \text{ GHz} \implies T \ll 240 \text{ mK}), \quad (125)$$

necessitating millikelvin operation ($T \sim 10 - 50 \text{ mK}$). Finally, spurious two-level systems (TLS) couple to ΔV_0 , inducing frequency shifts $\delta\omega_r \propto \Delta V_0$. High- Z_r designs amplify this noise, requiring mitigation strategies like atomic-layer-deposited dielectrics and superconducting shielding.

C Telegrapher's equations

The telegrapher's equations are a pair of linear differential equations which describe the voltage and current on an electrical TL with distance and time, in the distributed element construction. To derive them, let's follow the procedure presented in Ref. [35]. By applying Kirchhoff's voltage law to the distributed circuit of the TL (see Fig. 8 (b)), we find:

$$0 = v(x, t) - R_1 i(x, t) \Delta x - L_1 \Delta x \frac{\partial i(x, t)}{\partial t} - v(x + \Delta x, t). \quad (126)$$

Similarly, using Kirchhoff's current law gives:

$$0 = i(x, t) - G_1 v(x, t) \Delta x - C_1 \Delta x \frac{\partial v(x + \Delta x, t)}{\partial t} - i(x + \Delta x, t). \quad (127)$$

Dividing Eq. [126] and Eq. [127] by Δx we obtain :

$$\frac{v(x, t) - v(x + \Delta x, t)}{\Delta x} = R_1 i(x, t) + L_1 \frac{\partial i(x, t)}{\partial t}, \quad (128)$$

$$\frac{i(x, t) - i(x + \Delta x, t)}{\Delta x} = G_1 v(x, t) + C_1 \frac{\partial v(x, t)}{\partial t}. \quad (129)$$

Using the distributed element model with infinitesimally small lumped elements ($\Delta x \rightarrow 0$):

$$\frac{\partial v(x, t)}{\partial x} = R_1 i(x, t) + L_1 \frac{\partial i(x, t)}{\partial t}, \quad (130)$$

$$\frac{\partial i(x, t)}{\partial x} = G_1 v(x, t) + C_1 \frac{\partial v(x, t)}{\partial t}. \quad (131)$$

Finally, Eq. [130] and Eq. [131] may be combined to get two decoupled partial differential equations, one for voltage and one for current. A similar result is attained from the current by differentiating Eq. [131] with respect to x and applying Eq. [130], giving:

$$\frac{\partial^2 v(x, t)}{\partial t^2} - c^2 \frac{\partial^2 v(x, t)}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^2 i(x, t)}{\partial t^2} - c^2 \frac{\partial^2 i(x, t)}{\partial x^2} = 0. \quad (132)$$

